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Quantitative detection and compensation of phase-shifting error
in two-step phase-shifting digital holography

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ABSTRACT
Phase-shifting digital holographic technique is a powerful tool for the measurement of various physical parameters, such as object deformation and liquid or cell’s refractive index change. However, for an accurate measurement, phase-shifting error in the reference wave path is still a major issue. In this paper, three novel and simple algorithms are proposed to quantitatively detect and correct phase-shifting error for a pure phase object in two-step phase-shifting digital holography. Influence of phase-shifting error is illustrated, and the effectiveness of the proposed algorithms is demonstrated by numerical simulation results.

Keywords: Phase-shifting digital holography; Phase-shifting error; Quantitative detection; Quantitative compensation

1. Introduction

Since holography was introduced by Gabor [1], it has been widely applied to practical measurements [2–6]. Some types of techniques, such as off-axis digital holography [7] and phase-shifting digital holography [8,9], have also been developed. Compared with wet chemical processing method in optical holography, off-axis digital holography is limited by the lower resolution of charge-coupled device (CCD), and object size cannot be too large. In the off-axis experimental arrangement, some algorithms, such as convolution method [7] and Hilbert transform [10], can be employed to obtain phase-contrast maps. Phase-shifting digital holography can overcome the above problems, and enhance the quality of reconstructed images [8,9]. Phase-shifting technique [11,12] usually employs two or more fringe patterns to extract phase-contrast maps, and has been widely applied to many fields [7,12]. However, with a phase-shifting operation, phase-shifting error in the reference wave path can not be completely avoided due to an imperfect calibration of a piezoelectric transducer. The phase-shifting error can make a reconstructed image blurred, especially for the reconstructed phase-contrast map. To eliminate the influence of phase-shifting error, some algorithms, such as frequency-shifting method [13] and an averaging method [14], have been developed. In the previous approaches [13,14], much more experimental effort should be made to eliminate or suppress phase-
shifting error. In Ref. [15], amplitude deviation summation of the reconstructed wave is calculated as an evaluation function to determine phase-shifting error.

In this paper, we propose three novel and simple algorithms to quantitatively detect and correct phase-shifting errors for a pure phase object in two-step phase-shifting digital holography. Influence of the phase-shifting errors is illustrated, and the effectiveness of the proposed approaches is demonstrated by numerical simulations.

2. Theoretical analysis

The in-line hologram intensities, denoted by $I_i(x, y)$, are obtained by the interference between an object wave $O(x, y; z)$ and a plane reference wave $R_i(x, y)$ in the hologram plane.

$$I_i(x, y) = O(x, y; z)O^*(x, y; z) + R_i(x, y)R_i^*(x, y) + R_i(x, y)O(x, y; z)O^*(x, y; z),$$

where $z$ denotes the distance between object plane and hologram plane, factor $i = 1, 2,$ and * denotes the complex conjugate.

Without phase-shifting error, the object wave in the hologram plane is expressed as

$$O(x, y; z) = \frac{I_1(x, y) - I_0(x, y) - I_R(x, y) + j[I_2(x, y) - I_0(x, y) - I_R(x, y)]}{2R}$$

$$= A(x, y) \exp[j\phi(x, y)],$$

where $R$ denotes real amplitude of the reference wave, $A(x, y)$ denotes real amplitude of the object wave, $\phi(x, y)$ denotes phase map of the object wave, $j = \sqrt{-1}, I_0 = |A|^2,$ and $I_R = |R|^2$. The intensities $I_1(x, y)$ and $I_2(x, y)$ are recorded when the phase shift of reference wave path is $0$ and $\pi/2$, respectively. In practice, $I_0(x, y)$ and $I_R(x, y)$ can be recorded by blocking the reference and object waves, respectively.

With a phase-shifting error of $\Delta \theta$ in the reference wave path, $I_1(x, y)$ and $I_2(x, y)$ are described by

$$I_1 = R^2 + A^2 + RA \exp(j\phi) + RA \exp(-j\phi),$$

$$I_2 = R^2 + A^2 - jRA \exp(j\phi) \exp(-j\Delta \theta) + jRA \exp(-j\phi) \exp(j\Delta \theta),$$

where for simplicity, the coordinate $(x, y)$ is omitted. Hence, the object wave in the hologram plane is given by
After the object wave is determined in the hologram plane, an object wave \( O(\xi, \eta; 0) \) in the image plane is determined by

\[
O(\xi, \eta; 0) = F^{-1}[H(f_x, f_y; -z)O(f_x, f_y; z)],
\]

where \( f_x \) and \( f_y \) are spatial frequencies, \( F^{-1} \) denotes 2D inverse Fourier transform, \( O(f_x, f_y; z) \) denotes 2D Fourier transform of \( O(x, y; z) \), and \( H(f_x, f_y; -z) \) is a transfer function described by

