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<td>Author(s)</td>
<td>Warbal, Pankaj; Pramanik, Manojit; Saha, Ratan K.</td>
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A robust modified delay-and-sum algorithm for photoacoustic tomography imaging with apodized sensors

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A robust modified delay-and-sum algorithm for photoacoustic tomography imaging with apodized transducers

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ABSTRACT

Role of sensor sensitivity in photoacoustic tomography (PAT) imaging is discussed. In this study, sensitivity profile (apodization) of finite size sensors was considered as axisymmetric and modelled by using a Gaussian function. The full width at half maximum (FWHM) of the Gaussian function was varied in order to investigate its effect on PAT image reconstruction. The images were reconstructed using conventional delay-and-sum (CDAS) and modified delay-and-sum (MDAS) algorithms. In case of the CDAS, a Gaussian function was used to weight the PA signals detected by different parts of the sensor and the resultant signal was computed by summing those signals. However, in case of the MDAS, the Gaussian weight was applied in both directions (signal acquisition and redistribution of the pressure values at different point locations on the aperture of the finite sensor). The performance of these algorithms was investigated with respect to ideal point detectors by conducting numerical experiments in the k-Wave toolbox. The results for the CDAS and the MDAS algorithms are found to be very close to that of ideal point detectors when FWHM is small. The MDAS technique appears to be much superior to the CDAS approach when FWHM is large. The MDAS method can be employed in practice for apodized transducers as well if the Gaussian weight is applied in both directions (signal acquisition and redistribution).

Keywords: Photoacoustic tomography, conventional delay-and-sum, modified delay-and-sum, Gaussian function, apodization.

1. INTRODUCTION

Photoacoustic tomography (PAT) exploits photoacoustic (PA) effect for image formation of biological tissue. The application of a short laser pulse on a biological tissue causes thermo-elastic expansion and acoustic waves (also known as PA signals) are produced. The detectors in a typical PA setup are placed around the tissue and record the PA signals. The recorded sensor data are processed to map initial pressure rise in a biological tissue. The reconstruction can be carried out using analytical approaches like backprojection or model based techniques. PAT gives the morphological information of the illuminated tissue and has applications in tumor angiogenesis,\textsuperscript{1,2} breast imaging,\textsuperscript{3,4} small animal brain imaging,\textsuperscript{5,6} vasculature imaging,\textsuperscript{7,8} molecular imaging,\textsuperscript{9,10} and sentinel lymph node imaging.\textsuperscript{11,12}

The scanning geometry is circular [see Fig. 1(a)] i.e. the sensor is positioned on the circumference of the circle at different angular positions to capture PA signals coming from the illuminated tissue.\textsuperscript{13} Finite aperture and limited bandwidth of the detector dictate the spatial resolutions (axial and tangential).\textsuperscript{14} Both the resolutions depend on the transducer bandwidth whereas the tangential resolution is affected by finite aperture size only. Axial resolution is constant throughout the imaging region but tangential resolution deteriorates as we move away from the scanning center.

So to improve the tangential resolution we can use small-sized detectors which increases the angle of acceptance for the incoming PA signals (leads to weaker sensitivity due to high thermal noise) or by placing the

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detectors at a large distance from the scanning center (contributes to poor signal-to-noise ratio). Some ways like use of virtual point detectors\textsuperscript{15} and negative acoustic lens\textsuperscript{16} have been tried to improve tangential resolution but they have their shortcomings. At the reconstruction level, algorithms have been developed to improve tangential resolution. The large detectors are treated as point detectors in conventional delay-and-sum (CDAS) algorithm where spatially averaged PA signal is backprojected from the centers of the detectors to recreate an image. In modified delay-and-sum (MDAS),\textsuperscript{17,18} the recorded signal for each transducer is uniformly divided and distributed across all the segments of the sensor. Then it is backprojected to produce an image.

