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Advances in dynamic metrology using in-line digital holographic interferometry

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ABSTRACT

In this paper a lensless in-line digital holographic microscope is presented as interferometric applications for dynamic measurements. A diverging beam is used to illuminate the object to get the required magnification. In particular, time average in-line digital holographic interferometry is studied for vibration measurements of a smaller membrane. The sensitivities of the numerically reconstructed amplitude and phase information are studied with the distance from object to the CCD, during hologram recording. It is shown that, the increase in object recording distance results the increase in the sensitivity of the Bessel type of fringes representing the vibration amplitude information, while it shows the opposite behaviour for phase information which represent the mean deformation fringes. To explain this phenomenon, the samplings of the interference of object & reference beams, and of the diffracted speckled wavefront from the object are individually studied. A double exposure approach is used for the suppression of noise from real image wave caused by zero-order term and twin image waves because of in-line geometry. The experiment is performed for the study of vibration behaviour of harmonically excited aluminium membranes of 5 mm in size and results are presented.

Keywords: digital holography, in-line geometry, lensless microscopy, time-average method, speckled wavefront.

1. INTRODUCTION

In digital holography (DH), holograms are recorded in digital form using CCD sensors and then numerically reconstructed by computers [1]. Numerical reconstruction of holograms provides the quantitative analysis of the amplitude and phase information of the object wavefield, this is the unique features of DH over other existing optical method. The wide range of applications of DH in metrology [2], micro-interferometry [3], 3-D imaging [4], particles analysis [5] have been presented. In-line digital holography is the preferred choice of DH, it is because of the poor resolution of commercially available CCD sensors, which restrict the angle between object and reference wave to few degrees only. Additionally, in-line digital holography utilized the full sensor area of the CCD for real image formation during the reconstruction process. The overlapping of zero-order term and twin image wave with real image wave are the two main problems with in-line geometry. These unwanted overlapping can be suppressed digitally [6-8]. One simple approach of zero order wave suppression is the subtracting the sum of the intensities of object and reference wave from the hologram, before reconstruction process. The effects of the twin image wave can be minimized by selecting the suitable optical geometry, with one approach is simply using a lensless microscopic set-up with geometrically magnified object wave [9].

Time average method can be used for vibration analysis of the objects using DH [10]. For the recording the time average digital holograms, the recording time of CCD sensor is much larger than the time period of object vibration. The numerically reconstructed amplitude information from time average hologram is modulated by the zero-order Bessel function, same as in conventional time average holographic interferometry. Time average off-axis digital holographic interferometry has been presented for vibration analysis, the high contrast fringe visualization in low resolution digital holography using the concept of zero crossing phase has also presented [11]. Based on quasi-fourier configuration time average DH is also presented for so-called hidden stationary deformations of vibrating surfaces [12]. Furthermore it is demonstrated as a low-cost device for dynamic model characterization of musical instruments [13]. All these methods use the off-axis geometry or the quasi-Fourier configuration which requires large object distances from the CCD during hologram recording and has been thus applied for relatively large objects (greater than 30mm in diameter). More recently
we have presented time average digital in-line holographic interferometry for vibration analysis of smaller object (10mm in size) [14]. A double exposure method has been offered for vibration amplitude and mean static deformation analysis of the objects. This method is used for simultaneous suppression the overlapping of zero-order and twin image wave from real image wave. To further extend the capability of lensless in-line digital holography for dynamic analysis of smaller object like MEMS diaphragm characterization, the object wave needs to be magnified. This magnification can be achieved by illuminating the objects with a diverging wave, which provides the geometrical magnification of the object wave. In this paper we have explored the effects of diverging object wave on the numerically reconstructed amplitude and phase information from the time average in-line averages holograms. It is found that the increase in the divergence of the object wave by increasing the distance between object and CCD results as increase in the sensitivity reconstructed amplitude information which representing the vibration amplitude of the object. At the same time the sensitivity of reconstructed phase information which represents the mean static deformation, reduces with increase the distance. The theoretical explanation of this contradictory phenomenon is presented in this paper and experimentally verified.

