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Energy Efficiency Analysis of Channel Aware Geographic-Informed Forwarding (CAGIF) for Wireless Sensor Networks

Lili Zhang, Member, IEEE, and Boon-Hee Soong, Senior Member, IEEE

Abstract

This letter analytically examines the achievable energy efficiency of Channel Aware Geographic-Informed Forwarding (CAGIF) algorithm, which chooses the next hop relay node by taking into consideration the underlying channel conditions, rather than purely maximizing the forwarding progress. The theoretical analysis in terms of the average forward distance is developed and the upper bound and lower bound of the average energy consumption are provided by referring to the retransmission techniques. Numerical results show that CAGIF significantly improves the energy efficiency over the previous geographic information based routing algorithms that ignore the channel conditions.

Index Terms

Wireless sensor networks, channel aware geographic-informed forwarding, energy efficiency.

I. INTRODUCTION

Geographic information based routing algorithms have been demonstrated to represent an effective way of finding the appropriate next hop relay nodes by utilizing the location information while avoiding the large number of control packets necessary for route discovery in wireless sensor networks. Existing literatures [1] [2] [3] usually concentrate on choosing the next hop relay nodes according to the maximum advance progress in order to minimize the number of hops from the source to the destination node. However, an inarguable fact is that there is often serious fading phenomena [4] [5] due to the radio wave propagation. Consequently, the performance gain of traditional approaches can be traded off for a bad channel condition that requires the repeated packet transmissions to ensure the correct packet reception and hence consumes more energy. Disseminating data information in the deep-fading channel state incurs additional energy burden and it is indispensable to reduce the unnecessary retransmission due to the propagation impairments from physical environment. A straightforward way is to choose the next hop relay nodes under good channel conditions to ensure the correct packet transmission and to avoid the relay nodes in the deep-fading channel state.

Recently, ExOR [7] chooses the forwarder with the lowest remaining cost to the ultimate destination and investigates its throughput performance. The work in [8] identifies the weak-link problem and introduces several new blacklisting and link-selection strategies to make localized geographic forwarding decision based on distance and packet reception ratio, as well as, the
combinations of both. The algorithm in [9] proposes the normalized advance for geographic routing as the metric to select neighbors with the optimal trade-off between proximity and link cost. The channel-adaptive routing [10] evaluates the information efficiency of the channel adaptive routing in a code division multiple access based multi-hop network. The solution in [11] demonstrates the expected transmission count metric to find the high-throughput paths for some classical routing protocols in a multi-hop wireless network. Moreover, although the work in [12] derives a recursion expression for the probability that there is no connection between any arbitrary pair of nodes of distance \( d \) in \( r \)-hop or less, and the work in [13] further provides the probability distribution of the minimal total transmit energy required per packet as a function of distance \( d \), they focus on investigating the minimal mean transmit power to keep the network connected in the presence of channel outage and the minimum number of hops, from the perspective of connectivity. Nevertheless, to the best of our knowledge, none of them has studied the achievable benefit of energy saving in a channel state aware geographic information based routing algorithm over those that ignore the channel conditions by using ARQ techniques [6].

In this letter, we propose a Channel Aware Geographic-Informed Forwarding (CAGIF) routing algorithm in a wireless sensor network and present an analytical framework to study the achievable energy efficiency improvements compared with Purely Geographic-Informed Forwarding (PGIF) regarding to the impact of fading on channel situations. This work differs from the previous conference version [17] in that it aims to explore the benefit of energy efficiency in a generic narrow-band wireless sensor network instead of finding a new energy efficient local metric for a CDMA-based wireless sensor network. By exploiting the inherent channel conditions, CAGIF chooses the next hop relay node from the links that not only maximize the forward progress but also with favorable channel state simultaneously. In contrast to the previous work, we consider from the perspective of retransmission to study the decreased average energy consumption when taking into account the channel conditions, as retransmission is mostly employed to ensure the successful packet arrival at the receiver. The contributions of this letter are as follows. Firstly, we provide the analysis of the average forward distance in our sense and show an interesting result that the average forward distance in CAGIF increases over that of PGIF. Based on it, we compute the upper bound and lower bound of the number of hops from the source to the destination node and further verify that the lower bound of the number of hops coincides with the result in GeRaF [3]. Finally, with respect to the ARQ techniques, we offer the analysis of the energy efficiency and show that CAGIF is significantly energy efficient than PGIF since it implicitly takes the channel condition into account.

