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Dielectric metamaterial magnifier
creating a virtual color image with
far-field subwavelength information

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Abstract: We propose an approach for far-field optical subwavelength imaging by using a dielectric metamaterial magnifier with gradient refractive index. Different from previous superlens and hyperlens that form a real image with subwavelength features within narrowband, this magnifier creates a virtual color image with sub-100 nm resolution over broadband that can be captured directly by a conventional microscope in the far field. Because the magnifier is made of isotropic dielectric materials, the fabrication will be greatly simplified with existing metamaterial technologies.

OCIS codes: (120.4570) Optical design of instruments; (110.0180) Microscopy; (160.3918) Metamaterials

References and links
1. Introduction

Biological imaging at the molecular and cellular scales requires resolving of features in the order of 10’s of nanometers. However, the fundamental diffraction limit discovered by Abbe in 1873 [1] restricts the resolution of a conventional optical microscope to about half a wavelength (≈200 nm in air) due to the loss of high spatial frequency information carried in the evanescent waves. To break this diffraction limit, near-field scanning microscopy and far-field time-sequential fluorescence nanoscopy have been developed [2], but a generally slow scanning or sequentially recording procedure in principle prevents users from observing dynamical processes which are of fundamental importance in many biological and medical studies. It is strongly desirable, therefore, to develop a real time imaging technique that can capture subwavelength information, especially on the scale of sub-100 nm.

Aiming toward this goal, the recent metamaterial-based superlens [3–5] and hyperlens [6–9] have provided promising results. For example, with illumination of the wavelength of 365 nm, experiments of the superlens made of a silver film [4] and the hyperlens made of cylindrical metal-dielectric stacks [8] have realized resolution of 60 nm and 130 nm, respectively. A planar plasmon-assisted hyperlens has achieved 70 nm resolution for distinguishing rows of polymer dots on top of a gold plane [9]. However, the original superlens did not provide any magnification, which has limited its wide application in microscopy. Although the hyperlens with a cylindrical structure was able to achieve magnification, the object to be imaged was required to have a curved surface or to be flexible so that it may conform to a curved surface. Moreover, the space-bandwidth product of the hyperlens is limited by the small curvature of the structure, meaning that the field of view is only moderately enhanced over scanning techniques. Some proposals of flat hyperlens [10, 11] have been reported in the literature while their fabrication complexity will be greatly increased when compared with the original hyperlens design. Both the superlens and the hyperlens suffer from material absorption, sensitivity to surface roughness, and narrowband operation. These limit their applicability to molecular and cellular imaging, where fluorescence labeling is usually used as a standard tool.

In this paper, we propose a novel gradient refractive index (GRIN) design that magnifies subwavelength details and deliver information into the far field. Different from the previous superlens and hyperlens, this magnifier produces a virtual image containing magnified subwavelength details, rather than a real image in front of the object. Our design does not rely on surface plasmon resonance between metal and dielectric, and thus isotropic dielectric materials with negligible loss can be used. Moreover, it works over broadband, creating a color image, which is suitable for molecular fluorescence imaging.

2. Design of magnifier with gradient refractive index

We consider the two-dimensional (2D) cylindrical geometry of Fig. 1(a) with polar coordinates \((r, \phi)\), where the \(E\) field is along the out-of-plane direction of \(z\), and the distribution of refractive index satisfies

\[
n(r) = \begin{cases} 
  b/a & r < a \\
  b/r & a < r < b \\
  1 & r > b
\end{cases}
\]

(1)

where \(a\) and \(b\) are the inner and outer radii, respectively. We can think of this geometry as an immersion lens with a tailored GRIN shell. We now proceed to justify this selection of GRIN shell using transformation optics [12–14].

