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An Adaptive Filtering Algorithm for Direct-Conversion Receivers: Architecture and Performance Analysis

Yuanjin Zheng, Member, IEEE, Mingzheng Cao, Student Member, IEEE, Edmund K. H. Teo, and Hari K. Garg, Senior Member, IEEE

Abstract—An adaptive filtering algorithm is proposed in this paper to remove \( I/Q \) mismatch, dc offsets, flicker noise, and intersymbol interference (ISI) simultaneously in a direct-conversion receiver. \( I/Q \) mismatch is cancelled by a real valued adaptive mismatch canceller, and dc offsets are removed with one complex tap. In addition, flicker noise is modeled as a complex autoregressive (AR) random process so the system to be identified transforms to an ARX model. After estimating the coefficients in the model during the training period, the desired signal can be estimated using the decision feedback method. To accelerate the convergence of the algorithm and to reduce the estimation variance, an internal iterative algorithm is introduced. The convergence analysis of the proposed algorithm is also given, and the closed form of the minimum mean square error of the proposed algorithm is derived. Simulation results are provided to verify the superior performance of the proposed algorithm.

Index Terms—ARX model, convergence, dc offsets, direct-conversion receivers, flicker noise, \( I/Q \) mismatch, minimum mean-square error (MMSE).

I. INTRODUCTION

In the past, direct-conversion receivers (DCRs), illustrated in Fig. 1, are rarely used due to issues such as \( I/Q \) mismatch, dc offsets, even-order distortion, and flicker noise [1]–[4]. Among these issues, dc offsets and flicker noise are more serious and challenging [2], [5].

Several methods have been proposed to solve \( I/Q \) mismatch, dc offsets, and flicker noise [2]–[4], [7]. In [2], the dc offset is removed by employing an ac coupling which may corrupt the signal’s dc components due to its high corner frequency [1], [7]; in addition, the flicker noise is regarded as equivalent in its effect on intersymbol interference (ISI) and is mitigated only by employing a finite-impulse response (FIR) minimum mean-square-error linear equalizer (MMSE-LE); it is also difficult to design a suitable ac coupling because the exact 1/f roll-off frequency is unknown in practice.

In this paper, a novel adaptive method to estimate the transmitted signal corrupted by ISI, \( I/Q \) mismatch, dc offsets, and flicker noise is proposed. A real valued adaptive mismatch canceller is employed to cancel \( I/Q \) mismatch, and the varying dc offsets are removed by one complex tap [3], [8]. In addition, the flicker noise is modeled as a complex AR random process (Fig. 2), so the system transforms to an ARX model [9], [10]. By estimating the coefficients in the model during the training period, the desired signal can then be estimated by the decision feedback method. To accelerate the convergence of the algorithm and to reduce the estimation variance, an internal iterative algorithm is introduced [8]. On top of that, the convergence analysis of the proposed algorithm is given, and the closed form of the MMSE of the algorithm is derived as the lower bound of its performance.

The received signal model, architecture of the proposed adaptive filters, detailed algorithm, convergence analysis, and simulation results are illustrated and explained in Sections II–VI, respectively. Finally, the conclusions are drawn in Section VII.

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II. SIGNAL MODEL

Let \( \mathbf{t}(n), \mathbf{d}(n), \mathbf{u}(n) \) and \( \mathbf{f}(n) \) denote the transmitted signal, constant dc offset, varying dc offset, and flicker noise, respectively. In this paper, the bold lower case letters represent complex variables, the upper case letters represent vectors, and bold
upper case letters represent matrices. Here, \( n \) denotes the discrete time index. Then, the received signal \( \tilde{x}(n) \) corrupted by ISI, dc offsets, and flicker noise can be written as

\[
\tilde{x}(n) = \sum_{i=0}^{N_h-1} h_i t(n - i) + \tilde{d}(n) + \tilde{d}(n) + f(n).
\]

Here, \((h_0 = 1), h_i (i = 0, 1, \ldots, N_h - 1)\) are the coefficients of the channel discrete-time impulse response (assuming minimum phase channel in this paper) and \( N_h \) is the order of channel response. Since \( f(n) \) is a highly colored noise, it can be modeled as an AR random process as \([9], [11]\)

\[
f(n) = \sum_{i=1}^{N_a-1} (-a_i) f(n - i) + \eta(n).
\]

Here, \( N_a - 1 \) is the order. \( \{a_i\} (i = 1, 2, \ldots, N_a - 1)\) are the coefficients, and \( \eta(n) \) is the innovation. \( a_i \) is chosen to make the power spectral density (PSD) of the generated \( f(n) \) close to the PSD of the flicker noise. Denote \( A(z) \) as

\[
A(z) = 1 + \sum_{i=1}^{N_a-1} a_i z^{-i}.
\]

Then, the \( z \)-transform of \( f(n) \) can be represented as

\[
F(z) = \frac{V(z)}{A(z)}.
\]

where \( V(z) \) is the \( z \)-transform of \( \eta(n) \) in (2).

