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## Influence of shear stress fluctuation on bed particle mobility

Nian-Sheng Cheng<sup>a)</sup>

School of Civil and Environmental Engineering, Nanyang Technological University,  
Nanyang Avenue, 639798 Singapore

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Whether or not a sediment particle is entrained from a channel bed is associated with both average bed shear stress and shear stress fluctuation, the latter being flow-dependent and also related to bed irregularities. In the first part of this study, a preliminary analysis of possible fluctuations induced by bed roughness is presented for the case of an immobile plane bed comprised of unisized sediments. The result shows that the roughness-induced variation is generally comparable to that associated with near-bed turbulence, and both variations can be approximated as log-normal in terms of probability density distribution. The bed particle mobility is then analyzed by considering the effects of shear stress fluctuations. The relevant computations demonstrate that with increasing shear stress fluctuations, the probability of the mobility of a bed particle may be enhanced or weakened. For the case of low sediment entrainment, the probability is increased by turbulence. However, the probability may be reduced by the shear stress fluctuation if the average bed shear stress becomes relatively higher. This study shows that the critical condition for initial sediment motion could be overestimated if the Shields diagram is applied for the condition of flows with high turbulence intensities. © 2006 American Institute of Physics. [DOI: 10.1063/1.2354434]

### INTRODUCTION

For a bed particle subject to turbulent flow, its mobility is random, varying with time and also location. This is due to the fact that driving forces such as shear stress acting at sediment bed fluctuate temporally and spatially even with constant time-mean magnitudes. Such random variations inherent in the motion of bed particles imply that sediment transport rate is also closely associated with characteristics of turbulent flows. However, relevant mechanisms are not well understood. For example, widely cited bedload functions such as those developed by Einstein,<sup>1</sup> Bagnold,<sup>2</sup> and Yalin<sup>3</sup> are all expressed by relating sediment transport rate to time-mean flow parameters. In these pioneering works that address sediment transport problems, either stochastically or deterministically, the information of turbulence was taken into account only at the first-order level, i.e., in the sense of time-mean velocity or shear stress. This limitation was substantiated by Grass and Ayoub<sup>4</sup> with their sediment transport data collected for laminar and turbulent flow conditions, which were designed at similar mean shear levels. Grass and Ayoub also presented an excellent conceptual explanation of their observation with a probability-based model that was proposed for delineating turbulence-induced variations in the instability of sediment bed.

A recent extension of Grass and Ayoub's effort has been done by Sumer *et al.*,<sup>5</sup> showing that a significant increase in the bedload transport rate could be induced by extra turbulent excitation exerting on a sediment bed. Their experiments were conducted for both plane bed and ripple-covered bed conditions, with turbulence being characterized by shear stress and velocity fluctuations, respectively. The plane bed

was prepared with fine sediment and thus served as a smooth flow boundary prior to initiation of ripples. For this plane-bed condition, it can be further shown that with reference to sediment transport in laminar flow, the turbulence-induced increment in the transport rate can be evaluated reasonably using a probabilistic model based on the log-normal function.<sup>6</sup>

Stochastic considerations generally facilitate realization of the fact that a turbulent flow quantity or sediment-related parameter with the same time-mean value may fluctuate with different amplitudes. Several such examples are available in the literature showing applications of stochastic theories to sediment transport. The earliest contribution is due to Einstein<sup>1</sup> in formulating bedload transport rate. Recent efforts have been provided in different aspects. For example, Cheng and Chiew<sup>7,8</sup> showed that both pickup probability and initial suspension can be reasonably described using normally distributed near-bed velocities. Parker *et al.*<sup>9</sup> reported that implementation of a probabilistic concept in reformulating the Exner equation of sediment continuity would offer a better understanding of how streams create a vertical structure of sediment bed. Kleinhans and van Rijn's work<sup>10</sup> demonstrated that a probability-based approach can significantly improve prediction of bedload transport near the incipient motion, which otherwise may encounter failure when employing classical deterministic predictors such as the Meyer-Peter and Mueller formula. It is also noted that various probability density functions (pdf) have been assumed in these research works, and most of them were used without theoretical interpretation or experimental verification. The Gaussian or normal function appears popular compared to others; it may apply to near-bed flows dominated largely by a combination of turbulent events with equivalent sweeps and ejections.<sup>7,8,10</sup> On the other hand, some experimental obser-

<sup>a)</sup>Electronic mail: cnscheng@ntu.edu.sg

vations also uncover skewed probability density distributions of some turbulent quantities. The skewed distribution is believed to be closely related to turbulent coherent structures or generated by nonequivalent contributions of sweeps and ejections. However, theoretical connections in this regard are lacking at present, which is why various approximations have been made in literature. For example, in investigating incipient motion for three representative bed packing conditions, Papanicolaou *et al.*<sup>11</sup> reported that the interrelationship of flow and sediment can be accounted for by the Chi-square distribution. Other examples are the log-normal function<sup>12-14</sup> and more complicated functions including the polynomial approximation<sup>15</sup> and the fourth-order Gram-Charlier function.<sup>16</sup>

A fluctuation in the bed shear stress is of course closely related to the condition of turbulent flow, but also being subject to the detail of bed particle configuration. The latter is particularly important in the presence of a sediment bed that is hydrodynamically rough. This study aims to perform an analysis of roughness-induced shear stress fluctuation for the case of an immobile plane bed comprised of unisized sediment particles. Then, a pdf is proposed to take into account random variations caused by both turbulent flow and bed roughness. With the derived pdf, the mobility of bed particles subject to turbulence is finally discussed.

