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<th>Determination of diffusion length from within a confined region with the use of EBIC.( Published )</th>
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<tr>
<td><strong>Author(s)</strong></td>
<td>Ong, Vincent K. S.; Wu, Dethau.</td>
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Determination of Diffusion Length from Within a Confined Region with the Use of EBIC

Vincent K. S. Ong, Member, IEEE, and Dethau Wu

Abstract—A new method of extracting minority carrier diffusion length from within a confined region of material is presented in this paper. This technique uses the finite difference method and can be used on samples where the diffusion lengths are longer than the width of the region. This cannot be achieved using the conventional method, which evaluates the negative reciprocal of the slope of the EBIC signals line scan plotted on a semi-logarithmic scale. A limitation of this method is that the beam entrance surface of the sample is assumed to have negligible surface recombination.

Index Terms—Electron beam applications, electron microscopy, finite difference methods, microscopy, semiconductor materials measurements, simulation.

NOTATION

\(d\) \hspace{1cm} \text{Sample width (refer to Fig. 1).}
\(d_b\) \hspace{1cm} \text{Beam diameter.}
\(D\) \hspace{1cm} \text{Minority carrier diffusion coefficient.}
\(E_b\) \hspace{1cm} \text{Beam energy.}
\(E_t\) \hspace{1cm} \text{Energy to create a pair of electron-hole.}
\(f\) \hspace{1cm} \text{Correction factor.}
\(I_b\) \hspace{1cm} \text{Beam current.}
\(I_{\text{EBIC}}\) \hspace{1cm} \text{EBIC current.}
\(J_e\) \hspace{1cm} \text{Electron current density.}
\(J_p\) \hspace{1cm} \text{Hole current density.}
\(k\) \hspace{1cm} \text{Proportionality constant defined in (1).}
\(L\) \hspace{1cm} \text{Minority carrier diffusion length.}
\(L_{\text{error}}\) \hspace{1cm} \text{Error in the extracted minority carrier diffusion length.}
\(L_{\text{ext}}\) \hspace{1cm} \text{Extracted minority carrier diffusion length.}
\(n\) \hspace{1cm} \text{Electron concentration.}
\(N_D^+\) \hspace{1cm} \text{Donor ionized impurity concentration.}
\(N_A^-\) \hspace{1cm} \text{Acceptor ionized impurity concentration.}
\(N_e\) \hspace{1cm} \text{Doping concentration.}
\(\phi\) \hspace{1cm} \text{Electrostatic potential.}
\(\rho\) \hspace{1cm} \text{Material density of silicon.}
\(\tau\) \hspace{1cm} \text{Minority carrier lifetime.}

NSRHN \hspace{1cm} \text{Shockley–Read–Hall concentration parameter for electron (equal to} \(5 \times 10^{16}\ \text{cm}^{-3}\) \text{for silicon).}
\(p\) \hspace{1cm} \text{Hole concentration.}
\(q\) \hspace{1cm} \text{Electronic charge (}1.602 \times 10^{-19}\ \text{C).}
\(R\) \hspace{1cm} \text{Electron range.}
\(R_n\) \hspace{1cm} \text{Net electron recombination rate.}
\(R_p\) \hspace{1cm} \text{Net hole recombination rate.}
\(x\) \hspace{1cm} \text{Distance between the junction and the generation volume (refer to Fig. 1).}
\(\varepsilon\) \hspace{1cm} \text{Dielectric permittivity.}

I. INTRODUCTION

The electron beam induced current (EBIC) mode of scanning electron microscope (SEM) has been extensively used as an analytic tool in the characterization of semiconductor material. This technique has gained much popularity because it requires minimal specimen preparation. Another advantage of the EBIC technique that has attracted the interest of researchers in the physical sciences is that the beam position and penetration depth can be easily and accurately controlled. The minority carrier diffusion length, surface recombination velocity and minority carrier lifetime are a few examples of the material parameters that can be extracted using the EBIC technique. The theory of this technique in semiconductor material characterization is well developed and has been reported in various literatures [1]–[7].

