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Theory and Analysis of Differentially-Driven Microstrip Antennas

Y. P. Zhang and J. J. Wang

Abstract—This paper studies differentially-driven microstrip antennas. The theory of microstrip antennas based on the improved cavity model is expanded to analyze the input impedance and radiation characteristics of the differentially-driven microstrip antennas. The differentially-driven microstrip antennas were fabricated. Their performances were experimentally verified. Results show that the occurrence of resonance for the differentially-driven microstrip antennas also depends on the ratio of the separation \( \xi \) of the dual feeds to the free-space wavelength \( \lambda_o \). When the dual feeds are located far from each other \( \xi / \lambda_o > 0.1 \), the resonance occurs, and the input resistance at resonance is rather large. However, when the dual feeds are located near to each other \( \xi / \lambda_o < 0.1 \), the resonance does not occur; the input resistance is quite small, and the input impedance is inductive. Compared with single-ended microstrip antennas, the differentially-driven microstrip antennas have larger resonant resistance, similar co-polar radiation patterns, and lower cross-polar radiation component.

Index Terms—Cavity model, input impedance, microstrip patch antenna, radiation pattern.

I. INTRODUCTION

MICROSTRIP antennas have many unique and attractive properties—low in profile, light in weight, compact and conformable in structure, and easy to fabricate and to be integrated with solid-state devices [1]. Microstrip antennas have found wide applications in radio systems with single-ended signal operation. Recently, microstrip antennas can be seen for use in radio systems with differential signal operation as well [2]–[5]. For example, a differentially-driven microstrip antenna integrated with a push-pull power amplifier in Gallium Arsenide semiconductor technology is reported in [2]. The differentially-driven microstrip antenna acts also as an output balun, which makes the push-pull power amplifier more compact and efficient. A second differentially-driven microstrip antenna integrated with a push-pull power amplifier in complementary metal oxide semiconductor technology is presented in [3]. The differentially-driven microstrip antenna takes full advantages of the standard surface-mounted ceramic ball grid array package; consequently, the system-level board space and the system-level assembly can be reduced and facilitated, respectively. A third differentially-driven microstrip antenna integrated with an oscillator including a buffer amplifier in Silicon Germanium semiconductor technology is proposed in [4]. The differentially-driven microstrip antenna introduces a virtual ground; accordingly, there is no need for connecting the ground of the solid-state devices and the ground plane of the antenna, the parasitic effect caused by interconnect is eliminated. Besides these differentially-driven microstrip antennas, a differential receiving microstrip antenna integrated with a differential low noise amplifier in Gallium Arsenide semiconductor technology is described in [5]. The differential receiving microstrip antenna acts also as an input balun, which offers a new possibility to build small, robust and cost-effective smart antenna arrays. The above works focus on integration with solid-state devices and indeed provide little information on the design of differential microstrip antennas. Furthermore, the state of the art single-chip and single-package solutions of radio systems call for differential antennas to reduce the bill of materials and to improve the receiver noise performance and transmitter power efficiency [6], [7]. In this paper we present a study of differentially-driven microstrip antennas. We describe the theory of differentially-driven microstrip antennas based on the improved cavity model in Section II. The improved cavity model offers both simplicity and physical insight. We validate the theory with experiments and HFSS simulations in Section III. We apply the validated theory to analyze the differentially-driven microstrip antennas to gain deep insight into the differential operation of the microstrip antennas in Section IV. Finally, we summarize the conclusions in Section V.

II. THEORY OF DIFFERENTIALLY-DRIVEN MICROSTRIP ANTENNAS

A microstrip antenna and a coordinate system are illustrated in Fig. 1. The microstrip antenna that has dimensions \( a \) along \( x\)-axis and \( b \) along \( y\)-axis located on the surface of a grounded dielectric substrate with thickness \( t \), dielectric constant \( \varepsilon_r \), and...
dielectric loss tangent $\delta$ is differentially driven at points $(x_1, y_1)$ and $(x_2, y_2)$.

