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<td>Rights</td>
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Deformation-Controlled Design of Reinforced Concrete Flexural Members Subjected to Blast Loadings

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Abstract

Both maximum displacement and displacement ductility factors should be considered in the design of a blast-resistant structure since both parameters correlate with an expected performance level of a reinforced concrete (RC) structural member during a blast event. The blast-resistant design procedure discussed in this paper takes into account both the maximum displacement and displacement ductility responses of an equivalent single-degree-of-freedom (SDOF) system, while the response of the SDOF system is made equivalent to the corresponding targets of design performance. Some approximate errors are present when comparing the actual responses of the structural member, which has been designed for blast loading, and their corresponding design performance targets. Two indices are defined to quantify the approximation errors, and their expressions are obtained through comprehensive numerical and statistical analyses. By using the error indices, the design procedure is then modified such that the approximate responses of the RC member are equivalent to the targets of the design performance. The modified procedure is implemented in three design examples and numerically evaluated. It is concluded that the modified procedure can be used more effectively in order to ensure that the actual responses of designed members reflect the respective targets of design performance.

Keywords: Displacement; Blast loads; Errors; Reinforced concrete; Deformation.

Introduction and Background

The design of reinforced concrete (RC) structural members against accidental or deliberate explosions is deemed necessary with an increasing emphasis on blast loading on structures. The blast load exerted on a structural member can be adequately simplified as a uniformly distributed dynamic loading, characterized by its peak pressure and duration, with the exception of very close-in explosion situations. Some level of inelastic deformation of the structural member is allowed in the blast-resistant design when subjected to severe blast loadings to dissipate energy, therefore, most of blast design guidelines (ASCE 1985; NFEC 1986; U.S. Army 1986, 1990; Biggs 1964), with explicit consideration of inelastic deformation, have been proposed in recent years. These design
guidelines help to ensure that a RC member is designed such that its response is equivalent to the predefined performance level under blast loading. Thus, the proposed design procedure seeks to reconcile the differences between a possibly lower response as determined by numerical methods and a possibly higher level response as predetermined by design performance.

In this paper, a new deformation-controlled blast-resistant design procedure using nondimensional energy spectra (NES) was developed. The effective depth \((d)\) and longitudinal reinforcement ratio \((\rho)\) of a RC member can be determined by representing a continuous RC member as an equivalent elastic–plastic single-degree-of-freedom (SDOF) system. Subsequently, the maximum displacement and displacement ductility responses \((y_{m}^{\text{eq}}\text{ and } \mu_{m}^{\text{eq}})\) can be made equivalent to the corresponding design performance targets, which are defined by its target displacement and target displacement ductility factor \((y_{t}\text{ and } \mu_{t})\) (ASCE 1985; NFEC 1986; U.S. Army 1986, 1990; Kottegoda and Rosso 1996). This procedure is based on an underlying assumption that the responses \((y_{m}^{\text{rc}}\text{ and } \mu_{m}^{\text{rc}})\) of the designed RC member at a critical location (for a fixed–fixed member, the critical location is the midspan; for a cantilever member, the critical location is the free end) can be represented by the responses of its equivalent SDOF system \((y_{m}^{\text{eq}}\text{ and } \mu_{m}^{\text{eq}})\) for a defined blast loading. However, the difference of \(y_{m}^{\text{rc}}\) with its respective \(y_{m}^{\text{eq}}\) and \(\mu_{m}^{\text{rc}}\) with \(\mu_{m}^{\text{eq}}\) can be expected due to the complexities of the nonlinear dynamic response of RC members under blast loading conditions, which cannot be captured by the equivalent elastic–plastic SDOF system. This manifests itself in the errors between the displacement ductility responses \((y_{m}^{\text{rc}}\text{ and } \mu_{m}^{\text{rc}})\) and the corresponding design performance targets \((y_{t}\text{ and } \mu_{t})\).

In principle, a response analysis should be capable of controlling the actual responses of a RC member \((y_{m}^{\text{rc}}\text{ and } \mu_{m}^{\text{rc}})\) rather than those of the corresponding equivalent SDOF system \((y_{m}^{\text{eq}}\text{ and } \mu_{m}^{\text{eq}})\), for a given design performance targets \((y_{t}\text{ and } \mu_{t})\). Thus, it is worthwhile determining the errors between the actual responses \((y_{m}^{\text{rc}}\text{ and } \mu_{m}^{\text{rc}})\) and the corresponding performance targets \((y_{t}\text{ and } \mu_{t})\) of a RC member. Application of these errors in modifying the deformation-controlled blast-resistant design procedure will actually help in controlling the responses of \(y_{m}^{\text{rc}}\) and \(\mu_{m}^{\text{rc}}\). This paper discusses some numerical examples to demonstrate the analysis procedure using the modified design method. Numerical simulations of the displacement and the displacement ductility responses under blast loading are performed, and the results are compared with the performance targets.

