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<td><strong>Author(s)</strong></td>
<td>Rong, Hai-Cheng.; Li, Bing.</td>
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<td><strong>Citation</strong></td>
<td>Rong, H. C., &amp; Li, B. (2007). Probabilistic response evaluation for RC flexural members subjected to blast loadings. Structural Safety, 29(2), 146-163.</td>
</tr>
<tr>
<td><strong>Date</strong></td>
<td>2006</td>
</tr>
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<td><strong>URL</strong></td>
<td><a href="http://hdl.handle.net/10220/8372">http://hdl.handle.net/10220/8372</a></td>
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Probabilistic response evaluation for RC flexural members subjected to blast loadings

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Received 3 March 2006; accepted 6 March 2006
Available online 6 June 2006

Abstract

The probabilistic responses of the maximum displacement and displacement ductility factor for a reinforced concrete (RC) flexural member against potential blast loadings are evaluated through a nonlinear dynamic analysis of its equivalent single-degree-of-freedom (SDOF) system. Monte-Carlo simulation is used in the analysis. Some differences are observed between the actual responses of the RC member and those of the equivalent SDOF system due to the complex behaviours of reinforced concrete structural members under blast conditions. Two non-dimensional indices are defined to quantify the differences and their expressions are generated through a large amount of numerical and statistical analyses. The approach of utilizing the indices into a probabilistic response assessment of RC flexural members accounting for different kinds of uncertainties is illustrated via four numerical examples which are verified through nonlinear dynamic finite element analysis. It is concluded that the probabilistic response of RC flexural members obtained from the developed approach have a similar distribution with those from probabilistic nonlinear finite element analysis.

Keywords: Equivalent SDOF system; Non-dimensional indices; Maximum displacement; Displacement ductility factor; Monte-Carlo simulation

1. Introduction and background

Information on the probabilistic responses of the maximum displacement and displacement ductility factor is of critical importance for the reliable design or analysis of reinforced concrete (RC) flexural members that might be affected by blast loadings [1–3]. The probabilistic analysis based on a combination of nonlinear dynamic finite element analysis of structural members under blast conditions and Monte-Carlo simulation is computationally considerably more expensive due to the significant geometric and material nonlinearity of RC members [4–6]. Therefore, a simple and efficient way for probabilistic evaluation of the blast responses for RC members is necessary.

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The maximum displacement and displacement ductility responses of RC flexural members under blast loadings that are simplified into triangular pulses can be determined in an approximate way by transforming the continuous members into equivalent single-degree-of-freedom (SDOF) systems by taking these responses of equivalent systems to be equal to those of the members [7]. However, considering the complicated material behaviours of the concrete and reinforcement under blast conditions, some differences do exist between the actual responses \(y_{rc}^m\) and \(y_{rc}^l\) of a RC structural member and those \(y_{eq}^m\) and \(\mu_{eq}^c\) of the equivalent SDOF system as indicated in Fig. 1. But if the indices quantifying the above differences can be obtained, they will provide a simpler way to the estimation of the deformation response of the RC flexural member under the blast conditions from those of the equivalent SDOF system and without performing nonlinear finite element analysis.

This paper aims to present a simple approach for the development of response surfaces for \(y_{rc}^m\) and \(\mu_{rc}^c\) of RC flexural members against blast loadings. For this purpose, the distribution of two non-dimensional indices quantifying the differences between the responses of RC members and those of equivalent SDOF systems are

<table>
<thead>
<tr>
<th>Nomenclature</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>(d)</td>
<td>effective depth of the RC member measured from the extreme compression fiber to the centroid tensile reinforcement</td>
</tr>
<tr>
<td>(E_{dc})</td>
<td>dynamic Young’s modulus of elasticity for concrete</td>
</tr>
<tr>
<td>(E_{ds})</td>
<td>dynamic Young’s modulus of elasticity for steel</td>
</tr>
<tr>
<td>(E_{\epsilon y}, E_{\epsilon l})</td>
<td>mean values of (\epsilon_y) and (\epsilon_l), respectively</td>
</tr>
<tr>
<td>(\epsilon_y, \epsilon_l)</td>
<td>nominal random variables</td>
</tr>
<tr>
<td>(F)</td>
<td>blast load from idealized triangular pulse</td>
</tr>
<tr>
<td>(f_{dc})</td>
<td>strength of concrete in compression in dynamic conditions</td>
</tr>
<tr>
<td>(f_{dv})</td>
<td>dynamic stirrup yield strength</td>
</tr>
<tr>
<td>(f_{ds})</td>
<td>strength of flexural steel in dynamic conditions</td>
</tr>
<tr>
<td>(i)</td>
<td>impulse</td>
</tr>
<tr>
<td>(K_{LE}, K_{ME})</td>
<td>transformation load and mass factor, respectively</td>
</tr>
<tr>
<td>(l)</td>
<td>length of the member</td>
</tr>
<tr>
<td>(P)</td>
<td>pressure</td>
</tr>
<tr>
<td>(R)</td>
<td>standoff distance</td>
</tr>
<tr>
<td>(t_d)</td>
<td>load duration</td>
</tr>
<tr>
<td>(W)</td>
<td>explosive charge mass expressed in kg of TNT</td>
</tr>
<tr>
<td>(y_{eq})</td>
<td>elastic displacement response of the equivalent SDOF system of the designed RC member due to blast loading</td>
</tr>
<tr>
<td>(y_{rc})</td>
<td>elastic displacement response of the designed RC member at the significant point due to blast loading</td>
</tr>
<tr>
<td>(y_{eq}^m)</td>
<td>maximum displacement response of the equivalent SDOF system of the designed RC member due to blast loading</td>
</tr>
<tr>
<td>(y_{rc}^m)</td>
<td>maximum displacement response of the designed RC member at the significant point due to blast loading</td>
</tr>
<tr>
<td>(y_t)</td>
<td>target displacement</td>
</tr>
<tr>
<td>(\zeta_{rc})</td>
<td>non-dimensional displacement index</td>
</tr>
<tr>
<td>(\zeta_{eq})</td>
<td>non-dimensional displacement ductility index</td>
</tr>
<tr>
<td>(\mu_{eq})</td>
<td>displacement ductility response of the equivalent SDOF system of the designed RC member under the given blast loading</td>
</tr>
<tr>
<td>(\mu_{rc})</td>
<td>displacement ductility response of the designed RC member at the significant point due to blast loading</td>
</tr>
<tr>
<td>(\mu_t)</td>
<td>target displacement ductility ratio</td>
</tr>
<tr>
<td>(\rho)</td>
<td>longitudinal tension reinforcement ratio</td>
</tr>
<tr>
<td>(\sigma_{\epsilon y}, \sigma_{\epsilon l})</td>
<td>standard deviations</td>
</tr>
</tbody>
</table>
firstly derived through extensive numerical studies and fitting. The utilization of these two indices for the probabilistic response assessment of $y_{rc}^m$ and $l_{rc}^m$ is further addressed by the combination of the Monte-Carlo simulation. As an illustration, the blast responses of four flexural members with different kinds of support conditions are probabilistically evaluated and numerically verified by comparing the results with those from probabilistically nonlinear finite element analysis.

