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Confining Reinforcement for High-Strength Concrete Columns

by Bing Li and R. Park

A comprehensive parametric study of reinforced high-strength concrete (HSC) columns is undertaken to verify the current ACI 318 and NZS 3101 code provisions of confining reinforcement in the potential plastic hinge regions of HSC columns. A full range of parameters that have significant influences on the available strength and curvature ductility factor in the potential plastic hinges of the columns during cyclic flexure are examined. These include the concrete compressive strength \( f'_c \), the yield strength of transverse reinforcement \( f_{yw} \), the volume ratio of the transverse reinforcement, the area ratio of the longitudinal reinforcement to the gross section area \( \rho \), and the ratio of \( A_g/A_c \). Previously derived stress-strain relationships for compressed HSC confined by various quantities, yield strengths, and arrangements of transverse reinforcement are used in cyclic moment-curvature analyses of a range of reinforced HSC columns to derive design equations. The proposed design equations are able to determine the quantities of transverse reinforcement required for the specified curvature ductility demands of HSC columns. The accuracy of the proposed design equations was checked by comparing with experimental data reported in previous literature, and a satisfactory correlation was found.

**Keywords:** column; ductility; high-strength concrete; reinforced concrete; reinforcement; steel.

**INTRODUCTION**

The increasing use of high-strength concrete (HSC) in construction in North America, Japan, and Australia has been due to a number of beneficial outcomes from its use in reinforced concrete building structures. One obvious use of HSC is in the columns of tall structures. Most of the structures that have been designed and constructed using HSC make use of that concrete in axial load-dominated members. For a given load, the HSC column has a smaller cross-sectional area, thus providing more floor space. The use of HSC together with high-yield-strength steel also appears to be an attractive proposition for the building structures. Many buildings throughout the world that are under design or construction incorporate HSC together with high yield strength reinforcement.

With the possible increased use of HSC for general structural applications, a concern is whether HSC columns can be designed to have a predictable strength and to behave in a ductile manner under severe seismic loading. Many current national seismic design code provisions are not intended for concrete with a compressive strength greater than 55 MPa. Figure 1 shows the relationship of the strength of materials and the fields of research and development. In the last three decades, most of the research conducted in New Zealand and the U.S. has been in the A region. Therefore, the present design provisions in the NZS 3101\(^1\) and ACI 318\(^2\) need to be revised considering the case of high-strength materials. HSC tends to be much more brittle and has much less postpeak deformability than normal-strength concrete. Previous tests on reinforced HSC columns by Li, Park, and Tanaka\(^3\) have confirmed that satisfactory ductile behavior is not achieved in the plastic hinge regions of HSC column units with confining reinforcement designed according to the NZS 3101\(^1\) and ACI 318.\(^2\) There are still many regions such as B, C, D, E, and F, shown in Fig. 1 that are worthy of much further research, for the current code to be modified so as to be applicable to all material regions. It is to be noted that reinforcing steel with yield strengths of up to 1300 MPa are available. Therefore, a critical examination of the validity of code provision when applied to HSC may lead to the development of more rational provisions as well as a better understanding of the current empirical formula.

The present paper describes an analytical procedure developed to derive equations for the quantities of transverse reinforcement required for HSC confinement to achieve specified strength and ductility levels in the potential plastic hinge regions of HSC columns. The derived equations are reasonably accurate, reflecting the influence of the significant variables such as the concrete’s compressive strength, the axial load level, and the yield strength of the transverse reinforcement.

**RESEARCH SIGNIFICANCE**

HSC columns require large quantities of transverse reinforcement to achieve ductile behavior. One obvious way of reducing
the congestion of the transverse reinforcement would be to use high-yield-strength steel. Current confinement requirements, which were originally derived from experimental and theoretical results on normal-strength concrete together with normal yield strength steel, may not be directly suited for HSC columns. In this paper, an analytical investigation to determine a more appropriate design equation for the quantities of confining reinforcement required in the potential plastic hinge regions of HSC columns is conducted. A designer will be able to select the quantities of transverse confining reinforcing reinforcement necessary to sustain the ultimate curvatures anticipated in plastic hinge regions during severe or moderate earthquake loading.

PREVIOUS DESIGN EQUATIONS FOR CONFINING REINFORCEMENT IN PLASTIC HINGE REGIONS OF HSC COLUMNS

While improvement in the strength and ductility of reinforced concrete columns as a result of transverse reinforcement have been confirmed by many investigators, the amount of confining steel required in the form of circular or rectangular transverse reinforcement is still the subject of controversy. Added to this is a lack of agreement on the criteria for satisfactory behavior of columns during an earthquake, which has resulted in vastly different requirements for confining reinforcement in the ACI 318 and NZS 3101 codes for the amount and distribution of confining reinforcement in columns necessary to ensure adequate ductility. The ACI 318 and NZS 3101 approaches are described as follows.

