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<td><strong>Citation</strong></td>
<td>Ge, T., &amp; Chang, J. S. (2008). Modeling and technique to improve PSRR and PS-IMD in analog PWM class-D amplifiers. IEEE Transactions on Circuits and Systems Part 2 Express Briefs, 55(6), 512-516.</td>
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<td><strong>Date</strong></td>
<td>2008</td>
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<td><a href="http://hdl.handle.net/10220/6307">http://hdl.handle.net/10220/6307</a></td>
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Modeling and Technique to Improve PSRR and PS-IMD in Analog PWM Class-D Amplifiers

Tong Ge and Joseph S. Chang

Abstract—Although power-supply noise, qualified by power-supply rejection ratio (PSRR), has been recognized as a potential drawback of Class-D amplifiers (CDAs) compared to linear amplifiers, the mechanisms of PSRR for CDAs are not well established. It is also not well recognized that the power-supply noise can intermodulate with the input signal, manifesting into power-supply induced intermodulation distortion (PS-IMD), and that the PS-IMD can be significantly larger than the output distortion component at supply noise frequency. Furthermore, techniques to improve PSRR and PS-IMD are largely unreported in literature. In this brief, by means of a linear model, the PSRR and PS-IMD of single-feedback and double-feedback CDAs are analyzed and analytical expressions derived. A simple method is proposed to improve PSRR and PS-IMD with very low hardware overheads, and the improvement is \(\sim 26\) dB. Analytical expressions for PSRR and PS-IMD of the improved design are derived and the pertinent parameters thereof are investigated. The model and analyses provide practical insight to the mechanisms of PSRR and PS-IMD, and how various parameters may be varied to meet a given specification.

Index Terms—Class-D amplifiers (CDAs), power-supply induced intermodulation distortion (PS-IMD), power-supply rejection ratio (PSRR).

I. INTRODUCTION

CLASS-D audio power amplifiers (CDAs) are increasingly ubiquitous largely because of their significantly higher power-efficiency attribute [1] compared to their linear counterparts. The modulation techniques employed in Analog CDAs include: 1) pulsewidth modulation (PWM) [2], [3]; 2) sigma-delta modulation [4] and 3) bang–bang control modulation [5]–[7]. Of these modulations, PWM is arguably most prevalent largely because of its hardware simplicity and low quiescent current (due to its lower switching frequency, \(f_{sw}\)). The analog PWM CDA is the CDA of interest in this brief.

Noise in the power-supply, qualified by power-supply rejection ratio (PSRR), is well recognized [8]–[10] to be a problem in some practical CDA realizations and may otherwise require expensive hardware solutions such as highly stabilized low-noise supply rails. Interestingly, it is not well known that the supply noise may intermodulate with the input signal and manifest into power-supply induced intermodulation distortion (PS-IMD), and that, in some instances, the PS-IMD can be significantly larger than the output distortion component at supply noise frequency.

The PSRR and PS-IMD are two (potential) drawbacks of CDAs compared to their linear counterparts. In spite of this, the mechanisms of PSRR and PS-IMD are not well understood with little, if any, reported literature. Current design techniques to improve PSRR and PS-IMD are largely empirical.

In this brief, we employ the linear model method applicable to switching power supplies [11], etc, for the analysis of PSRR and PS-IMD of CDAs embodying a feedback structure. We derive mathematical expressions for PSRR and PS-IMD of the prevalent single-feedback CDA and for the higher fidelity double-feedback CDA. These expressions are particularly significant as they depict the primary parameters that affect PSRR and PS-IMD, and provide insight to a CDA designer on how various parameters may be varied and/or compromised to meet a given set of specifications.

We also propose a very simple design technique with very low hardware overheads to improve PSRR and PS-IMD and demonstrate its efficacy by its application to the single-feedback CDA, and we denote this CDA as the “improved single-feedback CDA.” We further derive the expressions for PSRR and PS-IMD of the improved single-feedback CDA. A comparison of the single-feedback CDA, double-feedback CDA and improved single-feedback CDA shows that the improved single-feedback CDA exhibits an \(\sim 26\) dB improvement for both the PSRR and PS-IMD over the single-feedback CDA. Compared to the improved single-feedback CDA, the double-feedback CDA, however, does exhibit \(\sim 20\) dB better PSRR and PS-IMD but only in the low frequencies, and at the expense of complex hardware and larger IC footprint (more I/O pins and external components). The derived analytical expressions are verified by HSPICE simulations and by measurements on a prototype CDA IC and on other CDAs realized using discrete components.