One of the common experimental configuration in demonstrating the EBIC theory is illustrated in Fig. 1. The sample consists of a p-n junction, which is perpendicular to the electron beam incident surface. The current signal induced within the sample due to electron beam bombardment is collected by the EBIC amplifier. If the surface recombination velocity is negligible, the collected current is a function of beam-junction distance and it is given as follows:

\[I_{\text{EBIC}}(x) = ke^{-\frac{x}{L}}\]  

where

\(I_{\text{EBIC}}\) \hspace{1cm} \text{collected current;}
\(x\) \hspace{1cm} \text{beam-junction distance;}
\(L\) \hspace{1cm} \text{diffusion length;}
\(k\) \hspace{1cm} \text{proportionality constant.}

To obtain the diffusion length, a series of \(I_{\text{EBIC}}\) measurements for various \(x\) are plotted on a semi-logarithmic graph. The diffusion length is then evaluated from the negative reciprocal of the slope of the graph. The above relationship is developed based on two assumptions. First, the carrier generation source is assumed to be a point source and secondly, the sample width is infinitely large with respect to the diffusion length. The sample width is defined as the distance between the junction and the device boundary. In practical experiment, the first assumption can be met when a low energy beam is used. The second assumption is approximately met when the distance between the beam and the device boundary is more than two diffusion lengths [8].
When EBIC measurements are made from regions where the beam and the device boundary is less than two diffusion lengths, the extracted diffusion length tends to be inaccurate. However, since the value of the diffusion length is initially unknown, it is not easy to determine the range of measurements, which should be excluded from the determination of diffusion length.

In cases where the sample width is shorter than its diffusion length, such as in high-power diodes, the above method cannot be used. The reason for this is that there is no range of in which decays exponentially with . Zimmerman [9] suggested a method to overcome this problem. He pointed out that when the beam position is kept constant and after the beam is turned off, the collected current decreases as

\[ I(t) = I_0 e^{-\frac{t}{\tau}} \]  

(2)

where \( I_0 \) is the steady state current and \( \tau \) is the minority carrier lifetime. By plotting \( I(t) \) on a semi-logarithmic graph, the minority carrier lifetime can be obtained from the negative reciprocal of the slope. Using this parameter, the diffusion length can be calculated using the following expression.

\[ L = \sqrt{D\tau} \]  

(3)

where \( D \) is the diffusion coefficient. The relationship in (2) is true even when the beam-junction or device boundary distance is less than two diffusion lengths. This method is useful in detecting variations in diffusion length only when the exact value of \( D \) is known. It is also a transient technique, which is inherently more difficult to carry out than steady-state techniques.

A method of extracting diffusion length within a confined region is proposed in this paper. It is based on the evaluation of the second order derivative of the EBIC signals using the finite difference method [10]. It will be shown that, using this method, EBIC signal measurements taken at regions less than two diffusion lengths away from the device boundary can be used in extracting the diffusion lengths. Hence, this method will be useful in extracting diffusion length of the materials in a confined region of an IC where the EBIC measurement has to be taken in a region close to the boundary. In addition, it will also be useful in extracting the diffusion length of power diodes where the diffusion length is longer than the width.

II. THEORY

For a sample with negligible surface recombination velocity and infinite width, the EBIC current \( I_{\text{EBIC}} \) is given in (1). The second order derivative of Eq. (1) with respect to \( x \) is

\[ I''_{\text{EBIC}}(x) = \frac{k e^{-\frac{x}{\tau}}}{L^2} \]  

(4)

Substituting (1) into (4), we obtain

\[ L = \sqrt{\frac{I_{\text{EBIC}}(x)}{I''_{\text{EBIC}}(x)}} \]  

(5)

This shows that the value of \( L \) can be determined if the value of \( I''_{\text{EBIC}}(x) \) at a particular \( x \) is known. This is true for all ranges of \( x \). It will be shown later that this relationship is also true in cases of samples with finite widths.

