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Drift-controlled design of reinforced concrete frame structures under distant blast conditions—Part I: Theoretical basis

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Abstract

Proper control levels of lateral drifts anticipated for reinforced concrete (RC) frame structures within the predefined performance level become crucial when the frame structure is subjected to distant intense surface explosions. For this purpose, a new design method is presented in a two-part paper based on the transformation of a blast loading into an equivalent static force (ESF). The ESF is calculated in such a manner that the same maximum inter-storey drift ratio (MIDR) under the blast loading will be reproduced. The first part of the two-part paper focuses on the computational model of ESF for a single-degree-of-freedom (SDOF) system and the design method based on ESF with the requirement for controlling its maximum displacement response to achieve the specified target displacement. Numerical examples have been included to illustrate the method while the verifications of the dynamic responses of the designed SDOF system are performed with nonlinear dynamic analyzes. The numerical results indicate that the target displacement is well met for the designed SDOF system in resisting a given blast loading. Extension of the computational model of ESF and the corresponding design method with ESF for a SDOF system into a RC frame structure will be further discussed in the companion paper.

Keywords: Maximum inter-storey drift ratio; Equivalent static force; Maximum displacement response; SDOF system

1. Introduction and background

Due to an accidental severe industrial explosion of petroleum refineries, chemical plants or ammunition storage areas, a surrounding civilian building may be subjected to a large-scale blast shock wave [1–3]. In these kinds of relatively distant explosion conditions, the hemispherical blast wave produced may be reasonably simplified to be a planar wave and essentially parallel to the front faces of the target structure by comparing the sizes of hemispherical blast wave with the target structure. The distant blast wave loading on the structure produces a uniform lateral blast pressure on the front and rear faces and a vertical pressure on the roof [1–3]. A significant side-way response may occur inducing a certain degree of damage for a reinforced concrete (RC) frame structure under such blast pressure [4,5,7]. As a result, controlling their maximum inter-storey drift ratios (MIDRs) within the predefined performance level becomes a crucial consideration in the blast resistant design of building structures. The vertical roof pressure has only a slight effect on the frame structure.
side-sway as discussed from the comparison study of a planar single-storey frame structure by Baker et al. [6], therefore it is ignored in this study considering that the limitations of side-sway are of main concern. In order to ensure the side-sway response within the expected performance level, different levels of side-sway limits are specified for the design of single-storey rigid frame structures in the current design guidance [7,8] according to the operational needs of the facility and the needs for reusability.

Blast loadings with short durations act dynamically on structures. However, the dynamic loadings are extremely difficult to handle within the structural design since they cannot be directly utilized for the calculation of the internal forces of the structural members. In the seismic/wind resistant design procedures, the dynamic seismic/wind actions are generally transformed into static loadings, which are deemed to be able to produce equivalent effects on structures [9–12]. Similarly, if an equivalent static force (ESF) applied to the structure with the magnitude and direction that could approximately represent the effects of the lateral blast loading can be found, it will greatly facilitate the blast resistant design of the building structures.

The implementation of ESF into the blast resistant design for a single-storey rigid frame subjected to a relatively low blast overpressure has been recommended in the current design guidance [7,17]. The dynamic load factors were provided for establishing the equivalent static loads for identifying the frame failure mechanism. Based upon the mechanism method, as employed in static-plastic design, estimations were made for the required plastic bending capacity. However, these dynamic load factors were approximate and made no distinction for different end conditions. They were only expected to result in the estimation of the required resistance for a trial design. In order to confirm that a trial design meets the recommended deformation criteria, a rigorous frame analysis has to be performed.

This two-part paper aims to present a new blast-resistant design method for RC frame structure for keeping its MIDR under proper control based on the ESF of the lateral blast force. For this purpose, this part of the two-part paper concentrates on the description of the computational model of the ESF and the corresponding design method based on the ESF for a single-degree-of-freedom (SDOF) system. Three numerical examples have been included to illustrate the implementation of the method while verifications of the dynamic responses of the designed SDOF systems are carried out through nonlinear dynamic analysis. The design method presented is further extended to the design of RC frame structures in the companion paper [13], where the frame structure could not be simply idealized as a SDOF system under the blast condition.

