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Influence of the Elastic Properties of the Spring Element on the Rotor Tuning Condition of a Rotor Vibratory Gyroscope

Kirill V. Poletkin, Alexsandr I. Chernomorsky, Christopher Shearwood

Abstract—This paper considers the dependence of the rotor tuning condition of a rotor-vibratory gyroscope on the elastic properties of the spring element which has, in particular, additional angular deflection along an axis perpendicular to the output axis of gyroscope. As a consequence of this dependence, the limitations to the performance of the rotor tuning condition are examined and a mathematical equation describing the rotor tuning error, dependent on variations in the gyroscope parameters, is obtained. Analysis of the equation of rotor tuning error allows us to formulate the condition needed for minimization of this error against temperature change.

Index Terms—Inertial Sensor, Gyroscope, Angular rate.

I. INTRODUCTION

GYROSCOPES for measuring rate or angle of rotation have attracted a lot of attention over the last decade for a wide range of applications such as ride stabilization and rollover detection in automotive area; consumer electronic applications, such as video-camera stabilization, virtual reality, and inertial mice for computers; robotics applications; children’s toys; and a wide range of military applications. In fact, the continued development is opening up an ever increasing list of applications, some of which require tactile or inertial grade performance [1], [2].

For autonomous navigation and orientation of many modern, highly dynamic movable objects Strapdown Inertial System (SIS), directly mounted onto the object, are used. Typically, the sensory element of the SIS that provides measurements of angular rate is a gyroscope. These gyroscopes have to meet a wide range of metrological, operational, technical as well as economical requirements which are often difficult to balance. An advanced gyroscope, frequently applied to the above mentioned scenarios is the Rotor Vibratory Gyroscope (RVG) [3]–[8], and [9]. Although not widely reported in the literature, it has the capability of fully satisfying the technical and economical requirements for SIS of such movable objects.

A schematic of a RVG is shown on Fig. 1. It consists of Motor Supply (MS), Signal Processing System (SPS), and a gyroscope. The gyroscope includes a Driving Motor (DM), a dual axis Angular displacement Sensor (AS), a dual channel electromagnetic Torque Sensor (TS), and a rotor. The rotor is mounted on the shaft of the DM by means of a Spring Element (SE) which includes a ring-type magnetic core and two magnets of the same geometry.

The operating principle of a RVG is described as follows: an electromotive force within each coil of the AS is induced by the vibratory motion of the rotor magnets, the deflection of this vibratory motion in one turn is proportional to the projections of the measured angular rate along axes O_ξ and O_η that are fixed to the gyroscopic casing. This induced voltage is then fed into the SPS. Each TS coil is connected to a load resister, R_L, and the potential drop, U_out, across R_L is proportional to the projections of the measured angular rates ω_ξ and ω_η on the respective measuring axes.

Presently, the performance level of the RVG has not yet achieved the necessary tactical-grade quality (1 deg/h) for application to near-term development of highly dynamical movable objects. The experimental investigation of the RVG bias drift indicates that the accuracy of the measured angular
rate is limited by the Thermal Bias Drift (TBD) [10].

The TBD may be represented by a heuristic model and can be written as [11]:

\[ U_{δω} = \frac{U_l}{K_{RVG}}, \]  
(1)

where \( U_{δω} \), the potential drop across \( R_L \), is equivalent to the RVG bias drift; \( K_{RVG} \) is the RVG gain and is equal to the product of TS, AS, and the mechanical part of gyroscope (\( K_g \)) gain; \( U_l \) is the source of thermal bias drift present as the bias of the AS output.

According to Eq. (1), to reduce the influence of \( U_l \) on RVG (the entire thermal bias drift) then \( K_{RVG} \) must be increased. Basically, the value of \( K_{RVG} \) depends on the mechanical part of gyroscope gain, \( K_g \). A maximum value of \( K_g \) is obtained by meeting the Rotor Tuning Condition (RTC) of the RVG. As shown both experimentally and theoretically (over the same time frame) the RTC of RVG depends sensitively on its relationship with the stiffness of the SE.

However, to minimize the influence of TBD on RVG, it is important not only to meet the requirement of the RTC but also to maintain this condition despite variations in gyroscope parameters, in particular, those induced by changes in temperature. This type of problem has previously been investigated [6], [7], [12] and a mathematical model that describes change of the rotor tuning condition with temperature has been formulated for an ideal SE, with only one degree of freedom to the rotor. But to obtain a more accurate model dependence of rotor tuning condition on the change in temperature and, finally, to increase the accuracy of RVG, the influence of the elastic properties of a real fabricated SE needs to be taken into consideration.

This paper considers the rotor tuning condition of a RVG taking into account the equatorial axis stiffness \( Oy \) that is perpendicular to the output axis stiffness \( Ox \), the limitation of this condition, and its dependence on temperature variations.