2. Response differences between RC flexural members and equivalent SDOF systems

2.1. Definition of the non-dimensional indices quantifying the differences

To obtain a consistent measurement of the degree of the differences between the actual responses $(y_{rc}^m$ and $\mu_{rc}^c)$ for the flexural member under a certain blast loading and the their respective equivalent SDOF system $(y_{eq}^m$ and $\mu_{eq}^c)$, two non-dimensional indices are defined as

$$\frac{\varepsilon_{y}^{rc}}{\varepsilon_{y}} = \frac{y_{eq}^m - y_{rc}^m}{y_{eq}^m}$$

$$\frac{\varepsilon_{\mu}^{rc}}{\varepsilon_{\mu}} = \frac{\mu_{eq} - \mu_{rc}^c}{\mu_{eq}^c}$$

where $\varepsilon_{y}^{rc}$ is the non-dimensional displacement index standing for the difference between $y_{rc}^m$ and $y_{eq}^m$; $\varepsilon_{\mu}^{rc}$ is the non-dimensional displacement ductility index representing the difference between $\mu_{rc}^c$ and $\mu_{eq}^c$. Eqs. (1) and (2) describe the basic relationships between $(y_{rc}^m, \mu_{rc}^c)$, $(y_{eq}^m, \mu_{eq}^c)$ and $(\varepsilon_{y}^{rc}, \varepsilon_{\mu}^{rc})$. With $\varepsilon_{y}^{rc}$ and $\varepsilon_{\mu}^{rc}$ initially known, values of $y_{eq}^m$ and $\mu_{eq}^c$ can be obtained from $y_{rc}^m$ and $\mu_{rc}^c$. However, depending on these two equations to estimate $\varepsilon_{y}^{rc}$ and $\varepsilon_{\mu}^{rc}$ seems to have few advantages since $y_{eq}^m$ and $\mu_{eq}^c$ have to be firstly computed. This entails explosion testing or numerical finite element analysis of the RC members under blast conditions, which would be sure to be costly or involve intensive computational effort. Therefore, simple alternative formulae for the determination of $\varepsilon_{y}^{rc}$ and $\varepsilon_{\mu}^{rc}$ rather than Eqs. (1) and (2) need to be developed.

2.2. Symbolical expressions of the non-dimensional indices

By controlling the responses of the equivalent SDOF system $(y_{eq}^m$ and $\mu_{eq}^c)$ to be exactly equal to their respective design performance targets ($y_t$ and $\mu_t$), the design solution of effective depth ($d$) and longitudinal reinforcement ratio ($\rho$) for a RC flexural member can be specifically gained with the given basic variables including the peak reflected pressure ($P_r$) and duration of blast loadings ($t_d$), the support condition, the length ($l$) and the properties of concrete and reinforcement under the dynamic conditions ($f_{dc}$, $E_c$, $f_{ds}$, $E_d$, and $f_{dv}$) [13]. Accordingly, the actual responses $(y_{rc}^m$ and $\mu_{rc}^c)$ of the specifically designed member under the given blast loading can be specifically determined with the nonlinear finite element analysis. Considering the complicated
behaviours of a RC member under the blast condition, some discrepancies occur between the actual responses \((y_m^e, \mu^e)\) and the design performance targets \((y_1, \mu_1)\), which also represent the differences between the responses of the designed RC member and those of the equivalent SDOF system as shown in Fig. 2. The responses \((y_m^e, \mu^e)\) of the designed member can be determined and then substituted into Eqs. (1) and (2), respectively, to produce \(\xi_y^e\) and \(\xi_\mu^e\) to be the symbolical functions of these variables as

\[
\begin{align*}
\xi_y^e &= \xi_y^e(P, t_d, y_1, \mu_1, l, SC, f_{dc}, E_c, f_{ds}, E_s, f_{dv}, d, \rho) \\
\xi_\mu^e &= \xi_\mu^e(P, t_d, y_1, \mu_1, l, SC, f_{dc}, E_c, f_{ds}, E_s, f_{dv}, d, \rho)
\end{align*}
\]