This brief is organized as follows. In Section II, the PSRR and PS-IMD of the single-feedback CDA and double-feedback CDA are analyzed. In Section III, the proposed design technique to improve PSRR and PS-IMD of CDAs is presented. In Section IV, the derived analytical equations are verified with HSPICE simulations and on the basis of hardware measurements. Finally, conclusions are drawn in Section V.

II. PSRR AND PS-IMD OF SINGLE-FEEDBACK AND DOUBLE-FEEDBACK CDAS

Fig. 1(a) depicts the prevalent single-feedback CDA. By straightforward linear control theory, the single-feedback CDA is modeled in Fig. 1(b) where \(G_1 = R_{fb1}/(R_1 + R_{fb1})\) is the transfer function from the input to the integrator inverting input, \(G_{int}\) the integrator gain, \(G_{PWM}\) the PWM stage gain, \(H_1\) the...
feedback factor, and \( N_D \) the added noise due to the supply noise, \( V_n(2) \). By inspection, the noise transfer function is

\[
\frac{V_{ai}}{N_D} = \left(1 + |G_{int}| G_{PWM} H_1 \right)^{-1}
\]  

(1)

where

\[
G_{int} = \frac{-A_{opA}}{1 + (2R_2C) + (A_{opA} + 1)(R_1/R_f)R_2C^2s^2}
\]

\[
G_{PWM} = \frac{V_{DD}}{V_C}
\]

\[
H_1 = \frac{R_1}{R_1 + R_f}
\]

\[
V_n = N \cos(2\pi f_n t)
\]

(2)

where \( A_{opA} \) is the op-amp open-loop gain, \( V_{DD} \) the supply voltage, \( V_C \) the amplitude of the carrier, \( N \) the amplitude of the supply noise and \( f_n \) the supply noise frequency.

A. PSRR and PS-IMD of the Single-Feedback CDA

To derive the expressions for PSRR and PS-IMD of the single-feedback CDA, the expression for \( N_D \) is first derived

\[
N_D = 0.5 N \cos(2\pi f_n t) + 0.25 MN \cos(2\pi f_n t + 2\pi f_n t)
\]

where \( M \) is the modulation index of the input signal and \( f_n \) the input frequency. The first term in (3) is the output distortion at \( f_n \) due to the supply noise, and the second term is the intermodulation distortion components at \( f_n \). Hence, from (1) and (3), the expressions for PSRR and PS-IMD for the single-feedback CDA are as shown in (4a)–(4b) at the bottom of the page, where \( LG_{1FB} \) is the loop gain of the single-feedback CDA

\[
LG_{1FB} = |G_{int}| |G_{PWM}| H_1.
\]

(4c)

The PSRR\(_{1FB} \) and PS-IMD\(_{1FB} \) are interpreted as follows. PSRR\(_{1FB} \) is independent of the input signal, whilst PS-IMD\(_{1FB} \) increases as the magnitude of the input signal increases. Both the PSRR and PS-IMD are largely determined by \( LG_{1FB} \). Specifically, the higher the \( LG_{1FB} \), the better is the PSRR\(_{1FB} \) and the PS-IMD\(_{1FB} \). Further, the PS-IMD components in the single-feedback CDA may be larger than the distortion component at \( f_n \) because \( LG_{1FB} \) at \( f_n \pm f_n \) can be significantly smaller that \( LG_{1FB} \) at \( f_n \). For example, for \( f_n = 100 \text{ Hz}, f_n = 15 \text{ kHz}, \) and \( M = 0.2 \) (a typical CDA power setting given the large crest factor of audio signals), the PSRR is unexpectedly better than the PS-IMD, specifically, PSRR \( \approx 47 \text{ dB} \) while PS-IMD \( \approx -20 \text{ dB} \). Put simply, in this condition, the PS-IMD is \( \approx 27 \text{ dB} \) larger than the distortion components at \( f_n \) (see Section IV later). It is apparent from (4a)–(4c) that to obtain a high PSRR and low PS-IMD, it is imperative that \( G_{int}, G_{PWM}, \) and \( H_1 \) are high. For completeness, note that the associated primary parameters (12) for high \( G_{int} \) are (high) \( A_{opA} \) and (high) \( f_{sw} \).