The value of \( I''_{\text{EBIC}}(x) \) can be calculated using finite difference method (FDM). Consider a series of EBIC signals, \( I_{\text{EBIC,1}}, I_{\text{EBIC,2}}, I_{\text{EBIC,3}}, \ldots, I_{\text{EBIC,n}} \), we define

\[ \Delta^2 I_{\text{EBIC}}(x)_p = \frac{I_{\text{EBIC}}(x)_{p+\frac{1}{2}} - I_{\text{EBIC}}(x)_{p-\frac{1}{2}}}{x_{p+\frac{1}{2}} - x_{p-\frac{1}{2}}} \]  

(6)

where \( p = 2, \ldots n - 1 \) and

\[ \Delta I_{\text{EBIC}}(x)_{p+\frac{1}{2}} = \frac{I_{\text{EBIC}}(x)_{p+1} - I_{\text{EBIC}}(x)_p}{x_{p+1} - x_p} \]  

(7)

If \( x_{p+1} - x_p = x_p - x_{p-1} = \Delta x \) and \( \Delta x \to 0 \), then (6) simplifies to (8), shown at the bottom of the page.

It can be seen in (8) that the value of a particular \( I''_{\text{EBIC}}(x) \) is estimated based on the values of three adjacent \( I_{\text{EBIC}} \) data points. The adjacent data points should be as close as possible, i.e., \( \Delta x \to 0 \), to obtain an accurate result.

A. Samples with Finite Widths

In the case of samples with finite widths, if the surface recombination velocity is negligible, then the expression for the normalized EBIC signals is given in [11] as

\[ I_{\text{EBIC,1}}(x) = \frac{\sinh[(d-x)/L]}{\sinh[d/L]} \]  

(9)

\[ I''_{\text{EBIC}}(x) = \frac{I_{\text{EBIC}}(x + \Delta x) - 2I_{\text{EBIC}}(x) + I_{\text{EBIC}}(x - \Delta x)}{\Delta x^2} \bigg|_{\Delta x \to 0} \]  

(8)
where \( d \) is the sample width. Using the conventional method, a series of \( I_{\text{EBIC}} \) measurements are plotted on the semi-logarithmic graph and the extracted diffusion length \( L_{\text{ext}} \) is obtained from the negative reciprocal of the slope of the graph, i.e.,

\[
L_{\text{ext}} = \frac{d}{dx} \left[ \ln \left( \frac{\sinh\left( \frac{d - x}{L} \right)}{\sinh\left( d/L \right)} \right) \right]^{-1}
\]

It can be seen in (10) that \( L_{\text{ext}} \to L \) when \( \cosh\left( \frac{d - x}{L} \right) \to 1 \), which implies that \( (d - x)/L \to \infty \). However, in order to extract \( L_{\text{ext}} \) with error of not more than 4%

\[
L_{\text{ext}} = \frac{L - L_{\text{ext}}}{L} \leq 0.04
\]

Solving the inequality we get

\[
x \leq d - 2L.
\]

This indicates that the extracted diffusion length using the conventional method depends on the range of \( x \) used. By maintaining the beam position to be more than two diffusion lengths from the sample boundary, theoretically, the maximum error in the extracted diffusion length is 4%. This also explains the necessity of keeping the beam position at a distance away from the sample boundary in order to extract \( L \) accurately.

It can be shown that, if the diffusion length of a sample is extracted using the FDM, all ranges of \( x \) can be used. The second order derivative of (9) with respect to \( x \) is

\[
\frac{d^2 I_{\text{EBIC}}}{dx^2} = \frac{1}{L^2} \frac{\sinh\left( \frac{d - x}{L} \right)}{\sinh\left( d/L \right)}.
\]

Substituting (9) into (13), we obtain (5), which we derived for samples with infinite widths. This shows that this method can also be used on samples with finite widths. The main advantage of this method is that all ranges of \( x \) can be used, since (13) and (5) are both true for \( 0 \leq x \leq d \). This also implies that the FDM method can be used on samples, which have diffusion lengths longer than their widths.

