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ANALYSIS ON INITIATION OF SEDIMENT SUSPENSION FROM BED-LOAD

By Nian-Sheng Cheng[1] and Yee-Meng Chiew[2]

ABSTRACT: A theoretical analysis on the initiation of sediment suspension is performed based on the concept of probability of suspension. The proposed condition for the initiation of sediment suspension from the top of the bed-load layer is comparable with most existing empirical relationships for large particle Reynolds numbers. The study shows that a constant Rouse parameter can be used as the critical condition for sediment suspension only for large dimensionless particle diameters. When the probability of suspension is taken to be an infinitesimal value ($P = 10^{-7}$), the derived relationship of the Shields parameter and the particle Reynolds number is in good agreement with the updated Shields diagram.

INTRODUCTION

In conducting numerical computations of suspended sediment transport, it is necessary to identify the condition of the mass exchange process of sediment particles at the lower boundary of the suspended load. One of the problems involved is therefore to determine the flow condition at which initiation of sediment suspension occurs. In spite of its importance, only few relevant studies can be found in the literature, e.g. Bagnold (1966), Xie (1981), van Rijn (1984), Sumer (1986) and Celik and Rodi (1991). Unfortunately, these studies often provides inconsistent results in determining the condition of the initiation of suspension, and thus more work need to be done.

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This paper first presents a review of the published studies outlined above. A theoretical derivation is then conducted to obtain a probability function for the initiation of suspension from the top of the bed-load layer. Finally, comparisons are made between the present study and published relationships for the initiation of sediment suspension, as well as that for incipient sediment motion.

PREVIOUS STUDIES

Bagnold (1966) stated that no particles remains in suspension unless the upward velocity of turbulent eddies, v'_{up}, exceeds the settling velocity of the particles, w. He related the upward velocity of the turbulent eddies to the shear velocity in the form

$$v'_{up} = 1.25\, u_* \tag{1}$$

This leads to the critical condition for sediment suspension to be written as

$$\frac{w}{u_*} = 1.25 \tag{2}$$

It means that the particles are kept in suspension if $w/u_* < 1.25$.

Xie (1981) derived the condition for the initiation of sediment suspension from the well-known Rouse equation for suspended sediment distributions. The profile of the relative concentration of suspended sediment is dependent on the Rouse parameter $z = w/(\kappa u_*)$, where κ = von Karman constant (= 0.4 for clear water). With increasing values of the Rouse parameter, the profile becomes more non-uniform and the amount of sediment transported in the form of suspended load decreases. At $z = 5$, the amount of suspended load becomes very small, and Xie defined the initiation of suspension to occur at this condition

$$\frac{w}{\kappa u_*} = 5 \tag{3}$$

The condition for the initiation of sediment suspension identified by van Rijn (1984) is that the instantaneous upward motion of the sediment particle has a jump length of 100 particle diameters. His experimental results were represented by the following relationships:

$$\frac{u_*}{w} = \frac{4}{d_*} \qquad \text{for } 1 < d_* \le 10 \tag{4}$$

$$\frac{u_*}{w} = 0.4 \qquad \text{for } d_* > 10 \tag{5}$$

where $d_* = (\Delta g/\nu^2)^{1/3} d$ = dimensionless diameter of particles; $\Delta = (\rho_s - \rho)/\rho$; ρ_s = density of particles; ρ = density of fluid; g = gravitational acceleration; ν = kinematic viscosity of fluid and d = diameter of particles.

Sumer (1986) formulated the condition for the initiation of sediment suspension from the bed as

$$\tau_* = \frac{17}{Re_*} \tag{6}$$

and

$$\tau_* = 0.27 \tag{7}$$

where $\tau_* = u_*^2/(\Delta g d)$ = Shields parameter and $Re_* = u_* d/\nu$.

Celik and Rodi (1991) used the following empirical relationship to determine the critical shear stress for suspension from the bed:

$$\tau_* = \frac{0.15}{Re_*} \qquad \text{for } Re_* \le 0.6 \tag{8}$$

and

$$\tau_* = 0.25 \qquad \text{for } Re_* > 0.6 \tag{9}$$

Eqs. (8) and (9) are the same as Sumer's equations (6) and (7) except for the different constants.

Even though various formulas for the initiation of sediment suspension have been proposed by different researchers, they can be plotted in Fig. 1 in the form of τ_* against Re_*. This is because the parameters τ_* and Re_* can be expressed using the parameters w/u_* and d_*, which are included in (2) through (5), with the following considerations:

$$Re_* = \frac{wd/\nu}{w/u_*} \tag{10}$$

$$\tau_* = \frac{Re_*^2}{d_*^3} = \frac{(wd/\nu)^2}{(w/u_*)^2 d_*^3} \tag{11}$$

Here the dimensionless parameter wd/ν can be expressed using d_* for natural sediment particles (Cheng, 1997):