## ROUGHNESS-INDUCED SHEAR STRESS FLUCTUATIONS

Generally, the near-bed flow condition is not only related to turbulence and its coherent structure but also subject to sediment-related irregularities. Both factors cause random fluctuations in the bed shear stress. The component associated with the bed particles may be defined as roughness-induced fluctuation  $\tau'_1$ , while that associated with turbulence may be referred to as turbulence-induced fluctuation  $\tau'_2$ . As a result, the bed shear stress fluctuation is generally given by

$$\tau' = \tau - \tau_m = \tau'_1 + \tau'_2, \quad (1)$$

where  $\tau_m$  is the mean bed shear stress being averaged both in time and space domains, or the double-average bed shear stress.<sup>17,18</sup> In comparison with  $\tau'_2$ , the fluctuation  $\tau'_1$  is negligible for a smooth bed comprised of fine particles but becomes dominantly important for laminar flow over a rough bed.

Here, it is assumed that open channel flows occur over an immobile flat bed comprised of unisized sediment particles. We first consider that the variation in the bed shear stress is only associated with the bed particle configuration. For this limited condition, turbulent effects can be excluded and thus the variation can be indirectly evaluated under the condition of laminar flow. In comparison with the temporal turbulence fluctuation normally considered at a certain point, the roughness-induced variation is considered here only in the spatial domain, i.e., a two-dimensional plane parallel to the average bed surface.

As sketched in Fig. 1, the presence of sediment particles can have considerable effects on the flow, but such effects are limited to the area near the bed. The affected flow layer

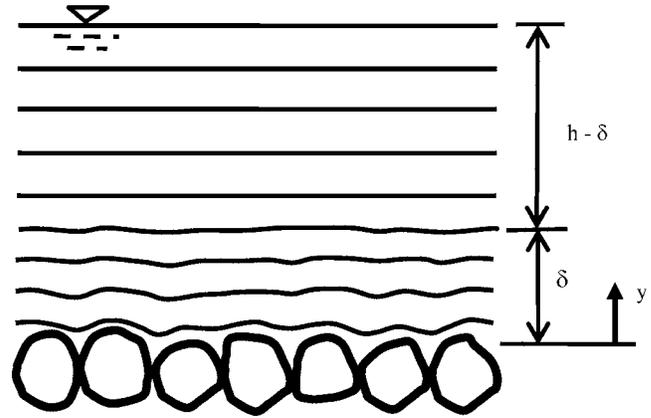


FIG. 1. Definition of roughness sublayer.

can be called the roughness sublayer,<sup>17,18</sup> with its thickness denoted by  $\delta$ . It can generally be expected that for a plane bed of unisized sediment,  $\delta$  is proportional to the average roughness size, the latter being taken to be the particle diameter  $D$ . Based on the results reported by Raupach *et al.*,<sup>17</sup>  $\delta = (2-5)D$  or on average  $\delta \approx 3.5D$ .

For the case of laminar flow, the velocity distribution is given by

$$u = \frac{u_*^2 h}{\nu} \left( \frac{y}{h} - \frac{y^2}{2h^2} \right), \quad (2)$$

where  $u_*$  is shear velocity,  $\nu$  is kinematic viscosity,  $h$  is flow depth, and  $y$  is measured from the average bed surface. The average velocity in the roughness sublayer is

$$U_\delta = \frac{1}{\delta} \int_0^\delta u dy = \frac{u_*^2 h}{\nu} \left( \frac{\delta}{2h} - \frac{\delta^2}{6h^2} \right). \quad (3)$$

If  $\delta$  is much smaller than  $h$ , Eq. (3) can be reduced to

$$U_\delta = \frac{u_*^2 \delta}{2\nu} = \frac{\tau_1 \delta}{2\rho\nu}, \quad (4)$$

where  $\tau_1 = \rho u_*^2$  is the local bed shear stress and  $\rho$  is the density of fluid. It is further assumed that the flow discharge per unit width within the roughness sublayer remains unchanged,

$$q = U_\delta \delta = \frac{\tau_1 \delta^2}{2\rho\nu} = \text{const.} \quad (5)$$

This discharge can also be characterized in terms of average parameters,

$$q = \frac{\tau_{1m} \delta_m^2}{2\rho\nu}, \quad (6)$$

where  $\tau_{1m}$  is the average bed shear stress and  $\delta_m$  is the average thickness.

