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Source Localization by TDOA with Random Sensor Position Errors - Part II: Mobile sensors

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Abstract—For the purpose of source localization, we have proposed a constrained weighted least squares (CWLS) source localization method in our companion paper, which uses static sensors by accounting for random uncertainties in sensor positions. This paper is devoted to developing two recursive algorithms to deal with the source localization problem by using time difference of arrival (TDOA) measurements received by mobile sensors. More specifically, the first one uses the current TDOA measurements to estimate the unknown source position and then treats the estimate as a measurement to update the source localization. For the second approach, we estimate an auxiliary variable with the current TDOA measurements and then rearrange the nonlinear TDOA equations into a set of linear measurement equations to update the source localization. An illustrative example is given to demonstrate that the second algorithm outperforms the first one.

Index Terms—Source localization, time difference of arrival, weighted least-squares, recursive localization algorithm.

I. INTRODUCTION

In our companion paper [1], we have proposed a constrained weighted least squares (CWLS) source localization method which uses time difference of arrival (TDOA) measurements received by a network of passive static sensors. Although the proposed CWLS source localization method is able to reach the optimal accuracy, i.e. the Cramer-Rao lower bound (CRLB), the performance of the localization still could be poor as the CRLB depends on the specific location geometry formed by sensors and the source. When the location geometry is not desirable, such as the positions of sensors are close to each other or the source is far away, the CRLB will be large.

Modern localization system often uses passive sensors which could be born by airplanes or unmanned aerial vehicles (UAVs). The positions of these mobile platforms will change dynamically and the information of their positions can be available with GPS [2]. So source localization based on mobile sensor network may be tracked over time from multiple TDOA measurements. The problem of tracking a stationary or moving source using multiple TDOA collected over time is not a trivial task due to the nonlinear nature of the TDOA measurements. Okello and Musicki [3] tackled this problem based on only two UAVs. They used the TDOA to define a hyperboloid on which the emitter must be located where the hyperbolic measurement error region is approximated by a sum of weighted Gaussian distributions. They applied unscented Kalman filters (UKFs) initiated with Gaussian mixture measurement (GMM) components. This method has been extended to include frequency difference of arrival (FDOA) in addition to TDOA in [4]. Fletcher et al. [5] have investigated the recursive localization problem using a simple extended Kalman filter (EKF) based on two UAVs. A comparison of these recursive algorithms for a pair of UAVs is given in [6].

All of the above mentioned methods are based on dealing with the nonlinear TDOA measurement equations directly. In our companion paper [1], we dealt with the nonlinearity of TDOA by rearranging the nonlinear TDOA equations into a set of linear equations. However, this rearrangement needs to introduce an auxiliary variable which depends on the source position. Actually, this reorganization idea was initially proposed in [7] for source localization with static sensors, and improved in [8]–[10] which make use of the relationship between the unknown source position and the auxiliary variable to achieve better localization performance. Inspired by this reorganization idea to remove the nonlinearity in the TDOA measurement equations, the present paper is devoted to developing two linear recursive source localization algorithms for a stationary target by using TDOA measurements received from a mobile sensor network.

The first recursive algorithm is referred to as “recursive localization algorithm” and the second as “improved recursive localization algorithm”. In the recursive localization algorithm, we rearrange the nonlinear TDOA measurement equations into a set of linear equations with the TDOA measurements at the current sampling step and use the weighted least-squares (WLS) method to estimate the unknown source position and its estimation error covariance. Then we treat the estimate as a measurement to update the source localization estimation obtained at the last sampling step. In the improved recursive localization algorithm, we use the weighted least-squares (WLS) method to estimate the auxiliary variable at the current sampling step firstly and then substitute it into the set of rearranged TDOA equations which gives rise to a new set of linear measurement equations with respect to the unknown source position only. We then use Kalman filter to update the
source localization estimate of the last sampling step.

The rest of the paper is organized as follows. Section II formulates the recursive source localization problem with mobile sensors and introduces the symbols and notations used. Section III presents the proposed two recursive source localization algorithms. Section IV contains the simulation results to demonstrate the performance of the two algorithms. Section V is the conclusion.

Throughout this paper, the transpose and inverse of matrix \( X \in \mathbb{R}^{m \times n} \) are denoted by \( X' \) and \( X^{-1} \) respectively, and \( X \preceq 0 \) (\( > 0 \)) means that \( X \in \mathbb{R}^{m \times n} \) is symmetric and positive semidefinite (positive definite). The symbols \( I \) and \( 0 \) represent the identity matrix and zero matrix with appropriate dimension, \( \| \cdot \| \) means the Euclidean norm, and \( E(\cdot) \) means the mathematical expectation.

