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CONTROL OF PARALLEL INVERTERS IN MICROGRID

LIU JIAZHE
SCHOOL OF ELECTRICAL & ELECTRONIC ENGINEERING
2019
CONTROL OF PARALLEL INVERTERS IN MICROGRID

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School of Electrical & Electronic Engineering

A thesis submitted to the Nanyang Technological University
in partial fulfillment of the requirement for the degree of
Master of Engineering

2019
Statement of Originality

I hereby certify that the work embodied in this thesis is the result of original research, is free of plagiarised materials, and has not been submitted for a higher degree to any other University or Institution.

\[26/03/2019\]  \hspace{2cm}  \text{Jiazhe Liu}

Date  \hspace{2cm}  Jiazhe Liu
Supervisor Declaration Statement

I have reviewed the content and presentation style of this thesis and declare it is free of plagiarism and of sufficient grammatical clarity to be examined. To the best of my knowledge, the research and writing are those of the candidate except as acknowledged in the Author Attribution Statement. I confirm that the investigations were conducted in accord with the ethics policies and integrity standards of Nanyang Technological University and that the research data are presented honestly and without prejudice.

26/3/2019
Date

Tang Yi

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Authorship Attribution Statement

This thesis contains material from 1 paper published in the following peer-reviewed journal and 1 paper accepted at a conference in which I am listed as an author.


The contributions of the co-authors are as follows:

- Assistant Prof Tang provided the initial project direction and edited the manuscript draft.
- I prepared the manuscript draft. The manuscript was revised by Assistant Prof Tang and Mr. Yang Qi.
- I co-designed the control strategies with Assistant Prof Tang and Mr. Yang Qi.
- I built up the models of the proposed method in PLECS with Mr. Yang Qi and carried out simulations.
- All the circuits used in the scale-down experiments were conducted by Mr. Yang Qi and me for analysis and testing.
- I analyzed data with Mr. Yang Qi.
- Mr. Yang Qi provided the guidance in the hardware experiments.

The contributions of the co-authors are as follows:

- Assistant Prof Tang provided the initial project direction and edited the manuscript draft.
- Mr. Yang Qi prepared the manuscript draft. The manuscript was revised by Assistant Prof Tang, Dr. Jingyang Fang and me.
- Mr. Yang Qi, Dr. Jingyang Fang, Assistant Prof Tang and I co-designed the control strategies.
- I built up the models of the proposed method in PLECS with Mr. Yang Qi and carried out simulations.
- All the circuits used in the scale-down experiments were conducted by Mr. Yang Qi and me for analysis and testing.
- Mr. Yang Qi and Dr. Jingyang Fang provided the guidance in the hardware experiments.

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Jiazhe Liu
04/03/2019
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Summary

With the development of renewable energy sources, the microgrid is widely implemented because of its flexibility and high-efficiency. It makes the most use of distributed generators and energy storage systems, which can supply the local load with minimum energy loss.

However, there are always two concerns in the microgrid. The first one is power sharing. Generally, power sharing is designed to be proportional to the generator’s capacity to reach the highest efficiency. Droop control, which is well-known for its non-communication control strategy, can achieve proportional active and reactive power sharing among parallel power converter systems. However, its feasibility can be seriously influenced by some additional factors, such as line impedance mismatch, different types of line impedance and sensor error. Thus, in this report, the power sharing of the droop control with various disturbances are comprehensively discussed and the corresponding method to mitigate the power sharing error is proposed.

The other concern is power quality. The droop control is only responsible for the power sharing in the fundamental domain but cannot control the currents in the harmonic domain. However, in the microgrid, the widely implemented nonlinear load can inject the harmonic currents in the microgrid and cause the voltage distortion at the point of common coupling (PCC). Besides, switching deadtime of the converter is another harmonic source, which can generate large harmonic circulating currents when the line impedance is small. To simultaneously suppress the circulating current harmonics and mitigate the PCC voltage harmonics, a novel control strategy is proposed based on virtual harmonic impedance controls.

The feasibility of the proposed control strategies is verified through the Matlab/Simulink simulations or scale-down experiments.
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<tr>
<td>PCC</td>
<td>Point of Common Coupling</td>
</tr>
<tr>
<td>RES</td>
<td>Renewable Energy Sources</td>
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<td>DG</td>
<td>Distributed Generator</td>
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<tr>
<td>PV</td>
<td>Photovoltaic</td>
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<tr>
<td>ESS</td>
<td>Energy Storage System</td>
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<td>VI</td>
<td>Virtual Impedances</td>
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<td>MR</td>
<td>Multi Resonance</td>
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<tr>
<td>SOGI</td>
<td>Second-Order Generalized Integrator</td>
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<tr>
<td>VHI</td>
<td>Virtual Harmonic Impedances</td>
</tr>
<tr>
<td>PCS</td>
<td>Power Converter System</td>
</tr>
<tr>
<td>SPWM</td>
<td>Sinusoidal Pulse Width Modulation</td>
</tr>
<tr>
<td>EMI</td>
<td>Electromagnetic Interference</td>
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<tr>
<td>FFT</td>
<td>Fast Fourier Transformation</td>
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<tr>
<td>SVPWM</td>
<td>Space Vector Pulse Width Modulation</td>
</tr>
<tr>
<td>FFT</td>
<td>Fast Fourier Transform</td>
</tr>
<tr>
<td>MSRF</td>
<td>Multiple Synchronous Reference Frames</td>
</tr>
<tr>
<td>NVHI</td>
<td>Negative Virtual Harmonic Impedance</td>
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<tr>
<td>PVHI</td>
<td>Positive Virtual Harmonic Impedance</td>
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Chapter 1 Introduction

1.1 Background

In the last decades, the trend of the power system transformed from the centralized generation to the distributed generation. The promotion of renewable energy sources (RES) generation in the power system has been done in many countries since it is environmental-harmless and sustainable. Fig. 1.1 shows the trend of the global renewable energy installed capacity from 2006 to 2017. Microgrid, which is generally considered as a small-scale power grid, can become more economical and flexible with the implementation of the distributed generator (DG) with RES[1-5]. Since DGs are normally connected to the local microgrid, the distributed RES such as photovoltaic (PV) power and wind power can be fully utilized with lower energy transmission losses. Besides, the energy storage system (ESS) works as another important part of the microgrid. It improves the reliability of the power supply in the microgrid, which is quite important to uninterrupted RES generation. Thus, because of its high efficiency and good performance, microgrid plays a necessary role in the modern power system.

Generally, DGs can be classified by the control strategy. There are two types of DGs, grid-feeding DG and grid-forming DG[1, 2, 6]. Grid-feeding DGs are usually applied in the grid-connected mode. They work as current sources to supply the power to the microgrid. On the other hand, Grid-feeding DGs are usually applied in the islanded mode. They work as voltage sources and regulate the voltage and the frequency of the islanded microgrid. Thus, the power quality and power sharing largely rely on the control performance of the grid-feeding DGs. The control strategy for the grid-forming DGs can be sorted into two types, i.e. control with communication and control without communication. The typical communication-based controls are concentrated control and Master/Slave [7-10]. The performance of these kinds of control greatly depends on the communication line. They have good performance on regulation the communication is ideal. However, the reliability of the communication-based controls is low because of the
communication interruption and failure. For communication-less controls, droop control can achieve the active and reactive power sharing by mimicking the droop characteristic of the synchronous generator, which has been widely implemented in the modern microgrid[1-3, 11].

Fig. 1.1. Trends in installed renewable energy capacity[47].

Though droop control is famous for its reliability, redundancy and simplicity, some new problems are raised because of the technical limitation. The first one is load sharing accuracy. Mismatched line impedances and sensor errors are the main reasons for the inaccurate power sharing in the droop control. Poor power sharing might cause the overload of the converter and cannot make full use of DG’s capacity. Another point of concern is the power quality[12, 13]. In islanded microgrids consisting of parallel voltage-controlled inverters, nonlinear loads and switching deadtime are the two main sources of the harmonic. They can degrade the quality of the output and reliability of the system. Specifically, the harmonics current generated from nonlinear loads can cause the harmonic voltage drop at PCC when they flow through line impedances[14]. The output
voltage distortions result from the witching deadtime can also lead to additional circulating harmonic currents among parallel inverters[15].

1.2 Objectives and Scopes

1.2.1 Power Sharing

The power sharing accuracy of droop control can be seriously influenced by the mismatched line impedance and complex types of line impedance. Normally, virtual impedance is implemented in the control, which adjusts the reference voltage to reshape the equivalent impedance of each line. Though virtual impedance can improve the power sharing in many previous studies, there is always a trade-off between accurate power sharing and voltage regulation.

Besides, the influences of DC offset and scaling error of the sensor on the control are rarely discussed, which can also seriously deteriorate the power sharing. However, they are inevitable in practical implementation. Thus, the impact of the sensor errors on voltage regulation and power sharing accuracy are analyzed in this report. The simulations of sensor error are also done to verify the analysis. To mitigate the disturbance of DC offset and scaling error on the power sharing accuracy, an advanced control method has been proposed.

1.2.2 Harmonic Mitigation

Generally, voltage harmonics at PCC caused by nonlinear loads can be mitigated through passive power filters or active power filters. For passive power filters, they can effectively suppress the harmonic. However, it can lead to resonance and the suppression is only effective to the harmonics with a certain order. For active power filter, the mitigating performance is excellent but the cost is much expensive because of the additional power devices. Some methods from control aspect are also investigated. Virtual harmonic impedance (VHI) is proposed in[16] to damp the PCC voltage harmonics by adjusting line impedances. However, apart from nonlinear loads, switching
deadtime of the inverter can also result in circulating current. The circulating harmonic can be largely amplified if VHI is applied in the control, which leads to additional energy loss and poor stability. Therefore, it is difficult to simultaneously attenuate the PCC voltage distortion and circulating current harmonics because of the inherent conflict.

To simultaneously mitigate the PCC voltage distortion caused by nonlinear loads and the harmonic circulating current introduced by the deadtime of the inverter, this thesis proposes a novel control strategy for a simplified microgrid with two parallel inverters. Different virtual harmonic impedances are adopted for two inverters to equivalently change the topology of the parallel inverter system. As a result, all the harmonic currents will only flow to one inverter without large harmonic voltage distortion at PCC and the circulating harmonic current can be suppressed. Note that multiple second-order generalized integrator (SOGI) blocks are applied in the method to selectively synthesize virtual harmonic impedances[17]. With Multi SOGI, the fundamental power sharing performance mentioned in chapter 2 will not be influenced.

1.3 Organization

The rest of paper is organized as follows:

In Chapter 2, the power sharing problem in the microgrid is discussed in detail, which includes the literature review of the control strategy and existing solutions to power sharing. An advanced control strategy based on droop control is proposed to achieve accurate power sharing with various disturbance.

In Chapter 3, the harmonic in the microgrid is analyzed through the harmonic sources, influence on the microgrid and solutions to the harmonics. A novel method to simultaneously mitigate the harmonics from the nonlinear load and converter deadtime is proposed.

In chapter 4, the works in this thesis are concluded and the future work is provided.
Chapter 2 Power Sharing

2.1 Overview

In this chapter, the concern of the power sharing problems in parallel inverter microgrids is discussed. In the first part, the literature review of the power sharing is provided, which consists of the introduction of the islanded parallel inverter microgrid model, classical control method of the power sharing and their limitations. Then, the different disturbances in droop control, such as mismatched line impedance, different types of line impedance and sensor errors, are analyzed and existing solutions to these problems are discussed. In the third part, a novel method to improve power sharing performance is proposed. The controller design and the simulation results are provided to show the feasibility of the proposed method. Finally, a brief conclusion of the work in this chapter is given.

2.2 Literature Review

2.2.1 System configuration of the microgrid

With the development of the RES and ESS, microgrid is widely applied in modern power systems. It integrates a number of parallel DGs to supply energy during peak load time with high operating efficiency and low transmission loss. At the same time, comparing to single DG, it has a larger capacity and higher stability. A general configuration of the microgrid is shown in Fig. 2.1. \( L_{fi} \) and \( C_{fi} \) are the filter inductor and capacitor of each voltage source, respectively. The combination of the DG and converter is named as the power converter system (PCS) in this thesis. \( R_{li} \) and \( L_{li} \) are the resistive and inductive line impedance between the PCC and PCS, respectively. The microgrid is connected to the utility grid at PCC.

Microgrid can be operated in two modes, grid-connecting mode and islanded mode. In the grid-connecting mode, the loads are fed by both DGs and the utility grid. Since DGs are located close to the local load, the energy loss during transmission can be reduced.
Chapter 2 Power Sharing

When a fault occurs in the utility grid, the microgrid will suddenly switch to the islanded mode. In this mode, DGs are controlled as a voltage source to establish voltage and supply all the local load. Once the fault removed, the microgrid will synchronize the output voltage with the voltage at the utility grid side. It switches back to the grid-connecting mode when the synchronization is finished.

2.2.2 Power Sharing Strategies

PCS control generally consists of inner voltage and current loop and external power loop. For inner loop, it regulates the output of the converter. The design of the inner loop will be discussed later in chapter 2.3.2. For external power loop, it provides a reference for inner loop according to the output power. In islanded microgrids, it is necessary to share the power delivered from DGs proportionally to their capacities to avoid overload and realize maximum capacity utilization. The control strategy for power sharing among DGs can be sorted into two types, communication-based power sharing control and wireless power sharing control. Central control, Master/Slave control and droop control are investigated in this chapter.

a) Central control

The structure of central control is shown in Fig. 2.2[7, 18]. To share equal power to each converter, all the currents are sent to the central controller first. The current reference is
Chapter 2 Power Sharing

derived by calculating the mean of the all PCS output currents. The control design in this method is simple and the sharing performance is good. However, the communication delay of this method is large because of the data sending and receiving. The current signal requires large bandwidth communication line, which sharply increases the cost. At the same time, communication can also decrease the reliability of the system.

