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Double-layer floor to mitigate in-structure shock of underground structures:

A conceptual design

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**Abstract:** In-structure shock poses a great threat to equipment and devices in underground protective structures, preventing them from accomplishing designed tasks. Unlike traditional mitigation methods such as backfilling soft soil or sand around the buried structures and adding shock-isolation cushions to the equipment, a new structural design of underground structures are proposed by constructing an isolation slab, to which the equipment is attached. The excitation mechanism for the equipment attached to the slab is altered from the combination of flexural deflection and rigid body motion of the traditional floor to the response of the isolation slab. Analysis indicates that vertical shock level experienced by the equipment is effectively reduced. The in-structure shock mitigation approach proposed in the present study provides a supplement to the existing shock mitigation methods for underground structures.

**Keywords:** In-structure shock, Shock mitigation, Blast load, Underground structure, Shock response spectra, Protective design.
Notation

\( A \)  
Area of the blast load on a structural member

\( c_s \)  
Acoustic velocity of soil

\( E \)  
Young’s modulus of concrete

\( f \)  
Coupling factor of the explosion energy to soil

\( h \)  
Thickness of floor and walls of the structure

\( h_s \)  
Thickness of the isolation slab

\( L_1, L_2, L_3 \)  
Length, width and height of the structure

\( k_s \)  
Stiffness of the isolation slab

\( K_L \)  
Load factor

\( K_M \)  
Mass factor

\( n \)  
Attenuation coefficient of stress wave in soil

\( m_s \)  
Mass of the isolation slab

\( M \)  
Mass of the whole underground structure

\( P_0 \)  
Free-field peak pressure

\( R \)  
Distance from explosion center to the structure

\( t \)  
Time starting at the arrival of the blast load

\( t_1 \)  
Time starting after the blast load

\( T_d \)  
Blast duration

\( u(t) \)  
Relative displacement between the isolation slab and whole structure

\( U(t) \)  
Rigid body motion displacement of the whole structure
\( W \) \quad \text{TNT equivalent charge weight}

\( x_s \) \quad \text{Absolute displacement of the isolation slab}

\( \alpha \) \quad \text{Reduction factor}

\( \beta \) \quad \text{A factor equal to 160 in imperial unit system}

\( \rho \) \quad \text{Mass density of concrete}

\( \rho_s \) \quad \text{Mass density of soil}

\( \tau \) \quad \text{Characteristic response time}

\( \omega \) \quad \text{Circular natural frequency of equivalent SDOF system representing isolation slab}

\( \varsigma \) \quad \text{Damping ratio of equivalent SDOF system representing isolation slab}

\( \nu \) \quad \text{Poisson’s ratio of concrete}
1. Introduction

Underground protective structures provide relatively effective protection for the contained personnel and equipment from being injured and damaged by blast loads compared to surface structures. With the development of penetration weapons, subsurface detonation posed greater threat to the buried structures which were considered safe decades ago. With a small standoff distance, the structures may experience major damage; however, with a mediate standoff distance, the whole structure may stay undamaged or experience minor damage, but the personnel inside are injured and the contained devices may not function properly due to the shock or severe vibration induced by this detonation. This kind of shock, not damaging the whole structure, but causing inside equipment to fail, is termed as in-structure shock.

Over the decades, research and discussions were done on strength and failure prevention of underground structures as well as the soil-structure interaction under various dynamic loads numerically [1-8], analytically [9-14] and experimentally [14,15]. On no account can we ignore the significance of the strength and failure-prevention of underground structures under dynamic loads. Nevertheless, the safety of the personnel and the serviceability of the inside equipment are equally important. Failure in either aspect will completely prevent the whole structure from performing its designed tasks. Only few works can be found on the vibration response of secondary or attached components of surface structures [16-18]. Therefore, it is necessary to discuss the serviceability of underground structures in terms of in-structure shock.