(3)

(4) 2.3. Analytical procedures

Establishing the explicit expressions of Eqs. (3) and (4) is almost impractical since RC members will exhibit a significantly complicated geometric and material nonlinearity under most blast conditions. A curve fitting technique with a large amount of reliable data for \(\xi_y^e\) and \(\xi_\mu^e\), which are determined according to Eqs. (1) and (2) with \(y_m^e\) and \(\mu^e\) obtained from nonlinear dynamic finite element analyses of the designed members, is executed together with the statistical analyses so as to find the simplified explicit expressions of Eqs. (3) and (4). The procedure is listed as follows:

1. Select the type of support conditions for the RC members to be designed.
2. Sample the design variable vector of \(\{P, t_d, y_1, \mu_1, l, SC, f_{dc}, E_c, f_{ds}, E_s, f_{dv}\}\), where 2000 samples are randomly taken to ensure the accuracy of the statistical analysis.
3. Design the member with each sample of the design variable vector to obtain \(d, \rho\) and further \(\rho', \rho_v\) and \(b_w\) of a ratio of \(d\).
4. Repeat step (3) until 2000 sampled design cases are accomplished.
5. Select 500 design cases with \(\rho\) ranging from 0.31% to 2.2%.
6. Perform numerical analysis on the selected 500 design cases to find \(y_m^e\) and \(\mu^e\) with ABAQUS [18].
7. Compute \(\xi_y^e\) and \(\xi_\mu^e\) of the 500 design cases with Eqs. (1) and (2).
8. Plot the distributions of \(\xi_y^e\) and \(\xi_\mu^e\) versus these basic design variables.
9. Carry out the curve fitting of the distributions of \(\xi_y^e\) and \(\xi_\mu^e\) followed by statistical analyses.
10. Recommend the simplified formulae to estimate \(\xi_y^e\) and \(\xi_\mu^e\).
11. Change the type of SCs of the members and repeat the above steps.

2.3.1. Random selection of basic variables

In order to perform the curve fitting of the distributions of \(\xi_y^e\) and \(\xi_\mu^e\) to find their simplified formulae, the design variables involved in the second step of the above procedure are randomly sampled within their concerned ranges and is discussed as follows.
The blast loadings that can be reasonably idealized into triangular pulses are basically dependent on the explosive mass expressed in kg of TNT \( W \) and the standoff distance \( R \) for external explosions \([9,10]\). Therefore, the random selections of \( W \) and \( R \) rather than \( P_r \) and \( t_d \) is executed in their respective ranges from 50 kg to 1000 kg equivalent TNT and from 4 m to 15 m standoff distance for the blast resistant design in the following analysis. This provides a significantly wide range of blast incidents from the most severe loadings produced by 1000 kg TNT at 4 m distance (a scaled distance \( Z = 0.4 \text{ m/kg}^{1/3} \)) to the slightest loadings by 50 kg TNT at 15 m distance (\( Z = 4.1 \text{ m/kg}^{1/3} \)). For the more severe blast situations, the design of the structural system with the alternate load path in redistributing loadings seems to be more reasonable and economic. In this case the structural members immediately opposite the blast are allowed to fail, while in the situations of the lighter blasts, the blast loadings become less critical in the structural design. From \( W \) and \( R \), the blast loadings of reflected pressure \( (P_r) \) and impulse \( (i) \) can be obtained with CONWEP \([9]\) which has been developed based on TM5-855-1 \([10]\) and generally used in the blast-resistant design, while the loadings durations \( (t_d) \) are approximately obtained with \( 2i/P_r \).

The determination of the reasonable design targets of \( y_t \) and \( l_t \) should account for possible blast intensities within the blast resistant design of a particular RC member. Hence, the random selection of design targets \( (y_t \) and \( \mu_t ) \) is executed in their different ranges according to the various scopes of the blast impulses as shown in Table 1. Some overlaps are allowed for the ranges of \( y_t \) and \( \mu_t \) considering that there is no strict watershed for them at a particular impulse within the design. The maximum allowable \( y_t \) and \( \mu_t \) are taken, respectively, to be larger than the limit of \( 4\theta \) for non-laced RC members as recommended in current design guidance \([1–3,10–12]\).

It is to cover the possibility of the actual responses of the designed members in blast conditions being located on the conservative side when compared with their respective performance targets of \( y_t \) and \( \mu_t \).

The parameter of \( l \) is selected in a range from 4.5 m to 9.0 m for the RC members with two ends constrained while from 2.5 m to 4.5 m for members with one end free, thus covering the most typical cases for RC structures. Due to strain rate effects, the parameters of material properties of concrete and reinforcement under the dynamic conditions are implemented with the possible ranges listed in Table 2. Since the structural members exist in the context of structural systems, their real support conditions are complex and generally depend on the connection with the adjacent members as well as the construction requirements. However, in the blast resistant design they are generally idealized into one of the four types as listed in Table 3. Considering that RC members should be designed to resist the negative deflection or rebound of the members subsequent to its maximum positive deflection and that the possibility of the explosion occurring in the two opposite sides of the members to be designed, the area of compression reinforcement employed is equal to that of the tensile reinforcement area.