Next, we take the transceiver \( I/Q \) mismatch into consideration. Assume that \( \alpha \) and \( \beta \) denote the amplitude gains of the \( I \) and \( Q \) channels, respectively; \( \phi \) denotes the phase splitter mismatch and is split equally between the \( I \) and \( Q \) channels for symmetry; \( \tilde{m}(n) \) denotes the equivalent transceiver \( I/Q \) mismatch induced signal, where \( \tilde{m}_I(n) \) and \( \tilde{m}_Q(n) \) are its real and imaginary parts, respectively. Then, \( \tilde{m}_I(n) \) and \( \tilde{m}_Q(n) \) can be modeled by using a matrix notation according to \([4]\) as

\[
\begin{pmatrix}
\tilde{m}_I(n) \\
\tilde{m}_Q(n)
\end{pmatrix}
= \begin{pmatrix}
\alpha \cos(\phi/2) & \beta \sin(\phi/2) \\
\alpha \sin(\phi/2) & \beta \cos(\phi/2)
\end{pmatrix}
\begin{pmatrix}
\tilde{x}_I(n) \\
\tilde{x}_Q(n)
\end{pmatrix}
= \tilde{M} \begin{pmatrix}
\tilde{x}_I(n) \\
\tilde{x}_Q(n)
\end{pmatrix},
\]

Here, \( \tilde{M} \) represents the mismatch matrix. The mismatch model in (5) is applied to an AR model in (1) through signal \( \tilde{x}(n) \).

Since all of the delay effect due to the channel is modeled in (1), the mismatch delay (if any) is also modeled in (5).

When \( \alpha = \beta \) and \( \phi = 0^\circ \), (5) becomes

\[
x(n) = \alpha \tilde{x}_I(n) + \beta \tilde{x}_Q(n),
\]

and there is no \( I/Q \) mismatch. When \( \alpha \neq \beta \), amplitude mismatch occurs. When \( \phi \neq 0^\circ \), phase mismatch occurs. If mismatch exists, it can be mitigated by inverse modeling (e.g., refer to \([11, \text{Sec. 3.6.4}]\)). Considering that there are four elements in the matrix \( \tilde{M} \), a four-real-tap equalizer is employed to model the inverse of it. According to the model in (5), a four-real-tap equalizer can remove both the amplitude mismatch and the phase mismatch simultaneously.

Furthermore, considering the effect of additive white Gaussian noise (AWGN) \( \tilde{n}(n) \), the digitalized baseband signal after the ADC can be represented as

\[
x(n) = x_I(n) + j x_Q(n) = \tilde{m}(n) + \tilde{n}(n).
\]

Note that this paper deals with the baseband equivalent model. It can be verified that the noise generated in the RF front-end can be modeled equivalently in (6), which is a digital baseband equivalent model \([13]\).

III. ARCHITECTURE OF THE ADAPTIVE FILTER

Based on the system model in Section II, an integrated adaptive filter and equalizer is proposed in Fig. 3. It is composed of, in sequence, a mismatch canceller, a dc offsets canceller, a complex feed-forward equalizer (CFFE), and a complex decision-feedback equalizer (CDFE). In this sequence, the mismatch matrix will also be applied to the dc offset and flicker noise. However, the model is still valid, as typically the dc offset is generated due to the self-mixing of the mixer and transistor mismatch in either mixer in the \( I \) or \( Q \) channel \([1]\). Thus, the equivalent dc offset in the \( I \) and \( Q \) channels can be different, and this can be modeled by the \( I/Q \) mismatch model. Similarly, flicker noise is mainly generated due to the usage of CMOS transistors in building the RF front-end (LNA and mixer). Since the transistors built into the \( I \) channel have some mismatch with those in the \( Q \) channel during fabrication, the equivalent total flicker noise generated in the \( I \) channel is different from that in the \( Q \) channel, and thus the \( I/Q \) mismatch model is also applied to flicker noise to account for this effect.

In the mismatch canceller, which is a four-real-tap equalizer, the taps \( m_1 \) and \( m_4 \) are used to cancel the amplitude imbalance while the taps \( m_2 \) and \( m_3 \) are used to cancel the component of \( \tilde{x}_Q(n) \) in \( x_I(n) \) and the component of \( \tilde{x}_I(n) \) in \( x_Q(n) \), respectively. The output of the mismatch canceller \( y(n) \) is treated as a complex valued signal with the signal in the \( I \) channel as its real part and the signal in the \( Q \) channel as its imaginary part. At
\( y(n) \), a complex valued tap cancels off the dc offsets [3]. After which, CFFE mitigates ISI followed by CDFE, which mitigates flicker noise and provides data estimation.