ACI 318 Code provisions

To assure adequate ductility of reinforced concrete columns subjected to severe seismic excitation, the ACI 318 Building Code stipulates that the amount of confining reinforcement be designed as follows.

For circular columns, the volume ratio of spirals is given by the larger of

$$\rho_s = 0.45 \left( \frac{A_g}{A_c} - 1 \right) \frac{f_{yh}'}{f_{yh}}$$  \hspace{1cm} (1)$$

and

$$\rho_s = 0.09 \frac{f_{yh}'}{f_{yh}}$$  \hspace{1cm} (2)$$

where $A_g$ and $A_c$ are the gross and the confined core cross-sectional areas, respectively.

For rectangular columns, the required area of hoop reinforcement is given by the larger of

$$A_{sh} = 0.3\pi h \frac{f_{yh}'}{f_{yh}} \left( \frac{A_g}{A_c} - 1 \right)$$  \hspace{1cm} (3)$$

where $A_{sh}$ is the total cross-sectional area of ties including cross ties.

The current ACI Code equations are based on a philosophy of preserving the ultimate strength of axially loaded columns after the spalling of the concrete cover rather than emphasizing the ultimate deformation of columns subject to combined axial load and bending. However, this will not necessarily result in adequate curvature ductility, and a more logical approach would be to relate the required amount of confining steel to the curvature ductility demand, using a moment-curvature analysis that takes into account the suitable stress-strain relation for the confined concrete. This approach would ensure a sufficient amount capacity at the required curvature ductility.

NZS 3101:1995 Code provision

The NZS 3101:1995 requires that the transverse reinforcing steel in the plastic hinge region be as follows. For columns with rectangular hoops with or without cross ties,

$$\frac{A_{sh}}{s_y h_c} = \left[ \frac{A_g \frac{1.3 - \rho_m f_{yh}'}{3.3}}{A_c} \right] \left( \frac{P_e}{\phi f_{cy} A_g} \right) - 0.006$$  \hspace{1cm} (5)$$

For columns with circular hoops or spirals,

$$\rho_s = \left[ \frac{A_g \frac{1.3 - \rho_m f_{yh}'}{2.4}}{A_c} \right] \left( \frac{P_e}{\phi f_{cy} A_g} \right) - 0.0084$$  \hspace{1cm} (6)$$

In Eq. (5) and (6), the maximum value of $\rho_m$ that can be substituted is 0.4. Also, $A_g/A_c$ is not permitted to exceed 1.5 unless it can be shown that the design strength of the core of the column can resist the design axial load applied concentrically. Equation (5) and (6) were derived through cyclic moment-curvature analysis by Watson, Zahn, and Park.4

It is noted that the NZS 3101 design equations for the quantities of transverse confining reinforcement derived by Watson, Zahn, and Park4 were for concrete with a compressive strength that ranged from 20 to 40 MPa, and for yield strength of confining reinforcement that was less than 400 MPa. A property of HSC is that at high stresses, it exhibits less internal microcracking than normal-strength concrete for a given imposed axial strain. The lower lateral dilation of HSC means that confinement from transverse reinforcement may be less effective. Li, Park, and Tanaka5 confirmed that the stress in steel helices at the peak load of a HSC column might be less than the yield strength of the transverse steel if high-yield strength of transverse reinforcement is used. Hence, NZS 3101 design equations might not be adequate for HSC columns confined by transverse reinforcement that had yield strength beyond 400 MPa. Therefore, NZS 3101’s design equations need to be modified to make them applicable for use with HSC and also for use high yield strength confining reinforcement.

PREVIOUS RESEARCH ON STRENGTH AND DUCTILITY OF HSC COLUMNS

Extensive experimental research on HSC columns conducted in different countries in the past decade has given a better understanding of the stress-strain relationship of confined HSC columns. Results from such research programs were summarized by Li, Park, and Tanaka.5 Unlike the large database of concentric
axial compression tests on confined HSC columns, only a limited number of experimental test results are available in the literature for the strength and ductility of HSC columns under combined cyclic flexure and axial load. The following section provides an overview of these experimental works on HSC columns.

Li, Park, and Tanaka\(^3\)

The experiments carried out by Li, Park, and Tanaka\(^3\) involved five large-scale HSC square columns to investigate the possibility of using HSC columns in regions of high seismic risk. Each specimen had a 350 x 350 mm cross section with a shear-span ratio of 11.0 that was subjected to combined flexure and constant axial loads of from between 0.3\(f_c\)\(A_g\) to 0.6\(f_c\)\(A_g\) with different yield strengths of confining reinforcement. The concrete strength varied from 93 to 98 MPa. Based on their test results, Li, Park, and Tanaka\(^3\) concluded that HSC concrete columns designed on the basis of the NZS 3101-82 were not adequate and that a higher amount of transverse reinforcement would be needed, especially when axial load levels are relatively high. Given the fact that at high axial load levels, ACI 318 seismic provisions require a lower amount of transverse reinforcement than that specified by NZS 3101-82 requirements, it could be concluded that for columns subjected to high axial load levels, the amount of transverse reinforcement specified by ACI 318-89 seismic provisions is inadequate.

