<table>
<thead>
<tr>
<th>Title</th>
<th>Throughput-Lifetime tradeoffs in multihop wireless networks under a realistic interference model.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Author(s)</td>
<td>Luo, Jun.; Iyer, Aravind.; Rosenberg, Catherine.</td>
</tr>
<tr>
<td>Date</td>
<td>2007</td>
</tr>
<tr>
<td>URL</td>
<td><a href="http://hdl.handle.net/10220/6163">http://hdl.handle.net/10220/6163</a></td>
</tr>
<tr>
<td>Rights</td>
<td></td>
</tr>
</tbody>
</table>
Throughput-Lifetime Tradeoffs in Multihop Wireless Networks under a Realistic Interference Model

Jun Luo*, Aravind Iyer† and Catherine Rosenberg*

Abstract—Throughput and lifetime are both crucial design objectives for multihop wireless networks. In general, it is not sufficient to optimize either of them separately. As these two objectives are often conflicting with each other, we can only hope to identify the tradeoffs between them. This entails a harder problem than dealing with either solely. In this paper, we propose a general framework for investigating the tradeoff between throughput and lifetime. We employ a utility-based tradeoff objective that allows us to identify tradeoffs that are of physical interest. We consider a scheduled network where link transmissions can be coordinated to be conflict-free. We use a realistic interference model owing to which we gain a deep understanding of the network configurations that achieve the optimal tradeoffs. Our analytical and numerical results provide instructive insights into the interplay between the configurations and the throughput/lifetime.

I. INTRODUCTION

Maximizing throughput (how fast data may be delivered) and lifetime (how long the network may last) are two important design objectives for multihop wireless networks. These two objectives are often conflicting with each other: higher throughput usually leads to faster (node) energy dissipation and hence reduces the network lifetime. In some special cases, one of these two objectives can be totally or partially ignored. For example in mesh networks nodes are usually tethered and thus do not care about (energy-related) lifetime. However, in general, the ability to identify optimal tradeoffs between throughput and lifetime based on the application requirements, is a more practical problem than optimizing either of them individually.

In the research community, maximizing throughput (or a network utility in general) [12], [19], [6], [7], [13] and lifetime [8], [18], [14], [16] in multihop wireless networks have long been treated as two separate problems. It is only very recently that the tradeoff between network utility and lifetime has been investigated [17], [20]. In these recent works, the tradeoff is identified only through scalarizing the two conflicting objectives. In practice, scalarization (i.e., linear combination through a weight vector) [5] yields results that are not always easy to interpret in the networking context: what does the weighted sum of throughput (in, e.g., bit per second) and lifetime (in, e.g., second) mean?

In the wireless networking context, both power control and scheduling have a significant impact on throughput and lifetime. However, existing approaches have certain limitations in investigating these mechanisms. Some studies only take into account power control or scheduling alone, as considering them jointly leads to a non-convex problem [17], [20]. Others use inadequate interference and link capacity models that obscure the effects of power control. For example, the conflict graph model used in [12] (commonly referred to as protocol interference model [10]) was shown in [11] to yield wrong insights. Also, as we will see later in our paper, the effect of power control takes place only at discrete points, as a consequence of discrete modulation levels. This will not be observed if we derive the link capacity from the Shannon capacity formula (e.g., [16]).

In this paper, we consider a generic network model that represents a broad category of practical multihop wireless networks. We assume all nodes are static and the channel gains are invariant. We consider the scenarios where an adequate throughput (not necessarily the maximum) as well as a sufficiently long lifetime are required. This could be the case for wireless sensor networks if, for example, they carry multimedia traffic [1]. Wireless mesh networks whose nodes have only occasional access to the power grid serve as another example. Our notion of network throughput is the max-min flow rate [13] and our notion of network lifetime is the max-min node lifetime [8]. In addition, we take into account a realistic interference model for scheduling the wireless links. The conflict set model (also used in [11], [13]) we use captures the fact that the interference to a certain link is the cumulative interference from multiple links that are activated during the same period of time. Finally, we accurately model practical modulation technologies: the link rate is a discontinuous function of the transmit power.

Based on the aforementioned network model, we are focusing on identifying the tradeoff between throughput and lifetime. Therefore, the problem we would like to solve is:

P: Maximize, over all possible network configurations (in terms of routing, link scheduling, transmit power, and modulation level), a tradeoff utility \( \Gamma \), which is a function of both the max-min throughput \( \lambda \) and the max-min lifetime \( T \). The utility \( \Gamma \) is assumed to be increasing in \( \lambda \) (resp. \( T \)) for fixed \( T \) (resp. \( \lambda \)).

Note that we are not aiming at a distributed (e.g., [20]) or dynamic (e.g., [9]) algorithm to solve the problems. Our solutions are centralized and provide static configurations. On one hand, our centralized approach intends to provide a
benchmark for dimensioning a network. On the other hand, our approach does have practical significance due to the upcoming standards such as IEEE 802.16 that enable scheduled network operation (we refer to our arguments in [13] for details).