2. TIME AVERAGE IN-LINE DIGITAL HOLOGRAPHY

Digital recording of time average in-line holograms

Let the object is placed at the plane \((x, y)\) and harmonically excited. The vibration displacement is \(\ddot{z} = \dot{z}(x, y)\cos \omega t\), here \(\omega\) is the frequency of vibration. The instantaneous wave scattered by the object can be written as,

\[
O(x, y, t) = O_0(x, y)\exp[i\phi_0(x, y)]\exp[i(\vec{K} \cdot \vec{z})]
\]  

(1)

\(O_0(x, y)\) is the complex amplitude, \(\phi_0(x, y)\) is the phase representing the mean static state of the object and \(\vec{K}\) is the sensitivity vector that depends on the angles of illumination and observation.

The object and reference wave lies in the same optical axis in in-line digital holography as shown in fig. (1). The interference of object wave \(O(\xi, \eta)\) and plane reference wave \(R(\xi, \eta)\) at the hologram plane \((\xi, \eta)\) can be written as,

\[
I(\xi, \eta) = |O(\xi, \eta)|^2 + |R(\xi, \eta)|^2 + O^*(\xi, \eta)R(\xi, \eta) + O(\xi, \eta)R^*(\xi, \eta)
\]

(2)

Here \(O^*\) and \(R^*\) are the complex conjugate of \(O\) and \(R\) respectively. For time average recording, the exposure time is much larger than the period of object vibration, \(\tau\). In case of time average recording the amplitude of object wave is modulated by the zero-order Bessel function, \(J_0\) and can be written as follows,

\[
O = O_0 \exp[i\phi_0]J_0[\vec{K} \cdot \vec{z}]
\]

(3)

A CCD, placed at the hologram plane, records this interference pattern. The recorded pattern by CCD is a two-dimensional array of discrete electrical signals which is converted in digital form by using the sampling theorem. If
$M \times N$ be the total number of pixels of the CCD with corresponding size $\Delta \xi$ and $\Delta \eta$, then the digitally sampled holograms $I(m,n)$, can be written as,

$$I(m,n) = [I(\xi, \eta) \otimes \text{rect}(\frac{\xi}{\alpha \Delta \xi}, \frac{\eta}{\beta \Delta \eta})] \times \text{rect}(\frac{\xi}{M \Delta \xi}, \frac{\eta}{N \Delta \eta}) \text{comb}(\frac{\xi}{\Delta \xi}, \frac{\eta}{\Delta \eta})$$

(4)

Where $\otimes$ represents the two-dimensional convolution and $(\alpha, \beta) \in [0,1]$ are the fill factors of the CCD pixels. Thus the time average in-line digital hologram can be written as,

$$H(m,n) = \int_{0}^{r} I(m,n)d\tau$$

(5)

### Numerical Reconstruction

The reconstruction geometry of in-line digital holographic system is shown in fig 2. For numerical reconstruction, the reconstruction wave is numerically defined which should be the same as the reference wave used during recording.

$$U(x', y', D') = e^{i \mu y'} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} H'_{\xi, \eta} R'(\xi, \eta) g(\xi, \eta) \exp \{-2\pi i (\xi y' + \eta \mu)\} d\xi d\eta$$

(6)

Here $\nu = \frac{x'}{\lambda D}$ & $\mu = \frac{y'}{\lambda D}$ are the spatial frequencies present in the hologram and $g(\xi, \eta)$ is the impulse response of the coherent optical system defined as $g(\xi, \eta) = e^{\frac{i \mu D}{\lambda D}} \exp \{-\frac{i \pi}{\lambda D} (\xi^2 + \eta^2)\}$. For numerical reconstruction process the above equation can be converted into discrete form and the wavefield is obtained by using the discrete Fresnel transformation. For the hologram recorded with CCD contains $M \times N$ pixels with pixel size $\Delta \xi$ and $\Delta \eta$ respectively, the reconstructed field defined by eqn. (6) can be converted to finite sums as follows [1],