$$\frac{wd}{\nu} = (\sqrt{25 + 1.2 d_*^2} - 5)^{1.5} \tag{12}$$

With (12), (10) and (11) change, respectively, to the form

$$Re_* = \frac{(\sqrt{25 + 1.2d_*^2} - 5)^{1.5}}{w / u_*} \tag{13}$$

$$\tau_* = \frac{(\sqrt{25 + 1.2d_*^2} - 5)^3}{(w / u_*)^2 d_*^3} \tag{14}$$

Eqs. (13) and (14) are the equations that can be used to relate the dimensionless parameters, Re_*, τ_*, d_* and z (= w/ku_*), showing that any two of these parameters can be expressed by the others. Using (13) and (14), (2) to (5) can thus be plotted as τ_* against Re_* as shown in Fig. 1. When examining the differences among the various formulas mentioned above, however, it should be noted that Sumer (1986) and Celik and Rodi (1991) considered the sediment particles to be suspended from the bed or bed material rather than from the top of the bed-load layer. For comparison purposes, the well-known Shields curve for incipient sediment motion is also superimposed in the figure. Fig. 1 shows that for large Re_*, all the formulas except for Bagnold's equation (2) predict nearly consistent values of the dimensionless shear stress at the initiation of sediment suspension. For example, the predicted dimensionless shear stress varies from 0.19 to 0.33 when $Re_* > 1000$. However, for small Re_*, these formulas yield very different critical conditions for sediment suspension. Moreover, the dimensionless shear stresses computed using (2) to (4) are less than that for the initial motion of sediment particles predicted using the Shields curve. This implies that suspension would occur before the threshold condition of sediment movement, which may not be in agreement with facts.

PROBABILITY ANALYSIS

The zone for suspended sediment particles varies from the water surface to the top of the bed-load layer. It is assumed in this paper that the suspended particles are exchangeable with the bed-load, and the wash load is not considered. That is to say, no particles other than those on or near the bed can be a source for the suspended load. Therefore, termination of suspension of sediment particles can occur at any depth within the suspension zone, while initiation of suspension can occur only at the lower boundary of the suspension zone, i.e., the top of the bed-load layer.

If a sediment particle is assumed to be suspended by a turbulent flow when the vertical velocity fluctuation v' of the flow exceeds the settling velocity w of the particle, the condition for initial suspension of sediment particles can be defined as

$$v' > w \tag{15}$$

It is also noted that the condition of $v' < w$ can be used for termination of suspension of sediment particles at any elevations through the suspension zone. Using the condition defined by (15), the probability of suspension P can be expressed as

$$P = P(v' > w) \tag{16}$$

With reference to published experimental investigations, such as Klebanoff (1953) and Nezu (1977), the probability of vertical turbulence fluctuations near the bed surface can be considered, as a first approximation, to follow a gaussian distribution. With this, (16) can be written as

$$P = \frac{1}{\sigma\sqrt{2\pi}} \int_w^\infty exp(-\frac{v'^2}{2\sigma^2}) dv' \tag{17}$$

where $\sigma = \sqrt{\overline{v'^2}}$. Using the approximate solution of the error function (see Cheng and Chiew, 1998), (17) can be approximated to

$$P = 0.5 - 0.5\sqrt{1 - exp(-\frac{2}{\pi}\frac{w^2}{\sigma^2})} \tag{18}$$

where the settling velocity w can be computed using (12) for natural sediment particles and the rms value of the vertical velocity fluctuation σ can be evaluated depending on the type of bed regime.

Types of Bed Regimes

Rough bed

Experimental studies conducted by Grass (1971), Nezu (1977) and Kironoto and Graf (1995) have shown that the rms value of the vertical velocity fluctuation near a rough bed is almost equal to the shear velocity. Taking $\sigma = u_*$, as a first approximation, (18) changes to

$$P = 0.5 - 0.5\sqrt{1 - exp(-\frac{2}{\pi}\frac{w^2}{u_*^2})} \tag{19}$$

Fig. 2 is a plot of P against w/u_* according to (19), showing that P decreases rapidly with increasing w/u_*. For example, $P \approx 16\%$ when $w/u_* = 1$; $P \approx 2\%$ when $w/u_* = 2$; and $P \approx 0.08\%$ when $w/u_* = 3$.

Smooth bed

With a smooth bed, the rms value of the vertical velocity fluctuation near the bed depends on the shear velocity as well as the distance from the bed. Fig. 3 shows the experimental data collected by Grass (1971) for such a condition. It shows that the rms value of the vertical velocity fluctuation is almost equal to the shear velocity when the distance from the bed is large, and decreases when the distance is reduced. Fig. 3 also shows that the data can generally be fitted to the following equation:

$$\frac{\sigma}{u_*} = 1 - exp[-0.025(\frac{u_* y}{\nu})^{1.3}] \qquad (20)$$

where y = distance from the bed. The use of the exponent function in (20) is to make the relative turbulent intensity σ/u_* almost equal to zero for distances very close to the bed, where the viscosity of the fluid has a dominant effect on both the flows and sediment transport.