\[
H(f_x, f_y; -z) = \begin{cases} 
\exp \left( jkz \left[ 1 - (f_x^2 + f_y^2)^{1/2} \right] \right) & \text{if } (f_x^2 + f_y^2)^{1/2} < (1/\lambda), \\
0 & \text{otherwise},
\end{cases}
\]

where \( \lambda \) denotes laser wavelength, and wave number \( k = 2\pi/\lambda \). In practice, the bandwidth is determined by some factors, such as CCD pixel size. It is assumed in this study that \( f_x \) and \( f_y \) are always within the above bandwidth disk. Hence, a discrete representation of Eq. (7) can be expressed as

\[
H(m, n; -z) = \exp \left( jkz \left[ 1 - \left( \frac{m-M/2}{M\Delta x} \right)^2 - \left( \frac{n-N/2}{N\Delta y} \right)^2 \right]^{1/2} \right),
\]

where \( m = 0, 1, \ldots, M-1, n = 0, 1, \ldots, N-1, M \) and \( N \) denote pixel numbers of the CCD, and \( \Delta x \) and \( \Delta y \) denote pixel sizes of the CCD. A significant advantage of the algorithm in Eq. (6) is no minimum reconstruction distance requirement.

The amplitude-contrast distribution \( A(\xi, \eta) \) is determined by

\[
A(\xi, \eta) = \sqrt{\text{Im}[O(\xi, \eta; 0)]^2 + \text{Re}[O(\xi, \eta; 0)]^2}.
\]

As illustrated in Eqs. (5) and (6), due to a phase-shifting error, the twin image cannot be fully eliminated after object reconstruction. The phase-shifting error would make the reconstructed phase-contrast map inaccurate. To eliminate or suppress the influence of phase-shifting error, in our study, two-step phase-shifting method is firstly rewritten as [15]

\[
O(x, y; z) = \frac{l_1 - l_0 - l_R - \exp \left[ -j\left( \frac{\xi}{2} + \Delta \theta' \right) \right] (l_2 - l_0 - l_R)}{R \left( 1 - \exp \left\{ -j \left[ 2\left( \frac{\xi}{2} + \Delta \theta' \right) \right] \right\} \right)},
\]
where $I_2$ contains a fixed phase-shifting error, and $\Delta \theta'$ denotes a phase-shifting error searching parameter.

Within a defined phase-shifting error detection range, each phase-shifting error $\Delta \theta'$ is used in Eq. (10), and a series of amplitude-contrast distributions is obtained using Eqs. (6) and (9). Three algorithms are proposed to evaluate the reconstructed amplitude-contrast maps and quantitatively detect the phase-shifting error. The first algorithm uses a Laplace operator:

$$S_1 = \int_\eta \int_\xi \left\{ \left[ \partial^2 A(\xi, \eta) / \partial \xi^2 \right] + \left[ \partial^2 A(\xi, \eta) / \partial \eta^2 \right] \right\}^2 d\xi d\eta,$$

(11)

The second algorithm uses a spectral analysis described by

$$S_2 = \int_\eta \int_\xi \log \{1 + |F[A(\xi, \eta)]| \} d\xi d\eta,$$

(12)

where $F$ denotes 2D Fourier transform, $f_\xi$ and $f_\eta$ are spatial frequencies, and $\| \|$ denotes the calculation of the magnitude. In certain cases, $F[A(\xi, \eta)]$ may be filtered before the integration.

The third algorithm uses a gradient operator described by

$$S_3 = \int_\eta \int_\xi \sqrt{\left( \partial A(\xi, \eta) / \partial \xi \right)^2 + \left( \partial A(\xi, \eta) / \partial \eta \right)^2} d\xi d\eta.$$

(13)

The above algorithms are carried out based on the reconstructed amplitude-contrast maps and image sharpness principle [16]. Phase-shifting error is quantitatively detected using a global minimum value of $S_1$, or $S_2$, or $S_3$, and the global minimum point is usually obtained as the slope signs change from negative to positive. A flow chart of the procedure to determine phase-shifting error is shown in Fig. 1.

3. Simulation and result discussions

We carry out a numerical simulation to illustrate the effectiveness of the proposed algorithms. In numerical simulation, the image size is $512 \times 512$ pixels, and the pixel size of CCD camera is $7.4$ $\mu$m. The laser wavelength is $635$ nm, and the distance between object plane and hologram plane is $26$ cm. The phase-contrast map of a sample phase object is shown in Fig. 2a. Typical holograms with phase shifts of 0 and $\pi/2$ are shown in Fig. 2b and c, respectively. Fig. 3a–c show the reconstructed phase-contrast maps with prescribed phase-shifting errors of 0.064 rad, 0.104 rad, and 0.124 rad, respectively. The transversal resolution in the image plane is equal to pixel size of CCD camera ($7.4$ $\mu$m). As can be seen in Fig. 3a–c, the quality of reconstructed phase-contrast maps is poor mainly due to the superposition of twin image.