Generally, devices and equipment are damaged by excessive high acceleration, whose tolerances can be found in various provisions and codes [e.g. 19]. To ensure the serviceability of the equipment attached to the buried structures, various measures should be taken. For horizontal
shock, steel shots are used to allow the equipment with support to roll freely and nylon ropes are used to prevent its excessive movement [20]. For vertical shock induced by incident floor motion, the general mitigation philosophy is to use a system to dissipate energy imparted by the relative motion between the moving floor and the equipment with supporting, i.e. adding a crushable foam layer; or provide a so-called ‘shock impedance mismatch’ which causes poor coupling between the floor and the equipment [20]. For certain device in a traditional underground structure, a separate isolator can be applied to mitigate in-structure shock. However, for different shock-sensitive devices in an underground structure, testing, design and installation of such separate isolators are expensive, labor-intensive and time-consuming. More important, to mitigate in-structure shock on personnel in underground structures, the only way is to reduce shock level, rather than add isolators. Another approach is to backfill soft soil or sand around the buried structure so that the load applied on the structure is reduced. In addition, modification of the protective structures may realize a situation that the load applied on the whole structure and the motion induced are the same while internally the shock on equipment is mitigated. In the present study, a simple design of underground structures to alleviate vertical in-structure shock is proposed by adding an additional slab, termed as isolation slab, onto which the devices are mounted, shown in Fig. 1.

When an underground structure is subjected to a deep subsurface explosion, the floor will experience rigid body motion (RBM) of the whole structure and deformation relative to RBM. Generally, responses of the deflection are significantly larger than those of the RBM if the structure is large and heavy [21]. Unlike traditional surface double-layer protective structures with outer layers used as sacrificial layers in viewpoint of strength, in the present study, the aim of adding a thin isolation slab internally near the floor is to mitigate vertical in-structure shock
applied on the contained equipment. By adding such an isolation slab, subjected to the same soil-transmitted blast load, the shock excitation on the devices is altered: from the floor deflection plus the whole structure RBM to the isolation slab vibration induced by the whole structure RBM. At the edges, the slab vibration is zero and the excitation is the whole structure RBM. For this design, on one hand, the isolation slab is not necessarily thick compared to the walls and floors of the protective structure; on the other hand, only a small amount of additional space is needed since the gap between the isolation slab and the floor is negligibly small provided that it is larger than the maximum deflection of the floor when subjected to a typical subsurface detonation. Therefore, obviously, this design is simple and economical.

In the present study, first the RBM responses of the whole structure subjected to a blast load induced by a subsurface explosion are determined, with soil-structure interaction taken into consideration. Then the isolation slab is simplified to a single-degree-of-freedom (SDOF) system and its motion time histories are derived with the previously obtained RBM responses as excitation. After that, the vertical shock level experienced by equipment is evaluated by adopting shock response spectra, which establish relationship between the maximum vertical shock level on a specific device and the corresponding natural frequency of the system consisting of the device and its support.

2. Rigid body motion of the whole structure

Consider a box-typed underground structure subjected to a blast load induced by a subsurface detonation. In the present study, all structural members are assumed rigid by ignoring their deformations since they are relatively thick and the explosion is relatively distant. By neglecting
the load alleviation effect due to the small deformation [12], the responses of the RBM such as displacement, velocity and acceleration are slightly over-predicted. In addition, among various detonation locations, the most dangerous scenario (with the same charge weigh and detonation depth), in which the explosion occurs beneath the buried structure center is studied. The structure is assumed to be deeply buried so that the reflection of stress wave from ground surface can be ignored. Further, the RBM responses are obtained by assuming no separation and re-contact between the structure and surrounding soil in all time, which may lead to a conservative prediction.

To analyze the underground structure RBM accurately, the dynamic soil-structure interaction is incorporated [11-13, 21]. The solution of the whole structure RBM is divided into two phases, i.e. loading phase and post-loading phase, respectively. The blast duration in fact is very short, and the RBM may last longer.