II. SYSTEM MODEL

Let us consider a generic narrow-band wireless network where nodes are randomly distributed according to a homogeneous two-dimensional Poisson point process with average density \( \lambda \). Then, the probability \( P(N, k) \) of finding \( k \) nodes within the region with radius \( R \) is

\[
P(N, k) = \frac{e^{-N}N^k}{k!}, \quad k \geq 0
\]

(1)

where the mean \( \tilde{N} = \lambda \pi R^2 \). Now as the nodes are uniformly distributed over the region, the probability density function (pdf) of the distance \( r \) between a transmitter-receiver pair follows
\[ f_R(r) = \frac{2r}{R^2}, \quad 0 \leq r \leq R \]  \hspace{1cm} (2)

Let \( P_t \) and \( P_r \) denote the transmission power and receiving signal strength respectively. Then, we have

\[ P_r = \frac{K}{r^\alpha} P_t \]  \hspace{1cm} (3)

as a simple channel model, where \( K \) is a constant determined by the parameters such as antenna height and gain etc. and \( \alpha \) is the path-loss exponent. The path loss exponent \( \alpha \) depends on the environment and can vary between 2 in free space and 6 in heavily built urban areas. Let \( \mu_t \) denote the threshold value of receiving signal strength for the successful packet reception. Then, two nodes can communicate directly with each other only when \( P_r \geq \mu_t \). Hence, given \( R_0 \) as the transmission range in the absence of fading, we have \( R_0 = \left( \frac{K P_t}{\mu_t} \right)^{1/\alpha} \). Thereafter, assume that the transmission range \( R_0 \) is normalized to 1 and \( D \) is the normalized euclidian distance from the source to the destination node with respect to \( R_0 \). Define \( N_0 = \lambda \pi R_0^2 \).

Consider a flat Rayleigh fading channel, then we have

\[ P_r = \beta^2 \frac{K}{r^\alpha} P_t \]  \hspace{1cm} (4)

where \( \beta \) is a Rayleigh-distributed random variable and its pdf is given by

\[ f_{\beta}(\beta) = \frac{\beta}{\sigma^2} \exp\left(-\frac{\beta^2}{2\sigma^2}\right), \quad 0 \leq \beta \leq \infty \]  \hspace{1cm}

with \( \sigma \) being the rms value of the received signal strength before the envelope detection. Assume that the fading between any two nodes are independent and identical distributed (i.i.d.) and consider a system where all the nodes have the same transmission power and the wireless link is symmetric.

### III. CAGIF ALGORITHM

Before studying the energy efficiency, we first introduce our geographic-informed forwarding algorithm. In the geographic-informed forwarding algorithm, there is a predefined forwarding area [14] [15] to choose the relay node. All nodes residing in this forwarding area contend to become the possible next hop relay node. In the study [15], various forwarding areas of contention-based geographic forwarding algorithms have been examined. In this letter, we adopt \( \phi \)-degree radian forwarding area and employ polar coordinate \((r, \theta)\) as shown in Fig. 1. \( r \) denotes the distance between the source node and the relay node. We suppose that there is a line to connect the source node and the relay node. \( \theta \) is the angle between this connection and the dashed line connecting the source node and the destination node. As shown in Fig. 1, the radian sector from \((r, -\phi/2)\) to \((r, \phi/2)\) is specified as the geographic region, which the relay nodes should be located in. Here, we have \(-\phi/2 \leq \theta \leq \phi/2\). Based on it, CAGIF algorithm chooses the next hop relay node within the forwarding area that may not only maximize the forward progress...
but also be in a good channel condition simultaneously. The chosen relay node becomes in turn the new source node in next hop and henceforth the new forwarding area is formulated. In such a way, CAGIF always picks a next hop node from closer neighbors to the destination node and thereby progressively routes it towards the target region. Next, we focus on the presentation of the analytical framework on the achievable energy efficiency improvements in CAGIF.