II. MATHEMATICAL MODEL DESCRIBING THE RVG ROTOR MOTION

The ideal construction of the SE has to provide only angular deflection along the \( Ox \) axis. However, in reality, a real SE [13]–[16] has additional angular variation, in particular, along the \( Oy \) axis (see Fig. 1). As stated previously, the rotor of the RVG has two degree of freedom relative to the shaft of the drive motor.

To obtain a mathematical model of the RVG rotor, it is assumed that the angular deflection of the rotor is small and Hookian, the angular rate of the motor shaft is constant and the static and dynamic unbalances of the rotor are negligible. The angle of deflection of the rotor along the \( Ox \) and \( Oy \) axes are denoted by \( α \) and \( β \) respectively. Consequently, an expression for the \( α \) and \( β \) deflections may be represented as:

\[
\begin{bmatrix}
\alpha \\
\beta
\end{bmatrix} = \begin{bmatrix}
\delta_{11} & \delta_{12} \\
\delta_{21} & \delta_{22}
\end{bmatrix} \begin{bmatrix}
X_1 \\
X_2
\end{bmatrix},
\]  
(2)

where \( \delta_{ij} \) (\( i = 1, 2; j = 1, 2 \)) - coefficients of flexibility matrix \( \delta \); \( X_1 \) and \( X_2 \) - components of moment vector \( \mathbf{X} \) which are the sums of the active and inertial torque moments applied to the rotor along the \( Ox \) and \( Oy \) axes respectively. Both \( X_1 \) and \( X_2 \) can be represented as follows:

\[
X_1 = -J_x (\ddot{\alpha} - \Omega \dot{β}) - (\ddot{β} + \Omega α)\Omega (J_z - J_y) - \mu \dot{α} + M_x;
\]

\[
X_2 = -J_y (\ddot{β} + \Omega α) - (\ddot{α} - \Omega β)\Omega (J_z - J_x) - \mu \dot{β} + M_y,
\]  
(3)

where \( J_x, J_y, J_z \) - denote the three central principal moments of inertia of the rotor; \( μ \) - damping coefficient of the SE; \( M_x, M_y \) - the sums of moments from measuring angular rate (gyroscope moments) and the torque coil.

It is assumed that the physical flexibility axes of the SE and geometrical axes \( Ox \) and \( Oy \) are coincident; hence, the units of displacement \( δ_{12} = δ_{21} = 0 \) can be neglected. A system of differential equations are generated by substituting equation (3) into (2). The solution of this system in operator form is as follow:

\[
\alpha(p) = \frac{\delta_{11} \left\{ \delta_{22} \left( J_y p^2 + μ p + Ω^2(J_z - J_y) \right) + 1 \right\} \times}{\Delta(p)} \rightarrow M_z(p) - \delta_{11} \delta_{22} \left( J_z - J_y - J_x \right) Ω p M_y(p),
\]  
(4)

where

\[
\Delta(p) = \left\{ \delta_{11} \left( J_x p^2 + μ p + Ω^2(J_z - J_y) \right) + 1 \right\} \times \left\{ \delta_{22} \left( J_y p^2 + μ p + Ω^2(J_z - J_x) \right) + 1 \right\} + Ω^2 \left( J_z - J_y - J_x \right)^2 p^2.
\]  
(5)

Notice that the quotient \( 1/Δ(p) \) is the mechanical part of the gyroscope gain \( K_g \) in operator form.

III. ROTOR TUNING CONDITION

A maximum in \( K_g \) can be achieved only when the RVG is tuned. By substituting the complex term \( p = jΩ \) into (5) and setting the determinant \( Δ(p) \) equal to zero, the following expression is obtained:

\[
\left\{ \delta_{11} \left( -J_x Ω^2 + μ j Ω + Ω^2(J_z - J_y) \right) + 1 \right\} \times \left\{ \delta_{22} \left( -J_y Ω^2 + μ j Ω + Ω^2(J_z - J_x) \right) + 1 \right\} = 0.
\]  
(6)

By neglecting internal friction, \( μ \), the RTC that determines the optimum value of the angular rate \( Ω \) of the drive motor shaft becomes:

\[
Ω^2 = \frac{\delta_{11} + \delta_{22} - \sqrt{\left( \delta_{11} + \delta_{22} \right)^2 - 8\delta_{11} \delta_{22}}}{4J_x (1 - \chi) \delta_{11} \delta_{22}},
\]  
(7)

where \( χ = (J_z - J_y)/J_x \) is the gyroscope constructive parameter.