Generally, a blast load induced by a subsurface detonation is a suddenly applied load attenuating quickly in an exponential manner. In engineering practice, it is further simplified to a triangular pulse while preserving the impulse and peak pressure [21]:

\[
P(t) = \begin{cases} 
P_0 \alpha \left(1 - \frac{t}{T_d}\right) & t \leq T_d \\ 0 & t > T_d \end{cases} \tag{1a}
\]

where \(P_0\) is the free-field peak pressure [19]:

\[
P_0 = \beta f \left(\rho_s c_s\right) \left(\frac{R}{W^{1/3}}\right)^m \tag{1b}
\]
$T_d$ the blast duration; $f$ a coupling factor of the explosion energy to soil, dimensionless; $\rho_s c_s$ the acoustic impedance of soil; $n$ an attenuation coefficient, dimensionless; $W$ the TNT equivalent charge weight; $R$ the distance measured from the explosion center to the structure; $\alpha$ a reduction factor, defined as ratio of the equivalent uniform pressure on a structural member to the maximum pressure of the actual load distribution; and $\beta$ a dimensionless factor equal to 160 in the imperial unit system [19]. It should be noted that the pressure calculated using Eq. (1b) in psi is converted to SI unit in Pa before further calculation.

2.1 Loading phase: in the blast duration

The soil-structure interaction is incorporated in the analysis, whose nature is the load applied onto the structure is further modified due to the structural response and it results in a load-mitigating effect. Taking this effect into consideration and in light of the Newton’s law, the governing equation, or equation of motion (EOM) for the whole structure RBM in the blast duration is:

$$\frac{M}{A} \frac{1}{2\rho_s c_s} \frac{d^2U(t)}{dt^2} + \frac{dU(t)}{dt} = \frac{P_0 \alpha \left(1 - \frac{t}{T_d} \right)}{\rho_s c_s}$$

(2)

where $U(t)$ is the whole structure RBM displacement; $M$ the mass of the whole structure; $A$ the area of the structural member on which the blast load is normally applied. Define characteristic response time:

$$\tau = \frac{M}{A} \frac{1}{2\rho_s c_s}$$

(3)
The EOM can be simplified as:

\[
\tau \frac{d^2 U(t)}{dt^2} + \frac{dU(t)}{dt} = \frac{P_0 \alpha \left(1 - \frac{t}{T_d}\right)}{\rho_s c_s}
\]  

(4)

At time zero, the whole structure is at rest with zero displacement and velocity. By combining the EOM and initial condition, the displacement, velocity and acceleration of the whole structure within the blast duration are:

\[
U(t) = \frac{P_0 \alpha}{\rho_s c_s} \left[ \left(1 + \frac{\tau}{T_d}\right) - \frac{t^2}{2T_d} + \frac{\tau}{T_d} \left(\tau + T_d\right) \left(e^{-\tau/T_d} - 1\right) \right]
\]  

(5a)

\[
\dot{U}(t) = \frac{P_0 \alpha}{\rho_s c_s} \left[ \left(1 + \frac{\tau}{T_d}\right) - \frac{t}{T_d} - \left(1 + \frac{\tau}{T_d}\right) e^{-t/T_d} \right]
\]  

(5b)

\[
\ddot{U}(t) = -\frac{P_0 \alpha}{\rho_s c_s} \left[ \frac{1}{T_d} - \left(1 + \frac{1}{\tau} \right) e^{-t/T_d} \right]
\]  

(5c)

2.2 Post-loading phase: after the blast duration

When the blast load decreases to zero, offset the time by blast duration, \(t_1 = t - T_d\), the EOM of the structure after the pulse is:

\[
\frac{M}{A} \frac{d^2 U(t_1)}{dt^2} + \rho_s c_s \frac{dU(t_1)}{dt} = 0
\]  

(6)

Combine the EOM and initial condition, in fact the terminal displacement and velocity in the previous phase denoted as \(U(t_1 = 0)\) and \(\dot{U}(t_1 = 0)\), the responses after the blast are:
\[ U(t_i) = U(t_i = 0) + 2\dot{U}(t_i = 0)\tau\left[1 - e^{-\frac{t_i}{(2\tau)}}\right] \]  \hspace{1cm} (7a)

\[ \dot{U}(t_i) = \dot{U}(t_i = 0)e^{-\frac{t_i}{(2\tau)}} \]  \hspace{1cm} (7b)

\[ \ddot{U}(t_i) = -\frac{\dot{U}(t_i = 0)}{2\tau}e^{-\frac{t_i}{(2\tau)}} \]  \hspace{1cm} (7c)