B. PSRR and PS-IMD of the Double-Feedback CDA

Fig. 2 depicts a double-feedback CDA. On the basis of the analysis method for PSRR and PS-IMD of the single-feedback CDA, the PSRR and PS-IMD of the double-feedback CDA can be similarly derived as shown in (5a)–(5b) at the bottom of the page, where \( LG_{2FB} \) is the loop gain of the double-feedback CDA

\[
LG_{2FB} = |G_{int2}| |G_{int1}| |G_{PWM}| H_2 G_1 + LG_{1FB}
\]

(5c)

where \( G_{int2} \) is the gain of the outer-loop integrator and \( H_2 \) is the outer-loop feedback factor. These equations can be interpreted
as follows. Similar to the single-feedback CDA, PSRR$_{	ext{2FB}}$ and PS-IMD$_{	ext{2FB}}$ are largely determined by the LG$_{	ext{2FB}}$—the higher the LG$_{	ext{2FB}}$, the better is the PSRR$_{	ext{2FB}}$ and the PS-IMD$_{	ext{2FB}}$. Hence, it is desirable that $G_{\text{int}2}$, $G_{\text{PWM}}$, and $H_3$ be high.

A comparison between LG$_{	ext{2FB}}$ and LG$_{	ext{1FB}}$ shows that LG$_{	ext{2FB}}$ is significantly higher. This significant improvement will consequently result in significant improvements in both PSRR and PS-IMD. Further, since both $G_{\text{int}1}$ and $G_{\text{int}2}$ reduce as frequency increases, the degree of loop-gain improvement also reduces as frequency increases. This reduction consequently reduces the improvement in both PSRR and PS-IMD.

### III. Technique to Improve PSRR and PS-IMD

In this section, a simple low hardware overhead technique to improve PSRR and PS-IMD is proposed. This technique involves replacing the conventional carrier generator with the improved generator depicted in Fig. 3. In a conventional generator, the non-inverting input of terminal of the comparator is tied to signal ground (i.e., in Fig. 3, $R_{C1}$ is open-circuit and $R_{C2}$ short-circuit) and its hysteresis is nearly independent of the supply noise. The hysteresis of the comparator used in the improved carrier generator, on the other hand, depends on the supply voltage and the ratio of $R_{C1}$ and $R_{C2}$.

The improved carrier generator introduces supply noise into the carrier such that the distortion components introduced by the supply noise are to a first order nullified by the distortion components in the carrier. Fig. 4 depicts the waveforms of the improved carrier generator, where $V_{h1}$ and $V_{h2}$ are the respective ideal upper and lower limits of the hysteresis, and $V_{h3}$ and $V_{h4}$ are, respectively, the actual upper and lower practical limits of the hysteresis range. Note that the difference between the ideal and practical hysteresis is primarily due to the propagation delay $t_D$ of the comparator.

### A. Open-Loop CDA Embodying Improved Carrier Generator

The output of an open-loop CDA $V_{\text{OLi}}$ embodying the improved carrier generator will now be derived. Based on Figs. 3 and 4, the expressions for $V_{h3}$ and $V_{h4}$ can be shown to be

$$V_{h3} = \frac{V_{DD}}{G_{\text{PWM}}} + \frac{4V_{DD}t_D}{G_{\text{PWM}}T} + \frac{V_n}{G_{\text{PWM}}} \quad (6a)$$

$$V_{h4} = -\frac{V_{DD}}{G_{\text{PWM}}} - \frac{4V_{DD}t_D}{G_{\text{PWM}}T} \quad (6b)$$

where $G_{\text{PWM}} = (R_{C1} + R_{C2})/R_{C2}$, and $T = 4C_CV_{DD}/(IG_{\text{PWM}})$ is one switching period when $V_n = 0$ and $t_D = 0$.

Note that the carrier frequency of the CDA embodying the improved carrier generator varies due to the supply noise. For instance, a 10% supply rail variation (i.e., $V_n = 10\% V_{DD}$) results in a 5% variation in the carrier frequency. The variation in the carrier frequency results in a slight variation in the power efficiency [11], but does not degrade the performance (e.g., Total Harmonic Distortion) of the CDA.