Based on the ranges of the variables discussed above, their random samplings are executed by assuming them equally frequent or equally likely to occur. This ensures the likelihood of a more general curve fitting of \( c_{y_t}^* \) and \( c_{\mu_t}^* \) which can incorporate the effects of these variables as adequately as possible.

### Table 1
Various design targets

<table>
<thead>
<tr>
<th>Impulse (kPa ms)</th>
<th>( \leq 10^3 )</th>
<th>( 10^3–10^4 )</th>
<th>( 10^4–2 \times 10^4 )</th>
<th>( 2 \times 10^4–3 \times 10^4 )</th>
<th>( \geq 3 \times 10^4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \theta_t (\degree) )</td>
<td>1–1.8</td>
<td>1.4–3.2</td>
<td>2.8–3.6</td>
<td>3.2–4.0</td>
<td>3.4–4.8</td>
</tr>
<tr>
<td>( \mu_t )</td>
<td>3.0–8.5</td>
<td>4.5–10.5</td>
<td>5.5–12.5</td>
<td>6.0–14.5</td>
<td>7.0–16.5</td>
</tr>
</tbody>
</table>

\( *_{y_t} = Itan(\theta_t)/2 \) for the member with two ends constrained.
\( *_{y_t} = Itan(\theta_t) \) for member with one end free.

### Table 2
Various material properties

<table>
<thead>
<tr>
<th>Variable</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_c^* ) (MPa)</td>
<td>30.0–40.0</td>
</tr>
<tr>
<td>( E_c ) (GPa)</td>
<td>25.5–30.5</td>
</tr>
<tr>
<td>( f_s^* ) (MPa)</td>
<td>460.0–560.0</td>
</tr>
<tr>
<td>( E_s ) (GPa)</td>
<td>190.0–210.0</td>
</tr>
<tr>
<td>( f_v^* ) (MPa)</td>
<td>250.0–300.0</td>
</tr>
</tbody>
</table>

\( *_{f_c} = 1.19 \times f_c \)
\( *_{f_s} = 1.17 \times f_s, f_{dv} = 1.17 \times f_v \).
2.3.2. Nonlinear finite element models

The sixth step of the above analytical procedure (the numerical analysis on the designed members to find $y_{rc}$ and $l_{rc}$) is executed with the program ABAQUS [18]. The failure of RC members under blast conditions is characterized by concrete crushing accompanied by concrete cracking, thus the smeared cracking model for concrete is utilized. In this model, cracking appears when the maximum principal tensile stresses reach a failure surface. To simulate the softening effect of the concrete in tension, a bilinear tension stress–strain curve is used after cracking as shown in Fig. 3, where the failure strain $e_{ur}^c$ is taken as $10^{-3}$. The selection of this value is based on the assumption that the strain softening after failure reduces the stress linearly to zero at a total strain of about 10 times the strain at failure of concrete in tension, which is typically $10^{-4}$ in the standard concretes [8]. The tensile strength $f_t$ is determined from the compressive strength $f_c$ as [19]

$$f_t = 0.30f_c^{2/3}$$

In the case of concrete in compression, concrete is simulated with an elastic–plastic mode and the elastic stress state is limited by a yield surface. Once yielding has occurred, an associated flow rule with isotropic hardening is used. The yield surface is determined in terms of the first two stress invariants

$$f = q - \sqrt{3} a_0 p - \sqrt{3} \sigma_c(e_{uniaxial}^p)$$

where $p$ is the effective pressure, $q$ is Mises equivalent deviatoric stress, and $a_0$ is a constant, which is chosen from the ratio of the ultimate stress reached in biaxial compression to the ultimate stress reached in uniaxial compression. $\sigma_c(e_{uniaxial}^p)$ is the hardening (and softening) parameter that is defined from the uniaxial compression data of the concrete as a function of the plastic strain [19]. The stiffness used in the analysis for unloaded concrete in tension and compression is also given in Fig. 3. When the cracked concrete is unloaded, the secant unloading modulus is utilized as stiffness so that the strain across the crack is reduced linearly to zero as the stress approaches zero. If the load is removed at some point after inelastic straining has occurred for the concrete in compression, the unloading response is softer than that of the initial elastic response, but this effect is ignored in this model. Thus initial elastic stiffness is used when the concrete in compression is unloaded.

The Von-Mises yield criterion is used to describe the constitutive behaviour of the reinforcement. The stress and strain relationship of reinforcement is shown in Fig. 3, where the reinforcement is modelled with an elastoplastic curve. The strain hardening of reinforcement is not considered in this analysis since it is hard to define under the blast conditions due to a lack of sufficient experimental data plus the ultimate strain value is often not reported in the current literatures due to the difficulty of determining exactly when the peak stress occurs as well as the confusion between ultimate strain and rupture strain.