**Deformation-controlled Design Procedure**

**Nondimensional Energy Spectrum**

A nondimensional energy spectrum is an important tool in the incorporation of the target displacements and target displacement ductility factors \((y_{t}\text{ and } \mu_{t})\), which allows for determining design parameters of \(d\) and \(\rho\) for RC members. The nondimensional energy factor \((C)\) is introduced into an elastic–plastic SDOF system (see Fig. 1),
expressing the ratio of maximum strain energy $E_{\text{max}}$ to the ultimate elastic energy $E_{\text{el}}$ and is given in Eq. (1)

$$C = \frac{E_{\text{max}}}{E_{\text{el}}}$$  \hspace{1cm} (1)

where $E_{\text{el}} = k_e y_e^2/2$; and $E_{\text{max}} = k_e y_e (y_{\text{max}} - y_e)/2$. Substituting them into Eq. (1), $C$ becomes

$$C = 2\mu - 1$$  \hspace{1cm} (2)

where $\mu = y_{\text{max}}/y_e$. The displacement ductility factor ($\mu$) is a function of $F_1/R_m$ (where $F_1 =$ peak value of the force); and $t_d/T$ and their distributions can be found in Biggs (1964). A group of curves, which represent the factor $C$ against the ratio $t_d/T$ with respect to various $F_1/R_m$, are defined as NES and are shown in Fig. 2.

**Design of RC Flexural Member Using NES**

A structural member with continuous mass and stiffness can be represented by an equivalent elastic–plastic SDOF system with an equivalent mass and stiffness (Biggs 1964). The equivalent SDOF system is such that the deformation response of the concentrated mass is assumed to be the same as that for the critical point on the structural member (e.g., the midspan of a member with two ends constrained or the free end of a cantilever member). Thus, the responses of the equivalent SDOF system under a given blast loading should achieve the expected performance level defined by $y_t$ and $\mu_t$. To achieve this objective, there exists only one solution in the form of the initial stiffness ($k_e$) and ultimate strength ($R_m$) for the equivalent system. Therefore, it is clear with the condition $y_e = y_t/\mu_t$ having been satisfied, and the maximum displacement response of the equivalent SDOF system $k_e$ having reached $y_e$ exactly, the maximum displacements of equivalent systems with initial stiffness are either larger or smaller than $k_e$ but not equal to $y_e$. The specific solution for $k_e$ and $R_m$ of the equivalent SDOF system can then be obtained by an iterative procedure. Assuming an initial stiffness $k_e$, the parameters $F_1/R_m$ and $t_d/T$ of the system are obtained and then the values, $C$ and $E_{\text{max}}$ are found by referring to the NES curves in Fig. 2. However, the value of $E_{\text{max}}$ will result in a new stiffness $k_{\text{el}}$ with

$$k_{\text{el}} = \frac{E_{\text{max}}}{y_e y_t - y_e^2/2}$$  \hspace{1cm} (3)

In order to make $k_e = k_{\text{el}}$, the above procedure is repeated until the convergent condition of $|k_{\text{el}} - k_e| < \epsilon$ (where $\epsilon =$ convergence tolerance) is fulfilled.

A few methods to compute $d$ and $\rho$ for a RC member from $k_e$ and $R_m$ of the corresponding equivalent SDOF system are discussed in blast design guidelines (ASCE 1985; NFEC 1986; U.S. Army 1986, 1990; Biggs 1964). These ways consider the
characteristics of concrete and the embedded reinforcements. The equations are summarized as follows (Li et al. 2006):

\[
\begin{align*}
    d &= \left( \frac{24l^3k_c}{(\gamma + 1)\alpha K_{LE}E_c b_w} \right)^{1/3} \\
    \rho &= \frac{\beta R_m l}{K_{LE} f_{ds} b_w (d - d')} \\
    \rho_v &= \frac{V_u - V_c}{\varphi f_{dv} b_w d}
\end{align*}
\]  

(4)  

(5)  

(6)

where \( l \) = length of member; \( K_{LE} \) = load transformation factor; \( \alpha \) and \( \beta \) = coefficients for different boundary conditions; \( E_c \) = Young’s modulus of concrete; \( b_w \) = member width; and the parameter of \( \gamma \) can be obtained from Fig. 3. It is observed that the coefficient varies with \( \rho \) as well as the modulus ratio \( (E_s/E_c) \), where \( E_s \) = Young’s modulus of steel (U.S. Army 1986, 1990); \( V_u \) = ultimate shear force; and \( V_c \) = shear capacity of the concrete. \( f_{ds} \) and \( f_{dv} \) = dynamic yield strength for longitudinal and shear reinforcements, respectively, in which the dynamic increase factors (DIFs) are used in the design according to TM5-1300 (U.S. Army 1990).

Eq. (4) demonstrates that \( d \) is a function of \( \gamma \). Since \( \gamma \) varies with \( \rho \) as shown by Fig. 3, \( d \) such that it considers \( \rho \). On the other hand, Eq. (5) indicates that \( \rho \) is determined by \( d \). Therefore, an iterative computational procedure needs to be employed to determine \( d \) and \( \rho \) of a RC member against the given blast loading. Combining it with the previous iterative procedure of determining \( k_c \) and \( R_m \) of the equivalent SDOF system for a given blast loading and the dual targets of \( \gamma_t \) and \( \mu_t \), the design flow chart for determining \( d \) and \( \rho \) based on NES is shown in Fig. 4. In the above developed design procedure, an initial data of \( d_0 \) and \( \rho_0 \) is assumed such that the value of \( E_{max} \) for the equivalent SDOF system can be obtained. However, this \( E_{max} \) will result in a new solution of \( d_1 \) and \( \rho_1 \) and the process must be repeated until \( d \) and \( \gamma \) are consistent.