A. Average Number of Nodes that a Node can Communicate

In the presence of Rayleigh fading, the probability that two nodes are within communication range can be derived as \(^1\).

\[
Prob(P_r \geq \mu_t) = \int_{0}^{\alpha} \frac{f_{\beta}(\beta)}{\mu_t} \int_{0}^{\alpha} \frac{\sqrt{K P_r \beta^2}}{\mu_t} f_R(r)drd\beta \\
\approx \frac{R^2}{\mu_t} \Gamma[1 + 2/\alpha] \\
= \frac{R^2}{\mu_t} \Gamma[1 + 2/\alpha] \\
\text{(5)}
\]

Define \(\omega = \frac{R^2}{\mu_t} \Gamma[1 + 2/\alpha] \). Let \(N_f\) denote the number of nodes with which a node could communicate in the presence of fading, then we have

\[
Prob(N_f = k) = \sum_{n=k}^{\infty} \mathcal{P}(N, n) C_n^k \omega^k (1 - \omega)^{n-k} \\
\text{(6)}
\]

When we let \(R \rightarrow \infty\) and \(i = n - k\), the upper equation can be further derived into

\[
Prob(N_f = k) = \lim_{R \rightarrow \infty} \sum_{n=k}^{\infty} \mathcal{P}(N, n) C_n^k \omega^k (1 - \omega)^{n-k} \\
= \lim_{R \rightarrow \infty} \frac{\exp(-\lambda \pi R^2)(\lambda \pi R^2)^k \omega^k}{k!} \sum_{i=0}^{\infty} \frac{1}{i!} \omega^i (1 - \omega)^i \\
= \lim_{R \rightarrow \infty} \frac{\exp(-\lambda \pi \omega R^2)}{k!} \cdot \frac{\exp(-\lambda \pi \omega R^2)}{k!} \cdot \frac{\exp(-\lambda \pi \omega R^2)}{k!} \\
= \mathcal{P}(N_f, k) \\
\text{(7)}
\]

where the average number of nodes \(\bar{N}_f = \lambda \pi \omega R^2\). Thus, we conclude that the number of nodes

\(^1\text{INF is set to be 9 to guarantee that 99.99\% of the pdf values are covered, i.e., } \int_{0}^{\text{INF}} f_{\beta}(\beta)d\beta \approx 1.\)
with which a node can have a communication in the presence of Rayleigh fading follows the Poisson distribution with $\bar{N}_f$. This is consistent with the results in [5] [16].

B. Average Forward Distance

We define the average forward distance as the expected value of the distance $r$ between the sender and relay node. This differs from the definition [3] that uses the orthogonal projection of the distance between the sender and relay node onto the line from the source to the destination node. Let $\zeta$ denote the average forward distance, then the probability of $\zeta \leq r_0$ is the probability that a node has no connection to the neighboring nodes with distance bigger than $r_0$. With the definition of $\phi$ being the angle that the relay node should stay within, then we have the ratio of the number of nodes within the specified angle to the number of nodes in the total area denoted as $\eta = \frac{\phi}{2\pi}$, since the nodes are uniformly distributed. Let $N'_f$ denote the average number of the nodes with which a node can have connection within the angle $\phi$ in the presence of fading, then we have the number of nodes with the radius larger than $r_0$ and within the specified angles is expressed as,

\[
N'_f(r_0 \leq r \leq 2DCos(\phi/2)) = \int_{r_0}^{2DCos(\phi/2)} \lambda r^2 \eta \int_0^{\infty} \frac{\mu_\text{th} r^\alpha}{(KP_\text{t})} \int_0^\infty \beta(\beta) d\beta dr
\]

(8)

where the upper limit of the integral is $2DCos(\phi/2)$ according to the criterion that the remaining distance between the relay and the destination node is less than $D$ (i.e. the value of $x$ that satisfies $x^2 + D^2 - 2xDCos(\phi/2) \leq D^2$). Thus, we have

\[
Prob(\zeta \leq r_0) = \exp(-N'_f(r_0 \leq r \leq 2DCos(\phi/2)))
\]

(9)

Hence, according to [3], the average forward distance towards the destination node is

\[
E[\zeta] = \int_0^D (1 - Prob(\zeta \leq r)) r dr
\]

(10)

C. Average Number of Hops

According to the specified route region in Fig. 1, one can compute the simple reasonable approximation of the upper bound $h_{\text{upper}}$ and lower bound $h_{\text{lower}}$ of the average number of hops that a packet has to travel in the worst case and best case, respectively, based on the average forward distance. The variable definitions and the outline of the computation are shown in Table I.

