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On thermal boundary layers on a flat plate subjected to a variable heat flux

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Abstract

The problem of a steady forced convection thermal boundary-layer past a flat plate with a prescribed surface heat flux proportional to $(1+x^2)^m$ (m a constant) is investigated both analytically and numerically. In view of the present formulation, the governing equations reduce to the well-known Blasius similarity equation and to the full boundary-layer energy equation with two parameters: the wall flux exponent m and Prandtl number Pr. The range of existence of solutions is considered, it being shown that solutions for both x small and x large exist only for $m > -1/2$. However, for $m \leq -1/2$ the asymptotic structure for x large is found to be different for $m < -1/2$ and $m = -1/2$, respectively. These asymptotic solutions for large x are derived and compared with numerical solutions of the full boundary-layer equation. A very good agreement between these asymptotic solutions and numerical simulations are found in the range of Prandtl numbers considered.

Notation

C	constant defined in Eq. (22)
f	reduced stream function
F	Gauss function
g, $\tilde{g}$, G, h	reduced temperature functions
I	dummy function defined in Eq. (25)
m	wall heat flux exponent defined in Eq. (1)
Pr	Prandtl number
r_m	function defined in Eq. (40)
x, y	Cartesian coordinates along and normal to the plate, respectively

Greek

Γ	Gamma function
δ	Kronecker delta function defined in Eq. (42)
θ	non-dimensional temperature
θ_w	non-dimensional wall temperature
ξ, η	non-similarity variables
v	alternative unit-step function defined in Eq. (41)
ψ	non-dimensional stream function

Superscripts

′	differentiation with respect to η

1. Introduction

Historically, the theoretical description of the thermal boundary-layer flow began with the analysis of Prandtl (1910) (see Tani, 1977), who applied the boundary-layer concept to the heat transfer problems. The work of Prandtl was especially noteworthy as it first introduced the mathematical technique of boundary-layer theory into the subject of heat transfer. Subsequently, Pohlhausen (1921) identified similarity solutions for the heat transfer part of the forced convection flow past a flat plate by introducing the dimensionless similarity profile for the boundary-layer energy equation. Pohlhausen's analysis has been very much refined and generalized since then. Among different methods, we mention that of scaling analysis proposed by Bejan (1985), which is a very efficient approach to obtaining useful engineering results. There have been very many advances made along the lines of thermal boundary- layer theory, and the most important of these are extensively reviewed in Kakac et al. (1987), Gebhart et al. (1988) and very recently by Bejan (1995).

It is well-established that convective heat transfer depends on the form of the thermal boundary conditions imposed, with it being usual to take either a prescribed temperature or a prescribed heat flux on the boundary surface. However, in many problems, particularly those involving the cooling of electrical and nuclear components, the wall heat flux is known. In such problems, overheating, burnout, and meltdown are very important issues; therefore, one of the objects of heat transfer theory is the prediction of the wall temperature as wall heat flux varies. The design objective is to control this wall temperature distribution (Bejan, 1995). However, the situation of a prescribed heat flux rate at a surface is often approximated in practical applications and is easier to measure in a laboratory than the case of a surface with prescribed wall temperature.

The main objective of this paper is to complete the existing solutions in the open literature of the forced convection thermal boundary-layer on a flat plate by considering the case when a prescribed wall heat flux is given and is of the form

$$\left(\frac{\partial\theta}{\partial y}\right)_{y=0} = -(1+x^2)^m, \qquad (1)$$

where x and y are the non-dimensional Cartesian coordinates along and normal to the plate, respectively, θ is the non-dimensional temperature and m is a constant. To the authors' best knowledge this situation has not been treated previously for the flow geometry considered in this paper. However the analogous problem of free convection boundary-layer flow on a vertical plate immersed in a viscous (Newtonian) fluid with the prescribed wall heat flux given by Eq. (1) has been discussed in detail first by Merkin and Mahmood (1990), and more recently by Wright et al. (1996) for a vertical flat plate embedded in a fluid-saturated porous medium. In what follows, we make use of the elegant work of Merkin and Mahmood (1990) to complete the solution of the present classical problem with the case given by Eq. (1) and to compare the nature of the solution with the corresponding results for the prescribed uniform wall heat flux case. It was shown that we must have $m > -1/2$ for a solution of the energy equation to exist. However, we found that this equation has a physically acceptable asymptotic solution for x large when $m \leq -1/2$, as well. The behaviour of this asymptotic solution for $m \leq -1/2$ is fully discussed. Finally, we compare the analytical solutions for various values of the parameter m and Prandtl number Pr with the results obtained using numerical techniques. The numerical results confirm the heat transfer features anticipated by the asymptotic solutions.

It is worth mentioning to this end that the precise form that the surface heat flux takes is not important, only that it has the functional forms for x small and x large given by Eq. (1).