With Eqs. (5) and (6), the relative bed shear stress can be associated with the relative thickness of the roughness sublayer,

$$\frac{\tau_1}{\tau_{1m}} = \left( \frac{\delta_m}{\delta} \right)^2. \quad (7)$$

For a flat sediment bed, the fluctuation in the bed surface elevation and thus  $\delta$  is random. With uniform sediment, it is reasonable to employ the Gaussian function for describing the resulting  $\delta$  distribution. Therefore, the pdf of the variable  $\delta$  is given by

$$f(\delta) = \frac{1}{\sqrt{2\pi}\sigma_\delta} \exp\left[-\frac{(\delta - \delta_m)^2}{2\sigma_\delta^2}\right], \quad (8)$$

where  $\sigma_\delta$  is the standard deviation that can be taken as the rms value of  $\delta$  (i.e.,  $\delta_{\text{rms}}$ ). Using probability transformation, the pdf of  $\tau$  can be derived from Eq. (8) based on the relation given by Eq. (7), yielding

$$f(\tau_1) = \frac{1}{\sqrt{8\pi}I_\delta} \sqrt{\frac{\tau_{1m}}{\tau_1^3}} \left[ \exp\left(-\frac{(1 - \sqrt{\tau_{1m}/\tau_1})^2}{2I_\delta^2}\right) + \exp\left(-\frac{(1 + \sqrt{\tau_{1m}/\tau_1})^2}{2I_\delta^2}\right) \right], \quad (9)$$

where  $I_\delta = \delta_{\text{rms}}/\delta_m$  is defined as the intensity of the variation in the bed surface elevation. A further probability transformation from Eq. (9) results in the following pdf, which applies for the normalized shear stress defined as  $\tau_1^+ = \tau_1/\tau_{1m}$ :

$$f(\tau_1^+) = \frac{1}{I_\delta \sqrt{8\pi}(\tau_1^+)^3} \left[ \exp\left(-\frac{(1 - \sqrt{\tau_1^+})^2}{2I_\delta^2 \tau_1^{+2}}\right) + \exp\left(-\frac{(1 + \sqrt{\tau_1^+})^2}{2I_\delta^2 \tau_1^{+2}}\right) \right]. \quad (10)$$

Equation (10) shows that the pdf of  $\tau_1^+$  varies with the intensity of  $I_\delta$  if the randomness of the bed shear stress is induced solely by the bed irregularity.

## COMPARISON WITH FLOW-INDUCED SHEAR STRESS FLUCTUATIONS

The above-derived probability density distribution may differ from that associated with near-bed turbulence. The latter has been attempted recently for unidirectional turbulent flows over a smooth boundary.<sup>13,14</sup> The relevant results demonstrated that the log-normal function can effectively represent the probability density distribution of the turbulence-induced bed shear stress (denoted by  $\tau_2$ ), which were measured for various flow conditions. The log-normal function used is expressed as

$$f(\tau_2) = \frac{1}{\sqrt{2\pi}\beta\tau_2} \exp\left[-\frac{[\beta + 2 \ln(\tau_2/\tau_{2m})]^2}{8\beta}\right] \quad (11)$$

or

$$f(\tau_2^+) = \frac{1}{\sqrt{2\pi}\beta\tau_2^+} \exp\left[-\frac{(\beta + 2 \ln \tau_2^+)^2}{8\beta}\right], \quad (12)$$

where  $\tau_2^+ = \tau_2/\tau_{2m}$  and  $\beta = \ln(1 + I_2^2)$  with  $I_2 = \tau_{2\text{rms}}/\tau_{2m}$  the bed shear stress intensity,  $\tau_{2m}$  the average bed shear stress, and  $\tau_{2\text{rms}}$  the rms value of  $\tau_2$ . [It should be mentioned that  $\sqrt{2\pi}\beta$

included in Eqs. (11) and (12) was printed by mistake as  $\sqrt{2\pi}(1 + I^2)$  in the previous works.<sup>6,13]</sup>

Equation (12) shows that for the case of a smooth bed, the probability function used for describing the turbulence-induced shear stress variation varies only with the near-bed turbulence intensity defined by  $I_2$ . Cheng and Law<sup>14</sup> also indicated that the value of  $I_2$  increases with reducing Reynolds number, and at higher  $I_2$  value, the stress distribution appears to be more skewed. In the extreme, a peaky distribution would occur at very low Reynolds number, which agrees with experimental observations.<sup>19</sup> The log-normal function is also characterized by its long ‘‘tail’’ for positive stresses, which serves to represent the intermittent nature of Reynolds stress contributions during bursting events.<sup>19</sup>

