<table>
<thead>
<tr>
<th>Title</th>
<th>Dynamic measurement of micro-components by image-plane digital holography</th>
</tr>
</thead>
<tbody>
<tr>
<td>Author(s)</td>
<td>Fu, Yu; Shi, Hongjian; Quan, Chenggen; Tay, Cho Jui</td>
</tr>
<tr>
<td>Date</td>
<td>2008</td>
</tr>
<tr>
<td>URL</td>
<td><a href="http://hdl.handle.net/10220/6479">http://hdl.handle.net/10220/6479</a></td>
</tr>
<tr>
<td>Rights</td>
<td>Copyright 2009 Society of Photo-Optical Instrumentation Engineers. One print or electronic copy may be made for personal use only. Systematic reproduction and distribution, duplication of any material in this paper for a fee or for commercial purposes, or modification of the content of the paper are prohibited.</td>
</tr>
</tbody>
</table>
Dynamic measurement of micro-components by image-plane digital holography

Y. Fu∗, H. Shi, C. Quan and C. J. Tay
Department of Mechanical Engineering, National University of Singapore,
9 Engineering Drive 1, Singapore 117576

Abstract

A method for whole-field non-contact measurement of displacement, velocity and acceleration of a vibrating micro-object based on digital holographic microscopy is presented. A micro-beam is excited by a fluctuating voltage with a sinusoidal configuration. A series of digital holograms are captured using a digital holographic microscope with a high-speed camera. The result of reconstruction is a three dimensional complex-valued matrix with noises. In this paper, Fourier analysis and windowed Fourier analysis are applied in both the spatial and temporal domains to extract the kinematic parameters. The instantaneous displacement is obtained by temporal phase unwrapping of the filtered wrapped phase map, while the velocity and acceleration are evaluated by windowed Fourier analysis along the time axis. The combination of digital holographic microscopy and temporal Fourier analyses is able to study the vibration without a phase ambiguity problem, and the instantaneous kinematic parameters on each point are obtained.

Keywords: Temporal analysis, phase extraction, vibration measurement, Fourier analysis, digital holography.

1. INTRODUCTION

Micro-electro Mechanical Systems (MEMS) is the integration of mechanical elements, sensors, actuators, and electronics on a common silicon substrate through microfabrication technology. MEMS devices provide significant cost and performance advantages in the area of aerospace, appliances, automotive, biotechnology, chemical systems, and communications. From mechanical viewpoint, there are a few typical components commonly employed in MEMS devices, such as membranes, beams, comb flats and spiral springs. These components are normally subjected to various mechanical (pressure, bending, compression and tension) and thermal loadings [1]. These devices generally range from a few ten microns to millimeters in dimensions. Measuring static and dynamic properties of micro-structure poses a number of difficult and interesting problems. Usually, the smaller these components become, the more challenging it is to measure their performance with accuracy. The optical methods enjoy the virtues of being whole-field, noninvasive and high-resolution (in both space and time), and are getting more widely used by industry for the measurement of shape, displacement, and mechanical properties of microelements [2-3]. In recent years, the dynamic behaviors of micro elements, such as vibration and transient deformation are more interested.

Due to the rapid development of high-speed digital recording devices, it is now possible to record interferograms with rates exceeding 100,000 frames per second (fps) under microscope. Retrieving precise spatial phase maps from these transient interferograms enables measurement of instantaneous 3-D profiles, deformations as well as dynamic responses. Different methods were reported to analyze this interferogram sequence. The instantaneous phase distributions can be retrieved by spatial frequency analysis [4], or by fast phase-shifting [5] followed by a phase unwrapping process along time axis [6]. Spatial carrier is necessary for the former method, while the latter requires a high-speed phase shifter in experiment. Other methods are based on temporal phase analysis techniques [7-8], where each pixel in the interferograms is analyzed independently as a function of time. The temporal phase analysis technique has the advantage of eliminating speckle noise, as it evaluates the phase pixel by pixel along the time axis. However, it does have a disadvantage: if only the intensity variations of pixels are analyzed, the determination of absolute sign of the phase is

* fuyuoptics@gmail.com; phone 65 65162230; fax 65 67791459; http://www.me.nus.edu.sg
impossible. This limits the technique to the measurement of deformation in one known direction. Adding a temporal carrier [9] to the image acquisition process is a method to overcome these problems. However, the carrier frequency limits the measurement range of phase variation due to the constraints of Nyquist sampling theorem and the acquisition speed of the high-speed camera [10].