2. Basic equations

Consider the laminar forced convection boundary-layer flow of a viscous incompressible fluid past a flat plate with a uniform free stream velocity U_∞ and constant temperature T_∞. The boundary-layer equations in terms of the stream function ψ and temperature θ can be written in non-dimensional form as

$$\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial x\partial y}-\frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2}=\frac{\partial^3\psi}{\partial y^3}, \tag{2}$$

$$\frac{\partial\psi}{\partial y}\frac{\partial\theta}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial\theta}{\partial y}=\frac{1}{\mathrm{Pr}}\frac{\partial^2\theta}{\partial y^2}, \tag{3}$$

where Pr is the Prandtl number. Eqs. (2) and (3) are subject to the boundary conditions

$$y=0:\quad \psi=\frac{\partial\psi}{\partial y}=0,\ \frac{\partial\theta}{\partial y}=-(1+x^2)^m,$$

$$y\to\infty:\quad \frac{\partial\psi}{\partial y}\to 1, \theta\to 0, \tag{4}$$

where m is a real constant. From Eq. (4) we see that for $x\ll 1$, $(\partial\theta/\partial y)_{y=0}=-1$, while, for $x\gg 1$, $(\partial\theta/\partial y)_{y=0}=-x^{2m}$, so that though it is possible to write down similarity equations for both x small and x large, in the latter case these possess a solution only if we take $m>-1/2$.

To facilitate the analysis of Eqs. (2)-(4) we follow Merkin and Mahmood (1990), and introduce the variables

$$\psi=x^{1/2}f(\eta),\ \theta=x^{1/2}(1+x^2)^m\mathrm{g}(x,\eta),\ \eta=y/x^{1/2}. \tag{5}$$

Substituting these variables into Eqs. (2) and (3) we get

$$f'''+\frac{1}{2}ff''=0, \tag{6}$$

$$\frac{1}{\mathrm{Pr}}\frac{\partial^2\mathrm{g}}{\partial\eta^2}+\frac{1}{2}f\frac{\partial\mathrm{g}}{\partial\eta}-\frac{1}{1+x^2}\left[\frac{1}{2}+\left(\frac{1}{2}+2m\right)x^2\right]f'\mathrm{g}=xf'\frac{\partial\mathrm{g}}{\partial x}. \tag{7}$$

along with the boundary conditions

$$f(0)=f'(0)=0,\ \frac{\partial\mathrm{g}}{\partial\eta}(x,0)=-1.$$

$$f'(\infty)=1,\ \mathrm{g}(x,\infty)=0, \tag{8}$$

where primes denote differentiation with respect to η.

Similarity solutions of Eq. (7) are possible for $x=0$, and they are given by

$$\mathrm{g}_0''+\frac{1}{2}\mathrm{Pr}f\mathrm{g}_0'-\frac{1}{2}\mathrm{Pr}f'\mathrm{g}_0=0, \tag{9}$$

$$\mathrm{g}_0'(0)=-1,\ \mathrm{g}_0(\infty)=0, \tag{10}$$

which correspond to the thermal boundary-layer on a flat plate with a prescribed uniform heat flux, see Bejan (1995), p. 80. On the other hand, on letting $x\to\infty$ in Eq. (7), we get

$$g_1'' + \frac{1}{2}\mathrm{Pr} f g_1' - (\frac{1}{2} + 2\mathrm{m})\mathrm{Pr} f' g_1 = 0, \quad (11)$$

$$g_1'(0) = -1,\ g_1(\infty) = 0. \quad (12)$$

These equations correspond to a prescribed wall heat flux $(\partial\theta/\partial y)_{y=0} = -x^{2m}$ (see Sparrow and Lin, 1965, for $m \geq 0$).

By integrating Eq. (11) and using the boundary conditions (8) for f and (12) for g_1, we obtain

$$(1 + 2m)\int_0^x f' g_1 d\eta = \mathrm{Pr}^{-1}. \quad (13)$$

Hence we must have

$$m > -1/2. \quad (14)$$

for a solution of Eq. (7) to exist for both x small and x large. However, there are two separate cases to consider for the present problem for large x, namely, $m < -1/2$ and $m = -1/2$, respectively.

Eqs. (6) and (7) have been solved using a standard Keller box method (Keller, 1971) and with the solution of Eqs. (9) and (10) used as the initial condition for $g(x,\eta)$ at $x = 0$. The results are shown in Table 1 giving the wall temperature distribution $\theta(x,0) = \theta_w(x)$ for different values of the parameter m and Prandtl number Pr. The wall temperature distribution for large x is given by

$$\theta_w(x) = g(x,0)x^{1/2}(1 + x^2)^m + \mathrm{O}(x^{-3/2}(1 + x^2)^m). \quad (15)$$

The variation of the wall temperature $\theta_w(x)$ is shown in Fig. 1 for Pr = 1. It can be seen from this figure that the numerical solution attains its asymptotic condition as given by Eq. (15) for the case $m = -\frac{1}{4}$, which corresponds to a uniform wall temperature for large x.

3. Asymptotic solution for x large when $m < -1/2$

To find an asymptotic solution of Eq. (7), which is valid for x large when $m < -1/2$, we make the transformation

$$\psi = x^{1/2} f(\eta),\ \theta = x^{1/2} G(x,\eta),\ \eta = y/x^{1/2}, \quad (16)$$

which is the one that gives Eq. (11) for the critical case $m = -1/2$.