The concept of Gaussian apodization has been introduced recently and the impact of using apodized transducers on the PAT imaging has been studied by us.\textsuperscript{19} Essentially, Gaussian weight was used to apodize the transducers and resultant PA signals were generated. These signals were then used for image reconstructions using the CDAS and the MDAS algorithms. Further, in the MDAS, the PA signals were redistributed uniformly in that work.\textsuperscript{19} The redistribution of the PA signal in the present work has been made in a Gaussian manner and its effect on image reconstruction has been investigated for different apodization conditions with respect to the ideal point detectors. Thus, we have obtained a robust MDAS algorithm.

This paper contains different sections. Section 2 gives the mathematical background and also describes the numerical methods for image reconstruction. Section 3 presents the results obtained in this study. The discussion along with the conclusions based on the results are provided in section 4.

2. MATERIALS AND METHODS

2.1 Equations Involved

When an acoustically homogeneous medium is illuminated by light, a pressure $p(r, t)$ is developed due to the absorption of electromagnetic radiation. Accordingly, a wave equation can be written as:\textsuperscript{20}

$$\nabla^2 p(r, t) - \frac{1}{v^2} \frac{\partial^2 p(r, t)}{\partial t^2} = -\beta \frac{\partial H(r, t)}{\partial t},$$  \hspace{1cm} (1)

where $\beta$, $C_p$, $v$, and $H(r, t)$ are the isobaric volume expansion coefficient, specific heat, speed of sound, and the heat deposition per unit time per unit volume, respectively. If the excitation laser pulse is very short and thus can be modeled as a $\delta(t)$ pulse. Then the heat function can be written as $H(r, t) = A(r)\delta(t)$, where $A(r)$ is the spatial light absorption function. If $p_0(r) = \Gamma(r)A(r)$ is the initial pressure then Eq. (1) becomes,\textsuperscript{21}

$$\nabla^2 p(r, t) - \frac{1}{v^2} \frac{\partial^2 p(r, t)}{\partial t^2} = -\frac{p_0(r)}{v^2} \frac{d\delta(t)}{dt},$$  \hspace{1cm} (2)

where, $\Gamma(r) = \frac{\rho \sigma^2}{C_p}$ is the Gr"uneisen parameter.

In PAT, one maps the initial pressure rise $p_0(r)$ in a tissue from the time series acoustic data $p(r_0, t)$ measured at $r_0$. The exact solution can be analytically obtained with Green’s function approach for various geometries.\textsuperscript{14}
Figure 2. (a) The PA set up for simulation, (b) five points phantom, (c) k-Wave phantom, (d) vasculature phantom. The initial pressure of the white region is 0 Pa and that of the black region is equal to 1 Pa.

However, this Fourier method may involve multiple integration and series summations and hence is complicated. In order to simplify the solution, Xu and Wang proposed a universal backprojection formula in the time domain given as,

\[ p'_b(r) = \int_{\Omega_0} b(r_0, t = \frac{|r - r_0|}{v}) d\Omega_0 / \Omega_0, \quad (3) \]

where

\[ b(r_0, t) = 2p(r_0, t) - 2t \frac{\partial}{\partial t} p(r_0, t), \quad (4) \]

is the backprojection term, \( d\Omega_0 \) is the solid angle subtended by the transducer element \( dS_0 \) at \( r \) and \( \Omega_0 \) is the total solid angle subtended by entire detection surface \( S_0 \) with respect to \( r \). For 2D, \( \Omega_0 = 2\pi \) and \( \Omega_0 = 4\pi \) for 3D.

The finite size detectors were used in this study to capture PA signals at different angular locations around the illuminated tissue [see Fig. 2(a)]. Such a transducer may have nonuniform aperture sensitivity and thus the resultant signal can be calculated by taking the weighted sum of the incident PA waves. So the detector output can be given as,

\[ p'(r_0, t) = \int p(r'_0, t) W(r'_0) d^2 r'_0, \quad (5) \]

where \( W(r'_0) \) is the sensitivity term or the weighting factor. In our study, we have varied transducer sensitivity in a Gaussian manner with respect to the center of the aperture. Thus Eq. (5) can be written as,

\[ p'(r_0, t) = \int p(r'_0, t) \exp \left( -\frac{|r'_0 - r_c'|^2}{2\sigma^2} \right) d^2 r'_0, \quad (6) \]

where \( \sigma \) is the standard deviation and \( r'_c \) is the center of the transducer. The above equation in this work was evaluated to compute the signal recorded by a sensor.
2.2 Numerical Simulation