Basically the design method developed with ESF is for the design of a RC frame structural system in controlling its MIDR response, which occurs at a time generally later than the blast loading duration \( t_d \) [6,15]. The descriptions of the model of ESF and the design method with ESF for a SDOF system herein are only to provide a theoretical basis. Therefore the assumption that the peak response (maximum displacement for a SDOF system or MIDR for a frame structure) takes place after the loading duration \( t_d \) is generally adopted in the study. Only the SDOF systems that satisfy this assumption are of concern in this paper.

2. ESF for a SDOF system

2.1. Process for the development of ESF

Because of the short duration of the blast loading, the vibration of a SDOF system after reaching the peak response will be limited within its elastic range inducing no further cumulative damage [16]. Therefore, for a well-defined SDOF system, the maximum displacement response can adequately characterize its damage status in blast events. Under such conditions, if there exists a force which makes it feasible for the SDOF system to experience exactly the same maximum displacement response when statically applied, this force is called the ESF of the blast loading. To calculate the ESF on a SDOF system, a model is presented herein with its process illustrated in Fig. 1.

The blast force can be rationally simplified into a triangular pulse only if the peak pressure and impulse are preserved [3,14], hence the function of the blast force can be written as

\[
F(t) = \begin{cases} 
F_1(1 - t/t_d), & t \leq t_d, \\
0, & t > t_d,
\end{cases}
\]  

(1)
where \( t_d \) is the duration and \( F_1 \) is the peak amplitude of the blast force. Since after \( t_d \), the blast force will be zero, no additional external energy is transferred to the system and the total energy within the SDOF system (consisting of the kinetic energy and the strain energy) will remain constant. This magnitude determines the maximum response if any of the damping effects are ignored [16]. The total energy can be obtained from the response state of the system at \( t_d \) plotted in Fig. 1(a), written as

\[
W = W_{k,t_d} + W_{s,t_d},
\]

where \( W \) is the total energy of the SDOF system at time \( t_d \); \( W_{k,t_d} \) and \( W_{s,t_d} \) are the kinetic and strain energies at \( t_d \), respectively. In order to simulate the energy components of the SDOF system at this instant of time, an equivalent static SDOF system is developed as shown in Fig. 1(b), where an additional elastic spring is added to the original SDOF system. It is proposed that under a certain external static force \( P \), the equivalent static system experiences the same displacement response as \( y_{t_d} \) (the dynamic response of the original SDOF system at \( t_d \)) and therefore the strain energies within the spring \( K_a \) in both systems are identical. In order to model the kinetic energy, the strain energy \( W_a \) within the additional spring \( K_a \) should be equal to \( W_{k,t_d} \), thus

\[
M y_{t_d}^2 / 2 = K_a y_{t_d}^2 / 2
\]
and

\[ K_a = M\frac{\dot{y}_{td}^2}{\dot{y}_{td}^2}, \quad (4) \]

where \( \dot{y}_{td} \) is the velocity of the original SDOF system at time \( t_{td} \) under \( f(t) \), \( K_a \) is the elastic stiffness of the additional spring. From the equilibrium of the equivalent static system

\[ P = F_s + F_a, \quad (5) \]

where \( F_s \) and \( F_a \) are the forces produced, respectively, by the original and additional springs (\( K_s \) and \( K_a \)) in the equivalent static system in Fig. 1(b). For the additional elastic spring, \( F_a \) is given as

\[ F_a = K_a\dot{y}_{td}. \quad (6) \]

Substituting Eq. (4) into Eq. (6) and further Eq. (5) induces

\[ F_a = M\frac{\dot{y}_{td}^2}{\dot{y}_{td}^2}, \quad (7) \]

\[ P = F_s + M\frac{\dot{y}_{td}^2}{\dot{y}_{td}^2}. \quad (8) \]