Fig. 4 shows a definition sketch of the lower boundary of a suspended sediment particle, which is taken to be at $y = 2.75d$. This approximation is based on the assumption that the reference level is half a diameter above the bed load layer. The thickness of the bed load layer is assumed to be two times the grain diameter and the theoretical bed level is at a distance $0.25d$ below the top of the bed particles. While there are researchers who had chosen a reference level that is much higher than the present definition, e.g., van Rijn (1984) and Garcia and Parker (1991), there are others who insisted that the reference level should just be a few grain diameters above the bed. In advocating such a choice, Zyserman and Fredsoe (1994) argued that at a distance a few grain diameters away from the bed, "the sediment particles are kept in suspension by the turbulence of the fluid rather than by grain-to-grain collisions, and should therefore be regarded as sediment suspension".

Using (18) with (20) for $y = 2.75d$, it can be shown that P depends on w/u_* and Re_* ($= u_* d/\nu$) for a smooth bed. The relationship computed using (18) and (20) is plotted in Fig. 5. It shows that P decreases more rapidly with increasing w/u_* for lower Re_*; and the relationship of P against w/u_* remains unchanged when Re_* is greater than approximately 20. The latter case is the same as that for a rough bed, as shown in Fig. 2.

Fig. 6 is an alternative presentation of Fig. 5 in the form of τ_* against Re_* with P as a third parameter. The related conversions among parameters w/u_*, τ_*, and Re_* are

made using (13) and (14). Also superimposed in Fig. 6 are the curves computed using (19) and the Shields diagram. Fig. 6 shows that the curves computed using (19) coincide with those computed using (18) and (20) for Re_* greater than approximately 20. By increasing the probability of suspension, the predicted dimensionless shear stress for the initiation of sediment suspension is increasingly greater than that for the initiation of sediment motion for a given particle Reynolds number. When the probability of suspension is very small, the computed curve of τ_* against Re_* approaches asymptotically towards the Shields diagram. This feature will further be discussed in a latter section.

As stated before, the definition of the elevation for the lower boundary of the suspension zone can be different based on different assumptions. To evaluate the effects of the lower boundary on the computed curves, different elevations for the boundary are chosen to conduct a sensitivity analysis. The result is plotted in Fig. 7, showing that the shape of the curve does not change for the different boundaries, but for low Reynolds numbers, the curve shifts up slightly when the boundary is lowered, and vice versa. It means that if the lower limit of the suspension zone is less than 2.75d, the critical shear stress would have to increase to achieve the same probability of suspension.

COMPARISONS

The comparison between Fig. 6 and Fig. 1 shows that the various probabilities have in effect been implied in the previous studies for the initiation of sediment suspension. For example, Bagnold's equation (2) is equivalent to a probability of 10%; Xie's equation (3) a probability of 2%; Sumer's equation (7) and Celik and Rodi's equation (9) a probability of 1%; and van Rijn's equation (5) a probability of 0.5%. In summary, all the equations are equivalent to a probability of about 1% except for Bagnold's prediction. In addition, most researchers have taken τ_* to be 0.2 ~ 0.3 for large Re_* for the initiation of suspension (see Fig. 1). Comparatively, the present study provides $\tau_* \approx 0.18 \sim 0.25$ for $Re_* = 100 \sim 10000$ when $P = 1\%$, as shown in Fig. 6. This indicates that the probability of 1% can be considered to be a reasonable index for the initiation of sediment suspension from the top of the bed-load layer.

Rouse Parameter as Criterion

As the Rouse parameter $z = w/(\kappa u_*)$ reflects the relative effect of gravity and turbulence on a sediment particle, it is often used to check the status of suspended load. The smaller the *z*-value, the more are the sediment particles in suspension. However, this argument cannot hold true for all cases.

Fig. 8 shows the effect of the dimensionless particle diameter, d_* on Rouse parameter, *z* based on the previous studies discussed earlier. The related conversions are made using (13) and (14). The computed curve using the present equations (18) and (20) for P = 1% is also plotted in the figure for comparison. Fig. 8 shows that when d_* is greater that approximately 50, each formula provides nearly the same constant Rouse parameter *z*, except for Bagnold's prediction. This constant z-value can be considered to be an index for the initiation of sediment suspension. The values, which are predicted using the formulas proposed, respectively, by Xie (1981), Sumer (1986), Celik and Rodi (1991) and the present study (P = 1%), vary from 5.0 to 5.6 when $d_* > 50$. Therefore, one can assume sediment particles to remain in suspension when $z < 5.0 \sim 5.6$. However, this is not true for sediments with smaller d_*. As shown in Fig. 8, all the formulas except for Bagnold (1966) and Xie (1981) suggest that the Rouse parameter is much less than 5.0 ~ 5.6 at the initiation for sediment suspension. For example, when $d_* = 1$, sediment particles cannot be suspended even for a small Rouse parameter, say, $z = 2$.