On the basis of (6a) and (6b), the output signal, $V_{\text{OLi}}$, can be derived by taking the average value over one carrier period

$$V_{\text{OLi}} \approx MV_{DD}\cos(2\pi f_{\text{int}}t) + \frac{Af_D}{T}N_D. \quad (7)$$

Note that the first term in (7) is the replica of the input signal, and the second term is the output distortion due to the supply noise. A comparison of the second term in (7) and (3) shows that by means of the improved carrier generator, the major distortion components in the improved open-loop CDA are significantly reduced from $N_D$ to $(4t_D/T)N_D$ and that the smaller the $t_D$, the lower is the distortion. Note that in an ideal case where $t_D = 0$, the distortion components are eliminated.

### B. PSRR and PS-IMD of the Improved Single-Feedback CDA

Consider now the embodiment of the improved carrier generator in the single-feedback CDA. Based on (7) and the linear model, the PSRR$_{1FBi}$ and PS-IMD$_{1FBi}$ of the improved single-feedback CDA can be shown to be as shown in (8a)–(8b) at the bottom of the page. Equations (8a) and (8b) can be interpreted as follows.

1) Similar to the single-feedback CDA and double-feedback CDA, PSRR$_{1FBi}$ and PS-IMD$_{1FBi}$ are largely affected by the loop-gain—the higher the LG$_{1FB}$, the better is the PSRR$_{1FBi}$ and the PS-IMD$_{1FBi}$.

2) Compared to the single-feedback CDA, PSRR$_{1FBi}$ and PS-IMD$_{1FBi}$ are improved by 20$\log(T/4t_D)$ dB. This improvement is usually significant as $t_D$ (typically from 10 ns to 100 ns) is much smaller than $T$ (typically from 1 $\mu$s to 10 $\mu$s). Obviously, a shorter $t_D$ (for example by

$$\text{PSRR}_{1FBi} = 20\log \left( \frac{T}{4t_D} \right) + 20\log \left( 2 + 2L_{G_{1FB}}|f_{\text{in}}| \right) \text{ dB} \quad (8a)$$

$$\text{PS-IMD}_{1FBi} = 20\log \frac{Af_D}{T} + 20\log \left( \frac{0.25M}{1 + L_{G_{1FB}}|f_{\text{in}} + f_n|} \right)^2 + \left( \frac{0.25M}{1 + L_{G_{1FB}}|f_{\text{in}} - f_n|} \right)^2 \text{ dB} \quad (8b)$$
increasing the gain of the comparator) and/or lower $f_{in}$ (hence longer $T$) results in better PSRR and PS-IMD.

3) A comparison of the improved single-feedback CDA and double-feedback CDA shows that the improved single-feedback CDA is likely to feature better PSRR and PS-IMD in the higher frequency range ($>2$ kHz) but worse PSRR and PS-IMD in the lower frequency range (e.g., 100 Hz). This is because the improvement of the improved single-feedback CDA is independent of frequency. In contrast, the improvement of the PSRR and PS-IMD of the double-feedback CDA reduces as frequency increases.

IV. VERIFICATION AND MEASUREMENTS

In this section, the analytical expressions for PSRR and PS-IMD are verified by HSPICE simulations and on the basis of measurements on a prototype CDA IC and on other CDAs constructed using discrete components. The microphotograph of the prototype IC embodying a single-feedback CDA is depicted in Fig. 5. The double-feedback CDA was constructed by adding an external loop to the single-feedback CDA IC using discrete components and the improved single-feedback CDA was similarly constructed using discrete components. The circuit parameters of these CDAs are tabulated in Table I.

Fig. 6 depicts the PSRR against $f_n$ obtained analytically using (4a), (5a), and (8a), by HSPICE simulations, and by practical measurements for the 3 CDAs. The following observations are made based on Fig. 6.

1) The derived analytical expression agrees well with the HSPICE simulations and practical measurements for all three designs, hence verifying (4a), (5a) and (8a). The PSRR of all CDAs reduces as the supply noise frequency reduces. This is as expected because the loop-gain reduces as frequency increases.

2) The PSRR of the double-feedback CDA is significantly better than the single-feedback CDA due to the additional feedback loop. The degree of improvement decreases as the supply noise frequency increases, e.g., the PSRR improvement reduces from $\sim 45$ dB @ 100 Hz to $\sim 17$ dB @ 15 kHz. This is because the improvement of the loop gain reduces as the frequency increases.

3) The PSRR of the improved single-feedback CDA is significantly better than the single-feedback CDA. The PSRR improvement here is nearly frequency independent. For example, the PSRR improvement is $\sim 26$ dB throughout the audio frequency range.