After \( t_{td} \), the kinetic energy \( W_{k,t_{td}} \) will be gradually transformed into the strain energy causing further displacement for the original SDOF system until the maximum response \( y_m \) is reached as shown in Fig. 1(a). For modeling this process with the equivalent static system, the strain energy \( W_a \) in the additional spring \( K_a \) needs to be released statically in such a way that this part of the energy is transferred to the original spring \( K_s \). With the support of the additional spring statically moving toward the spring until the support reaction force RF decreases to zero as shown in Fig. 1(c) and (d), the strain energy of the additional spring \( K_a \) is gradually transformed into that of the original spring \( K_s \) and the equivalent static system is finally recovered to the original SDOF system with the static force \( P \) exerting on it. However, it should be noted that during this process, besides the strain energy transformation from the additional spring \( K_a \) to the original spring \( K_s \), an extra positive external energy is produced by \( P \) together with the declining RF. Accordingly a relatively larger displacement response \( y_{np} \) will be induced than \( y_m \) of the original SDOF system under the blast condition. Thus an ESF factor \((\lambda)\) less than one is introduced to decrease \( P \) producing the ESF \( (P_{st}) \), which creates the same maximum response as \( y_m \) when statically applied to the original SDOF system as shown in Fig. 1(e)

\[ P_{st} = \lambda P. \quad (9) \]

2.2. ESF factor

For a SDOF system with the elastic–perfectly plastic resistance function, the closed-form solution of the ESF factor \( \lambda \) is derived with respect to three different cases according to the response states at times \( t_{td} \) and \( t_m \) (the time for maximum displacement response) as shown in Fig. 2(a)–(c). They are:

– Case I: In the elastic state at \( t = t_{td} \) as well as \( t = t_m \);
– Case II: In the elastic state at \( t = t_{td} \), while in the plastic state at \( t = t_m \);
– Case III: In the plastic state at \( t = t_{td} \) and \( t = t_m \).

Case I: Since the SDOF system is still within its elastic limit during the whole response range as shown in Fig. 2(a), the total energy at \( t_{td} \) and \( t_m \) can be obtained as

\[ W = K_s\dot{y}_{td}^2/2 + M\dot{y}_{td}^2/2 \quad (10) \]

and

\[ W = K_s\dot{y}_{m}^2/2, \quad (11) \]

where \( K_s \) is the initial stiffness of the elastic–perfectly plastic SDOF system. Since no external blast force is applied to the system after \( t_{td} \), Eqs. (10) and (11) can be equated leading to

\[ M\dot{y}_{td}^2 = K_s\left(y_{m}^2 - \dot{y}_{td}^2\right). \quad (12) \]
Substituting Eq. (12) into Eq. (8) and considering $F_s = K_s y_t$ (the spring $K_s$ in its elastic range at $t_d$)

$$P = K_s y_m \frac{y_m}{y_{t_d}}.$$  \hfill (13)

To meet the requirement that the same $y_m$ appears for the SDOF system under ESF, $P_{st}$ should be equal to its resistance at the displacement $y_m$, which in the elastic range is given as

$$P_{st} = K_s y_m.$$  \hfill (14)

Substituting Eqs. (13) and (14) into Eq. (9), the ESF factor $\lambda$ in this case is determined as

$$\lambda = \frac{y_{t_d}}{y_m}.$$  \hfill (15)
Case II: For the second case, Eq. (10) is still valid for computing the total energy at \( t_d \) since the SDOF system at this moment is within its elastic limit. However, the system has entered the plastic response stage at \( t_m \), hence Eq. (11) for the calculation of the total energy at \( t_m \) changes into

\[
W = \frac{1}{2} K_s y_e (2y_m - y_e),
\]

where \( y_e \) is the elastic limit displacement of the SDOF system. Equating Eq. (10) with Eq. (16) yields

\[
M y_{td}^2 = K_s \left( 2y_e y_m - y_e^2 - y_{td}^2 \right).
\]

By substituting Eq. (17) into Eq. (8) and considering \( F_s = K_s y_e \), \( P \) is given as

\[
P = K_s y_e \left( \frac{2y_m - y_e}{y_{td}} \right).
\]

In this case, in order to statically produce the same maximum displacement as \( y_m \) in the blast condition, which is beyond the elastic limit of the SDOF system with an elastic–perfectly plastic resistance function, the ESF \( (P_{st}) \) should be identical with the ultimate strength. Thus

\[
P_{st} = K_s y_e.
\]

The value of \( \lambda \) is finally obtained by substituting Eqs. (18) and (19) into Eq. (9) as

\[
\lambda = \frac{y_{td}}{2y_m - y_e}.
\]