Azizinamini et al.\(^5\)

Azizinamini et al.\(^6\) conducted tests on seven full-scale HSC square columns to evaluate the strength and ductility of HSC columns designed according to ACI 318-89. Each specimen had a 305 x 305 mm cross section with a shear-span ratio of 8.0 and was subjected to combined flexure and constant axial loads of 0.2\(f_c\)\(A_g\) with different yield strengths of transverse reinforcement. The concrete strength varied from 54 to 104 MPa. Azizinamini et al.\(^6\) concluded that HSC columns designed to the seismic provisions of ACI 318 possessed conservative strength and adequate displacement ductility when subjected to axial loads of 0.2\(f_c\)\(A_g\) or less. The use of high-strength steel in HSC columns results in larger spacing than that recommended by ACI 318. This leads to the early buckling of the longitudinal bars.

Khoury and Sheikh\(^7\)

Khoury and Sheikh\(^7\) carried out tests on four large-scale HSC square columns. Each specimen had a 305 x 305 mm cross section with a shear-span ratio of 4.8 and was subjected to combined flexure and constant axial loads of 0.62\(f_c\)\(A_g\) to 0.65\(f_c\)\(A_g\) with different yield strengths in the transverse reinforcement. The concrete compressive strength varied from 54 to 60 MPa. Relevant seismic provisions of ACI 318 were evaluated in light of these test results. It was concluded that confining steel designed according to the seismic provisions of ACI 318 could provide satisfactory column behavior for a certain situation, but that for other situations, the design may be either unnecessarily conservative or unsafe. They also indicated that the required amount of confining reinforcement was proportionate to the concrete compressive strength.

Bayrak\(^8\)

An extensive experimental program giving attention to the amount of transverse reinforcement was conducted by Bayrak\(^8\) with 24 large-scale HSC square columns recently tested. Each specimen had a 305 x 305 mm cross section with a shear-span ratio of 4.8 and was subjected to combined flexure and constant axial loads of 0.31\(f_c\)\(A_g\) to 0.53\(f_c\)\(A_g\) with different yield strengths for the transverse reinforcement. The concrete strength varied from 56 to 102 MPa. Test results indicated that columns that are made with very high-strength concrete could be made to behave in a ductile manner under high levels of axial load, provided that a sufficient amount of transverse reinforcement is used in an efficient configuration. The increase in axial load level was observed to reduce column ductility and increase stiffness degradation. The transverse reinforcement configuration and the level of axial load must therefore be considered in the design of confinement reinforcement in the seismic provisions of ACI 318 Code.

Saatcioglu and Baingo\(^9\)

Saatcioglu and Baingo\(^9\) reported a series of nine tests on large-scale HSC circular columns with varying levels of column axial load to investigate the strength and ductility of HSC columns. Each specimen had a 250 mm-diameter circular cross section with a shear-span ratio of 5.5 and was subjected to combined flexure and constant axial loads of 0.22\(f_c\)\(A_g\) to 0.43\(f_c\)\(A_g\) with different yield strengths for the transverse reinforcement. The concrete strength varied from 65 to 90 MPa. The report indicates that HSC columns confined with high-yield-strength transverse reinforcement can be designed to perform in a ductile manner under simulated seismic loading as compared with those confined with normal yield strength reinforcement. The volumetric ratio of the confinement steel required for HSC, however, is higher than that commonly used for normal-strength concrete columns. Saatcioglu and Baingo also pointed out that if the ACI 318 Code requirement is extended to HSC columns, it might result in unrealistically high volumetric ratios of steel that may not be placed without encountering construction problems.

Saatcioglu and Lipien\(^10\)

Saatcioglu and Lipien\(^10\) conducted tests on nine large-scale HSC square columns. Each specimen had a 250 x 250 mm cross section with a shear-span ratio of 5.5 and was subjected to combined flexure and constant axial loads of 0.28\(f_c\)\(A_g\) to 0.31\(f_c\)\(A_g\) with different yield strengths for the transverse reinforcement. The concrete strength varied from 64 to 104 MPa. Similar conclusions to those of Saatcioglu and Baingo were reached. It was concluded that the ductile behavior of HSC columns could be achieved when a sufficient amount of confinement steel was used. The use of higher-yield-strength transverse reinforcement did not have a significant effect on column ductility when the columns were subjected to low axial load.