Based on the above considerations, it appears that the equations proposed by van Rijn (1984), Sumer (1986) and Celik and Rodi (1991) can be acceptable in most cases for the prediction of the initiation of sediment suspension; while the predictive method formulated by Bagnold (1966) and Xie (1981) are only applicable to the case for large d_*-values.

Furthermore, using (14), Fig. 9 shows the relationship between *z* and d_* for P = 1% is replotted as *z* against τ_*. This figure clearly shows that given a z-value, the sediment particles in a flow can either be bed-load or suspended load depending on how high the Shields parameter is. For example, the particles can never be suspended for a small z-value, say z = 1.0, if the Shields parameter is not greater than 0.16.

Extension to Initial Sediment Motion

With reference to Fig. 6, the computed relationship of τ_* and Re_* using (18) and (20) approaches the Shields curve with reducing probabilities, P. In Fig. 10, the original

Shields diagram (Vanoni 1975) is plotted together with the updated curves proposed, respectively, by Yalin and Karahan (1979) and Chien and Wan (1983). Based on the available experimental data in the literature, Yalin and Karahan (1979) defined an averaged curve in place of the Shields curve while Chien and Wan (1983) furnished a band to modify the original Shields diagram. Fig. 10 shows that the computed relationship of τ_* and Re_* is in good agreement with the updated Shields curves when the probability of suspension is taken to be 10^{-7}. This indicates that the probability of suspension may reflect not only the ability of sediment particles to be suspended but also the intensity of the effect of the near-bed flow on sediment transport. When P is large, sediment particles are transported in the form of both bed load and suspended load. When P is small, the particles move only within the near-bed layer and the amount of suspended load is approximately zero. When P is further reduced, the amount of bed load decreases and even some particles in motion revert to the bed material.

Figs. 11 shows the theoretical divisions among suspended load, bed load and bed material according to computations using (18) and (20), where the probabilities are taken to be 0.01 and 10^{-7}, respectively, for the initiation of sediment suspension and sediment motion. It is obvious that the divisions shown in Fig. 11 will change when different probabilities from those used above are chosen. In fact, there is no absolute division between bed load and suspended load. Due to the unsatisfactory understanding of sediment movement near the bed, however, the division of total sediment load into bed load and suspended load is still a common method in dealing with sediment transport. Fig. 11 can be expected to be used for such a purpose.

CONCLUSIONS

This paper presents a probability analysis on the initiation of sediment suspension for both smooth and rough bed conditions. Different critical conditions for suspension predicted by previous studies result mainly from different probabilities implied in these studies.

A constant Rouse parameter can be used as an indicator for the initiation of sediment suspension only for large dimensionless particle diameters. Otherwise, the critical value of the Rouse parameter varies depending on the dimensionless particle

diameters. It is possible for the sediment particles never to be suspended from the bed load, even for a small Rouse parameter, if the Shields parameter is not large enough.

When the probability of suspension approaches an infinitesimal value of 10^{-7}, the proposed relationship of the Shields parameter and the particle Reynolds number approximates very well to the updated Shields diagram for the incipient sediment motion. Finally, the study proposes a criterion for the divisions among suspended load, bed load and bed material.

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APPENDIX II. NOTATIONS

The following symbols are used in this paper:

C_1 = constant;

C_2 = constant;

C_3 = constant;

C_4 = constant;

d = grain diameter;

d_* = dimensionless particle diameter = $(\Delta g/\nu^2)^{1/3} d$;

g = gravitational acceleration;

k_s = mean height of roughness;

P = probability of suspension;

Re_* = $u_* d/\nu$;

u_* = shear velocity;

v' = velocity fluctuation;

v'_{up} = upward velocity of eddies;

w = settling velocity of sediment;

y = vertical distance from lower boundary;

z = Rouse parameter = $w/(\kappa u_*)$;

Δ = $(\rho_s - \rho)/\rho$;

ρ = density of fluid;

ρ_s = density of sediment;

σ = $\sqrt{\overline{v'^2}}$;

ν = viscosity of fluid; and

τ_* = Shields parameter = $u_*^2/(\Delta g d)$.

CAPTION FOR FIGURES

Fig. 1. Previous relationships for initiation of sediment suspension

Fig. 2. Effect of w/u_* on probability of suspension P for a rough bed

Fig. 3. rms values of vertical velocity fluctuation over a smooth bed

Fig. 4. Schematic diagram of lower boundary of a suspended particle

Fig. 5. Effect of w/u_* on probability of suspension P for different Re_*-values over a smooth bed

Fig. 6. Computed relationship of τ_* and Re_* for different probabilities

Fig. 7. Sensitivity of relationship of τ_* and Re_* to lower boundary of suspension zone

Fig. 8. Effect of dimensionless particle diameter, d_* on Rouse parameter, z

Fig. 9. Suspension criterion expressed using Rouse parameter, z and Shields parameter, τ_* for P = 1%