D. Average Energy Consumption

The average energy consumption is related with the number of hops from the source to the
destination node. Hence, we have the upper and lower bound of the average energy consumption respectively expressed as

\[ E_u = h_{\text{upper}} \times P_t \]  \hspace{1cm} (11)

and

\[ E_l = h_{\text{lower}} \times P_t \]  \hspace{1cm} (12)

### IV. PGIF ALGORITHM

PGIF algorithm chooses the next hop relay node only according to the maximal forward progress while ignores the channel condition and hence employs the ARQ [6] techniques to ensure the correct packet reception.

#### A. Average Forward Distance

The average number of nodes \( N' \) with the radius larger than \( r_0 \) and within the specified angles is expressed as

\[
N'(r_0 \leq r \leq R_0) = \int_{r_0}^{2D \cos(\phi/2)} 2\pi \eta R f_R(r)dr = \int_{r_0}^{2D \cos(\phi/2)} \lambda \phi r dr
\]  \hspace{1cm} (13)

Accordingly, we have

\[ \text{Prob}(\zeta \leq r_0) = \exp(-N'(r_0 \leq r \leq 2D \cos(\phi/2))) \]

Hence, the average forward distance towards the destination node is

\[ E[\zeta] = \int_0^D (1 - \text{Prob}(\zeta \leq r))r dr \]  \hspace{1cm} (14)

Correspondingly, we can compute \( h_{\text{lower}} \) and \( h_{\text{upper}} \) respectively according to Table I.

#### B. Average Number of Packet Retransmissions for a Correct Packet Reception

There are various retransmission techniques and we consider here the stop and wait (SAW) and selective retransmission (SR) schemes. According to [6], the average number of packet retransmission \( \kappa_{\text{SAW}} \) and \( \kappa_{\text{SR}} \) for a correct packet reception in SAW and SR scheme can be computed as
and
\[ \kappa_{SR} = \sum_{i=1}^{\infty} (1 - (1 - P_f^i) \xi) \]
respectively, where \( P_f \) is the block error probability and \( \xi \) is the block message length.

C. Average Energy Consumption

The average energy consumption is related with the number of hops from the source to the destination node and the adopted retransmission scheme. Thus, the upper and lower bound of the average energy consumption are \( \kappa_x \cdot E_u \) and \( \kappa_x \cdot E_l \) respectively, where \( x \) can be SAW or SR scheme.

V. NUMERICAL RESULTS

For simplification, we choose the system parameters \( K = 1 \), \( \mu_t = 1 \) and \( \sigma = 1/\sqrt{2} \) in the following numerical results. Fig. 2 shows the average forward distance versus average number of nodes of \( N_0 \) within unit circle for different \( \phi \) when \( \alpha = 4 \) and \( D = 10 \). Average forward distance arises when there is more number of nodes in connection with a sender. The average forward distance in CAGIF exceeds 1 while that in PGIF only approaches 1 with an increase of \( N_0 \), since a node that is very far away from the sender may be in connection with it in presence of fading. As \( \phi \) becomes larger, there is increasing average forward distance, which is attributed to the fact that more number of nodes come into connection with the sender with a larger specified angle.

We observe that average forward distance in GeRaF is larger than that in PGIF with \( \phi = \pi/3 \) when \( N \leq 18 \), while smaller than that in PGIF with \( \phi = 2\pi/3 \). This suggests that the geographic specification of forwarding area of \( \theta \leq \phi \) in PGIF that reduces the number of nodes in connection dominates the forward distance when \( N \leq 18 \). Average forward distance in PGIF with \( \phi = \pi/3 \) is larger than that in GeRaF when \( N > 18 \) as the forward distance in PGIF refers to the direct advancement rather than orthogonal projection, which counteracts the impact of the specification of the forwarding area and thereby dominates the forward distance.

Fig. 3 shows the average number of hops versus average number of nodes of \( N_0 \) within unit circle when \( D = 10 \), \( \alpha = 4 \) and \( \phi = \pi/3 \). The average number of hops decreases with average number of nodes within unit circle, due to the increasing average forward distance with more number of nodes within connection. The upper bound and lower bound in CAGIF are slightly larger than those in PGIF when \( N \leq 10 \), while smaller than those in PGIF when \( N \geq 10 \). This can be explained as follows. For a small \( N \), the decreased number of nodes in connection due to fading plays a dominant role. However, for a large \( N \), the phenomena that the further nodes away from the sender come into connection due to fading attains a larger average forward distance and hence results in the decrease of the average number of hops. We observe that the lower bound of the average number of hops in PGIF coincides with that of GeRaF when \( N \geq 20 \), which verifies our derivation. When \( N \) is small, the specification of the forwarding area in PGIF reduces the
forwarding distance and thereby results in an increase of the average number of hops in PGIF compared with GeRaF.