To address \( P \), we face two difficulties. First, the conflicting structure results in a combinatorial optimization that is shown to be NP-hard. Secondly, jointly considering throughput and lifetime in general entails non-convex programming. While the first difficulty can be partially overcome by a smart enumerative technique [13], we avoid treating the non-convexity issue by focusing on two special tradeoff utilities that are of practical interests. For each utility, we first study the problem by fixing the transmit power. We then analyze the trend of the solutions as functions of the power.

We give numerical results for networks whose nodes are deployed on square grids. In order to make the effects highlighted by our analysis visible for small scale networks, we deliberately consider traffic patterns that converge to a sink placed at a corner. Examples of such patterns can be easily found in wireless mesh or sensor networks, where only a few base stations or sinks attract or initiate traffic. Both analytical and numerical results show that the optimal tradeoffs between \( \lambda \) and \( T \) are usually obtained at high transmit power. In addition, our results show that, by fixing the throughput requirement, the lifetime is not a monotonically decreasing function of transmit power. On one hand, our results extend the result proved in [2, 13] that increasing transmit power always improves the throughput: this result is a special case of ours with the tradeoff objective used in the existing literature. The intuition is again related to the degree of scheduling freedom, which is augmented (to some extent) by less stringent demand in throughput.

The main contributions of this paper are:

1) We generalize the framework proposed in [13] to identify the tradeoffs between throughput and lifetime in an arbitrary multihop wireless network. Our utility-based tradeoff objective greatly extends the scalarization objective used in the existing literature.

2) We propose and study two tradeoff utilities that are of practical interests.

3) We characterize, for these two utilities, the transmit power levels and corresponding network configurations that achieve the optimal tradeoffs through analytical results.

4) We bring insights on the interplay between configurations and throughput/lifetime, which contradict a common belief that higher power causes shorter lifetime.

The rest of the paper is organized as follows. We first describe our network model and formulate the general throughput-lifetime tradeoff problem in Sec. II. Then, we focus on two utility functions and address them separately in Sec. III and IV. In Sec. V, we report the numerical results. Finally, we conclude our paper in Sec. VI. Due to space constraints, we omit all the proofs. Readers are referred to [15] for details.

II. NETWORK MODEL AND PROBLEM FORMULATION

A. Network Model

We model the network as a set \( \mathcal{N} \) of nodes and a set \( \mathcal{L} \) of links, with \( |\mathcal{N}| = N \) and \( |\mathcal{L}| = L \). Each node \( n \in \mathcal{N} \) is associated with a geographical location. We actually take \( \mathcal{L} \) as the set of links in the complete directed graph with node set \( \mathcal{N} \); it is a superset of all feasible links. A link \( l = (i, j) \in \mathcal{L} \) denotes that the link is outgoing from node \( i \) and incoming to node \( j \). The feasibility of each link depends on our physical layer model defined below along with its parameters.

Physical Layer Model: We associate with all links the same transmit power \( P_{tx} \) that may vary in \([0, \bar{P}]\). Let \( z_l \) (in bits per symbol) denote a modulation scheme feasible on link \( l \) and let \( Z_l(P_{tx}) \) denote the (discrete) set of all available modulation schemes under \( P_{tx} \), \( z := (z_1, \cdots, z_L) \) and \( Z(P_{tx}) := \prod_{l \in \mathcal{L}} Z_l(P_{tx}) \). The link rate (in bits per second) is \( c_l z_l \) where \( c_l \) refers to the baud rate (in symbols per second). We assume that a link \( l \) requires an SINR of at least \( \beta(z_l) \) to be able to use modulation \( z_l \). Since the SINR for link \( l \) is \( \gamma_l = \frac{G_{tl}P_{tx}}{N_0 + \sum_{k: k \neq l} G_{kl}P_{tx}} \), where \( G_{tl} \) (resp. \( G_{kl} \)) denotes the channel gain on \( l \) (resp. from the transmitter of \( l \) to the receiver of \( l \) and \( N_0 \) is the average thermal noise power in the operating frequency band, \( z_l \) is feasible if \( \gamma_l \geq \beta(z_l) \). The channel gain between two points with distance \( d \) is given by \((\frac{d}{d_0})^{-\eta}\) where \( d_0 \) is the close-in reference distance and \( \eta \) is the path loss exponent.

Link Conflict Structure: We only briefly review the concepts here; readers are referred to our previous work [13] for detailed discussions. Our physical model results in a more comprehensive conflict structure than the commonly used conflict graph model. The avatar of this structure is the concept of a conflict set: let \( \zeta \) be an \( L \)-dimensional \( \{0, 1\} \) vector; \( \zeta_l = 1 \) if \( l \in \zeta \) and \( \zeta_l = 0 \) otherwise. The conflict set \( \mathcal{D}(z_l, P_{tx}) \) is defined as follows for each link \( l \) that applies modulation scheme \( z_l \):

\[
\left\{ \zeta: \zeta_l = 1 \text{ and } \zeta_k = 0 \text{ for } k \neq l, G_{kl}P_{tx} \geq \frac{G_{tl}P_{tx}}{N_0 + \sum_{k: k \neq l} G_{kl}P_{tx}} \beta(z_l) \right\}
\]