Considering that both the concrete and reinforcement exhibit increased strengths under higher loading rates, the expressions of dynamic increase factors (DIFs) by Malvar and Crawford [20,21] are adopted. These
expressions are derived from the literature review of the extensive test data concerning the effects of strain rate on the strength of concrete and reinforcement. For the concrete compressive strength, DIF is given as

\[
DIF = \begin{cases} 
(\dot{\varepsilon}/\dot{\varepsilon}_a)^{0.026x_a}, & \dot{\varepsilon} \leq 30 \text{ s}^{-1} \\
\gamma_s(\dot{\varepsilon}/\dot{\varepsilon}_a)^{1/3}, & \dot{\varepsilon} > 30 \text{ s}^{-1}
\end{cases}
\]

where \(\dot{\varepsilon}\) is the strain rate in the range of \(30 \times 10^{-6} - 300 \text{ s}^{-1}\); \(\dot{\varepsilon}_a = 30 \times 10^{-6} \text{ s}^{-1}\) (static strain rate); \(\log_{10}\gamma_s = 6.156 x_a - 2;\) \(x_a = 1/(5 + 9 f_c/f_{co})\); \(f_{co} = 10 \text{ MPa}\); \(f_c\) is the static compressive strength of concrete. For the concrete in tension, the formula is:

\[
DIF = \begin{cases} 
(\dot{\varepsilon}/\dot{\varepsilon}_a)^{\delta}, & \dot{\varepsilon} \leq 1.0 \text{ s}^{-1} \\
\beta(\dot{\varepsilon}/\dot{\varepsilon}_a)^{1/3}, & \dot{\varepsilon} > 1.0 \text{ s}^{-1}
\end{cases}
\]
where \( \dot{\varepsilon} \) is the strain rate in the range of \( 10^{-6} \text{--} 160 \text{ s}^{-1} \); \( \dot{\varepsilon}_s = 10^{-6} \text{ s}^{-1} \); \( \log \beta = 6 \dot{\varepsilon} - 2 \); \( \delta = 1/(1 + 8 f_c/f_{co}) \); \( f_{co} = 10 \text{ MPa} \). The DIF formula for the yield stress of reinforcement is:

\[
\text{DIF} = (\dot{\varepsilon}/10^{-4})^\alpha
\]  

where \( \alpha = \alpha_{fs} \) and \( \alpha_{fs} = 0.074 - 0.04f_{/414} \). This formula is valid for reinforcement with yield stresses between 290 and 710 MPa and for strain rates between \( 10^{-4} \text{ and } 225 \text{ s}^{-1} \). To integrate Eqs. (7)–(9) into the analysis, the user subroutine USDFLD in ABAQUS is used which allows the user to define the field variable at a material point as a function of any of the available material point quantities. Thus by taking the strain rate as a field variable, the strain rate-dependent material properties can be introduced in the analysis since such properties can be easily defined as functions of strain rate with Eqs. (7)–(9). Timoshenko beam elements are assigned to model the members while the rebar option is utilized to place each reinforcement at its exact location while perfect bond is assumed between the reinforcement and the concrete.

![Fig. 4. The details of the simply-supported beam tested by Seabold [22].](image)

![Fig. 5. Verification of finite element models showing comparison between the numerical and experimental results.](image)
The verification of the finite element models as mentioned above is carried out by implementing it into the analysis of a simply supported RC beam subjected to blast loadings tested by Seabold [22] as shown in Fig. 4. The computed and experimental displacement–time history at mid-span is shown for a comparison in Fig. 5. It can be observed that the peak experimental response of 28.5 mm was recorded at the time of 19.5 ms, which closely agreed with the analytical results for which the computed peak displacement of 28.8 mm is reached at the time of 19.2 ms. The recorded permanent deformation of the RC beam was 20.8 mm and it also matched well with the predicted deflection of 21.7 mm. In addition, the analytical and experimental results of the mid-span velocity history, the top compressive strain history of concrete, plus the strain history of the main reinforcement of the beam are also compared in Fig. 5. It is demonstrated that the numerical analysis has a good agreement with the observed experimental behaviours and thus the numerical model has the ability to simulate the failure process of concrete and reinforcement.

2.4. Distributions of the non-dimensional indices

Based on the above analytical procedure, the distributions of $\gamma^c_\mu$ and $\gamma^c_\mu$ for the designed RC members versus $\rho$ under various support conditions are obtained in Figs. 6–9. It can be observed that the distributions of $\gamma^c_\mu$ are generally concentrated in the strips that are much narrower and closer to the X-axis as compared to those of $\gamma^c_\mu$ especially in the situation of the first three types of support conditions. This indicates that the discrepancies between $\gamma^c_\mu$ and $\gamma^c_\mu$ are much smaller than those of $\mu^c$ with $\mu^c$. This result is reasonable since $\mu^c$ is determined by not only $\gamma^c_\mu$ but also the elastic displacement of $\gamma^c_\mu$ of the member. As a continuous RC member is converted into an equivalent SDOF system with its resistance function simplified by a bilinear curve, the elastic displacement of the member in a blast incident is significantly underestimated, which leads to the relatively large positive values of $\gamma^c_\mu$.

The convergent distributions of $\gamma^c_\mu$ and $\gamma^c_\mu$ with respect to $\rho$ under any support condition as shown in Figs. 6–9 indicate that the importance of parameter $\rho$, which can be conceptually accounted for from two aspects. In the calculation of the longitudinal reinforcement ratio ($\rho$) from the ultimate strength $R_m$, the contribution of concrete to $R_m$ is ignored within the design. For the members with a small value of $\rho$ where the concrete becomes predominant in resisting external force, this lack of attention will lead to a significantly conservative design and therefore higher conservative values for $\gamma^c_\mu$ and $\gamma^c_\mu$. With an increase in $\rho$, the contribution of concrete to $R_m$ becomes smaller and so does the conservative degree of design. It also agrees quite well with the trend of $\gamma^c_\mu$ with respect to $\rho$ under all support conditions and $\gamma^c_\mu$ to $\rho$ under the fourth support conditions. The second reason that makes $\rho$ important to $\gamma^c_\mu$ and $\gamma^c_\mu$, is related to the calculation of the stiffness of RC members. The stiffness calculated are influenced by a coefficient determined by $\rho$ from the empirical data fitting curve, applied to obtain the cross-sectional moment of inertia along RC members.