Fig. 2(a) shows a typical PA set up used in this work. The size of the computational domain consisted of $341 \times 341$ grid points with each point separated from the next point by a distance of 0.1 mm. The computational domain is surrounded by a 2 mm thick absorbing layer known as the perfectly matched layer (PML). The medium properties such as speed of sound (1500 m/s) and density (1000 Kg/m$^3$) were considered to be homogeneous throughout the acoustically lossless medium. The imaging region of size 201 $\times$ 201 was placed at the centre of the computational domain. The imaging domain was surrounded by 200 detectors placed in a circular manner at a distance of 15 mm from its center to capture the PA signals as shown in the figure. Flat detectors of 12 mm aperture diameter were considered to study the effect of Gaussian distribution being applied during signal acquisition as well as redistribution. Three Gaussian widths were considered i.e., $\sigma = 0.8, 2.4$ and 4.8 mm and accordingly, full width at half maximum (FWHM) can be calculated. These finite size detectors were broken into 101 points and were located within the computational domain. Further, the time domain pressure fields were recorded at these points. The sampling interval was 20 ns and the number of time points was 1608 for each detector. The sensors had center frequency of 2.25 MHz and 70% bandwidth. A 40 dB noise was also added.

In this work, we used three numerical phantoms which were taken as binary images (1 for the source points and 0 outside). These were five points, k-Wave, and vasculature phantoms. The first phantom consisted of five point sources located at a distance of 0, 2.4, 4.8, 7.2 and 9.6 mm from the center (see Fig. 2(b)). This allowed us to study variations in axial and tangential resolutions under different apodization conditions. The second phantom used was a text k-Wave [see Fig. 2(c)] and the third one was a vasculature which mimicked the human blood vessel network as shown in Fig. 2(d).

![Point detector](https://www.spiedigitallibrary.org/conference-proceedings-of-spie)

**Figure 3.** (a) Reconstructed image of the five points phantom using point detectors. A color bar on the left indicates gray levels in the image. (a1)-(a5) Enlarged images of those point sources in the phantom. (b)-(d) Reconstructed images for the same phantom when the CDAS algorithm is employed at $\sigma = 0.8, 2.4$ and 4.8 mm, respectively for a 12 mm diameter transducer. The corresponding zoomed images are also shown. (e)-(g) Same as (b)-(d) but reconstructed using the MDAS algorithm. The PA signals were recorded at 200 different sensor locations in each case.
2.3 Image Reconstruction

The forward simulation was performed in the k-Wave toolbox. The PA signals were recorded at the point locations for each detector. After that the Gaussian weighting was applied and a resultant signal was produced as given in Eq. (6). The backprojection term in Eq. (4) was computed for each detector. Here we omitted the second part of the equation for simplicity. Image reconstructions were performed using the CDAS [see Fig. 1(b)] and the MDAS [shown in Fig. 1(c)] algorithms. For the CDAS, the Gaussian weight was applied only during signal capture and then backprojection algorithm was implemented. However, for the MDAS protocol, the weight was applied in both directions meaning during signal acquisition and redistribution of the pressure values at the point locations on the surface of the transducer. The Pearson correlation coefficient (PCC) was computed with respect to the original image for each reconstructed image to evaluate the performance of an algorithm quantitatively. All the codes were written in MATLAB and executed using a 64 bit personal computer with i5 processor, 12 GB RAM and 3.50 GHz clock speed.

3. SIMULATION RESULTS

Fig. 3 exhibits the reconstructed image of the five points phantom. The images are normalized and gray scale values are displayed using a color bar. Figs. 3(a1)-(a5) are the enlarged point sources. These are included for better visualization. The reconstructed image using ideal point detectors [see Fig. 3(a)] has retained its shape but the points now appear as blurred circles because of band-limited nature of the detectors. All the points are identical in size and shape, hence, tangential and axial resolution remain spatially invariant in this case.