To understand how Eq. (10) differs from Eq. (12), one of the simple ways is to find first the relation of  $I_\delta$  and  $I$  for the case considered in this study. Using Eq. (7), the shear stress fluctuation  $\tau_1'$  is

$$\tau_1' = \tau_1 - \tau_{1m} = \left[ \left( \frac{\delta_m}{\delta_m + \delta'} \right)^2 - 1 \right] \tau_{1m}. \quad (13)$$

Equation (13) indicates that an increase in the thickness of the roughness sublayer results in a decrease in the local bed shear stress. This equation can be simplified to  $\tau_1'/\tau_{1m} \approx -2\delta'/\delta_m$  by applying a power series expansion for the case of  $\delta'$  much smaller than  $\delta_m$ . Furthermore, by taking the root mean square, one gets that  $I_1 \approx 2I_\delta$  with  $I_1 = \tau_{1\text{rms}}/\tau_{1m}$ . Using this approximation, Eq. (10) can be compared to Eq. (12) by plotting them as pdf( $\tau_s$ ) against  $\tau_s$  for various  $I$  values, where  $\tau_s = (\tau - \tau_m)/\tau_{\text{rms}}$  is defined as the standardized bed shear stress. Note that the probability transformation used for plotting the graphs is based on the relation of pdf( $\tau_s$ ) =  $I f(\tau^+)$ . As shown in Fig. 2, the results suggest that both pdfs are very close, particularly for small shear stress intensities. In other words, the log-normal function can also be used as a good alternative to Eq. (9) or (10) for describing the probability density distribution associated with roughness-induced shear stress fluctuations. The advantage of the use of this similarity will be demonstrated in the subsequent derivation for evaluating bed particle mobility.

As given in Eq. (1), the bed shear stress fluctuation for turbulent flows over a sediment bed can be generally considered as a sum of two components,  $\tau_1'$  and  $\tau_2'$ . Since both components can be approximated as log-normal variables, the log-normal pdf can be further used for describing the variable  $\tau$ . This is because a sum of log-normal random variables is also log-normally distributed, which appears as a reasonable assumption with many successful applications.<sup>20</sup> However, the extended use of the log-normal function is based on the approximation that the two components are independent. The limitation of this approximation will be discussed later in this paper. From Eq. (1), it can be derived that

$$\tau_{\text{rms}}^2 = \tau_{1\text{rms}}^2 + \tau_{2\text{rms}}^2. \quad (14)$$

Dividing by  $\tau_m^2$ , Eq. (14) is rewritten as

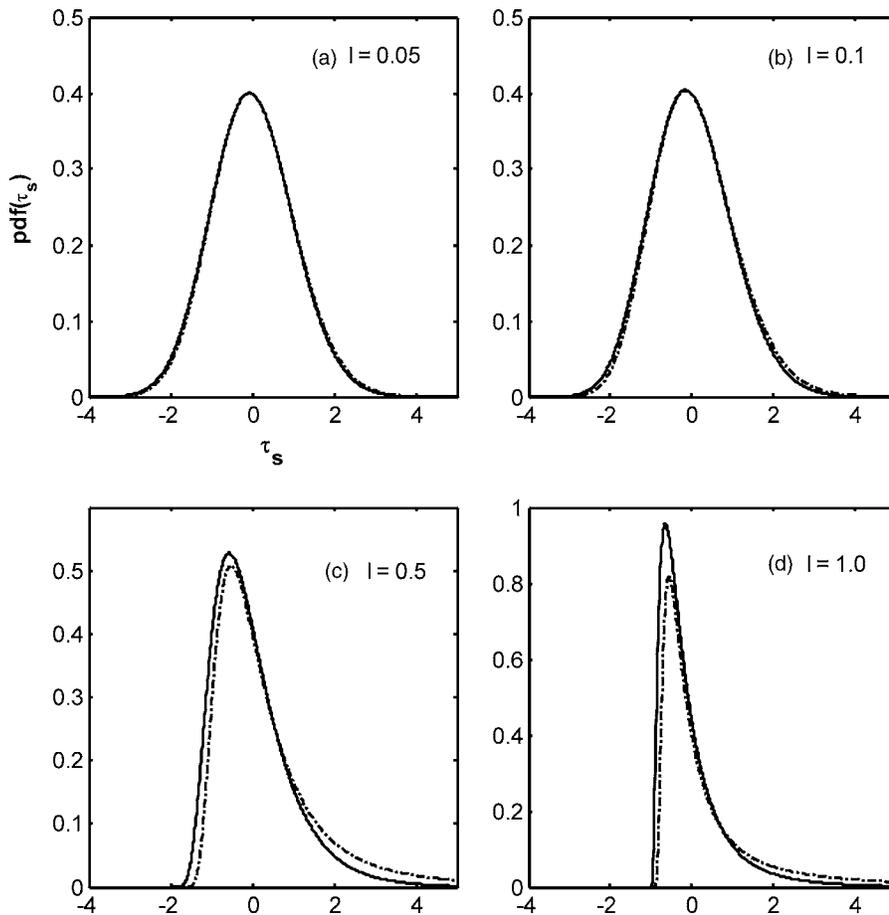


FIG. 2. Comparisons of Eq. (12) with Eq. (10) for different turbulence intensities. The solid lines are computed using Eq. (12) and the dash-dot lines are computed using Eq. (10).

$$I^2 = I_1^2 + I_2^2, \quad (15)$$

where  $I = \tau_{\text{rms}}/\tau_m$ ,  $I_1 = \tau_{1\text{rms}}/\tau_m$ , and  $I_2 = \tau_{2\text{rms}}/\tau_m$ . In the following, the two kinds of fluctuation intensities are compared based on limited information available in the literature.