The differentially-driven microstrip antenna can be treated as a two-port network. With reference to the ground plane, the driving point at $(x_1, y_1)$ is defined as port 1 and the driving point at $(x_2, y_2)$ as port 2. Using the $Z$ parameters for ports 1 and 2; one can express the differential voltage $V_d$ as

$$V_d = V_1 - V_2 = (Z_{11} - Z_{21})I_1 - (Z_{22} - Z_{12})I_2 \tag{1}$$

where $V_1$ and $V_2$ are the driving point voltages and $I_1$ and $I_2$ are the driving point currents of ports 1 and 2, respectively. Since for the differentially-driven microstrip antenna

$$I_1 = -I_2 = I. \tag{2}$$

Equation (1) simplifies to

$$Z_d = \frac{V_d}{I} = 2(Z_{11} - Z_{21}) = 2(Z_{22} - Z_{12}) \tag{3}$$

where $Z_d$ is the input impedance of the differentially-driven microstrip antenna. The value of $Z_d$ is required in the design of matching network between the differentially-driven microstrip antenna and the differential active circuitry in a radio system. The $Z$ parameters for the microstrip antenna shown in Fig. 1 are calculated using the improved cavity model as follows [8]:

$$Z_{11} = j\omega \mu_0 d \sum_{m,n=0}^{\infty} \frac{\phi_{mn}(x_1, y_1) \phi_{mn}(x_2, y_2)}{k_{mn}^2 - k_e^2} \left( \frac{\mu_0 d}{2\varepsilon_e} \right) \tag{4}$$

$$Z_{12} = j\omega \mu_0 d \sum_{m,n=0}^{\infty} \frac{\phi_{mn}(x_1, y_1) \phi_{mn}(x_2, y_2)}{k_{mn}^2 - k_e^2} \left( \frac{\mu_0 d}{2\varepsilon_e} \right) \tag{5}$$

where $\omega$ is angular frequency, $\mu_0$ is the permeability of vacuum, $d_e$ is the effective width of a uniform strip of $z$-directed source current of one amp

$$k_e^2 = \varepsilon_e (1 - j\delta_e)k_0^2 \tag{6}$$

$$k_0 = \omega / \nu \tag{7}$$

$$k_{mn}^2 = (m\pi / a_e)^2 + (n\pi / b_e)^2 \tag{8}$$

$$j_0(x) = \sin(x) / x \tag{9}$$

$$\phi_{mn}(x, y) = \sqrt{\varepsilon_0 \varepsilon_m / \alpha_e b_e} \cos(m\pi x / a_e) \cos(n\pi y / b_e) \tag{10}$$

where $\varepsilon_m = 1$ for $m = 0$ and 2 for $m \neq 0$, $m$ and $n$ are the mode indices in the $x$ and $y$ directions, respectively, $\delta_e$ is the effective loss tangent, $\nu$ is the speed of light, $a_e$ and $b_e$ are the effective dimensions, taking into account the fringing fields at the edge of the microstrip patch. Similarly, $Z_{21}$ and $Z_{22}$ can be computed. $a_e$ may be approximated as follows [9]:

$$a_e = a + \left( \frac{b_{eq} - b}{2} \right) \frac{\varepsilon_e(b) + 0.3}{\varepsilon_e(b) - 0.258} \tag{11}$$

where $\varepsilon_e(b)$ is the effective dielectric constant and $b_{eq}$ is the equivalent width calculated from the planar waveguide model. $\varepsilon_e(b)$ can be written as

$$\varepsilon_e(b) = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2} \left( 1 + 10 \frac{k}{b} \right)^{-1/2} \tag{12}$$

and $b_{eq}$ as

$$b_{eq} = \frac{120\pi t}{Z(b)\sqrt{\varepsilon_e(b)}} \tag{13}$$

where $Z(b)$, the impedance of a microstrip line of width $b$ and thickness $t$, is given by

$$Z(b) = \frac{60\pi}{\sqrt{\varepsilon_r}} \left\{ \frac{b}{\nu} + 0.441 + 0.082 \left[ \frac{\varepsilon_r - 1}{\varepsilon_r} \right] \left[ \frac{1.415 + 0.94}{\nu} \right] \right\}^{-1} \tag{14}$$

Similarly, $b_e$ can be calculated by replacing $a_e, b_{eq}, b, Z(b)$, and $\varepsilon_e(b)$ in (11)–(14) with $b_e, b_{eq}, a_e, Z(a_e)$, and $\varepsilon_e(a)$, respectively.