IV. ALGORITHM

Using the model in Section II, the output of the mismatch canceller, as in Fig. 3, is

\[
\begin{align*}
\hat{y}(n) &= y(n) + jy_2(n) \\
y_1(n) &= m_1(n) \times x_1(n) + m_2(n) \times x_2(n) \\
y_2(n) &= m_3(n) \times x_1(n) + m_4(n) \times x_2(n).
\end{align*}
\]

(7) (8) (9)

Here, \( \{m_i\} \) \( (i = 1, \ldots, 4) \) denotes the taps of the mismatch canceller, \( \hat{w}^*(n) \) denotes the tap of the dc offsets canceller, and then the output \( \hat{y}(n) \) can be represented as

\[
\hat{y}(n) = y(n) + \hat{w}^*(n).
\]

(10)

The superscript "\*" denotes complex conjugation, and all of the taps of the filters are given in their complex conjugation forms in Fig. 3. During the derivation of the algorithm, AWGN is dropped. Despite this, the simulated results (with AWGN) using the derived algorithm are still very good. In addition, in practical situations, the input SNR for receiver is normally high (around 20 dB), so neglecting AWGN here is reasonable. Referring to Fig. 3, \( \hat{y}(n) \) is \( \tilde{x}(n) \) with mismatch and dc offsets cancelled.

Drop \( \tilde{x}(n), \tilde{\hat{y}}(n) \) and replace \( x(n) \) in (1) with \( y(n) \). Take the \( z \) transform and, according to (2)-(4), we obtain

\[
A(z)Y(z) = B(z)T(z) + V(z).
\]

(11)

Here, \( B(z) = A(z)H(z) \). \( Y(z), T(z) \) and \( H(z) \) are the \( z \)-transforms of \( \hat{y}(n), \tilde{x}(n) \) and \( \tilde{\hat{y}}(n) \), respectively. \( B(z) \) can also be represented as

\[
B(z) = 1 - \sum_{i=1}^{N_a-1} b_iz^{-i} (N_b = N_a + N_b - 1). \]

(12)

In this paper, we assume that the channel is minimum phase. Since (11) is an ARX model, given \( \hat{y}(n) \) and \( \tilde{x}(n) \), the coefficients of \( A(z) \) and \( B(z) \) can be estimated as \( \{\hat{a}_i\} (i = 1, \ldots, N_a - 1) \) and \( \{\hat{b}_i\} (i = 1, \ldots, N_b - 1) \), respectively, during training period. Then, the desired signal can be estimated during decision period by \[8, \[10\]

\[
\hat{t}(n) = \left( \hat{y}(n) + \sum_{i=1}^{N_a-1} \hat{a}_i\hat{y}(n-i) \right) + \sum_{i=1}^{N_b-1} \hat{b}_i\tilde{t}(n-i).
\]

(13)

Here, \( \hat{t}(n) \) denotes the estimated transmitted sequence \( \tilde{t}(n) \). The output \( z(n) \) and the filtering error signal \( e(n) \) during training period are

\[
z(n) = \sum_{j=0}^{k_1-1} \tilde{w}_j(n)y(n-j) + \sum_{k=1}^{k_2-1} \tilde{w}_k(n)t(n-k) \]

(14)

\[
e(n) = \hat{t}(n) - z(n)
\]

(15)

where \( k_1 \) and \( k_2 \) denote the orders of the CFFE and CDFE taps, respectively. \( \tilde{w}_j(n) (j = 0, 1, \ldots, k_1-1) \) and \( \tilde{w}_k(n) (k = 0, 1, \ldots, k_2-1) \) and \( \hat{t}(n) \) denote the CFFE tap values, CDFE tap values, and the training signal, respectively. The coefficients of \( A(z) \) and \( B(z) \) in (11) are estimated by the taps \( \{\tilde{w}_j\} \) and \( \{\tilde{w}_k\} \) in (14), respectively, by using the least-mean-square (LMS) algorithm [11, [12]. Without loss of generality, assume \( k_1 = N_a \) and \( k_2 = N_b \). In this case, the estimated coefficients \( \{\hat{a}_i\} \) and \( \{\hat{b}_i\} \) will be obtained by the adaptations of \( \{\tilde{w}_j\} \) and \( \{\tilde{w}_k\} \), respectively.