Case III: Since the SDOF system has entered its plastic response stage before \( t_d \), the total energy for the elastic–perfectly plastic SDOF system at \( t_d \) is given by

\[
W = \frac{1}{2} K_s y_e (2y_m - y_e) + \frac{1}{2} M y_{td}^2.
\]

In this case, Eq. (16) is also valid for expressing the total energy at \( t_m \), therefore equating Eq. (16) with Eq. (21) yields

\[
M y_{td}^2 = 2K_s y_e (y_m - y_{td}).
\]

By taking \( F_s = K_s y_e \) and substituting Eq. (22) into Eq. (8), \( P \) is obtained as

\[
P = K_s y_e \left( \frac{2y_m - y_{td}}{y_{td}} \right).
\]

With Eq. (19) for the evaluation of \( P_{st} \), \( \lambda \) is derived from Eqs. (9) and (23) as

\[
\lambda = \frac{y_{td}}{2y_m - y_e}.
\]

2.3. Computational model for ESF

By summarizing the above analyzes, a model to calculate the ESF for an elastic–perfectly plastic SDOF system is given as below:

\[
\begin{cases}
P_{st} = \lambda P, \\
P = F_s + F_a, \\
\lambda = X, \\
X = \begin{cases} \\
y_{td}/y_m, & y_m \leq y_e, \\
y_{td}/(2y_m - y_e), & y_{td} \leq y_e \text{ and } y_m \geq y_e, \\
y_{td}/(2y_m - y_{td}), & y_{td} \geq y_e, \\end{cases}
\end{cases}
\]

\[\text{(25)}\]
An extra variable $X$ is introduced in Eq. (25) for the convenience of extending this model to RC frame structures as discussed in the companion paper [13]. The physical meanings for the other variables have been well defined previously.

It should be pointed out that this model does not attempt to assess the maximum response of a particular SDOF system under the blast condition through the ESF, but provides a powerful tool in designing the ultimate strength of the system to achieve the specified target displacement. The design method based on this model is addressed in the following section.

3. Design of a SDOF system with the ESF

For an elastic–perfectly plastic SDOF system with stiffness $K_s$, mass $M$ and a blast loading of peak force $F_1$, and a duration $t_d$ acting on them, the relationship between their ultimate strengths $R_m$ and the corresponding maximum displacement responses $y_m$ can be explained as follows. Supposing that there are two such SDOF systems but with different ultimate strengths where $R_{m1} < R_{m2}$, it should be noted that:

1. During any short period of time $\Delta t$ within $t_d$, the displacement response for the first system with $R_{m2}$ will not exceed that of the second system with $R_{m1}$, accordingly the total external work done by the blast loading on the first system will be equal to or less than that on the second system.
2. With the same displacement response, the energy absorbing capacity by the deformation of the first system will not be less than that of the second one.

Accordingly, it can be derived that, with the increase in the ultimate strength of the system, the maximum displacement response will decrease gradually as shown in Fig. 3, where $R_{m1} < R_{m2} < R_{m3}$ and $y_{m1} > y_{m2} > y_{m3}$. However, when $R_m$ reaches the critical value of $R_{mref}$ and $y_m$ declines to $y_{mref}$, the whole response of the system is located in its elastic range. A further increase in $R_m$ will have little effect on its maximum displacement response. This means that for the SDOF system with $K_s$, $M$, $F_1$ and $t_d$, its maximum response will never be smaller than $y_{mref}$, which can be obtained from dynamic analysis by assuming the SDOF system to be fully elastic.

Design of the ultimate strength for a SDOF system satisfying the given target maximum displacement $y_t$ is to find the value of $R_m$ corresponding to $y_m = y_t$. If $y_t \geq y_{mref}$, then the maximum response of the designed SDOF system will coincide with or be beyond its elastic limit state as shown in Fig. 3. In such a case the ESF defined above is equal to the ultimate strength, i.e., $R_m = P_{st}$. Therefore $R_m$ and $y_t$ should satisfy Eq. (25) by
replacing \( P_{at} \) and \( y_{m} \) with \( R_{m} \) and \( y_{t} \), respectively

\[
\begin{align*}
R_{m} &= \lambda P, \\
P &= F_{s} + F_{a}, \\
\lambda &= X, \\
X &= \begin{cases} 
    y_{td}/y_{t}, & y_{t} \leq y_{e}, \\
    y_{td}/(2y_{t} - y_{e}), & y_{td} \leq y_{e} \text{ and } y_{t} \geq y_{e}, \\
    y_{td}/(2y_{e} - y_{td}), & y_{td} \geq y_{e}.
\end{cases}
\end{align*}
\tag{26}
\]