![Fig. 2- In line digital holographic reconstruction geometry](image-url)
The eqn. (7) is the discrete Fresnel transformation. The matrix $U(k,l)$ is the discrete Fourier transform of the product of $H(m,n)$, $R(m,n)$ and $\exp\left(\frac{i\pi}{\lambda D} (m^2 \Delta \xi^2 + n^2 \Delta \eta^2)\right)$. Thus calculation of reconstructed wavefield can be done effectively by using the fast Fourier transform (FFT) algorithm. When reconstructed wave is the complex conjugate of the reference wave the real image of the object is formed at the reconstruction distance $D$ same as recording distance the hologram. The intensity $I(x', y')$ and phase $\phi(x', y')$ of the reconstructed real image wave can be written as,

\[ I = |U_{\text{real}}(x', y')|^2 = O_0^2(x', y') J_0^2(\tilde{K} \cdot \tilde{z}(x', y')) \]

\[ \phi_0(x', y') = \arctan \left( \frac{\text{Im}[U_{\text{real}}(x', y')]}{\text{Re}[U_{\text{real}}(x', y')]} \right) \]

The traditional time average holographic interferometry gives the mode shapes and amplitudes of vibration, and the phase term is not useful information and a source of speckle noise. For the suppression of the overlapping of zero-order and twin image wave two time average in-line digital holograms recorded at the same vibration frequency and at different amplitudes of vibration. Then the subtraction of the individually reconstructed amplitudes provides $J_0$ type fringes, which represents the vibration of the object with different visibility. The reconstructed phase information represent two parts, the first part is the time average phase (also called zero- crossing phase) and the second part represent the mean deformation of the object. Thus phase subtraction gives the difference in mean deformation of the object during its vibration cycle as in conventional digital holographic interferometry.

3. ANALYSIS OF DIVERGING OBJECT WAVE WITH RECORDING DISTANCE

In order to magnify the object wave in lens-less geometry, there is a need to illuminate the object with diverging wave as shown in fig. 3. The object wave is geometrically magnified by the factor $(D + d)/d$. Thus higher magnification is achieved at larger distance between object and CCD.

To record the digital hologram, the sampling theorem requires that the fringe spacing of the interference pattern between object and reference wave must be larger than the size of two pixels. Let $L$ is size of the object, then for a CCD sensor containing $N$ pixels, $\Delta \xi$ in size, the smallest distance $(D_{\text{min}})$ between the object and CCD sensor for proper sampling of hologram is [16],

\[ D_{\text{min}} \geq \frac{L}{\Delta \xi} \]
\[ D_{\text{min}} = \frac{\Delta \xi}{\lambda} (N \Delta \xi + L) \]

where \( \lambda \) is the wavelength of light. Increase in object to CCD distance increases the fringe spacing of the interference pattern this results as the better sampling of the holograms at the distances larger than \( D_{\text{min}} \).

The size of the objective speckles at the CCD plane \( \frac{\lambda D}{L} \), which is linearly proportional to the distance between object to CCD. Increase in objective speckle size results as the decrease in the object spatial frequencies. The reduction in the object spatial frequency will reduces the sensitivity of the information represented by the speckles. The mean deformation information of the object wave is modulated in the speckle pattern and thus smaller the object to CCD distance, smaller the speckle size and larger the phase sensitivity. Fig. 4 plot the variation of objective speckle size, spatial frequencies of the speckles and spatial frequencies interference of object and reference wave (hologram) as a function of object to CCD distance. The parameters considered for this plot are, object size 5mm, CCD array size: 2044x2024, pixel size: 9 \( \mu \)m, wavelength: 533nm.

![Fig 4 Variation of Speckle size and spatial frequencies of speckles & hologram as a function of object to CCD distance](chart.png)

**4. EXPERIMENTAL RESULTS**

The schematic of in-line digital holographic experimental set-up is shown in Fig. 5. The output beam of a frequency doubled Nd-YAG laser operating at wavelength 532 nm is coupled into a single mode fiber coupler and split into two beams. The beam coming from one fiber illuminates the vibrating object with the diverging beam and the other is collimated to form the in-line reference beam. Thus the diverging object wave interferes with the collimated reference beam and forms the hologram which is recorded by the CCD.

The object is a thin aluminium circular membrane of 5 mm in size. The membrane is stuck on a metal circular ring and attached to an earphone as shown in fig. 5. The earphone is used to provide sinusoidal harmonic excitation to the membrane, for this purpose a frequency generator is connected to the earphone that can be tuned to different frequencies.
and amplitudes. An 8-bit CCD sensor (from Kodak mega plus) containing 2029x2044 square pixels \( 9 \mu m \) in size is used to record the holograms at the speed of 30 frames per second. Thus time-averaged hologram can be recorded when the frequency of object vibration is much larger than 30 Hz. The numerical reconstruction process is performed by using fast Fourier Transform (FFT) algorithm (eqn. 7) and simulated using MATLAB\textsuperscript{TM} software.

