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A CHANCE-CONSTRAINED MODEL FOR REGIONAL AIR QUALITY MANAGEMENT

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A CHANCE-CONSTRAINED MODEL FOR REGIONAL AIR QUALITY MANAGEMENT

Submitted by
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ABSTRACT

Regional air pollution has been a major global problem due to its health-associated risks together with its economic and environmental impacts. Though there have been significant advances in air pollution control technologies, the implementation of these control strategies are costly. It is thus desirable for effective air quality planning and management to be undertaken to identify and implement cost-effective strategies, ensuring local air quality at safe levels.

In this study, an inexact chance-constrained optimization model (ICCLP) was developed for air quality management under uncertainty. The ICCLP was formulated by integrating the inexact linear programming (ILP) and the chance-constrained programming model (CCP). The ICCLP allows the left-hand side (LHS) random variables to be expressed as interval numbers while letting the right-hand side (RHS) constraints to be expressed as probabilistic functions. In this way, the highly random RHS constraints will be satisfied at predetermined confidence levels, providing a more flexible and in-depth tradeoff analysis when applied to air quality management.

To determine the applicability of the proposed ICCLP to regional air quality management, it was applied to a hypothetical case study, where the results were analyzed and compared with those from the ILP and CCP models.

It was observed that the ICCLP could incorporate more uncertain information within its modeling framework. In addition, the method provides not only decision variable solutions presented as intervals but also the associated risk levels in violating the system constraints. It can therefore support an elaborate analysis of the tradeoff between system cost and system-failure risk. Hence, it is a useful tool for generating decision alternatives and thus helps policy makers identify desired policies under various environmental, economic, and system-reliability constraints.
ACKNOWLEDGEMENT

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The paper would not have been possible without their contributions and inspiration along the way.

Michele
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*School of Civil & Environmental Engineering
# Final Year Project Report

A CHANCE-CONSTRAINED MODEL FOR REGIONAL AIR QUALITY MANAGEMENT

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CHAPTER 1. INTRODUCTION

Regional air pollution has been a major global problem due to its health-associated risks together with its economic and environmental impacts [2]. Though there have been significant advances in air pollution control technologies, the implementation of these control strategies are costly. It is thus desirable for effective air quality planning and management to be undertaken to identify and implement cost-effective strategies, ensuring local air quality at safe levels.

Figure 1 shows a complete representation of an air quality management system, where assessments of air quality, environmental damage and abatement options are inputs into a cost-benefit or a cost-effectiveness analysis. These analyses are guided by established air quality objectives (i.e., guidelines, standards) and economic objectives (i.e., reduction of damage costs) with the main aim of developing an optimum control strategy in the form of an action plan, with prioritized abatement measures.

Figure 1: System for air quality management [1]
To facilitate the analytical process, optimization tools can be used to analyze tradeoffs between system economy and reliability [3]. The deterministic models have been widely used to perform such analysis. However, air pollution control involves several processes (i.e., pollution generation, mitigation, transport, and impact) that are associated with extensive uncertainties due to their interactive, dynamic and multi-objective features, causing deterministic models to be inadequate when dealing with air pollution management problems.

Previous works have thus ventured the use of inexact models such as interval linear programming (ILP), fuzzy, mathematical programming (FMP) and chance-constrained programming (CCP) to deal with the uncertainty of air quality management systems. However, these models each have their respective limitations, affecting practical applicability. Hence, the objective of this study is to develop a time-efficient and cost-effective air quality optimization model, which would lead to an efficient and accurate analysis of tradeoffs. This will help policy makers and industrial companies in deciding the type of polices and strategies to implement. The model will allow uncertainties in simulation and optimization parameters to be considered explicitly in the design of least-cost strategies and its applicability will be demonstrated through the study of a hypothetical management case.
CHAPTER 2. LITERATURE REVIEW

2.1. Optimization models

Over the last few decades, optimization tools have been applied to air pollution control planning systems to analyze tradeoffs between system economy and reliability. Yet, air pollution control involves several processes (i.e., pollution generation, mitigation, transport, and impact) that are associated with extensive uncertainties due to their interactive, dynamic and multi-objective features. These uncertainties affect relevant optimization, complicating air quality planning efforts. Figure 2 illustrates the development of optimization models research in the last few decades.

Figure 2: Development of optimization models research
2.1.1. Early disciplinary studies

2.1.1.1. Deterministic models

Deterministic models are the first optimization models developed to handle optimization problems for various management systems. These models use deterministic coefficient values and hence can only generate a single-scenario solution. Due to its inability to account for the high stochastic modeling parameters found in air quality control systems, its application to air quality management problems is relatively limited.

2.1.1.2. Inexact models

Development of inexact models started with the main objective of tackling optimization problems with random parameters. Some common inexact models developed are fuzzy mathematical programming (FMP), interval mathematical programming (ILP) and stochastic mathematical programming (SMP). With the ability to account for the uncertainties in management problems, inexact models are extensively applied in environmental management problems, including air quality management problems as reported in [4-7]. These models provide a more accurate alternative to the deterministic models in tackling air quality management problems.

**Fuzzy mathematical programming (FMP)**

FMP is a model developed from fuzzy set theory. It delimits an uncertain decision space through the specification of uncertainties by dimensional enlargement of original fuzzy constraints [8]. Using this method, the FMP is able to handle the vagueness of environmental management problems. However when applied to air quality management, despite its excellencies in accounting for the highly imprecise modeling parameters, the method is difficult to obtain due to its complex mathematical manipulations and intensive computer burden [2, 9].

**Interval linear programming (ILP)**

ILP is an inexact optimization model that deals with parameter uncertainties through the use of interval numbers. By transforming the uncertainties which are expressed as interval numbers into 2 deterministic sub models that correspond to upper and lower boundaries of the desired objective function, the model can be solved [2, 10-13].
The main advantages of the ILP are the relatively low computational requirement [2, 14] and its ability to proceed with optimization, without requiring distributional information of model parameters. These advantages are particularly meaningful for practical application as it is more convenient for planners and engineers to allocate data into intervals instead of distribution functions or membership functions [15], increasing solving efficiencies. Owing to these advantages, ILP applications have been reported in many environmental problems including air quality management problems [16-20].

However, the ILP poses one main limitation when applied to the case of air quality management problems. It does not allow the violation of constraints and may become infeasible when dealing with the highly uncertain right-hand side (RHS) parameters of air quality management optimization models [2, 14].

**Stochastic mathematical programming (CCP)**

Chance-constrained programming (CCP) is one stochastic optimization tool that can deal with the randomness of the RHS parameters in optimization models unlike the FMP and ILP. It is able to effectively reflect system reliability of satisfying system constraints under uncertainty and thus does not require all constraints to be strictly satisfied. Instead, these constraints can be satisfied in a proportion of case with the given probabilities [2, 13, 21, 22].

With that advantage, CCP has gained its popularity over other inexact models and has been applied to many environmental management problems [9, 11, 13, 21-27]. Early applications of CCP to air quality management problems have also been reported in Ellis et al. and Guldmanl [28, 29].

However, the CCP has also shown several limitations when applied to air quality modeling. This arises as the quality of many uncertainties are often not good enough to be expressed as probability distribution functions [7]. In addition, the many air quality optimization parameters expressed as PDFs tend to lead to a high computational difficulty when CCP is applied. These limitations have restricted the practical applicability of CCP in tackling air quality management problems [5].
2.1.2. Recent studies

Recent research has ventured into the possibility of integrating inexact optimization models to counter their limitations.

Liu [30-32] proposed a CCP integrated with fuzzy parameters (FCCP), improving the conventional FMP method by incorporating confidence levels of constraint satisfaction into optimization models [2]. Although it has gained in popularity in many fields, application is relatively limited in the environmental field, including air quality management problems. This is due to the model limited ability to handle parameter uncertainties in the objective function of an optimization model and the left-hand side (LHS) of the model constraints.

As an improvement to the FCCP, Ye Xe proposed a chance-constrained model integrated with ILP and FMP – an inexact fuzzy chance-constrained programming (IFCCP) and applied it to an air quality management problem. This model surpasses the FCCP and existing ILP in dealing with air quality management problems by allowing uncertainties to be presented as discrete intervals and possibilistic distributions. In addition, the constraints with fuzzy variables can be satisfied at different confidence levels such that various solutions can be obtained [2].

However, the fuzzy integration in the IFCCP still poses the problem of complex mathematical manipulation and intensive computer burden and hence is not often desired by policy makers and industrial engineers.

2.2. Modelling of atmospheric dispersion of pollutants

For optimization models to effectively carry out tradeoff analysis in air quality management systems, an estimation of the amount of emission pollutants reaching various receptor zones is required. This is often done through atmospheric dispersion modeling where mathematical simulation of air pollutants dispersion pathways in the atmosphere is used to determine the amount of pollutants at various receptor zones. Nonetheless, there are many factors influencing air pollutants dispersion, complicating the modeling process [33, 34]. These factors include:

i. Meteorological conditions
ii. Emissions or release parameters
iii. Surrounding topography
iv. Possible reactions of pollutants in the atmosphere

2.2.1. Early disciplinary research

Early disciplinary research on atmospheric dispersion of pollutants has identified four basic types of air pollution dispersion models, as well as their hybrids. They include the box model, the gaussian model, the lagrangian model and the eularian model [34].

Box model

Box model is the most elementary of the model types. It assumes that airshed (i.e., a given volume of atmospheric air in a geographical region) is in the shape of a box in which emissions are mixed uniformly. This assumption allows the estimation of average pollutant concentrations anywhere within the airshed through the use of mass balance equations.

Although useful, this model is limited in accurately predicting dispersion of air pollutants over an airshed as most pollutants distributions are not homogenous.

Gaussian model

The gaussian model is the oldest and the most commonly-used technique for dispersion calculations. Its approximation is based on the assumption that air pollutant dispersion follows a gaussian distribution, where pollutant concentrations model a normal distribution curve. The primary algorithm used in gaussian modeling is the gaussian plume model for a continuous point source plume as shown in Equation 1 [35]. This model is often used for predicting the dispersion of continuous and buoyant air pollution plumes originating from ground-level or elevated sources.

\[
C(x, y) = \frac{Q}{\pi \mu \sigma_y \sigma_z} \exp \left[ -\frac{y^2}{2\sigma_y^2} - \left( \frac{H^2}{2\sigma_z^2} \right) \right]
\]

(1)

where

- \(C\) = ground pollutant concentration at arbitrary down-wind location \((x, y)\) (mg/m);
- \(x\) and \(y\) = relative horizontal and vertical coordinates of the receptor zone with the location of emission source being the origin of the coordinates;
\[ H = \text{average effective stack height (m)}; \]
\[ Q = \text{pollutant emission rate (mg/s)}; \]
\[ u = \text{wind velocity (m/s)}; \]
\[ \sigma_y \text{ and } \sigma_z = \text{standard deviations of the plume in the } y \text{ and } z \text{ direction (m)}. \]

The standard deviations, \( \sigma_y \) and \( \sigma_z \), in Equation 1 are defined by dispersion coefficients, a measure of turbulence in the atmosphere [33, 36]. To determine dispersion coefficients, Pasquill (1961) introduced a method to estimate atmospheric stability by developing a stability classification where atmospheric turbulence is categorized into 6 stability classes - A, B, C, D, E and F. The progressive alphabets represent the increasing stability of atmospheric turbulence with Class A being the most unstable and turbulent while Class F being the most stable or least turbulent. In some cases, a seventh stability class, G, is used to represent very stable conditions. With this stability classification, the appropriate dispersion coefficients can be specified and used in the gaussian model [37]. Tables 1 and 2 below list the six classes and the related meteorological conditions for each class.

Table 1: The pasquill stability class

<table>
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<th>Stability class</th>
<th>Definition</th>
<th>Stability class</th>
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<tr>
<td>A</td>
<td>very unstable</td>
<td>D</td>
<td>neutral</td>
</tr>
<tr>
<td>B</td>
<td>unstable</td>
<td>E</td>
<td>slightly stable</td>
</tr>
<tr>
<td>C</td>
<td>slightly unstable</td>
<td>F</td>
<td>stable</td>
</tr>
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Table 2: Meteorological conditions that define the pasquill stability classes

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<th>Surface wind speed</th>
<th>Daytime incoming solar radiation</th>
<th>Nighttime cloud cover</th>
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<tr>
<td>m/s</td>
<td>mi/h</td>
<td>Strong</td>
</tr>
<tr>
<td>&lt; 2</td>
<td>&lt; 5</td>
<td>A</td>
</tr>
<tr>
<td>2 – 3</td>
<td>5 – 7</td>
<td>A – B</td>
</tr>
<tr>
<td>3 – 5</td>
<td>7 – 11</td>
<td>B</td>
</tr>
<tr>
<td>5 – 6</td>
<td>11 – 13</td>
<td>C</td>
</tr>
<tr>
<td>&gt; 6</td>
<td>&gt; 13</td>
<td>C</td>
</tr>
</tbody>
</table>

Note: Class D applies to heavily overcast skies, at any wind speed day or night
Extended gaussian models can also be used for predicting the dispersion of non-continuous air pollution plumes (i.e., gaussian puff model) or take into consideration removal processes such as dry deposition of particles and chemical transformation assuming exponential decay.

Till date, the gaussian plume model is currently the most commonly-used dispersion model in air quality optimization models dealing with continuous point sources to determine the amount of emission pollutants reaching various receptor zones. This is due to its easier computational methods, generating results of relatively high accuracy when dealing with continuous point sources [33, 38].

**Lagrangian model**

The lagrangian approach models the behavior of representative fluid particles or pollution plume parcels using random walk process. By computing the statistics of the trajectories of a large number of the pollution plume parcels, the lagrangian model is then able to calculate the air pollution dispersion. A moving frame of reference is used in the lagrangian model as the parcels move from their initial location [39, 40].

One major drawback of this approach is the limited applicability of the resulting equations due to the difficulty of accurately determining the required particle statistics. In addition, the equations are not directly applicable to problems involving nonlinear chemical reactions [39].

**Eulerian model**

Eulerian dispersion models, similar to that of lagrangian models also tracks the movement of a large number of pollution plume parcels as they move from their initial location. However, eulerian methods uses a fixed three-dimensional cartesian grid by attempting to formulate the concentration statistics in terms of the stastical properties of the eulerian velocities, i.e. the velocities measured at fixed points in the fluid instead [41].

Unfortunately, the eulerian approaches tend to lead to serious mathematical unstable known as the closure problem. A number of approximate solutions have been proposed but each of them leads to an equation which gives accurate results for only a limited class of problems [41].

In attempt to deal with the constraints of each model, researchers have proposed the integration of some of these models. The applications of these hybrids are reported in [34].
2.2.2. Recent research

One drawback of the models mentioned in chapter 2.2.1 is the inability to account for the topographical impacts [34]. These limitations led to the research and development of computer modeling systems with the ability to account for these factors.

Computer models are an extension of the four basis models. They are built on highly sophisticated numerical models that use several numerical techniques to stimulate pollutant dispersion. It takes into consideration the meteorological conditions, topographical conditions, emission sources type and parameters, and removal processes by solving mathematical equations and algorithms. Most of these models do not categorize atmospheric turbulence by using the simple meteorological parameters commonly used in defining the six pasquill classes as shown in Table 2, but instead some forms of the complex monin-obukhov similarity theory [42-44].

Most modern, advanced dispersion modeling programs include a pre-processor module for the input of meteorological, topographical, emission sources type and parameters and other data.

Two of such computer modeling systems highly recommend by the United States Environmental Protection Agency (EPA) are the CALPUFF and the AERMOD modeling system [45].
CHAPTER 3. METHODOLOGY

As it is economically infeasible or technically impossible to design a process leading to zero emission of air pollutants, local authorities and decision makers always seek to control the emissions to safe levels at the minimum cost. Thus, air pollution optimization problems are focused on selecting the type of abatement measures that will ensure emission sources standards and receptor zone environmental loading capacities are met at a minimum cost.

In this study, the use of ILP and CCP in air pollutant optimization models are first applied to account for the stochastic parameters found in air quality management problems. This results in the development of 2 different models - (1) the inexact air quality optimization model and (2) the chance-constrained air quality optimization model. FMP is not considered in this study due to its limited practical applicability owing to its complex mathematical manipulations and intensive computer burden.

To counter the limitations of the ILP and CCP model (as stated in the literature review), the ILP model is proposed to be integrated with the CCP model, developing an inexact chance-constrained linear programming model (ICCLP). The ICCLP model will allow the LHS random variables to be expressed as interval sets while letting the highly random RHS constraints to be satisfied at predetermined confidence levels. In this way, the ICCLP is able to provide a more flexible and in-depth tradeoff analysis when applied to air quality management.

Following which the developed models (IPL, CCP and ICCLP) are finally applied to a case study to demonstrate its applicability where the results will be generated and compared. The below sections entail the development of these models.
3.1. Development of air quality optimization models

a) Deterministic air pollution optimization model

Below shows a general air pollution optimization model [2].

