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Influence of secondary flow on distribution of suspended sediment concentration

Influence de l’écoulement secondaire sur la distribution de la concentration des sédiments en suspension

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ABSTRACT

This research attempts to qualitatively evaluate the effects of secondary flow on the distribution of suspended sediment concentration. The analysis is simplified using a special mode of secondary flows generated by longitudinal bedforms. In the presence of secondary flows, the distribution may be significantly distorted in the cross-sectional plane. Vertically, the concentration gradient increases in the upflow zone; laterally, the concentration distribution may vary periodically. The motion of suspended sediment particles is associated with the ratio of the maximum upwelling velocity to the sediment settling velocity. If the ratio is greater than unity, sediment particles are trapped in the retention zone and the concentration is homogenized.

RÉSUMÉ

Cette recherche essaye d’évaluer qualitativement les effets de l’écoulement secondaire sur la distribution de la concentration des sédiments en suspension. L’analyse est simplifiée en utilisant un mode spécial des écoulements secondaires produits par des formes longitudinales du lit. En présence d’écoulements secondaires, la distribution peut être sensiblement distordue dans la section transversale. Verticalement, le gradient de concentration augmente dans la zone de flux ascendant; latéralement, la distribution de concentration peut changer périodiquement. Le mouvement des particules en suspension est associé au rapport entre la vitesse maximum du courant ascendant et la vitesse de dépôt du sédiment. Si le rapport est plus grand que un, des particules de sédiment sont emprisonnées dans la zone de rétention et la concentration est homogénéisée.

Keywords: Open channel flow, secondary flow, sediment concentration, settling velocity, suspension, trapping
1. Introduction

The temporal-average river flow circulation induced by transverse motions is often referred to as secondary flow. Their turbulence structure differs from those of two-dimensional (2D) flows (Bradshaw, 1987), thereby modifying the distribution of suspended sediment. The relevant literature is limited.

Vanoni (1946) was the first to link the secondary flow structure with the suspended sediment distribution pattern. From the laterally periodic variations of the measured sediment concentration of a natural river, he deduced the possible existence of the associated secondary flow (Fig. 1). Powell (1946) argued that it was the cellular secondary flow that caused the lateral variation in the sediment concentration. Allen (1984) observed that the transverse variation in the bed conditions induced secondary flows resulting in variations of the sediment concentration.

This work attempts to qualitatively interpret possible effects of secondary flows on the distribution of suspended sediment concentration. To facilitate considerations, the secondary flow chosen for this study is limited to that generated by longitudinal bedforms (Wang and Cheng 2006). The problem is considered from two aspects: (1) Modification of the settling behavior of sediment particles, and (2) Effect of the cross-sectional convection.

2. Modified settling velocity

The sediment settling velocity is affected by flow turbulence and particle-particle interactions. Since the latter effect is significant only for a high sediment concentration (Cheng, 1997a), the settling velocity is often replaced by the falling speed of sediment particles in still fluid (Chien and Wan, 1999). This substitution is applicable for 2D flows, but may be subject to modifications in the presence of secondary flows.

It is assumed that the flow acceleration is negligible and the “relative” velocity between a sediment particle and the ambient fluid is everywhere equal to the still water settling velocity \( \omega_0 \) (Nielsen, 1992), i.e.

\[
\omega = U_f + \omega_0
\]  

where \( \omega \) is the vector of the modified settling velocity, \( U_f \) is the velocity vector of the secondary flow and \( \omega_0 \) and settling velocity vector in still fluid expressed as

\[
\omega_0 = -\omega_0 j
\]

in which \( j \) is the unit vector of the vertical coordinate \( z \). The value of \( \omega_0 \) can be estimated using empirical formulas (e.g. Cheng, 1997b).

Secondary flows can be associated with many factors such as the presence of side walls or compound channels. They usually exhibit a similar mode with closed streamlines in the cross-sectional plane. For simplicity, the secondary flows analyzed here are those generated by
longitudinal bedforms (Figure 2). According to Wang and Cheng (2006), the velocity vector of the secondary flow cells can be described by

\[
U_i = Vj + Wk = -V_{\text{max}} \sin \left( \frac{\pi z}{h} \right) \cos \left( \frac{\pi y}{\lambda} \right) j
\]

\[
+ \frac{\lambda V_{\text{max}}}{h} \cos \left( \frac{\pi z}{h} \right) \sin \left( \frac{\pi y}{\lambda} \right) k
\]

where \( W \) and \( V \) are temporally-averaged spanwise and vertical velocities, respectively, \( k \) is the unit vector of the transverse coordinate \( z \), \( V_{\text{max}} \) is the maximum upwelling velocity; \( z \) is the vertical coordinate; \( h \) is the flow depth; \( y \) is the transverse coordinate with the origin at the centerline of the rough strip and \( \lambda \) is the average bed strip width, which is usually equal to \( h \). Substituting Eqs (2) and (3) into Eq. (1) and manipulating with \( \omega_0 \) yields