In the last decade, a novel computer-aided optical technology, digital holography [11], has been successfully applied in different types of measurement. This is due to the development of CCD, or CMOS sensors, and the capabilities of computers. It is now possible to record holograms directly by a camera and to reconstruct the object digitally by computer simulation. The result of the digital reconstruction is not only the amplitude of the object, but the phase of the object as well. This advantage makes digital holography more suitable for dynamic measurement. In addition, no phase ambiguity problem exists in further processing of the reconstructed phase. Pedrini [12] described a method for measuring dynamic events in which digital holograms of an object are recorded on a high-speed sensor and the phase of the wavefront recorded at different instants is calculated from the recorded intensity, by use of a 2-D digital Fourier transform method [13]. By unwrapping the phase in the temporal domain it is possible to get the displacement, including the direction of a vibrating object as a function of time. In this paper a high-speed digital holography is combined with microscopy to capture a sequence of digital holograms during the vibration of a micro-beam. After reconstruction, a 3-D matrix of complex amplitude is obtained. Further processing of this matrix makes it possible to evaluate the displacement, velocity and acceleration simultaneously. In this investigation, windowed Fourier transform (WFT) [14] is selected as a main processing technique to extract the displacement, velocity and acceleration in time axis. The results show the instantaneous kinematic parameters of a micro-component can be measured by image-plane digital holographic microscopy.

![Experimental setup](image)

**Fig. 1** (a) Schematic layout of experimental setup; (b) Experimental setup in experimental mechanics lab, NUS.

### 2. THEORETICAL ANALYSIS

#### 2.1 Image-plane digital holography

Figure 1 shows a schematic layout of the image-plane digital holographic microscopy which is sensitive to the out-of-plane displacement. The laser light is split into a beam for illumination of the object and a reference beam. The object beam illuminates a specimen with diffuse surface along a direction $e_i$. Some light is scattered in the observation direction $e_o$ where an image-plane hologram is formed on the CCD sensor as a result of the interference between the reference beam and the object beam. The reference beam is introduced by a single-mode optical fiber and is diverged toward the detector. Let $R(x, y)$ and $U(x, y)$ denote the reference and object waves, the intensity recorded on the CCD sensor can be expressed as

$$I_{I}(x, y) = \left| R_{H}(x, y) \right|^2 + \left| U_{H}(x, y) \right|^2 + R_{I}(x, y)U_{H}(x, y) + R_{H}(x, y)U_{I}(x, y)$$

(1)
where the subscript $H$ indicated that these values are in the image plane and * denotes the complex conjugate. If $\theta_{\text{max}}$ is the maximum angle between the object and reference beams, the maximum spatial frequency in the hologram is

$$f_{\text{max}} = \frac{2}{\lambda} \sin \left( \frac{\theta_{\text{max}}}{2} \right)$$  \hspace{1cm} (2)$$

where $\lambda$ is the wavelength of the laser. The experimental setup should be properly designed so that the Nyquist sampling theorem is satisfied over the full area of the detector, and in our case it is $f_{\text{max}} < 1/(2\Delta)$, where $\Delta$ denotes the pixel size of the camera.

The last two terms of Eq.(1) contain information of the amplitude and phase of the object wave. This information can be obtained by spatial filtering using the Fourier-transform method. The detailed description of Fourier evaluation of digital image-plane hologram can be found in Ref. 12. By taking the Fourier transform of the recorded intensity it is possible to filter out one of the last two terms in Eq.(1). After an inversed Fourier transform, the complex amplitude of the wavefront $U_H(x,y)$ is obtained. When a series of digital hologram is captured during the deformation or vibration of an object, a sequence of $U_H(x,y,t)$ is obtained. In this investigation, the obtained 3-D complex matrix is the original measurement data from which the displacement, velocity and acceleration of the testing object are evaluated.