Paultre, Legeron, and Mongeau\(^11\)

Paultre, Legeron, and Mongeau\(^11\) have reported that the results of tests conducted on 11 full-scale HSC square columns. Each specimen had a 305 x 305 mm cross section with a shear-span ratio of 7.0 and was subjected to combined flexure and constant axial loads of 0.26\(f_c\)\(A_g\) to 0.53\(f_c\)\(A_g\) with different yield strengths for the transverse reinforcement. The concrete strength varied from 79 to 109 MPa. They drew the following conclusion: the concrete compressive strength significantly influences the flexural behavior of HSC columns at a constant level of axial load compression. High-yield-strength reinforcement may be used to effectively confine HSC while reducing the volumetric ratio of lateral transverse reinforcement. They also indicated that the confinement reinforcement requirements
of the ACI 318 and NZS 3101 codes are not directly applicable to columns confined with high-yield-strength steel.

Several experimental investigations on the behavior of reinforced HSC columns under simulated seismic loading have been conducted in different countries. There is still insufficient information, however, regarding the postelastic behavior of HSC columns with various quantities and yield strengths of confining reinforcement as well as other variables. The ACI 318 Code requirements tend to be conservative for columns subjected to low axial load and unsafe for columns under high axial load.

PARAMETERS INVESTIGATED

The design equations for the quantities of transverse confining reinforcement derived by Watson, Zahn, and Park, were for concrete with a compressive strength ranging between 20 and 40 MPa, and for yield strength of the confining reinforcement that was less than 400 MPa. Hence, Watson, Zahn, and Park’s equations might not be adequate for HSC columns confined by transverse reinforcement with yield strength higher than 400 MPa. Variables expected to have a significant influence on the available curvature ductility factors of HSC columns are: the axial load level ratio $P_e / f_c'$, the concrete compressive strength $f_c'$, the yield strength of confining reinforcement $f_{yh}$, the mechanical reinforcing ratio $\rho_{m}$, and the ratio of core sectional area to the gross sectional area $A_c / A_g$. The variables used and the ranges over which they are considered in this study are shown in Table 1.

Curvature ductility factor

It is widely known that the quantity of confining reinforcement provided in the potential plastic hinge regions of columns has a significant effect on the available curvature ductility factors of columns. In this analytical study, curvature ductility factors $\phi_p / \phi_y$, between 0 and 20 are investigated. A commonly quoted criterion for adequate ductility of columns is the ability to sustain a curvature ductility factor of approximately 20. This order of curvature ductility should enable the plastic hinges at the bases of columns of building structures to undergo sufficient plastic rotation for the building structures to reach a displacement ductility factor of 4 to 6, as is implied by the level of seismic design loading for ductile structures. $\phi_p / \phi_y$ of 10 is considered for columns of frames where limited ductility would be sufficient.

Axial load level

The axial load level is expected to have a major effect on moment-curvature behavior and available ultimate curvature. Axial load levels of columns between 0.1$P_e / f_c'$ and 0.7$P_e / f_c'$ are investigated. The upper limit is approximately the maximum axial load level allowed by design codes. Axial tension is not investigated. It is expected that the moment-curvature response of members subject to axial tensions would be dominated by the behavior of the longitudinal reinforcement, and therefore, large ultimate curvatures could normally be sustained.

### Table 1—Parameters investigated

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<th>Description</th>
<th>Range investigated</th>
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<tr>
<td>$P_e / f_c'$</td>
<td>0.2 to 0.7</td>
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<tr>
<td>$f_c'$</td>
<td>50 to 100 MPa</td>
</tr>
<tr>
<td>$f_{yh}$</td>
<td>430 or 1318 MPa</td>
</tr>
<tr>
<td>$\rho_{m}$</td>
<td>0.06 to 0.32</td>
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<tr>
<td>$A_c / A_g$</td>
<td>0.7, 0.75, and 0.81</td>
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Concrete compressive strength

Concrete compressive cylinder strengths between 60 to 100 MPa are considered. These strengths of concrete could be widely used in the future.

Mechanical reinforcement ratio

A range of values of the variable $\rho_{m}$ between 0.06 and 0.32 is considered. In some cases, for HSC columns with $\rho_{m}$ less than 0.06, a huge amount of confining reinforcement must be provided in the potential plastic hinge regions to achieve adequate ductility because the ductility of columns depends significantly on the concrete that has brittle characteristics unless the concrete is well confined by closely spaced transverse reinforcement. When the value of $\rho_{m}$ of the column is very high, steel congestion problems may arise, and a larger size of column may be preferable to reduce the value of $\rho_{m}$. Moreover, the ductility of columns with large $\rho_{m}$ depends more on the longitudinal reinforcement provided and thus the role of the confining reinforcement is not as critical as that for columns with smaller values of $\rho_{m}$. Hence, it is decided to investigate only the range of $\rho_{m}$ listed in Table 1.

Yield strength of confining reinforcement

The stress-strain curves for HSC confined by normal- and high-yield-strength steel are significantly different for a similar spiral pitch and steel volume. The greater the yield strength, the smaller the quantity of confining reinforcement required. This has been emphasized by the results of experimental tests conducted on HSC prisms and cylinders by Li, Park, and Tanaka. In this study, the yield strengths of confining reinforcement investigated are 430 and 1318 MPa. Typical stress-strain curves for the two steels are shown in Fig. 2.