Denote \( \mu_m, \mu_d, \mu_f \) and \( \mu_s \) as the step sizes for adjusting \( \{m_i\} (i = 1, \ldots, 4) \), \( \{\tilde{w}_j\} (j = 0, 1, \ldots, k_1-1) \) and \( \{\tilde{w}_k\} (k = 2, \ldots, k_2) \), respectively. According to the LMS algorithm, the closed forms of the taps adaptations are given as

\[
m_i(n+1) = m_i(n) - \mu_m \times \nabla m_i(n), \quad i = 1, \ldots, 4
\]

(16)

\[
\tilde{w}_j^*(n+1) = \tilde{w}_j^*(n) + 2\mu_d \sum_{j=0}^{k_1-1} \tilde{w}_j(n)e(n)
\]

(17)

\[
\tilde{w}_j^*(n+1) = \tilde{w}_j^*(n) + 2\mu_f \tilde{w}_j^*(n-j)e(n),
\]

(18)

\[
\tilde{w}_k^*(n+1) = \tilde{w}_k^*(n) + 2\mu_s \tilde{w}_k^*(n-k)e(n),
\]

(19)
where \( \{ \nabla m_k \} \) in (16) can be derived by taking partial derivatives of \( |e(n)|^2 \) with respect to \( m_k \).

It is obvious that the adaptations of \( \{ m_k \}, \{ \theta^* \}, \{ w^*_j \} \) and \( \{ \hat{w}^*_k \} \) influence mutually. Hence, we propose an internal iterative algorithm below to accelerate the convergence of the algorithm and to reduce the estimation variance [8].

Let \( \tilde{N} \) denote the total number of internal iterations. \( m_i^{(l)}(n), \bar{w}^{(l)}(n), \tilde{W}^{(l)}(n), w_i(n), \tilde{W}^{(l)}(n) \) denote the updated values of \( m_i(n), \bar{w}^{(l)}(n), \tilde{W}^{(l)}(n), w_i(n) \) and \( e(n) \), respectively, after \( l = 0, 1, \ldots, \tilde{N} - 1 \) internal iterations at time \( n \). To reduce the complexity of the algorithm, \( m_i(n - j) \) and \( \bar{w}^{(l)}(n - j) \) \((j = 1, 2, \ldots, k_1 - 1) \) are only updated and do not take part in the internal iterations at time \( n \). Simulation results show that whether or not \( m_i(n - j) \) and \( \bar{w}^{(l)}(n - j) \) \((j = 1, 2, \ldots, k_1 - 1) \) take part in the internal iterations makes little difference to the performance.

Hence, the internal iterations for \( l = 0, 1, \ldots, \tilde{N} - 1 \) are as follows:

\[
m_i^{(l+1)}(n) = m_i^{(l)}(n) - \mu_m \times H \left( X(n), e^{(l)}(n), \tilde{W}^{(l)}(n) \right)
\]

\[
\bar{w}^{(l+1)}(n) - \mu_d \times I \left( \tilde{W}^{(l)}(n), e^{(l)}(n) \right)
\]

\[
\tilde{W}^{(l+1)}(n) = \tilde{W}^{(l)}(n) - \mu_f \times J \left( X(n), m_i^{(l)}(n), \bar{w}^{(l)}(n), e^{(l)}(n) \right)
\]

\[
\tilde{W}^{(l+1)}(n) = \tilde{W}^{(l)}(n) - \mu_d \times K \left( T^{(l)}(n), e^{(l)}(n) \right)
\]

\[
e^{(l+1)}(n) = G \left( X(n), m_i^{(l+1)}(n), \bar{w}^{(l+1)}(n), \tilde{W}^{(l+1)}(n) \right)
\]

where \( G(\bullet), H(\bullet), I(\bullet), J(\bullet) \) and \( K(\bullet) \) above are general symbolic functions.

At time \( n + 1 \), the initial tap values are the latest updated tap values at time \( n \). The next internal iteration goes so forth. After all of the taps converged to their optimum values, the training sequence \( f(n) \) is replaced by the estimated sequence \( \hat{f}(n) \).

V. PERFORMANCE ANALYSIS

A. Convergence Analysis

Denote

\[
A = \begin{bmatrix} a_0 & a_1 & \cdots & a_{N_k - 1} \end{bmatrix}^H(a_0 = 1) \quad (25)
\]

\[
B' = \begin{bmatrix} b_1 & b_2 & \cdots & b_{N_k - 1} \end{bmatrix}^H \quad (26)
\]

\[
Y'(n) = \begin{bmatrix} y^*_1(n) & y^*_1(n - 1) & \cdots & y^*_1(n - N_a + 1) \end{bmatrix}^H \quad (27)
\]

\[
\begin{bmatrix} R_Y & R_T \\
R_T & R_R \end{bmatrix} = E \left[ Y'(n)Y'^H(n) \right] \quad (28)
\]