![Diagram](image)

**Fig. 2.2. Control strategy of general central control with current reference.**

**b) Master/Slave control**

Master/Slave control strategies[19, 20] can be applied to achieve power sharing among parallel inverters. The general control strategy is depicted in Fig. 2.3(a)[19]. In this figure, the power sharing is designed to be equal. When the power system works, the first operating PCS is chosen as the master unit. The control of the master unit contains a voltage controller and a current controller to regulate the voltage of the AC bus. Other units running later are slave units. Their current controllers track the current reference provided by the master unit, which achieves precise power sharing among all the units. All the reference is delivered by the CAN bus, which reduces the communication delay and the amount of the communication bus.

In [20], an auto master/slave control based on tracking active power and reactive power is proposed. The control strategy is displayed in Fig. 2.3(b). The unit with maximum power will automatically be selected as the master unit and send its active power and
reactive power to the CAN as reference. Other output power of the slave units will track the master unit and the power can be evenly shared among converters. Comparing to the master/slave control with current reference, this method reduces the requirement of the communication band. However, the reliability problem still remains and sharing performance can be seriously degraded when sensor errors occur in the master units. Besides, the controller design is complicated since the control loop of the master unit and slave units are coupled.

Fig. 2.3. Control strategy of the general master/slave; (a) with current reference, (b) with power reference.
Chapter 2 Power Sharing

The reliability of the master/slave power sharing control is a great concern. Since only one master module is responsible for establishing the voltage, the overall system is prone to a single point of failure. In other words, if the master module fails, the entire system will break down. To address this issue, enhanced master/slave power sharing methods have been proposed in [20-23]. In [21], a rotating priority window is designed, which allows the random selection of a new master module and hence results in true redundancy. A similar approach is proposed in [20], which selects the PCS with the highest output power to be the master module. In [22], the utility interface is regarded as the master module, while the distributed energy resources are integrated through slave-controlled PCSs. Moreover, a droop and master/slave hybrid power sharing strategy has been proposed in [23]. The entire large grid is divided into small blocks with every single block containing a master module and slave modules. By doing so, the power sharing issue within a block can be addressed by the master/slave control, while the power sharing among different blocks is achieved by the conventional droop control. The redundancy caused by multiple master modules can, therefore, improve system reliability.

Master/Slave control has excellent performance on the power sharing and control dynamic. However, the requirement of the communication line introduces lots of additional troubles, such as high cost, low reliability and vulnerability to the sensor error and communication interference.

c) Droop control

Droop control is one of the well-known communication-less control strategies. The idea of droop control derived from the droop characteristic of the synchronous generator, whose output frequency and voltage magnitude change according to the output active and reactive powers. To analyze the relationships between the output powers and the output characteristics of frequency and voltage magnitude, an equivalent model and a phasor diagram describing a PCS connected to the PCC are shown in Fig. 2.4 and Fig. 2.5, respectively.
The active power and reactive power of the PCS can be expressed as:

\[
\begin{align*}
P &= \frac{U}{Z} [(E \cos \delta - U) \cos \theta + E \sin \theta \sin \delta] \\
Q &= \frac{U}{Z} [(U - E \cos \delta) \sin \theta - E \cos \theta \sin \delta]
\end{align*}
\]  

(2.1)

where \( E \) and \( U \) are the RMS values of the output voltage of the PCS and PCC, respectively. \( \delta \) is the angle difference between the PCS voltage and the PCC voltage (\( E \) leading \( U \)), \( Z \angle \theta = R + jX \) is the line impedance. Considering that the angle difference \( \delta \) is usually small (\( \sin \delta = \delta \)) and the line impedance is mainly inductive in conventional power systems (\( Z \angle \theta = Z \angle 90^\circ = jX \)), (2.1) can be simplified as:

\[
\begin{align*}
P &\approx \frac{UE \delta}{X} \\
Q &\approx \frac{U(E - U)}{X}
\end{align*}
\]  

(2.2)

As it is shown in (2.2), the active power is proportional to the angle difference \( \delta \), while the reactive power is determined by the voltage difference (\( E - U \)). Therefore, the active power and reactive power are completely decoupled. Specifically, the active power and reactive power of the PCS can be regulated by controlling the frequency and voltage.
magnitude, respectively. Based on this principle, conventional droop control is designed as:

\[
\begin{align*}
\omega &= \omega^* - m(P^* - P) \\
E &= E^* - n(Q^* - Q)
\end{align*}
\] (2.3)

where \( \omega^* \) and \( E^* \) are the operating frequency and reference voltage magnitude, respectively. \( m \) and \( n \) are droop coefficients. \( P^* \) and \( Q^* \) are the dispatched active power and reference reactive power in the grid-connected mode, respectively. The relationship between these characteristics is graphically shown in Fig. 2.6.

![Fig. 2.6. Characteristics of the conventional droop control.](image)

Droop coefficients \( m \) and \( n \) determine the PCS power sharing ratio and are designed inversely proportional to the PCS power rating. Moreover, the design of droop coefficients should also ensure that the maximum voltage/frequency deviations are within acceptable ranges [24, 25], i.e.:

\[
\begin{align*}
m &= \frac{\omega^* - \omega_{\text{min}}}{P^* - P_{\text{max}}} \\
n &= \frac{E^* - E_{\text{min}}}{Q_{\text{max}} - Q^*}
\end{align*}
\] (2.4)

where \( \omega_{\text{min}} \) and \( E_{\text{min}} \) are the minimum permitted operating frequency and output voltage, respectively. \( P_{\text{max}} \) and \( Q_{\text{max}} \) are the maximum PCS output active and reactive power, respectively.

Fig. 2.7 shows the circuit and control block diagram of a droop-controlled parallel converter system. An \( LC \) filter (\( L_f \) represents the filter inductor and \( C_f \) represents the filter
capacitor) is adopted to attenuate the high-frequency switching noise. $Z_l$ denotes the line impedance between the PCS and the PCC. To perform a grid-forming converter, the overall controller consists of a primary power controller, an outer voltage control loop, and an inner current control loop. The power controller generates the reference voltage according to the droop equations in (2.3), whereas the voltage controller eliminates the tracking error between the reference voltage and the measured capacitor output voltage rapidly, with the help from the inner current control loop. Moreover, from the outside to the inside, the dynamic responses of control loops are usually designed to slow down gradually.

Droop control can be operated in both grid-connected mode and islanded mode. When a fault occurs in the utility grid, static transfer switch (STS) will be switched off. Microgrid supplies all the local load to ensure the consistency of the power supply. The output power of each converter will increase and the operating frequency and the output voltage will decrease according to the droop characteristic. Once the fault in the utility grid is removed, the synchronization controller in Fig. 2.7 will be activated to force the voltage at the microgrid side equal to that at the grid side. After the synchronization is completed and STS is switched on, the synchronization controller should be disabled immediately to avoid the influence on grid-connected control.

![Fig. 2.7. Circuit and control block diagram of a droop-controlled PCS.](image-url)
Although droop control is preferred due to the simple implementation and communication-free characteristic, it is also limited by some disadvantages:

(a) Droop control is implemented under the assumption of inductive line impedances. However, in low-voltage microgrids, line impedances may be resistive or complex. In such cases, the former simplification cannot be made, and the coupling between the active power and reactive power becomes strong.

(b) The reactive power sharing among parallel PCSs can be seriously deteriorated due to mismatched line impedances.

(c) Droop control can only realize the fundamental power sharing among parallel PCSs. It cannot handle the harmonic power sharing or the unbalanced load power sharing.

(d) The steady-state voltage and frequency will deviate from nominal values, especially at heavy load situations.

**d) Comparison of power sharing control with and w/o communication**

The salient advantage of communication-based control is the improved power sharing performance. For the conventional droop control, the reactive power sharing error can hardly be eliminated due to inevitable mismatched line impedances and measuring errors. In contrast, with the central control or master/slave configuration, since the current sharing among the PCSs is directly controlled by the master module, the power sharing accuracy can always be guaranteed.

In addition, the central control and master/slave control can achieve instantaneous power sharing, since the current control loop dynamics can be very fast for the slave modules. As a contrary, the dynamics of the conventional droop control is relatively slow (e.g. in terms of several seconds). Because of the low pass power filters employed [26], the power distribution in droop control takes a much longer time to settle down.
However, as the master/slave power sharing control requires the external reference, communication links are necessary if PCSs are not located nearby. Communication links produce additional cost and degrade the system expandability. Besides, the delay in communication can also deteriorate sharing performance.

Besides, with the implementation of the master/slave power sharing strategy, all the instantaneous imbalance between power generations and load consumptions has to be compensated by the voltage-controlled master module. Therefore, some problems may occur in the Master/Slave control. Considerable current overshoot may probably occur during transients such as sudden load changes and shut down of a slave unit[2]. Such high output current may trigger current protection and jeopardize the lifetime of the master module.

Comparing these three power sharing methods, droop control is more preferred for its high reliability and flexibility. Thus, the research and analysis later are based on droop method.

### 2.2.3 Sharing Error

Ideally, droop control can achieve equal or proportional active power and reactive power sharing. However, in practice, some additional factors such as line impedance, different types of line impedance and sensor error can lead to power sharing error among PCSs.

**a) Mismatched line impedance and output impedance**

In the ideal case, the active power sharing and reactive power sharing of droop control are decided by droop coefficients, $m_i$ and $n_i$. In the power system with inductive line impedance, the active power can always be shared accurately because it is only related to the frequency and the frequency of parallel PCSs is the same. On the contrary, reactive power sharing is not only determined by the droop coefficients. The voltage drop on line impedance can be simplified to the following equation:
where \( E_i \) and \( U_{PCC} \) are the output voltage of the \( i \)th PCS and the voltage at PCC, respectively. \( X_{gi} \) and \( X_{ci} \) are the line impedance and equivalent converter output impedance of the \( i \)th PCS. \( Q_i \) presents the output reactive power of the \( i \)th PCS.

For simplicity, \( U_{PCC} \) is considered as \( U_{ref} \) when the system is stable because the voltage deviation is in the acceptable range. \( U_{ref} \) is the reference voltage in droop control. Thus, the reactive power sharing can be derived from (2.5):

\[
\frac{Q_i}{Q_j} = \frac{n_j + (X_{gi} + X_{cj})}{n_i + (X_{gi} + X_{cj})} \frac{U_{ref}}{U_{ref}}
\]

Clearly, the reactive power sharing is inversely proportional to the combination of droop coefficient, line impedance and PCS output impedance. In practice, the line impedance of each PCS cannot be exactly the same because it depends on the length of the transmission lines and the location of the DG. Besides, the PCS output impedance depends on the control strategy of the converter, which can also result in the impedance mismatch. Thus, it difficult to ensure the accurate reactive power sharing of the droop control.

b) Complex line impedance

As discussed before, the conventional droop control is based on the assumption that line impedance is purely inductive. Actually, the real network is mainly mixed resistive and inductive. Especially, in the low-voltage microgrid, the R/X ratio of the line impedance is very large[27]. When the line impedance is complex or resistive, (2.1) cannot be derived and active power and reactive power are coupled with each other. When the line impedances of all PCSs are complex and have the same, (2.5) can be rewritten to:
$U_{PCC} = E_i - n_i Q_i - \frac{|Z_{gi} + Z_{ci}| \cos \varphi_i P_i + |Z_{gi} + Z_{ci}| \sin \varphi_i Q_i}{E_i}$  \hspace{1cm} (2.7)

where $\varphi_i$ presents the impedance angle of the sum of line impedance. $Z_{gi}$ and $Z_{ci}$ present the line impedance and the converter output impedance, respectively. To analyze the influence of the line impedance angle on power sharing, some assumptions have been made:

\[
\begin{align*}
Z_{ui} &= Z_{gi} + Z_{ci} \\
|Z_{ui}| &= |Z_{uj}| = |Z_s| \\
\varphi_i &\neq \varphi_j \\
\frac{P_i}{P_j} &= \frac{m_j}{m_i}
\end{align*}
\hspace{1cm} (2.8)
\]

$Z_{ui}$ is the sum of the line impedance and PCS output impedance of $i$th PCS. The magnitudes of the line impedance of each PCS are the same but the line impedance angles are different. Because the frequencies of all PCS are the same, the active power $P_i$ is inversely proportional to the droop coefficient $m_i$. Based on these assumptions, the reactive power sharing can be derived as:

\[
Q_i = n_i + \frac{|Z_s| \sin \varphi_j}{n_j + \frac{|Z_s| \sin \varphi_i}{U_{ref}}}
\hspace{1cm} (2.9)
\]

It is obvious that the reactive power sharing is decided by the droop coefficients $m$ and $n$ and the impedance angle $\varphi_i$. When the line impedance is complex, the reactive power sharing can be greatly disturbed by the impedance angle. Even the impedance angles of two PCS are the same, the droop coefficient $m$, which is supposed to determine the active power sharing, can also affect the reactive power sharing. Thus, it is difficult to apply droop control to the mainly resistive low-voltage microgrid and complex medium-voltage microgrid.
c) Measurement errors

Generally, sensor errors are unavoidable in practice, which can greatly affect the control of the converter. DC offset and scaling error are two common sensor errors.