![Diagram of Digital in-line holographic set-up](image)

In order to satisfy the Nyquest sampling criteria for digital sampling of holograms by the CCD sensor, the minimum object distance from the CCD should be 388 mm. The effect of object distance with CCD on numerically reconstructed amplitude and phase information is studied. Three different distances 250mm, 380mm and 420mm were selected and their effects on numerical reconstruction are presented. The reconstructed amplitude from time average hologram is modulated by the zero-order Bessel function and represents the vibration behaviour of the object. The larger object to CCD distance provides the better sampling of the interference fringes of the hologram and thus will increase the sensitivity of vibration amplitude information. The reconstructed amplitude images from the time averaged holograms recorded at the vibration frequency 2.6 KHz with excitation voltage of 2.5 volts are shown in Fig. 6(a), 6(b) and 6(c). The fringes represent vibration mode of the membrane. It can be clearly observed that the sensitivity of the vibration fringes increases with increase in distances.

![Vibration fringes behaviour with divergence of object wave](image)

The phase information of the reconstructed wavefield is basically contains two parts, the first part contains information about the mean static deformation of the object and the second part called the zero crossing phase (or time average phase)
that can be used for the better determination of the Bessel fringes. The mean static deformation in the vibrating membrane can be introduced by an offset voltage or can occur naturally due the non-linear mechanical effects. The objective speckles in the object wavefront contain information about the object mean deformation and also a cause of speckle noise. These speckles can be used for mean deformation of the object same as in conventional speckle interferometry. In our case the subtraction of reconstructed phases during double exposure represents the mean static deformation fringes. Only the mean deformation fringes are observed at lower amplitudes of vibrations, and as vibration amplitude increases it mixes with time average phase information caused by vibrations. We observed the reconstructed phase information at the above mentioned three distances between object to CCD. As discussed earlier, the sensitivity of the mean deformation is higher at smaller distances because of the higher spatial frequencies of objective speckles.

Figure 7(a-c) shows the subtraction of reconstructed phases from two time average holograms recorded at the frequency 2.6 kHz and at different amplitudes of vibration (excitation voltages are 1.5 volts and 1.5 volts with offset voltage 2.5 volts) corresponding to the three distances respectively. The mean deformation fringes are clearly observed. The number of fringes is larger at the smaller recording distance and as the distance increases the number of fringes reduces. Since at the larger distances the sensitivity of amplitude information increases, this simultaneously increases the sensitivity of zero crossing phase information. Thus the effect of the mixing of the mean deformation phase information and zero crossing phase information becomes more effective at larger distances.

Figure 8 Mixing of phase information (time average and mean deformation)

At the resonant frequencies, the inspection of the fringe pattern obtained by the phase subtraction from two time average holograms indicates, what appear to be additional phase effects due to the fact that the membrane is vibrating at resonance. These additional phase changes mask the mean deformation fringes in these regions. This is because of the amplitudes at resonant frequencies are much higher than for static deformation if the plate was not vibrating, as is to be
expected. It is clearly shown in fig. 8, here phase subtraction of the holograms recorded at the resonant frequency 2.6 KHz and exciting amplitudes are 2.5 and 1.5Volts and the object to CCD distance is 420mm. The zero crossing phase can be used to identify the location of the nodal lines of the mode pattern, thus also define the location of the phase jump region.

5. CONCLUSIONS

In conclusion, the amplitude and phase information of lensless in-line digital holographic microscopy interferometry is presented for vibration analysis of smaller objects. The effects of a magnified object speckled wavefront on reconstructed amplitude and phase information of the vibrating objects are studied for the variation in the recording distances. The sampling of interference of object and reference wave (hologram) and of individual speckles is independently studied and their effects during reconstruction process are presented. The contradict behaviour of the sensitivity of time averaged vibration and mean static deformation fringes are presented with a change in the recording distance. Finally, the analysis of the lensless in-line digital holographic microscopic geometry is very useful for the vibration analysis of smaller objects and particularly can be readily extended for smaller objects like MEMS diaphragms.

References