**Implementation of the Design Procedure**

To evaluate the effectiveness of applying the presented design procedure, a demonstration is given by implementing it on the design of a RC wall subjected to blast loading. The RC wall is designed to resist the blast loading perpendicular to its plane. The blast loading is simplified into a triangular pulse with the peak pressure and duration as shown in Fig. 5. The design is required to achieve the expected performance level defined by \( \mu_e = 9 \) and \( \theta_t = 4^\circ \). Thus, \( \gamma_t \) of the wall at the free end under the given blast condition can be obtained with an approximate expression of \( \gamma_t \approx l \tan(\theta_t) \). The area of compression reinforcement is taken to be equal to that of the tension reinforcement. This equivalent of reinforcement would consider the rebound effect of the member subsequent to its maximum displacement response, and makes provision for the possibility of the explosion occurring in the opposite side of the wall.
The design procedures for the iterative step are illustrated in Table 1. During the design process, the value of the initial effective depth \( (d_0) \) is taken as 2.0 m and the initial \( \rho_0 \) to be 3.0\%. The convergence limit employed in the design is 0.001. It can be seen from Table 1 that the design procedure is insensitive to the initial values of \( d_0 \) and \( \rho_0 \). Therefore, there is no difficulty in achieving computational convergence during the iterations. It takes only seven iterative steps in this example to find the solution and satisfy the condition of \(|(d_1-d_0)/d_0| < 0.001 \) and \(|(\rho_1-\rho_0)/\rho_0| < 0.001 \). The time taken for the computation of this design example is only 0.12 s using a program written with MATLAB (2001).

**Numerical Verification**

**Verification of the Responses for the Equivalent SDOF System**

To assess whether the responses of the equivalent SDOF system for the designed RC member under the given blast loading are controlled to be exactly equal to their performance targets, the nonlinear time–history analysis is carried out upon the equivalent SDOF system having \( k_e \) and \( R_m \). The displacement responses are shown in Fig. 6. It is observed that \( y_m^{eq} \) and \( \mu^{eq} \) of the equivalent SDOF system meet their targets exactly.

**Verification of the Actual Responses of RC Member**

Due to the complicated behaviors of continuous RC members, some differences between the responses of the equivalent SDOF system \( \langle y_m^{eq} \text{ and } \mu^{eq} \rangle \) and those of the designed member \( \langle y_m^{rc} \text{ and } \mu^{rc} \rangle \) are conceivable, as shown in Fig. 7. Therefore, some approximate errors are noticed between the actual response \( \langle y_m^{rc} \text{ and } \mu^{rc} \rangle \) and the corresponding targets \( \langle y_t \text{ and } \mu_t \rangle \). To evaluate the effectiveness of the presented design procedure in controlling the responses of the designed member, \( y_m^{rc} \) and \( \mu^{rc} \) of the member under the same blast loading were determined using the ABAQUS software (AB AQUS 2003). The values of \( y_m^{rc} \) and \( \mu^{rc} \) were compared with their respective design performance targets \( y_t \) and \( \mu_t \).

The smeared cracking model for concrete is utilized considering that the failure of RC members under blast conditions is characterized by concrete crushing accompanied by concrete cracking. In this model, a crack appears when the maximum principal tensile stresses reach a failure surface. The von Mises yield criterion is used to describe the constitutive behavior of the reinforcement. The stress–strain relationship of reinforcement is modeled with an elastoplastic curve. The strain hardening of reinforcement is not considered in this analysis since it is hard to define under the blast conditions due to lack of experimental data. The ultimate strain value is never reported in the current literature due to the difficulty of determining exactly when the peak stress occurs as well as the confusion between ultimate strain and rupture strain. To simulate the softening effect of the concrete in tension, a bilinear tension stress–strain curve is used after cracking, where the failure strain \( \varepsilon_u^{cr} \) is taken as \( 10^{-3} \). The selection of this value is based on the
assumption that the strain softening after failure reduces the stress linearly to zero at a total strain of about 10 times of the strain at failure of concrete in tension, which is, typically, $10^{-4}$ in standard concretes (Hilleborg et al. 1976). For strain failure of concrete in compression, it is simulated with an elastic–plastic mode and the elastic stress state is limited by a yield surface. Once yielding had occurred, an associated flow rule with isotropic hardening is used.

Considering that both concrete and reinforcement exhibit increased strength under higher loading rates, the expressions of DIFs (Malvar and Crawford 1998a,b) are adopted. The user subroutine was developed to consider DIFs in the analysis, which allows the user to define the field variable of a material at any point as a function of any available material point quantity. Thus, by making the strain rate a variable, the strain rate-dependent material properties can be introduced in the analysis. Timoshenko beam elements were assigned to model the members while the rebar option was utilized to place each reinforcement at its exact location. A perfect bond between rebar and concrete was assumed. The finite-element models have been verified for a simply supported RC beam and a slab subjected to blast loading, where numerically determined responses are similar to experimental ones (Rong 2005; Rong and Li 2007).

Nonlinear dynamic analysis was performed on a design of a cantilever RC wall designed for a given blast loading. The plot of free-end responses of the wall in terms of displacement versus time is illustrated in Fig. 8. It is observed that $y_m^{rc}$ for the designed RC walls under the given blast loading is slightly less than $y_t$. A deviation of about 10% of $y_t$ was observed while $\mu^{rc}$ was equal to 5.18 as compared to $\mu_t = 9$. The difference between $\mu^{rc}$ and $\mu_t$ was about 42.4% of $\mu_t$, and the error between $\mu^{rc}$ and $\mu_t$ was larger than that between $y_m^{rc}$ and $y_t$.