A set of links represented by \( \zeta \) in \( \mathcal{D}(z_l, P_{tx}) \) are conflicting with \( l \) in the sense that if all the links in it are scheduled together with \( l \), \( z_l \) becomes infeasible for \( l \). A dual concept of conflict set is the independent set (not in the conventional graph-theoretical sense). The following defines the set of independent sets:

\[
\mathcal{I}(z, P_{tx}) = \left\{ \zeta: \sum_{k: k \neq l} G_{kl}P_{tx} \geq \frac{G_{tl}P_{tx}}{N_0 + \sum_{k: k \neq l} G_{kl}P_{tx}} \beta(z_l) \text{ for all } l \right\}
\]

Links belonging to an element of \( \mathcal{I}(z, P_{tx}) \) can operate at the same time without conflicts (in particular, with link \( l \) using modulation level \( z_l \)). Let \( \mathcal{I}_l(z, P_{tx}) \) refer to the set of independent sets that contain link \( l \).
Maximize \[ \Gamma(\lambda, T) \] Subject to: \[ \sum_{f \in \mathcal{F}} \lambda_f \left( \sum_{r \in R_f^3} \phi_f^r \right) - c_l z_l \sum_{k \in \mathcal{I}_f(\alpha, P_{tx})} \alpha_k \leq 0 \quad \forall \ l \in \mathcal{L} \] \[ \sum_{r \in R_f} \phi_f^r - 1 = 0 \quad \forall \ f \in \mathcal{F} \] \[ \sum_{k=1}^{S} \alpha_k = 1 \] \[ \lambda - \lambda_f \leq 0 \quad \forall \ f \in \mathcal{F} \] \[ \phi_f^r \geq 0, \ \ z \in \mathcal{Z}(P_{tx}), \ P_{tx} \in [0, \tilde{P}] \]

Routing and Scheduling: Let \( \mathcal{F} \) denote the set of flows. A flow \( f \in \mathcal{F} \) is scheduled by its source-destination pair. Let \( R_f \) be the set of all routes used by \( f \) and \( R_f^3 \) be the set of routes of \( f \) going through link \( l \). The fraction of \( f \) routed on \( r \in R_f \) is \( \phi_f^r \), hence \( \sum_{r \in R_f} \phi_f^r = 1 \). Let \( S \) denote the power set of \( \mathcal{L} \). A transmission schedule is an \(|S|\)-dimensional vector \( \alpha = \{\alpha_1, \alpha_2, \cdots, \alpha_{|S|}\} \) such that \( \alpha_k \geq 0 \) only if the \( k \)th set in \( S \) is an independent set (otherwise \( \alpha_k = 0 \)) and \( \sum_{k=1}^{|S|} \alpha_k = 1 \). We can interpret \( \alpha_k \) as the fraction of time allocated to the \( k \)th link set in \( S \). For scheduling purpose, we assume that the time is slotted.

Energy Model: To incorporate lifetime into the optimization problem, we need an energy model to describe the way energy is dissipated during the network operations. We assume a simplified model: as mentioned earlier the power \( P_{tx} \) is expended by all the nodes during transmission; we also assume that a fixed power \( P_{rx} \) is consumed by all the nodes during reception. Finally, we assume that nodes have an initial energy capacity denoted by \( \bar{E} = \{E_i\}_{i \in \mathcal{N}} \).

B. The General Problem

Given the network model, we formulate the problem that maximizes a certain tradeoff utility \( \Gamma \) (a function of both throughput and lifetime) as (1)–(6). The maximization is with respect to the routing \( \phi_f^r \), the link scheduling \( \alpha_k \), the transmit power \( P_{tx} \), and the modulation level vector \( z \). We refer to this problem as General throughput-lifetime TradeOff (GTO). We explain the objective and the constraints in the following:

- The objective (1) can be any function \( \Gamma \) that represents a reasonable tradeoff between the lifetime and throughput. The only assumption we make on \( \Gamma \) is that it is increasing in \( \lambda \) (resp. \( T \)) for fixed \( T \) (resp. \( \lambda \)). The particular utilities on which we are going to focus are
  \[ \Gamma_1(\lambda, T) = \lambda \mathbf{1}_{\{T \geq T_{\text{min}}\}} \]  \[ \Gamma_2(\lambda, T) = T \mathbf{1}_{\{\lambda \geq \lambda_{\text{min}}\}} \]  \[ \phi_f^r \geq 0, \ z \in \mathcal{Z}(P_{tx}), \ P_{tx} \in [0, \tilde{P}] \]
- The constraint (2) expresses the link capacity bound: the amount of flow going through a link is bounded from above by the link capacity (represented by the product of the link rate and the scheduled time), \( \lambda_f \) denotes the rate of flow \( f \).
- The constraint (3) is actually a flow conservation law at the source node: the flow out of the source balances the flow injected to the source.
- The constraint (4) defines a repetitive pattern of link scheduling, with \( \alpha_k \) being the time fraction spent scheduling the \( k \)th independent set during a unit time frame.
- The constraint (5) defines the throughput as the max-min flow rate. Constraints (2), (4) and (5) distinguish our problem from a pure lifetime maximization (e.g., [18]).
- The constraint (6) defines the (network) lifetime as the max-min node lifetime through the energy conservation: since transmitting and receiving data consume nodes’ energy, the amount of flow that can be carried by a given node is bounded from above by the initial energy. Note that only feasible links (i.e., \( z_{(i,j)} > 0 \)) are included in the summation. This is the constraint that distinguishes our problem from a pure throughput maximization problem (e.g., [13]). In order to compute the energy needed to operate a link \( l \), we need the time spent for the operation. We compute this time as the ratio of the amount of flow carried by a link to the link rate (i.e., \( T \sum_{f \in \mathcal{F}} \lambda_f \left( \sum_{r \in R_f} \phi_f^r \right) \)). Note that the operation time of a link is bounded from above by the sum of all scheduled times (i.e., \( T \sum_{k \in \mathcal{I}_f(\alpha, P_{tx})} \alpha_k \)), as indicated by (2).
- Without loss of generality, we assume in the remainder of our paper that there is only one level of modulation (i.e., \( \mathcal{Z}(P_{tx}) \) is a singleton). As we will explain later, adding extra modulation levels will not fundamentally change our results. As a consequence, we denote the link rate by \( c_l \) with a harmless abuse of notation.