It is well known that deformation of an object involves a change in the phase of the object beam. The relationship between the phase change $\Delta \phi = \phi_i - \phi_o$ and the out-of-plane displacement $z$ is given by

$$\Delta \phi = \frac{2\pi z}{\lambda} \cdot S$$  \hspace{1cm} (3)$$

where $S = e_i - e_o$ is the sensitivity vector given by the geometry of the setup, and $e_i$ and $e_o$ are the unit vectors of illumination and observation, respectively. The phase difference between two digital holograms recorded at $t_i$ and $t_n$ can be calculated by

$$\Delta \phi = \text{arctan} \left( \frac{\text{Im}(U(x,y,t_i) \cdot U^*(x,y,t_n))}{\text{Re}(U(x,y,t_i) \cdot U^*(x,y,t_n))} \right)$$  \hspace{1cm} (4)$$

where Re and Im denote the real and imaginary parts respectively of the complex value. The $\Delta \phi$ obtained from Eq. (4) is within $[0,2\pi)$, and phase unwrapping in time axis allows the determination of the total displacement since the first frame of the measurement process, as opposed to the displacement relative to some other points in the field of view, which is all that can be achieved with spatial unwrapping. If the first hologram is considered as a reference, we will have a series of $U(x,y,t_n) \cdot U^*(x,y,t_1)$ whose phase values after temporal unwrapping $\varphi(x,y;t_n)$ are proportional to the displacement at the instant $t_n$. In this application, two algorithms based on Fourier transform filtering and windowed Fourier ridge methods are applied temporally to evaluate the velocity and acceleration pixel by pixel.

### 2.2 Windowed Fourier analysis

A one-dimensional windowed Fourier transform (WFT) of a temporal signal $f(t)$ and can be written as[15]:

$$Sf(u,\xi) = \int_{-\infty}^{\infty} f(t) g_{u,\xi}^*(t) \, dt$$  \hspace{1cm} (5)$$

where $Sf(u,\xi)$ denotes the spectrum of WFT; and $g_{u,\xi}(t)$ is the WFT kernel, which can be expressed as $g_{u,\xi}(t) = g(t-u)\exp(j\xi t)$. The window $g(t)$ is usually chosen as the Gaussian function: $g(t) = \exp(-t^2/2\sigma^2)$ which permits the best time-frequency localization in analysis. $\sigma$ is a parameter to control the expansion of the window size. On the point $P(x,y)$ of the object, we calculate a series of phase variations $\Delta \phi(x,y;t_n) = \text{arg}(U(x,y,t_n) \cdot U^*(x,y,t_1))$, $n = 1, 2, \ldots, N$, where $N$ is the total frame number captured, and we convert the unwrapped phase $\Delta \phi(x,y;t_n)$ to an exponential signal $C_p = \exp(j \cdot \Delta \phi)$. The windowed Fourier transform of this complex signal will be [15]:

---

*Downloaded from SPIE Digital Library on 25 Aug 2010 to 155.69.4.4. Terms of Use: http://spiedl.org/terms*
\[ SC_p(u, \xi) = \frac{\sqrt{s}}{2} A(u) \exp \left( j \left( \varphi(u) - \tilde{u} \right) \right) \left( g(s \xi - \varphi(u)) + \varepsilon(u, \xi) \right) \]  

(6)

where \( u \) and \( \xi \) represent the time and frequency, respectively. \( s \) is a scaling factor. For a fixed \( s \), \( g_p(t) = s^{-1/2} g(t/s) \) has a support size of \( s \). \( A(u) \) is the modulus of \( C_p \) and in this case, \( A(u) = 1 \). \( \varepsilon(u, \xi) \) is a corrective term which can be neglected if \( A(u) \) and \( \varphi'(u) \) have small relative variations over the support of window \( g \). \( \hat{g}(\omega) \) denotes the Fourier transform of \( g(t) \). The trajectory of maximum \( |SC_p(u, \xi)|^2 \) on the \( u - \xi \) plane is called a windowed Fourier ‘ridge’. Since \( |\hat{g}(\omega)| \) is maximum at \( \omega = 0 \), and if \( \varepsilon \) is negligible, \( |SC_p(u, \xi)|^2 \) reaches maximum when

\[ \xi(u) = \varphi'(u) \]  

(7)

where \( \varphi'(u) \) is defined as instantaneous frequency of the signal, which is proportional to the velocity of \( P(x, y) \). Eq.(7) is valid based on the assumption that the phase of the signal is linear in the local area determined by the extension of \( g \). For nonlinear signal, the assumption is better satisfied when the window size is small. A windowed Fourier transform maps a 1-D temporal signal to a 2-D time-frequency plane, and extracts the signal’s instantaneous frequency. Thus it is more effective to remove the noise within the frequency band of the signal. This is the advantage of WFT over Fourier transform. However, the time-frequency uncertainty principle affects the resolution, which leads to trade-off between time and frequency localization. Once the window size is determined, WFT has a uniform resolution at different frequencies.