Cheng *et al.*<sup>13</sup> observed that for unidirectional flows over a smooth boundary, the bed shear stress fluctuates depending on local flow conditions and also upstream effects. Their observations also showed that  $I_2$  ranged from 0.212 to 1.015 for open channel flows subject to extra turbulent eddies, which were artificially generated by superimposing external structures including horizontally installed transverse pipe and grids above the channel bed. This range covers the regular case of uniform open channel flows without any disturbances imposed, of which  $I_2 \approx 0.3-0.5$ .

To estimate  $I_1$ , the approximation of  $I_1 = 2I_\delta$  is again invoked here. Since  $\delta_{\text{rms}}$  is the same as the rms value of the fluctuation in the bed surface elevation,  $I_1$  can then be evaluated based on the bed configuration. To perform such calculations for illustration purposes, we consider here three bed conditions with regular particle arrangement. The first scenario, case I, is that the bed surface varies in one direction only, with bed particles being represented by semicircular cylinders closely connected. The second case concerns a two-dimensional bed surface, which is comprised of hemispheres arranged in a square matrix. The last situation, case III, is similar to the second, but the hemispheres are positioned in a rhombus fashion, which allows the least interparticle space. A plan view of cases I–III is provided in Fig. 3.

For the above three cases, a bed surface displacement is first computed for the determination of the average bed level. This level is defined in such a way that the interstitial space of the sediment bed below it is equivalent to the particle volume above it. The rms value of the fluctuation in the bed surface elevation is then computed by the area-weighted average with reference to the average bed level. The results are summarized in Table I, showing that  $\delta_{\text{rms}} = (0.112-0.172)D$  for the simplified bed conditions. Obviously, the ratio of  $\delta_{\text{rms}}/D$  would increase for a bed comprised of irregular sediment. For example, Nikora *et al.*<sup>18</sup> reported that  $\delta_{\text{rms}} \approx 0.375D$  for a gravel-roughened bed. However, such information is generally not available for sediment beds observed in laboratory channels and natural rivers.

If taking  $\delta_{\text{rms}} = (0.1-0.4)D$  and  $\delta = (2-5)D$ , then  $I_1 = 2I_\delta = 0.04-0.4$ . Such  $I_1$  values can cause an increase of up to 110% in the total fluctuation given in Eq. (15), which is estimated based on the observed  $I_2$  range.<sup>13</sup> This result is rough but implies that the roughness-induced fluctuation in the bed shear stress is generally considerable in comparison

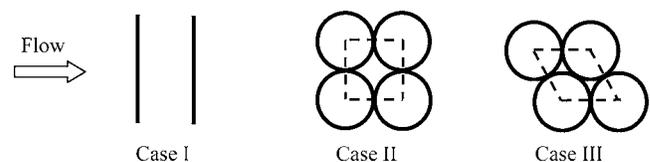


FIG. 3. Plan view of particle configurations.

TABLE I. Estimated rms values of roughness sublayer thickness.

Case	Average bed level measured downward from the top of roughness elements	$\delta_{rms}/D$
I	0.107D	0.112
II	0.238D	0.172
III	0.198D	0.148

with the turbulence-associated counterpart. Therefore, to take into account the effects of bed shear stress fluctuation on sediment transport, a double-average technique should be generally applied for the case of turbulent flows over a sediment bed.

### PROBABILITY OF BED PARTICLE MOBILITY

The mobility of bed particles can be quantified by its probability defined as<sup>21</sup>

$$p = \int_{\tau_{min}}^{\tau_{max}} f(\tau) p_e(\tau) d\tau, \quad (16)$$

where the bed shear stress  $\tau$  varies from  $\tau_{min}$  to  $\tau_{max}$ , and  $p_e$  is the probability of mobility associated only with random bed particle configuration. For a plane bed comprised of uniform sediment, the random variation of the bed mobility can be approximated as a hypothetical wave process,<sup>21</sup> which yields

$$p_e(\tau) = 1 - \exp\left(-\frac{\pi}{4} \frac{\tau^2}{\tau_{em}^2}\right), \quad (17)$$

where  $\tau_{em}$  is the average value of the minimum bed shear stress (called effective shear stress) that is required for a bed particle to move. It can be shown that  $\tau_{em}$  is similar to the critical shear stress defined by the Shields diagram but associated with sediment entrainment of higher intensity.<sup>22</sup>

