<table>
<thead>
<tr>
<th>Title</th>
<th>Vibration measurement of miniature component by high-speed image-plane digital holographic microscopy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Author(s)</td>
<td>Fu, Yu; Shi, Hongjian; Miao, Hong</td>
</tr>
<tr>
<td>Date</td>
<td>2009</td>
</tr>
<tr>
<td>URL</td>
<td><a href="http://hdl.handle.net/10220/6473">http://hdl.handle.net/10220/6473</a></td>
</tr>
<tr>
<td>Rights</td>
<td>This paper was published in [Applied Optics] and is made available as an electronic reprint with the permission of OSA. The paper can be found at the following URL on the OSA website: [<a href="http://www.opticsinfobase.org/abstract.cfm?URI=ao-48-11-1990">http://www.opticsinfobase.org/abstract.cfm?URI=ao-48-11-1990</a>]. Systematic or multiple reproduction or distribution to multiple reproduction or distribution to multiple locations via electronic or other means is prohibited and is subject to penalties under law.</td>
</tr>
</tbody>
</table>
Vibration measurement of a miniature component by high-speed image-plane digital holographic microscopy

Yu Fu,1,* Hongjian Shi,1 and Hong Miao2

1Department of Mechanical Engineering, National University of Singapore, 9 Engineering Drive 1, Singapore 117576
2Department of Modern Mechanics, University of Science and Technology of China, Hefei 230027, China
*Corresponding author: fuyuoptics@gmail.com

Received 2 January 2009; revised 18 March 2009; accepted 20 March 2009; posted 20 March 2009 (Doc. ID 105861); published 1 April 2009

Measuring deformation of vibrating specimens whose dimensions are in the submillimeter range introduces a number of difficulties using laser interferometry. Normal interferometry is not suitable because of a phase ambiguity problem. In addition, the noise effect is much more serious in the measurement of small objects because a high-magnification lens is used. We present a method for full-field measurement of displacement, velocity, and acceleration of a vibrating miniature object based on image-plane digital holographic microscopy. A miniature cantilever beam is excited by a piezoelectric transducer stage with a sinusoidal configuration. A sequence of digital holograms is captured using a high-speed digital holographic microscope. Windowed Fourier analysis is applied in the spatial and spatiotemporal domains to extract the displacement, velocity and acceleration. The result shows that a combination of image-plane digital holographic microscopy and windowed Fourier analyses can be used to study vibration without encountering a phase ambiguity problem, and one can obtain instantaneous kinematic parameters on each point. © 2009 Optical Society of America

1. Introduction

In pace with the fast-growing industries of microelectromechanical systems (MEMS) and micro-optical electromechanical systems (MOEMS), the demands for higher product quality and reliability generate new challenges for measurement techniques. Generally, the smaller these components become, the more challenging it is to measure their performance with accuracy. Optical methods [1] enjoy the virtues of being full field, noninvasive, and high resolution (in both space and time) and are widely used by industry for the measurement of shape, displacement, and mechanical properties of microelements [2–7]. Nonetheless, the following are some challenges to be addressed for successful implementation of optical assessment of microscale objects. (1) The optical setup is constrained by use of a microscope, and the constraints include a short working distance, limited depth of field, limited numerical aperture, a light source, and the arrangement of the setup. (2) For macroscale specimens, the sensitivity of optical interferometry is high enough; but for microscale objects, the measurement precision required is likely to exceed the capability of optical interferometry with a visible light source. (3) Because of the high magnification of the microscopic system, the signal-to-noise ratio is reduced by environmental factors such as vibration. A more powerful algorithm is thus needed to extract the useful information from a noisy signal.