Objective Function

Minimise $F = \sum_{i=1}^{r} \sum_{j=1}^{m} \sum_{k=1}^{n} c_{jk} x_{ijk}$

Subjected to

1) Constraints of pollution generation:

$\sum_{j=1}^{m} x_{ijk} \geq s_{ik}, \ \forall i, k$ (2b)

2) Constraints of emission standards:

$\sum_{j=1}^{m} (1-\eta_j)x_{ijk} \leq e_{ik}, \ \forall i, k$ (2c)

3) Constraints of environmental loading capacities:

$\sum_{i=1}^{r} \sum_{j=1}^{m} t_{ip} (1-\eta_j)x_{ijk} \leq a_{pk}, \ p = 1, 2..., r; \ \forall k$ (2d)

4) Non-negativity constraints:

$x_{ijk} \geq 0, \ \forall i, j, k$ (2e)

*Refer to page 28 for the nomenclature of the terms above

To apply the deterministic model, the random parameters in the optimization model are allocated deterministic values, and are thus solved linearly. These deterministic values can be obtained by taking the mean of each random parameter provided.
b) **Interval linear programming air quality optimization model (ILP)**

When applying the ILP model to the air quality optimization model, the random parameters in the optimization model are expressed as interval numbers without the need for any distributional information. This allows the random variables to be directly transferred into the optimization model.

To solve the ILP optimization problem, the interval numbers are expressed into 2 deterministic sub models, corresponding to upper and lower boundaries of the desired objective function. From which, they may be solved linearly [21, 22, 25] and the final solutions of the objective value \( F^0 = [F_{opt}^-, F_{opt}^+] \) and decision variables \( X^0 = [X_{opt}^-, X_{opt}^+] \) can be obtained.

According to Huang et al [46], an interactive solution algorithm can be used to solve the ILP model through analyzing the relationships among the parameters, variables, objective function, and constraints. The details are shown in Appendix A. With reference to the algorithm, the inexact air quality optimization model can formulated and solved as follows:

**Step 1: Formulate the \( F^- \) submodel**

Objective Function

\[
\text{Minimise } F^- = \sum_{i=1}^{r} \sum_{j=1}^{m} \sum_{k=1}^{n} c_{jk}^- x_{ijk}^-
\]  \hspace{1cm} (3a)

Subjected to

1) Constraints of pollution generation:

\[
\sum_{j=1}^{m} x_{ijk}^- \geq s_{ik}^-, \quad \forall i, k
\]  \hspace{1cm} (3b)

2) Constraints of emission standards:

\[
\sum_{j=1}^{m} \eta_{ij}^- x_{ijk}^- \leq e_{ik}^+, \quad \forall i, k
\]  \hspace{1cm} (3c)

3) Constraints of environmental loading capacities:

\[
\sum_{i=1}^{r} \sum_{j=1}^{m} \eta_{ij}^+ (1-\eta_{ij}^-) x_{ijk}^- \leq a_{ik}^+, \quad p = 1, 2, \ldots, r, \ \forall i, k
\]  \hspace{1cm} (3d)
4) Non-negativity constraints:
\[ x_{ijk}^{-} \geq 0, \ \forall i, j, k \]  
(3e)

*Refer to page 28 for the nomenclature of the terms above

**Step 2: Solve for F-**

**Step 3: Formulate the F* model based on the solution of the F- model**

Objective Function

\[
\text{Minimise } F^{+} = \sum_{i=1}^{r} \sum_{j=1}^{m} \sum_{k=1}^{n} c_{jk} x_{jk}^{+}
\]

(4a)

Subjected to

5) Constraints of pollution generation:

\[ \sum_{j=1}^{m} x_{jk}^{+} \geq s_{ik}^{+}, \ \forall i, k \]

(4b)

6) Constraints of emission standards:

\[ \sum_{j=1}^{m} (1 - \eta_{ij}^{+}) x_{jk}^{+} \leq e_{ik}^{-}, \ \forall i, k \]

(4c)

7) Constraints of environmental loading capacities:

\[ \sum_{j=1}^{m} t_{ip}^{-} (1 - \eta_{ij}^{+}) x_{jk}^{+} \leq a_{pk}^{-}, \ p = 1, 2, ..., r; \ \forall k \]

(4d)

8) Model Constraints:

\[ x_{ijk}^{+} \geq x_{ijk}^{-}, \ \forall i, j, k \]

(4e)

*Refer to page 28 for the nomenclature of the terms above

**Step 4: Obtain the solutions for the F* model**
c) **Chance-constrained air quality optimization model**

When applying the CCP to the air quality optimization model, the random LHS parameters in the optimization model are first allocated deterministic values by taking the mean of each random parameter provided.

The random RHS constraints on the other hand, are expressed as probabilistic functions. This allows for constraints to be satisfied under pre-defined probability levels \( (1-q_i) \) instead of being strictly satisfied. \( q_i \) is defined as the probability of violating the RHS constraint where \( (q_i \in [0,1]) \).

Equations 5 and 6 can be used to translate the random RHS constraints of the air quality optimization model which are expressed in interval sets model into probabilistic functions [47].

Assuming that:

\[
\mu_i - 3\sigma_i \leq e_i^{+} \leq \mu_i + 3\sigma_i, \quad \forall i \tag{5}
\]

where

\( e_i^{\pm} = \text{random RHS constraint, expressed as interval numbers;} \)

\( \mu_i = \text{mean of probabilistic function of random RHS constraint;} \)

\( \sigma_i = \text{standard deviation of probabilistic function random RHS constraint.} \)

The random RHS constraints can be expressed as probabilistic functions, where the mean and the variance of the probabilistic functions can be defined as follows:

\[
\mu_i = \frac{e_i^{+} + e_i^{-}}{2}, \quad \forall i \tag{6a}
\]

\[
\sigma_i = \frac{e_i^{+} - e_i^{-}}{6}, \quad \forall i \tag{6b}
\]
The CCP model is hence formulated as:

**Objective Function**

\[
\text{Minimise } F = \sum_{i=1}^{r} \sum_{j=1}^{m} \sum_{k=1}^{n} c_{jik} x_{ijk}
\]  

(7a)

Subjected to

1) Constraints of pollution generation:

\[
\sum_{j=1}^{m} x_{ijk} \geq s_{ik}, \forall i, k
\]

(7b)

2) Constraints of emission standards:

\[
\Pr\{ \sum_{j=1}^{m} (1-\eta_{j}) x_{ijk} \leq e_{ik} \} \geq 1 - q_{i}, \forall i, x_{ijk} \geq 0, \forall i, k
\]

(7c)

3) Constraints of environmental loading capacities:

\[
\Pr\{ \sum_{i=1}^{r} \sum_{j=1}^{m} \tau_{ip} (1-\eta_{j}) x_{ijk} \leq a_{pk} \} \geq 1 - q_{i}, \ p = 1, 2..., r; \forall k
\]

(7d)

4) Non-negativity constraints:

\[
x_{ijk} \geq 0, \forall i, j, k
\]

(7e)

*Refer to page 28 for the nomenclature of the terms above

To solve the chance-constrained optimization problem, constraints 2 and 3 can be transformed into a deterministic linear model by first initializing the probability level of \( q_{i} \) of constraint \( ( q_{i} \in [0,1] ) \) before converting the chance-constraints into the following equation \([13, 22] \):

\[
A_{i} x \leq b_{i}^{a}, \forall i \text{ and } b_{i}^{a} = F_{i}^{-1} (q_{i})
\]

(8)

where

- \( b_{i}^{a} \) = RHS constraint of air quality optimization model;
- \( F_{i} \) = cumulative function of \( b_{i}^{a}; \)
- \( q_{i} \) = probability of violating constraint \( b_{i}. \)
The CCP air quality optimization model can thus be switched to a deterministic linear model as follows, from which it can be solved.

Objective Function

\[
\text{Minimise } F = \sum_{i=1}^{k} \sum_{j=1}^{m} \sum_{k=1}^{n} c_{jk}x_{ijk} \quad (9a)
\]

Subjected to

5) Constraints of pollution generation:

\[
\sum_{j=1}^{m} x_{ijk} \geq s_{ik}, \quad \forall i, \ k \quad (9b)
\]

6) Constraints of emission standards:

\[
\sum_{j=1}^{m} (1-\eta_{ij})x_{ijk} \leq e_{ik}^{q}, \quad \forall i, \ k \quad (9c)
\]

7) Constraints of environmental loading capacities:

\[
\sum_{i=1}^{r} \sum_{j=1}^{m} f_{ip} (1-\eta_{ij})x_{ijk} \leq a_{pk}^{q}, \quad p = 1, 2..., r; \quad \forall k \quad (9d)
\]

8) Non-negativity constraints:

\[
x_{ijk} \geq 0, \quad \forall i, j, k \quad (9e)
\]

*Refer to page 28 for the nomenclature of the terms above*
d) Inexact chance-constrained air quality optimization model (ICCLP)

To counter the limitations of the ILP and CCP air quality optimization model, an integration of the two models is proposed. The resulting hybrid model - an inexact chance-constrained model (ICCLP) can be devised as follows. The ICCLP allows the LHS random variables to be expressed as interval sets while letting the RHS constraints be expressed as probabilistic functions. In this way, the highly random RHS constraints to be satisfied at predetermined confidence levels, providing a more flexible and in-depth tradeoff analysis when applied to air quality management. Below illustrates the formulation of the ICCLP model.

Objective Function

\[
\text{Minimise } F = \sum_{i=1}^{r} \sum_{j=1}^{m} \sum_{k=1}^{n} c_{jk}^\pm x_{ijk}^\pm
\]

(10a)

Subjected to

1) Constraints of pollution generation:

\[
\sum_{j=1}^{m} x_{ijk}^\pm \geq s_{ik}^\pm, \forall i, k
\]

(10b)

2) Constraints of emission standards:

\[
\sum_{j=1}^{m} (1 - \eta_{ij}^\pm) x_{ijk}^\pm \leq (e_{i}^{q})_{ik}^\pm, \forall i, k
\]

(10c)

3) Constraints of environmental loading capacities:

\[
\sum_{i=1}^{r} \sum_{j=1}^{m} \sum_{p=1}^{r} \sum_{q=1}^{n} t_{ip}^\pm (1 - \eta_{ij}^\pm) x_{ijk}^\pm \leq (a_{pk}^{q})_{ik}^\pm, \ p = 1, 2..., r; \forall k
\]

(10d)

4) Non-negativity constraints:

\[
x_{ijk}^\pm \geq 0, \forall i, j, k
\]

(10e)

*Refer to page 28 for the nomenclature of the terms above

To solve the ICCLP model, it can be transformed into two deterministic sub models corresponding to the upper and lower bounds of the objective equation. Solving the ICCLP model can be an integration of the two submodels. The solutions of the objective and decision variables for the proposed model are the optimal ones in the sub models as the upper and lower bounds. The framework of the ICCLP is shown in Figure 3 and the detailed solution processes are listed as follows [13, 21, 22]:

*School of Civil & Environmental Engineering
Step 1: Obtain the parameter values in the constrained models
Step 2: Analyze the study system and formulate the ICCLP model
Step 3: Transform the ICCLP model into two sub models according to the parameter intervals
Step 4: Solve the sub model in different $q_i$ levels based on the following procedures;
   a) Formulate the $F$ submodel;
   b) Solve for $F$;
   c) Formulate the $F^+$ model based on the solution of the $F^-$ model;
   d) Obtain the solutions for the $F^+$ model;
Step 5: Acquire the optimal results under different $q_i$ levels, such as $F = [F_{opt}, F_i^{opt}]$, $x = [x_{ijk}^{opt}, x_{ijk}^{+opt}]$.

![Diagram](image-url)

Figure 3: Framework of ICCLP air quality optimization model
Nomenclature

\( i \) = name of SO\(_2\) emission source;
\( j \) = type of SO\(_2\) control method;
\( x_{i,j,k} \) = amount of SO\(_2\) generated from source \( i \) allocated to control measure \( j \) (t/d) during period \( k \);
\( F \) = daily operational cost ($/d);
\( m \) = number of pollution control measures;
\( p \) = name of receptor zone;
\( r \) = number of emission sources;
\( a_{p,k} \) = environmental loading capacity of receptor zone \( p \) during period \( k \) (µg/m\(^3\));
\( c_{j,k} \) = operating cost of control measure \( j \) during period \( k \) ($/t);
\( e_{i,k} \) = SO\(_2\) emission standard for source \( i \) during period \( k \) (t/d);
\( S_{i,k} \) = SO\(_2\) generation rate from source \( i \) during period \( k \) (t/d);
\( t_{i,p} \) = transfer factor from emission source \( i \) to receptor zone \( p \), which is functions of stochastic parameters \( \tilde{\gamma}_1, \tilde{\beta}_1, \tilde{\gamma}_2, \tilde{\beta}_2 \) and \( \tilde{u} \);
\( \eta_{i,j} \) = efficiency of control measure \( j \) applied to source \( i \) (%);
\( + \) = upper quartile number of interval set;
\( - \) = lower quartile number of interval set;
\( q_i \) = probability of violating RHS constraint
3.2. Determination of transfer coefficients

In this study, the gaussian plume model will be used to determine the transfer coefficients used in the optimization models above.

As the main objective of this study, is to develop and demonstrate the applicability of an optimization model for regional air quality management involving only continuous point-sources emissions, where the pollutant is assumed to undergo negligible removal processes, the use of the gaussian plume model (Equation 1) is sufficient in modeling the atmospheric dispersion of the pollutant, SO$_2$. The $\sigma_y$ and $\sigma_z$ values in Equation 1 are dependent on wind speed, solar heating, local turbulence, and can be estimated by the following empirical equations [37, 48].

$$\sigma_y = \gamma_1 x^\beta_1$$  \hspace{1cm} (11a)  
$$\sigma_z = \gamma_2 x^\beta_2$$  \hspace{1cm} (11b)

Where
\[ \gamma_1 \text{ and } \beta_1 = \text{regression coefficient and exponent along the } y \text{ direction, respectively;} \]
\[ \gamma_2 \text{ and } \beta_2 = \text{regression coefficient and exponent along the } z \text{ direction, respectively.} \]

The values of $\gamma_1, \beta_1, \gamma_2$ and $\beta_2$ are related to the degree of atmospheric stability and the downwind distance. [47] By integrating Equation 1 with Equations 11a and 11b, the transfer factor $t_{ip}$ the contribution of pollutant emission rate at source $i$ to the ground concentration of receptor zone $p$ can be defined as follows:

$$t_{ip} = \frac{1}{\pi u \gamma_1 \gamma_2 \sigma_x^{2\beta_1+2\beta_2}} \exp \left[ \left( \frac{-y_{ip}^2}{2\gamma_1^2 x_{ip}^{2\beta_1}} \right) - \left( \frac{H_x^2}{2\gamma_2^2 x_{ip}^{2\beta_2}} \right) \right] \hspace{1cm} (12)$$

where
\[ t_{ip} = \text{transfer coefficient (the contribution of pollutant emission rate at source } i \text{ to the ground concentration of receptor zone } p); \]
\[ x_{ip} \text{ and } y_{ip} = \text{relative horizontal and vertical coordinates of the receptor zone } p, \text{ with the location of emission source } i \text{ being the origin of the coordinates;} \]
$H$ = average effective stack height;

$\gamma_1$ and $\beta_1$ = regression coefficient and exponent along the $y$ direction, respectively;

$\gamma_2$, and $\beta_2$ = regression coefficient and exponent along the $z$ direction, respectively.
CHAPTER 4. CASE STUDY

In this study, a hypothetical regional air quality management case proposed by Liu et al [7] will be modified and used for demonstrating the applicability of the models proposed above.

Table 3: Case study - Emission sources and receptor zones coordinates

<table>
<thead>
<tr>
<th></th>
<th>$p = 1$</th>
<th>$p = 2$</th>
<th>$p = 3$</th>
<th>$p = 4$</th>
<th>$p = 5$</th>
<th>$i = 1$</th>
<th>$i = 2$</th>
<th>$i = 3$</th>
<th>$i = 4$</th>
<th>$i = 5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$ (km)</td>
<td>55.2</td>
<td>64.6</td>
<td>42.5</td>
<td>33.3</td>
<td>51.2</td>
<td>12.5</td>
<td>8.3</td>
<td>22.1</td>
<td>16.2</td>
<td>5.5</td>
</tr>
<tr>
<td>$y$ (km)</td>
<td>34.8</td>
<td>23</td>
<td>43.9</td>
<td>21.2</td>
<td>8.6</td>
<td>40.5</td>
<td>21.3</td>
<td>32.8</td>
<td>4.4</td>
<td>13.2</td>
</tr>
</tbody>
</table>

Five SO$_2$ emission sources are in the study region, consisting of 2 power plants ($i = 1$, $i = 2$), a chemical industry ($i = 3$), a petroleum refinery industry ($i = 4$), and a steel-milling industry ($i = 5$).

The SO$_2$ pollutant emissions are transported downwind, adversely impacting the region which constitutes of four land-use types - 1 residential ($p = 1$), 1 scenic ($p = 2$), 1 industrial ($p = 3$), and 2 agricultural zones ($p = 4, 5$) which are known as receptor zones in this study system. These receptor zones have different loading environmental capabilities depending on their land-use type.
The detailed locations and specific positions of the sources and receptors (i.e., the affected zones) are presented in Figure 4 and Table 3, respectively while the relative positions of each emission source to the various receptors are shown in Tables 4 and 5.