![Figure 4](https://example.com/figure4.png)

Figure 4. (a) Reconstructed image of the k-Wave phantom for point detectors. (b)-(d) Images reconstructed using the CDAS algorithm at $\sigma = 0.8, 2.4$ and $4.8$ mm, respectively for a 12 mm diameter transducer. (e)-(g) Same as (b)-(d) but for the MDAS algorithm.

Figs. 3(b)-(d) show the images reconstructed using the CDAS method for a 12 mm diameter sensor under different apodization conditions. At the scanning center, the reconstruction seems to be perfect [Fig. 3(b1)] but as we go away from the scanning center an arch is formed from a blob. The rate of increase of arch length is maximum for $\sigma = 4.8$ mm [Fig. 3(d)] and minimum for $\sigma = 0.8$ mm [Fig. 3(b)]. In the MDAS technique, the images [see Fig. 3(e)-(g)] look identical for all apodization conditions and the results are comparable to that of ideal point detectors. Similar results were produced for k-Wave and vasculature phantoms (see Fig. 4 and Fig. 5, respectively).

Table 1 presents the numerical values for the PCC for a 12 mm sensor for various reconstruction methods. It can be seen that the CDAS performs well when apodization is strong ($\sigma = 0.8$ mm) whereas the MDAS can
Figure 5. (a) Reconstructed image of the vasculature phantom using point detectors. (b)-(d) Images reconstructed by employing the CDAS algorithm at $\sigma = 0.8, 2.4$ and $4.8$ mm, respectively for a $12$ mm sensor. (e)-(g) Similar to (b)-(d) but for the MDAS algorithm.

produce images similar to ideal point detectors for all apodization conditions for all phantoms considered in this study.

Table 1. PCC values for reconstructed images based on the CDAS and the MDAS algorithms at different FWHM values. Results for point detectors (PD) are also included here.

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<tr>
<th>Aperture diameter</th>
<th>$\sigma$ values</th>
<th>Five points</th>
<th>k-Wave</th>
<th>Vasculature</th>
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<tr>
<td></td>
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<td>MDAS</td>
<td>CDAS</td>
<td>MDAS</td>
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<tr>
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4. DISCUSSION AND CONCLUSIONS

Single element transducers are generally used in PAT set ups as they have good signal-to-noise ratio. Moreover, such transducers can be purchased from vendors easily. Because of their finite size, the PA waves striking the surface get spatially averaged (i.e. higher frequency components are filtered out). Thus, reconstructed images appear blurry. In this study, we used apodized transducers for signal collection and subsequent image reconstruction. These transducers have higher sensitivity at the central region of the aperture and it decays towards the periphery in a Gaussian manner. This suppresses side lobes reducing image artifacts.

The CDAS algorithm is a widely used and a simple method for image reconstruction. We can see in Fig. 3(b) that the results for the CDAS are comparable to those of point detectors when the FWHM width is small ($\sigma = 0.8$ mm) i.e, strong apodization for a $12$ mm sensor. The MDAS algorithm was also implemented for image reconstruction. In our previous work, the MDAS showed an opposite trend to that of the CDAS when we redistributed the captured signal equally among all the points lying on the $12$ mm sensor surface.\(^{19}\) Nevertheless, in the present study, we redistributed the captured signal in a Gaussian manner with highest value at the center of the transducer. The simulation results show that if this procedure (Gaussian acquisition and redistribution) is adapted then the MDAS algorithm provides results comparable to that of the CDAS even in case of strong
apodization [compare Fig. 3(b) and 3(e)]. The PCC values for $\sigma = 0.8$ mm for both the algorithms have now become comparable for all phantoms. It may be noted that the CDAS performs better than the MDAS when the Gaussian redistribution is not incorporated.

The MDAS method in both the cases (strong and weak apodizations) works faithfully and proves to be a robust algorithm if the Gaussian weight is applied during signal capture and redistribution of pressure values at the point locations on the surface of the detector.

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REFERENCES