Using transformation (16), Eq. (3) and boundary conditions (4) become

$$\frac{1}{Pr}\frac{\partial^2 G}{\partial\eta^2} + \frac{1}{2} f \frac{\partial G}{\partial\eta} + \frac{1}{2} f' G = x f' \frac{\partial G}{\partial x} \quad (17)$$

subject to the boundary conditions

$$\frac{\partial G}{\partial\eta}(x,0) = -x^{1+2m}(1 + \frac{1}{x^2})^m,\ G(x,\infty) = 0. \quad (18)$$

We look for a solution of Eq. (17) subject to the boundary conditions (18) by expanding G in powers of x^{1+2m} $(1+2m<0)$, of the form

$$G(x,\eta)=G_0(\eta)+x^{1+2m}G_1(\eta)+\mathrm{O}(x^{-1+2m}), \tag{19}$$

where G_0 satisfies the equation

$$G_0'+\frac{1}{2}\mathrm{Pr}fG_0=0, \tag{20}$$

$$G_0(\infty)=0. \tag{21}$$

Eq. (20) has been obtained by integrating the equation derived from (17) once, with boundary conditions (8) for f and (21) for G_0 satisfied. The required solution to the homogeneous problems given by Eq. (20) and boundary condition (21) can be found from a particular solution $\bar{G}_0(\eta)$ by the transformation

$$G_0(\eta)=C\bar{G}_0(\eta), \tag{22}$$

where the solution $\bar{G}_0(\eta)$ is such that $\bar{G}_0(0)=1$.

To determine the value of the constant C, we integrate Eq. (3) and apply boundary conditions (4) to get

$$\int_0^x \theta\frac{\partial\psi}{\partial y}\,dy=\frac{1}{\mathrm{Pr}}\int_0^x(1+s^2)^m\,ds=\frac{x}{\mathrm{Pr}}F\left(-m,\frac{1}{2};\frac{3}{2};-x^2\right)$$

$$=\frac{\sqrt{\pi}\,\Gamma(-m-\frac{1}{2})}{2\mathrm{Pr}\,\Gamma(-m)}-\frac{1}{\mathrm{Pr}}x^{1+2m}+\mathrm{O}(x^{-1+2m})\quad as\ x\to\infty\,, \tag{23}$$

where $\Gamma(z)$ and $F(a,b;c;z)$ are the Gamma function and the Gauss function respectively (Spanier and Oldham, 1987).

So that for the leading-order solution in expansion (19), Eq. (23) gives, using Eq. (22),

$$C\int_0^x \bar{G}_0 f'\,d\eta=\frac{\sqrt{\pi}\,\Gamma(-m-\frac{1}{2})}{2\mathrm{Pr}\,\Gamma(-m)}. \tag{24}$$

Introducing the integral

$$I(\eta)=\int_0^\eta \bar{G}_0 f'\,d\eta. \tag{25}$$

the leading-order solution can be written as

$$G_0(\eta)=\frac{\sqrt{\pi}\,\Gamma(-m-\frac{1}{2})}{2\mathrm{Pr}\,\Gamma(-m)I(\infty)}\bar{G}_0(\eta). \tag{26}$$

Now the full numerical solution of $G_0(\eta)$ can be determined from the basic variables $(\bar{G}_0,I)$, which satisfies the following equations,

$$\bar{G}'_0+\frac{1}{2}\mathrm{Pr}f\bar{G}_0=0. \tag{27}$$

$$I' = \bar{G}_0 f' \tag{28}$$

subject to the boundary conditions

$$\bar{G}_0(0) = 1,\ I(0) = 0. \tag{29}$$

Table 2 lists the values of $I(\infty)$ for different Prandtl number Pr. The wall temperature distribution for large x and $m < -1/2$ can then be expressed as

$$\theta_w(x) = \frac{\sqrt{\pi}\,\Gamma(-m-\frac{1}{2})}{2\mathrm{Pr}\,\Gamma(-m)I(\infty)} x^{1/2} + \mathrm{O}(x^{1/2+2m}). \tag{30}$$

A graph of asymptotic expression (30) is shown in Fig. 2 for $\mathrm{Pr} = 1$ where we can see that it is in very good agreement with the value obtained from the numerical solution of Eq. (7). In the paper by Merkin and Mahmood (1990), the case of $m = -1$ was treated as an exception due to existence of eigensolutions. No such eigensolution appears in our case because the momentum equation (Blasius equation) is decoupled from the energy equation.

4. Asymptotic solution for *x* large when $\mathbf{m = -1/2}$

It is clear from Eq. (19) that the expansion of G in powers of x^{1+2m} for $m < -1/2$ breaks down when $m = -1/2$ and an alternative approach is required. In this case relation (5) is replaced by

$$\psi = x^{1/2} f(\eta),\ \ \theta = x^{-1/2}(\ln x) h(x,\eta),\ \ \eta = y/x^{1/2} \tag{31}$$

so that Eq. (23) reduces to

$$\begin{aligned}\int_0^x \theta \frac{\partial \psi}{\partial y} dy &= \frac{1}{\mathrm{Pr}} \ln\left(x + \sqrt{x^2+1}\right) \\ &= \frac{1}{\mathrm{Pr}} \ln x + \frac{1}{\mathrm{Pr}} \ln 2 + \mathrm{O}(x^{-2}) \ \text{ as } x \to \infty.\end{aligned} \tag{32}$$

