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<th>Insertion loss of parallel wide barriers.</th>
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<td>Date</td>
<td>2008</td>
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ABSTRACT

A ray method is proposed for predicting the insertion loss of parallel wide barriers, where some neighboring ones have same height. The method is based on the geometrical theory of diffraction and extended from an exact boundary solution for a rigid wedge. The model is available for arbitrary receiver locations, even on the shadow or reflection boundaries from the barrier top edges or being very close to the edges. The results from the simulations and the boundary element method are compared to validate the ray model, which shows that the model is considerably accurate for engineering applications.

1 INTRODUCTION

To assess the impacts of noise on multiple residential buildings along highways, a quantitative description of sound diffraction around these buildings is required. Sometimes these buildings are similar with same height and the receiving points are close to the top edges of these buildings compared to wavelength, such as top floor windows.

There are many previous studies on predicting sound diffraction around similar obstacles. Keller [1-3] presented the geometrical theory of diffraction (GTD) to efficiently describe the diffraction in terms of rays, which is accurate enough for most practical cases. Pierce [4, 5] presented solutions for single diffraction around a wedge and for double diffraction over a single wide barrier. Chu et al. [6] further extended Pierce’s formulas [4] for a wide barrier with finite thickness. Kwai [7] developed a method for diffraction around a multi-sided barrier, which was later modified by Kim et al. [8] for many extended cases such as polygonal-like shapes. Salomons [9] presented a model for sound propagation over several wedges with receiver far from the edge. Using statistical energy analysis, Reboul et al. [10] recently proposed an equation able to predict the multiple diffraction around several diffracting edges.

However all these reviewed previous models either are hard to be used for sound diffraction over obstacles with more than four edges or lead to singularities for wide barriers of equal height. In this paper, a method convenient for engineering application is derived to predict the insertion loss of separate parallel wide barriers with some neighboring ones of equal height for arbitrary receiver locations.
2 THEORETICAL FORMULAS

Two wide barriers of different heights shown in Figure 1 are taken for illustrating the geometry and overall sound diffraction of the current problem, where source is a point source or coherent line one and the ground is infinite and rigid. Only the incident sound normal to the lengthwise axis of barriers is considered, thereby the geometry is simplified to a plane perpendicular to lengthwise axis including the receiver and source shown in Figure 1.

Sound diffraction over these wide barriers is simplified and illustrated by arrowed lines in Figure 1. There are 12 main diffraction rays in energy by geometric rules: \( S(S') \rightarrow 1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow R(R') \), \( S(S') \rightarrow 1 \rightarrow 2 \rightarrow M \rightarrow 3 \rightarrow 4 \rightarrow R(R') \) and \( S(S') \rightarrow 1 \rightarrow 3 \rightarrow 4 \rightarrow R(R') \). The brackets mean optional. When the heights of two barriers become identical, every edge is assumed to be able to observe all the others from its location when searching diffraction rays similarly in Figure 1.

![Figure 1: Overall sound diffraction over wide barriers](image)

2.1 Diffraction Coefficient

Diffraction coefficient is introduced to solve the field of an individual diffraction ray. In Figure 1, the single sound diffraction field \( \phi_d \) at \( R \) can be written as [1]

\[
\phi_d = \phi_{inc} \cdot D_E ,
\]

where \( \phi_{inc} \) is the incident sound to edge \( E \) and \( D_E \) is a complex coefficient named after diffraction coefficient at edge \( E \).

![Figure 2: Single diffraction by a wedge.](image)
$D_E$ is deduced from Eq. (1) and the boundary solution of a rigid wedge with a point source [5] and can be expressed as

$$D_E = -\frac{\sum_{i=1}^{4} A(\zeta_i) \cdot F_v(\omega, R_i, \beta)}{\pi \cdot G_j(S \mid E)}, \quad (2)$$

where $\omega$ is the angular frequency of sound, $G_j(S \mid E)$ denotes the direct sound in free field at edge $E$, parameters $\zeta_i$ and $\beta$ are the angles illustrated in Figure 2. $F_v(\omega, R_i, \beta)$ is an integral

$$F_v(\omega, R_i, \beta) = \int_0^1 I(q) dq, \quad (3)$$

where parameters $R_i$ are illustrated in Figure 2. Integrand function $I(q)$ is

$$I(q) = \begin{cases} e^{ikR_i}/R_i, & \text{for the point source with unit power} \\ (-j/4)H_0^2(kR_i), & \text{for the coherent line source with unit power} \end{cases}, \quad (4)$$

where $k$ is the wave number and $j = \sqrt{-1}$.

The detailed function form dependent on the integrant $q$ in Eq. (3) of parameter $R_i$ is given by [5]

$$R_i = \left[ L^2 + rSr_r(Y - Y^{-1})^2 \right]^{1/2}, \quad (5)$$

in which

$$Y = \frac{\tan(A(\zeta_i)) + \tan(qA(\zeta_i))^{\beta/(2\pi)}}{\tan(A(\zeta_i)) - \tan(qA(\zeta_i))}. \quad (6)$$

$A(\zeta_i)$ is an angular function and can be expressed as

$$A(\zeta_i) = \frac{\pi}{2\beta}(-\beta - \pi + \zeta_i) + \pi U(\pi - \zeta_i), \quad (7)$$

and

$$U(\theta) = \begin{cases} 1 & \text{if } \theta \geq 0 \\ 0 & \text{if } \theta < 0 \end{cases}. \quad (8)$$

In Eq. (5), the parameters $r_S$ and $r_R$ are the distances shown in Figure 2, and the quantity $L$ is defined as the total distance along the path of diffraction ray from $S$ to edge $E$ and then to $R$, which equals $r_s + r_r$ in Figure 2.
When $R$ is on the shadow or reflection boundary in Figure 2, $\Re_i$ should be calculated by

$$\Re_i = \left( r_s^2 + r_R^2 - 2r_s \cdot r_R \cdot \cos \xi \right)^{1/2}. \quad (9)$$