Next, let us examine the radiation characteristics of the differentially-driven microstrip antenna. It is known that the radiation of the microstrip antenna originates from two slots and each slot can be thought of as radiating the same field as a magnetic dipole with a magnetic current of

$$M = 2\pi \times 2E_z \tag{15}$$

where the factor of 2 comes from the image of the magnetic current in the ground plane, $\hat{n}$ is the outward normal to the magnetic wall, $\hat{z}$ is the unit vector in the $z$-direction, and $E_z$ is the $z$ component of electric field in the cavity region. For the differentially-driven microstrip antenna, $E_z$ can be written as

$$E_z = j\mu_0 \sum_{m,n=0}^{\infty} I \phi_{mn}(x, y) j_0 \left( \frac{\mu_0 d_e}{2\varepsilon_e} \right) \left[ \phi_{mn}(x_1, y_1) \phi_{mn}(x_2, y_2) \right], \tag{16}$$

where $I$ is the source current of one amp in our calculation. Equation (16) reveals that the differentially-driven microstrip antenna introduces the cancellation mechanism, which can be explored to suppress some higher-order modes to reduce the cross polarization radiation. The electric vector potential of $M$ is given by

$$F(r) = \frac{\varepsilon_0}{4\pi} \int \frac{M(r')}{|r - r'|} \exp(-jk_0 |r - r'|) d\Omega(r') \tag{17}$$

where $\varepsilon_0$ is the permittivity of vacuum and integration is over the perimeter of the patch. The far field radiation components at a point $P(r, \theta, \phi)$ can be expressed as

$$E_{\theta} = j\omega \eta_0 (-F_x \sin \phi + F_y \cos \phi) \tag{18}$$

$$E_{\phi} = -j\omega \eta_0 (F_x \cos \theta \cos \phi + F_y \cos \theta \sin \phi) \tag{19}$$

where $\eta_0 = 377 \Omega$. 

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In the implementation of the improved cavity model, the determination of the effective loss tangent \( \delta_e \) is critical. This is because the effective loss tangent \( \delta_e \) is introduced to account for the total losses in the cavity. If the dielectric substrate is thin so that surface wave loss can be negligible, \( \delta_e \) is given by

\[
\delta_e = \frac{P_r + P_d + P_c}{2\omega W_e}
\]  

(20)

where \( P_r, P_d, \) and \( P_c \) are radiation, dielectric, and conductor losses, respectively, and \( W_e \) is electric energy stored in the cavity under the patch at resonance. The effective loss tangent \( \delta_e \) can be written, near the resonant frequency, according to (20), as [10]

\[
\delta_e = \frac{P_r}{2\omega W_e} + \delta + \frac{s}{\delta}
\]  

(21)

where \( s = 0.029\sqrt{\lambda_0/\sigma} \) is the skin depth in the metal cladding with conductivity \( \sigma \) (with units of Siemens per meter). The radiation loss by the space wave is

\[
P_r = \frac{1}{2\omega} \int_0^{2\pi} \int_0^{\pi/2} (|E_{\theta}|^2 + |E_{\phi}|^2)^2 \sin \theta d\theta d\phi
\]  

(22)

the stored electric energy is

\[
W_e = \frac{\varepsilon_0 \varepsilon_r}{4} \int \int_{\text{cavity}} |E_z|^2 dV.
\]  

(23)

The calculation of \( \delta_e \) in (21) requires an iterative procedure. It is found that the final value of \( \delta_e \) is quite sensitive to the value chosen at the start of the iterative process [11]. We begin by setting \( \delta_e = \delta_0 \) instead of usual \( \delta \) to shorten the iterative procedure. \( \delta_0 \) is given by

\[
\delta_0 = \frac{P_{r0}}{2\omega W_{e0}} + \delta + \frac{s}{\delta},
\]  

(24)

where \( P_{r0} \) can be expressed as [12]

\[
P_{r0} = \frac{V_0^2 A_0^4}{20340} \left[ (1 - B) \left( 1 - \frac{A}{15} + \frac{A^2}{420} \right) + \frac{B^2}{5} \left( 2 - \frac{A}{7} + \frac{A^2}{180} \right) \right]
\]  

(25)

with \( A = (\pi a/\lambda_0)^2 \) and \( B = (2b/\lambda_0)^2 \). There appears to be an algebraic error. Working from the approximations indicated in [12], one obtains 40 680 instead of 20 340. \( W_{e0} \), the electric energy stored for dominant mode at resonance, can be derived to be

\[
W_{e0} = \frac{\varepsilon_0 \varepsilon_r abV_0^2}{8\delta}.
\]  

(26)