Fig. 10. Approximation to the Shields diagram

Fig. 11. Division among suspended load, bed load and bed material

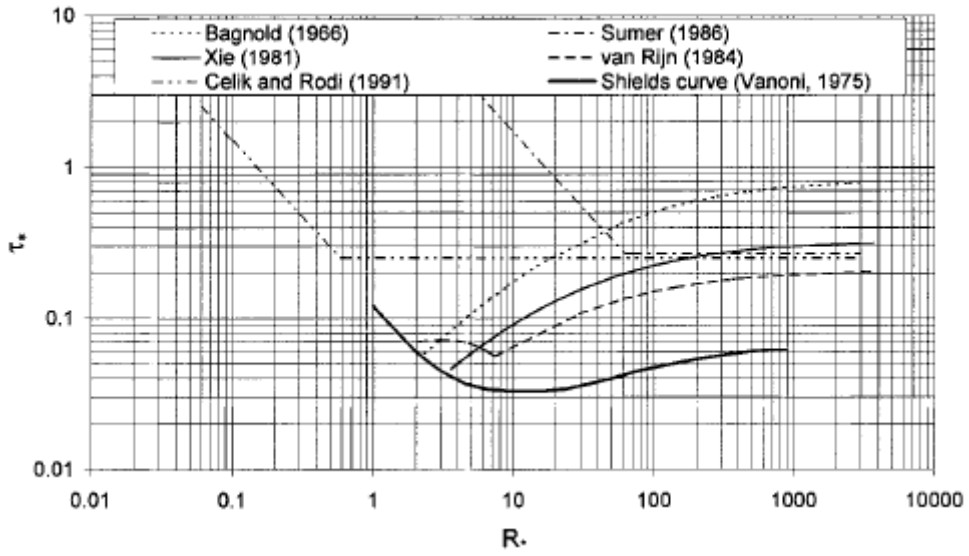
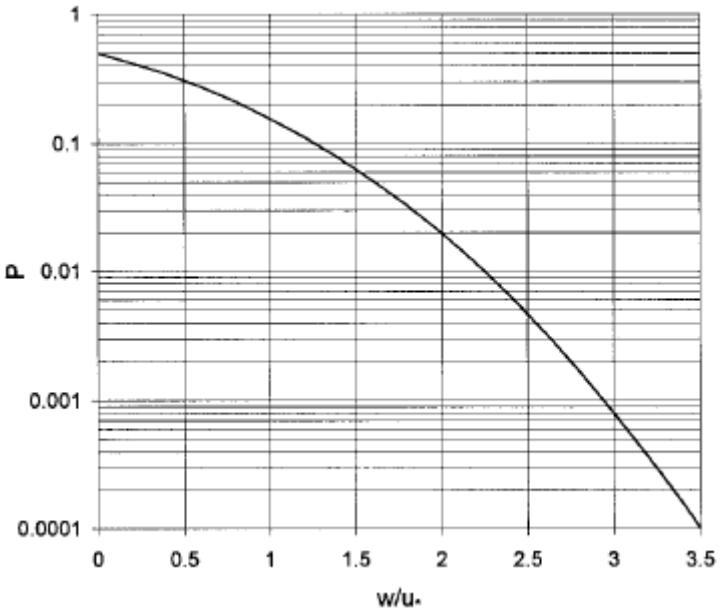

Bagnold (1966)
Xie (1981)
Celik and Rodi (1991)
Sumer (1986)
van Rijn (1984)
Shields curve (Vanoni, 1975)
10
1
0.1
0.01
τ*
0.01
0.1
1
10
100
1000
10000
R*

Fig. 1



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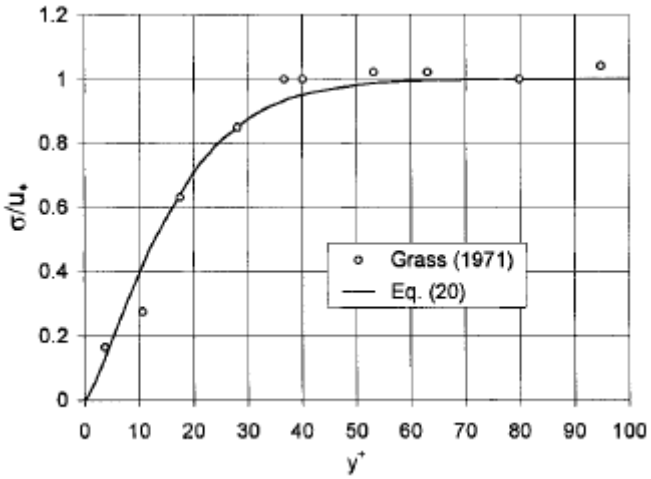
1.2
1
0.8
0.6
0.4
0.2
0
σ/u*
0
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20
30
40
50
60
70
80
90
100
y+
Grass (1971)
Eq. (20)