Using Eq. (31), Eq. (3) can now be transformed into the following form,

$$\frac{1}{\mathrm{Pr}} \frac{\partial^2 h}{\partial \eta^2} + \frac{1}{2} f \frac{\partial h}{\partial \eta} + \left(\frac{1}{2} - \frac{1}{\ln x}\right) f' h = x \frac{\partial h}{\partial x} \tag{33}$$

subject to the boundary conditions

$$\frac{\partial h}{\partial \eta}(x,0) = -\frac{1}{\ln x}\left(1 + \frac{1}{x^2}\right)^{-1/2},\ h(x,\infty) = 0. \tag{34}$$

We now look for a solution of Eq. (33) by expanding h in the form of series

$$h(x,\eta) = h_0(\eta) + (\ln x)^{-1} h_1(\eta) + \mathrm{O}\left(\frac{1}{x^2 \ln x}\right). \tag{35}$$

where function h_0 is given by

$$h_0' + \frac{1}{2} \mathrm{Pr} f h_0 = 0, \tag{36}$$

$$h_0(\infty) = 0. \tag{37}$$

Eqs. (36) and (37) are identical with Eqs. (20) and (21) in Section 3, except for the different notation. As discussed previously, the asymptotic expression for the wall temperature distribution can be found by comparing the leading-order terms in the integral condition (32), i.e.

$$\theta_w(x) = \frac{1}{\Pr I(\infty)} x^{-1/2} (\ln x) + O(x^{-1/2}). \tag{38}$$

Fig. 3 shows, for this case $m = -1/2$ and Pr = 1, that the numerical solution did not quite settle onto its asymptotic limit as it did for the values of $m > -1/2$. This must arise from the asymptotic expansions (30) and (38), which show that the expression 30 is algebraic in x for $m < -1/2$, whereas the expression (38) is only logarithmic in x for $m = -1/2$. Thus, much larger values of x will be needed to achieve the expression (38) than to achieve the expression (30). This point will be shown also numerically in the next section using the method of continuous transformation proposed by Hunt and Wilks (1981). This method allows the numerical solution of Eqs. (2) and (3) to proceed accurately to very large values of x for any given parameter m. The advantage of this method is that it displays the evaluation of the boundary layer between the similarity regimes. When a prior knowledge of the final similarity regime is available, Hunt and Wilks (1981) have demonstrated that a continuous transformation can successfully be invoked which follows closely the natural evolution of the boundary layer flow. Accordingly, full numerical solutions of the governing boundary layer equations may be obtained in the context of a single transformed system of equations.

5. Continuous transformation

A x-dependency for θ is introduced into transformation as follows.

$$\psi = \xi^{1/4} f(\eta), \qquad \theta = \xi^{1/4} r_m(\xi) \tilde{g}(\xi, \eta).$$

$$\xi = x^2, \ \eta = y/x^{1/2}. \tag{39}$$

Without loss of generality we can prescribe

$$r_m(\xi) = (1+\xi)^{m-(1/2+m)v(-1-2m)} \left[1 + \frac{1}{2}\delta(1+2m)\ln(1+\xi)\right]. \tag{40}$$

where

$$v(z) = \begin{cases} 0, & z < 0, \\ 1, & z \geq 0, \end{cases} \tag{41}$$

and

$$\delta(z) = \begin{cases} 0, & z \neq 0, \\ 1, & z = 0, \end{cases} \tag{42}$$

are called the alternative unit-step function and the Kronecker delta function respectively (Spanier and Oldham, 1987).

Substituting variables (39) into Eqs. (2) and (3), we get

$$f''' + \frac{1}{2} f f'' = 0, \tag{43}$$

$$\frac{1}{\mathrm{Pr}}\frac{\partial^2 \tilde{g}}{\partial \eta^2}+\frac{1}{2}f\frac{\partial \tilde{g}}{\partial \eta}-[\frac{1}{2}+2\xi\frac{r_m^{'}(\xi)}{r_m(\xi)}]f^{'}\tilde{g}=2\xi f'\frac{\partial \tilde{g}}{\partial \xi}. \qquad (44)$$

along with the boundary conditions

$$f(0)=f^{'}(0)=0,\ \frac{\partial \tilde{g}}{\partial \eta}(\xi,0)=-\frac{(1+\xi)^m}{r_m(\xi)},$$

$$f^{'}(\infty)=1,\ \tilde{g}(\xi,\infty)=0. \qquad (45)$$

The wall temperature distribution is given now by

$$\theta_w(x)=x^{1/2}r_m(x^2)\tilde{g}(x,0). \qquad (46)$$

Again a standard Keller box method can be adapted to solve Eqs. (43) and (44) numerically. The results for the wall temperature distribution $\theta_w(x)$ given by Eq. (46) are presented in Figs. 4-7 for various parameter m and Prandtl number Pr. As maybe expected $\theta_w(x)$ decreases as Pr increases, i.e. the thermal boundary-layer becomes thinner when Pr is increased. It is also seen from these figures that for highly conductive fluids (Pr<<1), the wall temperature $\theta_w(x)$ is much higher than for low conductive fluids (Pr >>1).

As a check we calculated $\theta_w(x)$ for the case $m=-1/2$. We found that the result do appear to be approaching the asymptotic limit as given by Eq. (38) as x is increased.