II. PROBLEM FORMULATION

The source localization scenario is 3-D where a point source at unknown position \( \mathbf{u} = [x, y, z]' \) radiates a signal to a network of \( n \) mobile passive sensors. At each sampling step, the TDOAs of the received signals with respect to the signal at a reference sensor are transmitted to the central processor. Without loss of generality, let the first sensor be the reference.

The true position of the \( i \)-th sensor at sampling time \( k \) is denoted as \( s_i^k = [x_i^k, y_i^k, z_i^k]' \), where \( i = 1, \ldots, n \). The TDOA measurement model between sensor pair \( i \) and \( j \) at step \( k \) is given by

\[
t_{ij}^k = t_{ij}^k + \Delta t_{ij}^k,
\]

where \( t_{ij}^k \) is the true TDOA, and

\[
t_{ij}^k = t_{ij}^0 - t_{ij}^0.
\]

The TDOA noise vector \( \Delta \mathbf{t}^k = [\Delta t_{i1}^k, \ldots, \Delta t_{in}^k]' \) is zero-mean Gaussian noise with covariance \( \mathbf{Q}_t^k \). After multiplying by the propagation speed \( c \), we have the range difference of arrival (RDOA) measurement

\[
r_{i1}^k = r_{i1}^0 + c\Delta t_{i1}^k,
\]

where \( r_{i1}^k = ct_{i1}^k, r_{i1}^0 = ct_{i1}^0 \). The true RDOA measurement \( r_{i1}^k \) is

\[
r_{i1}^k = r_{i1}^0 - r_{i1}^0,
\]

where \( r_{i1}^0 \) is the distance between the source to the true position of sensor \( i \), i.e.,

\[
r_{i1}^0 = \| \mathbf{u} - s_i^k \|.
\]

In the sequel of this paper, TDOA and RDOA will be used interchangeably. The collection of all RDOA measurements at sampling time \( k \) is denoted by an \((n - 1) \times 1\) vector as

\[
\mathbf{r}^k = [r_{21}^k, r_{31}^k, \ldots, r_{n1}^k]' = \mathbf{r}^k + \Delta \mathbf{r}^k,
\]

where the RDOA error vector \( \Delta \mathbf{r}^k = c\Delta \mathbf{t}^k \) is zero-mean Gaussian noise with covariance \( c^2 \mathbf{Q}_t^k \).

The positions of the sensors change dynamically at each step to collect multiple TDOA measurements over time. However, at sampling step \( k \), the true sensor positions \( s_i^k \) are not available but only noisy versions of them are known, which are denoted by \( \mathbf{s}_i^k = [x_i^k, y_i^k, z_i^k]' \), where

\[
s_i^k = s_i^0 + \Delta s_i^k,
\]

with \( \Delta s_i^k \) being the random uncertainty in \( s_i^k \). The distance between the source and the available position of \( i \)-th sensor is denoted as \( r_i^k \), i.e.,

\[
r_i^k = \| \mathbf{u} - s_i^k \|.
\]

We collect the available sensor positions as a vector \( \mathbf{s}^k = [s_1^k, s_2^k, \ldots, s_n^k]' \) and the corresponding uncertainty vector \( \Delta \mathbf{s}^k = [\Delta s_1^k, \Delta s_2^k, \ldots, \Delta s_n^k]' \) is zero-mean Gaussian with covariance matrix \( \mathbf{Q}_s^k \).

As in our companion paper [1], we also adopt the assumption that the sensor position uncertainty \( \Delta \mathbf{s}^k \) is independent of the TDOA noise \( \Delta \mathbf{t}^k \) in this paper for ease of illustration. In addition, we ignore synchronization, quantization and delays of data in this work.

The objective of this paper is to answer the second question proposed in companion paper [1], i.e., “for a mobile sensor network, given the noisy TDOA measurements \( \mathbf{r}^k \) together with the noisy sensor positions \( \mathbf{s}^k \) in each sampling time step \( k = 1, 2, \ldots \) how to fuse these information efficiently to estimate the source location \( \mathbf{u} \)?”

III. RECURSIVE LOCALIZATION ALGORITHMS FOR MOBILE SENSOR NETWORK

In this section, we shall extend the CWLS source localization method for static sensor networks proposed in [1] to mobile sensor networks.