### 3. Vibration of the isolation slab

With the same surrounding soil, structural configuration and loading condition, the traditional single-floor structure is modified to a double-floor one by adding an isolation slab as the second floor, shown in Fig. 1. These two structures undergo exactly the same responses provided that the relatively small mass of isolation slab relative to that of the whole structure is ignored. Rather than directly mounted to and excited by the floor in traditional structures, the equipment in the double-floor structure is attached to the isolation slab, excited by the RBM-induced slab response.

From the viewpoint of vibration theory, the advantage of the new design is the excitation source shift: for a traditional structure, shock imposed on the equipment is directly from the combination of global RBM of the whole structure plus some orders of deflection modes of the floor; in contrast, for the new designed structure with isolation slab, the excitation to the equipment is the response of the isolation slab as a secondary system thus the equipment attached becomes a tertiary system.

For practical design purpose, rigorous analysis of structures with continuous mass distribution under dynamic loads is not efficient, even not possible when the loads applied and boundary conditions are difficult to be expressed in manageable mathematical functions.
Approximate design method using SDOF idealization is established and well documented [22].

With a negligibly small mass compared to the whole structure, the isolation slab is modeled as an SDOF spring-mass system by applying the load factor and mass factor to modify its real stiffness and mass [22], subjected to the excitation induced by the whole structure RBM. It is worth noting that the isolation slab is built-in to the adjacent structural members. The rotation at the slab boundary is constraint to some degree, although neither fixed nor simply supported, it is closer to fixed boundary. In the present study, it is treated in two steps: first, subjected to a uniform load, the real stiffness is calculated with fixed boundary; second, when this real stiffness is idealized to that of the equivalent system, the constraint is released to some degree by apply the average of the simply supported and fixed load factors.

The equipment mounted onto the isolation slab, together with its support, is simplified as another SDOF system, attached to the SDOF system representing the isolation slab. Thus, a 2-DOF spring-mass system model without damping is established, as shown in Fig. 1.

In engineering practice, the mass of the equipment is always significantly smaller than that of the isolation slab. Therefore the 2-DOF system consisting of isolation slab and equipment with support can be decoupled while retaining adequate accuracy. The procedure is: first remove the equipment and analyze the SDOF system representing the isolation slab. Then the response of the SDOF system representing the equipment with support can be obtained with the isolation slab response as excitation. That is, when the mass of the device is significantly smaller than that of the isolation slab, the 2-DOF system becomes an SDOF system. Corresponding to the RBM responses in section 2 as excitation, the isolation slab responses are derived separately in two phases, i.e., loading and post-loading phases.
3.1 Phase I: loading phase

The EOM of the isolation slab equivalent SDOF system can be written as:

\[
K_m m_s \ddot{u}(t) + K_L k_s u(t) = K_m m_s \frac{P \alpha}{\rho_s c_s} \left[ \frac{1}{T_d} - \left( \frac{1}{\tau} + \frac{1}{T_d} \right) e^{-\frac{t}{\tau}} \right]
\]  

(8)

where \(u(t) = x_s(t) - U(t)\) is the relative displacement between the isolation slab and the whole structure. Define circular natural frequency of the equivalent system:

\[
\omega = \sqrt{\frac{K_s k_s}{K_m m_s}}
\]  

(9)

In structural engineering, a typical damping ratio, \(\zeta=0.05\), is used [23]. Then, adding such a damping and the EOM is simplified as:

\[
\ddot{u}(t) + 2\zeta \omega \dot{u}(t) + \omega^2 u(t) = \frac{P \alpha}{\rho_s c_s} \left[ \frac{1}{T_d} - \left( \frac{1}{\tau} + \frac{1}{T_d} \right) e^{-\frac{t}{\tau}} \right]
\]  

(10)