As mentioned earlier,  $\tau$  can be generally approximated as a log-normal variable, and therefore its pdf is given by

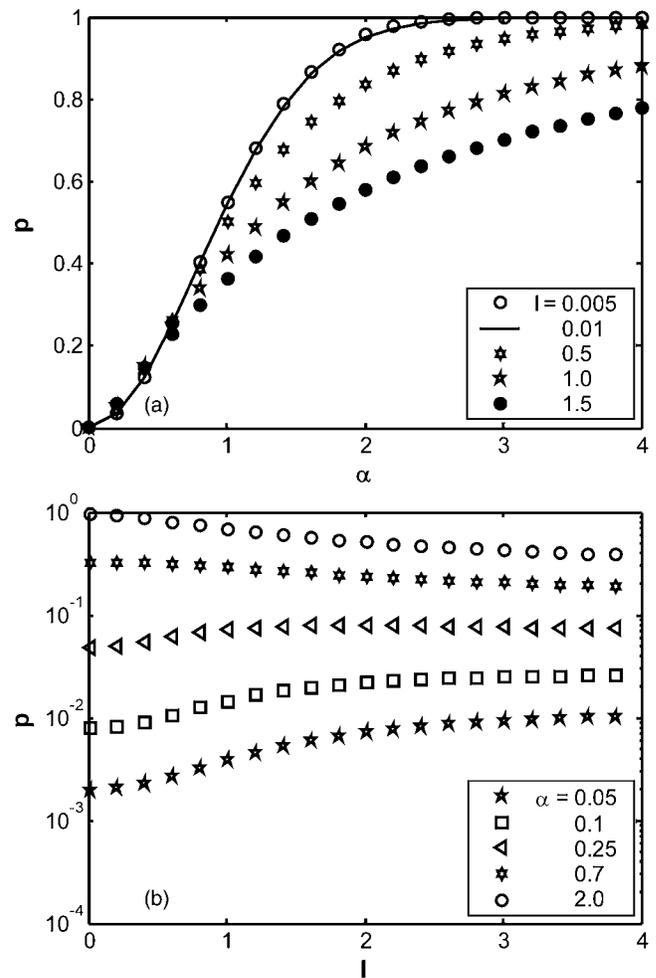
$$f(\tau) = \frac{1}{\sqrt{2\pi\beta}\tau} \exp\left[-\frac{[\beta + 2 \ln(\tau/\tau_m)]^2}{8\beta}\right], \quad (18)$$

where  $\beta = \ln(1+I^2)$  and  $I = \tau_{rms}/\tau_m$ . Substituting Eqs. (17) and (18) into Eq. (16) and then integrating from  $\tau=0$  to  $\tau=\infty$  provides an expression for evaluating the probability of the bed particle mobility subject to turbulent flow. It reads

$$p = 1 - \int_0^\infty \frac{1}{\sqrt{2\pi\beta}t} \exp\left[-\frac{[\beta + 2 \ln(t)]^2}{8\beta} - \frac{\pi}{4} \alpha^2 t^2\right] dt, \quad (19)$$

where  $\alpha = \tau_m/\tau_{em}$ . It is noted that two important parameters are involved in this evaluation, namely the turbulence intensity ( $I$ ) and the relative bed shear stress ( $\alpha$ ).

The variations of  $p$  with the two parameters for typical situations are presented in Fig. 4. It can be observed from Fig. 4(a) that a continuous increase in the probability occurs for various turbulence intensities when the relative shear

FIG. 4. Variations of  $p$  with two parameters,  $\alpha$  and  $I$ .

stress is gradually increased. However, an increase in turbulence intensity may not always result in an enhanced probability of the bed particle mobility. As shown in Fig. 4(b), for relatively low shear stress (e.g.,  $\alpha=0.1$ ), a bed particle becomes more unstable if the near-bed turbulence is enhanced. However, the opposite case occurs for higher shear stress (e.g.,  $\alpha=2.0$ ), where a bed particle would become more stable from increasing near-bed turbulence. Figure 4(b) also indicates that changes in  $p$  are not considerable for very high turbulence intensities, say  $I > 2$ .