6. Conclusion

We have considered the behavior of the solution of the equations for the forced convection thermal boundary-layer on a flat plate with a prescribed heating rate proportional to $(1+x^2)^m$. By solving the governing energy equation both analytically and numerically using the Keller box scheme in combination with the continuous transformation method, it has been possible to provide a detailed description of the solutions for large x in the range $m\le -1/2$ and different values of the Prandtl number Pr. In both cases $m<-1/2$ and $m=-1/2$ asymptotic analysis results in an altogether simpler formulation, giving equations whose analytical solution proves to be more easily tractable than the original full boundary-layer equation. Agreement between the asymptotic and numerical solutions for the two regimes $m<-1/2$ and $m=-1/2$ proved to be very good, leading us to believe that the asymptotic approach, although simple in nature, was successful in capturing the essential features of the heat transfer characteristics. The exponent m is, therefore, observed to influence the heat transfer characteristics significantly. A similar situation was found by Merkin and Mahmood (1990) in their solution of the free convection boundary-layer flow on a vertical flat plate.

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List of tables

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Pr	$g(x,0)$	
	$m = 0$	$m = -1/4$
0.001	9.85265	9.95016
0.01	8.74748	9.52395
0.1	4.93984	6.79432
1	2.17879	3.01153
10	1.00212	1.37336
100	0.46469	0.63620
1000	0.21567	0.29524

Table 1.

Pr	$I(\infty)$
0.001	56.03064
0.01	17.66650
0.1	5.45053
1	1.50576
10	0.34919
100	0.07596
1000	0.01638

Table 2.

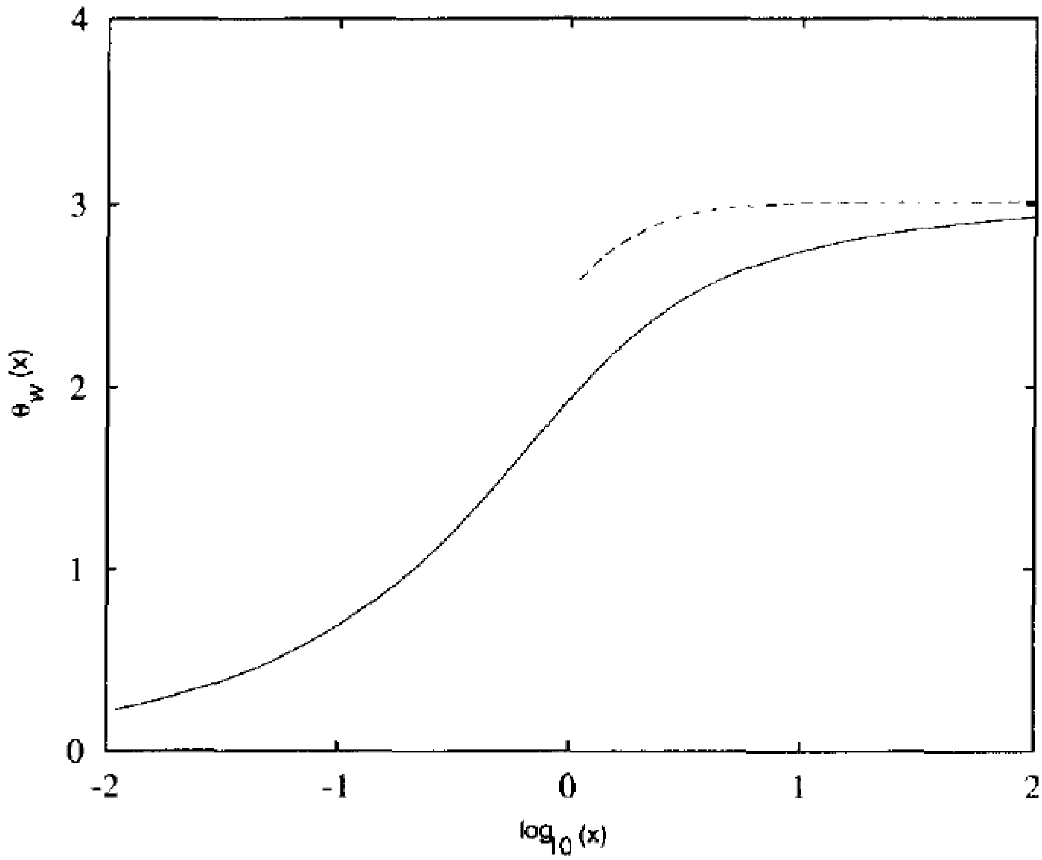
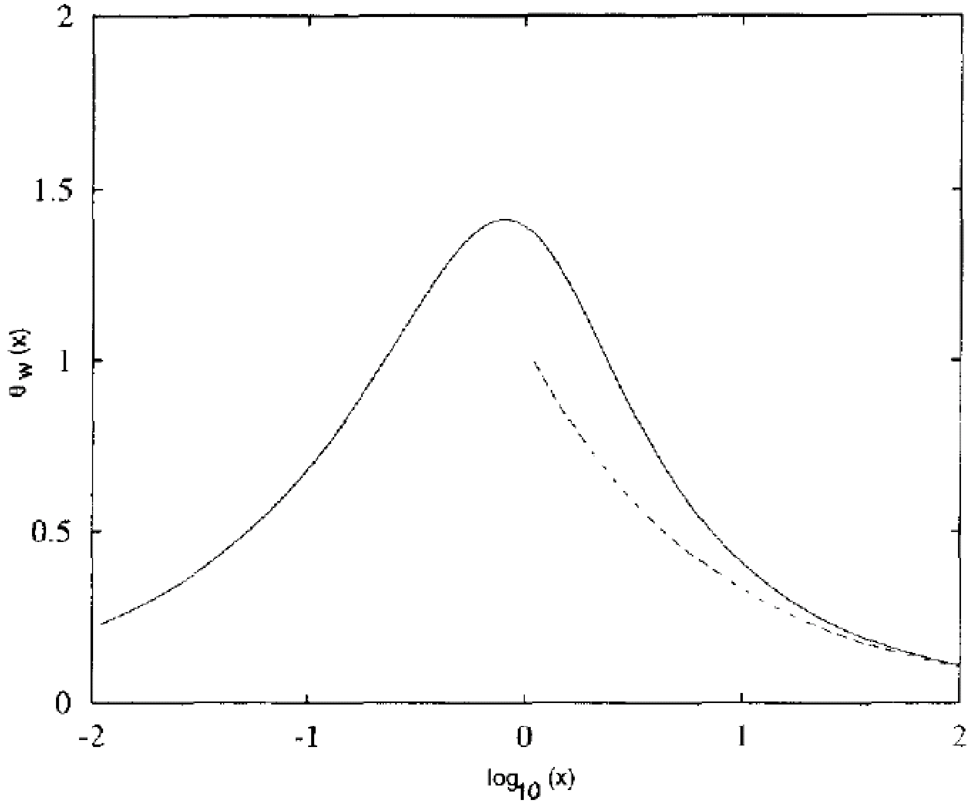