We then calculate \( E_z \) with (16). This is followed by the calculation of \( (\delta_e)_{i+1} \) with (21). The iterative process continues until the test \( (\delta_e)_{i+1} - (\delta_e)_i \) is smaller than a percentage, which is usually chosen to be 2%.
III. VALIDATION OF THEORY OF DIFFERENTIALLY-DRIVEN MICROSTRIP ANTENNAS

Theoretical calculations, numerical simulations, and physical experiments were made for a differentially-driven microstrip antenna. The antenna was constructed using Taconic TLY-5 with a relative dielectric constant of $2.2 \pm 0.02$ and a loss tangent of approximately $0.0009$ for the substrate and a conductivity of $3.7 \times 10^7$ S/m for the conductor. It had dimensions $a = 18.522$ mm, $b = 18$ mm, and $t = 0.635$ mm. The substrate size is $25 \times 25$ mm$^2$. The antenna was driven differentially at both radiating edges at $(x_1, y_1) = (0.0054a, 0.5b)$ and $(x_2, y_2) = (0.9946a, 0.5b)$ to excite the TM$_{10}$ mode. This differentially-driven scheme was implemented in Matlab. We first calculated the effective loss tangent $\delta_e$ and then the input impedance $Z_{dd}$. It was found $\delta_e$ to be 0.02 at 5.2 GHz. The simulations were executed using the HFSS. The measurements were conducted with an HP 8510C network analyzer. The measured $S$ parameters were converted to the differential input impedance by

$$Z_{dd} = 2Z_0 \left(1 - S_{11}^2 + S_{21}^2 - 2S_{21}\right) \left(1 - S_{11}^2 - S_{21}^2\right)^{-1} \tag{27}$$

where $Z_0$ is the reference impedance of 50 $\Omega$ [13].

Fig. 2 compares the calculated, simulated, and measured input impedance of the differentially-driven microstrip antenna. The impedance characteristics give insight on how the antenna must be modified to achieve a specified resonant frequency. Here the resonant frequency is defined as where the reactance of the input impedance is equal to zero. According to this definition, one can see that both calculated and simulated resonant frequencies are at 5.17 GHz and the measured resonant frequency is at 5.14 GHz. One also can see that the agreement between the calculated and simulated input impedance is excellent. The agreement between the calculated and measured input impedance is in general acceptable. The discrepancy is likely due to the fabrication tolerance and the experimental error of our measuring system. It was found that any small phase difference in measured $S_{11}$ and $S_{22}$ would cause a large error in the measured $Z_{dd}$. It should be also noted that in any 50 $\Omega$ system, high resistance measurements are sensitive to errors.

Fig. 3 compares the calculated, simulated, and measured co-polar radiation patterns of the differentially-driven microstrip antenna at 5.25 GHz. The comparison shows that excellent agreement is obtained between the theory, simulation, and experiment in the broadside direction of the forward region. The difference in the side lobe region between the theory and simulation and experiment is because the radiation pattern is calculated for an infinite ground plane while the simulation...
and experiment are performed on a finite ground plane. We do not compare the cross-polar radiation patterns because the cross-polar radiation is quite weak and hard to measure accurately due to the cancellation mechanism introduced by the differentially-driven technique. Fig. 4 compares the calculated radiation patterns of the microstrip antenna driven for differential and single-ended operations at 5.25 GHz. It is evident from the figure that the cross-polar radiation from the differentially-driven microstrip antenna is much weaker than that of the single-ended microstrip antenna. In addition, the calculation shows that the gain of the differentially-driven microstrip antenna is almost the same as that of the single-ended microstrip antenna.

IV. THEORETICAL ANALYSIS OF DIFFERENTIALLY-DRIVEN MICROSTRIP ANTENNAS

Having validated the theory, we now proceed to apply it to analyze the microstrip antenna to gain deep insight into its differential operation in this Section.

First, consider the differentially-driven points at one radiating edge: \((x_1, y_1) = (0.005\lambda_0, 0.005\lambda_0)\) and \((x_2, y_2) = (0.005\lambda_0, 0.994\lambda_0)\) to excite the \(TM_{01}\) mode. This differentially-driven scheme was used in [4]. Fig. 5 shows the calculated and simulated input impedance of the differentially-driven microstrip antenna. It is evident from the figure that the microstrip antenna resonates at 5.3 GHz and excellent agreement is obtained between the theory and simulation. Fig. 6 illustrates the calculated and simulated co-polar radiation patterns of the differentially-driven microstrip antenna at 5.45 GHz. Once again, the cross-polar radiation is negligible.