![Fig. 6. Distributions of non-dimensional indices with longitudinal reinforcement ratio for simply supported members.](image-url)
The support condition of a structural member also plays an important role in producing $\gamma_{rc}$ and $\mu_{rc}$, which is demonstrated by the trends of $\gamma_{rc}$ and $\mu_{rc}$ with $\rho$ under various support conditions. Comparing Figs. 6–8 with Fig. 9 indicates that both $\gamma_{rc}$ and $\mu_{rc}$ for fixed/fixed members are generally larger than those of members under other support conditions with the same $\rho$. This difference is induced by the application of the unreasonable transformation factors $K_{LE}$ and $K_{ME}$ in the design of fixed/fixed members. Fig. 10 shows the transformation of this type of continuous RC member into the equivalent SDOF system can be divided into three stages with different values of transformation factors at each stage: the elastic stage with no plastic hinges, the elastic-plastic stage when two plastic hinges occurs at the two ends of the member and the purely plastic stage where a third plastic hinge appears in the mid-span. In the second stage, transformation factors are taken with the same values as those for a simply supported member. However, the responses of a member with two ends pinned (plastic hinges formed at both ends) will be surely smaller than those of a simply supported member. This means the responses of fixed/fixed members are overestimated during the design thus leading to relatively larger $\gamma_{rc}$ and $\mu_{rc}$.

The variables other than $\rho$ and support conditions have some minor effects, which make the values of $\gamma_{rc}$ and $\mu_{rc}$ slightly scattered as $\rho$ and support condition are certain. Although these effects of other variables are not significant, they are also incorporated in the recommended formulae while considering all variable effects.
2.5. Formulae for the non-dimensional indices

In order to obtain the simplified expressions of Eqs. (3) and (4), the relevant variables are divided into two groups: the important variables of $q$ and support condition, and the secondary variables including $P_r$, $l$, $y$, $\mu$, and $d$. To incorporate the influences of $q$ and support condition into the expressions, the nonlinear curve fittings of $\zeta_{rc}^y$ and $\zeta_{rc}^\mu$ versus $q$ for various support conditions as plotted in Figs. 6–9, are carried out where the functions are expressed as $f(q, SC)$ and $g(q, SC)$, respectively. The effects of the secondary variables are dealt with by introducing the two nominal random variables of $e_y$ and $e_\mu$ that are assumed to represent the deviation of $\zeta_{rc}^y$ and $\zeta_{rc}^\mu$ around the fitting curves. As a consequence, Eqs. (3) and (4) are simplified into

\[
\begin{align*}
L_{plastic} - L_{elastic} & - 25.0 \times K_0 + K_L - K_M = 0.41, \\
L_{plastic} - L_{elastic} & - 25.0 \times K_0 + K_L - K_M = 0.50, \\
L_{plastic} - L_{elastic} & - 25.0 \times K_0 + K_L - K_M = 0.33
\end{align*}
\]

where

\[
K_0 = \left( K_{L,\text{elastic}} + K_{L,\text{plastic}} \right) \times 0.5 + K_{L,\text{plastic}},

K_0^L = \left( K_{L,\text{elastic}} + K_{L,\text{plastic}} \right) \times 0.5 + K_{L,\text{plastic}}/2
\]

\[
k_0 = \frac{E}{I} K_{L,\text{elastic}},

M = K_{M,\text{elastic}} l b d
\]

Fig. 10. Stages of fixed/fixed members in the determination of transformation factors [7,11,12].
Fig. 11. The distributions of $e_y$ and $e_\mu$ for RC members with different support conditions.

\[ \gamma_{rc} = f(\rho, SC) + e_y \]  
\[ \gamma_{rc} = g(\rho, SC) + e_\mu \]
A quadratic function is selected for the curve fitting given by

\[
f(q, SC) = a_1 + a_2 q + a_3 q^2
\]

With the result functions of \( f(q, SC) \) and \( g(q, SC) \), the nominal random variables of \( e_y \) and \( e_\mu \) can be obtained with

\[
e_y = \xi_y^{rc} - f(q, SC)
\]

\[
e_\mu = \xi_\mu^{rc} - g(q, SC)
\]

For members with various support conditions, the results of the parameters \( a_1, a_2 \) and \( a_3 \) are listed in Table 4 while the histograms of \( e_y \) and \( e_\mu \) are shown in Fig. 11. It is demonstrated that \( e_y \) and \( e_\mu \) follow normal distribution with the mean values (\( E_y \) and \( E_\mu \)) of \( e_y \) and \( e_\mu \), respectively, approximate to zero. The standard deviations (\( \sigma_y \) and \( \sigma_\mu \)) of \( e_y \) and \( e_\mu \), respectively, observed are relatively minor thus indicating the limited effect of the variables other than \( q \) and support condition.