Table 4: Relative x-positions of emission sources to receptor zones

<table>
<thead>
<tr>
<th>Relative (x)</th>
<th>p = 1</th>
<th>p = 2</th>
<th>p = 3</th>
<th>p = 4</th>
<th>p = 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>i = 1</td>
<td>42.7</td>
<td>52.1</td>
<td>30</td>
<td>20.8</td>
<td>38.7</td>
</tr>
<tr>
<td>i = 2</td>
<td>46.9</td>
<td>56.3</td>
<td>34.2</td>
<td>25</td>
<td>42.9</td>
</tr>
<tr>
<td>i = 3</td>
<td>33.1</td>
<td>42.5</td>
<td>20.4</td>
<td>11.2</td>
<td>29.1</td>
</tr>
<tr>
<td>i = 4</td>
<td>39</td>
<td>48.4</td>
<td>26.3</td>
<td>17.1</td>
<td>35</td>
</tr>
<tr>
<td>i = 5</td>
<td>49.7</td>
<td>59.1</td>
<td>37</td>
<td>27.8</td>
<td>45.7</td>
</tr>
</tbody>
</table>

Table 5: Case Study - Relative y-positions of emission sources to receptor zones

<table>
<thead>
<tr>
<th>Relative (y)</th>
<th>p = 1</th>
<th>p = 2</th>
<th>p = 3</th>
<th>p = 4</th>
<th>p = 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>i = 1</td>
<td>-5.7</td>
<td>-17.5</td>
<td>3.4</td>
<td>-19.3</td>
<td>-31.9</td>
</tr>
<tr>
<td>i = 2</td>
<td>13.5</td>
<td>1.7</td>
<td>22.6</td>
<td>-0.1</td>
<td>-12.7</td>
</tr>
<tr>
<td>i = 3</td>
<td>2</td>
<td>-9.8</td>
<td>11.1</td>
<td>-11.6</td>
<td>-24.2</td>
</tr>
<tr>
<td>i = 4</td>
<td>30.4</td>
<td>18.6</td>
<td>39.5</td>
<td>16.8</td>
<td>4.2</td>
</tr>
<tr>
<td>i = 5</td>
<td>21.6</td>
<td>9.8</td>
<td>30.7</td>
<td>8</td>
<td>-4.6</td>
</tr>
</tbody>
</table>

To ensure that ambient air quality at each receptor zone is kept at a safe level, standards are in place, with a predetermined emission limits and indicated receptor zones’ environmental loading capabilities.

For emission sources to avoid penalties, 3 different mitigation measures - Oxidation Packed Absorption (OPA) (j=1), Alkali Adsorption (AA) (j=2) and Limestone Wet Scrubbing (LWS) (j=3) will be considered to keep emission level below the emission standards set. Taking into consideration: (1) the revolution of environmental policies, (2) future increase of energy needs, and (3) future increase in operating costs, the planning horizon is divided into 3 periods of 5 years each (as shown in Table 6). These periods illustrate the trends stated above by reflecting (1) increasing
mitigation measures operating costs, (2) growing SO$_2$ emissions and (3) increasingly stricter levels of government policies.

Table 6: Case study - Planning period

<table>
<thead>
<tr>
<th>K</th>
<th>Time Interval (year)</th>
<th>Period</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>1998-2003</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>2003-2008</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>2008-2013</td>
</tr>
</tbody>
</table>

Meanwhile, the air quality management system is found to be associated with many uncertainties. These include the SO$_2$ generation rates, mitigation measures operational efficiencies and meteorological condition. Based on the quality of data, these uncertainties can be reflected as intervals or probability distribution functions.

Tables 7, 8 and 9 below show the parameters for different emission sources, control measures and receptor zones for each planning period respectively, while Table 10 displays the meteorological conditions.

Table 7: Case Study - Parameters for different emission sources for each planning period

<table>
<thead>
<tr>
<th></th>
<th>Power plant 1</th>
<th>Chemical industry</th>
<th>Petroleum refinery</th>
<th>Steel milling</th>
<th>Power plant 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(i = 1)</td>
<td>(i = 2)</td>
<td>(i = 3)</td>
<td>(i = 4)</td>
<td>(i = 5)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Period 1</th>
<th>(t/d)</th>
<th>(t/d)</th>
<th>(t/d)</th>
<th>(t/d)</th>
<th>(t/d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SO$_2$ generation rate</td>
<td>[93.50, 260.50]</td>
<td>[9.90, 22.50]</td>
<td>[17.00, 40.00]</td>
<td>[3.00, 11.00]</td>
<td>[89.80, 220.80]</td>
</tr>
<tr>
<td>SO$_2$ emission standard, $e_i$ (t/d)</td>
<td>[46.50, 69.50]</td>
<td>[5.50, 6.20]</td>
<td>[7.20, 14.40]</td>
<td>[1.60, 2.80]</td>
<td>[42.50, 65.50]</td>
</tr>
<tr>
<td>Stack height (m)</td>
<td>252</td>
<td>150</td>
<td>110</td>
<td>[1.60, 2.80]</td>
<td>265</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Period 2</th>
<th>(t/d)</th>
<th>(t/d)</th>
<th>(t/d)</th>
<th>(t/d)</th>
<th>(t/d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SO$_2$ generation rate</td>
<td>[110.50, 329.50]</td>
<td>[11.20, 24.80]</td>
<td>[20.72, 49.30]</td>
<td>[3.90,12.50]</td>
<td>[99.50, 290.50]</td>
</tr>
<tr>
<td>SO$_2$ emission standard, $e_i$ (t/d)</td>
<td>[41.00, 62.00]</td>
<td>[4.00, 5.60]</td>
<td>[7.00, 13.40]</td>
<td>[1.40, 2.40]</td>
<td>[38.00, 57.00]</td>
</tr>
<tr>
<td>Stack height (m)</td>
<td>252</td>
<td>150</td>
<td>110</td>
<td>100</td>
<td>265</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Period 3</th>
<th>(t/d)</th>
<th>(t/d)</th>
<th>(t/d)</th>
<th>(t/d)</th>
<th>(t/d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SO$_2$ generation rate</td>
<td>[136.00, 384.00]</td>
<td>[11.90, 29.10]</td>
<td>[20.40, 60.40]</td>
<td>[4.20,15.20]</td>
<td>[119.00, 384.00]</td>
</tr>
<tr>
<td>SO$_2$ emission standard, $e_i$ (t/d)</td>
<td>[36.50, 55.50]</td>
<td>[3.10, 4.90]</td>
<td>[6.80, 13.20]</td>
<td>[1.20, 2.20]</td>
<td>[33.00, 51.00]</td>
</tr>
<tr>
<td>Stack height (m)</td>
<td>252</td>
<td>150</td>
<td>110</td>
<td>100</td>
<td>265</td>
</tr>
</tbody>
</table>
Table 8: Case study - Parameters for different control measures for each planning period

<table>
<thead>
<tr>
<th></th>
<th>Oxidation Packed Absorption (OPA)</th>
<th>Alkali Absorption (AA)</th>
<th>Limestone Wet Scrubbing (LWS)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>j=1</td>
<td>j=2</td>
<td>j=3</td>
</tr>
<tr>
<td>Period 1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unit operating cost, $c_j$ ($/t$)</td>
<td>[47.00, 55.00]</td>
<td>[65.00, 73.00]</td>
<td>[32.00, 40.00]</td>
</tr>
<tr>
<td>Control efficiency, (%)</td>
<td>[72,88]</td>
<td>[81,99]</td>
<td>[63,77]</td>
</tr>
<tr>
<td>Period 2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unit operating cost, $c_j$ ($/t$)</td>
<td>[54.00, 62.00]</td>
<td>[71.50, 79.50]</td>
<td>[36.80, 45.00]</td>
</tr>
<tr>
<td>Control efficiency, (%)</td>
<td>[72,88]</td>
<td>[81,99]</td>
<td>[63,77]</td>
</tr>
<tr>
<td>Period 3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unit operating cost, $c_j$ ($/t$)</td>
<td>[62.20, 72.00]</td>
<td>[78.60, 87.00]</td>
<td>[42.30, 52.00]</td>
</tr>
<tr>
<td>Control efficiency, (%)</td>
<td>[72,88]</td>
<td>[81,99]</td>
<td>[63,77]</td>
</tr>
</tbody>
</table>

Table 9: Case Study - Parameters for different receptor zones for each period

<table>
<thead>
<tr>
<th>Residential</th>
<th>Scenic</th>
<th>Farmland 1</th>
<th>Industrial</th>
<th>Farmland 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(p = 1)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Loading capacity, $a_p$ ($\mu g/d$)</td>
<td>[36.00, 69.00]</td>
<td>[9.70, 14.00]</td>
<td>[21.00, 33.50]</td>
<td>[161.00, 239.00]</td>
</tr>
<tr>
<td>$(p = 2)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Loading capacity, $a_p$ ($\mu g/d$)</td>
<td>[30.00, 63.00]</td>
<td>[9.00, 13.20]</td>
<td>[19.00, 31.00]</td>
<td>[153.00, 231.00]</td>
</tr>
<tr>
<td>$(p = 3)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Loading capacity, $a_p$ ($\mu g/d$)</td>
<td>[22.40, 55.50]</td>
<td>[7.50, 12.30]</td>
<td>[17.00, 29.00]</td>
<td>[145.20, 224.80]</td>
</tr>
<tr>
<td>$(p = 4)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$(p = 5)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 10: Case study - Parameters for calculating transfer factors

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average wind speed, $u$, (m/s)</td>
<td>[1.6, 3.3]</td>
</tr>
<tr>
<td>Regression exponent, $\theta_2$</td>
<td>$LogN (-0.1180, 0.0035)$</td>
</tr>
<tr>
<td>Regression coefficient, $\gamma_2$</td>
<td>$LogN (-1.9196, 0.2131)$</td>
</tr>
<tr>
<td>Regression exponent, $\theta_2$</td>
<td>$LogN (-0.5863, 0.2130)$</td>
</tr>
<tr>
<td>Regression coefficient, $\gamma_2$</td>
<td>$LogN (-0.2098, 0.6349)$</td>
</tr>
</tbody>
</table>

The problem under consideration is to determine the allocation of the produced pollutant stream to the available $SO_2$ abatement facilities for obtaining maximized economic and environmental efficiencies under uncertainties, where policies in terms of allowable risks of violating environmental regulations need to be formulated.
CHAPTER 5. DATA HANDLING

A series of data manipulation has to be performed before various optimisation models can be applied to the case study.

5.1. Transfer coefficients

In attempt to find the interval set for transfer coefficients, Morte Carlo simulation (based on 10000 tries) was used. Below shows the various steps taken:

Step 1: Transfer factor parameters were randomly chosen from their respective probabilistic function (as provided in Table 10) and transferred into the Equation 13 to determine the transfer factor, \( t_{ip} \), the contribution of pollutant emission rate at source \( i \) to the ground concentration of receptor zone \( p \). This was repeated 10 000 times, generating 10 000 transfer factors.

Step 2: The 10000 transfer factors were fitted into a best-fit graph (gamma distribution). The mean and sigma of the gamma distribution are as shown in Tables 11 and 12 respectively.

Table 11: The mean value of the transfer factors gamma distributions

<table>
<thead>
<tr>
<th>Mean</th>
<th>( p = 1 )</th>
<th>( p = 2 )</th>
<th>( p = 3 )</th>
<th>( p = 4 )</th>
<th>( p = 5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( i = 1 )</td>
<td>0.3218</td>
<td>0.0023</td>
<td>0.5803</td>
<td>0.0001</td>
<td>0.0001</td>
</tr>
<tr>
<td>( i = 2 )</td>
<td>0.0045</td>
<td>1.6673</td>
<td>0.0002</td>
<td>1.6313</td>
<td>0.0041</td>
</tr>
<tr>
<td>( i = 3 )</td>
<td>1.672</td>
<td>0.0198</td>
<td>0.0008</td>
<td>0.0001</td>
<td>0.0001</td>
</tr>
<tr>
<td>( i = 4 )</td>
<td>0.0001</td>
<td>0.0011</td>
<td>0.0001</td>
<td>0.0001</td>
<td>0.5199</td>
</tr>
<tr>
<td>( i = 5 )</td>
<td>0.0011</td>
<td>0.1112</td>
<td>0.0001</td>
<td>0.0072</td>
<td>0.7057</td>
</tr>
</tbody>
</table>

Table 12: The sigma of the transfer factors gamma distributions

<table>
<thead>
<tr>
<th>Sigma</th>
<th>( p = 1 )</th>
<th>( p = 2 )</th>
<th>( p = 3 )</th>
<th>( p = 4 )</th>
<th>( p = 5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( i = 1 )</td>
<td>0.0660</td>
<td>0.0006</td>
<td>0.1418</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>( i = 2 )</td>
<td>0.0057</td>
<td>0.5598</td>
<td>0.0000</td>
<td>1.6657</td>
<td>0.0053</td>
</tr>
<tr>
<td>( i = 3 )</td>
<td>0.6435</td>
<td>0.0265</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>( i = 4 )</td>
<td>0.0000</td>
<td>0.0003</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.2373</td>
</tr>
<tr>
<td>( i = 5 )</td>
<td>0.0000</td>
<td>0.0237</td>
<td>0.0000</td>
<td>0.0064</td>
<td>0.1043</td>
</tr>
</tbody>
</table>
Matlab programming software was used for Steps 2 and 3. The related program code can be found in Appendix B.

**Step 3:** To translate the transfer factors random distribution into interval sets, a ±10% derivation from the mean was used to determine the upper and lower bound of the interval sets. The results are shown in Table 13.

Table 13: Transfer factors as interval sets

<table>
<thead>
<tr>
<th></th>
<th>$p = 1$</th>
<th>$p = 2$</th>
<th>$p = 3$</th>
<th>$p = 4$</th>
<th>$p = 5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$i = 1$</td>
<td>[0.28962, 0.35398]</td>
<td>[0.00207, 0.00253]</td>
<td>[0.52227, 0.63833]</td>
<td>[0.00009, 0.00011]</td>
<td>[0.00009, 0.00011]</td>
</tr>
<tr>
<td>$i = 2$</td>
<td>[0.00405, 0.00495]</td>
<td>[1.50057, 1.83403]</td>
<td>[0.00018, 0.00022]</td>
<td>[1.46817, 1.79443]</td>
<td>[0.00369, 0.00451]</td>
</tr>
<tr>
<td>$i = 3$</td>
<td>[1.50480, 1.83920]</td>
<td>[0.01782, 0.02178]</td>
<td>[0.00072, 0.00088]</td>
<td>[0.00009, 0.00011]</td>
<td>[0.00009, 0.00011]</td>
</tr>
<tr>
<td>$i = 4$</td>
<td>[0.00009, 0.00011]</td>
<td>[0.00099, 0.00121]</td>
<td>[0.00009, 0.00011]</td>
<td>[0.00009, 0.00011]</td>
<td>[0.46791, 0.57189]</td>
</tr>
<tr>
<td>$i = 5$</td>
<td>[0.00099, 0.00121]</td>
<td>[0.10008, 0.12232]</td>
<td>[0.00009, 0.00011]</td>
<td>[0.00648, 0.00792]</td>
<td>[0.63513, 0.77627]</td>
</tr>
</tbody>
</table>

**5.2. Data input**

In order to apply the various optimization models to the case, the random parameters in the case study data are manipulated as discussed in the methodology before communicated into the respective optimization models.
CHAPTER 6. RESULTS ANALYSIS

The case study was solved through the methods discussed in Section 3. Under each model, the total SO$_2$ amount at each emission source would be allocated to various facilities in order to obtain a minimum system cost. The related computer code for the respective optimization models were programmed in Lingo version 12 and can be found in Appendix C, D, E and F.

The analysis of the solutions of the deterministic, ILP, CCP and ICCLP air quality optimization model is provided in the sections 6.1 - 6.4.

6.1. Deterministic model

Appendix G shows the full solutions of the deterministic model. Figure 5 illustrates the treated amounts from the five emission sources by the abatement methods (OPA, AA, LWS) for period 1, 2 & 3.

![Figure 5: Deterministic model results for treated amounts from the five emission sources by each abatement method across period 1, 2 & 3](image)

6.2. ILP model

The ILP model considers the probabilistic nature of emissions and provides a more robust solution. The results from the ILP model are shown in Appendix H.

6.3. CCP model

The CCP model is a chance-constrained programming problem that takes into account the uncertainty in emissions. The solutions for the CCP model are presented in Appendix I.

6.4. ICCLP model

The ICCLP model incorporates both chance-constrained programming and robust optimization. The results from the ICCLP model are detailed in Appendix J.
In period 1, SO$_2$ amounts are observed to be treated mainly by OPA and LWS. The treated amounts from different sources by OPA are 70.61, 3.20, 0.00, 0.00, and 106.2 t/day respectively. Those by LWS are 109.39, 13.00, 28.50, 7.00, and 49.10 t/day respectively.

In period 2, a shift in allocation of SO$_2$ emissions to LWS from OPA was observed. In addition, for some cases (i.e., powerplants 1 and 2), AA, which has higher efficiency and abatement cost is used to replace LWS. The treated amounts by LWS are 0.00, 6.56, 32.00, 2.60, 0.00 t/day respectively. Those by OPA are 210.60, 11.44, 3.00, 5.60, and 137.85 t/day respectively.