On the other hand if \( y_{t} < y_{\text{mref}} \), there exists no solution for \( R_{m} \) and in order to reach such a \( y_{t} \), it needs to increase \( K_{s} \) or \( M \). According to the discussion, an iterative procedure is presented to determine this unique \( R_{m} \) based on the model of Eq. (26). The flowchart is shown in Fig. 4.

4. Illustrative examples

To illustrate the design procedures with ESF, three numerical examples are presented as shown in Table 1 where a variety of specified target displacements are required to be satisfied for the SDOF systems in resisting different blast forces. In these examples, \( t_{d} \) is taken in the range from 40 to 250 ms considering that the distant blast wave in this paper is defined by comparing the size of hemispherical blast wave produced by an industrial explosion with that of a mutli-storey frame structure. The iterative procedures at each step of these numerical
examples are demonstrated in Tables 2–4, where the initial ultimate strength $R_{m0}$ is taken as the multiplication of $K_s$ with $y_{mref}$. The convergence condition of $\left(\frac{R_{m1}}{R_{m0}}\right) - 1 \leq 0.001$ is employed.

In numerical examples I and II, $y_t$ is greater than $y_{mref}$ thus $R_m$ can be iteratively determined with respective to $K_s$ listed in Table 1. However $y_t$ is less than $y_{mref}$ in numerical example III where to find the solution of $R_m$ of the SDOF system, $K_s$ given in Table 1 should be enlarged. By taking the stiffness twice that of the initial
stiffness, the design is carried out and the results are shown in Table 4. It is indicated from the iterative procedures that there is no difficulty in reaching convergence for the design of the SDOF system with the ESF in finding the unique $R_m$.

To evaluate the displacement responses for the SDOF systems designed based on ESF, nonlinear time history analyzes are performed and the results are shown in Figs. 5–7. It is obvious that the maximum displacements of the designed SDOF systems reach exactly their respective objective values. Thus the design
based on the ESF can effectively control the maximum displacement of the SDOF system in resisting the given blast force.

5. Summary and conclusions

A computational model for the ESF and the design method using ESF for the elastic–perfectly plastic SDOF systems are discussed in this part of the two-part paper. This provides the theoretical basis for the blast-resistant design of RC frame structures in terms of controlling its MIDR within the predefined performance level under blast conditions from distant surface explosion, which appears in the companion paper [13].

The model for the computation of ESF is based on the assumption that the peak response of the SDOF system occurs after the blast loading duration \( t_d \). An equivalent static SDOF system is derived which simulates the energy components of the original SDOF system at time \( t_d \). Based on the equilibrium of an equivalent static system, an external static force is computed. However this force tends to produce a larger maximum displacement demand than that of the original SDOF system under the blast force. As a result, an ESF factor is introduced to reduce the external static force to obtain the final ESF. With respect to three different response states at time \( t_d \) and \( t_m \), a closed-form solution of the ESF factor is obtained for the elastic–perfectly plastic SDOF system.

By replacing the ESF with the ultimate strength as well as the maximum displacement with the required design target displacement, the computational model of the ESF is implemented for the design of an elastic–perfectly plastic SDOF system. The ultimate strength of the SDOF system is solved iteratively during the design process so as to satisfy the specified target displacement. The presented design procedures have been illustrated via three numerical examples, which show no difficulty in convergence of the iterative procedures. The maximum displacement responses of the designed SDOF systems under the given blast conditions can be controlled to be exactly equal to their corresponding targets, as verified from nonlinear dynamic analysis.

Acknowledgments

This research was supported by a research grant LEO 99.05 provided by the Defense Science & Technology Agency (DSTA), Singapore, under the Protective Technology Research Center, Nanyang Technological
University, Singapore. Any opinions, findings and conclusions expressed in this paper are those of the writers and do not necessarily reflect the view of DSTA, Singapore.

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