Comparisons of Figs. 6 and 8 demonstrate that values of $y_m^{eq}$ and $\mu^{eq}$ of the equivalent SDOF system met their targets. There still existed some errors between the actual responses ($y_m^{rc}$ and $\mu^{rc}$) of the RC member and the corresponding design performance targets ($y_t$ and $\mu_t$). This indicated that the errors occurred due to the derivation of $d$ and $\rho$ from $k_e$ and $R_m$ in the design process. Several points accounting for the errors are explained as follows:

• The load and mass factors ($K_{LE}$ and $K_{ME}$) are necessary in considering the continuous RC member as the equivalent elastic–perfectly plastic SDOF system, whereby these factors are obtained in an approximate way, which is because the continuous resistance function was represented in two or three independent linear stages during the design process as a simplification. Thus, the utilization of $K_{LE}$ and $K_{ME}$ will cause some errors between the responses of the designed member and their design targets.

• It is known that the embedded reinforcement and the cracking propagation of the concrete have a great effect on the value of $I$. To simplify this problem in the design process, $I$ obtained from the expression of $(\gamma+1)b_wd^3/24$ was used to calculate the RC member stiffness ($k_e$) and the corresponding deformation. However, the adoption of $\gamma$, which is dependent on the fitting experimental data, will incur some error in the design of RC members.
• It is difficult to determine $\rho$ accurately, from which the ultimate strength of the designed member equates to the anticipated value ($R_m$). The use of Eq. (5) to determine $\rho$ is quite conservative since it is assumed that concrete in the compression zone did not contribute towards the ultimate strength of a RC structural member. Thus, the reinforcement ratio $\rho$ tends to be enlarged in the design, which causes some errors in fulfilling the design targets.

• DIFs are employed for concrete and reinforcement during the design process while the varying DIFs with the strain rate are used in the numerical analysis. The inconsistent usage of DIF, in the design and analysis produces errors between the actual responses of the designed member and their design targets.

The deformation-controlled design procedure presented attempts to equate the responses of the equivalent SDOF system to the design performance targets (i.e., $\mu_{m}^{eq} = \mu_t$ and $y_{m}^{eq} = y_t$). However, the numerical verification indicated that some errors existed between the responses of the designed RC member ($y_{m}^{rc}$ and $\mu^{rc}$) and the corresponding design targets ($y_t$ and $\mu_t$). Since the RC member was specifically designed under a given blast loading condition, the following part shows the derivation of the formulas for quantifying the errors. Also, the method used to combine the formulas for the iterative design procedure to achieve $y_{m}^{rc}$ and $\mu^{rc}$ is shown.

**Error Analysis**

**Definition of Error Indices**

To obtain a consistent measurement of the degree of the errors between the actual responses ($y_{m}^{rc}$ and $\mu^{rc}$) and their respective performance targets ($y_t$ and $\mu_t$), two nondimensional error indices are defined as

$$s_y^{rc} = \frac{y_t - y_{m}^{rc}}{y_t}$$  \hspace{1cm} (7)

$$s_{\mu}^{rc} = \frac{\mu_t - \mu^{rc}}{\mu_t}$$  \hspace{1cm} (8)

where $s_y^{rc}$ = displacement error index for the error between $y_{m}^{rc}$ and $y_t$; and $s_{\mu}^{rc}$ = displacement ductility error index representing the error between $\mu^{rc}$ and $\mu_t$. With $s_y^{rc}$ and $s_{\mu}^{rc}$ initially known, controlling of $y_{m}^{rc}$ and $\mu^{rc}$ is possible in the design of blast-resistant structural members.

**Analytical Approach**

It is almost impractical to derive explicit expressions of Eqs. (7) and (8) since the behaviors of RC members will exhibit significantly complicated geometric and material
nonlinearity under most blast conditions. A curve fitting technique with a large amount of reliable data for \( \xi_{ym}^{rc} \) and \( \xi_{ym}^{rc} \), which are determined according to Eqs. (7) and (8), was used. \( y_{ym}^{rc} \) and \( \mu^{rc} \) obtained through nonlinear finite-element analyses of the designed members was executed together with the statistical analyses so as to find simplified explicit expressions of \( \xi_{ym}^{rc} \) and \( \xi_{ym}^{rc} \). The procedure is listed as follows:

1. Select the type of support conditions (SCs) of RC members to be designed;
2. Sample the design variable vector of \( \{P, t_d, Y_d, \mu, l, f_{dc}, E_c, f_{ds}, E_s, f_{du}\} \), where 2,000 samples are randomly taken to ensure the accuracy of the statistical analysis;
3. Design an RC member using the above procedure with each sample of the design variable vector to obtain \( d, \rho, \) and \( \rho' \), \( \rho'' \), and \( b_w \) as a ratio of \( d \);
4. Repeat Step 3 until 2,000 sampled design cases are accomplished;
5. Select 500 design cases with \( \rho \) ranging from 0.31 to 2.2%;
6. Perform the numerical analyses on the selected 500 design cases to find \( y_{ym}^{rc} \) and \( \mu^{rc} \) using ABAQUS (2003);
7. Compute \( \xi_{ym}^{rc} \) and \( \xi_{ym}^{rc} \) of the 500 design cases with Eqs. (7) and (8);
8. Plot the distributions of \( \xi_{ym}^{rc} \) and \( \xi_{ym}^{rc} \) with the basic design variables;
9. Carry out the curve fitting of the distributions of \( \xi_{ym}^{rc} \) and \( \xi_{ym}^{rc} \), followed by statistical analyses;
10. Establish the simplified formulas to estimate \( \xi_{ym}^{rc} \) and \( \xi_{ym}^{rc} \); and
11. Change the type of SCs of the members and repeat the above steps.