Concrete cover thickness

An $A_c / A_g$ ratio appears in the NZS 3101’s design equation. The original reason for excluding this ratio in equations for confining reinforcement was an attempt to ensure that the increase in strength of the core concrete of the column due to confinement would offset the loss of strength due to the spalling of the unconfined cover concrete. The previous research by Li, Park, and Tanaka indicated that the $A_c / A_g$ ratio needs to be limited more explicitly when HSC is used for columns to ensure that any strength reduction is acceptable and that reasonable ductility can be maintained after the spalling of the cover concrete of the column. Taking into account the inherent explosive behavior of HSC when the extreme fiber reached the spalling strain, it was proposed by Li, Park, and Tanaka that the $A_c / A_g$ should not be less than approximately 0.7 for HSC. Thus, in this study, the range of the lower and
upper limits of the $A_c/A_g$ ratio used is 0.7 to 0.81. These $A_c/A_g$ values are likely to be commonly used in design.

**Types of column sections**

The types of section investigated in this study are shown in Fig. 3, being circular and square sections. Two common arrangements of overlapping hoops are used.

**ANALYTICAL METHOD AND MODEL USED**

In this study, the analytical procedure proposed by Zahn, Park, and Priestley is employed. The ultimate curvature $\phi_u$ is obtained by imposing four identical cycles of bending moment to peak curvatures and of equal magnitude in each direction. The criterion that ductile members shall preserve four loading cycles to a particular curvature ductility level without the flexural capacity reducing by more than 20% is adopted. In the analytical procedure proposed by Zahn, Park, and Priestley, there is no need to determine the maximum concrete strain $\varepsilon_{cu}$ to be used in cyclic moment-curvature analysis. Knowledge of ultimate concrete strain is not essential because the deformation capacity of the column is controlled by other factors such as strength degradation. In this study, the section is considered to have achieved its ultimate curvature when the first of the following ultimate limit state conditions is reached:

1. The moment reached at either the positive or negative curvature peak of the last loading cycle has reduced to 0.8 $M_f$, where $M_f$ is the ideal flexure strength of the section (refer to Fig. 4); and

2. The longitudinal reinforcing steel fractures or buckles.

The definitions of ideal moment and yield curvature as defined by Zahn, Park, and Priestley, shown in Fig. 5, is used in this study. In Case 1, the moment drops after the spalling of the cover concrete and rises only very slowly (due to strain hardening of the tensile steel) with further increase in curvature, so that at a curvature of $5\phi_y$, the moment has not regained the maximum value attained before the spalling of the cover concrete. In that case, $M_f$ is given by that maximum moment. The final value of $\phi_u$ thus obtained is defined as the available ultimate curvature $\phi_u$.

**Constitutive HSC model**

The generalized monotonic stress-strain model for confined HSC used in this study was developed by Li, Park, and Tanaka (refer to Fig. 6) and incorporated in the cyclic moment-curvature analysis. The model was developed on the basis of a large volume of test data covering a wide range of concrete strengths. Different sets of expressions were suggested for HSC confined by either circular normal-yield-strength steel or rectilinear normal-yield-strength steel, and concrete confined by either circular high-yield-strength steel or rectilinear high-yield-strength steel. The model has been found to achieve satisfactory agreement with experimentally measured uniaxial stress-strain response. Li, Park, and Tanaka also found that the assumption that the monotonic loading curve represents the skeleton curve of the stress-strain curves under cyclic loading is still valid, regardless of the concrete compressive strength and the yield strength of the transverse reinforcement. Therefore, the cyclic stress-strain model proposed by Mander, Priestley, and Park later modified by Dodd and Cooke (refer to Fig. 7), is used in this analysis.

**Constitutive steel model**

A cyclic stress-strain model for steel proposed by Dodd and Restrepo-Posada is used in the cyclic moment-curvature analysis. A complete description of the model can be found elsewhere.

**Identification of analytical model**

If the applicability of the cyclic moment-curvature analysis program as well as the accuracy of the analytical model for
reinforced HSC columns can be verified, it can be confidently used for further parametric study. In this regard, 10 HSC columns tested by Li, Park, and Tanaka; Bayrak; and Saatcioglu and Baingo have been studied. The experimental lateral load-deflection hysteresis loops for the HSC columns are shown and compared with the theoretical predictions obtained from the cyclic moment-curvature theory that uses the stress-strain models for confined and unconfined HSC models proposed by Li, Park, and Tanaka. The comparisons revealed that the stress-strain relationships for confined and unconfined HSC models proposed by Li et al. enabled the lateral load versus deflection hysteresis loops of the HSC columns to be reasonably well predicted (refer to Fig. 8).