A. Review of CWLS source localization

The CWLS source localization has two stages. For ease of notation, we simply drop the time step \( k \) in \( \mathbf{r}^k, \mathbf{s}^k \) and their corresponding components in this subsection.

First Stage: Reorganize the nonlinear TDOA equations into a set of linear equations. This process is achieved by squaring both sides of the RDOA equations \( r_{i1}^0 + r_{i1}^0 = r_{i1}^0 \) and introducing an auxiliary variable \( r_{i1}^0 \) that depends on the source position. The RDOA measurement equations can be simplified as:

\[
r_{i1}^0 - r_{i1}^0 + r_{i1}^0 = -2(s_i^0 - s_i^0)' \mathbf{u} - 2r_{i1}^0 r_{i1}^0,
\]

where \( R_0^i = \sqrt{x_i^0 + y_i^0 + z_i^0} \), and \( i = 2, \ldots, n \).

Because only the noisy values of \( r_{i1}^0 \) and \( s_i^0 \) are available, we express them as \( r_{i1}^0 = r_{i1} - c\Delta t_{i1} \) and \( s_i^0 = s_i - \Delta s_i \), \( i = 2, \ldots, n \). Using the Taylor-series expansion to expand \( r_{i1}^0 \) around the noisy sensor position \( s_i \) up to linear error term, we have

\[
r_{i1}^0 \approx r_{i1} + g_{i1, s_i}^r \Delta s_i
\]

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where \( r_1 = \|u - s_1\| \) and \( g_{u,s_1} = \|u - s_1\|/\|u - s_1\|.

By defining the unknown vector as \( u_1 = [x, y, z, r_1]^T \), the set of equations (8) can be rewritten as

\[
\epsilon = h - Gu_1 \tag{9}
\]

where

\[
h = \begin{pmatrix} r_2^2 - R_2^2 + R_1^2 \\ \vdots \\ r_{n1}^2 - R_2^2 + R_1^2 \\ (s_2 - s_1)^T r_{21} \\ \vdots \\ (s_{n1} - s_1)^T r_{n1} \\ R_i = \sqrt{x_i^2 + y_i^2 + z_i^2}. \end{pmatrix}
\]

\[
G = -2 \begin{pmatrix} s_2 - s_1 \\ \vdots \\ s_{n1} - s_1 \\ r_{21} \\ \vdots \\ r_{n1} \end{pmatrix},
\]

\[
W = \text{diag}\left(\frac{1}{u_1^*(1) - x_1}, \frac{1}{u_1^*(2) - y_1}, \frac{1}{u_1^*(3) - z_1}\right).
\]

\[
\epsilon = cB\Delta t + c^2\Delta t \otimes \Delta t + D\Delta s + \Delta s \otimes \Delta s \tag{10}
\]

\[
\approx cB\Delta t + D\Delta s \tag{11}
\]

where the second order error terms have been ignored and \( \otimes \) represents the element by element multiplication. The matrix \( B \) is given by

\[
B = 2 \begin{pmatrix} r_2^0 & 0 & \cdots & 0 \\ 0 & r_3^0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & r_n^0 \end{pmatrix}, \tag{12}
\]

and \( D \) is shown at the bottom of the next page in (13).

Second Stage: Incorporate the relationship between \( u \) and \( r_1 \) as a second order equality constraint in the weighted LS estimation strategy, which can be formulated as the following optimization problem:

\[
\min_{u_1} \langle h - Gu_1\rangle^TW\langle h - Gu_1\rangle \\
\text{s.t. } \langle u_1 - \tilde{s}_1\rangle^TC\langle u_1 - \tilde{s}_1\rangle = 0 \tag{14}
\]

where \( C = \text{diag}(\{1, 1, 1, -1\}, \tilde{s}_1 = [x_1, y_1, z_1, 0]^T \) and \( W \) is the weighting matrix defined as

\[
W = E[\epsilon\epsilon^T]^{-1}. \tag{15}
\]

Although the CWLS source localization problem (14) is actually a nonconvex quadratically constrained quadratic programming. We have investigated the hidden convexity of the optimization problem (14) in our companion paper [1] and the global optimal source location estimate \( u_1^\ast \) can be efficiently obtained. Meanwhile, as a byproduct, an estimation of error covariance \( P \) of the CWLS localization is given by:

\[
P = V_1(V'G'WG'V)^{-1}V_1', \tag{16}
\]

where

\[
V_1 = \text{diag}\left(\frac{1}{u_1^*(1) - x_1}, \frac{1}{u_1^*(2) - y_1}, \frac{1}{u_1^*(3) - z_1}\right);
\]

\[
\epsilon = \sqrt{(u_1^*(1) - x_1)^2 + (u_1^*(2) - y_1)^2 + (u_1^*(3) - z_1)^2}.
\]