### 3. Response evaluation depending on formulae of indices

The importance of applying \( \xi_y^{rc} \) and \( \xi_\mu^{rc} \) is that they provide a simpler way to predicting of the responses \( y_m^{rc} \) and \( \mu^{rc} \) for RC flexural member under a possible blast loading without carrying out nonlinear finite element analysis. Substituting Eqs. (10) and (11) into Eqs. (1) and (2), respectively, and rearranging the terms, the response surface of \( y_m^{rc} \) and \( \mu^{rc} \) are obtained as

\[
y_m^{rc} = y_m^{eq}(1 - f(q, SC) - e_y)
\]

\[
\mu^{rc} = \mu^{eq}(1 - g(q, SC) - e_\mu)
\]
Table 5
Details of four RC flexural members

<table>
<thead>
<tr>
<th>Example</th>
<th>SC</th>
<th>$l$ (m)</th>
<th>$d$ (mm)</th>
<th>$\rho'$ (%)</th>
<th>$f_{dc}$ (MPa)</th>
<th>$f_{ds}$ (MPa)</th>
<th>$E_c$ (GPa)</th>
<th>$E_s$ (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>Simply-supported</td>
<td>5.0</td>
<td>450</td>
<td>0.70</td>
<td>36</td>
<td>520</td>
<td>28</td>
<td>210</td>
</tr>
<tr>
<td>II</td>
<td>Fixed/free</td>
<td>4.0</td>
<td>400</td>
<td>1.60</td>
<td>40</td>
<td>506</td>
<td>30</td>
<td>200</td>
</tr>
<tr>
<td>III</td>
<td>Fixed/roller-supported</td>
<td>4.5</td>
<td>350</td>
<td>2.00</td>
<td>40</td>
<td>550</td>
<td>30</td>
<td>200</td>
</tr>
<tr>
<td>IV</td>
<td>Fixed/fixed</td>
<td>6.0</td>
<td>300</td>
<td>1.00</td>
<td>36</td>
<td>520</td>
<td>28</td>
<td>210</td>
</tr>
</tbody>
</table>

Table 6
Random variables for the members

<table>
<thead>
<tr>
<th>Variables</th>
<th>Distribution</th>
<th>COV</th>
<th>Lower and upper limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{dc}$</td>
<td>Normal</td>
<td>0.08</td>
<td></td>
</tr>
<tr>
<td>$f_{ds}$</td>
<td>Normal</td>
<td>0.08</td>
<td></td>
</tr>
<tr>
<td>$E_c$</td>
<td>Normal</td>
<td>0.15</td>
<td></td>
</tr>
<tr>
<td>$E_s$</td>
<td>Normal</td>
<td>0.15</td>
<td></td>
</tr>
<tr>
<td>$A_s$</td>
<td>Uniform</td>
<td>0.05</td>
<td></td>
</tr>
<tr>
<td>$d$</td>
<td>Uniform</td>
<td>0.05</td>
<td></td>
</tr>
<tr>
<td>$l$</td>
<td>Uniform</td>
<td>0.05</td>
<td></td>
</tr>
<tr>
<td>$W$</td>
<td>Uniform</td>
<td></td>
<td>50–500 kg</td>
</tr>
<tr>
<td>$R$</td>
<td>Uniform</td>
<td></td>
<td>5–15 m</td>
</tr>
</tbody>
</table>

Fig. 13. Probabilistic results of the simply supported RC member (Example I).
According to Eqs. (15) and (16), $y_{rc}^m$ and $\mu_{rc}$ of the member under a blast loading can be estimated from the corresponding $y_{eq}^m$ and $\mu_{eq}$, which are computationally simple and straightforward. By converting the continuous member into an equivalent SDOF system [7], these parameters including equivalent mass ($m$), equivalent force ($F_1$), equivalent initial stiffness ($k_e$), equivalent ultimate strength ($R_m$), natural period ($T$), elastic displacement ($y_{eq}^e = R_m/k_e$), and elastic energy ($E_{el} = ky_{eq}^e/2$) can be obtained. Accordingly, $y_{eq}^m$ and $y_{eq}^e$ are effortlessly solved with

$$y_{eq}^m = \frac{E_{max}}{k_e y_{eq}^e} + \frac{y_{eq}^e}{2} \quad \text{and} \quad \mu_{eq} = \frac{y_{eq}^m}{y_{eq}^e}$$

(17)

where $E_{max}$ is computed with the non-dimensional energy factor ($C = E_{max}/E_{el}$) found from the energy spectra according to the ratios $t_d/T$ and $F_1/R_m$ [13]. The combination of Eqs. (15)–(17) with the probabilistic methods such as the Monte-Carlo simulation provides a simpler probabilistic solution for $y_{rc}^m$ and $\mu_{rc}$ of RC flexural members in blast conditions taking different uncertainties into account.