From a mechanical viewpoint, there are a few typical components commonly employed in microsystems, such as membranes, beams, comb flats and spiral springs, which are normally subjected to various mechanical (pressure, bending, compression,
and tension) and thermal loads [8]. In many cases, the dynamic behavior of these microelements, such as vibration and transient deformation, are of more interest. For periodic high-frequency vibration measurements, a combination of laser Doppler vibrometry [9] and microscopy is the predominant method. However, this technique has an important limitation: it is a pointwise measurement and a 2-D scanning mechanism with a time-consuming scanning procedure that cannot be avoided to obtain full-field information, which means that nonrepeatable events cannot be studied in detail. In addition, the spatial resolution is limited by the size of the laser spot, especially for the measurement of small components.

Because of the rapid development of high-speed digital recording devices, it is now possible to record interferograms with rates that exceed 100,000 frames/s under a microscope. Retrieving precise spatial phase maps from these transient interferograms enables measurement of instantaneous 3-D profiles, deformations as well as dynamic responses. Different methods have been reported to analyze this interferogram sequence. The instantaneous phase distributions can be retrieved by spatial frequency analysis [10] or by fast phase shifting [11] followed by a phase unwrapping process along a time axis [12]. A spatial carrier is necessary for the former method, while the latter requires a high-speed phase shifter for the experiment. Other methods are based on temporal phase analysis techniques [13–17] in which each pixel in the interferograms is analyzed independently as a function of time. The temporal phase analysis technique has the advantage of eliminating speckle noise as it evaluates the phase pixel by pixel along the time axis. However, it does have a disadvantage: if only the intensity variations of pixels are analyzed, determination of the absolute sign of the phase is impossible, which limits the technique to the measurement of deformation in one known direction. The addition of a temporal carrier [18] to the image acquisition process is one way to overcome this problem. However, the carrier frequency limits the measurement range of the phase variation that is due to the constraints of the Nyquist sampling theorem and the acquisition speed of a high-speed camera [19].

Because of the increment of the resolution of imaging sensors, a novel computer-aided optical technology, digital holography [20], has been successfully applied in different types of measurement in the past decade. It is now possible to record holograms directly by a camera and to reconstruct the object digitally by computer simulation. The result of digital reconstruction is not only the amplitude of the object wave, but the phase value as well. This advantage makes digital holography more suitable for dynamic measurements. In addition, no phase ambiguity problem exists during additional processing of the reconstructed phase. Different vibration measurement techniques based on digital holography have been reported in combination with a pulsed ruby laser [21] and time average method [22]. Previous research [23,24] has shown that it is possible to measure the kinematic and deformation parameters by detecting the windowed Fourier ridge [25,26] of the phase difference in spatial and temporal axes.

In recent years, digital holography has also been applied with a microscope to obtain the quantitative amplitude and phase-contrast image of a micro-object [27,28]. Detection of refractive-index changes and variations in the shape of biological objects, such as undyed cellular samples, have also been reported [29–31]. We apply high-speed image-plane digital holographic microscopy to capture a sequence of digital holograms during the vibration of a miniature metal beam. These holograms are evaluated by a 2-D digital Fourier transform method [32]. The result is a 3-D matrix of complex amplitude. Further processing of this matrix makes it possible to evaluate the displacement, velocity, and acceleration simultaneously. For this investigation we selected a windowed Fourier transform (WPT) as the main processing technique to extract displacement, velocity, and acceleration in a 2-D spatiotemporal plane. The results show that the instantaneous kinematic parameters of a vibrating miniature component can be measured with image-plane digital holographic microscopy.

2. Theoretical Analysis

A. Image-Plane Digital Holographic Microscopy

Figure 1(a) shows a schematic layout of the image-plane digital holographic microscopy which is sensitive to out-of-plane displacement. The laser light is split into a beam for illumination of the object and a reference beam. The specimen with a diffuse surface is illuminated in a direction $e_i$. Some light is scattered in the observation direction $e_o$, where an image-plane hologram is formed by a long working distance microscope as a result of interference between the reference beam and the object beam. The reference beam is introduced by a single-mode optical fiber and a beam splitter, and it diverges toward the detector. It is worth noting that this setup also works on highly reflective objects by introducing the object beam at a right angle using a beam splitter. Let $R(x,y)$ and $U(x,y)$ denote the reference and object waves; the intensity recorded on the imaging sensor can be expressed as

$$I_H(x,y) = |H(x,y)|^2 + |U_H(x,y)|^2 + R_H(x,y)U_H^*(x,y) + R_H^*(x,y)U_H(x,y),$$