To further simplify this expression, a dimensionless parameter is introduced:

\[
λ = δ_{22}/δ_{11}.
\]  
(8)
By substituting Eq. (8) into Eq. (7), the RTC can be rewritten as:
\[
\Omega^2 = \frac{1 + \lambda - \sqrt{1 - 6\lambda + \lambda^2}}{4J_x(1-\chi)\delta_1\lambda}.
\] (9)

In particular, according to (9), if \(\delta_{11} >> \delta_{22}(\lambda << 1)\), then the classical rotor tuning equation of a RVG is obtained [5]:
\[
\Omega^2 = \frac{1}{J_x(1-\chi)\delta_1}.
\] (10)

An analysis of Eq. (9) reveals the relationships between flexibilities \(\delta_{11}\) and \(\delta_{22}\), under which the RVG cannot be tuned. Values of \(\lambda\) that correspond to a positive value of the state discriminant \(1 - 6\lambda + \lambda^2\), need to be found. For this reason, the state discriminant is equated to zero:
\[
1 - 6\lambda + \lambda^2 = 0.
\] (11)

The roots of the quadratic Eq. (11) are \(\lambda_{1,2} = 3 \pm 2\sqrt{2}\). The sign of the discriminant over the following three intervals: I (\(0 < \lambda < \lambda_2\)), II (\(\lambda_2 < \lambda < \lambda_1\)), III (\(\lambda_1 < \lambda\)) can then be obtained and are tabulated in the Table I.

<table>
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<th>Interval</th>
<th>I</th>
<th>II</th>
<th>III</th>
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<tbody>
<tr>
<td>Sign of discriminant</td>
<td>+</td>
<td>−</td>
<td>+</td>
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It can be seen from Table I that the discriminant is less than zero only in the interval II; hence, for this case, rotor tuning is impossible. Accordingly, to achieve rotor tuning of the RVG, the coefficient of flexibility \(\delta_{22}\) of the SE along \(Oy\) axis should be less than \((3 - 2\sqrt{2})\delta_{11}\) or approximately 0.172\(\delta_{11}\). Mathematically, the relationship between the flexibilities of SE may be represented as follows:
\[
\delta_{22} < (3 - 2\sqrt{2})\delta_{11} \quad \text{or} \quad \lambda < (3 - 2\sqrt{2}).
\] (12)

Only under the condition described by equation (12) is rotor tuning possible and, as a result, \(K_g\) maximized.

For practical reason, an analysis of interval III is not necessary because in this interval the \(Oy\) and \(Ox\) axes only change their functions (i.e. the \(Oy\) axis becomes the output axis of the SE).

To understand the behavior of the rotor tuning dependence on the parameter \(\lambda\), let’s first define the concept of relative rotor tuning, \(\tilde{\Omega}\). Both sides of Eq. (9) are divided by \(1/(J_x(1-\chi)\delta_1)\) after which the square root is taken. The relative rotor tuning can be written as:
\[
\tilde{\Omega} = \sqrt{\frac{1 + \lambda - \sqrt{1 - 6\lambda + \lambda^2}}{4\lambda}}.
\] (13)

Using Eq. (13), the \(\tilde{\Omega}(\lambda)\) dependence can be graphically represented as in Fig. 2. The function \(\tilde{\Omega}(\lambda)\) is limited by two asymptotes: a vertical asymptote perpendicular to the \(\lambda\) - axis and passes through the point where \(\lambda\) is equal to \(3 - 2\sqrt{2}\) or around 0.172 and a skew asymptote \(1 + \lambda/2\).

An estimate of the relative error of rotor tuning \(\Delta\tilde{\Omega}(\lambda)\) can be found by subtracting 1 from \(\tilde{\Omega}(\lambda)\) in the range of \(\lambda\) from 0 to 0.01; that is the practical range of interest. In this range \(\Delta\tilde{\Omega}(\lambda)\) has linear characteristics, close to \(\lambda/2\) and changes in magnitude from 0 to 0.6 % (see Fig. 3). This error in the rotor tuning (as verified by experimental work) gives rise to a reduction in \(K_g\) of up to 50%.

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IV. DEPENDENCE OF ROTOR TUNING CONDITION ERROR ON VARIATION IN PARAMETERS OF ROTOR VIBRATORY GYROSCOPE

Within the literature [6], [12] the dependence of RTC on the variation in gyroscope parameters has only been obtained for a SE which supplies one degree of freedom to the gyroscope rotor, in other words for a RTC that is described by Eq. (10). However, practically, the RTC is best described by Eq. (9) and the influence of variations in \(\lambda\) on the change in RTC needs to be taken into consideration.