The FDM method, however, requires that the three EBIC signal measurements to be taken in very small increments of \( x \), i.e., \( \Delta x \to 0 \). The error in the extracted diffusion length \( L_{\text{error}} \) which comes from \( \Delta x \neq 0 \) can be calculated as follows:

\[
L_{\text{error}} = \frac{L - L_{\text{ext}}}{L} \leq 0.04
\]

Equation (15) can be simplified as

\[
L_{\text{error}} = \frac{[\Delta x]^2}{2 \sinh \left( \frac{\Delta x}{2L} \right)}.
\]

Rearranging (17) gives

\[
\frac{\Delta x}{2L_{\text{error}}} = \sinh \left( \frac{\Delta x}{2L} \right) \to 0.
\]

This shows that \( L_{\text{error}} \) is accurate when the \( \Delta x/L \) ratio is small. This also implies that, for a fixed spatial resolution, samples with longer diffusion lengths will allow for more accurate extraction of this parameter than samples with shorter diffusion length.

We can derive the expression for the \( L_{\text{error}} \) in terms of \( \Delta x \) by substituting (17) into (14).

\[
L_{\text{error}} = \frac{\Delta x}{2L \sinh \left( \frac{\Delta x}{2L} \right)}.
\]

From (20), it can be observed that the error in the extracted diffusion length is a function of the \( \Delta x/L \) ratio rather than that of \( \Delta x \) alone. This also justifies the above discussion that the extracted diffusion length is accurate when \( \Delta x/L \to 0 \). The plot of \( L_{\text{error}} \) against the \( \Delta x/L \) ratio is shown in Fig. 2. It can be seen in Fig. 2 that \( L_{\text{error}} \) increases rapidly with the \( \Delta x/L \) ratio. To maintain \( L_{\text{error}} \) to within 1%, the \( \Delta x/L \) ratio has to be kept at smaller than 0.5.

Rearranging (18) in terms of \( \Delta x/L_{\text{ext}} \), we obtain

\[
L = \frac{\Delta x}{2 \sinh^{-1} \left( \frac{\Delta x}{2L_{\text{ext}}} \right)}.
\]

This is a very useful equation, which enables us to determine the true diffusion length from \( \Delta x \) and the extracted diffusion length. The errors in \( L_{\text{ext}} \), which arises from large \( \Delta x/L \) ratios, can be eliminated using this equation. This also eliminates the experimental difficulty of the FDM method where the EBIC measurements have to be taken in small increments of \( x \).
III. VERIFICATION

A computer simulation package, MEDICI was used to generate EBIC signals of various sample configurations. MEDICI [13] is a commercially available 2-D device simulation software which solves semiconductor equations using the finite difference method. The fundamental semiconductor equations, which describe the bulk behavior of the semiconductor device, are the Poisson’s equation and the continuity equations for electrons and holes. The Poisson’s equation, which is derived from Maxwell’s electrostatic equation, describes the electrostatic behavior of the semiconductor device and is given as follows:

$$\varepsilon \nabla^2 \varphi = -q(p - n + N_D^+ - N_A^-)$$  \hspace{1cm} (22)

where

- $\varepsilon$: dielectric permittivity;
- $\varphi$: electrostatic potential;
- $q$: is the electronic charge ($1.602 \times 10^{-19}$ C);
- $p$ and $n$: hole and electron concentrations, respectively;
- $N_D^+$ and $N_A^-$: ionized donor and acceptor impurity concentrations, respectively.

The continuity equations which govern the carriers concentration are given below. For the electrons

$$\frac{\partial n}{\partial t} = \frac{1}{q} \nabla \cdot J_n - U_n$$  \hspace{1cm} (23)

where $J_n$ is the electron current density and the $U_n$ is the net electron recombination rate. For the holes,

$$\frac{\partial p}{\partial t} = -\frac{1}{q} \nabla \cdot J_p - U_p$$  \hspace{1cm} (24)

where $J_p$ is the hole current density and $U_p$ is the net hole recombination rate.