Based on the above analytical procedure, the distributions of \( \xi_{ym}^{rc} \) and \( \xi_{ym}^{rc} \) versus \( \rho \) for the designed RC members under various support conditions are shown in Fig. 9.

**Formulas of Error Indices**

Nonlinear curve fittings of \( \xi_{ym}^{rc} \) and \( \xi_{ym}^{rc} \) versus \( \rho \) for various SCs are carried out as shown in Fig. 9, where the functions are expressed as \( f(\rho, SC) \) and \( g(\rho, SC) \), respectively. The effects of variables other than \( \rho \) and SC are dealt with by introducing two nominal random variables of \( e_{y} \) and \( e_{\mu} \), which are assumed to represent the deviation of \( \xi_{ym}^{rc} \) and \( \xi_{ym}^{rc} \) around the fitting curves. As a result, \( \xi_{ym}^{rc} \) and \( \xi_{ym}^{rc} \) are written as

\[
\xi_{ym}^{rc} = f(\rho, SC) + e_{y} \\
\xi_{ym}^{rc} = g(\rho, SC) + e_{\mu} \tag{9,10}
\]

A second-order polynomial function is selected for the curve fitting given by

\[
f(\rho, SC) \text{ and } g(\rho, SC) = a_1 + a_2 \rho + a_3 \rho^2 \tag{11}
\]

With the result functions of \( f(\rho, SC) \) and \( g(\rho, SC) \), the nominal random variables of \( e_{y} \) and \( e_{\mu} \) can be obtained with
For members with various SCs, the results of the parameters \( a_1, a_2, \) and \( a_3 \) are listed in Table 2. The histograms of \( e_y \) and \( e_\mu \), which demonstrated that \( e_y \) and \( e_\mu \) (\( E_{e_y} \) and \( E_{e_\mu} \)) approximated to zero. Also, the standard deviations of \( e_y \) and \( e_\mu \) (\( \sigma_{e_y} \) and \( \sigma_{e_\mu} \)) being relatively minor, which indicates that the least effect of the variables other than \( \rho \) and SC, can be found in the reference (Rong 2005).

**Modification of the Design Procedure with Error Indices**

Eqs. (7) and (8) provide a valuable tool for modifying the design procedure in order to keep the actual responses (\( y_{m}^{rc} \) and \( \mu^{rc} \)) under control rather than those of equivalent SDOF system response (\( y_{m}^{eq} \) and \( \mu^{eq} \)), with respect to the design performance targets (\( y_t \) and \( \mu_t \)). For the convenience of the following discussion, the design targets are distinguished from two different viewpoints. From a physical viewpoint, design targets are the final goals for designed members under certain blast loading conditions and should remain unchanged within the design. They are called the physical design targets (PHY-DTs). However, from another point of view, design targets are only part of the primary parameters involved in the design to control the responses of the designed members in reaching their PHY-DTs. In this sense, they are named the parametric design targets (PAR-DTs). For the blast design procedure presented above, the values of PAR-DTs are simply fixed to be equal to those of PHY-DTs. However, this action induces some inevitable errors between the member’s responses (\( y_{m}^{rc} \) and \( \mu^{rc} \)) and the PHY-DTs. By properly adjusting the values of PAR-DTs within the design, such errors can easily be eliminated.

Denoting the PHY-DTs for maximum displacement and displacement ductility responses as \( y_t \) and \( \mu_t \) while those for PAR-DTs as \( y_{te} \) and \( \mu_{te} \), the two error indices can be expressed another form

\[
\xi_y = \frac{y_{te} - y_{m}^{rc}}{y_{te}} = f(\rho, SC) + e_y
\]

\[
\xi_\mu = \frac{\mu_{te} - \mu^{rc}}{\mu_{te}} = g(\rho, SC) + e_\mu
\]

In the design process, the responses of \( y_{m}^{rc} \) and \( \mu^{rc} \) are required to be controlled for achieving the PHY-DTs of \( y_t \) and \( \mu_t \) (i.e., \( y_{m}^{rc} = y_t \) and \( \mu^{rc} = \mu_t \)). By substituting them into Eqs. (14) and (15), the PAR-DTs of \( y_{te} \) and \( \mu_{te} \) are obtained as
Eqs. (16) and (17) can be utilized to adjust the PAR-DTs so as to gain better control of the responses of $y^{rc}_m$ and $\mu^{rc}$ within the design. The random variables $e_y$ and $e_\mu$, indicate the uncertain influences of the design variables other than $\rho$ and SC on error indices. Certain quantities of $e_{y,n}$ and $e_{\mu,m}$ corresponding to $n$ and $m$ percentages of nonexceedance probabilities for $e_y$ and $e_\mu$, have to be selected (Kottegoda and Rosso (1996)). Hence, Eqs. (16) and (17) are modified into

\begin{align}
y_{te} &= \frac{y_t}{1 - f(\rho, SC) - e_{y,n}} \\
\mu_{te} &= \frac{\mu_t}{1 - g(\rho, SC) - e_{\mu,m}}
\end{align}