In period 3, AA is observed to replace OPA for all emission sources, becoming the main abatement measure used to mitigate SO$_2$. The treated amounts by LWS are 68.07, 8.36, 29.80, 3.65, and 27.26 t/day respectively. Those by AA are 191.93, 12.14, 10.60, 6.05, and 224.24 t/day respectively.

The gradual shift in the allocation of SO$_2$ emissions from abatement methods with lower cost yet lower efficiency to abatement measures with higher cost and higher efficiency (LWS to OPA and then to AA) is due to the increased generation rate and stricter emission standards and receptor zone loading capacities of SO$_2$ across the periods. In period 1, due to the relatively large allowed SO$_2$ emission amounts and environmental loading capacities, LWS and OPA, which have a lower treatment cost than those of other control techniques, are suitable for controlling SO$_2$ emissions. However, with the increasing generation rate and stricter emission standards and receptor zone loading capacities of SO$_2$ in period 2 and period 3, abatement options with higher efficiency and hence higher costs (OPA & AA) is needed to replace the abatement measures used in period 1.

Figure 6 below shows the increasingly total system costs from period 1 to period 3.
6.2. Interval linear programming model

Appendix H shows the full solutions obtained through the ILP model.

The results indicate that the solutions for the objective function and decision variables are expressed as interval numbers $F^+$ and $F^-$, and $x_{ijk}^{-}$ and $x_{ijk}^{+}$ which correspond to the upper and lower bound solutions of the ILP method. Figure 7 below displays the upper and lower bound solutions for the total management system cost across 3 periods.

![Figure 7: ILP model solutions for the total management system cost](image)

The trends of the individual upper and lower bound solutions are similar to those of the deterministic model as mentioned in chapter 6.1. However when placed together, they illustrate two extreme decisions regarding environment/economic tradeoffs.

The lower bound of the objective function $F^-$ corresponding to $x_{ijk}^{-}$ represents an optimistic decision alternative for air quality control. This decision is made when the conditions are advantageous (i.e., when the emission source emission and receptor zone loading capacities standards are lenient). Planning for the lower bound of $F^-$ will lead to the decreased likelihood of achieving the air quality objectives, implying a higher risk of violating the relevant air quality standards under disadvantageous conditions. Conversely, the upper bound of the objective function $F^+$ corresponding to $x_{ijk}^{+}$ represents a conservative decision alternative. This decision is
made under disadvantageous conditions (i.e., when the emission source emission and receptor zone loading capacities standards are strict). Planning for the upper bound will guarantee that air quality standards are met.

In other words, when decision makers aims towards a lower cost, emissions are allocated to abatement options with lower cost yet lower efficiency (such as OPA and LWS). When decision makers aim towards a safer environment, emissions are allocated to abatement measures with higher cost yet higher efficiency such as AA and OPA will be chosen.

A comparison between the total management system cost for the deterministic solution and ILP model solution is illustrated in Figure 8 below.

![Figure 8: Comparison between IPL and the deterministic models’ total management system cost](image)

Compared to the deterministic model, the ILP flexibility in reflecting potential system condition variations caused by the existence of input uncertainties allows it to provide a more accurate representation of the situation. It is also able to display the trade-off between environmental pollution risks and economic objectives. Decision alternatives can be generated by adjusting the decision variable values with their solution intervals according to projected planning situations. This solution feature may be favoured by decision makers in comparison to the deterministic model due to the increased flexibility and applicability for determining final decision schemes.
6.3. Chance-constrained programming model

Appendix I shows the full solutions of the CCP model under various $q_i$ values.

Although the trends of the solutions under each $q_i$ are observed to be similar to the deterministic model as mentioned in chapter 6.1, the increasingly $q_i$ values are able to demonstrate the effects of the decreasing strictness in the environmental requirements. Figure 9 below shows the CCP model solutions for the total amount of SO$_2$ allocated to different abatement under different $q_i$ values.

![Figure 9: CCP model solutions for the total amount of SO$_2$ allocated to different abatement measures under different $q_i$ values](image)

In period 1, it is noticed that the total amount of SO$_2$ allocated to the higher efficiency abatement method, OPA decreases from 319.04 to 125.83 t/day when the $q_i$ value changes from 0.01 to 0.8. On the other hand, the percentage of SO$_2$ allocated to the lower efficiency abatement method (LWS), increases from 59.36 to 255.18 t/day as the $q_i$ value increases from 0.01 to 0.8. A similar trend is observed across the 3 periods. Such a trend is due to the varying $q_i$ value which indicates
an increase in constraint-violation with increasingly value. When the environmental requirement is less strict, (i.e., a higher $q_i$ value), a higher percentage of SO$_2$ can be allocated to lower efficiency and lower cost abatement methods as they are sufficient to meet the less strict environmental requirement. Conversely, when a lower risk of violating the ambient air quality standard is desired (i.e., a lower $q_i$ value), an increase in percentage of SO$_2$ allocated to the higher efficiency yet more costly abatement measures is necessary to ensure that the specified emission standards and receptor zone loading capacities are met.

According to the above analysis, the system cost is thus expected to decrease with the increasing $q_i$ values. This is validated by solutions as shown in the Appendix I. For example, when $\alpha$ level changes from 0.01 to 0.1, 0.25 to 0.5, and finally to 0.8, the total system management system cost (the sum of the system management cost for periods 1, 2 and 3) would decrease from 96,419.34 ($/d) to 92,586.63 ($/d), 90,422.05 ($/d) to 88,021.38 ($/d), and then to 85,313.72 ($/d) correspondingly. The relationship between the system cost and the $q_i$ values for each respective period is illustrated in Figure 10 below.

![Figure 10: Relationship between management system cost and $q_i$ values for CCP model](image)

Conclusively, the increased $q_i$ indicates a raised risk of constraint violation, which leads to a decreased strictness for the constraints and thus a decreased system cost. Such a decreased cost will be linked to a potentially increased amount of untreated SO$_2$ emissions and hence a raised...
risk of constraint violation. Hence the relationship between the system costs and $q_i$ values of the CCP model demonstrates a tradeoff between cost efficiency and constraint-violation risk.

Comparing with the total management system costs (sum of period 1, 2 and 3) for the deterministic model and CCP model solutions, it is noticed that the deterministic model solutions is equivalent to that of the CCP model when constraint-violation risk is 0.5. Figure 11 below shows the comparison between the solutions of the deterministic model with that of the CCP model.

Figure 11: Comparison of deterministic and CCP models’ solutions

This suggests that the solution from a deterministic model could be highly unreliable under the impacts of uncertainties, and may easily lead to a system failure. The results demonstrate that the CCP model could effectively communicate the highly random RHS uncertainties into the optimization process, and generate solutions that contain a spectrum of potential air-pollutant treatment options with both risk and cost information compared to that of the deterministic model. Decision alternatives can be obtained by analyzing tradeoffs between the overall pollutant treatment cost and the system-failure risk due to inherent uncertainties. Willingness to pay higher operating costs will guarantee meeting environmental objectives. A desire to reduce the costs will run into the risk of potentially violating the related environmental standards.
Similarly to that of the ILP model, the CCP model may be favoured by decision makers in comparison to the deterministic model due to the increased flexibility and applicability for determining final decision schemes.

### 6.4. Inexact chance-constrained programming model

Appendix J presents the full solutions obtained through the ICCLP model under different $q_i$ levels.

The solutions of the ICCLP model reflect the solution characteristics of both the CCP and ILP models. Figures 12 and 13 show the total amount of $\text{SO}_2$ allocated to the abatement measures.

![Figure 12: ICCLP model solutions for the total amount of $\text{SO}_2$ allocated to different abatement measures under different $q_i$ values (lower bound).](image)
Figure 13: ICCLP model solutions for the total amount of SO$_2$ allocated to different abatement measures under different $q_i$ values (upper bound)

It is indicated that, as the actual values of the decision variables vary within their two bounds, the expected system cost will change correspondingly between $x_{ijk}^{\text{opt}}$ and $x_{ijk}^{+\text{opt}}$ with different reliability levels. Figure 14 shows the effects of varied $q_i$ level on the system cost.
Figure 14: Total management system cost under different $q_i$ values for ICCP model

Similar to that observed in the CCP model solutions, decisions at a lower $q_i$ level will lead to an increased reliability in fulfilling the system requirements and hence a higher cost; whereas, decisions at a higher $q_i$ level will result in a lower cost, yet the risk of violating the constraints will be increased.

### 6.5. Comparison of ILP, CCP & ICCLP models

Solutions of the ILP model (Appendix H) provide two extremes of the expected system cost. Decisions for a lower cost may correspond to advantageous system conditions (e.g. more lenient emission source and receptor zone standards), while those with a higher cost correspond to more demanding conditions.

However, compared with ICCLP, the ILP has the following limitations. Firstly, it can only generate one interval solution without information about the risk of violating the capacity constraints. Secondly, the system cost obtained through the ILP model is generally higher than those through the ICLP method (under a range of $q_i$ levels) since no relaxation on capacity constraints is allowed in the ILP. Without the chance constraints, the ILP is unable to support in-depth analysis of the tradeoff between system cost and system-failure risk.
Using the CCP model, the left-hand-side interval coefficients (including the cost coefficients) are allocated deterministic values (equal to their mean values). The solutions (Appendix I) are found to be within the ICCLP solution intervals, demonstrating the stability of the CCP solutions. With the CCP, only one deterministic solution corresponding to each \( q_i \) level is generated, since the model’s left-hand-side coefficients) are all assumed to be deterministic.

Compared with the CCP, the ICCLP method can incorporate more uncertain information within its modeling framework. This is particularly useful in situations where the qualities of many uncertainties are often not good enough to be expressed as probability distribution function. The obtained interval solutions under different risk levels of violating the capacity constraints can be used to generate decision alternatives and help air quality planners to identify desired policies under various environmental, economic, and system-reliability constraints.

The relation between \( F^* \) and \( q_i \) demonstrates a more elaborate tradeoff analysis between system cost and constraint-violation risk compared to CCP or ILP model alone. These demonstrate a tradeoff between the waste-management cost and the system-failure risk due to the dual uncertainties that exist in various system components (i.e., interval and probabilistic information).
CHAPTER 7. CONCLUSION

An inexact chance-constrained linear programming (ICCLP) method has been proposed for air quality management systems. The method improves upon the existing ILP and CCP approaches by allowing uncertainties to be presented as both probability distributions and discrete intervals to be effectively incorporated within the optimization framework, without unrealistic simplifications.

The developed method was applied to a case where violations for capacity constraints were allowed under a range of significance levels. Interval solutions associated different risk levels of constraint violation have been obtained. These results demonstrated the applicability of the ICCLP method in the planning regional air quality management.

Compared with the ILP and CCP methods, the ICCLP can incorporate more uncertain information within its modeling framework. In addition, the method provides not only decision variable solutions presented as intervals but also the associated risk levels in violating the system constraints. It can therefore support an elaborate analysis of the tradeoff between system cost and system-failure risk. Hence, it is a useful tool for generating decision alternatives and thus helps policy makers identify desired policies under various environmental, economic, and system-reliability constraints.
REFERENCES


APPENDIX A: ILP MODEL ALGORITHM

1. MODELLING FORMULATION [46]

Definition 1. Let $x$ denote a closed and bounded set of real numbers. An inexact number $x^\pm$ is defined as an interval with known upper and lower bounds but unknown distribution information for $x$:

$$x^\pm = [x^-, x^+],$$

where $x^-$ and $x^+$ are the lower and upper bounds of $x^\pm$, respectively. When $x^\pm = x^+$, $x^-$ becomes a deterministic number.

Definition 2. For $x^\pm$, the following relationships hold:

$$x^\pm \geq 0, \quad \text{if } x^- \geq 0 \text{ and } x^+ \geq 0,$$

$$x^\pm \leq 0, \quad \text{if } x^- \leq 0 \text{ and } x^+ \leq 0. \quad (A1.2)$$

Definition 3. For $x^\pm$ and $y^\pm$, their order relations are as follows:

$$x^\pm \leq y^\pm, \quad \text{if } x^- \leq y^- \text{ and } x^+ \leq y^+,$$

$$x^\pm < y^\pm, \quad \text{if } x^- < y^- \text{ and } x^+ < y^+. \quad (A1.5)$$

Definition 4. For $x^\pm$, $\text{Sign}(x^\pm)$ is defined as follows:

$$\text{Sign}(x^\pm) = 1, \quad \text{if } x^\pm \geq 0,$$

$$-1, \quad \text{if } x^\pm < 0. \quad (A1.7)$$

Definition 5. For $x^\pm$, its absolute value $|x|^\pm$ as defined as follows:

$$|x|^\pm = x^\pm, \quad \text{if } x^\pm \geq 0,$$

$$-x^\pm, \quad \text{if } x^\pm < 0. \quad (A1.8)$$

Thus we have:

$$|x|^\pm = x^\pm, \quad \text{if } x^\pm \geq 0,$$

$$-x^\pm, \quad \text{if } x^\pm < 0; \quad (A1.9)$$

and
\[ |x^+| = x^+ \text{, if } x^\pm \geq 0, \]
\[ -x^- \text{, if } x^\pm < 0. \]  
(A1.10)

**Definition 6.** Let \( \mathbb{R}^\pm \) denote a set of inexact numbers. An inexact vector \( X^\pm \) is a tuple of inexact numbers, and an inexact matrix \( X^\pm \) is a matrix whose elements are inexact numbers:

\[
X^+ = [x^+] = [x_i^+, x_i^-] \; \forall \; i, \; X^\pm \in \{\mathbb{R}^\pm\}^{1 \times n}, \\
X^\pm = [x^\pm] = [x_{ij}^-, x_{ij}^+] \; \forall \; i, j, \; X^\pm \in \{\mathbb{R}^\pm\}^{m \times n}. 
\]  
(A1.11, A1.12)

**Definition 7.** In this paper, for inexact vectors and matrices:

\[
X^+ \geq 0, \text{ if } x_{ij}^\pm \geq 0, \forall i, j, \; X^\pm \in \{\mathbb{R}^\pm\}^{m \times n}, \; m \geq 1 \text{ and } m = \text{integer}, \\
X^+ \leq 0, \text{ if } x_{ij}^\pm \leq 0, \forall i, j, \; X^\pm \in \{\mathbb{R}^\pm\}^{m \times n}, \; m \geq 1 \text{ and } m = \text{integer}. 
\]  
(A1.13, A1.14)

**Definition 8.** Let \( * \in \{+, -, \times, \div\} \) be a binary operation on inexact numbers. For \( x^\pm \) and \( y^\pm \):

\[
x^\pm * y^\pm = [\text{min} \{x * y\}, \text{max} \{x * y\}] , x^- \leq x \leq x^+, y^- \leq y \leq y^+. 
\]  
(A1.15)

In the case of division, it is assumed that \( y^\pm \) does not contain a zero. Hence:

\[
x^\pm + y^\pm = [x^- + y^-, x^+ + y^+], \\
x^\pm - y^\pm = [x^- + y^+, x^+ + y^-], \\
x^\pm \times y^\pm = [\text{min} \{x \times y\}, \text{max} \{x \times y\}] , x^- \leq x \leq x^+, y^- \leq y \leq y^+, \\
x^\pm \div y^\pm = [\text{min} \{x \div y\}, \text{max} \{x \div y\}] , x^- \leq x \leq x^+, y^- \leq y \leq y^+. 
\]  

**Definition 9.** Let \( \mathbb{R}^\pm \) denote a set of inexact numbers. An inexact linear programming (ILP) model can be defined as follows:

\[
\text{max } f^\pm = C^\pm X^\pm, \\
\text{s.t. } A^\pm X^\pm \leq B^\pm, \\
X^\pm \geq 0, 
\]  
(A1.20a, A1.20b, A1.20c)

where \( A^\pm \in \{\mathbb{R}^\pm\}^{m \times n}, B^\pm \in \{\mathbb{R}^\pm\}^{m \times 1}, C^\pm \in \{\mathbb{R}^\pm\}^{1 \times n}, \) and \( X^\pm \in \{\mathbb{R}^\pm\}^{n \times 1}. \) Section 2 and 3 below shows the interactive solution algorithm developed to solve the inexact linear programming program.
2. SOLUTION ALGORITHM [46]

(1) Based on model A1.20:

Lemma 1. For \( A \in [A^-, A^+] \) and \( B \in [B^-, B^+] \), denoting \( Q = \{ X \mid A X \leq B, X \geq 0 \} \), \( Q^+ = \{ X \mid A^+ X \leq B, X \geq 0 \} \), and \( Q^- = \{ X \mid A^- X \leq B^+, X \geq 0 \} \), we have: \( Q^+ \supseteq Q \supseteq Q^- \).

Proof: If both \( X \in Q^- \) and \( X \geq 0 \) hold, we have \( A X \leq A^- X \leq B^+ \leq B \), such that \( X \in Q \) holds. Furthermore, if both \( X \in Q \) and \( X \geq 0 \) hold, we have \( A^- X \leq A X \leq B \leq B^+ \), such that \( X \in Q^+ \) holds. Hence, \( Q^+ \supseteq Q \supseteq Q^- \).

Remark 1: Lemma 1 provides the relationships between feasible \( X \) sets under two extreme constraints (best and worst cases).