Fig. 3

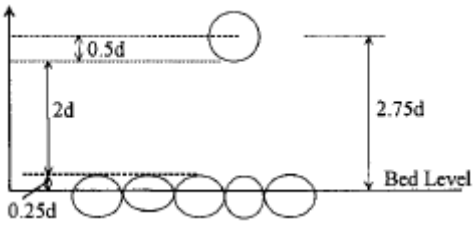

Fig. 4

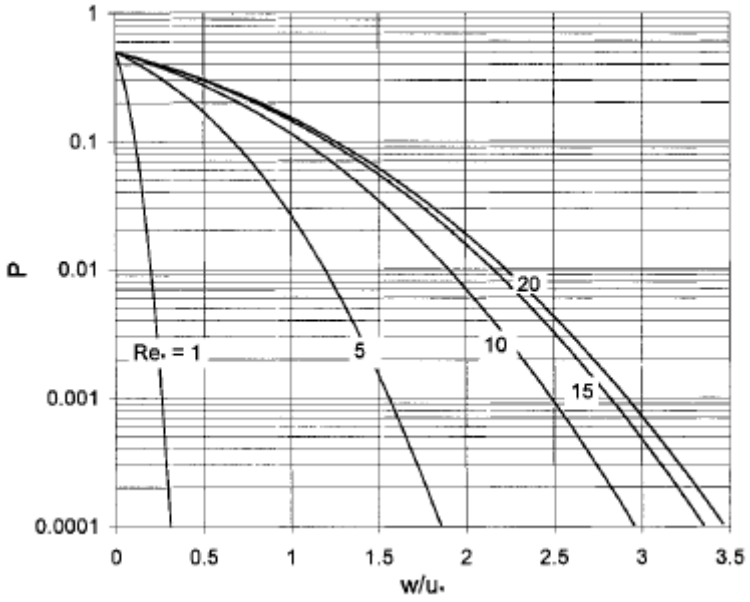

Fig. 5

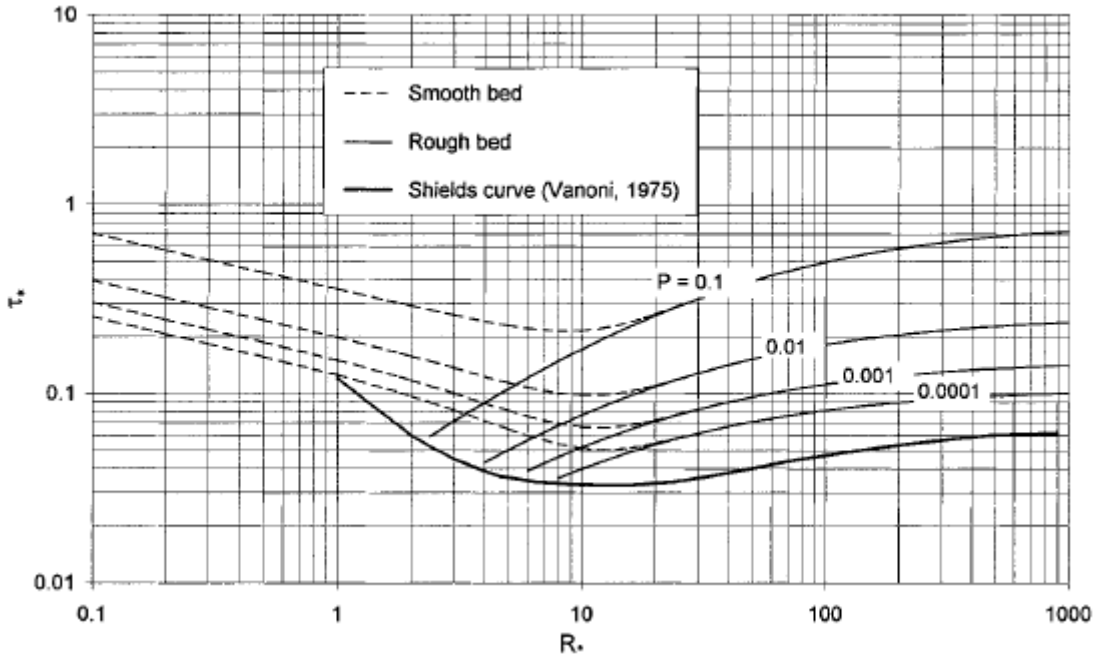
Smooth bed
Rough bed
Shields curve (Vanoni, 1975)
P = 0.1
0.01
0.001
0.0001
τ*
R*
10
1
0.1
0.01
0.1
1
10
100
1000