The physical meaning of such an action can be explained as follows. Subtracting Eqs. (18) and (19) from Eqs. (14) and (15), respectively, and rearranging them leads to

\begin{align}
y_t - y^{rc}_m = y_{te}(e_y - e_{y,n}) \\
\mu_t - \mu^{rc} = \mu_{te}(e_\mu - e_{\mu,m})
\end{align}

The item $e_y - e_{y,n}$ in the bracket provides a random variable with a probability of $n\%$, whose value is less than zero as shown in Fig. 10. Since $y_{te}$ is always positive, it is concluded from Eq. (20) that the maximum displacement response $y^{rc}_m$ of the members designed according to Eqs. (18) and (19) will have a probability of $(1-n)\%$ not exceeding the PHY-DT of $y_t$ (or a probability of $n\%$). Eq. (21) can be explained in the same way where the displacement ductility response $\mu^{rc}$ will have a probability of $(1-m)\%$ not exceeding the PHY-DT of $\mu_t$.

Another point to be emphasized is that the determination of PAR-DTs of $y_{te}$ and $\mu_{te}$ from Eqs. (18) and (19) is dependent on $\rho$. Therefore, an iterative procedure is necessary in the design. After assuming $d$ and $\rho$ values, the PAR-DTs of $y_{te}$ and $\mu_{te}$ are adjusted from the given $\alpha$ and PHY-DTs of $y_t$ and $\mu_t$. However, to attain $y_{te}$ and $\mu_{te}$, a new set of $d$ and $\rho$ will have to be established for the member. Hence, the process must be iterated until $d$ and $\rho$ are consistent. The flowchart of the modified blast resistant design depending on $c^{rc}_y$ and $c^{rc}_\mu$ is shown in Fig. 11.

**Illustrative Examples**
By incorporating error indices, the modified blast resistant design procedure is applied to three numerical examples. This demonstrates its use in the design practices. Fig. 12 shows a simply supported beam, a cantilever wall, and a fixed/roller-supported beam subjected to a variety of blast loadings. Examples I and III tend to control the member’s responses at a very low performance level (Schmidt 2003). Thus, a relatively higher target support rotation of $\theta_t = 4^\circ \left[ y_t \approx l \tan(\theta_t)/2 \right]$ and $\mu_t = 10$ are used. However, to control the response at a low performance level (Schmidt 2003), Example II adopts the values of $\theta_t = 2^\circ \left[ y_t \approx l \tan(\theta_t) \right]$ and $\mu_t = 6$. Taking into account of the rebound effect of the structural member subsequent to its maximum displacement response, the area of compression reinforcement is taken to be equal to that of tension reinforcement. In these numerical examples, the values for $n$ and $m$ are taken to be 5 so as to ensure a 95% probability for $y_m^{rc}$ and $\mu^{rc}$ of the designed members not exceeding the PHY-DTs of $y_t$ and $\mu_t$. The convergence conditions for $\rho$ and $d$ are defined as $|\left((\rho_1 - \rho_0)/\rho_0\right)| \leqslant 0.001$ and $|\left((d_1 - d_0)/d_0\right)| \leqslant 0.001$, respectively.

The initial values of $\rho_0$ and $d_0$ are taken as 1.00% and 1,000 mm, respectively. Iterative values of key terms during the design process for these three numerical examples, with the modified procedure depending on the error indices, are listed in Tables 3–5. For comparison, the design of the members with the original procedure is also listed in Tables 3–5. It is noticed that there is no difficulty in reaching convergence with the modified design procedure. However, due to the adjustment of PAR-DTs from Eqs. (15) and (16) within the modified design procedure, more iterative steps and computation time are needed to reach the convergence as compared to those from the original design procedure. Also, the member design shows an obvious decline in $\rho$ and some increase in $d$ with the modified procedure.

**Numerical Verification**

In order to check whether $y_m^{rc}$ and $\mu^{rc}$ of the designed RC members under the given blast loadings are controlled effectively by the modified procedure, nonlinear finite-element analysis of the members is performed. The simulation results are demonstrated in Fig. 13, which shows the comparison between the modified and the original design procedure $y_m^{rc}$ and $\mu^{rc}$ values of the members. Results indicate that the modified design procedure has overcome the disadvantages existing in the control of $y_m^{rc}$ and $\mu^{rc}$ for both the simply supported beam and the cantilever wall. Using the original design procedure $y_m^{rc}$ slightly exceeds the target of $y_t$, and $\mu^{rc}$ is too conservative. The modified design procedure values of $y_m^{rc}$ and $\mu^{rc}$ for these two members both are approximated to their respective targets and restricted to be a little on the conservative side. Besides, $y_m^{rc}$ seems to be controlled closer to its target than $\mu^{rc}$ due to the smaller standard deviation of $e_y$. As for the fixed/roller-supported beam, due to the strict requirement of 95% probability for $y_m^{rc}$ not exceeding $y_t$, the control of $y_m^{rc}$ seems to be slightly more conservative than that by the original design procedure. However, in this example, $\mu^{rc}$ controlled is still much closer to $\mu_t$ by the modified procedure. Therefore, comparison of these results demonstrates that the modified
Conclusions

A blast-resistant design procedure for RC flexural members has been presented in this paper. For blast-resistant designs, it would be more ideal that the RC member does not exhibit a brittle failure associated with shear failure during loading, and is allowed to experience flexural deformation. For this, adequate shear reinforcement should be present to mitigate against the brittle failure associated with shear failure. The proposed design method aims to provide an iterative procedure for the design of the longitudinal reinforcement and effective depth more closely related to the design of members for flexure. It has been demonstrated that the proposed procedure could incorporate the design performance criteria of maximum displacement and displacement ductility simultaneously to give a unique design of a RC member under a given blast loading on the basis of nondimensional energy spectra. Thus, the design values of \( d \) and \( \rho \) of the RC member can be specifically determined. It could also keep the actual deformation responses of the designed member under control in meeting design performance criteria.