The determination of  $\alpha$  depends on how to evaluate  $\tau_{em}$ . It can be shown that  $\tau_{em}$  could be computed indirectly using the Shields critical shear stress  $\tau_c$ .<sup>22</sup> For example, for bed-load transport in laminar flow, the dimensionless critical shear stress can be empirically related to shear Reynolds number in the power form of  $\tau_{*c} = \beta_c D_*^{m_c}$ , where  $\beta_c$  is a coefficient,  $m_c$  is an exponent,  $\tau_{*c} = \tau_c/(\Delta\rho g D)$  is the dimensionless shear stress used for the critical condition of initial sediment motion, and  $D_* = (\Delta g/\nu^2)^{1/3} D$  is the dimensionless particle diameter, with  $D$  the sediment particle diameter,  $\Delta = (\rho_s - \rho)/\rho$ ,  $\rho_s$  the sediment density,  $\rho$  the fluid density,  $\nu$  the kinematic viscosity of fluid, and  $g$  the gravitational acceleration. With a modified coefficient, the power relation may also be applied for the effective shear stress, which yields  $\tau_{*e} = \beta_e D_*^{m_e}$  with  $\tau_{*e} = \tau_{em}/(\rho g \Delta D)$  and  $u_{*e} = \sqrt{\tau_{em}/\rho}$ . To

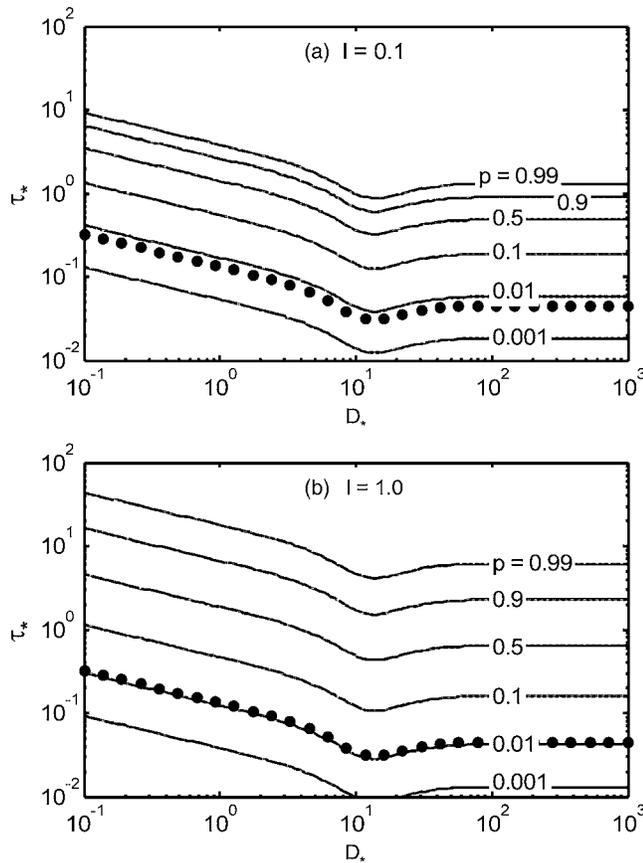


FIG. 5. Probability of bed particle mobility plotted in  $D_*-\tau_*$  plane. The Shields critical condition for initial sediment motion is denoted by black dots.

achieve the same pickup probability for the case of the initiation of sediment motion, by assuming that  $m_c = m_e$ , Cheng<sup>22</sup> finally obtained that  $\beta_e/\beta_c = 11.42$ . For simplicity, the same ratio of  $\beta_e/\beta_c$  is also used here in this study. Therefore,

$$\alpha = \frac{\tau_*}{\tau_{*c}} = \frac{\tau_*}{11.42\tau_{*c}}, \tag{20}$$

where  $\tau_{*c}$  can be computed using the following expression:<sup>23</sup>

$$\tau_{*c} = 0.13D_*^{-0.392}\exp(-0.015D_*^2) + 0.045[1 - \exp(-0.068D_*)]. \tag{21}$$

Plotted in Fig. 5 are a series of contours of  $p$  values computed using Eq. (19) for two different turbulence intensities. As a reference curve, the Shields critical condition given by Eq. (21) is also superimposed in this figure. It can be seen that the probability of bed particle mobility varies, which generally follows the trend of the Shields relation but with modifications to a certain extent, depending on bed shear stress fluctuations. As an example, it is noted that the probability that fits to the Shields curve is approximately 0.6% for  $I=0.1$ , while it increases to 1.1% for  $I=1.0$ . This difference suggests that the critical condition for the initial sediment motion should be altered for different turbulence intensities. In other words, the critical condition could be

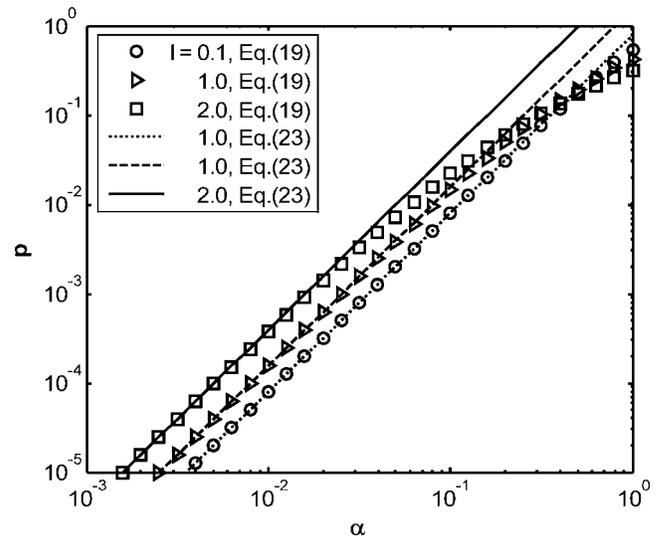


FIG. 6. Simplification of Eq. (19) for low bed shear stresses.

overestimated when applying the conventional Shields diagram to flow conditions with strong turbulence.