(1)

where subscript $H$ indicates that these values are in the image plane and the * denotes the complex conjugate. If $\theta_{\text{max}}$ denotes the maximum angle between the object and the reference beams, the maximum spatial frequency in the hologram is [33]

$$f_{\text{max}} = \frac{2}{\lambda} \sin \left( \frac{\theta_{\text{max}}}{2} \right),$$

(2)
where \( \lambda \) is the wavelength of the laser. The position and direction of the fiber end should be properly designed in the setup so that the Nyquist sampling theorem is satisfied over the full area of the detector; in our case it is \( f_{\text{max}} < 1/(2\Delta) \), where \( \Delta \) denotes the pixel size of the camera. For a normal imaging lens, an aperture is needed just behind the lens to limit the spatial frequencies of the interference pattern [24]. However, in our case, the spatial frequency bandwidth is already limited by the microscopic lens. The amplitude and phase of the object wave can be obtained by spatial filtering with the Fourier-transform method. Reference [32] gives a detailed description of spatial Fourier evaluation of a digital image-plane hologram. Figure 2(a) shows a typical digital hologram of a vibrating cantilever beam [shown in Fig. 1(b)] captured by the high-speed camera shown in Fig. 1(a). It is possible to filter out one of the last two terms in Eq. (1) on the Fourier spectrum of the recorded intensity. The Fourier spectrum and filtering window are shown in Fig. 2(b). After inverse Fourier transformation, the complex amplitude of wavefront \( U_H(x, y) \) is obtained. Similar processing yields a 3-D complex matrix of \( U_H(x, y; t) \) when a series of digital holograms is captured during the vibration of an object. From this matrix we evaluated the kinematic parameters, such as displacement, velocity, and acceleration.

In optical interferometry, the deformation of an object involves a change in the phase of the object beam. The relationship between the phase change \( \Delta \phi = \phi_t - \phi_i \) and the out-of-plane displacement \( z \) is given by

\[
\Delta \phi = \frac{2\pi z}{\lambda} S,
\]

where \( S = e_i - e_o \) is the sensitivity vector given by the geometry of the setup, and \( e_i \) and \( e_o \) are the unit vectors of illumination and observation, respectively. The phase difference between two digital holograms recorded at \( t_1 \) and \( t_n \) can be calculated by

\[
\Delta \phi = \arctan \frac{\text{Im}(U(x, y; t_n)U^*(x, y; t_1))}{\text{Re}(U(x, y; t_n)U^*(x, y; t_1))},
\]

where \( \text{Re} \) and \( \text{Im} \) denote the real and imaginary parts of the complex value, respectively. A typical instantaneous phase difference on a vibrating cantilever beam is shown in Fig. 2(c). The temporal phase unwrapping yields the displacement relative to the first frame of the measurement process. If the first hologram is considered as a reference, we will have a series of \( U(x, y; t_n)U^*(x, y; t_1) \) whose phase values of \( \phi(x, y; t_n) \) after temporal unwrapping are proportional to the displacement at instant \( t_n \), and a series of \( U(x, y; t_n)U^*(x, y; t_{n-1}) \) whose phase values are pro-

Fig. 1. (a) Schematic layout of the experimental setup and (b) specimen of a miniature cantilever beam with white-light illumination and the imaging area.

Fig. 2. (a) Typical digital hologram obtained on a cantilever beam; (b) spectrum of the obtained digital hologram; (c) typical original wrapped phase map indicating the relative displacement of the beam and the area of interest.
porportional to the numerical differentiation of the displacement. In this application, the WFT is applied in a spatiotemporal plane to extract the ridge of the phase variation, so that the velocity and acceleration of the cantilever beam can be evaluated.