During training period, according to (14), \( \hat{z}(n) \) can be written as

\[
\hat{z}(n) = \tilde{W}^H(n)Y'(n) + \tilde{W}^H(n)T'(n) \quad (29)
\]

It is obvious that the algorithm will converge only when the taps of all of the equalizers converge and the convergence of CFFE and CDFE plays a dominant role. Denote \( \lambda_{T_{	ext{max}}}^{\text{CFFE}} \) and \( \lambda_{T_{	ext{max}}}^{\text{CDFE}} \) as the maximum eigenvalues of \( R_Y \) and \( R_T \), respectively. When \( \{ m_i \} \) \((i = 1, 4) \) and \( \bar{w}^* \) are kept fixed, \( \gamma(n) \) can be regarded as a channel response of input \( \hat{x}(n) \) and \( 1 \). Therefore, when only considering CFFE and CDFE, they will converge when \( \mu_f \) and \( \mu_d \) meet the following requirements (see [11, Sec. 6.2]):

\[
0 < \mu_f < 1/\lambda_{T_{	ext{max}}}^{\text{CFFE}} \quad (30)
\]

\[
0 < \mu_d < 1/\lambda_{T_{	ext{max}}}^{\text{CDFE}} \quad (31)
\]

Eqs. (30) and (31) only guarantee the convergence of CFFE and CDFE. When \( \tilde{W}^H(n) \) and \( \tilde{W}^H(n) \) are adapted to two sets of values, e.g., \( \tilde{W}_c^H \) and \( \tilde{W}_c^H \) (not necessarily the optimum convergence values), the algorithm will converge when \( \{ m_i \} \) \((i = 1, 4) \) and \( \bar{w}^* \) converge as well. In the following convergence analysis of \( \{ m_i \} \) \((i = 1, 4) \) and \( \bar{w}^* \), \( \tilde{W}^H(n) \) and \( \tilde{W}^H(n) \) are regarded as constant values \( \tilde{W}_c^H \) and \( \tilde{W}_c^H \).

Denote

\[
X_f(n) = [x_f(n) \ x_f(n - 1) \ \cdots \ x_f(n - N_a + 1)]^T \quad (32)
\]

\[
X_Q(n) = [x_Q(n) \ x_Q(n - 1) \ \cdots x_Q(n - N_a + 1)]^T \quad (33)
\]

\[
P = \begin{bmatrix} E[Y'(n)e(n)] & E[T'(n)e(n)] \end{bmatrix} \quad (34)
\]

\[
R = E \left[ \begin{bmatrix} Y'(n) \ T'(n) \end{bmatrix} \begin{bmatrix} Y'^H(n) & T'^H(n) \end{bmatrix} \right] \quad (35)
\]

Denote \( \xi \) as the mean square error. Then

\[
\xi = E[|e(n)|^2] = E[|e(n)|^2] - \left( \begin{bmatrix} \tilde{W}_c^H \ \tilde{W}_c^H \end{bmatrix} \right)^H X_f(n)X_f(n) + \left( \begin{bmatrix} \tilde{W}_c^H \ \tilde{W}_c^H \end{bmatrix} \right)^H R \left( \begin{bmatrix} \tilde{W}_c^H \ \tilde{W}_c^H \end{bmatrix} \right) \quad (36)
\]

Denote

\[
\begin{bmatrix} R_{X_f} \\
R_{X_Q} \quad \underline{R} \end{bmatrix} = E \left[ X_f(n)X_f(n)^T \right] \quad (37)
\]

According to the LMS algorithm

\[
m_1(n + 1) = m_1(n) - \mu_m \frac{\partial E[|e(n)|^2]}{\partial m_1} \quad (38)
\]

Denote \( \nu_{m_1}(n) = m_1(n) - m_{10} \), where the subscript "o" represents the optimum value. By taking the expectation of (38) and setting \( \partial \xi/\partial m_1 \) to zero, it can be obtained that

\[
E[\nu_{m_1}(n + 1)] = (1 - 2\mu_m \tilde{W}_c^H R_{X_f} \tilde{W}_c)E[\nu_{m_1}(n)] \quad (39)
\]
Therefore, the convergence condition for \( m_3 \) is
\[
0 < \mu_m < \left( \hat{W}_c^H \cdot R_{XQ} \cdot \hat{W}_c \right)^{-1}. \tag{40}
\]

By similar derivations, it can be obtained that (40) is also the convergence condition for \( m_3 \). The convergence condition for \( m_2 \) and \( m_4 \) is
\[
0 < \mu_m < \left( \hat{W}_c^H \cdot R_{XI} \cdot \hat{W}_c \right)^{-1}. \tag{41}
\]