At the beginning, there is no relative displacement and velocity between the whole structure and isolation slab:

\[
u(0) = 0, \dot{u}(0) = 0
\]  

(11)

Combine the EOM and the initial conditions, the responses of the equivalent SDOF system in the blast duration are:

\[
u(t) = e^{-\zeta \omega t} \left( F \sin Dt + P \cos Dt \right) + \frac{S}{\omega^2} - \frac{B \tau^2}{1 - 2\zeta \omega \tau + \omega^2 \tau^2} e^{-\frac{t}{\tau}}
\]  

(12a)

where
\[B = \frac{P_d \alpha}{\rho_s c_s \left(\frac{1}{\tau} + \frac{1}{T_d}\right)}, \quad D = \sqrt{1 - \zeta^2 \omega}, \quad S = \frac{P_d \alpha}{\rho_s c_s T_d}\]

\[F = -\frac{\zeta S}{D \omega} + \frac{B \tau (\zeta \omega \tau - 1)}{D (1 - 2 \zeta \omega \tau + \omega^2 \tau^2)}, \quad P = -\frac{S}{\omega^2} + \frac{B \tau^2}{1 - 2 \zeta \omega \tau + \omega^2 \tau^2}\]

Then velocity and acceleration can be readily obtained:

\[
\dot{u}(t) = e^{-\zeta \omega t} \left[ - (\zeta \omega F + PD) \sin D t + (-\zeta \omega P + FD) \cos D t \right] + \frac{B \tau}{1 - 2 \zeta \omega \tau + \omega^2 \tau^2} e^{-\zeta \omega t}
\]

\[
\ddot{u}(t) = e^{-\zeta \omega t} \left[ (\zeta^2 \omega^2 F + 2 \zeta \omega PD - FD^2) \sin D t + (\zeta^2 \omega^2 P - 2 \zeta \omega FD - PD^2) \cos D t \right] - \frac{B}{1 - 2 \zeta \omega \tau + \omega^2 \tau^2} e^{-\zeta \omega t}
\]

### 3.2 Phase II: post-loading phase

Similarly, the governing equation of the isolation slab equivalent SDOF system after the blast load is:

\[
\ddot{u}(t_i) + 2 \zeta \omega u(t_i) + \omega^2 u(t_i) = C e^{-\zeta \omega t_i / (2 \tau_o)}
\]

where

\[t_i = t - T_d, \quad C = \frac{\hat{U}(t = T_d)}{2 \tau_o}\]

Combine the governing equation and initial conditions, the displacement of the equivalent SDOF system after the blast is:

\[
u(t_i) = e^{-\zeta \omega t_i} \left( G \sin D t_i + Q \cos D t_i \right) + \frac{4 \tau^2 C}{1 - 4 \zeta \omega \tau + 4 \omega^2 \tau^2} e^{-\zeta \omega t_i / (2 \tau)}
\]
where

\[ G = \frac{\omega H + I}{D} + \frac{2\tau \left(1 - 2\tau^3\omega\right) C}{D \left(1 - 4\zeta\omega\tau + 4\omega^2 \tau^2\right)} \]

\[ Q = H - \frac{4\tau^2 C}{1 - 4\zeta\omega\tau + 4\omega^2 \tau^2} \]

and

\[ H = u(t = T_d) = e^{-\omega t_d} \left( F \sin DT_d + P \cos DT_d \right) + \frac{S}{\omega^2} - \frac{B\tau^2}{1 - 2\zeta\omega\tau + \omega^2 \tau^2} e^{-T_d/\tau} \]

\[ I = \dot{u}(t = T_d) = e^{-\omega t_d} \left[-(\zeta\omega F + PD) \sin DT_d + (\zeta\omega P + FD) \cos DT_d \right] + \frac{B\tau}{1 - 2\zeta\omega\tau + \omega^2 \tau^2} e^{-T_d/\tau} \]

are the initial conditions of the time starting at the end of blast duration, in fact, the terminal response in the blast duration.