Fig. 4a–c show the proposed algorithms as a phase-shifting error is 0.064 rad. The phase-shifting error detection range is defined from $-0.032$ rad to $0.160$ rad, and the iterative
step is 0.008 rad. It is noteworthy that with an unknown phase-shifting error, a larger phase-shifting error detection range can be defined. As illustrated in Fig. 4a–c, all the proposed algorithms can effectively and accurately detect the phase-shifting error of 0.064 rad. The values of $S_1$, $S_2$, and $S_3$ at the global minimum points are $5.41 \times 10^{-25}$, 12.48, and $1.48 \times 10^{-10}$, respectively. Average computing time in Fig. 4a–c is 125 s, 96 s, and 121 s using a computer with Pentium IV 3.0 GHz, respectively. The time does not contain the procedures of original object loading and holograms generation. As can be seen in Fig. 4a–c, in this investigation, spectral analysis and gradient operator algorithms reach much sharper global minimums compared with Laplace operator algorithm. It is worth noting that a suitable iterative step in the proposed algorithms should be selected. As the iterative step is 0.01 rad and a prescribed phase-shifting error is also 0.064 rad, a phase-shifting error of 0.068 rad is determined by the proposed algorithms as shown in Fig. 5a–c. In this case, a relatively accurate phase-shifting error is also obtained, and error detection precision is high.

**Fig. 6a** shows a reconstructed phase-contrast map of the sample object after the compensation is introduced, as iterative step is 0.008 rad. It is shown that a high-quality phase-contrast map with the twin image removed is obtained. Fig. 6b shows a phase-contrast map after compensation using the detected phase-shifting error of 0.068 rad, as iterative step is 0.01 rad. Since the iterative step is still low and error detection algorithms possess high accuracy, a high-quality reconstructed phase-contrast map is also obtained in Fig. 6b. As error detection precision of three decimal places after the radix point is required, an iterative step of 0.001 rad can be chosen. To reduce computing time, a fast method with gradually decreasing of the iterative step can be used. In practice, error detection precision can be largely ensured within the resolution of the phase-shifting modulators by use of the proposed algorithms.

An experiment can be further conducted in the future, and a study on the influence of shot and thermal noise may be an important future issue. However, it is worth noting that phase images are much less sensitive to coherent noise [17]. Moreover, careful quality control of optical components, laser beam and CCD camera in the experiment can suppress the influence of noise on the phase-shifting error detection results, and a standard pure-phase mask [15] can also be used in practice.

**4. Concluding remarks**

In this paper, three novel algorithms are proposed to quantitatively detect and correct the phase-shifting error for a pure phase object in two-step phase-shifting digital holography. The influence of phase-shifting error is illustrated, and the results show that the proposed algorithms are effective. It is also illustrated that with the proposed algorithms, high quality of phase-contrast maps after compensation can be obtained. The proposed algorithms would be particularly useful in the study of biology [10,18,19] and micro-optical components [20] in two-step phase-shifting digital holography.
References

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Fig. 1 A flow chart of the procedure to determine phase-shifting error using the proposed algorithms.

Fig. 2 (a) Phase-contrast map of original pure phase object; (b) typical hologram with phase shift of 0; and (c) typical hologram with phase shift of $\pi/2$.

Fig. 3 Reconstructed phase-contrast map with phase-shifting errors of: (a) 0.064 rad; (b) 0.104 rad; and (c) 0.124 rad.

Fig. 4 An iterative step of 0.008 rad is selected: (a) Laplace operator algorithm; (b) spectral analysis algorithm; and (c) gradient operator algorithm within a defined phase-shifting error detection range.

Fig. 5 An iterative step of 0.01 rad is selected: (a) Laplace operator algorithm; (b) spectral analysis algorithm; and (c) gradient operator algorithm within a defined phase-shifting error detection range.

Fig. 6 For all the proposed algorithms, the reconstructed phase-contrast maps after compensation as iterative steps are: (a) 0.008 rad; and (b) 0.01 rad.
Fig. 1

Recording of intensities $I_1, I_2, I_0,$ and $I_G$

Definitions of phase-shifting error detection range $[\Delta \theta_1', \Delta \theta_2']$ and iterative step $\delta$

Initial phase-shifting error $\Delta \theta_1'$

Applied to Eq. (10)

Numerical reconstruction [Eq. (6)] and calculation of amplitude contrast distributions [Eq. (9)]

Detection algorithms (1), (2), or (3)

Iterative step addition

$\Delta \theta_1' = \Delta \theta_1' + \delta$

Modification of the defined phase-shifting error detection range or iterative step

End of phase-shifting error detection range $\Delta \theta_2'$?

Yes

Global minimum point is found?

Yes

Quantitative detection of phase-shifting error and corresponding compensation

Phase-contrast map

No

No
Fig. 3
Fig. 4
Fig. 5

(a) Laplace operator method

(b) Spectral analysis method

(c) Gradient operator method