B. Windowed Fourier Analysis

The WFT is a well-known signal processing algorithm that has been presented in different resources [23,24,26,34]. We will, therefore, omit the fundamental equation of the WFT. However, it is noteworthy that in the WFT the time-frequency uncertainty principle affects the resolution, which leads to a trade-off between time and frequency localization. The narrower the time window, the better the temporal resolution at the cost of a poorer resolution in frequency and vice versa. However, once the window size is determined, the WFT has a uniform resolution at different frequencies.

1. Windowed Fourier Ridge

On point $P(x, y)$ of the object, a sequence of phase variations is calculated as $\Delta \phi(x, y; t_n) = \arg(U(x, y, t_n)U^*(x, y, t_1))$. In this case, $N$ is the total frame number captured. These unwrapped phase values $\Delta \phi(x, y; t_n)$ are converted to an exponential signal, $C_p = \exp(j \cdot \Delta \phi)$. The WFT of this complex signal is [34]

$$SC_p(u, \xi) = \frac{\sqrt{8}}{2} A(u) \exp(j \phi(u) - \xi u) \{g(s|\xi - \phi'(u)) + \epsilon(u, \xi)\},$$

(5)

where $u$ and $\xi$ represent the time and frequency, respectively. The support size of Gaussian window $g(t)$ varies according to the scaling factor $s$. $A(u)$ is the modulus of $C_p$ and, in this case, $A(u) = 1$ as $C_p$ is an exponential signal. $\epsilon(u, \xi)$ is a corrective term that can be neglected if $\phi'(u)$ has small relative variations over the support of window $g(t)$. $g(\omega)$ denotes the Fourier transform of $g(t)$. The trajectory of maximum $|SC_p(u, \xi)|^2$ on the $u - \xi$ plane is called a windowed Fourier ridge. Since $|g(\omega)|$ is maximum at $\omega = 0$, and if $\epsilon$ is negligible, $|SC_p(u, \xi)|^2$ reaches maximum when

$$\xi(u) = \phi'(u),$$

(6)

where $\phi'(u)$ is defined as the instantaneous frequency of the signal, which is proportional to the velocity of $P(x, y)$ in this case. Equation (6) is valid when $\epsilon$ is negligible, which means that the phase of the signal is linear in the local area determined by the extension of $g$. For a nonlinear signal, the assumption is better satisfied when the window size is small. However, a large window size is more robust against noise. The choice of window size should be based on a trade-off between accuracy of the phase approximation and immunity to noise. In this study, the phase variation along the time axis is highly nonlinear, but the noise effect along the time axis is not obvious; a small window size is a better choice. In contrast, phase nonlinearity is not serious in spatial axes, but the noise effect is significant. Hence, a relatively large window size should be selected in the spatial domain. A detailed analysis of system errors generated by different window sizes is presented in Refs. [23,24]. In this application, the system errors of phase extraction by WFT in velocity and acceleration measurements are less than 2% and 7%, respectively. Similar to the Fourier transform, the 1-D windowed Fourier ridge method can be extended to a 2-D or 3-D case. Here we apply it to a 2-D spatiotemporal plane $(x - t)$ to extract the first (velocity) and the second (acceleration) derivatives of a cantilever beam.

2. Windowed Fourier Filtering

In digital holography, the phase variation can be extracted by Eq. (4). However, the result is a series of wrapped phase with speckle noise. In this study, each instantaneous wrapped phase map is filtered by a 2-D WFT. Details of the filtering are given in Refs. [25,26]. A comparison of WFT and FT has already shown that the WFT performs better than the FT with regard to noise elimination in 2-D and 3-D cases [25,35], because it can remove the noise within the FT bandpass filter window. Its efficiency is more obvious when the spectrum of the signal is broad. These filtered phase maps are then unwrapped temporally to obtain absolute displacement. It is noteworthy that the system error that is due to linear phase assumption no longer exists in the windowed Fourier filtering process. Hence, the selection of window size is mainly decided by the severity of the noise effect and the border effect of the WFT. Other filtering algorithms, such as the sine and cosine average filter (SCAF) [36] are also adopted by other researchers. The SCAF is often used because of its simplicity. However, it does not perform well when the fringe patterns have varying spatial frequencies; it can process only the wrapped phase maps with small slopes [37].