Let’s consider a construction of the SE in which the change of parameter \(\lambda\) lies in the range of 0 to 0.01. In this case, the
dependence of rotor tuning on \( \lambda \) can be estimated by the skew asymptote as follows (see Fig. 3)

\[
\Omega = \Omega_0 (1 + \lambda / 2),
\]

where \( \Omega_0 = 1/\sqrt{J_x(1-\chi)\delta_{11}} \). Hence, the dependence of RTC on variations in parameters \( \lambda \) and \( \Omega_0 \) can be written as:

\[
\Delta \Omega = \Delta \Omega_0 (1 + \lambda / 2) + \Omega_0 \Delta \lambda / 2.
\]  
(15)

Since \( \lambda \ll 1 \), equation (15) can be expressed as

\[
\Delta \Omega \approx \Delta \Omega_0 + \Omega_0 \Delta \lambda / 2.
\]  
(16)

After simplification, the variation of \( \Delta \Omega_0 \) with \( J_x, \chi \), and \( \delta_{11} \) can be rewritten as:

\[
\Delta \Omega_0 = \Omega_0 \frac{1}{2} \left\{ -\frac{\Delta J_x}{J_x} + \frac{\Delta \chi}{1 - \chi} - \frac{\Delta \delta_{11}}{\delta_{11}} \right\},
\]  
(17)

and the variation of \( \Delta \lambda \) against \( \delta_{11} \) and \( \delta_{22} \) can be expressed as:

\[
\Delta \lambda = \frac{\Delta \delta_{22}}{\delta_{11}} - \lambda \frac{\Delta \delta_{11}}{\delta_{11}}.
\]  
(18)

By substituting equations (17) and (18) into equation (16) and making the approximation \( \lambda \ll 1 \), the dependence of the rotor tuning condition on the variation in the gyroscopic parameters can be written as:

\[
\Delta \Omega = \Omega_0 \frac{1}{2} \left\{ -\frac{\Delta J_x}{J_x} + \frac{\Delta \chi}{1 - \chi} - \frac{\Delta \delta_{11}}{\delta_{11}} + \frac{\Delta \delta_{22}}{\delta_{11}} \right\}.
\]  
(19)

Under quasi-static thermal process [12], [17] the relative variation in the moments of inertia \( \Delta J_x/J_x \) and \( \Delta \chi/(1 - \chi) \) depends on the Temperature Coefficient of Density (TCD) and the Thermal Linear Expansion Coefficient (TCLE) of the rotor material. At the same time, the relative variation in the coefficients of flexibility \( \Delta \delta_{11}/\delta_{11} \) and \( \Delta \delta_{22}/\delta_{11} \) depend on the TCLE and the Temperature Coefficients of the Modulus of Elasticity (TCME). Also, the relative variation in \( \Delta \delta_{22}/\delta_{11} \) depends on the ratio of geometrical dimension elastic elements and both the modulus of elasticity in tension and shear. As a rule, the value of TCME is greater by one order of magnitude compared to the values of TCD and TCLE. For instance, the TCME of silicon varies in the range of \(-80 \) to \(-40 \times 10^{-6} K^{-1} \) whereas its TCLE varies only in the range fo \( 2 \) to \( 7 \times 10^{-6} K^{-1} \). For phosphor bronze, widely used in the construction of spring elements, TCME and TCLE are equal to \(-33.5 \times 10^{-6} K^{-1} \) and \( 1 \times 10^{-5} K^{-1} \), respectively.

Now the major source of RTC variation, for an ideal SE, is due to temperature variations in the materials modulus of elasticity [6]. Thus, the major source of rotor tuning error (against temperature changes) for the condition which is described by equation (7), is due to changes in the modulus of elasticity with temperature, which is taken into account by the relative variation in coefficients of flexibility \( \Delta \delta_{11}/\delta_{11} \) and \( \Delta \delta_{22}/\delta_{11} \).

Based on this inference, the condition necessary to minimize the rotor tuning error under temperature variation can be formulated as follows:

\[
\frac{\Delta \delta_{22} - \Delta \delta_{11}}{\delta_{11}} \rightarrow \min.
\]  
(20)

The minimum described by equation (20) can be achieved by the appropriate dimension selection of the elastic elements and the elastic properties of the SE material.

V. Conclusion

The dependence of rotor tuning condition of a rotor vibratory gyroscope on the elastic properties of spring elements and discussed limitation of this rotor tuning condition allow, on the one hand, to achieve a maximum value of \( K_g \) and in this way to reduce the influence of the thermal bias drift. On the other hand, to regulate the rational selection of angular equatorial stiffness of the rotor spring element contraction.

At the same time, based on this dependence, a mathematical model of the error of rotor tuning is obtained. Analysis of this model show us that the major source of error in rotor tuning under temperature variation is due to changes in the modulus of elasticity with temperature, which are taken into account by relative variation in the coefficients of flexibility. As a result of this analysis, the condition for minimization of the error of rotor tuning under temperature variation is formulated. Achieving this condition allows the mechanical part of gyroscope gain, \( K_g \), to be close to a maximum as temperature is varied and hence minimization of the thermal bias drift of rotor vibratory gyroscope.

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