RESULTS OF CYCLIC MOMENT-CURVATURE ANALYSIS

Approximately 300 separate cyclic moment-curvature analyses have been carried out to provide the results that are presented in this paper. Although this did not allow for every possible combination of variables to be considered, it did enable the important trends to be identified. The predicted results obtained from the cyclic moment-curvature analyses of reinforced HSC columns confined by two different types of confining reinforcement, namely Grade 430 and Grade 1318 steel, are presented in the following. In the case of HSC confined by high-yield-strength steel ($f_{yh} > 1150$ MPa), the NZS 3101's equation for the usable yield strength of high-yield-strength steel is assumed to be 800 MPa. This value is based on a study done by Li, Park, and Tanaka that suggested that up to 400 MPa could be used. The analysis results are presented graphically in Fig. 9 to 12 and are compared with the ACI 3182 and NZS 3101 code requirements.

In this study, it is clearly demonstrated that the concrete compressive strength $f'_{c}$ and ratio of $A_e/A_g$ have considerable influence on the quantity of confining reinforcement. It is also found that the required transverse reinforcement for concrete confinement to achieve a particular curvature ductility factor increases significantly as the axial load ratio increases, and that the required confining reinforcement increases when the flexure steel content $\rho_t$ decreases.

In the case of HSC confined by normal-yield-strength steel ($f_{yh} < 500$ MPa), the comparisons showed that the NZS 3101.95's equations for rectilinear confining reinforcement for HSC columns give reasonable good predictions for the amounts of confining reinforcement required to ensure an available curvature ductility factor of $\phi_u/\phi_y = 10$ and 20. For circular confining reinforcement in HSC columns, however, the comparisons indicated that the NZS 3101 equation gives generally conservative predictions for the quantities of confining reinforcement. The comparisons also indicated that the ACI 318 Code expressions for confinement reinforcement are reasonable for low levels of axial load but are unconservative for high levels of axial load. The amount of transverse confining reinforcement required according to the ACI 318 Code is inadequate to achieve $\phi_u/\phi_y = 10$ under high axial load.

The beneficial effects of high-yield-strength transverse confining reinforcement can be seen in this parametric study.
The verification indicated that the NZS 3101:95’s equations for rectilinear confining reinforcement in HSC columns gives nonconservative predictions of the amount of high-strength confining reinforcement required to ensure an available curvature ductility factor of $\frac{\phi_u}{\phi_y} = 10$ and 20. It is also demonstrated that the ACI code expression is not applicable to HSC columns confined by the high-yield-strength steels.

DERIVATION OF EQUATIONS FOR QUANTITIES OF CONFINING REINFORCEMENT REQUIRED IN HSC COLUMNS FOR ADEQUATE DUCTILITY

The following is based on the comparisons conducted in this study between the cyclic moment-curvature analysis and the NZS 3101 and ACI 318 design equations. Some modifications were derived for the refined equation for application to HSC using either normal-yield-strength steel or high-yield-strength steel. Multivariable regression analysis is employed to derive the refined design equations for confined HSC column. These refined equations can correlate the attainable curvature ductility to the amount of transverse reinforcement in the plastic hinge region of HSC columns.

For columns confined by rectilinear normal yield strength steel

\[
A_{sh} = \frac{s_y h_c}{\phi_y} \left( \frac{A_c}{A_c} \times \left( \frac{\phi_u}{\phi_y} - 33 \rho_m - 22 \right) \times \frac{P_c}{P_{c'h}} \times \frac{f_{y''}}{f_{c'y''} A_c} \right) - 0.006
\]

where $\lambda = 117$ when $f_c < 70$ MPa, and $\lambda = 0.05 (f_c')^2 - 9.54 f_c' + 539.4$ when $f_c' \geq 70$ MPa.
For columns confined by circular normal yield strength steel

\[
\frac{A_{sh}}{s_h h_c} = \left( \frac{A_g}{A_c} \times \frac{(\phi_u/\phi_y) - 33 \rho_t m + 22}{111} \times \frac{f'_{yc} \times P_c}{f_{yc}' A_g} \right) - 0.006 \times \alpha (8)
\]

where \( \alpha = 1.1 \) when \( f_c < 80 \) MPa and \( \alpha = 1.0 \) when \( f_c \geq 80 \) MPa.

For columns confined by rectilinear high-yield-strength steel

\[
\frac{A_{sh}}{s_h h_c} = \left( \frac{A_g}{A_c} \times \frac{(\phi_u/\phi_y) - 30 \rho_t m + 22}{\lambda} \times \frac{f'_{yc} \times P_c}{f_{yc}' A_g} \right) (9)
\]

where \( \lambda = 91 - 0.1 f_c' \).

For columns confined by circular high-yield-strength steel

\[
\frac{A_{sh}}{s_h h_c} = \left( \frac{A_g}{A_c} \times \frac{(\phi_u/\phi_y) - 55 \rho_t m + 25}{f_{yc}} \times \frac{P_c}{\phi f'_{yc} A_g} \right) (10)
\]

In Eq. (7) to (10), the maximum value of \( \rho_t m \) that can be substituted is 0.4. Also, \( A_c/A_g \) is not permitted to exceed 1.5 unless it can be shown that the design strength of the core of the column can resist the design axial load applied concentrically.