One of the major difficulties in solving GTO lies in the fact that it is a non-convex programming. The non-convexity comes from the constraint (6), where a product of three variables \( \lambda_f \), \( T \) and \( P_{tx} \) is involved. In the following, we will focus on two utilities, i.e., we separate the throughput maximization from
the lifetime optimization through $\Gamma_1$ (9) and $\Gamma_2$ (10). We first investigate the two problems under a fixed transmit power, and then we study the trends of their solutions as functions of the transmit power.

III. Maximizing Throughput with Lower Bounded Lifetime (MaxT-LBL)

In this section, we consider the case where we try to maximize the throughput given a lower bound for the lifetime, which we refer to as MaxT-LBL. Taking $\Gamma(\lambda, T) = \lambda \mathbf{1}_{\{T \geq T_{\text{req}}\}}$, the objective becomes

$$\text{Maximize } \lambda$$

and the constraint (6) becomes

$$\sum \sum_j \frac{\lambda_f}{\sum_{i \in q} \phi_f^{(i, j)}} \left( P_{xz} \sum_{r \in R_f^{(j)}} \phi_r^{(f)} + P_{rx} \sum_{r \in R_r^{(j)}} \phi_r^{(r)} \right) \leq \hat{E}_i \quad \forall i \in \mathcal{N}$$

(11)

where $\hat{E}_i = E_i / T_{\text{req}}$ is the bound on the energy consumed per unit time for node $i$. This constraint absorbs the indicator function in $\Gamma$. Note that we directly take $T = T_{\text{req}}$ in the formulation, because the maximum $\lambda$ obtained for $T = T_{\text{req}}$ is no less than that obtained for $T > T_{\text{req}}$.

A. The Throughput Optimal Configuration

We first fix the transmit power $P_{xz} = P$. Let the optimal solution of MaxT-LBL at this power be $\lambda^*(P, T_{\text{req}})$. In order to treat the problem under a graph theoretical framework, we have to embed the link conflict structure (defined in Sec. II) into a graph. Such a graph is termed extended conflict graph (ECG) [13]. Since every link set $\zeta \in D_1(z_1, P)$ (7) conflicts with a certain physical link $l$ using modulation $z_2$, we have to represent all these constraints. Therefore, in an ECG, each physical link is replicated into multiple copies of “virtual” links, with each copy realizing one scheduling constraint given by $\zeta \in D_1(z_2, P)$. This extension allows us to apply graph theoretical results. For example, an independent set in the ECG does corresponds to the definition (8). In the following, we will need the clique $q$ defined in terms of the ECG. A node $i \in q$ means that at least one “virtual” copy of $(i, j)$ or $(j, i)$ belongs to $q$ for some node $j \in \mathcal{N}$.

A special case of interest is where $T_{\text{req}} = 0$. In this case, the energy bound become infinity, which annihilates the constraint (11) and thus reduces the MaxT-LBL problem to the Throughput Optimization (TO) problem described in [13]. Due to this close relation between MaxT-LBL and TO, we can have the following characterization for $\lambda^*(P, T_{\text{req}})$.

Proposition 1: For some $\kappa \in (0, 1]$ which depends on the conflict structure and a certain transmit power $P$, we have

$$\lambda^*(P, T_{\text{req}}) \leq \max_{\phi} \left\{ \min_{q, i \in q} \frac{1}{w_q(\phi)} + \frac{1}{w_i(\phi)} \right\}$$

(12)

$$\lambda^*(P, T_{\text{req}}) \geq \min_{\phi} \left\{ \frac{\kappa}{w_q(\phi)} + \frac{1}{w_i(\phi)} \right\}$$

(13)

where $w_q(\phi)$ and $w_i(\phi)$ are given by:

$$w_q(\phi) = \sum_{l \in q} \sum_{r \in R_r^{(j)}} \phi_r^{(r)}$$

$$w_i(\phi) = \sum_{\phi_f^{(i, j)}} \sum_{f \in F} \phi_f^{(f)}$$

with $\phi_f^{(f)} = P_{xz} \sum_{r \in R_r^{(j)}} \phi_r^{(r)} + P_{rx} \sum_{r \in R_r^{(j)}} \phi_r^{(r)}$.