Although the design procedure presented tries to keep the responses of the equivalent SDOF system under control such that the design performance targets are met (i.e., \( \mu^\text{eq} = \mu_t \) and \( y^\text{eq}_m = y_t \)), numerical verification indicates that some errors do exist between the responses of designed RC member \( (y^\text{rc}_m \text{ and } \mu^\text{rc}) \) and their respective design targets \( (y_t \text{ and } \mu_t) \). This is due to some simplifying assumptions made in the derivation of \( d \) and \( \rho \) by converting a continuous RC member into its equivalent SDOF system. However, since the RC member specifically is designed under a given blast loading condition by the modified procedure, the formulas for quantifying the approximate errors are derived from extensive numerical analysis and used for modifying the design procedures.

Through the adjustment of parametric design targets using the formulas for \( c^\text{rc}_y \) and \( c^\text{rc}_\mu \), the modification of the design procedure is accomplished to keep \( y^\text{rc}_m \text{ and } \mu^\text{rc} \) under control. The implementation of the modified design procedure into three numerical examples indicates that more iterative steps are needed to reach convergence as compared to those of the original design procedure. However, the responses of \( y^\text{rc}_m \) and \( \mu^\text{rc} \) for the member designed by the modified procedure from the nonlinear numerical analysis are controlled. These are controlled approximately similarly to the design performance targets in a conservative manner.

Acknowledgments

This research was supported by Research Grant LEO 99.05 provided by the Defense Science and Technology Agency (DSTA), Singapore. Special thanks are due to John Crawford, President of Karagozian and Case for his critical reading of the paper and many invaluable suggestions for improvement.
Notation

The following symbols are used in this paper:

- $C =$ Nondimensional energy factor;
- $DIF =$ dynamic increase factor;
- $d =$ effective depth of the element measured from the extreme compression fiber to the centroid tensile reinforcement;
- $d' =$ distance from the extreme compression fiber to the centroid of compression reinforcement;
- $E_c =$ Young's modulus of elasticity for concrete;
- $E_{el} =$ ultimate elastic energy;
- $E_{max} =$ maximum strain energy;
- $E_s =$ Young's modulus of elasticity for steel;
- $F_1 =$ peak value of the force;
- $f_{cs}, f_{dc} =$ strength of concrete in compression for the static and dynamic conditions;
- $f_{st}, f_{ds} =$ strength of flexural steel in static and dynamic conditions, respectively;
- $f_{it}, f_{dt} =$ tensile strength of concrete in static and dynamic conditions, respectively;
- $K_{LE}, K_{ME} =$ transformation load and mass factors, respectively;
- $k_a =$ equivalent elastic stiffness for the RC elements;
- $k_e =$ initial stiffness of the equivalent SDOF system of the designed RC member;
- $l =$ length of the member;
- $m =$ equivalent mass of the equivalent SDOF system;
- $m_a =$ mass of the RC member;
- $R_m =$ ultimate strength of the equivalent SDOF system;
- $T =$ period;
- $t_d =$ load duration;
- $V_c =$ shear capacity;
- $V_u =$ ultimate shear force;
- $y_{eq} =$ elastic displacement response of the equivalent SDOF system of the designed RC member under the given blast loading;
- $y_{eq} =$ Elastic displacement response of the designed RC member at the significant point under the given blast loading;
\[ y_{m}^{\text{eq}} = \text{maximum displacement response of the equivalent SDOF system of the designed RC member under the given blast loading;} \]

\[ y_{m}^{\text{rc}} = \text{maximum displacement response of the designed RC member at the significant point under the given blast loading;} \]

\[ y_{t} = \text{target maximum displacement;} \]

\[ y_{te} = \text{Parametric design target of displacement;} \]

\[ \varepsilon = \text{convergence tolerance;} \]

\[ \gamma = \text{reduction coefficient;} \]

\[ \mu_{m}^{\text{eq}} = \text{displacement ductility response of the equivalent SDOF system of the designed RC member under the given blast loading;} \]

\[ \mu_{m}^{\text{rc}} = \text{displacement ductility response of the designed RC member at the significant point under the given blast loading;} \]

\[ \mu_{t} = \text{target displacement ductility factor;} \]

\[ \mu_{te} = \text{Parametric design target of displacement ductility factor;} \]

\[ \rho, \rho' = \text{longitudinal tension and compression reinforcement ratio, respectively; and} \]

\[ \sigma_{e}, \sigma_{e_{\mu}} = \text{standard deviations.} \]
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<th>$k_e$ (N/m)</th>
<th>$T$ (s)</th>
<th>$R_a$ (N)</th>
<th>$E_{el}$ (N/m)</th>
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Note: Design results: $d=299.2$ mm; $\gamma=1.551$%; $p_0=0.15\%$. Initial data: $E_e=200$ GPa; $E_c=28$ GPa; $f_s=460$ MPa; $f_{el}=506$ MPa; $f_c=33$ MPa; $f_{el}=40$ MPa; $f_s=250$ MPa; $f_{el}=275$ MPa; $l=3.0$ m; $b_e=1.0$ m; $P_0=2.080$ KPa; $t_d=4.83$ ms; and density of concrete $\rho_{c}=1.500$ kg/m$^3$. 