To further understand Eq. (19), an extreme case is also considered here for which weak sediment transport occurs at small bed shear stresses. This makes possible the following approximation for small  $\alpha$  values:

$$\exp\left(-\frac{\pi}{4}\alpha^2 I^2\right) \approx 1 - \frac{1}{4}\pi\alpha^2 I^2. \tag{22}$$

Substituting into Eq. (19) and integrating yields

$$p = \frac{\pi\alpha^2}{4}(1 + I^2). \tag{23}$$

Equation (23) shows that turbulence-enhanced probability of mobility is proportional to the squared turbulent intensity for the condition of low bed shear stresses. In addition, it is also noted from the derivation that for this extreme case, the result given by Eq. (23) also holds even if other pdfs than Eq. (18) are used for this simplification. Figure 6 shows how well Eq. (19) can be approximated by Eq. (23) for low bed shear stresses. For example, if  $I=1.0$ , the simplification given by Eq. (23) can be considered very reasonable for  $\alpha < 0.1$ .

### DISCUSSIONS

The analyses performed in this study are limited to the condition of a plane bed comprised of uniform sediment. The bed failure is described in terms of mobility probability but for flow conditions prior to initiation of bed forms such as ripples or dunes.

This work can be improved by including the possible interrelation of the two fluctuation components, one being related to bed roughness and the other due to flow variation. However, relevant information is not available at this stage. Consider the bed roughness being represented in two extreme ways, i.e., (i) by individual sediment particles and (ii) by large-scale bedforms such as dunes. In this study, we are concerned with case I, which is relatively simple. Because of

the consideration being also limited to a flat, immobile, unisized sediment bed, it is expected that there would be a minor interaction between the two fluctuation components.

In contrast, for case II, the dimension of large-scale bedforms could be in the order of flow depth, which is much larger than the size of sediment particles. In addition, it is noted that the large-scale bedform is closely associated with the generation of coherent flow structures, which may thus dominantly affect the stress fluctuation. For this extreme condition, any ignoring of the roughness-flow interrelation would definitely compromise the applicability of analysis. If a similar analysis is performed, it would then require an effective capture of coherent flow structures, for example, by employing the decomposition technique proposed by Reynolds and Hussain<sup>24</sup> and subsequently applied by other researchers including Tamburrino and Gulliver.<sup>25</sup> Furthermore, the presence of large-scale roughness also modifies the characteristics and thus probability density distributions of near-bed turbulence. For such conditions, information is needed to verify the reasonability of the assumption of log-normal variables even in the sense of double-average.

For the case of high shear stresses, this study shows that an increase in the shear stress fluctuation may yield a reduction in the probability of bed particle mobility. This is an ideal case considered here because high shear stress is often associated with high sediment transport rate with bedforms presented. A physical interpretation of this result is not available at present.

In this study, the analysis is performed in terms of the bed shear stress rather than flow velocities. This is largely because most of the existing sediment transport functions are presented in terms of the time-mean bed shear stress. Many failures in applying traditional bedload formulas could be due to the ignoring of the bed shear stress fluctuation. Some experimental evidence was recently presented by Sumer *et al.*<sup>5</sup> If knowledge of the fluctuation of the bed shear stress is available for various cases, then the computation of sediment discharge can be improved if the traditional formulas are still engaged. On the other hand, it is noted that flow velocities could be more suitable parameters for exploring turbulence-related events. However, it should be mentioned that a quantitative link of sediment discharge and turbulence events is unknown at this stage. This would limit any further application of the analysis if it is being performed based on flow velocities.

## CONCLUSIONS

For the case of plane sediment bed, it is shown in this study that the distribution of the roughness-induced bed shear stress variations can be approximated as log-normal. Such variations are generally comparable to those associated with near-bed turbulence, of which much information is available in the literature for the case of unidirectional flows over smooth boundaries. To consider the effects of shear stress fluctuation on sediment transport, a double-average technique should be generally applied for the case of turbulent flows over a sediment bed.

With the log-normal pdf, the probability of bed particle

mobility is then analyzed, showing that the mobility may be enhanced or weakened by the bed shear stress fluctuation depending on the average shear stress. If the average shear stress is low, an increase in the shear stress fluctuation may result in larger probabilities; while the probability could be reduced with increasing fluctuations if the average shear stress becomes higher. This result further implies that the Shields diagram may overestimate the critical condition for initial sediment motion for flows subject to strong turbulence.

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