3. Experimental Illustration

Figure 1(a) shows the experimental setup. The specimens tested in this study is a steel cantilever beam with a diffuse surface as shown in Fig. 1(b). The length, width, and thickness of the beam are 1000, 200, and 100 $\mu$m, respectively. The beam is subjected to a preload and vibration with sinusoidal configuration at the free end by a piezoelectric transducer (PZT) stage. The frequency of the vibration can be controlled by the PZT stage and is around 50 Hz in this case. The beam of a He–Ne laser (Melles Griot, 05-LHP-927, maximum 75 mW at $\lambda = 632.8$ nm) is divided into an object beam and a reference beam. The cantilever beam is illuminated at an angle of 28° and is imaged at a right angle by a long working distance microscopic lens (Optem Zoom 100C, numerical aperture of 0.016 at the image side with 6x magnification) by the sensor of a high-speed Kodak
camera (Motion Corder Analyzer, SR-Ultra) with a pixel size of 7.4 \mu m \times 7.4 \mu m. With this arrangement, a 1 rad phase change is equivalent to a 51.9 nm displacement in the x direction. Since the illumination angle is relatively large, this value is calculated by vector analysis [38]. The interference between the reference and the object beams generates an image-plane hologram on the camera sensor. In Ref. [24], an aperture is needed behind the imaging lens to limit the spatial frequencies of the hologram. In this experiment, it is not necessary because of the low numerical aperture of the microscopic lens. Two hundred holograms are captured at an imaging rate of 500 frames/s. The image size is 512 \times 240 pixels and the imaging area is near the clamping end as shown in Fig. 1(b). Figure 2(c) is a typical wrapped phase map obtained by Eq. (4). The phase values of the area with a size of 480 \times 130 pixels, as shown in Fig. 2(c), have been selected to be processed.

It is worth noting that the specimen we measured has a diffuse surface. The imaging direction is not critical. Since the microscopic lens has a limited depth of field, it is more reasonable to image the specimen by the right angle. The illuminating light can also be introduced to the specimen at the right angle through the beam splitter and the microscopic lens. However, because of the low numerical aperture of the microscopic lens and high capture rate, high-intensity illumination is needed. To avoid the unnecessary loss of the illumination power, it is more reasonable in practice to illuminate the specimen directly.

### 4. Results and Discussion

The middle column in Fig. 3 shows a series of original wrapped phase \Delta \phi at different instants. The phase maps after 2-D WFT filtering are shown in the right-hand column. \sigma_x = 20 and \sigma_y = 10 were selected as the window size in the x and y directions, respectively. Hence, the filtering window size is 41 in the x axis, and 21 in the y axis with a Gaussian envelope [25]. We observed that the WFT filtering performs very well and a smooth wrapped phase is obtained. It can also be applied to all original wrapped phase maps. Since the measured specimen is a cantilever beam, and the displacements in the y direction vary only slightly, the phase values on the A–A cross section as shown in in Fig. 2(c) were selected to be processed. The temporal phase variations of the A–A section generate a spatiotemporal distribution as shown in Fig. 4(a). Phase unwrapping along the time axis yields a continuous phase map [Fig. 4(b)] from which the instantaneous displacement along the A–A section can be calculated by use of the above-mentioned sensitivity factor. In Fig. 4(b) the minimum and maximum phase values are represented by gray values of 0 and 255, respectively. Figure 4(c) shows the displacement of the beam at different instants of time t_a. Figure 4(d) shows the displacement variation of two points, x_1 = 100 \mu m and x_2 = 500 \mu m from the left-hand side of the image on the A–A line. In Fig. 4(a) a slight misalignment of phase value in the t axis indicates that the amplitude of the vibration might not be constant, which is obvious only on a wrapped phase map but inconspicuous in the final results. It is noteworthy that, when the object is more complex in shape or there is a sudden change of phase along spatial coordinates, the spatial WFT will generate large errors. In this case, we can rely on only the temporal analysis pixel by pixel.