Second, consider the differentially-driven points at \((x_1, y_1) = (0.005\lambda_0, 0.005\lambda_0)\) and \((x_2, y_2) = (0.994\lambda_0, 0.994\lambda_0)\) to excite both \(TM_{10}\) and \(TM_{01}\) modes to realize circular polarization. This differentially-driven scheme has never been used. Fig. 7 shows the calculated and simulated input impedance of the differentially-driven microstrip antenna. It is evident from the figure that the microstrip antenna resonates at 5.17 GHz of the \(TM_{10}\) mode and 5.3 GHz of the \(TM_{01}\) mode and excellent agreement is obtained between the theory and simulation. Fig. 8 illustrates the calculated and simulated co-polar radiation patterns of the differentially-driven microstrip antenna at 5.45 GHz. Once again, the cross-polar radiation is negligible.

As in microstrip antennas for single-ended signal operation and shown in the above three driven cases, the radiation patterns of differentially-driven microstrip antennas are not sensitive to
the locations of the driven points. However, the physical symmetry of the locations of the driven points should be maintained for better differential performance.

Third, we analyze the differential resonant resistance as a function of $\xi/\lambda_0$ for the above three differentially-driven schemes to excite the TM$_{10}$, TM$_{01}$, and both TM$_{10}$ and TM$_{01}$ modes, respectively. The resonant resistance for the single-ended signal operation is included for comparison. For single-ended operation, the microstrip antenna is fed at $(x_1,y_1)$, $\xi$ is calculated with respect to a virtual feed located at $(x_2,y_2)$. Fig. 9 shows the resonant resistance for the first scheme to excite the TM$_{10}$ mode. It is seen that the resonant resistance increase with $\xi/\lambda_0$ and the differential resonant resistance is larger than the single-ended resonant resistance. It is found that when $\xi/\lambda_0$ is smaller than 0.1, no resonance for differential operation will occur any more, the differential input impedance is inductive. Fig. 10 shows the resonant resistance for the second scheme to excite the TM$_{01}$ mode. Similarly, the resonant resistance increases with $\xi/\lambda_0$ and the differential resonant resistance is also larger than the single-ended resonant resistance. Fig. 11 shows the resonant resistance for the third scheme to excite the TM$_{10}$ and TM$_{01}$ modes for circular polarization. As can be seen from the figure, the resonant resistance increases with $\xi/\lambda_0$ and the differential resonant resistance is larger than the single-ended resonant resistance. It is also found that when $\xi/\lambda_0$ is smaller than 0.1, no resonance for differential operation will occur any more, the differential input impedance is inductive. The dependence of resonance for the differentially-driven microstrip antennas on the separation of the dual feeds can be explained as follows. When the dual feeds are located near to each other $\xi/\lambda_0 < 0.1$, the differential
signal applied to the dual feeds cancels, hence the resonance does not occur and the feeds themselves make the differential input impedance exhibit inductive.

Finally, it should be mentioned that there exists a virtual AC ground line between the two feeds in the middle of the differentially-driven microstrip patch. Fig. 12 shows the normalized $z$ component of electric field $E_z$ in dB on the surface of the differentially-driven microstrip patch. The existence of the virtual ac ground line can be clearly identified from Fig. 12. The virtual ac ground line guarantees the isolation between the two feeds.

V. CONCLUSION

We have studied differentially-driven microstrip antennas theoretically and experimentally in this paper. The improved cavity model of single-ended microstrip antennas was expanded to analyze the input impedance and radiation characteristics of the differentially-driven microstrip antennas. The differentially-driven microstrip antennas were fabricated using Taconic TLY-5 and measured with the HP 8510C network analyzer. Theoretical and experimental results were found to be in acceptable agreement. It was shown from the analysis that the occurrence of resonance for the differentially-driven microstrip antennas also depends on the ratio of the separation $\xi$ of the dual feeds to the free-space wavelength $\lambda_0$. When the dual feeds are located far from each other $\xi/\lambda_0 < 0.1$, the resonance occurs, and the input resistance at resonance is rather large. However, when the dual feeds are located near to each other $\xi/\lambda_0 < 0.1$, the resonance does not occur, the input resistance is quite small, and the input impedance is inductive. The comparison with single-ended microstrip antennas was made. It was found that the differentially-driven microstrip antennas have larger resonant resistance, similar co-polar radiation patterns, and lower cross-polar radiation component.

REFERENCES

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