a) Remark: By interpreting $w_q(\phi)$ and $w_i(\phi)$ as “costs” of routing through a clique $q$ and a node $i$, respectively, it is not difficult to see that MaxT-LBL yields an interpretation similar to that of TO, i.e., flow $f$ goes through a route that is influenced by the costs of the cliques along the route. Whereas the cost is only about air-time for TO, the cost of energy should also be taken into account for MaxT-LBL. If $T_{\text{req}}$ is below the lifetime achievable by a throughput optimal configuration, the second term in both bounds is not active and the bounds are the same as those of TO. Otherwise the configuration is forced to take into account the tradeoff between choosing capacity or energy abundant cliques. For a chosen clique, traffic has to be balanced among nodes in the clique. Another important implication is that, if we want to maintain the throughput in the face of large $T_{\text{req}}$, we could identify those cliques constrained by energy and assign higher energy storage for them. Based on Proposition 1, our previous results on throughput optimal routing described in [13] can be extended to MaxT-LBL. Readers are referred there for details.

B. Transmit Power vs. Optimal Throughput

We now look at how changing $P$ may affect $\lambda^*(P, T_{\text{req}})$. We first consider TO (MaxT-LBL with $T_{\text{req}} = 0$) and then MaxT-LBL in general.

Proposition 2: Let $P_{xz} = P \in [0, \bar{P}]$. The optimal throughput $\lambda^*(P, 0)$ is a step (or piecewise constant) function of $P$. Let $N_p$ be the point process on $[0, \bar{P}]$ that counts the events of new independent set appearing and let $\{P_n\}$ be its point sequence, we have

$$\lambda^*(P, 0) = \int_0^P \Delta \lambda(p) N_p(dp), \quad P \leq \bar{P}$$

where $\Delta \lambda(p)$ is the throughput variation at power level $p$.

a) Remark: Note that, for known network topology, the point process $N_p$ and its mark $\Delta \lambda(p)$ are deterministic, which can be computed a priori. In general, $\Delta \lambda(p)$, as the solution of TO, does not have a closed-form expression.

The increasing trend of $\lambda^*(P, 0)$ is made clear by the next proposition.

Proposition 3: The function $\lambda^*(P, 0)$ has only positive variation. In other words, $\Delta \lambda(p) \geq 0$.

Now let us look at the general case where $T_{\text{req}} > 0$. We have the following proposition that partially characterizes the optimal throughput $\lambda^*(P, T_{\text{req}})$ as a function of $P$; it extends the result of Proposition 2.

Proposition 4: Let $P_{xz} = P \in [0, \bar{P}]$. The optimal throughput $\lambda^*(P, T_{\text{req}})$ is a piecewise continuous function of $P$. Let $N_p$ be the same point process as defined in Proposition 2,

$$\lambda^*(P, T_{\text{req}}) = \int_0^P \Delta \lambda(p, T_{\text{req}}) N_p(dp) + \int_0^P g(p, T_{\text{req}}) dp$$
for $P \leq \bar{P}$, where $\Delta \lambda(p, T_{req}) \geq 0$ is the throughput variation at power level $p$, and $g(p, T_{req}) \leq 0$ is the derivative of the continuous part at $p$.

We illustrate $\lambda^*(P, 0)$ and $\lambda^*(P, T_{req})$ in Fig. 1.

![Fig. 1. The optimal throughput as function of $P$ given $T_{req} = 0$ (a) and $T_{req} > 0$ (b).](image)

**b) Remark:** The results presented in this section are based on the assumption of one modulation level. With more modulation levels, we can replace the physical link between two nodes with multiple “artificial links”; each representing a certain modulation level [13]. By redefining the independent two nodes with multiple “artificial links”, the constraint (15) is formulated as follows:

$$\sum_{\{i,j\} \in \mathcal{L}} x_{\{i,j\}}^f - \sum_{\{j,i\} \in \mathcal{L}} x_{\{j,i\}}^f \geq \lambda_{\text{req}} T_{1_i = f_o} - \lambda_{\text{req}} T_{1_j = f_d} \quad \forall \ f \in \mathcal{F} \quad \forall \ i \in \mathcal{N}$$

We slightly change the formulation by considering the aggregated flow pattern and by absorbing the indicator functions in the objective into the constraints. The resulting MaxLT-LSR is formulated as follows:

$$\text{Maximize} \quad T \quad \text{(14)}$$

$$\sum_{\{i,j\} \in \mathcal{L}} x_{\{i,j\}}^f - \sum_{\{j,i\} \in \mathcal{L}} x_{\{j,i\}}^f \geq \lambda_{\text{req}} T_{1_i = f_o} - \lambda_{\text{req}} T_{1_j = f_d} \quad \forall \ f \in \mathcal{F} \quad \forall \ i \in \mathcal{N} \quad (15)$$

$$\sum_{f \in \mathcal{F}} x_{\{i,j\}}^f c_{\{i,j\}} \sum_{\alpha_k} \alpha_k \leq 0 \quad \forall \ l \in \mathcal{L} \quad (16)$$