**B. The recursive localization algorithm**

Assume at the every sampling time step \( k = 1, 2, \cdots \), we can get all the noisy TDOA measurements \( r^k \) together with the noisy sensor positions \( s^k \) of the sensor network. At the first sampling time step \( k = 1 \), we can use the CWLS source localization method to get an initial estimate of the unknown source position and its estimation error covariance, which is denoted as \( \hat{u}^1 \) and \( P^1 \). However, because the following TDOA measurements \( r^k, (k = 2, 3, \cdots) \) are nonlinear with respect to \( u \), it is impossible to use them directly to update source localization.

In this recursive source localization algorithm, we firstly use the current TDOA measurements \( r^k \) and noisy sensor positions \( s^k \) to estimate the unknown source position \( u \), and then treat it as a linear measurement on \( u \), so the optimal recursive WLS algorithm is applicable. The following Algorithm 1 summarizes the recursive localization algorithm.

**Algorithm 1: Recursive localization algorithm**

1. Let \( k = 1 \), and calculate the initial localization \( \hat{u}^1 \) and the initial localization error covariance \( P^1 \).
2. Set \( k = k + 1 \), and calculate the current localization denoted as \( \hat{u}^k \) and its error covariance denoted as \( P^k \).
3. Treat the current location estimate \( \hat{u}^k \) as a measurement \( y^k \) of the unknown source position \( u \) at time step \( k \):

\[
y^k = u + \Delta u^k,
\]

where the covariance of measurement noise \( \Delta u^k \) is \( P^\ast \).
4. Update the source localization and its error covariance as follows:

\[
\hat{u}^k = \hat{u}^{k-1} + K(y^k - \hat{u}^{k-1})
\]

\[
K = P^{k-1}(P^{k-1} + P^\ast)^{-1}
\]

\[
P^k = (I - K)P^{k-1}.
\]

Go back to step 2.)

In step 2) of Algorithm 1, we can use the CWLS source localization method to get the current localization \( \hat{u}^k \) and its error covariance \( P^k \). However, this needs to solve a convex optimization problem in each sampling time step \( k \), which has a large computational load. Instead of this, we can also neglect the constraint and directly use WLS method to get the current localization, which has a closed-form solution [11] and can be calculated much faster. Our extensive simulation results indicate that the localization accuracy is relatively insensitive
to ignore the side information between source position and the auxiliary variable in the recursive localization algorithm.

**C. An improved recursive localization algorithm**

In the improved recursive localization algorithm, the initial step \( k = 1 \) is the same as in Algorithm 1. At sampling time step \( k \geq 2 \), instead of treating the current localization based on only the current TDOA measurements as a new measurement of \( u \), we use the current TDOA measurements to estimate the auxiliary variable \( r^k_1 \) using WLS method first, and then substitute the estimation into the TDOA equation to rearrange it as a linear measurement of the unknown source position \( u \) which can be used to update the source localization.

More specifically, we rearrange the nonlinear RDOA measurement equations into linear equations (8) at sampling time step \( k \) with respect to \( u \) and \( r^k_1 \) as

\[
 r^k_{i1} + R^0_{i1} - 2(\hat{s}^k_i - s^k_i)T u = 2r^0_{i1} r^k_1 \quad (17)
\]

where \( R^0_{i1} = \sqrt{x_i^2 + y_i^2 + z_i^2} \), \( i = 2, \ldots, n \).

In the presence of TDOA measurement noise in \( r^k_{i1} \), the sensor position errors \( \Delta s^k_i \) and the first-order approximation of \( r^k_{i1} \), the equation (17) could be viewed as a linear measurement equation on unknown parameter \( u \) as in (9) of CWLS source localization method:

\[
 r^k_{i1} - R^k_{i1} + 2r^k_{i1}r^k_1 = -2(\hat{s}^k_i - s^k_i)T u + e^k_{i1} \quad (18)
\]

where \( R^k_{i1} = \sqrt{x_i^2 + y_i^2 + z_i^2} \) and \( e^k_{i1} \) is the measurement error. We can apply the WLS estimation method to get an estimate of the auxiliary variable \( r^k_1 \) denoted as \( \hat{r}^k_1 \), where the estimation error is denoted as \( \Delta r^k_1 = r^k_1 - \hat{r}^k_1 \) and the estimation error covariance is \( Q^k_1 \).