Fig. 6

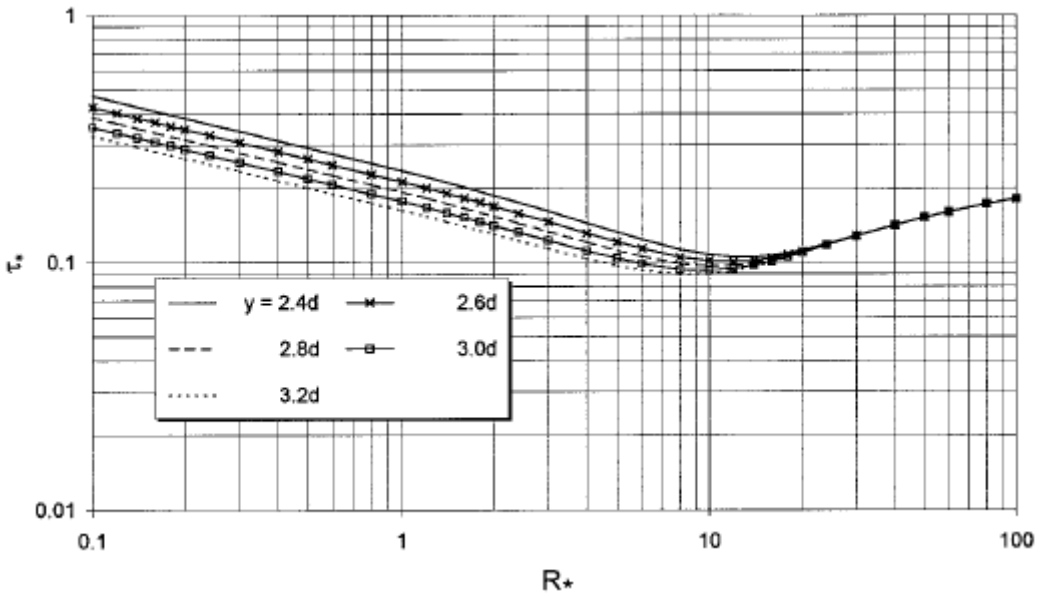

Fig. 7

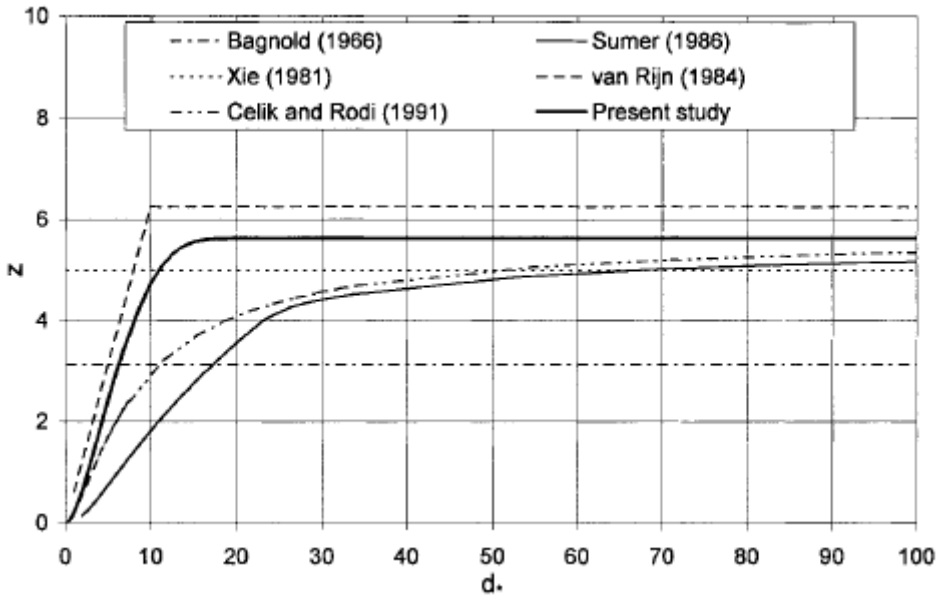
Bagnold (1966)
Sumer (1986)
Xie (1981)
van Rijn (1984)
Celik and Rodi (1991)
Present study
z
d*
0
2
4
6
8
10
0
10
20
30
40
50
60
70
80
90
100

Fig. 8

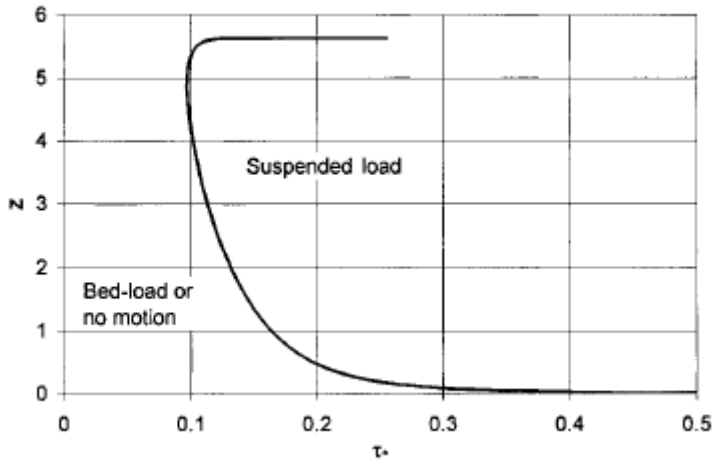
6
5
4
3
2
1
0
z
Suspended load
Bed-load or
no motion
0
0.1
0.2
0.3
0.4
0.5
τ*