Theorem 1: Model (A1.20) can have inexact solutions, which are composed of interval numbers, as follows:

\[
\{ X^+ \}_{\text{opt}} = \{ x^+_j \mid j = 1, 2, \ldots, n \}, \quad (A2.1)
\]

\[
x^+_j = [x^-_j, x^+_j], \quad \forall j, \quad (A2.2)
\]

\[
f^+_{\text{opt}} = [f^-_{\text{opt}}, f^+_{\text{opt}}]. \quad (A2.3)
\]

Proof 1: For Eqs. (A2.1) and (A2.2), consider two feasible solutions for model 1.20:

\[
X^{(1)} \in \{ X \mid X \in Q^+ \},
\]

\[
X^{(2)} \in \{ X \mid X \in Q^- \},
\]

where \( Q^- = \{ X \mid A^- X \leq B, X \geq 0 \} \), and \( Q^+ = \{ X \mid A^+ X \leq B^+, X \geq 0 \} \).

From Lemma 1, \( Q^+ \supseteq Q^- \) holds. Hence, for any \( X^{(2)} \) from \( Q^- \), including optimal solution \( X^{(2)}_{\text{opt}} \) which corresponds to \( f^+_{\text{opt}} = C^+_{\text{opt}} X^{(2)}_{\text{opt}} = \max \{ C X \mid X \in Q^+ \} \), \( \exists X^{(1)} \in Q^+ \) such that \( x^+_j \geq x^-_j \) holds. where \( x^+_j \in X^{(1)} \), and \( x^-_j \in X^{(2)}, \forall j. \)

Proof 2: For Eq. (A2.3), from Lemma 1:

\[
f^+_{\text{opt}} = C^+ X^{(1)}_{\text{opt}} = \max \{ C^+ X \mid X \in Q^+, X \geq 0 \}.
\]

Let

\[
\max \{ C^+ X \mid X \in Q^+, X \geq 0 \} = \max \{ C^+ (C^+ - C) X \mid X \in Q^+, X \geq 0 \}.
\]
Since \( C^+ - C^- \geq 0 \),
\[
\max \{ C^+ X^+ (C^+ - C^-) X \mid X \in Q^+, X \geq 0 \} \geq \max \{ C^+ X \mid X \in Q^+, X \geq 0 \} \geq \max \{ C^+ X \mid X \in Q^-, X \geq 0 \} = C^+ X^{(2)}_{opt} = f^+_{opt}.
\]
Thus, \( f^+_{opt} \geq f^-_{opt} \).

**Remark:** Theorem 1 demonstrates that inexact solutions exist and can be obtained by solving the ILP problems.

**(2) Interactive Relationships between Model Parameters and Decision Variables**

For \( n \) inexact coefficients \( c_j^\pm (j = 1, 2, \ldots, n) \) in the objective function of model (A1.20), if \( k_1 \) of them are positive, and \( k_2 \) are negative, let the former \( k_1 \) coefficients be positive, i.e., \( c_j^+ \geq 0 \) (\( j = 1, 2, \ldots, k_1 \)), and the latter \( k_2 \) coefficients be negative, i.e., \( c_j^- < 0 \) (\( j = k_1+1, k_1+2, \ldots, n \)), where \( k_1 + k_2 = n \) (the model does not include the situation when the two bounds of \( c_j^\pm \) have different signs). Thus, the following expressions for the upper and lower bounds of \( f^\pm \) can be developed:

\[
f^+ = \sum_{j=1}^{k_1} c_j^+ x_j^+ + \sum_{j=k_1+1}^{n} c_j^- x_j^-,
\]
(A2.4)

\[
f^- = \sum_{j=1}^{k_1} c_j^- x_j^- + \sum_{j=k_1+1}^{n} c_j^+ x_j^+,
\]
(A2.5)

For the constraints corresponding to the upper and lower bounds of the objective function value, the following can be proved.

**Lemma 2:** To obtain solutions as shown in Eqs. (A2.1) and (A2.3), constraints corresponding to \( f^- \) can be developed as follows, based on Eq. (A2.4) and the interactive relationships between model parameters and decision variables based on equations can be developed as follows:

\[
\sum_{j=1}^{k_1} |a_{ij}| \text{Sign}(a_{ij}) x_j^+ + \sum_{j=k_1+1}^{n} |a_{ij}| \text{Sign}(a_{ij}) x_j^- \leq b_i^\pm, \forall i,
\]
(A2.6)

Similarly, based on Eq. (A2.5), the constraints can be expressed as follows:
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\[ \sum_{j=1}^{k_i} |a_{ij}| \text{Sign}(a_{ij}^*) x_j^i + \sum_{j=k_i+1}^{n} |a_{ij}| \text{Sign}(a_{ij}^*) x_j^i \leq b_i^\pm, \forall i, \quad (A2.7) \]

Proof 3 below proves that models based on constraints \( AX \leq B^\pm \) and \( AX \leq B^- \) in Lemma 1 can only provide interval solutions with \( f_{opt}^\pm = [f_{opt}^-, (f_{opt}^+)^*] \), where \( (f_{opt}^+)^* \leq f_{opt}^+ \) and hence are unable to generate solutions with good quality.

**Proof 3:** From Lemma 1, \( Q^+ \supseteq Q^- \) holds. Thus, for \( X_{opt}^{(2)} \in Q^- \), \( \exists X_{opt}^{(1)} \in Q^+ \), such that:

\[
x_j^{(1)} \geq x_j^{(2)\text{opt}}, \text{ for } j = 1, 2, \ldots, k_1, \quad (A2.8a)
\]
\[
x_j^{(1)} \leq x_j^{(2)\text{opt}}, \text{ for } j = k_1+1, k_1+2, \ldots, n, \quad (A2.8b)
\]

where \( x_j^{(1)} \in X^{(1)} \), and \( x_j^{(2)\text{opt}} \in X^{(2)\text{opt}}, \forall j. \)

However, for \( X_{opt}^{(1)} \in Q^- \), it is not necessary that \( X^{(2)} \in Q^- \), such that:

\[
x_j^{(2)} \leq x_j^{(1)\text{opt}}, \text{ for } j = 1, 2, \ldots, k_1, \quad (A2.8c)
\]
\[
x_j^{(2)} \geq x_j^{(1)\text{opt}}, \text{ for } j = k_1+1, k_1+2, \ldots, n, \quad (A2.8d)
\]

because \( Q^+ \supseteq Q^- \), where \( x_j^{(1)\text{opt}} \in X^{(1)\text{opt}} \), and \( x_j^{(2)\text{opt}} \in X^{(2)\text{opt}}, \forall j. \)

Therefore, to obtain good quality solutions as shown in Eqs. (A2.1) to (A2.3), the lower bound submodel (when the objective is to be minimized) have to be solved first, while the upper bound submodel (when the objective is to be maximised) have to be solved first. Proof 4 below proves that constraints 2.6 and 2.7 lead to solutions with good quality.

**Proof 4:** Denote the upper bound \( Q^{(u)} \) and lower bound \( Q^{(d)} \) of \( Q \) as follows:

\[
Q^{(u)} = \{X^{(u)} | \sum_{j=1}^{k_i} |a_{ij}| \text{Sign}(a_{ij}^+) x_j^{(u)} + \sum_{j=k_i+1}^{n} |a_{ij}| \text{Sign}(a_{ij}^+) x_j^{(u)} \leq b_j^\pm, \forall j, \}
\]

\[
Q^{(d)} = \{X^{(d)} | \sum_{j=1}^{k_i} |a_{ij}| \text{Sign}(a_{ij}^+) x_j^{(d)} + \sum_{j=k_i+1}^{n} |a_{ij}| \text{Sign}(a_{ij}^+) x_j^{(d)} \leq b_j^\pm, \forall j, \}
\]

If the objective function is to be maximized, the upper bound submodel with the following constraints should be solved first:
\[ \sum_{j=1}^{k_i} \left| a_{ij} \right| \text{Sign}(a_{ij}) x_j^{(u)} + \sum_{j=k_i+1}^{n} \left| a_{ij} \right| \text{Sign}(a_{ij}) x_j^{(d)} \leq b_{i}^{\pm}, \forall i, \]  \hspace{1cm} (A2.11)

Conversely, if the objective function is to be minimized, the lower bound submodel with the following constraints should be solved first:

\[ \sum_{j=1}^{k_i} \left| a_{ij} \right| \text{Sign}(a_{ij}) x_j^{(d)} + \sum_{j=k_i+1}^{n} \left| a_{ij} \right| \text{Sign}(a_{ij}) x_j^{(u)} \leq b_{i}^{\pm}, \forall i, \]  \hspace{1cm} (A2.12)

As a comparison, when the two constraints in Lemma 1 are used, the lower bound submodel has to be first solved even through the objective is maximized.

Assuming that the upper bound solutions are \( X^{(u)}_{\text{opt}} \) and \( t^{(u)}_{\text{opt}} \) then:

\[ \sum_{j=1}^{k_i} \left| a_{ij} \right| \text{Sign}(a_{ij}) x_j^{(u)} + \sum_{j=k_i+1}^{n} \left| a_{ij} \right| \text{Sign}(a_{ij}) x_j^{(d)} \leq b_{i}^{\pm}, x_j^{(u)}_{\text{opt}} \in X^{(u)}_{\text{opt}} \text{ for } j = 1, 2, \ldots, n, \forall i \]  \hspace{1cm} (A2.13)

For \( j = 1, 2, \ldots, k_1 \), since \( |a_{ij}|^+ \leq |a_{ij}|^- \), \( \exists \ x_j^{(d)} \leq x_j^{(u)}_{\text{opt}} \), such that \( |a_{ij}|^- x_j^{(d)} = |a_{ij}|^+ x_j^{(u)}_{\text{opt}} \), \( \forall i \). Similarly, for \( j = k_1+1, k_1+2, \ldots, n \), since \( |a_{ij}|^- \leq |a_{ij}|^+ \), \( \exists \ x_j^{(d)} \geq x_j^{(u)}_{\text{opt}} \), such that \( |a_{ij}|^- x_j^{(d)} = |a_{ij}|^+ x_j^{(u)}_{\text{opt}} \), \( \forall i \). Thus, we have:

\[ \sum_{j=1}^{k_i} \left| a_{ij} \right| \text{Sign}(a_{ij}) x_j^{(d)} + \sum_{j=k_i+1}^{n} \left| a_{ij} \right| \text{Sign}(a_{ij}) x_j^{(u)}_{\text{opt}} \leq b_{i}^{\pm}, \forall i. \]  \hspace{1cm} (A2.14)

Therefore, for \( j = 1, 2, \ldots, k_1 \), we have:

\[ x_j^{+}_{\text{opt}} = x_j^{(u)}_{\text{opt}}, \]  \hspace{1cm} (A2.15a)

\[ x_j^{\pm}_{\text{opt}} = \min \{ x_i^{(d)} \mid i \}, \max \{ x_i^{(d)} \mid i \}; \]  \hspace{1cm} (A2.15b)

for \( j = k_1+1, k_1+2, \ldots, n \), we have:

\[ x_j^{+}_{\text{opt}} = x_j^{(u)}_{\text{opt}}, \]  \hspace{1cm} (A2.16a)

\[ x_j^{\pm}_{\text{opt}} = \min \{ x_i^{(d)} \mid i \}, \max \{ x_i^{(d)} \mid i \}. \]  \hspace{1cm} (A2.16b)

Therefore, for \( X^{(u)}_{\text{opt}} \), \( Q^{(u)} \), \( X^{(d)} \), \( Q^{(d)} \), such that \( x_j^{(d)} \leq x_j^{(u)}_{\text{opt}} \) for \( j = 1, 2, \ldots, k_1 \), and \( x_j^{(d)} \geq x_j^{(u)}_{\text{opt}} \) for \( j = k_1+1, k_1+2, \ldots, n \), where \( x_j^{(d)} \), \( X^{(d)} \), and \( x_j^{(u)}_{\text{opt}} \), \( X^{(u)}_{\text{opt}} \). According to Eqns. (A2.4) and (A2.5), we have:
\[ f^{(u)}_{\text{opt}} = \sum_{j=1}^{n} c_j^+ x_j^{(u)}_{\text{opt}} \quad (A2.17) \]

\[ f^{(d)}_{\text{opt}} = \sum_{j=1}^{n} c_j^- x_j^{(d)}_{\text{opt}} \quad (A2.18) \]

where \( c_j^+ \geq 0 \) when \( j = 1, 2, \ldots, k_1 \), and \( c_j^- < 0 \) when \( j = k_1+1, k_1+2, \ldots, n \). Obviously, \( f^{(u)}_{\text{opt}} \geq f^{(d)}_{\text{opt}} \).

**Proof 5:** From Lemma 1, \( Q^+ \supseteq Q^{(u)} \supseteq Q \), and \( Q^- \supseteq Q^{(d)} \supseteq Q \). Thus, we have: \( X^{(u)}_{\text{opt}} \in Q^+ \) since \( Q^+ \supseteq Q^{(u)} \) holds, and \( X^{(2)}_{\text{opt}} \in Q^{(d)} \) since \( Q^{(d)} \supseteq Q^- \) holds. Thus, for \( f^+_{\text{opt}} = C^+ X^{(2)}_{\text{opt}} = \max \{ C X | X \in Q^+, X \geq 0 \} \), and \( f^-(d)_{\text{opt}} = C^- X^{(d)}_{\text{opt}} = \max \{ C X | X \in Q^{(d)}, X \geq 0 \} \), we have \( f^{(d)}_{\text{opt}} \geq f^-_{\text{opt}} \). This means that higher system benefits may be achieved through Eqs. 2.11 and 2.12 as constraints.

Generally, constraints (2.11) and (2.12) allow submodel for \( f^+ \), corresponding to the upper bound of the system objective \( X^{(u)}_{\text{opt}} \), to be solved first when the objective is to be maximised, while the relevant submodel for \( f^- \), corresponding to the lower bound of the system objective \( X^{(d)}_{\text{opt}} \), is proven to have feasible solutions. Thus, solutions for \( x^+_j (j = 1, 2, \ldots, k_1) \), \( x^-_j (j = k_1+1, k_1+2, \ldots, n) \), and \( f^+ \) can be obtained by solving the submodel for \( f^+ \), and those for \( x^+_j (j = 1, 2, \ldots, k_1) \), \( x^-_j (k_1+1, k_1+2, \ldots, n) \), and \( f^- \) can then be from the submodel for \( f^- \). Since both the upper and lower bound solutions are optimal under constraints (2.6) and (2.7) corresponding to the two stable extremes for given system condition variations, a complete set of feasible or softly-feasible decision alternatives are contained within the two bounds. They reflect the potential variations of \( a_{ij}^\pm \) and \( c_{ij}^\pm \), \( \forall i,j \), and correspond to objective function values lying between \( f^{(d)}_{\text{opt}} \) and \( f^{(u)}_{\text{opt}} \). Therefore, solutions \( x^{\pm}_{\text{opt}} = [ x_j^{\pm}_{\text{opt}}, x_j^{+}_{\text{opt}} ] \) and \( f^{\pm}_{\text{opt}} = [ x_j^{(d)}_{\text{opt}}, x_j^{(u)}_{\text{opt}} ] \) obtained by using constraints (2.6) and (2.7) will provide optimal and stable results.