DC offset means that an additional DC injection is superposed on the real measuring signal, which is resulted from the mis-zeroing of the sensors before using.

Assuming the voltage controller of the PCS are well-designed and can track the reference precisely, the actual output voltage will deviate from the reference value, if the DC offset occurs in the output voltage sensor. The stable output voltage can be expressed as:

\[ U_o = U_{\text{ref}} - e_{dc} \]  

where \( U_o \) is the real output voltage, \( U_{\text{ref}} \) is the reference from the droop module and \( e_{dc} \) is the additional DC offset.

Scaling error presents that the magnitude of the measuring signal varies from the real value. Large sharing error can be introduced even if the scaling error is very small. Assuming the voltage controller of the PCS is well-designed and can track the reference precisely. When there is \( a\% \) scaling error in the voltage sensor. The stable output voltage can be expressed as:

\[ U_o = \frac{1}{(1 + a\%)} U_{\text{ref}} \]

Obviously, DC offset and scaling error can both be reflected in the output voltage and lead to circulating current among PCSs.

2.2.4 Existing Control Strategies for Sharing Error

a) Adaptive droop control

To realize more accurate reactive power sharing in droop control, adaptive droop control is proposed in [28]. The proposed droop function is expressed as:
\[
\begin{align*}
\omega_{\text{ref}} &= \omega^* - mP - m_1 \frac{dP}{dt} \\
E_{\text{ref}} &= E^* - nQ - n_1 \frac{dQ}{dt}
\end{align*}
\]  
(2.12)

where \(m_1\) and \(n_1\) present the transient droop coefficients, respectively. The transient droop coefficients is adaptively changed through analyzing the small-signal of the controller to get better reactive power sharing and desired transient and steady-state response. At the same time, the proposed adaptive droop can also enhance the resonance damping, which improves the stability of the system.

b) Virtual Impedance (VI)

Generally, virtual impedance control \([12, 17, 29]\) is implemented to mitigate the reactive power sharing inaccuracy. Virtual impedance can adjust the output impedance of a PCS by changing the reference voltage. Fig. 2.9 shows the control block of the virtual impedance, where the PCS output current is fed to the reference voltage \(V_{\text{ref}}\) through a transfer function:

\[
V_{\text{virtual}} = i_v \cdot Z_v = i_v \cdot (R_v + jX_v)
\]

(2.13)

where \(R_v\) and \(X_v\) are virtual resistive impedance and inductive impedance, respectively.

![Control block diagram of the virtual impedance.](image)

In \([29]\), the effect of virtual impedance on the power sharing is discussed and verified. Considering that line impedances are usually mismatched, virtual impedance control can enhance the reactive power sharing accuracy by modifying line impedances to be equal. Besides, larger virtual impedance values can adjust the output impedance of PCS to be
mainly inductive, which makes the system satisfy the assumption of the conventional
droop control and lead to better reactive power sharing results.

However, virtual impedances can also lead to severe voltage drop, especially under heavy
load situations. Therefore, there is always a trade-off between the reactive power sharing
accuracy and the steady-state voltage deviation. At the same time, the stability and
dynamic of the system can be influenced by employing virtual impedance loop. Due to
these reasons, the virtual impedance value needs to be carefully designed in order to
satisfy all the requirements.

c) Virtual frame transformation

(i) P-Q frame transformation

To decouple the active power and reactive power and apply the droop control in the
power system with different line impedance, rotating transformation methods[6, 30] was
proposed. Fig. 2.10 displays the diagram of droop with virtual frame transformation. In
the virtual frame transformation, a rotating matrix is introduced to modify the active
power P and reactive power Q as it shows in (2.14).

\[
\begin{bmatrix}
P_r \\
Q_r
\end{bmatrix} = \begin{bmatrix}
\cos \theta & -\sin \theta \\
\sin \theta & \cos \theta
\end{bmatrix}\begin{bmatrix}
P \\
Q
\end{bmatrix}
\]

(2.14)

where \(P_r\) and \(Q_r\) are the active power and reactive power after rotating transformation,
respectively. \(\theta\) is the angle of the line impedance.

Fig. 2.11 shows the rotating transformation for different kinds of line impedance.
Applying \(P_r\) and \(Q_r\) into the conventional droop control, the result shows that the power
angle is only related to \(P_r\), whereas the voltage difference is only related to \(Q_r\), which
means the system can be regarded as a pure inductive system after transformation. Thus,
the active power and reactive power are completely decoupled after transformation.
However, virtual P-Q frame transformation cannot directly control the actual output of the real power and reactive power of the converter, which means the proper power sharing among PCSs cannot be achieved. Besides, because of the PQ transformation, the real output power may deviate from the required output power and the power operation range should be carefully designed[31].

![Diagram of droop with virtual frame transformation](image)

**Fig. 2.9. Diagram of droop with virtual frame transformation.**

![P-Q frame transformation for different line impedance](image)

(a) inductive; (b) complex; (c) resistive.

(ii) **ω-E frame transformation**

Fig. 2.12 shows the principle of ω-E frame transformation. Similar to the P-Q frame transformation, it rotates the ω-E frame by the rotating matrix:

\[
\begin{bmatrix}
\omega_r \\
E_r
\end{bmatrix} =
\begin{bmatrix}
\cos \theta & \sin \theta \\
-\sin \theta & \cos \theta
\end{bmatrix}
\begin{bmatrix}
\omega \\
E
\end{bmatrix}
\]

where \(\omega_r\) and \(E_r\) are the frequency and voltage magnitude after rotating transformation, respectively. \(\theta\) is the angle of the line impedance. By regulating the voltage in the new frame, the output power can be accurately shared among PCS without output power deviation. However, since the converted is regulated in a new frequency and voltage frame, the output voltage operating range should be designed to ensure that the actual
output voltage is in the reasonable range. Furthermore, both P-Q frame transformation and ω-E frame transformation need precise line impedance information to realize good performance, which is difficult to obtain. When the line impedance of each PCS differs from each other, the rotating angle cannot be set as a different value because of the nature of the microgrid. Thus, accurate power sharing cannot be ensured.

![ω-E frame transformation](image)

Fig. 2.11. ω-E frame transformation.

### 2.3 Proposed Power Sharing Method

#### 2.3.1 System Configuration

The schematic diagram of the single-phase PCS is shown in Fig. 2.13. Each PCS contains a DC voltage source and an inverter with an LC filter. The inverter is connected to the PCC through a transmission/distribution line, represented by $L_g$ and $R_g$. The inductor current $i_l$, output current $i_o$ and output voltage $v_o$ are measured by the sensor units. The system parameters and sensor specification are listed in TABLE I.

The overall controller consists of an external power controller and inner voltage control loop and current control loop. The external power controller regulates the PCS output voltage and frequency according to the droop equations by generating an instantaneous voltage reference $v_{ref}$. A multi-resonant (MR) controller is employed to regulate the voltage with multiple harmonics. The order of the MR controller is designed from the 3rd order to the 13th order. The control signal generated by the multi-resonant controller is then transmitted to the current loop, which includes a proportional controller. An
inductive and adaptive VI is applied in this controller to adjust the output voltage. Its impedance is proportional to the output reactive power of the PCS.

![Schematic of a single-phase PCS and its control scheme.]

**Fig. 2.12.** The schematic of a single-phase PCS and its control scheme.

**TABLE I.** System parameters of the droop system.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{on}$</td>
<td>Rated output voltage</td>
<td>285.6 V</td>
</tr>
<tr>
<td>$V_{dc}$</td>
<td>DC-link voltage</td>
<td>350 V</td>
</tr>
<tr>
<td>$S_n$</td>
<td>Rated output power</td>
<td>1 kW</td>
</tr>
<tr>
<td>$f_n$</td>
<td>Rated output frequency</td>
<td>50 Hz</td>
</tr>
<tr>
<td>$f_s$</td>
<td>Switching frequency</td>
<td>20 kHz</td>
</tr>
<tr>
<td>$L_f$</td>
<td>Filter inductance</td>
<td>0.6 mH</td>
</tr>
<tr>
<td>$C_f$</td>
<td>Filter capacitance</td>
<td>45 µF</td>
</tr>
<tr>
<td>$L_g + R_g$</td>
<td>Line impedances</td>
<td>0.23 mH + 0.19 Ω</td>
</tr>
</tbody>
</table>

The proposed controller can effectively mitigate the power sharing error caused by different types of line impedance, line impedance mismatch and sensor errors. For the
system with resistive or complex line impedance, the large inductive virtual impedance turns the equivalent line impedance become mainly inductive. For the mismatch line impedance in the microgrid, the adaptive virtual impedance balances the equivalent line impedance of each PCS to reduce the power sharing error. Since the inductive VI is proportional to the reactive power, it achieves accurate sharing with small line impedance. The sharing accuracy is much improved comparing with fixed VI. For the DC offset in the sensor, a high-pass filter is applied at the output of the controller to eliminate the DC injection. For the scaling error, a special controller, which is separately designed in different frequency domains is implemented. In the fundamental domain, the feedforward voltage controller and notch filter make the control equivalent open-loop. Thus, the disturbance of the sensor can be suppressed. In the harmonic domain, the voltage control is regularly designed to track the harmonics and ensure the quality of the output.

2.3.2 Controller Design

a) Current controller

Fig. 2.14 displays the control diagram of the voltage control loop (including the nested current controller). The load impedance is not considered here for the following reasons: firstly, this controller design is for the converter itself, not for the whole system; secondly, there are various types of load, including linear loads and nonlinear loads. There is no unified load model can be presented in this control diagram.

![Control block diagram for single-phase droop system.](image)

The current controller $G_i(s)$ is a simple proportional controller, which is expressed as:

$$G_i(s) = k_r$$  \hspace{1cm} (2.16)
In order to suppress the transient voltage fluctuations under a sudden load change, the capacitor current is utilized as the feedback signal for the current controller. The controller gain $k_r$ is designed to attenuate the resonance of the LC filter and improve the dynamic of the controller.

$$G_n(s) = \frac{s^2 + \omega_n^2}{s^2 + \frac{\omega_n}{Q_n}s + \omega_n^2}$$  \hspace{1cm} (2.17)

where $\omega_n$ is the frequency of the suppressed wave. The parameter $Q_n$ determinates the bandwidth of the notch filter. The bode plot of $G_n(s)$ with different $Q_n$ is shown in Fig. 2.15. To reduce the damping of the notch filter in irrelevant frequency domain, the gain of notch filter at 150Hz should close to 0dB (Note the gains of higher harmonics are naturally guaranteed to 0dB because of the characteristic of $G_n$). Thus, the $Q_n$ is chosen as 3.14.

$G_h(s)$ represents the high-pass filter, which is used for the suppression of DC offset and phase compensation. The transfer function of $G_h(s)$ can be expressed as:
where $\omega_h$ is the cut-off frequency of the high-pass filter. In this model, it is set as $20\pi$ to avoid the influence on signal at the fundamental frequency.

$G_d(s)$ represents the control system delay, which contains one sampling period computation delay and a half sampling period PWM delay. $G_d(s)$ can be approximated as:

$$G_d(s) = e^{-1.5sT_s}$$  \hspace{1cm} (2.19)

To design the $k_r$, the open-loop transfer function $H_{io}(s)$ and closed-loop transfer function $H_{ic}(s)$ of the current loop are derived from Fig. 2.14:

$$H_{io}(s) = \frac{G_i(s)G_e(s)G_d(s)}{Z_i(s) + Z_e(s)}$$  \hspace{1cm} (2.20)

$$H_{ic}(s) = \frac{G_i(s)G_e(s)G_d(s)G_d(s)}{Z_i(s) + Z_e(s) + G_i(s)G_e(s)G_d(s)}$$  \hspace{1cm} (2.21)

The bode plot of $H_{ic}(s)$ is pictured in Fig. 2.16. When $k_r$ is very small, the controller cannot effectively damp LC resonance. However, when $k_r$ is too large, the magnitude gain is larger than 0dB when the phase response approaches $-180^\circ$, which can affect the stability of the voltage loop. Thus, $k_r$ in this model is set as 1.5.

Fig. 2.15. Bode plot of the closed-loop transfer function $H_{ic}(s)$. 

$$G_h(s) = \frac{s^2}{s^2 + \frac{\omega_h}{Q_h} s + \omega_h^2}$$  \hspace{1cm} (2.18)
b) Voltage controller

A cascaded control structure has been adopted for the secondary voltage control loop. $G_v(s)$ is the transfer function of the voltage controller. In this project, the transfer function of $G_v(s)$ contains a multi-resonant (MR) controller in harmonic frequency domain and can be expressed as:

$$G_v(s) = \sum_{i=3,5,7,9,11,13} \frac{2k_{pri}\omega_i s}{s^2 + 2\omega_i s + \omega_i^2}$$

(2.22)

where the resonant gains $k_{pri}$ are selected to ensure the open-loop voltage transfer function has high gains at the target frequencies $\omega_i$, while the resonant cut-off frequency $\omega_{ci}$ is designed 0.1% of the $\omega_i$.

Fig. 2.16. Modified control block diagram for the single-phase droop system.