Now we substitute \( \hat{r}^k_1 \) into (18) and rearrange the liner equation only with respect to \( u \) as:

\[
 r^k_{i1} - R^k_{i1} + 2r^k_{i1}r^k_1 = -2(\hat{s}^k_i - s^k_i)T u + e^k_{i1} \quad (19)
\]

The corresponding linear vector equation is

\[
 \hat{e}^k = h^k - G^k u \quad (20)
\]

where

\[
 h^k = \begin{pmatrix}
 r^k_{21} - R^k_{21} + 2r^k_1 \hat{r}^k_1 \\
 \vdots \\
 r^k_{n1} - R^k_{n1} + 2r^k_1 \hat{r}^k_1 \\
 \end{pmatrix},
\]

\[
 G^k = -2 \begin{pmatrix}
 (s_2 - s^k_1)^T \\
 \vdots \\
 (s_n - s^k_1)^T \\
 \end{pmatrix}
\]

The measurement noise vector \( \hat{e}^k \) can be written in terms of \( \Delta t^k \) and \( \Delta s^k \) as

\[
 \hat{e}^k = c B^k \Delta t^k + c^2 \Delta t^k \otimes \Delta t^k + D^k \Delta s^k + \Delta s^k \otimes \Delta s^k + E^k \Delta r^k_1 + c \Delta t^k \Delta r^k_1 + \Delta r^k_1 \\
 \approx c B^k \Delta t^k + D^k \Delta s^k + E^k \Delta r^k_1,
\]

where

\[
 E^k = 2 \begin{pmatrix}
 r^k_{21} & 0 & \cdots & 0 \\
 0 & r^k_{31} & \cdots & 0 \\
 \vdots & \vdots & \ddots & \vdots \\
 0 & 0 & \cdots & r^k_{n1} \\
 \end{pmatrix},
\]

\[
 B^k \text{ and } D^k \text{ are similar with } B \text{ and } D \text{ in (12) and (13).}
\]

Generally, \( \Delta r^k_1 \) is correlated with \( \Delta t^k \) and \( \Delta s^k \), here we neglect this correlation and the corresponding covariance of measurement noise \( \hat{e}^k \) can be approximated as

\[
 Q^k_\hat{e} = c^2 B^k Q^k_\Delta t \otimes B^k \Delta t + D^k Q^k_\Delta s \otimes D^k + Q^k_\Delta r \otimes E^k \\
 \approx c^2 B^k Q^k_\Delta t \otimes B^k \Delta t + D^k Q^k_\Delta s \otimes D^k + Q^k_\Delta r \otimes E^k. 
\]

The resulted measurement equation (20) is a linear model only with respect to the unknown parameter \( u \), so the Kalman filtering can be applied to recursively update the source localization. The following Algorithm 2 summarizes the improved recursive localization algorithm.

**Algorithm 2: Improved recursive localization algorithm**

1. Let \( k = 1 \), and calculate the initial localization \( \hat{u}^1 \) and the initial localization error covariance \( P^1 \).
2. Set \( k = k + 1 \), and use the WLS method to calculate the estimate of current auxiliary variable \( r^k_1 \) as \( \hat{r}^k_1 \) and its error covariance denoted as \( Q^k_1 \) with current TDOA measurements.
3. Rearrange the nonlinear TDOA measurement equations into a set of linear equations only with respect to the unknown source position \( u \) as in (19)-(20).
4. Update the source localization and its error covariance with Kalman filtering as follows:

\[
 \hat{u}^k = \hat{u}^{k-1} + K(h^k - G^k u^{k-1})
\]

\[
 K = P^{k-1} G^k T (G^k P^{k-1} G^k T + Q^k_\hat{e})^{-1}
\]

\[
 P^k = (I - K G^k) P^{k-1}.
\]

Go back to step 2).

**Remark 1:** It is noted that the measurement equation (20) is an approximation of the actual TDOA measurement equation.
Meanwhile, in comparison with the standard Kalman filtering, $Q^k_0$ in (22) is not the true covariance of measurement noise $\tilde{e}^k$, so generally the update in step 4) of the improved recursive localization algorithm is not the optimal update with all available TDOA and sensor position information. However, this is an efficient sub-optimal recursive algorithm to overcome the nonlinearity of TDOA measurement equation and simulation results indicate that the performance degradation due to the approximation of $Q^k_0$ is insignificant.