**Remark 3:** Theorem 2 presents the interactive relationships between different model components based on integrated analysis of interactions between system objective and constraints, and between model parameters and variables. The theorem is important for the formulation of ILP solution algorithms.
% calculate the stochastic transfer factor
\[ N_{\text{max}} = 10000; \]
\[ Np = 5; Ni = 5; \]
\[ t(N_{\text{max}}, Ni, Np) = 0; \]
\[ r = 0; \]
\[ \text{for } r = 1: N_{\text{max}} \]
\[ r = r + 1 \]

% Parameters Value of Gassian Equation
% Average wind speed, (m/s)
% softwind 0.3-1.5 m/s light wind 1.6-3.3 m/s breeze 3.4-5.4 m/s
\[ u = \text{unifrnd}(1.6, 3.3); \]

% Regression exponent ?1 LogN (-0.1180, 0.0035)
\[ \beta_1 = \text{lognrnd}(-0.1180, 0.0035); \]

% Regression coefficient ?1 LogN (-1.9196, 0.2131)
\[ \gamma_1 = \text{lognrnd}(-1.9196, 0.2131); \]

% Regression exponent ?2 LogN (-0.5863, 0.2130)
\[ \beta_2 = \text{lognrnd}(-0.5863, 0.2130); \]

% Regression coefficient ?2 LogN (-0.2098, 0.6349)
\[ \gamma_2 = \text{lognrnd}(-0.2098, 0.6349); \]

% minimum treatment efficiency
% \[ U(72, 88) \ U(81, 99) \ U(63, 77) \]
\[ \text{efficiency} = 0.63; \]

% Relative (x) - km
% \[ p = 1 \ p = 2 \ p = 3 \ p = 4 \ p = 5 \]
% \[ i = 1 \ 42.7 \ 52.1 \ 30 \ 20.8 \ 38.7 \]
% \[ i = 2 \ 46.9 \ 56.3 \ 34.2 \ 25 \ 42.9 \]
% \[ i = 3 \ 33.1 \ 42.5 \ 20.4 \ 11.2 \ 29.1 \]
% \[ i = 4 \ 39 \ 48.4 \ 26.3 \ 17.1 \ 35 \]
\[ \text{clc; } \]
\[ \text{Relative}_x = 1000\*[ \]
\[ \begin{array}{cccccc}
42.7 & 52.1 & 30 & 20.8 & 38.7 \\
46.9 & 56.3 & 34.2 & 25 & 42.9 \\
33.1 & 42.5 & 20.4 & 11.2 & 29.1 \\
39 & 48.4 & 26.3 & 17.1 & 35 \\
49.7 & 59.1 & 37 & 27.8 & 45.7
\end{array} \]
\[ \text{clc; } \]

% Relative (y) - km
% \[ p = 1 \ p = 2 \ p = 3 \ p = 4 \ p = 5 \]
% \[ i = 1 \ -5.7 \ -18.5 \ 4.9 \ -19.3 \ -31.9 \]
% \[ i = 2 \ 12.2 \ -0.6 & 22.8 & -1.4 & -14 \]
% \[ i = 3 \ 2 \ -10.8 & 12.6 & -11.6 & -24.2 \]
% \[ i = 4 \ 30.4 & 17.6 & 41 & 16.8 & 4.2 \]
Relative_y = 1000*[ -5.7  -17.5   3.4  -19.3  -31.9; 13.5  1.7  22.6  -0.1  -12.7; 2  -9.8  11.1  -11.6  -24.2; 30.4  18.6  39.5  16.8   4.2; 21.6  9.8  30.7   8  -4.6];

% stack height, m
H=[252 150 110 100 265];

for i=1:Ni
    for p=1:Np
        x(i,p)=Relative_x(i,p);
        y(i,p)=Relative_y(i,p);

        % Transfer coefficient
        temp1=1/(pi*u*gamma1*gamma2*x(i,p)^(beta1+beta2));
        temp2=-y(i,p)^2/(2*gamma1^2*x(i,p)^(2*beta1));
        temp3= H(i)^2/(2*gamma2^2*x(i,p)^(2*beta2));

        % mg/m3, assume the source unit is mg/s
        tt(i,p)=temp1*exp(temp2-temp3);
        % 1 tonne/day = 1000*1000*1000/(3600*24)=11574
        t(r,i,p)=11574*1000*tt(i,p); % miu g/m3
    end
end

for i=1:Ni
    for j=1:Np
        [phat]=gamfit(t(1:N_max,i,j)); % micro g/m3
        a(i,j)=phat(1);
        b(i,j)=phat(2);

        % [miu sigma]=normfit(t(1:N_max,i,j));
        %a(i,j)=miu;
        %b(i,j)=sigma;
    end
end

a  
b

% check accuracy
i=4;
p=5;
prediction=gamrnd(a(i,p),b(i,p),N_max,1);% or gamrnd
real=t(1:N_max,i,p);
hold on
h = findobj(gca,'Type','patch');
set(h,'FaceColor','r','EdgeColor','w')
hist(prediction,10)

% check feasibility
% for optimization model
    for r=1:N_max
\[ \begin{align*}
rr_{11}(r) &= t(r, 1, 1) \times (1 - \text{efficiency}) \times 160; \\
rr_{13}(r) &= t(r, 1, 3) \times (1 - \text{efficiency}) \times 160; \\
rr_{22}(r) &= t(r, 2, 2) \times (1 - \text{efficiency}) \times 14; \\
rr_{24}(r) &= t(r, 2, 4) \times (1 - \text{efficiency}) \times 14; \\
rr_{31}(r) &= t(r, 3, 1) \times (1 - \text{efficiency}) \times 24; \\
rr_{32}(r) &= t(r, 3, 2) \times (1 - \text{efficiency}) \times 24; \\
rr_{45}(r) &= t(r, 4, 5) \times (1 - \text{efficiency}) \times 5; \\
rr_{52}(r) &= t(r, 5, 2) \times (1 - \text{efficiency}) \times 140; \\
rr_{55}(r) &= t(r, 5, 5) \times (1 - \text{efficiency}) \times 140; 
\end{align*} \]
APPENDIX C: DETERMINISTIC MODEL PROGRAMMING LANGUAGE (LINGO)

model:
sets:
emissionsource/1..5/:g;
controlmethod/1..3/:o,efficiency;
time/1..3/:k;
receptorzone/1..5/:m;
operationalcost(controlmethod,time):c;
allocation(emissionsource,controlmethod):x_1,x_2,x_3;
generation(emissionsource,time):s;
standard(emissionsource,time):e;
capacity(receptorzone,time):a;
ratio(emissionsource,receptorzone):t;
link(emissionsource,controlmethod):p;
endsets

data:
efficiency=0.80,0.90,0.70;

e= 58.00 51.50 46.00
   5.85 4.80 4.00
  10.80 10.20 10.00
   2.20 1.90 1.70
  54.00 47.50 42.00;

a= 52.50 46.50 38.95
  11.85 11.10 9.90
  27.25 25.00 23.00
  200.00 192.00 185.00
  26.50 24.50 22.50;

s= 180.00 220.00 260.00
  16.20 18.00 20.50
  28.50 35.00 40.40
  7.00 8.20 9.70
  155.30 195.00 251.50;

t= 0.3218,0.0023,0.5803,0.0001,0.0001
   0.0045,1.6673,0.0002,1.6313,0.0041
   1.6720,0.0198,0.0008,0.0001,0.0001
   0.0001,0.0011,0.0001,0.0001,0.5199
   0.0011,0.1112,0.0001,0.0072,0.7057;

c= 51.00 58.00 66.60
   69.00 75.50 82.80
   36.00 40.90 47.15;
enddata

!objective function;
min=@sum(link (i,j) : c(j,1)*x_1(i,j)+c(j,2)*x_2(i,j)+c(j,3)*x_3(i,j));
!constraints of pollution generation;
@for(emissionsource(i):[a_1]
@sum(controlmethod(j):x_1(i,j))>=s(i,1);)

@for(emissionsource(i):[a_2]
@sum(controlmethod(j):x_2(i,j))>=s(i,2);)

@for(emissionsource(i):[a_3]
@sum(controlmethod(j):x_3(i,j))>=s(i,3);)

!constraints of emission standards;
@for(emissionsource(i):[a_4]
@sum(controlmethod(j):(1-efficiency(j))*x_1(i,j))<=e(i,1);)

@for(emissionsource(i):[a_5]
@sum(controlmethod(j):(1-efficiency(j))*x_2(i,j))<=e(i,2);)

@for(emissionsource(i):[a_6]
@sum(controlmethod(j):(1-efficiency(j))*x_3(i,j))<=e(i,3);)

!constraints of environmental loading capacities;
@for(receptorzone(l):[a_7]
@sum(link(i,j):t(i,l)*(1-efficiency(j))*x_1(i,j))<=a(l,1);)

@for(receptorzone(l):[a_8]
@sum(link(i,j):t(i,l)*(1-efficiency(j))*x_2(i,j))<=a(l,2);)

@for(receptorzone(l):[a_9]
@sum(link(i,j):t(i,l)*(1-efficiency(j))*x_3(i,j))<=a(l,3);)

!non-negativity constraints;
@for(link(i,j):x_1(i,j)>=0);
@for(link(i,j):x_2(i,j)>=0);
@for(link(i,j):x_3(i,j)>=0); end
APPENDIX D: ILP MODEL PROGRAMMING LANGUAGE (LINGO)

D1: Lower quartile:

model:
sets:
  emissionsource/1..5/:g;
  controlmethod/1..3/:o,efficiency;
  time/1..3/:k;
  receptorzone/1..5/:m;
operationalcost(controlmethod,time):c;
allocation(emissionsource,controlmethod):x_1,x_2,x_3;
generation(emissionsource,time):s;
standard(emissionsource,time):e;
capacity(receptorzone,time):a;
ratio(emissionsource,receptorzone):t;
link(emissionsource,controlmethod):p;
endsets

data:
  efficiency=0.72,0.81,0.63;
  e= 69.50 62.00 55.50
     6.20 5.60 4.90
     14.40 13.40 13.20
     2.80 2.40 2.20
     65.50 57.00 51.00;
  a= 69.00 63.00 55.50
     14.00 13.20 12.30
     33.50 31.00 29.00
     239.00 231.00 224.80
     32.00 30.00 27.70;
  c= 47.00 54.00 62.20
     65.00 71.50 78.60
     32.00 36.80 42.30;
  s= 99.50 110.50 136.00
     9.90 11.20 11.90
     17.00 20.70 20.40
     3.00 3.90 4.20
     89.80 99.50 119.00;
  t= 0.35398,0.00253,0.63833,0.00011,0.00011
     0.00495,1.83403,0.00022,1.79443,0.00451
     1.8392,0.02178,0.00088,0.00011,0.00011
     0.00011,0.00121,0.00011,0.00001,0.57189
     0.00121,0.12232,0.00011,0.00792,0.77627;
enddata

!objective function;
min=@sum(link (i,j) : c(j,1)*x_1(i,j)+c(j,2)*x_2(i,j)+c(j,3)*x_3(i,j));

!constraints of pollution generation;
@for(emissionsource(i):[a_1]
@sum(controlmethod(j):x_1(i,j))>=s(i,1);)

@for(emissionsource(i):[a_2]
@sum(controlmethod(j):x_2(i,j))>=s(i,2);)

@for(emissionsource(i):[a_3]
@sum(controlmethod(j):x_3(i,j))>=s(i,3);)

!constraints of emission standards;
@for(emissionsource(i):[a_4]
@sum(controlmethod(j):(1-efficiency(j))*x_1(i,j))<=e(i,1);)

@for(emissionsource(i):[a_5]
@sum(controlmethod(j):(1-efficiency(j))*x_2(i,j))<=e(i,2);)

@for(emissionsource(i):[a_6]
@sum(controlmethod(j):(1-efficiency(j))*x_3(i,j))<=e(i,3);)

!constraints of environmental loading capacities;
@for(receptorzone(l):[a_7]
@sum(link(i,j):(1-l)*(1-efficiency(j))*x_1(i,j))<=a(l,1);)

@for(receptorzone(l):[a_8]
@sum(link(i,j):(1-l)*(1-efficiency(j))*x_2(i,j))<=a(l,2);)

@for(receptorzone(l):[a_9]
@sum(link(i,j):(1-l)*(1-efficiency(j))*x_3(i,j))<=a(l,3);)

!non-negativity constraints;
@for(link(i,j):x_1(i,j)>=0);
@for(link(i,j):x_2(i,j)>=0);
@for(link(i,j):x_3(i,j)>=0);

end
**D2: Upper quartile model:**

model:
sets:
emissionsource/1..5/:g;
controlmethod/1..3/:o,efficiency;
time/1..3/:k;
receptorzone/1..5/:m;
operationalcost(controlmethod,time):c;
allocation(emissionsource,controlmethod):x_1,x_2,x_3;
generation(emissionsource,time):s;
standard(emissionsource,time):e;
capacity(receptorzone,time):a;
ratio(emissionsource,receptorzone):t;
link(emissionsource,controlmethod):p;
lowerquartile1(emissionsource,controlmethod):d_1;
lowerquartile2(emissionsource,controlmethod):d_2;
lowerquartile3(emissionsource,controlmethod):d_3;
endsets

data:
efficiency=0.88,0.99,0.77;

e= 46.50 41.00 36.50
5.50 4.00 3.10
7.20 7.00 6.80
1.60 1.40 1.20
42.50 38.00 33.00;

a= 36.00 30.00 22.40
9.70 9.00 7.50
21.00 19.00 17.00
161.00 153.00 145.20
21.00 19.00 17.30;

s= 260.50 329.50 384.00
22.50 24.80 29.10
40.00 49.30 60.40
11.00 12.50 15.20
220.80 290.50 384.00;

t= 0.28962 0.00207 0.52227 0.00009 0.00009
0.00405 1.50057 0.00018 0.00018 0.00009
1.5048 0.01782 0.00072 0.00009 0.00009
0.00099 0.00099 0.00009 0.00009 0.46791
0.00099 0.10008 0.00009 0.00648 0.63513;

d_1 = 0.0,99.50
0.0,9,9
0.0,17
0.0,3
0.0,89.80;

d_2 = 0.0,110.5
\begin{align*}
d_3 &= 0.2726, 108.74\, 0.84, 11.06\, 0.42, 52.91, 66.09; \\
c &= 55.00, 62.00, 71.00\, 73.00, 79.50, 87.00\, 40.00, 45.00, 52.00; 
\end{align*}

elemental function;
min = @sum(link (i, j) : c(j, 1) * x_1(i, j) + c(j, 2) * x_2(i, j) + c(j, 3) * x_3(i, j));

! constraints of pollution generation;
\begin{align*}
\text{for} &\text{emissionsource}(i): \&[a_1] \\
@sum(\text{controlmethod}(j): x_1(i, j)) &\geq s(i, 1); \\
\text{for} &\text{emissionsource}(i): \&[a_2] \\
@sum(\text{controlmethod}(j): x_2(i, j)) &\geq s(i, 2); \\
\text{for} &\text{emissionsource}(i): \&[a_3] \\
@sum(\text{controlmethod}(j): x_3(i, j)) &\geq s(i, 3); \\
\end{align*}

! constraints of emission standards;
\begin{align*}
\text{for} &\text{emissionsource}(i): \&[a_4] \\
@sum(\text{controlmethod}(j): (1 - efficiency(j)) * x_1(i, j)) &\leq e(i, 1); \\
\text{for} &\text{emissionsource}(i): \&[a_5] \\
@sum(\text{controlmethod}(j): (1 - efficiency(j)) * x_2(i, j)) &\leq e(i, 2); \\
\text{for} &\text{emissionsource}(i): \&[a_6] \\
@sum(\text{controlmethod}(j): (1 - efficiency(j)) * x_3(i, j)) &\leq e(i, 3); \\
\end{align*}

! constraints of environmental loading capacities;
\begin{align*}
\text{for} &\text{receptorzone}(l): \&[a_7] \\
@sum(link(i, j)): (1 - efficiency(j)) * x_1(i, j)) &\leq a(l, 1); \\
\text{for} &\text{receptorzone}(l): \&[a_8] \\
@sum(link(i, j)): (1 - efficiency(j)) * x_2(i, j)) &\leq a(l, 2); \\
\text{for} &\text{receptorzone}(l): \&[a_9] \\
@sum(link(i, j)): (1 - efficiency(j)) * x_3(i, j)) &\leq a(l, 3); \\
\end{align*}

! non-negativity constraints;
\begin{align*}
\text{for} &\text{link}(i, j): x_1(i, j) \geq d_1(i, j); \\
\text{for} &\text{link}(i, j): x_2(i, j) \geq d_2(i, j); \\
\text{for} &\text{link}(i, j): x_3(i, j) \geq d_3(i, j); \\
\end{align*}

end
APPENDIX E: CCP MODEL PROGRAMMING LANGUAGE (LINGO)

model:
sets:
emissionsource/1..5/:g;
controlmethod/1..3/:o,efficiency;
time/1..3/:k;
receptorzone/1..5/:m;
operationalcost(controlmethod,time):c;
allocation(emissionsource,controlmethod):x_1,x_2,x_3;
generation(emissionsource,time):s;
standard(emissionsource,time):e;
capacity(receptorzone,time):a;
ratio(emissionsource,receptorzone):t;
link(emissionsource,controlmethod):p;
endsets

data:
efficiency=0.8,0.9,0.70;
s=
180.00 220.00 260.00
16.20 18.00 20.50
28.50 35.00 40.40
7.00 8.20 9.70
155.30 195.00 251.50; !tons/d;
!0.01;
e= 49.0824 43.3578 38.6332
5.5785 4.1796 3.3021
8.0084 7.7185 7.5185
1.7347 1.5122 1.3122
45.0824 40.1332 35.0210;
!a= 39.7051 33.7051 26.1162
10.1827 9.4716 8.0389
22.4035 20.3473 18.3473
169.7575 161.7575 154.1370
22.2351 20.2351 18.4677;
!0.1;
e= 53.0874 47.0146 41.9417
5.7004 4.4582 3.6155
9.2621 8.8330 8.6330
1.9437 1.6864 1.4864
49.0874 43.4417 38.1553;
!a= 45.4515 39.4515 31.8801
10.9315 10.2029 8.8748
24.5801 22.4369 20.4369
183.3398 175.3398 167.9980
24.1505 22.1505 20.2787;
$0.25$

\[
\begin{array}{ccc}
55.4145 & 49.1393 & 43.8641 \\
5.7713 & 4.6201 & 3.7977 \\
2.0651 & 1.7876 & 1.5876 \\
51.4145 & 45.3641 & 39.9765 \\
\end{array}
\]

\[
\begin{array}{ccc}
48.7903 & 42.7903 & 35.2290 \\
11.3666 & 10.6279 & 9.3604 \\
25.8448 & 23.6510 & 21.6510 \\
191.2316 & 183.2316 & 176.0517 \\
25.2635 & 23.2635 & 21.3309 \\
\end{array}
\]