Combining (40) and (41), the I/Q mismatch canceller \((m_i)\) will converge when
\[
0 < \mu_m \text{min} \left( \left( \hat{W}_c^H \cdot R_{XQ} \cdot \hat{W}_c \right)^{-1}, \left( \hat{W}_c^H \cdot R_{XI} \cdot \hat{W}_c \right)^{-1} \right). \tag{42}
\]

Denote
\[
\hat{\hat{W}}_c = \left[ \hat{\mu}_{c,0}^*, \hat{\mu}_{c,1}^*, \ldots, \hat{\mu}_{c,N_c-1}^* \right]^T, \tag{43}
\]

\[
\hat{W}_c = \left[ \hat{\mu}_{c,0}, \hat{\mu}_{c,1}, \ldots, \hat{\mu}_{c,N_c-1} \right]^T. \tag{44}
\]

It can be obtained that
\[
E \left[ \psi_{n}(n+1) \right] = E \left[ \psi_{n}(n) \right] - 2 \mu_d [ \hat{W}_c^H \cdot I_{N_c \times N_c} \cdot \hat{W}_c ] E \left[ \psi_{n}(n) \right] \left\{ 1 - 2 \mu_d \sum_{i=0}^{N_c-1} \sum_{i=0}^{N_c-1} [ \hat{\mu}_{c,i}^* ] \cdot \hat{\mu}_{c,i} \right\}. \tag{45}
\]

Here, \( \psi_{n}(n) = \bar{u}_{n} - \hat{\psi}_{n} \), \( \bar{u}_{n} \) is obtained by setting \((\partial \xi / \partial \mu)^*\) to zero. Then, the convergence condition of the dc offset canceller \((\psi_{n})\) is
\[
0 < \mu_d \left[ \sum_{i=0}^{N_c-1} \sum_{i=0}^{N_c-1} [ \hat{\mu}_{c,i}^* ] \cdot \hat{\mu}_{c,i} \right]. \tag{46}
\]

Because we can always find \( \mu_f, \mu_t, \mu_m \) and \( \mu_d \) to satisfy (30), (31), (42), and (46) simultaneously, the mean convergence of the algorithm is guaranteed.

B. MMSE Analysis

According to (11), \( \hat{t}(n) \) in time domain can be written as
\[
\hat{t}(n) = [A^H \cdot B'^H] \cdot Y'(n) - \psi(n). \tag{47}
\]

Here, \( Y'(n) \) represents the received signal vector with mismatch and dc offsets fully cancelled, i.e., \( \{ m_c \} \) and \( \bar{u}_o \) are at their optimum values \( \{ m_o \} \) and \( \bar{u}_o \), respectively. Let \( \hat{\hat{W}}_c \) and \( \hat{W}_c \) denote the estimated vectors of \( A \) and \( B' \), respectively. \( \Delta \hat{W}_c \) and \( \Delta \hat{W}_c \) represent the weight-error vectors, and \( \Delta Y'(n) \) represents the error vector caused by the estimation error of \( \{ m_c \} \) and \( \bar{u}_o \). Then, after all of the taps of the mismatch canceller, dc canceller, CFFE, and CDFE are close to their optimum, according to (14), the output \( \hat{z}(n) \) can be represented as
\[
\hat{z}(n) = \hat{W}_c (n) Y'(n) + \hat{\hat{W}}_c (n) T'(n)
\]

\[
= [A^H \cdot B'^H] \cdot Y'(n) + \Delta \hat{W}_c^H \cdot \Delta Y'(n), \tag{48}
\]

From (15), (46), and (48), it can be obtained that
\[
e(n) = - \Delta \hat{W}_c^H (n) - \Delta \hat{\hat{W}}_c^H (n) \cdot Y'(n) \cdot T'(n)
\]

\[
- A^H \cdot \Delta Y'(n) = - \Delta \hat{W}_c^H (n) \cdot \Delta Y'(n) - u(n). \tag{49}
\]

Recall that \( \xi = E ||e(n)||^2 \). Assume that \( u(n) \) is white, zero mean with covariance \( \sigma_u^2 \), \( \sigma_u(n), \Delta \hat{W}_c (n), \Delta Y'(n), Y'(n), \) and \( \Delta \hat{W}_c (n) \) are mutually independent; \( \hat{W}_c (n), \Delta Y'(n) \) and \( \Delta \hat{W}_c (n) \) are zero mean; and \( \Delta \hat{W}_c (n) \) and \( \Delta \hat{W}_c (n) \) are independent of \( Y'(n) \) and \( T'(n) \), respectively. Then
\[
E \left[ \psi(n) \right] \left( \Delta \hat{W}_c (n) \cdot \Delta Y'(n) \right) = 0 \tag{50}
\]

\[
E \left[ \Delta \hat{W}_c (n) \cdot \Delta Y'(n) \right] \cdot \psi(n) = 0. \tag{51}
\]