The control block in Fig. 2.14 can be modified to Fig. 2.17 to get the open-loop transfer function easily. According to Mason's gain formula, the expressions for the open-loop transfer function $H_{vo}(s)$, closed-loop transfer function $H_v(s)$, and the output impedance $Z_o(s)$ can be derived as:

$$v_o = H_{vo}(s)v_{ref} + Z_{wo}(s)i_o$$

(2.23)

$$H_{vo}(s) = \frac{G_v(s)G_i(s)G_d(s)G_n(s)Z_c(s) + G_v(s)G_d(s)Z_c(s)}{Z_i(s) + Z_c(s) + G_i(s)G_n(s)G_d(s) - G_i(s)G_d(s)Z_c(s)}$$

(2.24)

$$H_{wc}(s) = \frac{G_i(s)G_n(s)G_d(s)Z_c(s) + G_i(s)G_d(s)Z_c(s)}{Z_i(s) + Z_c(s) + G_i(s)G_n(s)G_d(s) + G_i(s)G_n(s)G_d(s)Z_c(s)}$$

(2.25)

$$Z_o(s) = \frac{Z_i(s)Z_c(s) + Z_c(s)G_i(s)G_n(s)G_d(s) + G_i(s)G_n(s)G_d(s)Z_c(s)}{Z_i(s) + Z_c(s) + G_i(s)G_n(s)G_d(s) + G_i(s)G_n(s)G_d(s)Z_c(s)}$$

(2.26)
Fig. 2.17. Bode plot of the open-loop transfer function $H_{vo}(s)$.

Fig. 2.18 shows the bode plot of the open-loop transfer function $H_o(s)$. To ensure the voltage controller has great regulation performance and fast dynamic, the open-loop gain at selected harmonic frequency should be larger than 15dB. Thus, $k_{pr3}$, $k_{pr5}$, $k_{pr7}$, $k_{pr9}$, $k_{pr11}$ and $k_{pr13}$ are set at 3, 3, 2, 1, 1 respectively. The cut-off frequency $\omega_{ci}$ is chosen as $0.001\omega_f$ according to [27]. With the proposed parameters, the phase margin of $H_o(s)$ is $27.4^\circ$, which ensures the stability of this system.

c) Droop control

In the droop control, because the angle difference $\delta$ is usually less than 5 degrees, so we assume that $\sin \delta = \delta$, and $\cos \delta = 1$. The active power and reactive power measured from the PCS output can be expressed as:

$$P = \frac{\omega_f}{s + \omega_f} \cdot \frac{E U \delta}{Z} \sin \theta + \frac{\omega_f}{s + \omega_f} \cdot \frac{E(E - U)}{Z} \cos \theta$$ \hspace{1cm} (2.27)

$$Q = -\frac{\omega_f}{s + \omega_f} \cdot \frac{E U \delta}{Z} \cos \theta + \frac{\omega_f}{s + \omega_f} \cdot \frac{E(E - U)}{Z} \sin \theta$$ \hspace{1cm} (2.28)

where $P$ and $Q$ are active and reactive power; $\omega_f$ is the cut-off frequency of the low-pass filter; $E$ and $U$ are the RMS values for the PCS voltage and the PCC voltage; $Z$ is the line
impedance magnitude, and \( \theta \) is the line impedance angle; \( \delta \) is the angle difference between the PCS voltage and the PCC voltage.

From (2.29) and (2.30) the linearization yields can be derived:

\[
\Delta P = \frac{\omega_f}{s + \omega_f} \left[ \frac{EU \sin \theta}{Z} \cdot \Delta \delta + \frac{(2E-U) \cos \theta}{Z} \cdot \Delta E \right] \tag{2.29}
\]

\[
\Delta Q = \frac{\omega_f}{s + \omega_f} \left[ -\frac{EU \cos \theta}{Z} \cdot \Delta \delta + \frac{(2E-U) \sin \theta}{Z} \cdot \Delta E \right] \tag{2.30}
\]

For the droop control:

\[
\Delta \delta = -\frac{m}{s} \cdot \Delta P \tag{2.31}
\]

\[
\Delta E = -n \cdot \Delta Q \tag{2.32}
\]

Combining (2.29) to (2.32), the system characteristic equation can be derived as:

\[
C = \left[1 + G_{11} G_{c1}(s)\right] \left[1 + G_{22} G_{c2}(s)\right] - G_{12} G_{21} G_{c1}(s) G_{c2}(s) \tag{2.33}
\]

\[
G_{c1}(s) = \frac{m \omega_f EU}{Zs(s + \omega_f)} \tag{2.34}
\]

\[
G_{c2}(s) = \frac{n \omega_f (2E-U)}{Zs(s + \omega_f)} \tag{2.35}
\]

\[
G_{11} = \sin \theta, G_{12} = -\cos \theta, G_{21} = \cos \theta, G_{22} = \sin \theta \tag{2.36}
\]

The dynamics and stability of the system can be evaluated by analyzing the zeros of the characteristic equation.

For this single-phase system, \( E = 202V \) and \( Z = 0.203, \theta=20^\circ \). Assuming that \( m = n \), the zero distribution of the system with a constant \( \omega_f (\omega_f=10\pi) \) but varying droop coefficients are plotted in Fig. 2.19.
Fig. 2.18. Zero distribution with droop coefficients $m$ and $n$ increasing from $0.5 \times 10^{-4}$ to $4 \times 10^{-4}$.

In the beginning, the increase of droop coefficients will improve the system dynamic performance (the dominant zero, $\lambda_1$, moves to left). However, when the droop coefficients $m$ and $n$ exceed a critical value ($1.15 \times 10^{-4}$ for this case study), the system becomes more oscillatory ($\lambda_3$ and $\lambda_4$ start to contain imaginary parts). Note that if the droop coefficients are much larger than the critical value, the small-signal stability analysis provided may not accurately evaluate the system stability, because the linearized model is no longer valid when the power-control bandwidth is near or even beyond 50Hz.

d) Adaptive virtual impedance

To mitigate the influence of mismatched filter output and mismatched line impedance, an inductive virtual impedance is applied to the controller. The diagram of the inductive virtual impedance is displayed in Fig. 2.20.
Second-order-generalized-integrator (SOGI) module is used as a band-pass filter to extract the fundamental current. The output signal $i_{oq}$ lags the input signal by 90°. The closed-loop transfer function of SOGI is:

$$H_q(s) = \frac{i_{oq}}{i_o} = \frac{k_q \omega_s^2}{s^2 + k_q \omega_s s + \omega_s^2}$$  \hspace{1cm} (2.37)$$

where $\omega_s$ is the selected frequency for extraction. Fig. 2.21 shows the bode plot of $H_q(s)$ when $k_q$ is 1, 0.1 and 0.01, respectively. The SOGI with a smaller $k_q$ has a narrower band-pass width, which means the signal with unexpected frequency can be largely damped and the wanted signal can be extracted more accurately. However, for the droop control, the frequency will slightly deviate from the reference value. Thus, the band-pass width should not be too narrow. With these considerations, the $k_q$ is set as 0.05.
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Assuming that the input current is:

\[ i_o = A \sin(\omega t) \] (2.38)

where \( A \) is the amplitude of the harmonic current, \( \omega \) is the fundamental frequency of the input current. According to the transfer function of the SOGI, the output signal \( i_{eq} \) can be described as:

\[ i_{eq} = -A \cos(\omega t) \] (2.39)

The value of the harmonic virtual impedances can then be presented as follow:

\[ v_v = L_v \frac{di_{eq}}{dt} = -L_v \omega i_{eq} \] (2.40)

To achieve better power sharing performance, the value of fundamental virtual impedance \( L_v \) is related to the output reactive power of the PCS, which can be expressed as:

\[ L_v = k_v Q_j \] (2.41)

where \( L_v \) and \( Q_j \) are the virtual impedance and reactive power of the \( j \)th PCS, respectively. \( k_v \) is the coefficient of the adaptive virtual impedance.

The coefficient \( k_v \) must be set according to the criterions of the power sharing error and output voltage drop. First, in this system, the reactive power sharing error among the PCS is expected to be suppressed lower than 3\%. The definition of the reactive power sharing error of the \( j \)th PCS can be expressed as:

\[ e_j = \left| \frac{Q_j - Q_s}{Q_s} \right| \times 100\% \] (2.42)

where \( Q_s \) is the rated reactive power and \( n \) is the number of PCS in the system.

Because of the desperately designed voltage controller, scaling error and DC offset has little influence on the power sharing. Mismatch of the line impedance is the main sources of the sharing error. Assuming that there is 10\% random error among the line impedance
of each PCSs. The worst power sharing case occurs when the equivalent output
impedance (sum of line impedance and filter impedance) of one PCS is 10% greater than
the rated value and the another is 10% smaller than the rated value. In this case, there
will be 10% sharing error in the system, and the relationship between the reactive power
of each PCSs can be presented as:

\[ \frac{Q_i}{Q_j} = \frac{X_{fi} + X_{gi}}{X_{fi} + X_{gi}} = \frac{0.9(L_{fn} + L_{gn})}{1.1(L_{fn} + L_{gn})} = \frac{9}{11} \]  

(2.43)

where \( X_{fi}, X_{gi} \) and \( X_{fj}, X_{gj} \) are the value of filter impedances and line impedances of the
\( i \)th PCS and the \( j \)th PCS, respectively, \( X_{fn} \) and \( X_{gn} \) are the rated value of filter impedance
and line impedance. The reactive powers of the \( i \)th PCS and \( j \)th PCS, namely \( Q_i \) and \( Q_j \),
are then 10% smaller and 10% greater than the average reactive power, respectively.

With the virtual impedance applied, the reactive power sharing can then be expressed as:

\[ \frac{Q_i}{Q_j} = \frac{X_{fi} + X_{gi} + X_{fj} + X_{gj}}{X_{fi} + X_{gi} + X_{fj} + X_{gj}} = \frac{0.9(L_{fn} + L_{gn}) + k_v Q_j}{1.1(L_{fn} + L_{gn}) + k_v Q_i} \]  

(2.44)

From equation (2.44), the expression of \( k_v \) can be derived:

\[ k_v = \frac{(L_{fn} + L_{gn})(0.9 Q_j - 1.1 Q_i)}{(Q_i^2 - Q_j^2)} \]  

(2.45)

Assuming that the power sharing error with virtual impedance is less than 2%, which
means \( Q_i \) should larger than \( 0.98 \frac{Q_n}{n} \) and \( Q_j \) should smaller than \( 1.02 \frac{Q_n}{n} \). Thus, in the
single-phase droop model, \( k_v \) should larger than \( 1.37 \times 10^{-6} \).

Second, the voltage drop caused by the virtual impedance should be limited. Supposing
that the voltage drop from the inverter output to PCC is smaller than 5% of the reference
voltage, we have:

\[ \Delta U_f = \frac{P_R + Q X_{fi}}{U_{PCC}} = \frac{P_R + Q \omega (L_{fi} + L_{gi} + k Q_j)}{U_{PCC}} < 0.05 U_{ref} \]  

(2.46)
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Note that the voltage here is all RMS values. To simplify the calculation, we assume that $U_{pcc} = U_{ref}$. Thus, based on the specification provided, $k_v$ should smaller than $1.1 \times 10^{-5}$. Combining these two virtual impedance design rules, $k_v$ is set as $5 \times 10^{-6}$ in the virtual impedance.

Fig. 2.21. Plot of the output inductance of the adaptive virtual impedance.

TABLE II. Controller parameter values.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_0$</td>
<td>Output voltage reference</td>
<td>286 V</td>
</tr>
<tr>
<td>$\omega_0$</td>
<td>Angular speed reference</td>
<td>$100\pi$ rad/s</td>
</tr>
<tr>
<td>$k_{pr3}$</td>
<td>3$^{rd}$ order resonance gain</td>
<td>3</td>
</tr>
<tr>
<td>$k_{pr5}$</td>
<td>5$^{th}$ order resonance gain</td>
<td>3</td>
</tr>
<tr>
<td>$k_{pr7}$</td>
<td>7$^{th}$ order resonance gain</td>
<td>3</td>
</tr>
<tr>
<td>$k_{pr9}$</td>
<td>9$^{th}$ order resonance gain</td>
<td>2</td>
</tr>
<tr>
<td>$k_{pr11}$</td>
<td>11$^{th}$ order resonance gain</td>
<td>1</td>
</tr>
<tr>
<td>$k_{pr13}$</td>
<td>13$^{th}$ order resonance gain</td>
<td>1</td>
</tr>
<tr>
<td>$k_r$</td>
<td>Gain of current loop</td>
<td>1.5</td>
</tr>
<tr>
<td>$k_v$</td>
<td>Virtual impedance coefficient</td>
<td>$5 \times 10^{-6}$</td>
</tr>
<tr>
<td>$L$</td>
<td>Virtual impedance limitation</td>
<td>200 Var</td>
</tr>
<tr>
<td>$m$ and $n$</td>
<td>Droop coefficients</td>
<td>$1 \times 10^{-4}$</td>
</tr>
</tbody>
</table>
Besides, considering occasions of the resistive load and no load, where the reactive power is close to zero, the limit is necessary for the adaptive virtual impedance. To ensure the equivalent line impedance is mainly inductive, the minimum limit is set as 1mH. Fig. 2.22 shows how adaptive virtual impedance changes with reactive power. When the reactive power is less than 200 Var, it is a 1mH fixed constant inductor. When the reactive power of PCS is greater than the limit, it is proportional to the reactive power and the slope is $5 \times 10^{-6}$.

Based on these analyses, the control parameters are listed in TABLE II.

### 2.4 Simulation Results

The model with the proposed system is built in Matlab/Simulink. Three PCSs are parallel connected to PCC and supply RLC load, whose power factor is 0.7 (lag). The load resistance is 9.54 Ω and the load inductance is 11.6mH. The proposed method is tested in several cases, including: 1) the situation with mismatched line impedance, 2) the situation with different types of line impedances, 3) the situation with sensor errors in one PCS, 4) the situation with sensor errors in multiple PCS.