And accordingly,

$$F_\nu(\omega, \Re_i, \beta) = G_f(r_s + r_R). \quad (10)$$

### 2.2 Double and Multiple Diffractions

For double diffraction shown in Figure 3, the ray can be viewed as the single diffraction by edge $E_2$ from incident ray $S \to E_1 \to E_2$. The latter can be treated as a single diffraction of $S$ incidence to $E_1$ with location $E_2$ being a virtual receiver $VR_1$, whose field is denoted by $\phi_{d,1}(S, VR_1 \mid E_1)$. By using Eq. (1), field of the double diffraction ray, $\phi_{d,2}(S, R \mid E_1, E_2)$, can be determined by

$$\phi_{d,2}(S, R \mid E_1, E_2) = \phi_{d,1}(S, VR_1 \mid E_1) \cdot D_{E_2}, \quad (11)$$

where $D_{E_2}$ is coefficient of the single diffraction by edge $E_2$, from a virtual source $VS_2$ and then to receiver $R$. $VS_2$ is defined on the reverse extension line of $E_1 \to E_2$ and is apart from $E_2$ for a distance equaling the total length of the ray $S \to E_1 \to E_2$.

![Figure 3: Double and multiple sound diffraction rays over wide barriers.](image)

The field $\phi_{d,1}(S, VR_1 \mid E_1)$ can also be expressed similarly as

$$\phi_{d,1}(S, VR_1 \mid E_1) = G_f(S \mid E_1) \cdot D_{E_1}, \quad (12)$$

where $D_{E_1}$ is the diffraction coefficient at edge $E_1$, with receiver $VR_1$ and source $S$. 
The double sound diffraction $\phi_{d,2}(S, R \mid E_1, E_2)$ can be determined by Eq. (11) and (12),
\[
\phi_{d,2}(S, R \mid E_1, E_2) = G_j (S \mid E_1) \cdot D_{E_1} \cdot D_{E_2} \cdot \alpha(E_1, E_2),
\]
(13)
where $\alpha(E_1, E_2)$ is a weighting factor and equals to 1/2 for $E_1$ and $E_2$ being connected and equals to unit if $E_1$ and $E_2$ are not connected.

Similarly, $\phi_{d,n}(S, R \mid E_1, E_2, E_3, ..., E_{n-1}, E_n)$, field of the multiple sound diffraction ray propagating along $n$ edges shown in Figure 3, can be expressed as
\[
\phi_{d,n}(S, R \mid E_1, E_2, E_3, ..., E_{n-1}, E_n) = G_j (S \mid E_1) \cdot \prod_{i=1}^{n} D_{E_i} \cdot \alpha(E_{i-1}, E_i),
\]
(14)
where
\[
\alpha(E_{i-1}, E_i) = \begin{cases} 
1 & \text{if the edge } E_{i-1} \text{ and } E_i \text{ are separate} \\
1/2 & \text{if the edge } E_{i-1} \text{ and } E_i \text{ are connected} 
\end{cases}
\]
(15)

Particularly for $D_{E_i}$ in Eq. (14), $VS_i$ represents the location of source $S$, $VR_n$ represents the location of receiver $R$ as shown in Figure 3 and $\alpha(E_0, E_1) \equiv 1$ in Eq. (15).

3 NUMERICAL VALIDATIONS

The proposed method is validated with the boundary element method (BEM). Insertion loss of two hard wide barriers of equal height is investigated in Figure 4, where the source is a coherent line one parallel to barriers and the receiver is located in the shadow zone apart from the nearest barrier top edge for 3.5 metres. Here the frequency whose wavelength equals the minimum edge-edge distance 0.6 m is 573 Hz. From Figure 4, over the frequency range down from eight times larger than 573 Hz to some few times smaller than 573 Hz, the predictions from the proposed method agree well with those from the BEM.

![Figure 4: Insertion loss (IL) at R (5.14m, 0.5m) of two rigid barriers with same height of 2.4 m.](image)
Another example is investigated in Figure 5, where receiver changes to location apart from the nearest edge for only 0.2 m, the source and the minimum edge-edge distance remain the same with the first case. Good agreement between the results with the proposed method and the BEM is observed again, over the frequency range down from 5000 Hz to about one half of 573 Hz.

![Figure 5: Insertion loss at R (3.97m, 2.9m) of three rigid barriers with 2.4 m or 3 m height.](image)

From the above results, with wavelength less than edge-edge distances, small discrepancies between the predictions from the proposed method and the BEM are observed on some tone frequencies, and the reason for this is that the diffraction rays searched in the proposed method for the cases are actually not complete in theory but main ones in energy, which means that some weak rays such as ones diffracted by barrier edges and the barrier foot corners on the ground are not included to simplify the evaluation of the total field. These neglected rays may cause interference exactly at these frequencies and then take noticeable influence on the total field. Generally for less than six diffracting edges, accuracy of the proposed method can be ensured with wavelength less than twice of edge-edge distances. Additionally, the proposed method depends little on the receiver-edge distance, even if the latter is quite small compared to wavelength.

4 CONCLUSION

A ray-based method is proposed to predict the insertion loss of parallel wide barriers with some neighboring ones of equal height for arbitrary receiver locations. Numerical simulations show that the method can predict the insertion loss with a certain accuracy. The proposed method is useful for engineering applications in predicting sound diffraction around variously configured shapes with many diffracting edges, such as buildings, convex or concave obstacles.
5 ACKNOWLEDGEMENTS
Project 10674068 supported by NSFC and NCET.

6 REFERENCES