Similarly the velocity and acceleration after the blast can be obtained:

\[ \dot{u}(t_i) = e^{-\omega t_i} \left[-(\zeta\omega G + QD) \sin Dt_i + (\zeta\omega Q + GD) \cos Dt_i \right] - \frac{2\tau C}{1 - 4\zeta\omega\tau + 4\omega^2 \tau^2} e^{-t_i/(2\tau)} \]  

\[ \ddot{u}(t_i) = e^{-\omega t_i} \left[ (\zeta^2 \omega^2 G + 2\zeta\omega GD - GD^2) \sin Dt_i + (\zeta^2 \omega^2 Q - 2\zeta\omega GD - QD^2) \cos Dt_i \right] + \frac{C}{1 - 4\zeta\omega\tau + 4\omega^2 \tau^2} e^{-t_i/(2\tau)} \]  

It is worth noting that the above solutions of both phases are relative displacement, velocity and acceleration between the isolation slab equivalent system and the whole structure. Finally, summation of these relative responses and the rigid body motion, yields the total responses at the isolation slab center.

In addition, the responses obtained above are for the isolation slab center while the vibrations in the four edges of the slab are the rigid body motion of the whole structure. In the elastic range, superposition principle holds. The response of the locations other than the slab center and edges
should be the sum of the whole structure RBM response and the relative deflection response between the isolation slab and the whole structure, which can be interpolated using the shape function and the slab center response. For instance, the approximate shape can be chosen as double sine function.

3.3 Shock of equipment attached to the isolation slab

Until now the acceleration time history of the isolation slab is obtained, which is in fact the excitation for the equipment. Further, with the same method, the responses of the equipment with support, modeled as another SDOF system, can be obtained. In engineering practice, to determine whether a device is damaged by a shock, the maximum acceleration regardless of direction is compared with the corresponding tolerance. If the acceleration applied on the device exceeds that of the tolerance (typical values are listed in [19]), damage occurs. Here, shock response spectra method [23], well documented, is employed to calculate the maximum acceleration of various devices with supports simplified as SDOF systems of various natural frequencies.

It is worth noting that the system consisting of the isolation slab and the equipment, a typical primary and secondary system, is analyzed with a decoupled method. For a primary and second system, decoupled analysis can be applied with adequate accuracy if the frequency of the primary system only changes slightly no matter the secondary system is coupled or not [24]. Therefore, in the present study, if the mass of the equipment is significantly smaller than that of the isolation slab, the accuracy of the decoupling analysis is adequate. Otherwise, a coupled analysis is required.
4. Case study and discussions

Consider a box-typed protective structure with an isolation slab, installed in dry sand (properties in Table 1 [25]), subjected to a blast load induced by a subsurface detonation. The detailed structural and loading parameters are:

- \( \rho = 2500 \text{ kg/m}^3 \)
- \( E = 30 \text{ GPa} \)
- \( v = 0.2 \)
- \( L_1 = 9 \text{ m} \) (length of the structure)
- \( L_2 = 7 \text{ m} \) (width of the structure)
- \( L_3 = 5 \text{ m} \) (height of the structure)
- \( h = 0.5 \text{ m} \) (uniform thickness of all structural members except the isolation slab)
- \( h_s = 0.2 \text{ m} \) (thickness of the isolation slab)

\[
\frac{R}{W^{1/3}} = 2 \text{ m/kg}^{1/3} \quad (R=10 \text{ m}, W=125 \text{ kg})
\]

- \( T_0 = 20 \text{ ms} \) (typical for underground detonation, [25])
- \( \alpha = 0.65 \)
- \( f = 1 \)

- \( K_L = 0.435 \) ([22])
- \( K_M = 0.31 \) ([22])

where \( K_L \) and \( K_M \) are obtained as follows: with the dimension ratio of the isolation slab about 0.75, the load factors are read 0.5 and 0.37 for simply supported and fixed boundaries; and the
mass factors are read 0.36 and 0.26 for simply supported and fixed boundaries. Then through averaging, the load factor and mass factor are taken as 0.435 and 0.31, respectively.