Limit usable yield strength of transverse reinforcement is assumed not to exceed 900 MPa in Eq. (9) and (10).

**VERIFICATION OF PROPOSED EQUATIONS**

The validity of the quantities of confining transverse reinforcement obtained from the proposed refined design equation,
with the quantities provided in 56 HSC column units tested by various researchers, is necessary to ensure that the proposed equations are not unduly conservative and yet are reasonably accurate. Five columns were tested in New Zealand, eight in the U.S., and 43 in Canada. The selected column specimens encompass a wide range of material properties, geometry, and reinforcement detailing, as summarized in Table 2 and 3 according to chronological order. At this point, it is also

<table>
<thead>
<tr>
<th>Specimen</th>
<th>$f'_{c}$, MPa</th>
<th>$S$, mm</th>
<th>$f_{yh}$, MPa</th>
<th>$A_{sh}$, mm$^2$</th>
<th>$P/(f'<em>{c}A</em>{g})$</th>
<th>$A_{sh}$-ACI, mm$^2$</th>
<th>$A_{sh}$-NZ, mm$^2$</th>
<th>$A_{sh}$-Li, mm$^2$</th>
<th>% ACI</th>
<th>% NZ</th>
<th>% Li</th>
<th>$\mu_{b}$</th>
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Table 2—Comparison among provided effective area of rectilinear confinement reinforcement and required rectilinear reinforcement by ACI 318$^2$ Code, NZS 3101$^1$ Code, and proposed equations

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<tr>
<th>Specimen</th>
<th>$f'_{c}$, MPa</th>
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<th>$f_{yh}$, MPa</th>
<th>$A_{sh}$, mm$^2$</th>
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<th>$A_{sh}$-NZ, mm$^2$</th>
<th>$A_{sh}$-Li, mm$^2$</th>
<th>% ACI</th>
<th>% NZ</th>
<th>% Li</th>
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| Bayrak$^8$ (1988)—square section
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<th>$A_{sh}$-NZ, mm$^2$</th>
<th>$A_{sh}$-Li, mm$^2$</th>
<th>% ACI</th>
<th>% NZ</th>
<th>% Li</th>
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Saatcioglu and Lipien$^{10}$ (1997)—square section

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<th>$A_{sh}$-NZ, mm$^2$</th>
<th>$A_{sh}$-Li, mm$^2$</th>
<th>% ACI</th>
<th>% NZ</th>
<th>% Li</th>
<th>$\mu_{b}$</th>
<th>$\mu_{b}$, L</th>
<th>$\mu_{A}$</th>
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</table>

Paultre, Legeron, and Mongeau$^{11}$ (2001)
Table 3—Comparison among provided effective area of circular confinement reinforcement and required rectilinear reinforcement by ACI 318\(^2\) Code, NZS 3101\(^1\) Code, and proposed equations

<table>
<thead>
<tr>
<th>Specimen</th>
<th>(f'_c), MPa</th>
<th>(d), mm</th>
<th>(f_{sh}), MPa</th>
<th>(\rho_{sh}), %</th>
<th>(P(R_{sh}))</th>
<th>(\rho_{sh-ACI}), %</th>
<th>(\rho_{sh-AZ}), %</th>
<th>(\rho_{sh-LS}), %</th>
<th>% ACI</th>
<th>% NZ</th>
<th>% Li</th>
<th>(\mu_p)</th>
<th>(\mu_{ph})</th>
<th>(H_A)</th>
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Interesting to consider all the available data on full-scale HSC columns to assess the use of the ACI 318 and NZS 3101 code equations. In Table 2 and 3, the provided effective area of transverse confining reinforcement \(A_{sh}\) is compared with the required transverse confinement by the ACI 318\(^2\) Code, the NZS 3101\(^1\) code and the proposed design equations in this paper.

Comparison of the experimental results and the theoretical predictions indicated that the refined cyclic moment-curvature theory and the proposed design equations for ductility give sufficiently accurate yet conservative predictions. This was further confirmed by the test results of Units AS-20H\(^7\), AS-18H\(^7\), AS-4HT\(^8\), RS-19HT\(^8\), RS-1\(^10\), RS-8\(^10\), RC-3\(^9\), RC-4\(^9\), RC-8\(^9\), and RC-9\(^9\) that were confined by two different types of confining reinforcements.

For rectilinear confining reinforcement used in HSC columns, the investigation revealed that Units AS-20H\(^7\), AS-18H\(^7\), AS-4HT\(^8\) and AS-6HT\(^8\) that had been confined by normal-yield-strength reinforcement that contained 97, 85, 103, and 92% of the confining transverse reinforcement proposed in this study and contained 200, 138, 180, and 167% of the ACI 318 Code-recommended quantity of confining reinforcement for ductile detailing, achieved curvature ductility of 114% of the ACI 318 Code-recommended quantity of confining transverse reinforcement required to enable the ductility and energy dissipation of transverse confining reinforcement required to enable the critical sections of HSC columns to sustain particular ductility demands. The design equations that are proposed in this study give only the transverse reinforcement required for concrete confinement. The transverse reinforcement provided must also be checked to ensure that the stability of the longitudinal compression bars, and the shear requirement, are satisfied.