Fig. 1.



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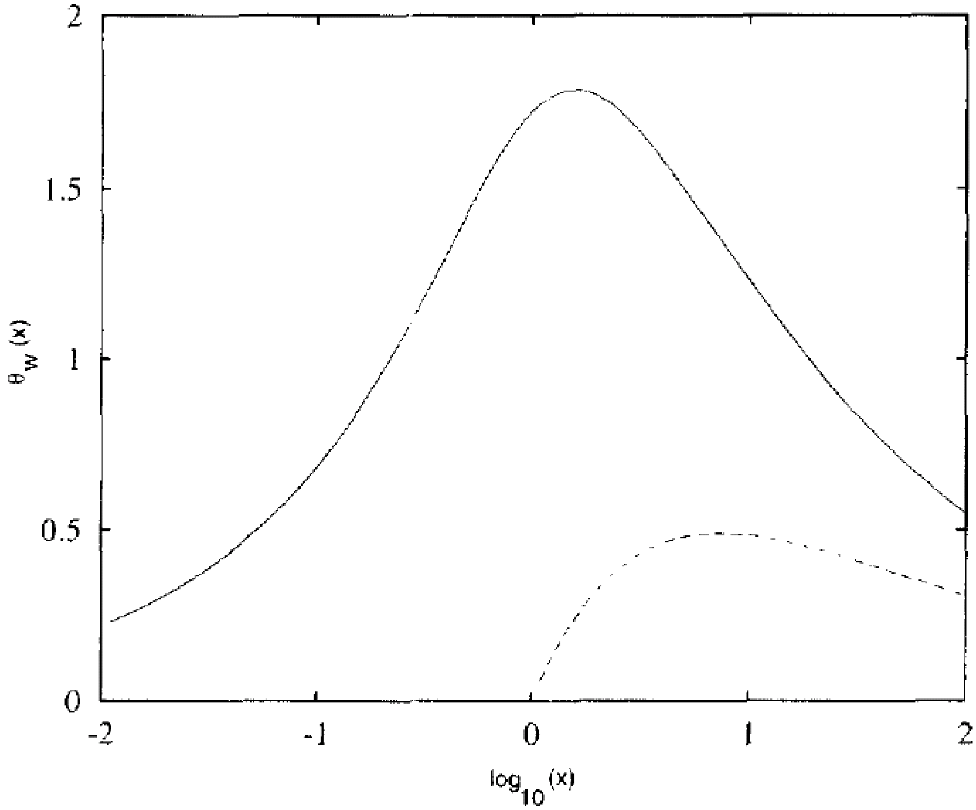