Table 1. Iterative Procedure for the Design of a Cantilever Wall Using NES.
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Table 2
Table 3

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| (b) With original procedure |
| 210.00     | 10.0     | 1,000.00  | 10.00     | 1,000.00   | 1.000    | -0.10966       | 0.36910        | 184.35     | 13.885    | 583.30    | 1.374    | 0.178          |
| 478.57     | 2.172    | 651.23    | 1.449     | 558.18     | 1.464    | 559.22         | 1.457          |
| 559.22     | 1.457    | 559.14    | 1.458     | 559.14     | 1.458    |                  |

Note: $f_{\mu_0}$=506 MPa; $f_{\mu_0}$=40 MPa; $E_1$=40 GPa; $E_2$=200 GPa; and $f_{\mu_0}$=275 MPa. Convergence conditions: $\|p_1-p_0\|/p_0 \leq 0.001$ and $\|d_1-d_0\|/d_0 \leq 0.001$. 
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<th>$\epsilon_{x,i}$</th>
<th>$\epsilon_{y,i}$</th>
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<th>$g_{(\rho_i, SC)}$</th>
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<th>$\mu_{w_i}$</th>
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(a) With modified procedure

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(b) With original procedure

Note: $f_{(\mu)}=506$ MPa, $f_{(\mu)}=40$ MPa, $E_{(\mu)}=40$ GPa, $E_{(\mu)}=200$ GPa, and $f_{(\mu)}=275$ MPa. Convergence conditions: $|\rho_i-\rho_0|/\rho_i<0.001$ and $|d_{1i}-d_{0}|/d_{0}<0.001$.

Table 4
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For the modified procedure.

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<th>$\epsilon_{p,5}$</th>
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Note: $f_{a}=506$ MPa; $f_{b}=40$ MPa; $E_{s}=40$ GPa; $E_{t}=200$ GPa; and $f_{a,b}=275$ MPa. Convergence conditions: $|\rho_t - \rho_0|/\rho_0 \leq 0.001$ and $|d_1 - d_0|/d_0 \leq 0.001$.  

Table 5
Fig. 1
Fig. 2
Fig. 3
Give peak pressure $P_0$, duration $t_0$, length $l$, target displacement $y$, and displacement ductility factor $\mu$, and compute elastic displacement $y_0$ with $y_0 / \mu$, 

Assume an initial reduction coefficient $\gamma$ of moment of inertia and a sectional effective depth $d_e$. 

Determine the mass $m$, stiffness $k$, of the assumed RC member 

Find the load and mass factor $(K_{ld} \text{ and } K_{md})$ and compute $m, F, k, T, R_n$, and $E_n$ for the equivalent SDOF system 

Compute $F_n/R_n$ and $t_0/T$ and find non-dimensional energy factor $C$ from NES in Fig. 2, and find maximum strain energy $E_{max}$ with Eq. (1) 

Determine a new initial stiffness $k_0$, and ultimate strength $R_u$, for equivalent SDOF system with Eq. (3) 

Compute a new $d_e$ and $\rho$ with Eqs. (4) and (5) 

If $|\rho_0 - \rho| / \rho_0 < \varepsilon_1$ and $|d_e - d_{e0}| / d_{e0} < \varepsilon_2$, then No 

Yes 

Determine the maximum shear force and the design stirrup with Eqs. (6) 

End 

Fig. 4
Fig. 6
Fig. 7
Fig. 8
Fig. 9
Fig. 10
Start

Give peak pressure $P_0$, duration $t_0$, length $l$ and all parameters related to the materials properties

Give PHY-DTs $(y_0, \mu_0)$, the percentages of non-exceedance probabilities for $e_0$ and $e_x$ $(n, m)$ and find $e_{x0}$ and $e_{xn}$

Assume an initial longitudinal reinforcement ratio $\rho_s$ and sectional effective depth $d_s$

Adjust the PAR-DTs $(y_0, \mu_0)$ from Eqs. (19) and (20)

Calculate a new longitudinal reinforcement ratio $\rho_f$ and sectional effective depth $d_f$

$\frac{|(\rho_f - \rho_s)/\rho_s| < \varepsilon_f}$  
and $|d_f - d_s)/d_s| < \varepsilon_d$

No

Yes

Determine the maximum shear force and the design stirrup

End

Fig. 11
(a). A simply supported beam under the blast loading (Design example I)

(b). A cantilever wall under the blast loading (Design example II)

(c). A fixed/roller-supported beam under the blast loading (Design example III)

Fig. 12
(a). Designed with the modified procedure

(b). Designed with the original procedure

(Example I)

(a). Designed with the modified procedure

(b). Designed with the original procedure

(Example II)

(a). Designed with the modified procedure

(b). Designed with the original procedure

Fig. 13