The equivalent mass of the isolation slab is:

\[ K_m m_s = 0.31 \times 2500 \times (0.2 \times 7 \times 9) = 9765 \text{ kg} \]

Obviously the mass of a typical device with support attached to the isolation slab is significantly smaller than that of the slab. Therefore it is reasonable to decouple the isolation slab response from that of the device with support.

Fig. 2 shows the RBM responses of the whole structure. In this case the RBM displacement continues to increase after the blast load vanished at 0.02 s and stops at about 0.05 s with a maximum value of approximate 4 mm. It is known that the RBM displacement is negligibly small due to the huge mass of the structure. The RBM velocity starts at zero and achieves its peak 0.225 m/s quickly at about 0.009 s then attenuation to zero around 0.05 s. Among these RBM responses, the acceleration is of most significance since it is the direct excitation for the isolation slab and subsequently indirect excitation for the equipment. At time zero, it achieves peak value, approximately 8g, and attenuates in a high rate to zero at about 0.05 s.

It is worth noting that when the soil particle velocity, calculated as the free field pressure divided by the soil acoustic impedance, is larger than the structure velocity, the blast load applies onto the structure. Otherwise, if the soil particle velocity is smaller than the structure velocity, the soil-transmitted load cannot apply on the structure. Possibly, according to this criteria, there may or may not be re-contact and re-separation between the soil and structure, depending on the blast load time history, soil and structure characteristics. In the present study, the velocity time histories with and without soil-structure separation are plotted, together with the soil particle
velocity time history. From Fig. 3, the difference is not significant, which implies neglecting the soil-structure separation is reasonable.

Further, with the RBM acceleration time history as excitation, the responses of the isolation slab are obtained, shown in Fig. 4. The maximum relative displacement between the isolation slab center and the floor is less than 2 mm. Compared to the thickness of 0.2 m, one can know the slab deflection is elastic and will not cause any damage. When it comes to the absolute acceleration at the isolation slab center, it is favorable to know the peak acceleration is reduced to 4 g.

It is obvious that the acceleration response at the isolation slab center depends on its natural frequency, which is determined by the stiffness and mass. To address this issue, in the present case, all the parameters including the geometrical dimensions and material of the structure as well as the blast load are fixed except the isolation slab equivalent natural frequency. Fig. 5 indicates that when its natural frequency is low, in other words, when the slab is relatively flexible, the maximum acceleration is low. The peak isolation slab acceleration increases with the natural frequency before certain value, e.g. 1600 Hz; after that the peak acceleration decreases. It is validated that if the natural frequency of the isolation slab becomes infinite, the acceleration time history of the slab center will exactly converge to that of the RBM of the whole structure. In the range of engineering application, theoretically, under the same blast, the more flexible the isolation slab is, the more effective the in-structure shock mitigation will be. However, this slab may not be too flexible since it will become unstable due to personnel movement and excessively large deflection may occur. Therefore based on these considerations, in design, the thickness of the isolation slab should be judiciously selected to balance the shock mitigation effect and stability.
Then shock response spectra of the equipment attached to the isolation slab subjected to vertical in-structure shock when the structure is buried in dry sand are established. The excitations for the equipment mounted near the edges and at the center of the isolation slab are shown in Fig. 2(c) and Fig. 4(b), respectively. It is obvious that these two excitations have different characteristics: the one at the isolation slab center is an attenuating oscillation with a lower peak value and a relatively long period while the one at the slab edges has a higher peak value and a relatively short duration. From Fig. 6, if the natural frequency of the SDOF system consisting of the equipment and support is less than 5 Hz, the peak acceleration on the equipment near the slab edge, excited in fact by the whole structure RBM, is higher than that of the equipment placed at the slab center; otherwise, shock on equipment at the slab center is severer. Further, from Fig. 6, it is worth noting that the equipment with support having a natural frequency of 20 Hz experiences highest acceleration level of approximately 16 g under this specific structural configuration and blast load. Therefore in engineering application, before constructing the protective structure and installing the equipment, shock spectra analysis should be conducted to select appropriate thickness and material of the isolation slab as well as the natural frequency of the equipment with support so that possible strong shock under certain blast load on equipment can be avoided.