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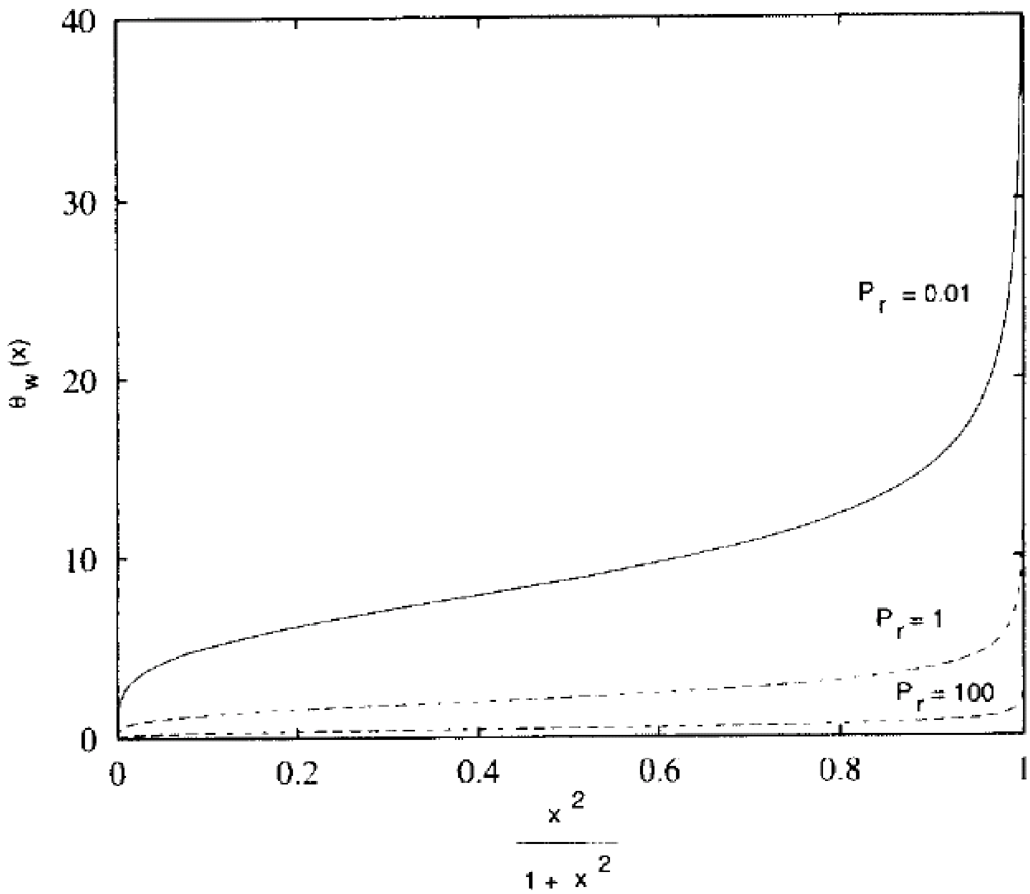

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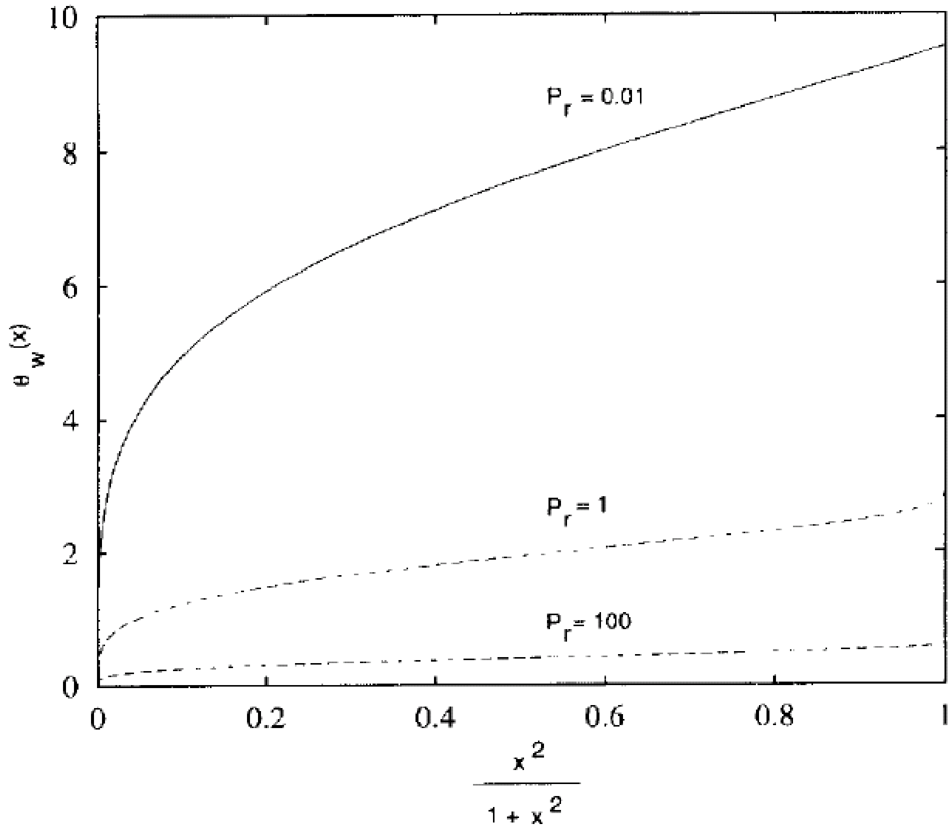