**Remark 2:** Both the two localization algorithms utilize the reorganization idea to rearrange the TDOA equations to deal with the nonlinearity of TDOA at each sampling time. The main difference is that in the improved recursive localization algorithm, the auxiliary variable is estimated in advance before using the TDOA equations to update the source localization, but in the recursive localization algorithm, the auxiliary variable is regarded as a completely unknown variable in the update of source localization. Intuitively, the improved recursive localization algorithm should have better performance since the measurement equation (19) makes use of more information on the auxiliary variable in each update of source localization, including the estimate of the auxiliary variable as well as its estimation error covariance. This is validated in the following simulations.

**IV. NUMERICAL SIMULATIONS**

This section contains simulation results of the proposed two recursive source localization algorithms. The simulation scenario contains $n = 8$ mobile sensors, and their nominal positions at sampling time step $k = 1$ are given in Table I. The sensor position noises at different coordinates and for different receivers are assumed to be independent Gaussian noises with variance $\sigma^2_s$, i.e. $Q^k_0 = \sigma^2_s I$, where $I$ is the $3n \times 3n$ identity matrix. We consider that the emitter source is far away from the sensor network and is located at $[20000, 50000, 0]'$m. The eight sensors move towards the target with a step size of $[40, 100, 0]'$m for each step. The TDOA measurements are obtained by adding Gaussian noise with covariance matrix $Q^k_0$ to the true values, where $Q^k_0 = \sigma^2_s T$ and $T$ is the $(n - 1) \times (n - 1)$ matrix with 1 in the diagonal elements and 0.5 otherwise. The TDOA noise and sensor position noise are independent.

![Table I](https://via.placeholder.com/150)

**TABLE I**

**Nominal positions (in meters) of sensors**

<table>
<thead>
<tr>
<th>sensor no.</th>
<th>$x_i$</th>
<th>$y_i$</th>
<th>$z_i$</th>
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<tr>
<td>1</td>
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We implement the proposed recursive localization algorithm and improved recursive algorithm following the steps as described in the Algorithm 1 and Algorithm 2. The localization accuracy is evaluated by the average range error (ARE) and the standard deviation (SD) of localization error which are defined as

$$\text{ARE}(u) = \frac{1}{L} \sum_{l=1}^{L} \| \hat{u}_l - u \| / L, \quad \text{SD}(u) = \sqrt{\frac{1}{L} \sum_{l=1}^{L} \| \hat{u}_l - u \|^2 / L},$$

where $\hat{u}_l$ denotes the unknown source position estimate at ensemble $l$ and $L = 1000$ is the number of ensemble runs.

Fig. 1 shows the localization accuracy of the proposed recursive localization algorithms for 100 sampling time steps when $\sigma_t = 0.1$ micro-second and $\sigma_s = 10/\sqrt{3}$ m. For comparison purpose, the performance of the recursive localization in Algorithm 1 is shown in dashed line, and that of the improved recursive localization in Algorithm 2 is shown in solid line. It is evident from the figure that both the two algorithms can significantly improve the location accuracy of the initial localization. Meanwhile, both the ARE and SD of the improved recursive algorithm is smaller than that of the recursive algorithm.

Fig. 2 is the corresponding result for 300 sampling time steps when $\sigma_t = 0.2$ micro-second and $\sigma_s = 20/\sqrt{3}$ m. As expected, the localization accuracy is generally worse in the case of a higher noise level. However, the proposed recursive localization algorithms can significantly improve the accuracy, especially in the first a few steps. Also, the performance of the improved recursive localization algorithm is better than that of the recursive algorithm.

**V. CONCLUSION**

In this work, we have provided two recursive source localization algorithms by using TDOA measurements received from mobile sensor network. Compared with centralized source localization with static sensors, the recursive localization algorithms can significantly improve the accuracy, especially by updating the localization at each sampling step. Simulation results showed that the improved recursive localization algorithm gives better localization performance.

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Fig. 1. Comparison of the localization accuracy of the proposed recursive localization algorithms. The left is the average range error of location when $\sigma_t = 0.1$ micro-second and $\sigma_x = \sigma_y = \sigma_z = 10/\sqrt{3}$ m, and the right is the corresponding SD.

Fig. 2. Comparison of the localization accuracy of the proposed recursive localization algorithms. The left is the average range error of location when $\sigma_t = 0.2$ micro-second and $\sigma_x = \sigma_y = \sigma_z = 20/\sqrt{3}$ m, and the right is the corresponding SD.