**Case 1: Mismatched line impedance**

In this case, the influence of mismatched line impedance is tested. The line impedance of PCS 1 is set at the rated value. According to the test requirement, the line impedance of PCS 2 is increased by 10% compared to the rated value and the line impedance of PCS 3 is decreased by 10% compared to the rated value. The line impedance of each PCS is listed in TABLE III.

<table>
<thead>
<tr>
<th>PCS 1</th>
<th>PCS 2</th>
<th>PCS 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Line impedance</td>
<td>$0.23\text{mH} + 0.19\Omega$</td>
<td>$(0.23\text{mH} + 0.19\Omega) \times 1.1$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$(0.23\text{mH} + 0.19\Omega) \times 0.9$</td>
</tr>
</tbody>
</table>
Fig. 3.18 shows the circulating currents and power sharing of three PCSs without virtual impedance. In this figure, the circulating current of the PCS is defined as:

\[
    i_{cir} = i_{oi} - \frac{i_{load}}{n}
\]

where \( i_{cir} \) indicates the circulating current of the \( i \)th PCS, \( i_{oi} \) indicates the current flow through the load, and \( n \) indicates the number of PCS.

The active power sharing error and reactive power sharing error are defined as:
where $e_P$ and $e_Q$ indicate the active power sharing error and reactive power sharing error among PCSs, respectively. $P_{\text{ave}}$ and $Q_{\text{ave}}$ indicate the average active power and average reactive power of three PCSs, respectively. $S_n$ indicates the rated power, which is set as 1 kW in this model.

As mentioned in chapter 2.2, the output voltage of the droop control is inversely proportional to the line impedance. The output voltage of PCS 2 is smaller than PCS 1 and the output voltage of PCS 3 is greater than PCS 2. These unbalanced output voltages result in a 0.5A circulating current among PCS 2 and PCS 3. The active power is equally shared among three PCSs since the frequency is almost the same. However, the sharing error of reactive power is 6.5%. Note that the neglectable circulating current in PCS 1 results from the symmetry of line impedance mismatch. If the mismatch is unsymmetrical, such as PCS 2 increases 10% and PCS 3 decreases 5%, the circulating currents will occur in all PCSs.

Then, the comparison between traditional constant virtual impedance and the proposed adaptive virtual impedance are tested. Fig. 2.23 shows the circulating currents and power sharing among three PCSs system with traditional constant virtual impedance. The line impedance mismatch is the same as the former simulation. The virtual impedance applied to each PCS is 3.5mH, which is equal to the adaptive virtual impedance with rated reactive power. Large virtual impedance makes the equivalent output impedance of three PCSs closer. Thus, the circulating current and reactive power sharing error decrease to 0.15A and 1.4%. The output voltage drops 1.4% because of the virtual impedance.
Fig. 2.22. Sharing performance of mismatched line impedance with constant virtual impedance; (a) circulating current, (b) power sharing

Fig. 2.23. Power sharing error and voltage drop with different constant virtual impedances
Chapter 2 Power Sharing

The reason why constant virtual impedance can suppress sharing errors is that it reduces the proportion of the line impedance difference by increasing the sum of the equivalent output impedance. Fig 2.24 shows the power sharing error and voltage drop with different constant virtual impedances. The slope of the sharing error decreases when the virtual impedance gets greater, which means the voltage drop is too serious to achieve accurate power sharing.

Fig. 2.25 shows the circulating currents and power sharing among three PCSs with the proposed adaptive virtual impedance. The virtual impedance on each PCS depends on its reactive power. With former line impedance mismatch, PCS 2 has greater line impedance but smaller reactive power. Thus, the additional virtual impedance on PCS2 becomes smaller. Vice versa, for PCS 3, the virtual impedance is greater than the other two PCSs because of the small reactive power. In short, the adaptive virtual impedance can automatically balance the equivalent impedance among three PCSs. The circulating current and power sharing error with the proposed method are 0.1A and 0.8%, respectively. The output voltage drop caused by virtual impedance is 1.2%. It is obvious that the performance of the proposed adaptive virtual impedance is better than traditional virtual impedance.
Case 2: Different types of line impedance

In this case, the performance of droop control with three kinds of line impedance, including inductive line impedance, complex line impedance and resistive line impedance, are tested firstly. The value of three types of line impedance is listed in TABLE IV. If the three PCSs are completely symmetry, power can be equally shared among them. However, the line impedances of three PCS are not completely equal in practice. Thus, 1% line impedance difference among three PCSs is considered in this case.

### TABLE IV. Line impedance parameters of case 2

<table>
<thead>
<tr>
<th>Line impedance</th>
<th>Value</th>
<th>PCS 1</th>
<th>PCS 2</th>
<th>PCS 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inductive</td>
<td>1 mH + 0.05 Ω</td>
<td>(1 mH + 0.05 Ω) ×</td>
<td>(1 mH + 0.05 Ω) ×</td>
<td>0.99</td>
</tr>
<tr>
<td></td>
<td>Ω</td>
<td>1.01</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Complex</td>
<td>0.23 mH + 0.19 Ω</td>
<td>(0.23 mH + 0.19 Ω) ×</td>
<td>(0.23 mH + 0.19 Ω) ×</td>
<td>0.99</td>
</tr>
<tr>
<td></td>
<td>Ω</td>
<td>×1.01</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Resistive</td>
<td>0.1 mH + 0.6 Ω</td>
<td>(0.1 mH + 0.6 Ω) ×</td>
<td>(0.1 mH + 0.6 Ω) ×</td>
<td>0.99</td>
</tr>
<tr>
<td></td>
<td>Ω</td>
<td>1.01</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Fig. 2.25. Power sharing with inductive line impedance conditions; (a) circulating current, (b) power sharing.
Fig. 2.26. Power sharing with complex line impedance conditions; (a) circulating current, (b) power sharing.

Fig. 2.27. Power sharing with resistive line impedance conditions; (a) circulating current, (b) power sharing.
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Fig. 2.26, Fig. 2.27 and Fig. 2.28 show the power sharing among three PCSs with different types of line impedance without proposed virtual impedance. The active power and reactive power can both equally shared among three PCSs when the line impedance is mainly inductive. However, when the line impedance is complex or resistive, even little line impedance difference among PCS can cause reactive power sharing error. The sharing error of complex line impedance and resistive line impedance are 0.9% and 2.1%, respectively.

Fig. 2.28. Power sharing of complex line impedance with inductive virtual impedance; (a) circulating current, (b) power sharing.
Virtual impedance can also be implemented to mitigate the sharing error caused by the
line impedance. By using virtual impedance, the total impedance is adjusted to be mainly
inductive and then the reactive power can be precisely shared. Fig. 2.29 and 2.30 show
accurate power sharing when the inductive virtual impedance is applied. The reactive
power sharing error decrease to 0.1% and 0.2%, respectively.

**Case 3: Sensor error**

Generally, there are two kinds of sensor errors: DC offset and scaling error. Both of them
can occur in voltage sensors and current sensors. The proposed feedforwarded voltage
control at the fundamental frequency and high-pass filter eliminate the influence of
sensor errors. To study the influences of sensor error on the power sharing accuracy and
verify the effectiveness of the designed controller against these influences, some simulations have been done in this case.

(a) DC offset

Fig. 2.31 shows the output current, circulating current and power sharing of the system when output voltage DC offset and inductor current DC offset coexist without a high-pass filter. The voltage DC offset at PCS 1 is +5 V and the current DC offset is +0.5 A. With these DC offsets, large DC injection occurs in the output current. Power sharing becomes inaccurate because the DC offset can also influence the power calculation in the droop control.
Fig. 2.30. Power sharing with DC offset errors but w/o the high-pass filter; (a) Output current, (b) Circulating current, (c) Power sharing.

The effect of the DC offset can be simply mitigated by the high-pass filter in the proposed method. Fig. 2.32 displays the corresponding simulation results. It is obvious that all the influence resulted from DC offset can be eliminated.
Fig. 2.31. Power sharing with DC offset errors and high-pass filter applied; (a) Output current, (b) Circulating current, (c) Power sharing.

(b) Scaling error

With scaling error in sensors, the amplitude of the PCS output voltages will deviate from the reference voltage magnitude $v_{ref}$, which can cause inaccurate fundamental power sharing among the parallel PCSs. In this case, the voltage scaling error at PCS 1 is $+2.1\%$ and current scaling error at PCS 1 is $+1.3\%$. The performance of power sharing with three control strategies, namely conventional MR closed-loop controller, conventional MR controller with virtual impedance and proposed control method, are compared.
Fig. 2.32. Power sharing with scaling errors; (a) Output voltage, (b) Output current, (c) Circulating current, (d) Power sharing.
Fig. 2.33 displays the output voltage, output current, circulating current and power sharing of the conventional closed-loop controller. It is clear that scaling error leads to a voltage drop at PCS 1. The output voltage difference between PCS 1 and other PCSs is 5V. Due to the small line impedance, 5V voltage difference results in a large circulating current among three PCSs. As a result, PCS 1 is charged by PCS 2 and PCS 3, which suggests that scaling error can seriously deteriorate the power sharing performance in parallel converter system.

Fig. 2.33. Power sharing with scaling errors when conventional closed-loop control applied; (a) Circulating current, (b) Power sharing.
The inaccurate power sharing is caused by the output voltage difference, which means it can also be suppressed by virtual impedance. Fig. 2.34 show the power sharing of the conventional closed-loop control strategy with 4mH virtual impedance. Though virtual impedance greatly suppresses the circulating current and sharing error, the sharing error still greater than 30% and cannot be ignored. There always exists a trade-off between acceptable sharing error and an acceptable voltage drop.

With the proposed control scheme, the sharing error is mitigated by the voltage feedforward control loop and the notch filter. Fig. 2.35 shows the power sharing when the scaling error occurs in the voltage sensor and current sensor simultaneously. With the
proposed method, the power sharing error decreases to 1.3%. The simulation results indicate that the proposed control can effectively suppress the inaccurate power sharing caused by the scaling error.

**Case 4: Multiple sensor errors**

The influence of multiple sensor errors in multi PCSs can be simply considered as the superposition of the influence of sensor error in single PCS. The solution to the multi sensor errors is as same as those methods in case 3. In this simulation, both errors of output voltage DC offset and inductor current DC offset occur in PCS 2 and PCS 3 (no measuring errors in PCS 1). Besides, both output voltage scaling error and inductor current scaling error occur in PCS 2 and PCS 3 (no measuring errors in PCS 1). The detail of DC offset and scaling error are listed in **TABLE V**.

<table>
<thead>
<tr>
<th>Sensor error</th>
<th>PCS 2</th>
<th>PCS 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output voltage DC offset</td>
<td>+5 V</td>
<td>-4 V</td>
</tr>
<tr>
<td>Inductor current DC offset</td>
<td>+0.5 A</td>
<td>-0.4 A</td>
</tr>
<tr>
<td>Output voltage scaling error</td>
<td>+2.1%</td>
<td>-2%</td>
</tr>
<tr>
<td>Inductor current scaling error</td>
<td>+1.3%</td>
<td>-1%</td>
</tr>
</tbody>
</table>
Fig. 2.35. Controller performance with multi sensor errors at two PCSs; (a) Circulating current, (b) Power sharing.

Fig. 2.36 shows the performance of the proposed method with multiple sensor errors in two PCSs. With the proposed method, the DC injection is eliminated and the power sharing error is mitigated to 2%. The simulation results prove that this method is applicable to the system with multiple sensor errors.

2.5 Conclusion

In this work, the control of the parallel converter in the microgrid is comprehensively discussed, including the power sharing strategies, power sharing errors and existing solutions. Then, a novel control strategy to deal with all sharing errors mentioned before is proposed. The sharing error caused by mismatch line impedance and different types of line impedance is suppressed by the adaptive virtual impedance. The influence of DC
offset is attenuated by the high-pass filter and the influence of the scaling error is mitigated by deigned feedforward controller. Besides, the controller parameters design and corresponding simulation results are provided to prove the feasibility of the proposed method.

The contribution of this work is that it perfectly integrates a control strategy to deal with most power sharing problems in the PCS system. Especially, the solution to the sharing error results from the sensor errors is included in this method, which is rarely discussed in previous studies.
Chapter 3 Harmonics Mitigation

3.1 Overview

In this chapter, the concern of the harmonic mitigation problem in parallel converter microgrid is discussed. In the first part, the literature review of the harmonics is provided, which consists of two main sources of the harmonic, the influence of the harmonics and existing methods to mitigate harmonics in the microgrid. In the second part, a novel method to simultaneously mitigate harmonics from the nonlinear load and converter deadtime is proposed. Corresponding controller design, simulation results and scaling-down experiment results are provided to prove the feasibility of this method. Finally, a brief conclusion of this work is summed up.

3.2 Literature Review

3.2.1 Source of the Harmonics

a) Harmonics introduced by nonlinear load

With the development of DG and microgrid, power electronic devices are widely implemented as the interface between RES, energy storage and microgrid. However, it also become one of the main sources of the harmonics in the microgrid. Nonlinear load, such as Uninterrupted Power Supply (UPS), switching power supply and power converter, can generate a large number of harmonics. Commonly, the nonlinear load can be modeled as a harmonic current source, which injects harmonic current to parallel inverter systems[32]. Fig. 3.1 shows the equivalent circuit of the two parallel inverters system in the harmonic domain.