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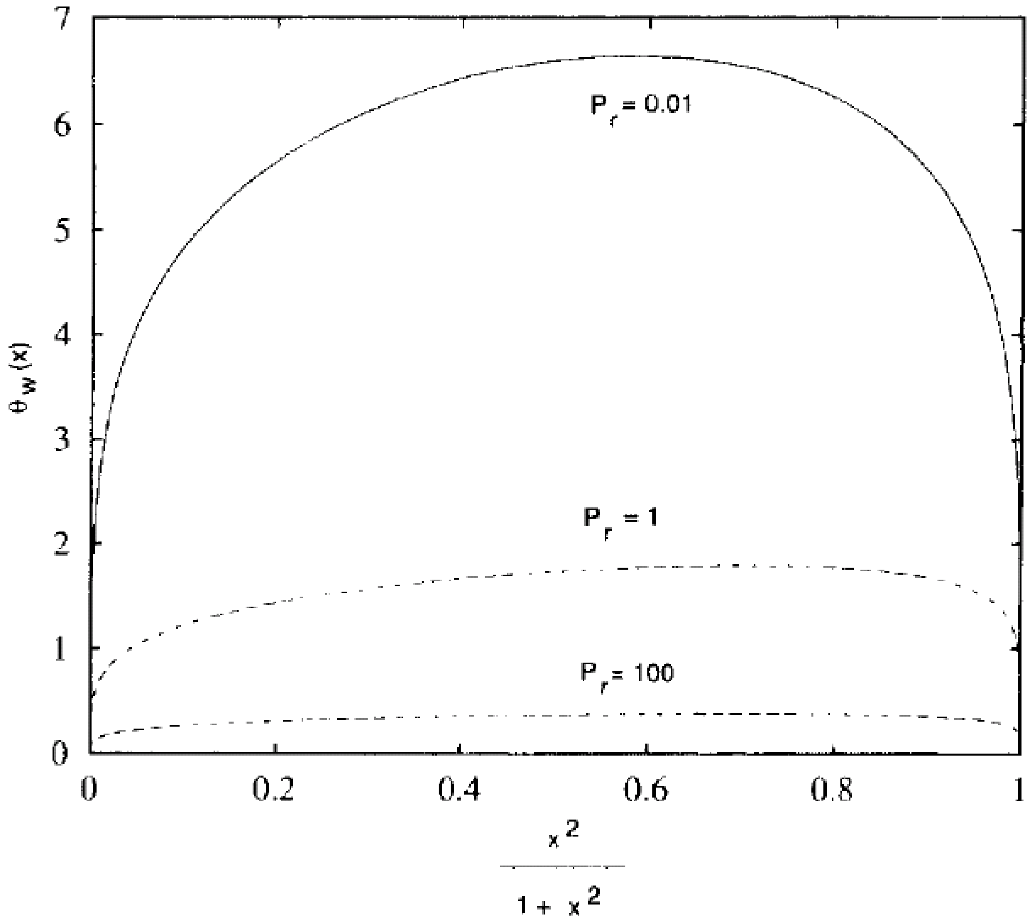

Fig. 6.

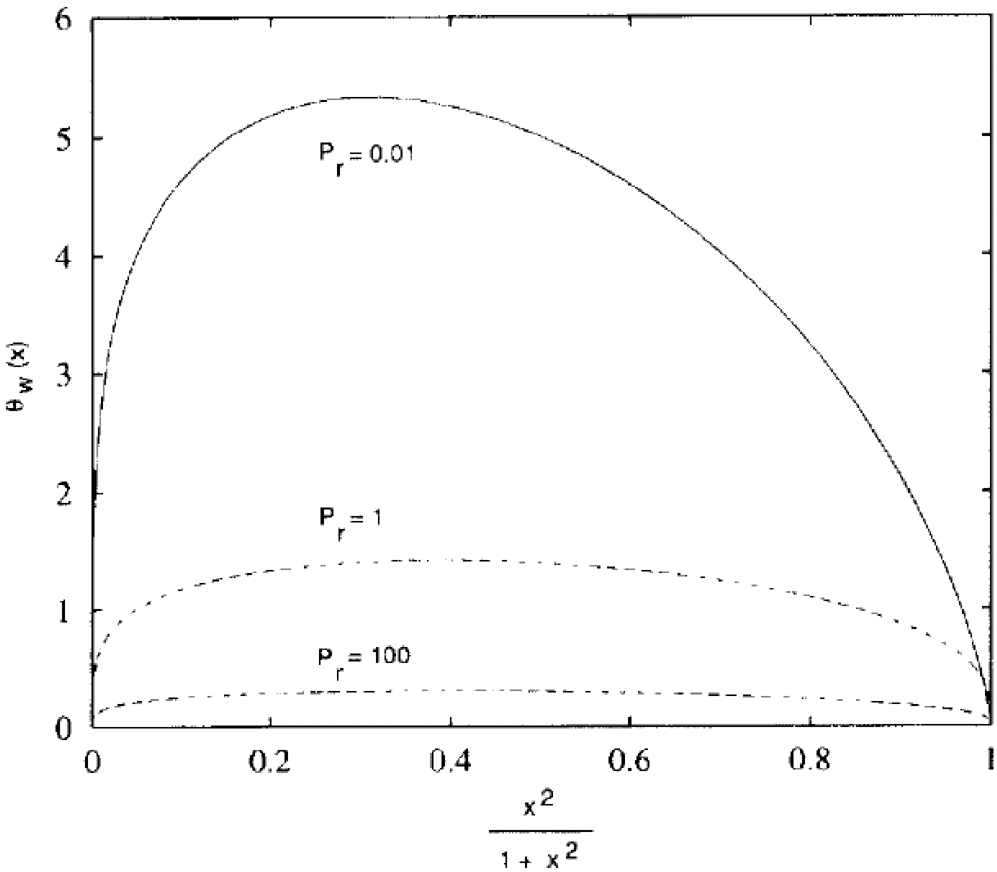

Fig. 7.