The phase values in Fig. 4(a) are then converted to the exponential values \exp(j \Delta \phi) and processed by the windowed Fourier ridge algorithm to extract the instantaneous velocity and acceleration. \sigma_t = 1 and \sigma_x = 5 were selected as the window size in the t and x axes, respectively, to minimize the system error that is due to the linear phase assumption. Figure 5(a) is a gray value distribution that shows the windowed Fourier ridge in the t axis. The velocity \frac{dx}{dt} can be obtained at any instant. Figure 5(b) shows two distributions of instantaneous velocities at t = 0.162 s and t = 0.172 s, respectively. Figure 5(c) shows the velocity variations of x_1 and x_2 on the A–A line. A similar process can be applied again in Fig. 5(a) to obtain the second derivatives in the time axis. Figure 6 shows the acceleration variations of points x_1 and x_2.

Similar to wavelet analysis, the windowed Fourier ridge method maps a 1-D signal to a 2-D plane of time and frequency and extracts the ridge with the highest energy. It is a time-consuming process and requires a high computing speed and large memory. It can take more than 1 h to calculate a matrix of
512 pixels × 240 pixels × 280 frames by use of a normal personal computer. Recently some fast converging iterative algorithms [39] were introduced to avoid calculation of the whole time-frequency plane.

In our investigation, we used the maximum value in the Fourier spectrum of $f(t)g(t-u)$ as an initial estimation of the ridge value at each instant and then searched the actual ridge with a preset accuracy. This method can be extended to 2-D and 3-D cases. The computation time depends on the accuracy required, but it is still much longer than that of the Fourier analysis. If the bandwidth of the frequency signal is not very broad, for example, in the evaluation of acceleration, the FT is still a good choice because of its speed.

It is noteworthy that the measurement range of the velocity and acceleration is limited because of the constraint of the Nyquist sampling theorem and imaging rate of the high-speed camera. The maximum phase change between two adjacent frames is $\pm \pi$, which is equivalent to the velocity of $\pm 81.52 \mu m/s$ with the experimental setup and imaging rate mentioned above. However, no phase ambiguity problem exists in digital holography, since it can extract the phase from one hologram. Compared with other optical interferometry based on an intensity variation signal, the measurement range of vibration frequency and amplitude are tremendously enlarged with the absence of a temporal carrier.
5. Concluding Remarks
We have presented a novel method for evaluating the transient displacement, velocity, and acceleration of a vibrating miniature object by image-plane digital holographic microscopy. It is an attempt to use a photodetector array with a microscope to evaluate full-field vibration parameters of a miniature object. Compared with pointwise laser vibrometric microscopy, the signal is processed in spatial and temporal domains as a 2-D and 3-D matrix. Windowed Fourier transform is selected to process a series of digital holograms. Two-dimensional windowed Fourier filtering and temporal unwrapping is applied to obtain a smooth distribution of displacement, while the windowed Fourier ridge method is applied in a spatiotemporal plane to evaluate the instantaneous velocity and acceleration of the beam. As digital holography can extract the instantaneous phase of an object, the phase ambiguity problem is avoided in vibration measurements. The results show that the combination of image-plane digital holographic microscopy and spatiotemporal windowed Fourier analyses allows the simultaneous evaluation of three important parameters of a low-frequency vibrating miniature component at different instants.

The authors gratefully acknowledge the financial support provided by the National Science Foundation of China (NSFC) under contracts 10772171 and 10732080.

References