In this figure, $Z_{lh1}$ and $Z_{lh2}$ are the line impedance in the harmonic domain for two inverters, respectively. $Z_{oh1}$ and $Z_{oh2}$ are the output impedance of two inverters, respectively. The harmonic current introduced by nonlinear load $i_{nonlinear}$ will be delivered
to each inverter. According to the Norton theorem, the sharing of the harmonic current is inversely proportional to the sum of the inverter output impedance and line impedance. These harmonic currents from the nonlinear load can result in the voltage distortion at PCC.

![System equivalent circuit with the nonlinear load in the harmonic domain.](image)

Fig. 3.1. System equivalent circuit with the nonlinear load in the harmonic domain.

### b) Circulating harmonics introduced by converter deadtime

Harmonics can also result from the switching deadtime of the converter, which is impossible to avoid. Some of the previous papers have done the investigation on the analysis of the converter deadtime[33]. Fig. 3.2 compares the output voltage of Sinusoidal Pulse Width Modulation (SPWM) without and with deadtime $t_d$. The four conduction conditions of the switches $Q_1-Q_4$ are shown in the right-hand side of the picture and the switching states are determined by comparing the modulation wave $u_r$ and carrier wave $u_c$. For example, in the interval $\Delta t_1$, the modulation wave $u_r$ is positive and greater than carrier wave $u_c$. Thus, switches $Q_1$ and $Q_4$ conduct and the output voltage of the inverter is positive. On the contrary, switches $Q_2$ and $Q_3$ conduct in interval $\Delta t_2$ and the output voltage turns out to be negative.

Generally, a fixed time interval $t_d$ is introduced to prevent the switches on the same bridge conduct at the same time. During the deadtime, the current will flow through the anti-parallel diodes on the switches. Thus, the output voltage of the converter is related to the
direction of the current, which is assumed to be in the same phase with modulation wave $u_r$.

Comparing the output voltage of the converter without deadtime $u_{o(\text{ideal})}$ and the output voltage with deadtime $u_o$, the voltage difference $\Delta u$ is pictured at the bottom of the Fig. 3.2. The percentage of voltage harmonics in $u_{ac}$ can be calculated through the Fourier transform\[33\]:

$$
\left| \frac{v_{hn}}{v_1} \right| = \frac{8}{n \cdot m \cdot \pi} \frac{t_d}{t_{sw}} \quad (n = 1, 3, 5, \ldots)
$$

(3.1)

where $m$ is the modulation index, $v_{hn}$ is the $n$th order voltage harmonics, and $v_1$ is the fundamental voltage. $t_{sw}$ is the switching time of the converter. Obviously, the harmonics are increased by the deadtime.

Fig. 3.2. PCS output voltage w/o and with deadtime.

Ideally, when the output voltages of parallel PCS are in the same phase with each other, $\Delta u$ cannot cause the harmonic circulating current because of the symmetry, which means the power quality at PCC cannot be affected. However, in practice, it is impossible to keep the output voltage of two inverters completely the same because of the imperfect
reactive power sharing. It leads to the voltage difference on the $\Delta u$ and the harmonic circulating current exists between two inverters.

In short, the harmonic current introduced by nonlinear loads can lead to serious PCC voltage distortions for large line impedances situations, and the voltage harmonics caused by deadtime can lead to considerable circulating current harmonics $i_{\text{circulating}}$ for small line impedances. Therefore, the trade-off always exists.

### 3.2.2 Influence of the Harmonics in power grid

**a) Less reliability**

Harmonics have great influences on the relay, breaker and communication line in the microgrid. First, harmonic currents with high frequency can cause electromagnetic interference to sensors, which may lead to the false operation of the relay. Second, electromagnetic interference (EMI) can also disturb the communication lines in the microgrid. The performance of some communication-based control strategies, such as Master/Slave control, can be deteriorated. Third, harmonics can be amplified by LC resonance in the microgrid, which leads to over current and over voltage. All these interferences seriously reduce the stability and reliability of the microgrid.

**b) Extra energy loss**

Harmonics can also cause the following energy loss in various types of electrical device. For the synchronous motor, it can generate the torque with inverse direction on the rotor, which lowers the output of the motor. At the same time, harmonics increases the copper loss on the rotor and stator. For the transmission line, energy consumption of the harmonic on its equivalent series resistance and inductance cannot be ignored.

**c) Deterioration of the devices’ lifetime**
The lifetime of some devices in the microgrid is enormously affected by harmonics. For capacitors in the microgrid, harmonics with high frequency can flow through the capacitor. It results in the overheat of the capacitor and accelerates the aging of the insulation. Besides, the output voltage with harmonics tends to have a sharp peak point, which can induce the electrical breakdown of the capacitor. For the generator or motor, when the frequency of the harmonic is close to the natural frequency of the hardware, the physical resonance can be caused. It is very harmful to the generator and motor. For the electrical appliance of the users, harmonics is regarded as electrical pollution. It can decrease the yield of the productions with precise manufacturing standard. In addition, harmonics can also lead to the poor performance of some domestic electrical appliances, such as television and personal computer.

3.2.3 Existing Harmonics Mitigation Methods

a) Passive compensation

Passive power filter is a traditional harmonic suppressing method. It is composed of power inductor and capacitor. Fig. 3.3 shows the structure of a passive power filter. Its impedance in the harmonic frequency domain can be expressed as:

\[ Z_n = j(n\omega L - \frac{1}{n\omega C}) \]  

(3.2)

where \( n \) and \( \omega \) indicate the order of the harmonic and fundamental frequency, separately. The resonance frequency of the filter is:

\[ f_n = \frac{1}{2\pi\sqrt{LC}} \]  

(3.3)

It provides a low-impedance pass-way for the harmonic to be suppressed. Thus, the Harmonic can be compensated before injected into the grid.
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Fig. 3.3. Structure of the passive filter.

The passive power filter is widely applied in the power grid due to its simple structure. However, it has some limits. First, passive power filter can only mitigate harmonic with a certain frequency. To mitigate multiple order harmonics, several passive power filters should work together, which will occupy a large space. Second, in practice, the resonance frequency will not be set as same as the frequency of the harmonic to avoid large harmonic current in the passive power filter. Thus, suppressing performance is limited. Third, parallel-connected passive power filter can influence the impedance of the whole system, which may result in resonance.

b) Control strategies for grid-connected mode

Though DG interfacing converter is one of the harmonic sources, it can also be implemented to suppress harmonic when the control strategy is properly designed. With the development of power electronic technology, active power filters (APF) become a popular method to attenuate harmonics[32, 34, 35]. Generally, active power filter in the microgrid can be categorized by the topology. There are three types of topology, shunt type, series type and hybrid type. The structure of the shunt active power filter is shown in Fig. 3.4 (a). It is mainly applied to mitigate the harmonic current and compensate the reactive power. The harmonics from the load is detected by sensors and applied as the current reference. Active power filter will generate the harmonic current with inverse direction and inject it into the microgrid to compensate the harmonic current. Commonly, it is set close to the harmonic source to minimize the influence of the harmonics. The structure of the series active power filter is shown in Fig. 3.4 (b). It connects to the
nonlinear load by a transformer to eliminate the voltage harmonic at PCC. Series active power filter works as a voltage source to compensate the voltage harmonics at PCC. The hybrid active power filter is a combination of active power filter and passive power filter, which can achieve better harmonics mitigation with a lower price. However, additional sensors are necessary for the active power filter to fulfill harmonic current track. Because of the distributed nonlinear load, the requirement of current sensors can be very large.

![Diagram](image1)

Fig. 3.4. Structure of the active filter; (a) shunt type, (b) series type.

The advantage of the active power filter is that it can mitigate the majority of the harmonics. The frequency of the mitigated harmonic is not limited. At the same time, as a power unit, it can compensate the reactive power for the microgrid and improve the voltage regulation. Besides, the active power filter is also not disturbed by the system structure and filter resonance. However, the active power filter is much expensive comparing to the passive filter and require high-cost maintenance.
To compensate the harmonic current proportional to the PCC voltage harmonic, the converter can be controlled as resistive active power filter (R-APF) in [36, 37]. The harmonic current reference can be expressed as:

$$i_{href} = G_h \cdot v_{hpcc}$$  \hspace{1cm} (3.4)

where $i_{href}$ and $v_{pcc}$ indicate harmonic current reference and PCC harmonic voltage, separately. $G_h$ indicates the gain of the controller. By applying this control loop, the output impedance of the converter is tuned to $1/G_h$ at harmonic frequency domain. Thus, the low-impedance converter can compensate harmonic current from PCC and the power quality at PCC can be improved. It does not need additional current sensors comparing to traditional APF. However, PCC voltage data should be sent to the converter through the communication line, which attenuates the reliability of the system. Besides, because of the existence of the complex line impedance, the performance of compensation can be limited.

c) Control strategies for islanded mode

(i) Improved voltage controller

In [38, 39], ideal proportional-resonant (PR) control is employed in the voltage controller. The transfer function of the ideal PR controller can be expressed as:

$$G_{pr}(s) = k_p + \frac{2k_s}{s^2 + \omega^2}$$  \hspace{1cm} (3.5)

It provides infinite control gain at the selected harmonic frequency, which ensures that output voltage can track the voltage reference with less harmonic. However, because of the infinite gain, some stability issues may occur when the resonant coefficient is set improperly. The control performance can be degraded in the droop control since the frequency deviates from the reference in the islanded mode. To improve the stability and performance of the RP control, non-ideal PR control is employed. The transfer function of the non-ideal PR controller can be expressed as:
where \( \omega_c \) indicates cut-off frequency. It reduces the sensitivity of the controller and improves the stability of the system. Generally, multi-resonant (MR) controller is applied to mitigate the low-order harmonics[40]. MR control simply consists of cascaded resonant controllers at the selected harmonic frequencies. The control performance of each resonant controller will not affect others. However, the stability issue can be caused when too many resonant controllers are cascaded.

Repetitive control, which can track the reference and mitigate the harmonic with every order, is proposed and applied in the microgrid[41-43]. The control diagram of a general repetitive control for the inverter is displayed in Fig 3.5. \( R(z) \) and \( Y(z) \) present the reference provided by the outer power loop and output signal, respectively. \( R(z) \) and \( Y(z) \) present the reference provided by the outer power loop and output signal, respectively. \( G_r(z) \), \( G_c(z) \) and \( G_p(z) \) present the repetitive controller, inner voltage and current controller and plant, respectively.

![Control Diagram](image)

The transfer function of repetitive controller \( G_r(z) \) is expressed as:

\[
G_r(z) = \frac{K_r G_f(z)}{Z^N - Q(z)} \tag{3.7}
\]

It can be designed to compensate both odd and even order harmonics by properly set \( N \), which is the number of samples in one repetitive period and \( G_f(z) \) phase lead filter. The low-pass filter \( Q(z) \) should be carefully designed to ensure the stability of the system.
(ii) Virtual harmonic impedance

In [16], harmonics sharing based on virtual harmonic impedance (VHI) method was proposed. It focuses on adjusting the equivalent output impedance of the converter to achieve harmonic current sharing. Fig. 3.6 illustrates the control structure of the VHI. The basic principle of the VHI is the same as the virtual impedance in the fundamental domain. The key point of this method is to use Fast Fourier Transformation (FFT) to extract fundamental current and harmonic current respectively. Note that to generate inductive VHI, FFT should be replaced by other extraction technology, such as multiple synchronous reference frames (MSRF). The virtual impedance in the different frequency domain is different and it is proportional to the frequency of the input current. Though VHI can realize accurate harmonic current sharing, it results in the voltage distortion at PCC.

![Fig. 3.6. Control structure with VHI.](image)

To solve this problem, a negative virtual harmonic impedance (NVHI) is proposed in [44]. By compensating line impedance in the harmonic frequency domain, it can eliminate the harmonic voltage drop on the line impedance. However, the small line impedance can amplify the circulating current caused by the deadtime of the PWM. Furthermore, the small equivalent impedance may also lead to resonance in the system.

(iii) Harmonic droop

Harmonic compensation can also be achieved by adjusting harmonic voltage reference through harmonic droop[45], whose principle is similar to droop in the fundamental
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frequency domain. Fig 3.7 shows the control diagram of the harmonic droop. By tuning harmonic voltage reference, the harmonic voltage drop on output impedance can be reduced and the harmonic current can be proportionally shared among converters without communication. However, control of the harmonic with different orders should be designed individually. The computation of the harmonic reference can be very complicated. Besides, same as the traditional droop, it can only compensate part of the harmonics.

A novel method combined with the harmonic droop and virtual harmonic conductance is proposed in [46]. Similar to virtual impedance, virtual conductance work as a conductance parallel connected to the converter, which can reduce the equivalent output impedance of the converter. The harmonic conductance-harmonic reactive power droop (G-H droop) realize the proportional harmonic current sharing by tuning output impedance according to the harmonic power. At the same time, it can provide harmonic damping without disturbing fundamental domain.

3.3 Proposed method

3.3.1 System Configuration

Fig 3.8 shows the model of the islanded microgrid system and its control strategy. Two single-phase inverters are parallel with each other. $L_{fi}$ and $C_{fi}$ ($i=1,2$) represent the inductor and capacitor of two LC filters and $Z_{li}$ ($i=1,2$) represents the line impedance of
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Each inverter. The load connected to the PCC includes the resistive load and nonlinear load. By calculating the active power and reactive power, droop control will provide the reference voltage. The output voltage is regulated by the voltage controller.

Fig. 3.8. The model of islanded microgrid system and its control strategy.