\[
\frac{\omega}{\omega_0} = V_s j + W_s k = \left[ -\frac{V_{\text{max}}}{\omega_0} \sin \left( \frac{\pi z}{h} \right) \cos \left( \frac{\pi y}{\lambda} \right) - 1 \right] j
\]

\[
+ \frac{\lambda V_{\text{max}}}{h \omega_0} \cos \left( \frac{\pi z}{h} \right) \sin \left( \frac{\pi y}{\lambda} \right) k
\]

where \( W_s \) and \( V_s \) are spanwise and vertical components of sediment velocity. Equation (4) predicts variations of the settling velocity in the \( yz \)-plane, including its magnitude and direction. Given \( h \) and \( \lambda \), the settling behavior is only controlled by the ratio \( V_{\text{max}} / \omega_0 \).

In Fig. 3, Eq. (4) is shown using the streamlines of the computed vector field of \( \omega \). Two typical scenarios are observed. If \( V_{\text{max}} / \omega_0 < 1 \), the \( \omega \)-streamlines appear vertical or curved but all open (Fig. 3a). If \( V_{\text{max}} / \omega_0 > 1 \), some of the \( \omega \)-streamlines appear closed (Fig. 3b). For the latter case, the secondary flow may carry sediment particles upwards in spite of gravity.

3. **Distributions of suspended sediment concentration**

3.1. **Homogenization in retention zone**

If \( V_{\text{max}} / \omega_0 > 1 \), the \( \omega \)-streamlines near the circulation centre is closed. This indicates that the secondary flow cells act as an effective trap to entrain sediment particles. The zone where sediment particles are trapped is referred to as retention zone (Nielsen, 1992). The concept of sediment retention was first presented by Stommel (1949). Tooby *et al.* (1977) subsequently observed that small particles in a vortex may closely follow circular paths. Figure 4 shows a typical retention zone created by the cellular secondary flows over longitudinal bedforms. Each retention zone consists of two counter-rotating sub-regions. The area of the retention zone is proportional to the magnitude of the ratio \( V_{\text{max}} / \omega_0 \).

It is known that the concentration of passive scalars or buoyant particles within a flow circulation tends to be uniform (Batchelor, 1956; Rhines and Young, 1983; Farmer and Li,
1994). In comparison, sediment particles are heavier than the ambient fluid and tend to settle downward such that they can only be trapped in the retention zone rather than the entire flow circulation. Despite of this difference, Wang (2006) has mathematically proven that sediment particles within the retention zone are also homogenized if the diffusion coefficient is assumed constant.

Relative to the homogenization in the retention zone, it is supposed that the vertical concentration profile tends to be more uniform than that in uniform flows. To verify this, a simple laboratory test was conducted (Wang and Cheng, 2006). Three longitudinal bedforms were set up. The first two were characterized by rough and smooth bed strips, in which the rough strip was prepared by small gravel with a median size of 2.55 mm and the smooth strip was made of poly vinyl chloride. The difference between the two bedforms was the width ratios of the rough to the smooth strip; one is 1:1 (Case S75) and the other is 1:2 (Case S50). The third case was characterized by bed ridges using a laterally wavy bed surface (Case WR). The flow velocity was measured with a Laser-Doppler, and an Ultrasonic-Doppler Velocimeter.

The sediment transport experiments were conducted under identical flow conditions. Refined Kaolin was chosen as the suspended load. The mean settling velocity for Kaolin flocs was $\omega_{F50} = 1.084 \text{ mm/s}$. The corresponding ratios $V_{mav}/\omega_{F50}$ for the three cases are thus about 10($> 1$), which means that Kaolin flocs could be effectively trapped by the secondary flow cells. An OBS-3 probe was used to measure the concentration at a frequency of 7 Hz. The sampling points were spaced 5 mm and 25 mm in the vertical and transverse directions, respectively. The average concentration $C_m$ was also determined from the samples collected at the flume tail tank. Figure 5 shows the laterally-averaged concentrations at different heights for Cases S75, S50 and WR, together with a reference case without obvious secondary flows. It is demonstrated that the relative concentration $C/C_m$ varies in the range of 0.9~1.05 for the three cases in the same vertical flow portion ($0.25 < z/h_m < 0.77$), but in the greater range of 0.8~1.15 for the reference case. It can also be seen that the concentration distributions for Cases S75, S50 and WR are more uniform than the Rouse profile, while the reference case is closer to the Rouse profile. It should be noted that the bulk flow conditions are almost the same for the four cases. These observations illustrate the possible homogenization trend associated with a secondary flow.