Denote
\[
\begin{bmatrix}
R_{\Delta Y'}(n)
\nR_{Y'}(n)
\nK_{\hat{W}}(n)
\nK_{\hat{W}}(n)
\end{bmatrix}
= E \left[ \begin{bmatrix}
\Delta Y'(n) \cdot Y'(n)
\n\Delta Y'(n) \cdot T'(n)
\n\Delta \hat{W}_c (n) \cdot \Delta W_c (n)
\n\Delta \hat{W}_c (n) \cdot \Delta W_c (n)
\end{bmatrix}
\right]. \tag{52}
\]

Denote \( \xi_1(n) \) as the error caused by the weight-error vectors \( \Delta \hat{W}_c (n) \) and \( \Delta \hat{W}_c (n) \), \( \xi_2(n) \) and \( \xi_3(n) \) as the errors caused by the \( A^H \cdot \Delta Y'(n) \) and \( \Delta \hat{W}_c (n) \cdot \Delta Y'(n) \), respectively. Then, according to the derivations in [11, Sec. 6.3.1], we have
\[
\xi_1(n) = \text{tr} \left[ \begin{bmatrix}
K_{\hat{W}}(n) & K_{\hat{W}}(n)
\end{bmatrix} \cdot \begin{bmatrix}
R_{Y'} & R_{T'}
\end{bmatrix} \right]. \tag{53}
\]

\[
\xi_2(n) = \text{tr} \left[ E \left[ A^H \cdot \Delta Y'(n) \cdot Y'(n) \right] \right]. \tag{54}
\]

\[
\xi_3(n) = \text{tr} \left[ K_{\hat{W}}(n) \cdot R_{\Delta Y'}(n) \right]. \tag{55}
\]

\[
\xi = \{ \xi_1(n) \}_{n=\infty} \tag{56}
\]
According to the misadjustment analysis of the LMS algorithm in [11, Sec. 6.3.3], it can be approximated that

\[ \xi_1(\infty) = \sigma_v^2 \left[ \mu \operatorname{tr}(\mathbf{R}_{\hat{z}z}) + \mu_d \operatorname{tr}(\mathbf{R}_z^\top) \right] \quad (57) \]

\[ \operatorname{tr}(\mathbf{R}_{\hat{z}z}) = \begin{bmatrix} m_{1o}^2 + m_{2o}^2 & m_{1o}^2 + m_{3o}^2 & 2(m_{1o}m_{2o} + m_{2o}m_{3o} + m_{3o}m_{4o}) \\ m_{1o}^2 + m_{3o}^2 & m_{2o}^2 + m_{4o}^2 & 2(m_{1o}m_{2o} + m_{2o}m_{3o} + m_{3o}m_{4o}) \end{bmatrix} \]

\[ \cdot \left[ \begin{bmatrix} \mathbf{R}_{\hat{x}x} \\ \mathbf{R}_{\hat{x}Q} \\ \mathbf{R}_{QQ} \end{bmatrix} \right] + N_0 C \quad (58) \]

where \( C \) is a constant that is determined by the optimum values of \( \{m_i\} (i = 1, \ldots, 4) \) and \( \varphi \).

Similarly, it can be obtained that

\[ \xi_2(\infty) \geq \sigma_v^2 \left[ \mu_m |a_{\min}|^2 \operatorname{tr}(\mathbf{R}_{\hat{x}z} + \mathbf{R}_{\hat{x}Q}) + \mu_d \sum_{i=0}^{N_z-1} |a_i|^2 \right] \quad (59) \]

Here, \( a_{\min} \) denotes the minimum element of \( A \) in (25).

The weight-error vector \( \Delta W(n) \) is much smaller than the true vector \( A \) when the taps \( \mathbf{w}_o(n) \) are near convergence. Assuming that the value of \( \Delta W(n) \) is 10% that of \( A \), then \( \xi_3(n) \) is only 1% of \( \xi_2(n) \). Therefore, \( \xi_3(n) \) can be ignored in (56). Thus, when \( n \to \infty \), (56) can be written as

\[ \xi(\infty) \approx \xi_1(\infty) + \xi_2(\infty) + \sigma_v^2. \quad (60) \]

When there is no mismatch canceller, dc offsets canceller, and CDFE, \( m_{1o} = m_{3o} = 1, m_{2o} = m_{4o} = 0, \varphi = 0, 1 \mu = 1 \mu_d = 0 \), then (60) becomes

\[ \xi(\infty) = \sigma_v^2 \mu \operatorname{tr}(\mathbf{R}_{\hat{z}z}) + \sigma_v^2. \quad (61) \]

If there is no I/Q mismatch, dc offsets, or flicker noise, then \( \{a_i\} = 0 (i = 1, 2, \ldots, N_z - 1) \). The received signal to be processed by the adaptive filter becomes the transmitted signal that is only corrupted by ISI and a white noise with variance of \( \sigma_v^2 \). The autocorrelation matrix of the transmitted signal corrupted by ISI in this case becomes \( \mathbf{R}_{\hat{z}z} \). Then, (61) becomes the MMSE of a standard LMS algorithm.