$$\sum_{k=1}^{S} \alpha_k = T \quad \text{(17)}$$

$$\sum_{j} \left( \sum_{f \in \mathcal{F}} \frac{x_{\{i,j\}}^f}{c_{\{i,j\}}} + \frac{p_{\text{txj}}}{c_{\{i,j\}}} \right) \leq E_i \quad \forall \ i \in \mathcal{N} \quad (18)$$

where $x_{\{i,j\}}^f = \lambda_{\text{req}} T (\sum_{r \in \mathcal{R}_{\{i,j\}}} \phi_r^f) \geq 0$ is the aggregated (over the lifetime $T$) flow over a link $(i, j)$ and $\alpha_k = T \alpha_k$ is the aggregated scheduling variable. The constraint (15) comes from flow conservation: the incoming flow should balance the outgoing flow for a given node. Since we require $\lambda_{\text{req}} \geq \lambda_{\text{req}}$, this constraint is an inequality rather than an equality suggested by the conservation law.

Note that, given an arbitrary $\lambda_{\text{req}}$, the problem might not be feasible. However, we can guarantee feasibility (and thus avoiding the complexity of feasibility verification) by requiring $\lambda_{\text{req}} = \rho \lambda^*(P, 0)$ where $\rho \leq 1$ and $\lambda^*(P, 0)$ is the optimal solution of the TO (MaxT-LBL with $T_{req} = 0$) at $P_{\text{tx}} = P$.

**A. Transmit Power vs. Optimal Lifetime**

We investigate the effect of transmit power on the optimal lifetime $T^*$ in this section. We want to characterize the trend of $T^*(P, \rho)$ as $P$ varies in $[0, \bar{P}]$. Note that the lower bound $\lambda_{\text{req}}(P) = \rho \lambda^*(P, 0)$ is also a function of $P$.

The following proposition shows that $T^*(P, \rho)$ has a similar trend (i.e., piecewise continuous) as $\lambda^*(P, T_{req})$ given by Proposition 4.

**Proposition 5:** Let $P_{\text{tx}} = P \in [0, \bar{P}]$. The optimal lifetime $T^*(P, \rho)$ is a piecewise continuous function $P$. Let $N_p$ be the same point process as defined in Proposition 2, we have

$$T^*(P, \rho) = \int_0^P \Delta T(p, \rho) N_p(dp) + \int_0^P h(p, \rho)dp$$

for $P \leq \bar{P}$, where $\Delta T(p, \rho)$ is the lifetime variation at $p$ and $h(p, \rho) \leq 0$ is the derivative of the continuous part at $p$.

**a) Remark:** Similar to $\Delta \lambda(p, T)$, $\Delta T(p, \rho)$ is deterministic for a known network topology.

As opposed to the decreasing trend between “jumps”, the following proposition shows that $T^*(P, \rho)$ is, surprisingly, not a decreasing function of $P$ as a whole.

**Proposition 6:** The function $T^*(P, \rho)$ can have either positive or negative variations at points of $N_p$. In other words, $\Delta T(P, \rho)$ can be of both signs.

**b) Remark:** As $\lambda^*(P, T_{req})$ is non-decreasing at points of $N_p$ (Proposition 3) and $T^*(P, \rho)$ may increase or decrease at the same points (Proposition 6), it is straightforward to see that lower power levels are not beneficial to either throughput or lifetime. Consequently, our results imply that the optimal tradeoffs between $\lambda$ and $T$ are usually obtained at high transmit powers.

**B. The Tradeoff between Throughput and Lifetime**

Unlike the MaxT-LBL problem (whose parameter $T_{req}$ is unbounded), the parameter $\lambda_{\text{req}}(P)$ of MaxLT-LSR is bounded within $[0, \lambda^*(P, 0)]$ for all $P \in [0, \bar{P}]$. This allows us to investigate the tradeoff between the throughput and lifetime.

We will need the following definition to facilitate the discussions.

**Definition 1:** Duty cycle scaling refers to a specific strategy of trading throughput for lifetime: it maintains the scheduled time while scaling the operation time by a factor.

When the operation time of a link (i.e., $\sum_{j} \alpha_j \frac{\sum_{r \in \mathcal{R}_{\{i,j\}}} \phi_r^f}{c_{\{i,j\}}}$) is strictly shorter than its scheduled time (i.e., $\sum_{k} \alpha_k \frac{\sum_{r \in \mathcal{R}_{\{i,j\}}} \phi_r^f}{c_{\{i,j\}}}$) in a unit time frame, we assume that both the transmitter and the receiver will switch to sleep...
mode during the spare time to conserve energy. The duty cycle scaling extends this mechanism to trade throughput for lifetime. The next proposition is a direct consequence of this definition. It shows that, if we accept a decrease in throughput, duty cycle scaling leads to a lower bound on the optimal lifetime. In other words, the tradeoff is biased towards the lifetime in proportional sense: sacrificing a certain fraction of throughput gains a larger or equal fraction of improvement in lifetime.