3. EXPERIMENTAL ILLUSTRATION

Figure 1(b) shows the experimental setup in the experimental mechanics laboratory of National University of Singapore. The specimen tested a micro-beam clamped at both sides and mounted over a substrate as shown in Fig. 2(a). The length, width and thickness of the beam are 550, 30 and 2.8 \( \mu \)m, respectively. When a voltage with a sinusoidal configuration is applied on the electrodes (shown in Fig. 2a) by two probes, a constrained vibration will be generated on the micro-beam. The frequency of the applied voltage is around 11Hz. The light beam of a He-Ne laser (Melles Groit, 05-LHP-927, Maximum 75mW at \( \lambda = 632.8nm \) ) is divided into the object beam and reference beam. The specimen is illuminated and imaged at an a right angle using a long working distance objective lens (Mitutoyo Apo, 20X) by the sensor of a high-speed camera (KODAK Motion Corder Analyzer, SR-Ultra) with pixel size of 7.4 \( \times \) 7.4 \( \mu \)m\(^2\). With this arrangement, a 1 \( \text{rad} \) phase change is equivalent to a 50.36 \( \text{nm} \) displacement in \( z \)-direction. The interference between the reference and object beams generates an image-plane hologram on the camera sensor. Two hundred holograms are captured with an imaging rate of 500 frames/s. The image size is 512 \( \times \) 240 pixels and the imaging area is shown in Fig. 2(a).

4. RESULTS AND DISCUSSIONS

Figure 2(b) shows a typical digital hologram of a vibrating micro-beam captured by the high-speed camera indicated in Fig. 1(a). The Fourier spectrum and a typical instantaneous phase difference map are also presented in the same figure.

Fig.2 (a) Micro-beam and area of inspection;  
(b) Digital hologram, spectrum and reconstructed wrapped phase difference of the micro-beam.
Two hundred reconstructed phase maps are obtained within a period of 0.4 second. On each pixel, the 1-D phase variation is processed by windowed Fourier transform. Figure 3(a) shows the out-of-plane displacement on point P (shown in Fig.2(b)) after windowed Fourier filtering. The ridge of the windowed Fourier transform is proportional to the velocity. Figure 3(b) shows the velocity variation on point P. The dash-line indicates the ridge. The window size is selected as $\sigma = 2$ to limit the errors due to linear phase approximation. Similarly the acceleration can also be obtained on each point. Figure 3(c) shows the acceleration on point P. At each instant, a spatial distribution of displacement, velocity and acceleration can be obtained by combining the values on each point.

![Fig.3 Instantaneous (a) displacement; (b) velocity (c) acceleration of point P on the micro-beam.](image)

The windowed Fourier ridge method maps a 1-D signal to a 2-D plane of time and frequency, and extracts the instantaneous frequencies with the highest energy. It is a time-consuming process and requires a high computing speed and large memory. In our investigation, we are using the maximum value in the Fourier spectrum of $f(t)g(t-u)$ as an initial estimation of the ridge value at each instant and then search the actual ridge with a preset accuracy. The computation time depends on the accuracy required, but it is still much longer than that of the Fourier analysis. However, this disadvantage of WFT has become inconspicuous due to the rapid improvement in the capacity of computers.

It is noteworthy that the measurement range of the velocity and acceleration is limited due to the constraint of Nyquist sampling theorem and imaging rate of the high-speed camera. The maximum phase change between two adjacent frames is $\pm \pi$. However digital holography can extract the phase from one hologram, no phase ambiguity problem exists. It is not necessary to introduce the temporal carrier. Comparing to other optical interferometries such as ESPI and shearography, the vibration measurement range is tremendously enlarged with the absence of temporal carrier frequency.

5. CONCLUSION

In this paper we have presented a novel method for evaluating the transient displacement, velocity and acceleration of a vibrating micro-object by image-plane digital holographic microscopy. Windowed Fourier transform is applied to the temporal processing of a series of digital holograms. One-dimensional windowed Fourier filtering and temporal unwrapping is applied to obtain a smooth temporal displacement, while windowed Fourier ridge method is applied to evaluate the instantaneous velocity and acceleration of the beam. As the digital holography can extract the instantaneous phase of an object, there is no phase ambiguity problem involved in vibration measurement. However, a wide spectrum including both negative and positive frequencies requires a robust processing algorithm to remove the noise and extract the useful information from the signal. Windowed Fourier analysis well performs in this case. The results show that the combination of image-plane digital holographic microscopy and temporal windowed Fourier analyses allows the simultaneous evaluation of three important vibration parameters of a micro-component at different instants.

REFERENCES