A maximum pitch of 4\(d_b\) for normal-yield-strength transverse reinforcement or 6\(d_b\) for ultra high-yield-strength steel is recommended by Li, Park, and Tanaka\(^5\).

CONCLUSIONS

An analytical investigation to determine a more appropriate design equation for the quantities of confining reinforcement required in the potential plastic hinge regions of HSC columns is conducted.

The parametric studies revealed that the ACI 318\(^2\) and the NZS 3101\(^1\) design provisions should be revised in some cases for HSC. Practical design equations for the confinement reinforcement in the plastic hinge regions of HSC columns are developed based on the results of the cyclic moment-curvature analysis. The tendency is well estimated by the proposed equations in this analytical study. The proposed equations were reproduced for 56 HSC column test results from the literature with reasonable accuracy. Therefore, the proposed design equations can be used to estimate the quantities of transverse confining reinforcement required to enable the critical sections of HSC columns to sustain particular ductility demands. The design equations that are proposed in this study give only the transverse reinforcement required for concrete confinement. The transverse reinforcement provided must also be checked to ensure that the stability of the longitudinal compression bars, and the shear requirement, are satisfied. A maximum pitch of 4\(d_b\) for normal-yield-strength transverse reinforcement or 6\(d_b\) for ultra high-yield-strength steel is recommended by Li, Park, and Tanaka\(^5\).

NOTATION

\(A_c\) = area of concrete core of column section measured to outside of peripheral spiral or hoop
\(A_t\) = cross area of column section
\(A_{sh}\) = area of hoop bars and supplementary cross-tie confining reinforcement in one principal direction of column section
\(E_c\) = Young’s modulus of elasticity for concrete
\(E_t\) = concrete stress at reloading reversal
\(E_{yc}\) = Young’s modulus of elasticity for steel
\(f_c\) = concrete stress
\(f_{sh}\) = specified concrete compressive strength
\(f_{csh}\) = confined concrete compressive strength
\(f_{c, in}\) = in-place unconfined concrete compressive strength
\(f_t\) = transverse confining stress
\(f_{new}\) = new concrete stress on reloading at strain of \(e_{cu}\)
\(f_{co}\) = concrete stress at reloading reversal
\(f_{ec}\) = effective transverse confining stress
\(f_s\) = yield strength of steel in tension
\(f_{sh}\) = yield strength of transverse reinforcing steel
\(h_c\) = concrete core dimension, measured to center line of perimeter hoop
\(K_{eff}\) = confinement effectiveness coefficient, based on area ratio
\(M_{max}\) = maximum bending moment to which column has been subjected
\(M'\) = ideal flexure strength of section, defined as maximum moment reached before curvature ductility factor \(\mu_p/\mu_{ph}\) = 5.0
\(M_{y}\) = moment calculated at first yield of longitudinal reinforcement, or when extreme compressive fiber strain reaches 0.002, whichever is smaller
\(P_e\) = axial compression load on column due to design gravity and seismic loading
\(s\) = clear spacing between circular hoops or spirals
\(s_b\) = center-to-center spacing of spiral or hoop sets
\(\alpha\) = section type factor
\(\varepsilon_c\) = strain at maximum confined strength of concrete \(f_{csh}\)
\(\varepsilon_{co}\) = compressive strain at maximum in-place unconfined concrete strength \(f_{c, in}\)
ε_{cu} = ultimate concrete compressive strain
ε_{pl} = plastic strain in concrete model
ε_{ro} = concrete strain at reloading reversal
ε_{un} = reversal strain when unloading from monotonic skeleton curve of concrete
φ_{pl} = plastic strain in concrete model
φ = yield strain of steel
φ_{y} = yield strain of steel
φ_{u} = ultimate curvature
φ_{y} = yield curvature
φ_{y}' = lower curvature of when yield in longitudinal steel is first reached and when extreme fiber concrete reaches compressive strain of 0.002
λ = modification factor
µΔ = measured displacement ductility
µφ = measured curvature ductility
µ_{p,t} = target curvature ductility based on Li and Park’s design equations
ρ_{x} = lateral confining steel parallel to x-axis
ρ_{y} = lateral confining steel parallel to y-axis
ρ_{m} = mechanical reinforcing ratio (m = f_{y}/0.85f'_{c})
ρ_{s} = volumetric ratio of confining reinforcement to core concrete
ρ_{t} = area of longitudinal reinforcement divided by gross area of column section
ρ_{t m} = mechanical reinforcing ratio (m = f_{y}/0.85f'_{c})

REFERENCES