**Proposition 7:** Given a fixed $P$ and two required source rates $\lambda_{req}(0) = \rho_1\lambda^*(P,0)$ and $\lambda_{req}(2) = \rho_2\lambda^*(P,0)$ with $\rho_1 > \rho_2$, we have

$$T^*(P, \rho_2) \geq \frac{\rho_1}{\rho_2} T^*(P, \rho_1)$$

An interesting question is whether the lower bound is tight. The answer is positive; we explain it in the remainder of this subsection. Usually, the network configuration that can produce $\lambda^*(P,0)$ is not unique; we identify these configurations by the independent sets involved. This is to say that there exists a configuration set $\mathcal{C}_f(P,\rho)$ of sets $\tilde{\mathcal{I}}(P,\rho)_i = \tilde{\mathcal{I}}(P,\rho)$ of independent sets, where $\tilde{\mathcal{I}}(P,\rho)_i \subseteq \tilde{\mathcal{I}}(P)$. For every set $\tilde{\mathcal{I}}^*_f(P,\rho)$ of independent sets, a schedule on the independent sets in $\tilde{\mathcal{I}}^*_f(P,\rho)$ exists such that $\rho_2\lambda^*(P,0)$ is achieved. The lifetime optimal configurations of MaxLT-LSR are chosen from this configuration set. Since all configurations achieving $\rho_1\lambda^*(P,0)$ can be duty-scaled to achieve $\rho_2\lambda^*(P,0)$, we have $\mathcal{C}_f(P,\rho_1) \subseteq \mathcal{C}_f(P,\rho_2)$, which accounts for the inequality in the Proposition 7. Therefore, $\mathcal{C}_f(P,\rho)$ grows with decreasing $\rho$. However, the size of $\mathcal{C}_f(P,\rho)$ is bounded from above by the size of the power set of $\mathcal{I}(P)$. As a consequence, $\mathcal{C}_f(P,\rho)$ will reach its maximum size for some $\rho_0 \in [0,1]$. We formally define this in the following:

**Definition 2:** We say $\mathcal{C}_f(P,\rho)$ is complete if $\mathcal{C}_f(P,\rho)$ includes all the elements of the power set of $\mathcal{I}(P)$ that yield a connected routing topology for $f \in \mathcal{F}$. Let $\rho_0$ be the largest $\rho$ for which $\mathcal{C}_f(P,\rho)$ is complete.

Based on Definition 2, we can state the sufficient condition for the equality in Proposition 7 to hold:

**Proposition 8:** If $\mathcal{C}_f(P,\rho_1)$ is complete and $\rho_1 > \rho_2$, then

$$T^*(P, \rho_2) = \frac{\rho_1}{\rho_2} T^*(P, \rho_1)$$

Literally, if the configuration set is sufficiently large, the tradeoff between throughput and lifetime is strictly proportional.

**V. NUMERICAL RESULTS**

We report numerical results in this section. To facilitate the interpretation of the results, we assign $E = 1$ mJ to all nodes and link rate $c = 1$ to all links. We also assume $P_{tx} = 0$ dBm for all nodes and the path loss exponent $\eta = 4$, unless otherwise specified. The numerical results are obtained for a network on a $5 \times 5$ square grid with a sink at the lower-left corner. We denote by $d_{\text{min}}$ the distance between the two closest nodes. We consider unidirectional traffic in the square grid network: the uplink traffic intended for the sink is originated from all nodes. For brevity, we only provide results for MaxLT-LSR ($\Gamma_2$) but omit the results for MaxT-LBL ($\Gamma_1$).

**A. Transmit Power vs. Optimal Throughput and Lifetime**

We first investigate the impacts of transmit power on both throughput and lifetime, as well as on the tradeoff between them. Fig. 2 shows $\lambda^*(P_{tx}, 0)$, the optimal throughput obtained without a lower bound on the lifetime, (resp. $0.75 \times \lambda^*(P_{tx}, 0)$) and the corresponding $T^*(P_{tx}, \rho = 1)$ (resp. $T^*(P_{tx}, \rho = 0.75)$) as functions of $P_{tx}$. Both throughput and lifetime are normalized to their maximum values. The trends described by Propositions 4, 5 and 6 are rather obvious: while $\lambda^*(P_{tx}, 0)$ is monotonic in $P_{tx}$, $T^*(P_{tx}, \rho)$ is not. The tradeoff is not “monotonic” in $P_{tx}$ either; it is proportional below $-7.5$ dBm but favors lifetime beyond that. This is suggested by the appearance of positive “jumps” of $T^*(P_{tx}, \rho)$ starting from $P_{tx} = -7.5$ dBm. It illustrates the fact that the optimal tradeoff points are usually obtained at relative high power levels. For example, the big jump of lifetime at $P_{tx} = 0$ dBm makes this point a favorable choice for obtaining optimal tradeoffs (see Sec. V-D for details).