Besides the scenario of the structure being installed in dry sand, more situations are also analyzed for structure buried in other soils (Kallang soil and Bukit Timah soil, two typical soils in Singapore, properties in Table 1 [25]), shown in Fig. 7 and Fig. 8. Similar conclusions can be made to the shock intensity on equipment near the isolation slab edge and at the center. With increase of acoustic impedance and decrease of attenuation coefficient of the surrounding soil, the shock applied on the equipment become severer no matter where the equipment is placed:
near slab edges or at center. Thus as expected, the protective structure should be installed in a soil with small acoustic impedance and large attenuation coefficient such as dry sand.

Finally the vertical in-structure shock mitigation effect of the new design is examined by comparing the shock level on equipment. A traditional protective structure of the same configuration but without isolation slab is considered to be installed in dry sand, Kallang soil and Bukit Timah soil, respectively and subjected to a blast load induced by the same subsurface detonation as in the present, the shock levels are evaluated [26]. Fig. 8 shows the comparison of the shock level on the equipment attached to the centers of the isolation slab of the new structure and the floor of the traditional structure, respectively and one can know that by adopting such a design, at least 50% and even higher reduction of the vertical in-structure acceleration is achieved in the present case. Further, it is worth noting that the vertical shock mitigation efficiency also depends on the dimensions of the underground structures. For a buried structure with large dimensions and relatively thin wall thickness, rather than a small one with relatively thick walls, the excitation difference for the equipment between the traditional and double-floor structures are significant: the floor deflection dominates while rigid body motion of the whole structure reduces significantly, resulting in a greater excitation for equipment in traditional protective structures and lower excitation for equipment in the newly proposed structure with isolation slab. Therefore the vertical shock mitigation efficiency of double-floor underground structures increases with structural dimensions. At the same time, the stability of the isolation slab should also be examined in design.

5. Conclusions
A simple mitigation method for in-structure shock of underground protective structures is proposed in the present study. By adding an isolation slab, the excitation for the equipment at the floor center is altered. The excitation source is changed from combination of floor deflection plus rigid body motion of the whole structure to that of the isolation slab. The excitation pattern for the equipment is altered from a shock with a higher peak acceleration and a shorter duration to an attenuation oscillation with a lower peak acceleration and a longer duration. Case study shows that at the isolation slab center, the vertical in-structure shock level on the equipment is effectively reduced while near the edges the shock intensity on equipment is not high. Therefore this design exhibits excellent capacity to reduce the vertical shock level on equipment thus it may provide a supplementary reference for further design and modification of underground structures. The proposed conceptual design of underground structure for mitigating vertical in-structure shock should be verified and validated experimentally.
References


Fig. 1 Illustration of the newly designed underground structure and the equivalent system of the isolation slab with attached equipment.
Fig. 2 The rigid body motion responses of the whole structure.
Fig. 3 Rigid body motion velocities: with and without soil-structure separation.
Fig. 4 The relative deflection and absolute acceleration time histories at the isolation slab center.
**Fig. 5** The maximum acceleration of the isolation slab center with respect to slab natural frequency.
Fig. 6 Comparison of shock response spectra at isolation slab center and edges: dry sand.
**Fig. 7** Comparison of shock response spectra for equipment near isolation slab edges: structure in different soils. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)
(a) In dry sand.

(b) In Kallang soil.
(c) In Bukit Timah soil.

**Fig. 8** Shock response spectra comparison: structure with and without isolation slab in three typical soils.
Table 1 Properties of dry sand and two typical soils in Singapore.

<table>
<thead>
<tr>
<th>Soil type</th>
<th>Density (kg/m³)</th>
<th>Acoustic velocity (m/s)</th>
<th>Attenuation coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dry sand</td>
<td>1633</td>
<td>305</td>
<td>2.75</td>
</tr>
<tr>
<td>Kallang soil</td>
<td>1420</td>
<td>1350</td>
<td>2.5</td>
</tr>
<tr>
<td>Bukit Timah soil</td>
<td>1800</td>
<td>1650</td>
<td>2.25</td>
</tr>
</tbody>
</table>