3.2. Laterally distorted distribution

If the diffusion is insignificant in the direction perpendicular to the $\omega$-streamlines, the contours of the sediment concentration would be perpendicular to the $\omega$-streamline. Figure 6(a) shows such an example with $V_{mav}/\omega_0 = 0.5$, where the concentration distribution becomes laterally non-uniform. Such a distribution pattern is similar to the observation reported by Vanoni (1946) (Fig. 6b). It is indicated that the largest non-uniformity occurs at the top and the bottom of flow cells while the concentration at the level of circulation center appears nearly uniform in the spanwise direction. For the upper flow portion, a higher concentration occurs in the downflow zone, and vice versa for the lower flow portion.

Figure 7(a) sketches two schematic profiles of the sediment concentration in the downflow and upflow zones, respectively. The vertical distribution in the downflow zone is more uniform than in the upflow zone. This differs from common believe that a downflow enhances the settling velocity and thus leads to a higher concentration gradient. However, in the presence of secondary flows, the
transverse convection may override the expected influence (Fig. 7b). The concentration decreases downwards when the gravitational effect is still dominant, i.e., the settling velocity is larger than the upflow velocity. However, due to the existence of a spanwise velocity, $W_s$, there must also be a concentration gradient along the spanwise direction. The possible pattern is that the concentration at point 2 is higher than at point 1, while the concentration at point 3 is lower than at point 4. As a result, the concentration increases sequentially from points 1 to 4. This yields a larger vertical concentration gradient in the upflow zone, where the highest and lowest concentrations in the flow cell occur near the bed and surface. In contrast, the lower vertical concentration gradient appears in the downflow zone. This again interprets the real case shown in Fig. 1.

4. Conclusions

A conceptual model of sediment settling and concentration distribution was presented for a simple mode of secondary flow pattern. In the presence of secondary flows, the settling of sediment particles is enhanced by downflow and depressed by upflow. Because of the modified settling velocity and the lateral convection, the distribution of sediment concentration may be significantly distorted in the cross-sectional plane. Vertically, the concentration gradient increases in the upflow zone; laterally, the concentration distribution becomes non-uniform. In addition, a retention zone that is specified by the closed $\omega$-streamlines may be formed to trap sediment particles in the middle of flow region. The analysis presented in this study is based only on a simplified case.

Notation

\[
\begin{align*}
    C &= \text{Time-mean sediment concentration} \\
    C_m &= \text{Bulk-averaged concentration} \\
    h &= \text{Flow depth} \\
    h_m &= \text{Average flow depth} \\
    j &= \text{Unit vector of vertical coordinate} \\
    k &= \text{Unit vector in the } z\text{-direction} \\
    V, W &= \text{Time-mean spanwise and vertical velocities, respectively} \\
    V_s, W_s &= \text{Vertical and transverse components of time-mean sediment particle velocity, respectively} \\
    U_t &= \text{Velocity vector of the secondary flow of fluid} \\
    V_{\text{max}} &= \text{Maximum upwelling velocity} \\
    y, z &= \text{Transverse and vertical coordinates, respectively} \\
    \lambda &= \text{Average bedform width (half bedform wave length)} \\
    \omega &= \text{Modified settling velocity vector} \\
    \omega_0 &= \text{Settling velocity in still fluid} \\
    \omega_0 &= \text{Vector of } \omega_0
\end{align*}
\]
References


**List of Figures**

**Figure 1**  Flow characteristics measured in River Po, Italy: (a) primary velocity distribution (m/s), (b) suspended load concentration distribution (g/m$^3$) in the same cross-sectional plane. Dashed “circles” plot the secondary flows inferred by Vanoni (1946).

**Figure 2**  Typical observation of cellular secondary flows over longitudinal bed strips in the cross-section plane (Wang and Cheng, 2006).

**Figure 3**  $\omega$-streamlines computed from Eq. (4) for (a) $V_{\text{max}}/\omega_0 = 0.5$ and (b) $V_{\text{max}}/\omega_0 = 5.0$.

**Figure 4**  Retention zone of sediment particles specified by the closed $\omega$-streamlines in the cross-sectional plane, for $V_{\text{max}}/\omega_0 = 2.5$.

**Figure 5**  Vertical distributions of laterally-averaged sediment concentration $C/C_m(z/h_m)$: (o) = measurements, (—) = Rouse profile (Chien and Wan, 1999).

**Figure 6**  (a) $\omega$-streamlines (solid lines) for $V_{\text{max}}/\omega_0 = 0.5$, and equi-potential lines (dashed lines) indicating the possible concentration contours; (b) field observation of suspended load concentration distribution (Vanoni, 1946).

**Figure 7**  (a) Schematic vertical profiles of sediment concentration in the upflow and downflow regions in the presence of secondary flows; and (b) conceptual explanation of possible distribution patterns of sediment concentration along a flow cell, where the numbers represent the low-to-high sequence of sediment concentration.
Figure 2
Figure 3
Figure 4
Figure 5
Figure 6
Figure 7