**B. Optimal Network Configuration**

We provide two examples in Fig. 3 to illustrate the optimal network configurations. This figure describes two configurations corresponding to two points at $P_{tx} = 4.88$ dBm on the curves $T^*(1, P_{tx})$ and $T^*(0.75, P_{tx})$ in Fig. 2. From Fig. 2, we see that sacrificing 25% in throughput yields a 4-time improvement in lifetime. Therefore, the configuration set $\mathcal{C}_f$ has definitely experienced a big change; this is clearly illustrated in Fig. 3. For $\rho = 1$, the configuration tends to use “short” links for improving spatial reuse. This gives high throughput but leads to an unbalanced use of the nodes that are directly connected to the sink (the links between these nodes and the sink constitutes the last hop clique). Reducing the throughput requirement ($\rho = 0.75$) allows a more balanced use of the last hop clique (actually of the nodes involved), because there is more freedom in choosing independent sets: “long” links, which leads to a reduced spatial reuse.
The optimal tradeoffs are always obtained at the power

A small reduction in throughput usually leads to drastic

\[ P_{tx} = \text{constant} \]

\[ \rho \]

\[ \eta \]

\[ \text{Fig. 4.} \quad \text{Local tradeoff curves for a 25-node square grid with} \quad d_{\text{min}} = 8, \eta = 4, P_{tx} = 4.88 \text{ dBm}. \quad (a) \rho = 1 \quad \text{and} \quad (b) \rho = 0.75. \]

### C. Tradeoff Curves at Fixed \( P_{tx} \)

Fig. 4 shows several lifetime curves as a function of \( \lambda_{\text{req}} \) (or \( \rho \)) for different values of \( P_{tx} \) in the 25-node square grid network. One can easily identify the existence of turning point between the proportional tradeoff region (left to the turning point) and the non-proportional tradeoff region. Note that a turning point is at least \( \rho_0 \); it can be larger than \( \rho_0 \) because the proportional tradeoff region may extend beyond \( \rho_0 \) where \( C_f(\rho) \) is not complete yet. Finally, it can be seen that, for very low or high \( P_{tx} \), \( \rho_0 = 1 \) (i.e., there is no turning point) because \( C_f(\rho) \) is complete for all \( \rho \).

![Fig. 4. Local tradeoff curves for a 25-node square grid with \( d_{\text{min}} = 8 \) and \( \eta = 4 \).](image)

### D. Tradeoff Curves under Variable \( P_{tx} \)

In this section, we study the lifetime as a function of both \( \lambda_{\text{req}} \) and \( P_{tx} \). We also look at the effect of \( \eta \) and \( P_{tx} \). Since the whole spectrum of \( P_{tx} \in [0, \bar{P}] \) is considered, we take \( \lambda_{\text{req}} = \theta \lambda^*(\bar{P}, 0) \) where \( \theta \leq 1 \) and \( \lambda^*(\bar{P}, 0) \) is the optimal solution of the TO at the maximum transmit power \( \bar{P} \) (it is the largest achievable throughput for a given network). Note that we should distinguish between \( \rho \) and \( \theta \): while the former is the “local” tradeoff parameter (with respect to a particular

\[ P_{tx} = \bar{P} \], the latter is “global” in the range of all achievable throughput.

Figures 5 and 6 show the lifetime curves as a function of \( \theta \) and the points \( P_n \) where the optimal lifetime is achieved, given different values for \( P_{tx} \) and \( \eta \).

![Fig. 5. Lifetime curves as a function of \( \theta \) for a 25-node square grid with \( d_{\text{min}} = 8 \).](image)

![Fig. 6. The power levels \( P_n \) where the optimal lifetime for a given \( \theta \) is obtained for a 25-node square grid with \( d_{\text{min}} = 8 \).](image)

The following observations are made:

- A small reduction in throughput usually leads to drastic improvement in lifetime. Taking the curve for \( P_{tx} = 0 \text{ dBm} \) and \( \eta = 4 \) in Fig. 5 as an example, sacrificing 30\% of throughput yields a lifetime more than 5 times higher. This suggests that we should not always “squeeze” the maximum throughput from a network.
- The optimal tradeoffs are always obtained at the power levels \( P_n \) where new independent sets are created. This can be shown by comparing Fig. 6 with Fig. 2. This is actually implied by Propositions 3 and 6.
• A smaller $\eta$ improves the optimal lifetime for a certain $\theta$, i.e., it raises the tradeoff curves. This could have been expected, because smaller $\eta$ has the same effect as higher power: they both reduce the relative noise floor.
• The non-smoothness between the throughput and lifetime is more pronounced for larger $\eta$. This is also reasonable: if $\eta \to 0$, links are all in the same conflicting region, which is equivalent to the case that every node applies a $P_{tx}$ that covers the whole network. As shown by Fig. 4 (the curve with $P_{tx} = 17.12$ dBm), the tradeoff between the throughput and lifetime is smooth.
• Changing $P_{tx}$ slightly does not dramatically change the tradeoff curves, although it may change a few power levels where the tradeoffs are obtained.

VI. CONCLUSION

In this paper, we have built a general framework to analyze the tradeoffs between throughput and lifetime in multihop wireless networks. Our framework employs a general tradeoff utility and a realistic interference model. Through our optimization framework, we are able to characterize the optimal tradeoffs and the corresponding network configurations in terms of power control, routing, and scheduling for two utilities of practical interests. We also provide numerical results to illustrate the analysis. Our investigations have revealed several intriguing phenomena, such as the non-monotonicity between the lifetime and transmit power, and the non-smooth tradeoff between the throughput and lifetime. In our future work, we would like to study the problem under more general utilities.

VII. ACKNOWLEDGEMENTS

The authors would like to thank Aditya Karnik for the computational tools that he developed along with Aravind Iyer; these tools serve as a basis of the computations performed in this paper.

REFERENCES