VI. SIMULATION RESULTS

In Fig. 4, five curves are illustrated to show the efficiency of the mismatch canceller when the dc offset canceller is absent. In the simulations, 16 quaternary amplitude modulation (QAM) signals are passed through a given channel and then corrupted by I/Q mismatch and AWGN. In condition 1, \( \alpha = 1.5, \beta = 0.5, \phi = 10^\circ, \hat{N} = 6 \) and, in condition 2, \( \alpha = 1.5, \beta = 0.5, \phi = 20^\circ, \hat{N} = 6 \) without the mismatch canceller, the bit error rates (BERs) of the two curves are around \( 10^{-1} \), but with the canceller they decreased dramatically.

The method to cancel dc offsets by one tap is discussed in many papers [3], [8]. Fig. 5 is obtained when quaternary phase-shift keying (QPSK) signals are passed through a given channel. When only flicker noise and ISI exist, and only CFFE is employed, the BER is around \( 10^{-1} \). However, after employing the CDFE, the BER reaches \( 10^{-1} \) at \( \text{SNR} = 20 \text{ dB} \). When ISI, flicker noise, mismatch (\( \alpha = 1.2, \beta = 0.8, \phi = 10^\circ \)), and varying dc offsets with the similar energy of the transmitted signal exist simultaneously and \( \hat{N} = 6 \), the BER reaches \( 10^{-1} \) at \( \text{SNR} = 25 \text{ dB} \). Fig. 6 is obtained when the QPSK signal is passed through a given channel and corrupted by I/Q mismatch, dc offsets, flicker noise, and AWGN as modeled in Section II. In the simulations, the internal iterative number \( \hat{N} = 6 \). The learning curve of the algorithm is obtained by an ensemble average of the sequence \( [e(n)]^2 \) over 100 independent runs. As shown in Fig. 6, the learning curve with internal iterations converges within 1000 iterations, which is much faster than that without internal iterations. The MSE of the algorithm with internal iterations is also smaller than that of the algorithm without internal iterations. Generally, the proposed algorithm is applied to a slow fading channel. For a fast fading channel, the proposed algorithm should improve convergence speed by incorporating some dedicated speeding algorithm.

To verify the proposed algorithm, comprehensive simulation results are given when binary phase-shift keying (BPSK), QPSK, and 16-QAM signals are passed through a given
channel and are corrupted by all of the distortions such as $I/Q$ mismatch, dc offsets, flicker noise, and AWGN, respectively. As shown in Figs. 7–9, the lower bound is obtained when the transmitted signal is only corrupted by ISI and AWGN. In the simulations, $\hat{N} = 6$, and SDR $= 10$ dB where SDR represents the power ratio of the desired signal to the dc offset. The flicker noise is generated as modeled in Section II, but its power is changed accordingly. In condition 1, $\alpha = 1.05, \beta = 0.95, \phi = 5^\circ$, and SFR $= 20$ dB. Here, SFR represents the power ratio of the desired signal to the flicker noise. In condition 2, $\alpha = 1.05, \beta = 0.95, \phi = 5^\circ$, and SFR $= 10$ dB. In condition 3, $\alpha = 1.1, \beta = 0.9, \phi = 10^\circ$, and SFR $= 10$ dB. As shown in Figs. 7–9, the BER curves of the three conditions are close to one another and are robust to mismatch and flicker noise.

Fig. 10 shows the learning curve of MSE and the derived MMSE. It is obtained when a QPSK signal is passed through a given channel and corrupted by $I/Q$ mismatch, dc offsets, and flicker noise. The learning curve of the algorithm is obtained by an ensemble average of the sequence $|e(n)|^2$ over 100 independent runs. As shown in Fig. 10, the derived MMSE provides a close lower bound for the steady-state MSE.
VII. CONCLUSION

In this paper, an adaptive algorithm has been proposed to cancel $I/Q$ mismatch, dc offsets, flicker noise, and ISI simultaneously in both the $I$ and $Q$ channels in DCRs by using adaptive filters. To accelerate the convergence of the algorithm and to reduce the estimation variance, an internal iterative algorithm is proposed. In addition, the convergence analysis of the proposed algorithm is given, and the closed form of the MMSE of the proposed algorithm is derived. Simulation results are provided to verify the superior performance of the proposed algorithm.

REFERENCES


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