Three silicon samples were created in this simulation, namely, sample A, sample B and sample C and the true diffusion lengths of these samples are 1 \(\mu\)m, 5 \(\mu\)m and 10 \(\mu\)m, respectively. The widths of these samples are identical, at 5 \(\mu\)m each. This means that the diffusion lengths of the latter two samples cannot be extracted using the conventional method. The doping concentrations of these samples are of the order of \(10^{19}\) to \(10^{16}\) cm\(^{-3}\), and the depletion widths are negligible compared to their respective diffusion lengths. The surface recombination velocities of these samples are set to zero. In practice, sample preparation techniques, e.g., with the use of etching, can be employed to produce samples with negligibly small surface recombination velocities [1].

In this simulation, the primary electron energy $E_b$ is 15 keV and the beam current $I_b$ is 1.5 nA. The electron penetration depth $R$ is calculated using the following expression [14]:

$$R = \frac{4.57 \times 10^{-6}}{\rho} E_b^{0.75}$$  \hspace{1cm} (25)

where $\rho$ is the density of silicon in gm/cm\(^3\) and the $R$ is given in cm. The distribution of the generated carrier is approximated by the use of the following expression [15].

$$G(x, z) = G_o F(x, z) h(z)$$  \hspace{1cm} (26)

IV. RESULTS AND DISCUSSION

It can be seen in Fig. 4 that the EBIC signal line scans lose their exponential behavior when their diffusion lengths are equal.
TABLE I

<table>
<thead>
<tr>
<th>Sample</th>
<th>True $L$ ($\mu$m)</th>
<th>$L_{\text{ext}}$ ($\mu$m)</th>
<th>$L_{\text{err}}$</th>
<th>$L_{\text{err}}$ %</th>
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</thead>
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<tr>
<td>A</td>
<td>1</td>
<td>0.985</td>
<td>1.5</td>
<td>1.004</td>
</tr>
<tr>
<td>B</td>
<td>5</td>
<td>3.01</td>
<td>39.8</td>
<td>5.0</td>
</tr>
<tr>
<td>C</td>
<td>10</td>
<td>3.53</td>
<td>78.48</td>
<td>10.027</td>
</tr>
</tbody>
</table>

Fig. 5. EBIC line scans for samples A, B, and C on the semilogarithmic scale.

Fig. 6 (a) Comparing the FDM method with the conventional method of extracting $L_{\text{ext}}$ for sample with $L = 1 \mu$m (sample A). (b) Comparing the FDM method with the conventional method of extracting $L_{\text{ext}}$ for sample with $L = 5 \mu$m (sample B). (c) Comparing the FDM method with the conventional method of extracting $L_{\text{ext}}$ for sample with $L = 10 \mu$m (sample C).
V. CONCLUSION

A new method of extracting minority carrier diffusion length is presented in this paper. The diffusion length can be estimated by evaluating the square root of the ratio of the EBIC signal to its second order derivative with respect to position. The second order derivative of the EBIC signal is evaluated using the finite difference method. It is shown that this method can be applied to EBIC signals collected from regions less than two diffusion lengths away from the sample boundaries. This means that diffusion lengths which are longer than the sample widths can also be extracted using this method with reasonable accuracy.

REFERENCES


Vincent K. S. Ong received the B.Eng. degree (with Honors) in electrical engineering and the M.Eng. and Ph.D. degrees in electronics in 1981, 1988, and 1995 respectively. He held a variety of positions in the manufacturing and testing of integrated circuits at the Hewlett Packard Company, both in Singapore and the United States, for 11 years between 1981 and 1992. In 1992, he joined the Faculty of Engineering of the National University of Singapore to manage the Centre for Integrated Circuit Failure Analysis and Reliability, and to work on research relating to electron beam effects on integrated circuits. In 1997, he joined Nanyang Technological University, Singapore, as Senior Lecturer in the School of Electrical and Electronic Engineering. He is now Associate Professor.

Dethau Wu received B.Eng. degree from the Nanyang Technological University, Singapore, in 1998. Upon graduation, he joined the Division of Circuits and Systems, School of Electrical and Electronic Engineering, Nanyang Technological University, as a research student. He is currently pursuing the Ph.D. degree.

Since 1998, he has been working on the image processing of EBIC images and the material characterization of semiconductor devices using the single contact EBIC. His research interests include imaging and characterization of semiconductor devices using EBIC mode of scanning electron microscope.