Fig. 9

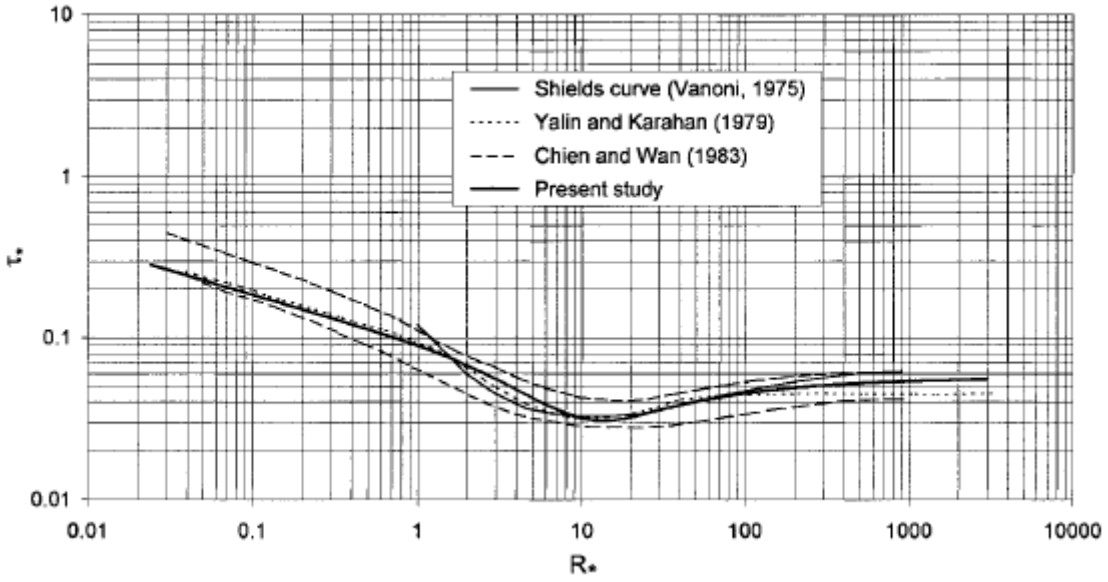
Shields curve (Vanoni, 1975)
Yalin and Karahan (1979)
Chien and Wan (1983)
Present study
10
1
0.1
0.01
τ*
0.01
0.1
1
10
100
1000
10000
R*

Fig. 10

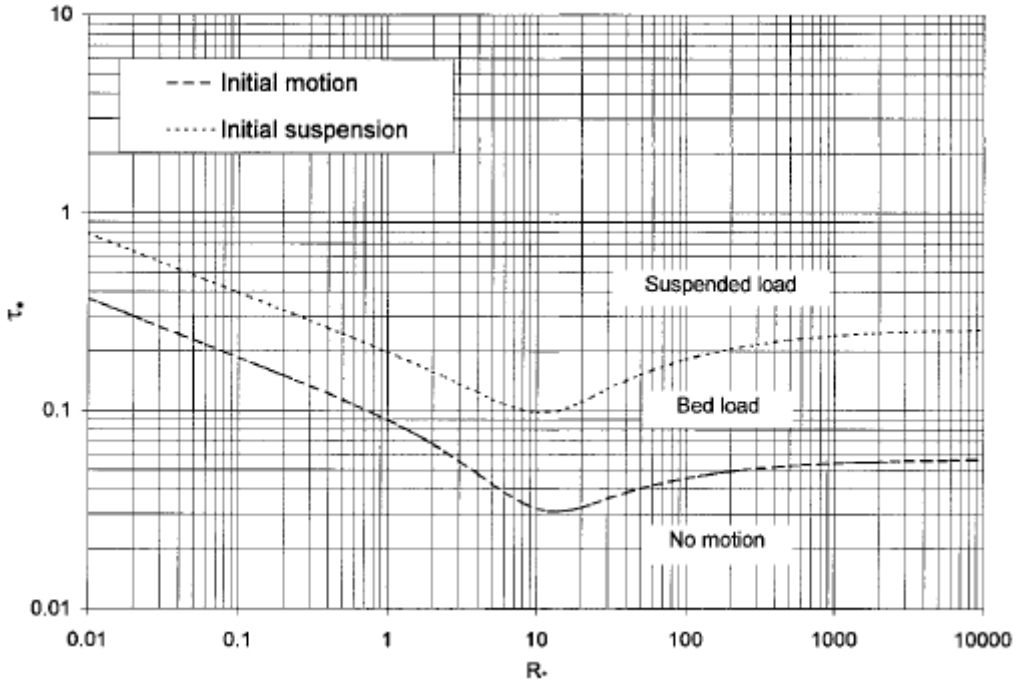
Initial motion
Initial suspension
Suspended load
Bed load
No motion
10
1
0.1
0.01
τ*
0.01
0.1
1
10
100
1000
10000
R*

Fig. 11