However, it should be pointed out that unlike the situation where the responses of $y_{rc}^m$ and $\mu_{rc}$ can be more accurately determined through nonlinear finite element analysis, Eqs. (15) and (16) only provide an approximate way to predict $y_{rc}^m$ and $\mu_{rc}$ from the respective $y_{eq}^m$ and $\mu_{eq}$ since $e_y$ and $e_\mu$ are two uncertain nominal random variables following normal distribution. Considering the random effects of $e_y$ and $e_\mu$, the coefficients of variation (COV) for $y_{rc}^m$ and $\mu_{rc}$ are obtained as

$$\text{COV}(y_{rc}^m) = \frac{\sigma_{e_y}}{1 - f(\rho, SC)}$$

(18)

$$\text{COV}(\mu_{rc}) = \frac{\sigma_{e_\mu}}{1 - g(\rho, SC)}$$

(19)

![Fig. 14. Probabilistic results of the fixed/free RC member (Example II).](image-url)
The COVs for $y_{rc}^m$ and $\mu_{rc}$ have no relationship with their respective $y_{eq}^m$ and $\mu_{eq}$ and they tend to change with $\rho$ and support condition of the member as shown in Fig. 12. It is shown that the maximum COVs of $y_{rc}^m$ for the simply supported, fixed/roller-supported and fixed/free members are generally less than 0.03, representing a significant concentration of $y_{rc}^m$ around their mean values. The COVs of $\mu_{rc}$ for these types of members range from 0.05 to 0.18 and rise gradually with the decline in $\rho$. For the fixed/free members, the predicted $y_{rc}^m$ is also concentrated around its mean values with the maximum COV less than 0.08, however, the $\mu_{rc}$ predicted from Eq. (16) has some deviation when $\rho$ is less than 1.0%.

4. Numerical illustration and verification

For illustration, $y_{rc}^m$ and $\mu_{rc}$ of four RC flexural members with different support conditions, whose details are shown in Table 5, are probabilistically evaluated using response surface with Eqs. (15) and (16) together with Monte-Carlo simulation [14]. The verifications of numerical results obtained are further performed as compared to the probabilistically nonlinear finite element analytical outcomes.

The uncertainties of different variables including $f_{ds}$, $f_{dc}$, $E_s$, $E_c$, $A_s$, $d$, and $l$ are considered in the probabilistic response assessment of these members. Their means values are taken as those employed in Table 5 while their COVs are listed in Table 6 [15–17]. The blast loading brings out the most uncertainty to the structural responses. However, to date not enough information is available on the random properties of blast loadings applied to a structure in its lifetime. In this example it is assumed that $W$ and $R$ follow uniform distributions with the ranges as shown in Table 6. The parameters of $\varepsilon_y$ and $\varepsilon_l$ are two additional independent random variables that link $y_{rc}^m$ and $\mu_{rc}$ with the respective $y_{eq}^m$ and $\mu_{eq}$. Their distributions can be found in Fig. 11 with the zeros of mean values and the corresponding standard deviations at various support conditions.

![Fig. 15. Probabilistic results of the fixed/roller-supported RC member (Example III).](image-url)
The probabilistic response assessment of the four RC members involving the uncertainty of the above variables is accomplished through Monte-Carlo simulation. The choice of Monte-Carlo simulation for this situation is justified because the approach to predict $y_{rc}^{m}$ and $\mu_{rc}^{l}$ using the response surface with Eqs. (15) and (16) is very simple. As a consequence, the simulation sample size can be selected relatively widely so as to achieve an accurate estimation without consuming much computational time and work. The probability density functions (PDFs) of $y_{rc}^{m}$ and $l_{rc}^{l}$ for these concerned members and the corresponding cumulative distribution functions (CDFs) are computed and shown in Figs. 13–16. To verify the numerical results obtained, the probabilistic nonlinear finite element analyses of these examples are also performed and compared with the probabilistic analysis as shown in Figs. 13–16. Summarizing the results, it can be seen that the PDFs and CDFs of $y_{rc}^{m}$ and $\mu_{rc}^{l}$ predicted using the response surface are in close agreement with those from the probabilistically nonlinear finite element analysis with the exception of $\mu_{rc}^{l}$ for the fixed/fixed member. The inconsistency in probabilistic distribution of $\mu_{rc}^{l}$ for the fixed/fixed member is mainly induced by the complicated resistance functions of fixed/fixed members. The CDFs of $\mu_{rc}^{l}$ predicted for the fixed/fixed member are on the conservative sides to some extent as compared to the probabilistically nonlinear finite element analysis.

5. Conclusions

A simple yet efficient approach of the probabilistic assessment of $y_{rc}^{m}$ and $\mu_{rc}^{l}$ for RC flexural members under blast conditions is presented without carrying out probabilistically nonlinear finite element analysis. This approach is accomplished depending on the formulae of two indices ($\gamma_{m}^{rc}$ and $\gamma_{p}^{rc}$) that quantify the difference between the responses of RC members with those of equivalent SDOF systems.
The distribution of \( r_c \) and \( r_{\mu} \) with different variables that have been selected randomly from their particular ranges within the blast-resistant design is generated through a large amount of numerical analyses. By comparing these distributions, it is found that the support condition and \( q \) have profound effects on \( r_c \) and \( r_{\mu} \), while the variables other than support condition and \( q \) have some minor influences. To incorporate all variables effects, the formulae of \( r_c \) and \( r_{\mu} \) are recommended including two items: a nonlinear fitting function correlated with \( q \) and support condition and a random variable (\( e_y \) and \( e_{\mu} \)) to consider the influence of the others.

Computation of \( r_c \) for RC flexural members under a blast loading becomes much simpler based on the formulae of \( r_c \) and \( r_{\mu} \) since they can be easily derived from the corresponding \( m_{eq} \) and \( e_{eq} \) that are computationally straightforward. By combining this approach with Monte-Carlo simulation, the probabilistic blast responses of RC members are conveniently evaluated accounting for different types of uncertainties. The PDFs and CDFs of \( r_c \) obtained from \( r_c \) and \( r_{\mu} \) have a good agreement with those from probabilistically nonlinear finite element analysis as indicated from numerical examples.

References

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