In this proposed method, a negative virtual harmonic impedance (NVHI) $Z_{vn1}$ is employed for inverter 1 to compensate for the line impedance. Another positive virtual harmonic impedance (PVHI) $Z_{vn2}$ is introduced into inverter 2 to reduce the circulating current caused by $V_{ih1}$ and $V_{ih2}$ in Fig. 3.1. Fig. 3.9 shows the system equivalent circuit with the proposed virtual harmonic impedances. Since the value of the line impedance can be approximately estimated by the parameter and the length of the transmission line, it is possible for the NVHI to compensate majority of the line impedance of the inverter 1. At the same time, the impedance in the harmonic domain at inverter 2 is increased by PVHI. The cooperation of $Z_{vn1}$ and $Z_{vn2}$ can nearly make the line 1 short circuit in the harmonic domain, and the majority of harmonic currents will flow through inverter 1. Since the equivalent output impedance of inverter 1 is close to zero, the voltage distortion at PCC can be effectively eliminated. At the same time, the circulating harmonic current caused by PWM deadtime is also suppressed by the large PVHI. The advantage of this harmonic mitigation method is that it simultaneously suppresses the harmonics caused
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by nonlinear load and deadtime. Besides, it uses the existing inverters to supply the harmonic current without voltage distortion, which means no additional devices is required.

![System equivalent circuit with the proposed method in the harmonic-frequency domain.](image)

### 3.3.2 Controller Design

Fig. 3.10 shows the voltage control loop and the plant model of the proposed method. The input $v_{ref}$ is provided by conventional droop control. $v_{vfi}$ denotes the output of the virtual impedance. MR voltage controller $G_v$ is implemented to track the reference voltage, which can also suppress voltage harmonics up to 9th order. Its transfer function $G_v(s)$ can be expressed as:

$$G_v(s) = \sum_{i} \left( \frac{2k_{ri}\omega_{ci}s}{s^2 + 2\omega_i + \omega_{ci}^2} \right)$$  \hspace{1cm} (3.8)

$k_{ri}$ ($i=1,3,5,7,9$) are the resonant gain, respectively. $\omega_i$ and $\omega_{ci}$ ($i=1,3,5,7,9$) are the resonant frequency and cut-off frequency, respectively.

A proportional current controller $G_i(s)$ is used to damp the LC resonance and improve dynamic response. The gain of the current controller is $k_i$.

$G_d(s)$ expresses the calculation and PWM delay, which can be expressed as:

$$G_d(s) = e^{-1.5Ts}$$  \hspace{1cm} (3.9)
For the plant, $Z_l(s)$ and $Z_c(s)$ indicate the impedance of filter inductor and filter capacitor, respectively.

![Diagram](image1.png)

Fig. 3.10. Voltage control loop and the plant model of the proposed method.

### a) Current controller

According to Fig. 3.10, the closed-loop transfer function of the current control can be derived as:

$$H_i(s) = \frac{G_i(s)}{Z_l(s) + Z_c(s) + G_i(s) + \frac{1}{sL} + \frac{1}{sC}} = \frac{k_iCs}{LCs^2 + k_iCs + 1} \quad (3.10)$$

The damp ration $\zeta$ of this second-order system is:

$$\zeta = \frac{k_i}{2} \sqrt{\frac{C}{L}} \quad (3.11)$$

![Diagram](image2.png)

Fig. 3.11. Bode plot of the closed-loop transfer function of the current controller with different gain $k_i$.

$\zeta$ should be set as 0.707 to ensure stability and effectively suppress the LC resonance. Thus, the gain of the current controller $k_i$ is set as 4. Fig. 3.11 compares the bode plot of
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$H_{vc}(s)$ with different $k_i$. It is obvious that the controller with large $k_i$ has greater performance on resonance damping.

b) Voltage controller

To ensure the stability and voltage regulation of the system, the voltage controller should be carefully designed. The open-loop transfer function and the closed-loop transfer function of the voltage controller can be expressed as:

$$H_{vo}(s) = \frac{G_v(s)G_i(s)G_d(s)Z_c(s)}{Z_L(s) + Z_c(s) + G_i(s)G_d(s)}$$  \hspace{1cm} (3.12)

$$H_{vc}(s) = \frac{G_v(s)G_i(s)G_d(s)Z_c(s)}{Z_L(s) + Z_c(s) + G_i(s)G_d(s) + G_v(s)G_i(s)G_d(s)Z_c(s)}$$  \hspace{1cm} (3.13)

The bode plots of $H_{vo}(s)$ and $H_{vc}(s)$ are displayed in Fig. 3.12. The resonant cut-off frequency $\omega_{ci}$ is designed as 0.1% of the $\omega_i$. To ensure that the output can track reference with few errors and good dynamic, the gain of the open-loop transfer function should be greater than 10dB. The gain of $G_v$ is set as $k_{r1}=50$, $k_{r3}=10$, $k_{r5}=5$, $k_{r7}=3$, $k_{r9}=1$. With these parameters, the phase margin of the open-loop transfer function is 49.4°, which ensures the stability of the system.
c) Virtual harmonic impedance

Based on the analysis of the voltage controller, the output voltage of the inverter can be expressed as:

\[ V_o = H_{vc}(s) \cdot (V_{ref} - Z_{vi}i_o) - Z_o(s)i_o \]  \hspace{1cm} (3.14)

\[ Z_o(s) = \frac{Z_c(s)Z_L(s) + Z_c(s)G_L(s)G_d(s)}{Z_L(s) + Z_c(s) + G_L(s)G_d(s) + G_v(s)G_f(s)G_d(s)Z_c(s)} \]  \hspace{1cm} (3.15)

where \( Z_o(s) \) is the output impedance of the inverter. \( Z_{vi} \) is the value of the virtual impedance. From (3.13) and (3.14), the output impedance with virtual impedance can be defined as:

\[ V_{o(c)} = H_{vc}(s)Z_{vi} + Z_o(s) \]  \hspace{1cm} (3.16)

Fig. 3.13 shows the bode plots of \( Z_o(s) \). The output impedance of inverter without virtual impedance \( Z_o(s) \) at selected harmonic frequencies is quite small. To some extent, the output impedance can be ignored. The gain of \( H_{vc}(s) \) at selected harmonic frequencies are almost 1 because of the MR voltage controller. Thus, (3.16) can be simplified as:
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\[ V_{o(v)} = Z_{vi} \]  

(3.17)

By tuning the value of virtual impedance can adjust the equivalent output harmonic impedance of the inverter.

![Bode plot of Z_o(s).](image)

Fig. 3.13. Bode plot of \( Z_o(s) \).

As mentioned in Chapter 2, SOGI can be applied to generate inductive virtual impedance. Two outputs of SOGI \( i_{od} \) and \( i_{oq} \) are orthogonal, where \( i_{oq} \) lags the input signal and \( i_{od} \) by 90°. and the closed-loop transfer function of \( i_{od} \) and \( i_{oq} \) are:

\[ H_d(s) = \frac{i_{od}}{i_o} = \frac{k\omega_s s}{s^2 + k\omega_s s + \omega_s^2} \]  

(3.18)

\[ H_q(s) = \frac{i_{oq}}{i_o} = \frac{k\omega_s^2}{s^2 + k\omega_s s + \omega_s^2} \]  

(3.19)

where \( \omega_s \) is the frequency of the required current. To generate an inductive virtual impedance, \( i_{oq} \) is used in the virtual impedance. In this method, the \( k \) is set as 0.01.

Assuming that the input is the \( i \)th harmonic current:

\[ i_{oi} = A \sin(i\omega t) \]  

(3.20)

where \( A \) is the amplitude of the \( i \)th order harmonic current. \( \omega \) is the fundamental frequency of the input current. \( i \) expresses the order of the harmonic. According to the transfer function of the SOGI, the output signal \( i_{oq} \) can be described as:
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\[ i_{q_{vi}} = -A \cos(\omega t) \]  \hspace{1cm} (3.21)

For the inductive virtual impedance, its impedance in different frequency domain varies from each other. Thus, it is necessary to extract the current with harmonic frequency. Fig. 3.14 shows the structure of the proposed harmonic virtual impedance control. Multi SOGIs are adopted to extract the harmonic currents from the 3\(^{rd}\) to the 9\(^{th}\) order, which contains the majority of the total harmonic.

Fig. 3.14. Structure of virtual impedance based on MSOGI.

The output of the harmonic virtual impedances can be presented as follow:

\[ v_{vi} = L_v \frac{di_{wi}}{dt} = -L_v n\omega i_{q_{vi}} \]  \hspace{1cm} (3.22)

The NVHI of inverter 1 is set as same as the line impedance to fulfill the one-side harmonic current sharing. The design of the PVHI of inverter 2 is based on the magnitude of the circulating current. The deadtime-induced circulating current harmonics can be expressed as [33]:

\[ |i_{hcd}| = \frac{8U_{dc}}{h\pi} \frac{t_d}{t_{sw}} G_i(s) \cdot \frac{1 - \cos(hA\Delta\delta)}{Z_{11} + Z_{12} + Z_{w} + Z_{vp}} \]  \hspace{1cm} (3.23)

where \(\Delta\delta\) is the angle difference between inverter output currents. \(h\) is the harmonic order. \(G_i(s)\) is the transfer function of the PVHI. In this controller, the PVHI is designed to make sure that the circulating current harmonics under the worst case will not exceed 2% of fundamental current.
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\[ |i_{hci}| \leq 0.02 |i_o| \]  

The harmonic circulating current reaches the maximum under the following situation:

1) The opposite phase between the \( \Delta v \) of two inverters at the certain harmonic frequency.

2) The total impedance without the PVHI is as small as possible.

3) The magnitude of the current is inversely proportional to the harmonic order.

Thus, the largest circulating harmonic current occurs when these three assumptions achieve at the same time: \( h \Delta \delta = \pi \), \( (Z_{l1} + Z_{l2} + Z_{vn}) = 0 \) and \( h = 3 \). Combining (3.23) and (3.24), to effectively mitigate the circulating harmonic current, the positive harmonic impedance should be designed as:

\[ z_{ip} \geq 4.24 \text{mH} \]  

Thus, in this experiment, the PVHI is set to 6mH to eliminate the distortion from the circulating current.

d) **Droop control**

The droop coefficient design principle can refer to the controller design in Chapter 2. The power sharing at the fundamental domain is inversely proportional to the droop coefficients. Generally, power sharing is designed to be equal or proportional to the generator capacitor. However, for this method, the power in the harmonic domain is completely supplied by inverter 1. Thus, it is better to make inverter 2 share more power in the fundamental domain. The droop coefficients of two inverters are finally designed as \( m_1, n_1 \) equal to \( 10^{-4} \) and \( m_2, n_2 \) equal to \( 5 \times 10^{-5} \). The power flows to inverter 2 is twice that of the inverter 1.

Based on all these analysis and design, the parameters of the controller are listed in Table VI.
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### TABLE VI. Parameters of the controller

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{\text{ref}}$</td>
<td>Output voltage reference</td>
<td>100 V</td>
</tr>
<tr>
<td>$\omega_{\text{ref}}$</td>
<td>Angular speed reference</td>
<td>$100\pi$ rad/s</td>
</tr>
<tr>
<td>$m_1$ and $n_1$</td>
<td>Droop coefficients</td>
<td>$10^{-4}$</td>
</tr>
<tr>
<td>$m_2$ and $n_2$</td>
<td>Droop coefficients</td>
<td>$5 \times 10^{-5}$</td>
</tr>
<tr>
<td>$k_{v1}$</td>
<td>1st order resonance gain</td>
<td>50</td>
</tr>
<tr>
<td>$k_{v3}$</td>
<td>3rd order resonance gain</td>
<td>10</td>
</tr>
<tr>
<td>$k_{v5}$</td>
<td>5th order resonance gain</td>
<td>5</td>
</tr>
<tr>
<td>$k_{v7}$</td>
<td>7th order resonance gain</td>
<td>3</td>
</tr>
<tr>
<td>$k_{v9}$</td>
<td>9th order resonance gain</td>
<td>1</td>
</tr>
<tr>
<td>$k_i$</td>
<td>Gain of the current control</td>
<td>4</td>
</tr>
<tr>
<td>$Z_{vl1}$</td>
<td>Virtual harmonic impedance for inverter 1</td>
<td>-1.6 mH</td>
</tr>
<tr>
<td>$Z_{vl2}$</td>
<td>Virtual harmonic impedance for inverter 2</td>
<td>6 mH</td>
</tr>
</tbody>
</table>

#### 3.4 Simulation and Experimental Result

### 3.4.1 Simulation in the PLECS

### TABLE VII. System parameters of the simulation and experiment.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{\text{dc}}$</td>
<td>DC-link voltage</td>
<td>140 V</td>
</tr>
<tr>
<td>$\omega_{\text{ref}}$</td>
<td>Nominal Frequency</td>
<td>$100\cdot\pi$ rad/s</td>
</tr>
<tr>
<td>$t_{\text{sw}}$</td>
<td>Switching time</td>
<td>1/20000 s</td>
</tr>
<tr>
<td>$L_f$</td>
<td>Filter inductance</td>
<td>1 mH</td>
</tr>
<tr>
<td>$C_f$</td>
<td>Filter capacitance</td>
<td>100 µF</td>
</tr>
<tr>
<td>$Z_l$</td>
<td>Line impedances</td>
<td>$1.6\text{mH} + 0.1\Omega$</td>
</tr>
<tr>
<td>$t_d$</td>
<td>Deadtime</td>
<td>1 µs</td>
</tr>
</tbody>
</table>
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The simulations of the experiment have been done first in the PLECS and the effect to suppress harmonic has been verified by several cases. The system parameters are provided in Table VII.