$0.5$

\[
\begin{array}{ccc}
58.0000 & 51.5000 & 46.0000 \\
5.8500 & 4.8000 & 4.0000 \\
10.8000 & 10.2000 & 10.0000 \\
2.2000 & 1.9000 & 1.7000 \\
54.0000 & 47.5000 & 42.0000 \\
\end{array}
\]

\[
\begin{array}{ccc}
52.5000 & 46.5000 & 38.9500 \\
11.8500 & 11.1000 & 9.9000 \\
27.2500 & 25.0000 & 23.0000 \\
200.0000 & 192.0000 & 185.0000 \\
26.5000 & 24.5000 & 22.5000 \\
\end{array}
\]

$0.8$

\[
\begin{array}{ccc}
61.2262 & 54.4457 & 48.6652 \\
5.9482 & 5.0245 & 4.2525 \\
11.8099 & 11.0978 & 10.8978 \\
2.3683 & 2.0403 & 1.8403 \\
57.2262 & 50.1652 & 44.5249 \\
\end{array}
\]

\[
\begin{array}{ccc}
57.1289 & 51.1289 & 43.5930 \\
12.4532 & 11.6891 & 10.5733 \\
29.0033 & 26.6832 & 24.6832 \\
210.9411 & 202.9411 & 196.1655 \\
28.0429 & 26.0429 & 23.9588 \\
\end{array}
\]

\[
\begin{array}{ccc}
0.3218,0.0023,0.5803,0.0001,0.0001 \\
0.0045,1.6673,0.0002,1.6313,0.0041 \\
1.6720,0.0198,0.0008,0.0001,0.0001 \\
0.0001,0.0011,0.0001,0.0001,0.5199 \\
0.0011,0.1112,0.0001,0.0072,0.7057 \\
\end{array}
\]

\[
\begin{array}{ccc}
51.00 & 58.00 & 66.60 \\
69.00 & 75.50 & 82.80 \\
36.00 & 40.90 & 47.15 \\
\end{array}
\]

enddata

!objective function;
\[
\text{min} = \sum (\text{link } (i,j) : c(j,1)*x_1(i,j)+c(j,2)*x_2(i,j)+c(j,3)*x_3(i,j));
\]

!constraints of pollution generation;
@for(emissionsource(i):[a_1]}
A CHANCE-CONSTRAINED MODEL FOR REGIONAL AIR QUALITY MANAGEMENT

@sum(controlmethod(j):x_1(i,j))>=s(i,1);
@for(emissionsource(i):[a_2]
@sum(controlmethod(j):x_2(i,j))>=s(i,2);
@for(emissionsource(i):[a_3]
@sum(controlmethod(j):x_3(i,j))>=s(i,3);

!constraints of emission standards;
@for(emissionsource(i):[a_4]
@sum(controlmethod(j):(1-efficiency(j))*x_1(i,j))<=e(i,1);

@for(emissionsource(i):[a_5]
@sum(controlmethod(j):(1-efficiency(j))*x_2(i,j))<=e(i,2);

@for(emissionsource(i):[a_6]
@sum(controlmethod(j):(1-efficiency(j))*x_3(i,j))<=e(i,3);

!constraints of environmental loading capacities;
@for(receptorzone(l):[a_7]
@sum(link(i,j):t(i,l)*(1-efficiency(j))*x_1(i,j))<=a(l,1);

@for(receptorzone(l):[a_8]
@sum(link(i,j):t(i,l)*(1-efficiency(j))*x_2(i,j))<=a(l,2);

@for(receptorzone(l):[a_9]
@sum(link(i,j):t(i,l)*(1-efficiency(j))*x_3(i,j))<=a(l,3);

!non-negativity constraints;
@for(link(i,j): x_1(i,j)>=0);
@for(link(i,j): x_2(i,j)>=0);
@for(link(i,j): x_3(i,j)>=0);
end
APPENDIX F: ICCLP PROGRAMMING LANGUAGE (LINGO)

F1: Lower quartile:

model:
sets:
emissionsource/1..5/:g;
controlmethod/1..3/:o,efficiency;
time/1..3/:k;
receptorzone/1..5/:m;
operationalcost(controlmethod,time):c;
allocation(emissionsource,controlmethod):x_1,x_2,x_3;
generation(emissionsource,time):s;
standard(emissionsource,time):e;
capacity(receptorzone,time):a;
ratio(emissionsource,receptorzone):t;
link(emissionsource,controlmethod):p;
endsets

data:
efficiency=0.72,0.81,0.63;

!0.01;
!e=  49.0824  43.3578  38.6332
     5.5785  4.1796  3.3021
     8.0084  7.7185  7.5185
    1.7347  1.5122  1.3122
   45.0824  40.1332  35.0210;
!a=  39.7051  33.7051  26.1162
   10.1827  9.4716  8.0389
  22.4035  20.3473  18.3473
 169.7575 161.7575 154.1370
 22.2351  20.2351  18.4677;
!0.1;
!e=  53.0874  47.0146  41.9417
     5.7004  4.4582  3.6155
     9.2621  8.8330  8.6330
    1.9437  1.6864  1.4864
   49.0874  43.4417  38.1553;
!a=  45.4515  39.4515  31.8801
  10.9315 10.2029  8.8748
  24.5801 22.4369  20.4369
 183.3398 175.3398 167.9980
 24.1505  22.1505  20.2787;
!0.25;
!e=  55.4145  49.1393  43.8641
     5.7713  4.6201  3.7977
     9.9906  9.4805  9.2805
    2.0651  1.7876  1.5876

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51.4145 45.3641 39.9765;

!a= 48.7903 42.7903 35.2290
11.3666 10.6279 9.3604
25.8448 23.6510 21.6510
191.2316 183.2316 176.0517
25.2635 23.2635 21.3309;

!0.5;

c= 58.0000 51.5000 46.0000
5.8500 4.8000 4.0000
10.8000 10.2000 10.0000
2.2000 1.9000 1.7000
54.0000 47.5000 42.0000;

a= 52.5000 46.5000 38.9500
11.8500 11.1000 9.9000
27.2500 25.0000 23.0000
200.0000 192.0000 185.0000
26.5000 24.5000 22.5000;

!0.8;

e= 61.2262 54.4457 48.6652
5.9482 5.0245 4.2525
11.8099 11.0978 10.8978
2.3683 2.0403 1.8403
57.2262 50.1652 44.5249;

!a= 57.1289 51.1289 43.5930
12.4532 11.6891 10.5733
29.0033 26.6832 24.6832
210.9411 202.9411 196.1655
28.0429 26.0429 23.9588;

c= 47.00 54.00 62.20
65.00 71.50 78.60
32.00 36.80 42.30;

s= 99.50 110.50 136.00
9.90 11.20 11.90
17.00 20.70 20.40
3.00 3.90 4.20
89.80 99.50 119.00;

t= 0.35398,0.00253,0.63833,0.00011,0.00011
0.00495,1.83403,0.00022,1.79443,0.00451
1.8392,0.02178,0.00088,0.00011,0.00011
0.00011,0.00121,0.00011,0.00011,0.57189
0.00121,0.12232,0.00011,0.00792,0.77627;
enddata

!objective function;
min=@sum(link (i,j) : c(j,1)*x_1(i,j)+c(j,2)*x_2(i,j)+c(j,3)*x_3(i,j));

!constraints of pollution generation;
A CHANCE-CONSTRAINED MODEL FOR REGIONAL AIR QUALITY MANAGEMENT

@for(emissionsource(i): [a_1]
@sum(controlmethod(j):x_1(i,j))>=s(i,1);

@for(emissionsource(i): [a_2]
@sum(controlmethod(j):x_2(i,j))>=s(i,2);

@for(emissionsource(i): [a_3]
@sum(controlmethod(j):x_3(i,j))>=s(i,3);

! constraints of emission standards;
@for(emissionsource(i): [a_4]
@sum(controlmethod(j):(1-efficiency(j))*x_1(i,j))<=e(i,1);

@for(emissionsource(i): [a_5]
@sum(controlmethod(j):(1-efficiency(j))*x_2(i,j))<=e(i,2);

@for(emissionsource(i): [a_6]
@sum(controlmethod(j):(1-efficiency(j))*x_3(i,j))<=e(i,3);

! constraints of environmental loading capacities;
@for(receptorzone(l): [a_7]
@sum(link(i,j):t(i,l)*(1-efficiency(j))*x_1(i,j))<=a(l,1);

@for(receptorzone(l): [a_8]
@sum(link(i,j):t(i,l)*(1-efficiency(j))*x_2(i,j))<=a(l,2);

@for(receptorzone(l): [a_9]
@sum(link(i,j):t(i,l)*(1-efficiency(j))*x_3(i,j))<=a(l,3);

! non-negativity constraints;
@for(link(i,j): x_1(i,j)>=0);
@for(link(i,j): x_2(i,j)>=0);
@for(link(i,j): x_3(i,j)>=0);
end
model:
sets:
emissionsource/1..5/:g;
controlmethod/1..3/:o,efficiency;
time/1..3/:k;
receptorzone/1..5/:m;
operationalcost(controlmethod,time):c;
allocation(emissionsource,controlmethod):x_1,x_2,x_3;
generation(emissionsource,time):s;
standard(emissionsource,time):e;
capacity(receptorzone,time):a;
ratio(emissionsource,receptorzone):t;
link(emissionsource,controlmethod):p;
lowerquartile1(emissionsource,controlmethod):d_1;
lowerquartile2(emissionsource,controlmethod):d_2;
lowerquartile3(emissionsource,controlmethod):d_3;
endsets

data:
efficiency=0.88,0.99,0.77;
!
10.01;
e= 49.0824 43.3578 38.6332
  5.5785 4.1796 3.3021
  8.0084 7.7185 7.5185
  1.7347 1.5122 1.3122
  45.0824 40.1332 35.0210;

a= 39.7051 33.7051 26.1162
  10.1827 9.4716 8.0389
  22.4035 20.3473 18.3473
  169.7575 161.7575 154.1370
  22.2351 20.2351 18.4677;

s= 260.50 329.50 384.00
  22.50 24.80 29.10
  40.00 49.30 60.40
  11.00 12.50 15.20
  220.80 290.50 384.00;

t= 0.28962 0.00207 0.52227 0.00009 0.00009
  0.00405 1.50057 0.00018 1.46817 0.00369
  1.5048 0.01782 0.00072 0.00009 0.00009
  0.00009 0.00099 0.00009 0.00009 0.46791
  0.00099 0.10008 0.00009 0.00648 0.63513;

d_1 = 19.25,0,80.25
  0.99,0,8.91
  0,0,17
  0,0,3
  60.3,0,29.50;
\[ d_2 = \begin{bmatrix} 100.28, 0, 10.22 \\ 8.68, 0.2, 5.2 \\ 0, 0.2, 0.7 \\ 0, 0.3, 9 \\ 67.49, 32.01, 0 \end{bmatrix}; \]

\[ d_3 = \begin{bmatrix} 0, 119.96, 16.04 \\ 0.9, 28.2, 62 \\ 0.0, 16, 20.24 \\ 0.1, 34.28, 66 \\ 0, 117.93, 1.07 \end{bmatrix}; \]

\[ c = \begin{bmatrix} 55.00 & 62.00 & 71.00 \\ 73.00 & 79.50 & 87.00 \\ 40.00 & 45.00 & 52.00 \end{bmatrix}; \]

\[ \text{enddata} \]

!objective function;
\[ \text{min} = \sum \text{link (i,j)} : c(j,1)\times x_1(i,j) + c(j,2)\times x_2(i,j) + c(j,3)\times x_3(i,j); \]

!constraints of pollution generation;
\[ \text{for (emissionsource(i):[a_1] } \]
\[ \sum \text{controlmethod(j):x_1(i,j)} > = s(i,1); \]
\[ \text{for (emissionsource(i):[a_2] } \]
\[ \sum \text{controlmethod(j):x_2(i,j)} > = s(i,2); \]
\[ \text{for (emissionsource(i):[a_3] } \]
\[ \sum \text{controlmethod(j):x_3(i,j)} > = s(i,3); \]

!constraints of emission standards;
\[ \text{for (emissionsource(i):[a_4] } \]
\[ \sum \text{controlmethod(j):x_1(i,j)} \times (1 - \text{efficiency(j)})} > = e(i,1); \]
\[ \text{for (emissionsource(i):[a_5] } \]
\[ \sum \text{controlmethod(j):x_2(i,j)} \times (1 - \text{efficiency(j)})} > = e(i,2); \]
\[ \text{for (emissionsource(i):[a_6] } \]
\[ \sum \text{controlmethod(j):x_3(i,j)} \times (1 - \text{efficiency(j)})} > = e(i,3); \]

!constraints of environmental loading capacities;
\[ \text{for (receptorzone(l):[a_7] } \]
\[ \sum \text{link(i,j):t(i,l)\times x_1(i,j)} > = a(l,1); \]
\[ \text{for (receptorzone(l):[a_8] } \]
\[ \sum \text{link(i,j):t(i,l)\times x_2(i,j)} > = a(l,2); \]
\[ \text{for (receptorzone(l):[a_9] } \]
\[ \sum \text{link(i,j):t(i,l)\times x_3(i,j)} > = a(l,3); \]

!non-negativity constraints;
\[ \text{for (link(i,j):x_1(i,j)} > = d_1(i,j); \]
\[ \text{for (link(i,j):x_2(i,j)} > = d_2(i,j); \]
\[ \text{for (link(i,j):x_3(i,j)} > = d_3(i,j); \]

\text{end}
**F3: Upper quartile for \( q_i = 0.1 \)**

model:
sets:
emissionsource/1..5/:g;
controlmethod/1..3/:o,efficiency;
time/1..3/:k;
receptorzone/1..5/:m;
operationalcost(controlmethod,time):c;
allocation(emissionsource,controlmethod):x_1,x_2,x_3;
generation(emissionsource,time):s;
standard(emissionsource,time):e;
capacity(receptorzone,time):a;
ratio(emissionsource,receptorzone):t;
link(emissionsource,controlmethod):p;
lowerquartile1(emissionsource,controlmethod):d_1;
lowerquartile2(emissionsource,controlmethod):d_2;
lowerquartile3(emissionsource,controlmethod):d_3;
endsets
data:
efficiency=0.88,0.99,0.77;
!
0.1;
e=
53.0874 47.0146 41.9417
5.7004 4.4582 3.6155
9.2621 8.8330 8.6330
1.9437 1.6864 1.4864
49.0874 43.4417 38.1553;

a=
45.4515 39.4515 31.8801
10.9315 10.2029 8.8748
24.5801 22.4369 20.4369
183.3398 175.3398 167.9980
24.1505 22.1505 20.2787;

s= 260.50 329.50 384.00
22.50 24.80 29.10
40.00 49.30 60.40
11.00 12.50 15.20
220.80 290.50 384.00;

t=
0.289620.002070.522270.000090.00009
0.004051.500570.00181.468170.00369
1.5048 0.017820.000720.000090.00009
0.000090.000090.000090.000090.46791
0.000990.100080.000090.006480.63513;

d_1 =
0.0,99.50
0.0,9.9
0.0,17
0.0,3
32.89,0.5691;
d_2 = 63.92,0,46.58
    6.13,0,5.07
    0,0,20.7
    0,0,3.9
    94.88,4.62,0;

d_3 = 0,101.78,34.22
    0,7.59,4.31
    0,0,20.4
    0,0.38,3.82
    0,105.69,13.31;

c= 55.00  62.00  71.00
    73.00  79.50  87.00
    40.00  45.00  52.00;
enddata

!objective function;
min=@sum(link (i,j) : c(j,1)*x_1(i,j)+c(j,2)*x_2(i,j)+c(j,3)*x_3(i,j));

!constraints of pollution generation;
@for(emissionsource(i):[a_1]
    @sum(controlmethod(j):x_1(i,j))>=s(i,1);)
@for(emissionsource(i):[a_2]
    @sum(controlmethod(j):x_2(i,j))>=s(i,2);)
@for(emissionsource(i):[a_3]
    @sum(controlmethod(j):x_3(i,j))>=s(i,3);)

!constraints of emission standards;
@for(emissionsource(i):[a_4]
    @sum(controlmethod(j):(1-efficiency(j))*x_1(i,j))<=e(i,1);)
@for(emissionsource(i):[a_5]
    @sum(controlmethod(j):(1-efficiency(j))*x_2(i,j))<=e(i,2);)
@for(emissionsource(i):[a_6]
    @sum(controlmethod(j):(1-efficiency(j))*x_3(i,j))<=e(i,3);)

!constraints of environmental loading capacities;
@for(receptorzone(l):[a_7]
    @sum(link(i,j):t(i,l)*(1-efficiency(j))*x_1(i,j))<=a(l,1);)
@for(receptorzone(l):[a_8]
    @sum(link(i,j):t(i,l)*(1-efficiency(j))*x_2(i,j))<=a(l,2);)
@for(receptorzone(l):[a_9]
    @sum(link(i,j):t(i,l)*(1-efficiency(j))*x_3(i,j))<=a(l,3);)