4) A comparison between the double-feedback CDA and improved single-feedback CDA shows that the double-feedback CDA has worthy advantages in the low frequency range (< 2 kHz) where the PSRR of the improved single-feedback CDA is substantially better than the double-feedback CDA. However, in the higher frequency range (2–20 kHz), the PSRR of the improved single-feedback CDA becomes better than the double-feedback CDA. In view of this, if PSRR in the lower frequency range is the major parameter of concern, the double-feedback CDA should be preferred. Otherwise, the improved single-feedback CDA is the better choice because it features a better PSRR in the higher frequencies and its simpler hardware. Furthermore, where it is necessary, to obtain an even higher PSRR (but at the increased hardware cost), the double-feedback CDA can embody the improved carrier generator.

Fig. 7(a)–(c), respectively, summarizes the PS-IMD of the single-feedback CDA, double-feedback CDA and improved single-feedback CDA obtained analytically using (4b), (5b), and (8b), by HSPICE simulations and by measurements. The noise frequency is $f_n = 100$ Hz, and the input frequency is $f_{in} = 2$ kHz in one case and $f_{in} = 15$ kHz in the second case. The following observations are made based on Fig. 7(a)–(c).

1) The derived analytical expression agrees well with the HSPICE simulations and practical measurements for all three designs, hence verifying (4b), (5b), and (8b). The PS-IMD, as expected, degrades for higher modulation indexes. The PS-IMD also degrades for higher input frequencies and this is due to the higher intermodulation frequencies where the CDA loop-gain is lower.

2) A comparison between Fig. 7(a) and (b) shows that, due to the additional feedback, the improvement of the PS-IMD of the double-feedback CDA over the single-feedback CDA is a significant $\sim 35$ dB and $\sim 17$ dB for $f_{in} = 2$ kHz and $f_{in} = 15$ kHz respectively. As expected, the degree of improvement reduces as the input frequency increases.

3) A comparison between Fig. 7(a) and (c) shows that, compared to the single-feedback CDA, the PS-IMD of the improved single-feedback CDA is significantly improved by $\sim 26$ dB for $f_{in} = 2$ kHz and $f_{in} = 15$ kHz. Note that unlike the case of the double-feedback CDA,
the PS-IMD improvement of the improved single-feedback CDA is independent of the input frequency.

4) In addition to 2) and 3), for $f_{in} = 2$ kHz, the improvement of PS-IMD$_{2FB}$ over PS-IMD$_{1FB}$ is $\sim 10$ dB higher than the improvement of PS-IMD$_{1FB}$ over PS-IMD$_{2FB}$. However, for $f_{in} = 15$ kHz, the improvement of PS-IMD$_{2FB}$ is $\sim 10$ dB lower than the improvement of PS-IMD$_{1FB}$. This is because the improvement of LG$_{2FB}$ over LG$_{1FB}$ reduces with frequency while the improvement of PS-IMD$_{1FB}$ over PS-IMD$_{2FB}$ is frequency independent as $f_D$ is nearly frequency independent (in the audio frequency range). Similar to the case of PSRR, if the low frequency PS-IMD is of primary interest, the double-feedback CDA would be a better choice, otherwise the improved single-feedback CDA would be preferred.

5) From Figs. 6(a) and 7(a), for the single-feedback CDA when $f_{in} = 100$ Hz, $f_{in} = 15$ kHz, and $M = 0.2$, note that the PS-IMD is substantially larger than the distortion component at $f_{in}$, specifically the PS-IMD $\leq -20$ dB while PSRR $\geq 47$ dB. This observation is not well known but can be easily explained from (4a) and (4b). Specifically, this is because LG$_{1FB}$ at $f_{in}$ is much larger than LG$_{2FB}$ at $f_{in} \pm f_{2FB}$. For the same reason, a similar observation can be made for the double-feedback CDA and the improved single-feedback CDA.

V. CONCLUSION

The PSRR and PS-IMD of the single-feedback, double-feedback, and improved single-feedback CDAs have been analyzed and analytical expressions derived. The proposed improved single-feedback CDA embodied a improved carrier generator that incorporated a simple design technique with very low hardware overheads. It has been shown that the proposed design technique has worthy advantages to improve PSRR and PS-IMD. The derived analytical expressions and the efficacy of the proposed technique have been verified by means of HSPICE simulations and on the basis of measurements on a prototype CDA IC and other CDAs realized using discrete components. The performance of the three CDAs have also been compared.

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