**Case 1: Mitigation of circulating harmonic current**

In the first case, only a 100Ω resistive load is connected to the PCC. Fig 3.15 shows the PCC voltage and output current and Table VIII shows the proportion of the harmonic in PCC voltage and output currents. \(i_{o1}\) and \(i_{o2}\) present the output current of the inverter 1 and inverter 2 without the proposed method, respectively. \(i_{o1}^*\) and \(i_{o2}^*\) present the output current of inverter 1 and inverter 2 with the proposed method, respectively. \(i_o\) and \(i_o^*\) present the current of the load without and with the proposed method, respectively.

![Diagram](image_url)
Fig. 3.15. Simulation system with resistive load: (a) without the proposed method, (b) with the proposed method.

From the output currents and output voltages in Fig. 3.15 and FFT analysis in Table VIII, it is clear that harmonic current circulates between inverter 1 and inverter 2 because of the deadtime. The THD of $i_{o1}$ is twice over that of $i_{o2}$ because the current in the fundamental domain is unevenly shared. With the proposed method, the circulating harmonic current between inverter 1 and inverter 2 is suppressed because of the positive virtual harmonic impedance.

**TABLE VIII. Simulation FFT analysis of PCC voltage and output current (resistive load).**

<table>
<thead>
<tr>
<th>Case</th>
<th>1st</th>
<th>3rd</th>
<th>5th</th>
<th>7th</th>
<th>9th</th>
<th>THD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$i_{o1}$</td>
<td>89.54%</td>
<td>5.43%</td>
<td>1.01%</td>
<td>0.89%</td>
<td>0.20%</td>
<td>10.46%</td>
</tr>
<tr>
<td>$i_{o2}$</td>
<td>94.22%</td>
<td>2.91%</td>
<td>0.57%</td>
<td>0.45%</td>
<td>0.12%</td>
<td>5.78%</td>
</tr>
<tr>
<td>$i_{o1}^*$</td>
<td>95.50%</td>
<td>1.33%</td>
<td>1.47%</td>
<td>0.78%</td>
<td>0.51%</td>
<td>4.50%</td>
</tr>
<tr>
<td>$i_{o2}^*$</td>
<td>97.64%</td>
<td>0.66%</td>
<td>0.74%</td>
<td>0.42%</td>
<td>0.28%</td>
<td>2.36%</td>
</tr>
<tr>
<td>$i_o$</td>
<td>99.53%</td>
<td>0.15%</td>
<td>0.12%</td>
<td>0.04%</td>
<td>0.06%</td>
<td>0.47%</td>
</tr>
<tr>
<td>$i_o^*$</td>
<td>99.73%</td>
<td>0.8%</td>
<td>0.06%</td>
<td>0.08%</td>
<td>0.04%</td>
<td>0.27%</td>
</tr>
</tbody>
</table>

Case 2: Mitigation of voltage harmonic at PCC

In the second case, two inverter supply a nonlinear load, which can be considered as a harmonic current source. Fig. 3.16 illustrates the harmonic of output current and PCC voltage and Table IX presents the FFT analysis of Fig. 3.16.

Without the proposed method, the harmonic current is evenly shared between two inverters since droop control has no control in the harmonic domain. The PCC voltage is distorted by the harmonic current. The top of the PCC voltage is flat.
Fig. 3.16. Simulation system with nonlinear load: (a) without the proposed method, (b) with the proposed method.

TABLE IX. Simulation FFT analysis of PCC voltage and output current (nonlinear load).

With the proposed method, the NVHI compensates the harmonic impedance between inverter 1 and PCC. The PVHI increases the harmonic impedance between inverter 2 and PCC, which further prevents the harmonic current from flowing to inverter 2. Because of the short circuit at the harmonic frequency, all the harmonic current from the nonlinear
load flows to inverter 1 and the harmonic voltage distortion at PCC is also eliminated. Thus, the power quality of PCC is largely improved.

<table>
<thead>
<tr>
<th></th>
<th>1st</th>
<th>3rd</th>
<th>5th</th>
<th>7th</th>
<th>9th</th>
<th>THD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$i_{o1}$</td>
<td>59.05%</td>
<td>26.97%</td>
<td>10.53%</td>
<td>2.71%</td>
<td>0.53%</td>
<td>41.83%</td>
</tr>
<tr>
<td>$i_{o2}$</td>
<td>72.68%</td>
<td>16.45%</td>
<td>6.70%</td>
<td>1.97%</td>
<td>0.84%</td>
<td>27.32%</td>
</tr>
<tr>
<td>$i_{o1}^*$</td>
<td>40.01%</td>
<td>37.05%</td>
<td>15.65%</td>
<td>5.20%</td>
<td>1.91%</td>
<td>59.99%</td>
</tr>
<tr>
<td>$i_{o2}^*$</td>
<td>91.18%</td>
<td>4.87%</td>
<td>1.56%</td>
<td>0.80%</td>
<td>0.90%</td>
<td>8.82%</td>
</tr>
<tr>
<td>$v_{pcc}$</td>
<td>95.06%</td>
<td>1.78%</td>
<td>1.57%</td>
<td>0.87%</td>
<td>0.68%</td>
<td>4.94%</td>
</tr>
<tr>
<td>$v_{pcc}^*$</td>
<td>98.28%</td>
<td>0.85%</td>
<td>0.49%</td>
<td>0.13%</td>
<td>0.20%</td>
<td>1.72%</td>
</tr>
</tbody>
</table>

3.4.2 Scale-down Experiment

Scaled-down experiments were carried out to test the effectiveness of the proposed control strategy in practice. Fig 3.17 shows the experimental platform, which is based on the structure shown in Fig. 3.8 and the parameters listed in Table VII.

Fig. 3.17. Platform of the scaled-down experiment.

Initially, a 50 Ω resistive load is connected to the PCC, and inverter output current waveforms for different control strategies are shown in Fig. 3.18.
Ideally, inverter output currents should not contain much harmonic. However, in Fig. 3.18 (a), inverter output currents are distorted. This is because the deadtime-introduced voltage harmonics cannot be damped by PR controllers and hence leads to considerable circulating current harmonics between the two inverters.

To suppress the circulating current harmonics, the proposed control strategy is applied to the platform with the result shown in Fig. 3.18 (b). Since the implementation of the positive virtual harmonic impedance increases the impedance of the circulating harmonic current path, the current waveforms are much more sinusoidal. The FFT analysis in Table X verified that the proposed method can effectively reduce the circulating harmonic currents.

Fig. 3.18. Scale-down experiment system with a 50 Ω resistive load: (a) without the proposed method, (b) with the proposed method.
TABLE X. Scale-down experiment FFT analysis of output current (resistive load).

<table>
<thead>
<tr>
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<th>1st</th>
<th>3rd</th>
<th>5th</th>
<th>7th</th>
<th>9th</th>
<th>THD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$i_o$</td>
<td>81.47%</td>
<td>14.01%</td>
<td>3.15%</td>
<td>0.60%</td>
<td>0.30%</td>
<td>18.53%</td>
</tr>
<tr>
<td>$i_o$</td>
<td>88.64%</td>
<td>8.86%</td>
<td>1.79%</td>
<td>0.51%</td>
<td>0.49%</td>
<td>11.36%</td>
</tr>
<tr>
<td>$i_o$</td>
<td>92.54%</td>
<td>2.54%</td>
<td>2.37%</td>
<td>1.86%</td>
<td>0.67%</td>
<td>7.46%</td>
</tr>
<tr>
<td>$i_o$</td>
<td>96.06%</td>
<td>1.43%</td>
<td>1.16%</td>
<td>0.98%</td>
<td>0.36%</td>
<td>3.94%</td>
</tr>
<tr>
<td>$i_o$</td>
<td>97.47%</td>
<td>1.80%</td>
<td>0.54%</td>
<td>0.12%</td>
<td>0.06%</td>
<td>2.53%</td>
</tr>
<tr>
<td>$i_o$</td>
<td>99.63%</td>
<td>0.06%</td>
<td>0.12%</td>
<td>0.12%</td>
<td>0.06%</td>
<td>0.37%</td>
</tr>
</tbody>
</table>

For the next case, a 100 Ω resistive load and a rectifier-type nonlinear load were connected to the PCC. Fig. 3.19 and Table XI show the output currents and PCC voltage without proposed control and with the proposed control method, respectively. It is obvious that the output voltage without the proposed method has a flat crest. With the proposed method, the output voltage becomes more sinusoidal. Harmonic voltage is clearly mitigated when the proposed control method is implemented. The Scaled-down experiments show the same results as the simulations, which indicates the feasibility of this method.
Chapter 3 Harmonics Mitigation

Fig. 3.19. Scale-down experiment system with nonlinear load: (a) without the proposed method, (b) with the proposed method.

TABLE XI. Scale-down experiment FFT analysis of PCC voltage (nonlinear load).

<table>
<thead>
<tr>
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<th>1st</th>
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<th>5th</th>
<th>7th</th>
<th>9th</th>
<th>THD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v_{pcc}$</td>
<td>89.64%</td>
<td>7.42%</td>
<td>1.65%</td>
<td>0.99%</td>
<td>0.27%</td>
<td>10.36%</td>
</tr>
<tr>
<td>$v_{pcc}^*$</td>
<td>96.29%</td>
<td>1.49%</td>
<td>1.35%</td>
<td>0.37%</td>
<td>0.48%</td>
<td>3.71%</td>
</tr>
</tbody>
</table>

3.5 Conclusion

In this work, the harmonic problem in the microgrid is comprehensively discussed, including the sources of the harmonic, influences of the harmonic and existing solutions to the harmonic. Based on that, a new control strategy to simultaneously mitigate harmonic from the nonlinear load and PWM deadtime is proposed. It realizes the harmonic mitigation by adjusting the equivalent output impedance of each inverter in the harmonics domain independently. Besides, the details of the controller design and the corresponding simulations and experimental results are provided to prove the feasibility of the proposed method.

The main contribution of this work is that it solves the conflict between harmonic mitigation of the nonlinear load and deadtime. Besides, as a harmonic mitigating control
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method, it needs no additional devices, which greatly decreases the cost of harmonic mitigation in the microgrid.
4.1 Conclusions

In this thesis, two concerns, power sharing and harmonic mitigation, in the microgrid have been discussed. At first, the background, scope and the objective of this project have been presented as the introduction of this thesis. Following that, power sharing problems and harmonic problem are analyzed in chapter 2 and chapter 3, respectively. In chapter 2, the existing power sharing strategies in the microgrid have been introduced and the reasons of the sharing error have been analyzed in detail. Then, in chapter 3, the problem of the harmonic in the microgrid is discussed from three parts, the sources of the harmonic, its impacts on the microgrid and existing solutions

The main contributions of this work are summarized:

(1) The accuracy of the power sharing in the parallel converter system is degraded by some inevitable interferences, such as mismatched line impedance, different types of line impedance and sensor errors. Especially, sensor error is hardly discussed in the previous research on power sharing. In this work, a novel control strategy that can simultaneously suppress multi sharing errors is proposed. The controller consists of two parts. Firstly, A well-designed adaptive inductive virtual impedance is applied for the sharing error suppression caused by mismatched and different types of line impedance. Secondly, a voltage controller that separately designed in fundamental frequency and harmonic frequency is applied to mitigate the influence of the sensor errors. The simulation results prove the feasibility of this method.

(2) Generally, the harmonics in the microgrid is mitigated by the passive power filter or active power filter, which can achieve good performance but has to be applied at every harmonic source. Thus, a method to mitigate the harmonics from the control aspect is proposed in this work. It realizes the unbalanced harmonic current sharing by compensating the line impedance of one inverter in the harmonic domain. With
the equivalent short circuit inverter, the voltage harmonic at PCC can be mitigated. The PVHI is also applied to mitigate the circulating harmonic current caused by the deadtime. With the proposed method, the harmonics caused by nonlinear load and deadtime can be effectively mitigated without any additional devices, which means the cost of this method is much lower than the conventional power filter.

4.2 Future Works

This thesis focuses on power sharing and harmonic issues in the microgrid and provides the corresponding methods on these problems. In the future, the proposed method can be further improved and some new ideas from other angles can be studied.

(1) Power sharing:

The power sharing control strategy in this work is based on traditional droop control. As mentioned in chapter 2, droop control has poor performance on voltage and frequency regulation. On the contrary, Master/Slave can achieve perfect control of its output under ideal condition. However, communication problems, such as communication delay, always deteriorate its performance. Thus, a new Master/Slave control with the compensation of the communication delay can be studied in the future.

Besides, in chapter 2, the Master/Slave with master unit current as the reference is introduced, which has less communication delay. However, the controls of all converter are coupled together, which is difficult to design. Further study can be done on the stability analysis and controller design of this kind of controller.

(2) Harmonics:

The proposed method in Chapter 4 has a limitation. The possibility of overload in the inverter with NVHI increases when all the heavy nonlinear load is shared by only one inverter. However, if the number of the inverter with NVHI is increased, a large harmonic current will circulate between these inverters with NVHI because of the
small impedance. Thus, a feasible solution should be proposed in the future. A possible solution is provided here. The inverters which shared nonlinear load can be set by location. There is only one inverter with NVHI in every region and it is only in charge of the local nonlinear load. Thus, the nonlinear load shared by each inverter is affordable. The line impedance among regions is large so that the harmonic circulating current is suppressed.

Bibliography