!non-negativity constraints;
@for(link(i,j):x_1(i,j)>=d_1(i,j));)
@for(link(i,j):x_2(i,j)>=d_2(i,j));
@for(link(i,j):x_3(i,j)>=d_3(i,j));
end
F4: Upper quartile for $q_i = 0.25$

model:
sets:
emissionsource/1..5/:g;
controlmethod/1..3/:o,efficiency;
time/1..3/:k;
receptorzone/1..5/:m;
operationalcost(controlmethod,time):c;
allocation(emissionsource,controlmethod):x_1,x_2,x_3;
generation(emissionsource,time):s;
standard(emissionsource,time):e;
capacity(receptorzone,time):a;
ratio(emissionsource,receptorzone):t;
link(emissionsource,controlmethod):p;
lowerquartile1(emissionsource,controlmethod):d_1;
lowerquartile2(emissionsource,controlmethod):d_2;
lowerquartile3(emissionsource,controlmethod):d_3;
endsets

data:
efficiency=0.88,0.99,0.77;

s = 260.50 329.50 384.00 22.50 24.80 29.10 40.00 49.30 60.40 11.00 12.50 15.20 220.80 290.50 384.00; 
t = 0.289620.002070.522270.000090.00009 0.004051.500570.00181.468170.00369 1.5048 0.017820.000720.000090.00009 0.000090.000990.000090.000090.46791 0.000990.100080.000090.006480.63513; 
d_1 = 0,0.99,50 0,0.9,9 0,0.17 0,0.3 16.96,0,72.84;
Final Year Project Report
A CHANCE-CONSTRAINED MODEL FOR REGIONAL AIR QUALITY MANAGEMENT

\[ d_2 = \begin{bmatrix} 42.79, 0, 67.71 \\ 4.65, 0, 6.55 \\ 0, 0.20.7 \\ 0, 0.39 \\ 88.2, 0, 11.3 \end{bmatrix}; \]

\[ d_3 = \begin{bmatrix} 0.91, 21, 44.79 \\ 0.62, 5.28 \\ 0, 0.20.4 \\ 0, 0.42 \\ 0, 98.45, 20.55 \end{bmatrix}; \]

\[ c = \begin{bmatrix} 55.00, 62.00, 71.00 \\ 73.00, 79.50, 87.00 \\ 40.00, 45.00, 52.00 \end{bmatrix}; \]

\[ \text{enddata} \]

!objective function;
\[ \text{min} = \sum (\text{link (i,j)} : c(j,1)*x_1(i,j)+c(j,2)*x_2(i,j)+c(j,3)*x_3(i,j)); \]

!constraints of pollution generation;
\[ @for(\text{emissionsource(i)}:[a_1]) \]
\[ @sum(\text{controlmethod(j)}:x_1(i,j)) = s(i,1); \]

\[ @for(\text{emissionsource(i)}:[a_2]) \]
\[ @sum(\text{controlmethod(j)}:x_2(i,j)) = s(i,2); \]

\[ @for(\text{emissionsource(i)}:[a_3]) \]
\[ @sum(\text{controlmethod(j)}:x_3(i,j)) = s(i,3); \]

!constraints of emission standards;
\[ @for(\text{emissionsource(i)}:[a_4]) \]
\[ @sum(\text{controlmethod(j)}:(1-\text{efficiency(j)})*x_1(i,j)) <= e(i,1); \]

\[ @for(\text{emissionsource(i)}:[a_5]) \]
\[ @sum(\text{controlmethod(j)}:(1-\text{efficiency(j)})*x_2(i,j)) <= e(i,2); \]

\[ @for(\text{emissionsource(i)}:[a_6]) \]
\[ @sum(\text{controlmethod(j)}:(1-\text{efficiency(j)})*x_3(i,j)) <= e(i,3); \]

!constraints of environmental loading capacities;
\[ @for(\text{receptorzone(l)}:[a_7]) \]
\[ @sum(\text{link(i,j)}:t(i,l)*(1-\text{efficiency(j)})*x_1(i,j)) <= a(l,1); \]

\[ @for(\text{receptorzone(l)}:[a_8]) \]
\[ @sum(\text{link(i,j)}:t(i,l)*(1-\text{efficiency(j)})*x_2(i,j)) <= a(l,2); \]

\[ @for(\text{receptorzone(l)}:[a_9]) \]
\[ @sum(\text{link(i,j)}:t(i,l)*(1-\text{efficiency(j)})*x_3(i,j)) <= a(l,3); \]

!non-negativity constraints;
\[ @for(\text{link(i,j)}):x_1(i,j) >= d_1(i,j); \]
\[ @for(\text{link(i,j)}):x_2(i,j) >= d_2(i,j); \]
\[ @for(\text{link(i,j)}):x_3(i,j) >= d_3(i,j); \]
end

*School of Civil & Environmental Engineering
**F5: Upper quartile for \( q_{i} = 0.5 \)**

model:

sets:
emissionsource/1..5/:g;
controlmethod/1..3/:o,efficiency;
time/1..3/:k;
receptorzone/1..5/:m;
operationalcost(controlmethod,time):c;
allocation(emissionsource,controlmethod):x_1,x_2,x_3;
generation(emissionsource,time):s;
standard(emissionsource,time):e;
capacity(receptorzone,time):a;
ratio(emissionsource,receptorzone):t;
link(emissionsource,controlmethod):p;
lowerquartile1(emissionsource,controlmethod):d_1;
lowerquartile2(emissionsource,controlmethod):d_2;
lowerquartile3(emissionsource,controlmethod):d_3;

endsets

data:
efficiency=0.88,0.99,0.77;

!0.5;

e= 58.0000 51.5000 46.0000  
   5.8500 4.8000 4.0000  
   10.8000 10.2000 10.0000  
   2.2000 1.9000 1.7000  
   54.0000 47.5000 42.0000;

a= 52.5000 46.5000 38.9500  
   11.8500 11.1000 9.9000  
   27.2500 25.0000 23.0000  
   200.0000 192.0000 185.0000  
   26.5000 24.5000 22.5000;

s= 260.50 329.50 384.00  
   22.50 24.80 29.10  
   40.00 49.30 60.40  
   11.00 12.50 15.20  
   220.80 290.50 384.00;

t= 0.289620.002070.522270.000090.00009  
   0.004051.500570.00181.468170.00369  
   1.5048 0.017820.000720.000090.00009  
   0.000090.000990.000090.000090.46791  
   0.000990.100080.00090.006480.63513;

d_1 = 0.0,99.50  
   0.0,9.9  
   0.0,17  
   0.0,3  
   0.0,89.8;
d_2 =  
3.082  
0.0207 
0.039  
70.51 0.2899;

d_3 =  
0.7947 56.53 
0.5663 4 
0.042 
0.9009 28.91;

c=  
55.00 62.00 71.00  
73.00 79.50 87.00  
40.00 45.00 52.00;

enddata

!objective function;
min=@sum(link (i,j) : c(j,1)*x_1(i,j)+c(j,2)*x_2(i,j)+c(j,3)*x_3(i,j));

!constraints of pollution generation;
@for(emissionsource(i):[a_1]
@sum(controlmethod(j):x_1(i,j))>=s(i,1);)

@for(emissionsource(i):[a_2]
@sum(controlmethod(j):x_2(i,j))>=s(i,2);)

@for(emissionsource(i):[a_3]
@sum(controlmethod(j):x_3(i,j))>=s(i,3);)

!constraints of emission standards;
@for(emissionsource(i):[a_4]
@sum(controlmethod(j):(1-efficiency(j))*x_1(i,j))<=e(i,1);)

@for(emissionsource(i):[a_5]
@sum(controlmethod(j):(1-efficiency(j))*x_2(i,j))<=e(i,2);)

@for(emissionsource(i):[a_6]
@sum(controlmethod(j):(1-efficiency(j))*x_3(i,j))<=e(i,3);)

!constraints of environmental loading capacities;
@for(receptorzone(l):[a_7]
@sum(link(i,j):t(i,l)*(1-efficiency(j))*x_1(i,j))<=a(l,1);)

@for(receptorzone(l):[a_8]
@sum(link(i,j):t(i,l)*(1-efficiency(j))*x_2(i,j))<=a(l,2);)

@for(receptorzone(l):[a_9]
@sum(link(i,j):t(i,l)*(1-efficiency(j))*x_3(i,j))<=a(l,3);)

!non-negativity constaints;
@for(link(i,j):x_1(i,j)>=d_1(i,j));
@for(link(i,j):x_2(i,j)>=d_2(i,j));
@for(link(i,j):x_3(i,j)>=d_3(i,j));
end
F6: Upper quartile for $q_i=0.8$

model:
sets:
emissionsource/1..5/:g;
controlmethod/1..3/:o,e,efficiency;
time/1..3/:k;
receptorzone/1..5/:m;
operationalcost(controlmethod,time):c;
allocation(emissionsource,controlmethod):x_1,x_2,x_3;
generation(emissionsource,time):s;
standard(emissionsource,time):e;
capacity(receptorzone,time):a;
ratio(emissionsource,receptorzone):t;
link(emissionsource,controlmethod):p;
lowerquartile1(emissionsource,controlmethod):d_1;
lowerquartile2(emissionsource,controlmethod):d_2;
lowerquartile3(emissionsource,controlmethod):d_3;
endsets

data:

!0.8;
e= 61.2262 54.4457 48.6652 5.9482 5.0245 4.2525 11.8099 11.0978 10.8978 2.3683 2.0403 1.8403 57.2262 50.1652 44.5249;
s= 260.50 329.50 384.00 22.50 24.80 29.10 40.00 49.30 60.40 11.00 12.50 15.20 220.80 290.50 384.00;
t= 0.289620.002070.522270.000090.00009 0.004051.500570.00181.468170.00369 1.5048 0.017820.000720.000090.00009 0.000090.000090.000090.000090.46791 0.000990.100080.000090.006480.63513;
d_1 = 0,0.99,50 0,0.99 0,0.17 0,0.3 0,0.89,8;
\[ \text{d}_2 = \begin{bmatrix} 0.0, 110.5 \\ 0.93, 0.1027 \\ 0.0, 20.7 \\ 0.0, 3.9 \\ 48.44, 0.5106; \end{bmatrix} \]

\[ \text{d}_3 = \begin{bmatrix} 0.64, 83.71.17 \\ 0.4237.67 \\ 0.0, 20.4 \\ 0.0, 4.2 \\ 0.79, 66.39.34; \end{bmatrix} \]

\[ \text{c} = \begin{bmatrix} 55.00, 62.00, 71.00 \\ 73.00, 79.50, 87.00 \\ 40.00, 45.00, 52.00; \end{bmatrix} \]

dendata

!objective function;
\[
\min = \sum \text{link (i,j) : c(j,1)*x_1(i,j)+c(j,2)*x_2(i,j)+c(j,3)*x_3(i,j)};\]

!constraints of pollution generation;
\[
\begin{align*}
\text{for (emissionsource(i):[a_1]} \\
\sum \text{controlmethod(j):x_1(i,j)} & \geq s(i,1); \end{align*} \]

\[
\begin{align*}
\text{for (emissionsource(i):[a_2]} \\
\sum \text{controlmethod(j):x_2(i,j)} & \geq s(i,2); \end{align*} \]

\[
\begin{align*}
\text{for (emissionsource(i):[a_3]} \\
\sum \text{controlmethod(j):x_3(i,j)} & \geq s(i,3); \end{align*} \]

!constraints of emission standards;
\[
\begin{align*}
\text{for (emissionsource(i):[a_4]} \\
\sum \text{controlmethod(j):(1-efficiency(j))*x_1(i,j)} & \leq e(i,1); \end{align*} \]

\[
\begin{align*}
\text{for (emissionsource(i):[a_5]} \\
\sum \text{controlmethod(j):(1-efficiency(j))*x_2(i,j)} & \leq e(i,2); \end{align*} \]

\[
\begin{align*}
\text{for (emissionsource(i):[a_6]} \\
\sum \text{controlmethod(j):(1-efficiency(j))*x_3(i,j)} & \leq e(i,3); \end{align*} \]

!constraints of environmental loading capacities;
\[
\begin{align*}
\text{for (receptorzone(l):[a_7]} \\
\sum \text{link(i,j):(l,1) *(1-efficiency(j))*x_1(i,j)} & \leq a(l,1); \end{align*} \]

\[
\begin{align*}
\text{for (receptorzone(l):[a_8]} \\
\sum \text{link(i,j):(l,1) *(1-efficiency(j))*x_2(i,j)} & \leq a(l,2); \end{align*} \]

\[
\begin{align*}
\text{for (receptorzone(l):[a_9]} \\
\sum \text{link(i,j):(l,1) *(1-efficiency(j))*x_3(i,j)} & \leq a(l,3); \end{align*} \]

!non-negativity constraints;
\[
\begin{align*}
\text{for (link(i,j):x_1(i,j)} & \geq d_1(i,j); \end{align*} \]

\[
\begin{align*}
\text{for (link(i,j):x_2(i,j)} & \geq d_2(i,j); \end{align*} \]

\[
\begin{align*}
\text{for (link(i,j):x_3(i,j)} & \geq d_3(i,j); \end{align*} \]
APPENDIX G: DETERMINISTIC MODEL SOLUTIONS

<table>
<thead>
<tr>
<th>Period</th>
<th>Source i</th>
<th>Control Measure j</th>
<th>Allocation (ton/d)</th>
<th>Cost ($)</th>
<th>Total Cost ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Power plant 1</td>
<td>OPA</td>
<td>70.61</td>
<td>3,601.11</td>
<td>7,539.15</td>
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<td></td>
<td>AA</td>
<td>109.39</td>
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<td>LWS</td>
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<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Chemical Industry</td>
<td>OPA</td>
<td>3.20</td>
<td>163.20</td>
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<td>468.00</td>
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<td>Petroleum Refinery</td>
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<td>1,026.00</td>
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<td>252.00</td>
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<td>LWS</td>
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<tr>
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<td>Steel Mining</td>
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<td>106.20</td>
<td>5,416.20</td>
<td>7,183.80</td>
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<td>Total Cost for Period 1</td>
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<td>Power plant 1</td>
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<td>9.40</td>
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<td>Chemical Industry</td>
<td>OPA</td>
<td>11.44</td>
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<td>Total Cost for Period 2</td>
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<td>LWS</td>
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<td>Steel Mining</td>
<td>OPA</td>
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<td>Total Cost for Period 3</td>
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<td>Grand Total</td>
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<td>88,021.38</td>
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### APPENDIX H: ILP MODEL SOLUTIONS

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<tr>
<th>Period</th>
<th>Source i</th>
<th>Control Measure j</th>
<th>Allocation (ton/d)</th>
<th>Cost ($)</th>
<th>Total Cost ($)</th>
</tr>
</thead>
<tbody>
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<td>1</td>
<td>Power plant 1</td>
<td>OPA</td>
<td>[0.00, 142.70]</td>
<td>[0.00, 7848.50]</td>
<td>3184.00, 13164.40</td>
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<td>AA</td>
<td>[0.00, 18.30]</td>
<td>[0.00, 1335.90]</td>
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<td>LWS</td>
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<td>[3184.00, 3980.00]</td>
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<td>Chemical Industry</td>
<td>OPA</td>
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<td>[0.00, 488.40]</td>
<td>316.80, 1033.20</td>
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<td>AA</td>
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<td>LWS</td>
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</tr>
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<td></td>
<td>Petroleum Refinery</td>
<td>OPA</td>
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<td>[0.00, 999.90]</td>
<td>544.00, 1872.70</td>
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<td>AA</td>
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<td>LWS</td>
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</tr>
<tr>
<td></td>
<td>Steel Mining</td>
<td>OPA</td>
<td>[0.00, 7.55]</td>
<td>[0.00, 415.25]</td>
<td>96.00, 568.10</td>
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<td>AA</td>
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'School of Civil & Environmental Engineering 86
### APPENDIX I: CCP MODEL SOLUTIONS

**I1: Solutions for \( q_c = 0.01 \)**

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**Total Cost for Period 1**

| Total Cost for Period 1 | 19,001.40 |

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**Total Cost for Period 2**

| Total Cost for Period 2 | 31,075.29 |

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**Total Cost for Period 3**

| Total Cost for Period 3 | 46,342.65 |

**Grand Total**

| Grand Total             | 96,419.34 |
### 12: Solutions for $q_i = 0.1$

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**Total Cost for Period 2**: 28,948.62

**Total Cost for Period 3**: 44,188.32

**Grand Total**: 90,422.05
## I4: Solutions for $q_i = 0.5$

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**APPENDIX J: ICCLP MODEL SOLUTIONS**

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### J3: Solutions for qi = 0.25

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'School of Civil & Environmental Engineering 94
## J4: Solutions for $q_i = 0.5$

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<td>AA</td>
<td>[20.4, 42.71]</td>
<td>[862.92, 2,220.92]</td>
<td>[143.52, 713.25]</td>
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<td>[20.4, 42.71]</td>
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<td>OPA</td>
<td>[0.00, 8.16]</td>
<td>[0.00, 709.92]</td>
<td>[177.66, 1,076.00]</td>
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<td>AA</td>
<td>[4.2, 7.04]</td>
<td>[177.66, 1,076.00]</td>
<td>[177.66, 1,076.00]</td>
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<td>[18,687.31, 62,381.82]</td>
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Grand Total

*School of Civil & Environmental Engineering*
### J5: Solutions for \( q_i = 0.8 \)

<table>
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<tr>
<th>Period</th>
<th>Source i</th>
<th>Control Measure j</th>
<th>Allocation (ton/d)</th>
<th>Cost ($)</th>
<th>Total Cost ($)</th>
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