Fig. 4 shows the average energy consumption versus the average number of nodes of $N_0$ within the unit circle in CAGIF with varying loss factor compared with PGIF with different retransmission schemes and $\alpha = 4$ when $D = 10$, $\phi = \pi/3$, $P_f = 0.2$ and $\zeta = 2$. For illustration, we choose $P_t = 1$. There is more energy consumption in PGIF compared with CAGIF, which is attributed to the fact that there is less additional packet retransmissions in CAGIF that chooses the favorable channel conditions. Even though with $P_f = 0.2$ and $x = 2$, the energy consumption in PGIF is significantly more than that in CAGIF. SAW scheme results in more energy consumption over SR scheme due to extra packet retransmissions. We observe that more average energy is consumed with a larger path loss exponent since the transmission range is reduced and thus there is an increased number of hops from the source to the destination node. Fig. 5 shows the average energy consumption versus block error probability $P_f$ when $N_0 = 20$, $D = 10$, $\alpha = 4$, $\phi = \pi/3$ and $\zeta = 2$. The average energy consumption increases with an incremental failure probability $P_f$. However, there is much more energy consumption in PGIF compared with CAGIF and the upward trend of energy consumption in PGIF is much larger than that in CAGIF.

VI. CONCLUSION

In wireless sensor network, fading seriously impairs the channel conditions, which results in the additional energy consumption due to packet retransmissions in the deep-fading channel state. This letter studies the energy efficiency benefit of CAGIF by taking into consideration the realistic channel conditions in wireless sensor networks and shows that CAGIF significantly outperforms PGIF.

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REFERENCES

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Figure 2  Average forward distance versus average number of nodes of $N_0$ within unit circle for different $\phi$ when $\alpha = 4$ and $D = 10$ in CAGIF and PGIF compared with GeRaF.

Figure 3  Average number of hops versus average number of nodes of $N_0$ within unit circle when $D = 10$, $\alpha = 4$ and $\phi = \pi/3$.

Figure 4  Average energy consumption versus average number of nodes of $N_0$ within unit circle in CAGIF with varying loss factor compared to PGIF with $\alpha = 4$ when $D = 10$, $\phi = \pi/3$, $P_f = 0.2$ and $\zeta = 2$.

Figure 5  Average energy consumption versus block error probability $P_f$ when $N_0 = 20$, $D = 10$, $\alpha = 4$, $\phi = \pi/3$ and $\zeta = 2$. 
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<th>Variable Definition</th>
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<td>$D$</td>
<td>the distance between the source and the destination node</td>
</tr>
<tr>
<td>$d_{h1}$</td>
<td>one hop distance</td>
</tr>
<tr>
<td>$x_f$</td>
<td>the forward distance on x axis per hop</td>
</tr>
<tr>
<td>$x_r$</td>
<td>the remaining distance on x axis</td>
</tr>
<tr>
<td>$d_{rd}$</td>
<td>the distance between the current relay node and the destination node</td>
</tr>
<tr>
<td>$d_{t}$</td>
<td>a temporary variable</td>
</tr>
<tr>
<td>$\theta'$</td>
<td>the angle that the chosen forward direction deviate from the x axis</td>
</tr>
<tr>
<td>$k$</td>
<td>a counting variable</td>
</tr>
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| Do Initialization  | $d_{h1} = E[\zeta]$; $d_{rd} = D$; $x_f = d_{h1} \cos(\phi/2)$; $x_r = D$; $k = 0$; |

| Computation of the Worst Case Route | $\text{While } (d_{rd} > d_{h1}) \text{ Do}$ |
|                                   |   $k = k + 1$; $d_t = d_{rd}$; $d_{rd} = \sqrt{d_{h1}^2 + d_t^2 - 2d_{h1}d_t \cos(\phi/2)}$; $x_r = x_r - x_f$; $\theta' = \frac{\phi}{2} + \arcsin(x_r/d_{rd}) - \pi/2$; $x_f = d_{h1} \cos(\theta')$; $h_{upper} = k + 1$; |

| Computation of the Best Case Route | $h_{lower} = D/E[\zeta]$ |

Table I
Figure 1
Figure 2
Figure 3
Figure 4
Figure 5