Consider two point sources located on opposite sides with respect to the center of the circle in Fig. 1(a) at coordinates \((r', \phi' = 0)\) and \((r', \phi' = \pi)\), respectively. For each point source at \(r'\) and \(\phi'\), its radiation is typically \(E_z = H_0^{(1)}(k|\vec{r} - \vec{r}'|)\), where \(H_0^{(1)}\) is the zeroth order Hankel
function of the first kind. From the addition theorem, we know

\[ E_z = \sum_{m=-\infty}^{\infty} J_m(kr_<)H_m^{(1)}(kr_>)e^{im(\phi-\phi')} , \]  

(2)

where \( J_m \) is the \( m \)th order Bessel function, \( r_< = \min\{r, r'\} \) and \( r_> = \max\{r, r'\} \). We can see that the radiation is decomposed into cylindrical waves with different angular variations.

![Diagram](image)

Fig. 1. (a) Geometry of the 2D GRIN magnifier. (b) A stratified medium created from transformation. (c) The layer with extended thickness pushes the left half part of the stratified medium to the left. (d) Profiles of refractive index in terms of \( e^\tilde{x} \). The blue solid line represents the case in (b). The red dotted line corresponds to the case in (c).

We first consider a transformation from \((r, \phi, z)\) to \((\tilde{x}, \tilde{y}, \tilde{z})\), where \( \tilde{x} = \ln r, \tilde{y} = \phi \) and \( \tilde{z} = z \). According to the formal invariance of Maxwell’s equations, in the transformed space of \((\tilde{x}, \tilde{y}, \tilde{z})\), \( \frac{\varepsilon}{\varepsilon_0} = \frac{\mu}{\mu_0} = \frac{\partial^2}{\partial r^2} = \text{diag}(1, 1, e^{2\tilde{x}}) \), where \( \tilde{J} = \frac{\partial(\tilde{x}, \tilde{y}, \tilde{z})}{\partial(r, \phi, z)} \) is the Jacobian matrix of the transformation [12–14]. When we consider TE waves, only \( \mu_\tilde{x}, \mu_\tilde{y} \) and \( \varepsilon_\tilde{z} \) enter into Maxwell’s equations, and thus the refractive index in the transformed space can be written as \( n(\tilde{x}) = e^{\tilde{x}} \), i.e. a one-dimensional (1D) inhomogeneous medium with refractive index varying along \( \tilde{x} \). The original \( m \)th order cylindrical wave becomes \( E_z = H_m^{(1)}(k_0 e^\tilde{x})e^{im\tilde{y}} \) in this transformed space. So now the propagation of a cylindrical wave away from the origin in the original space is transformed to propagation of a plane wave from left to right in a 1D inhomogeneous medium with refractive index \( n = e^{\tilde{x}} \), as shown in Fig. 1(b). The spatial frequency \( k_\tilde{y} \) of this plane wave equals \( m \), the angular momentum of the original cylindrical wave. Similar 1D exponential profiles that lead to solutions with expressions of Hankel functions were discussed previously in [15].

Propagation in the transformed space can be treated as propagation in a 1D continuously stratified medium as illustrated in Fig. 1(b). Inside each infinitesimally thin layer, there are an incident wave and a reflected wave. Due to the continuity of refractive index, the incident wave is dominant while the reflected wave is negligible in most cases. When a wave propagates with nonzero transverse wave number of \( m \) from left to right, because of the low starting value of refractive index, it is evanescent. After propagating for some distance, the refractive index is sufficiently increased, and converts the evanescent wave to a propagating one. The reflected
wave, although very small, is still important in explaining the physical picture. For example, the explanation of the energy tunneling of evanescent waves to propagating waves in the original cylindrical space will be difficult, if one approximates the cylindrical wave as a single plane wave locally [6]. In our case, since only dielectric materials are used, we will focus on the propagating plane waves in the transformed space which compose most of the far-field radiation.

We plot the distribution of the refractive index $n$ of the stratified medium as the blue solid line in Fig. 1(d) in terms of $\tilde{x}$. The horizontal lines in the background represent different $k_0 = m$, each of which has a height of $\frac{m}{k_0}$, where $k_0$ is the wave number in the free space. It can be seen that the horizontal lines in the region above the blue solid line are evanescent waves, while those in the region below the blue solid line are propagating waves. The two point sources in Fig. 1(a) are located at the same plane of $\tilde{x} = \ln r'$ in the transformed space. As $\tilde{x}$ increases, the number of propagating waves increases as well. This is intuitively satisfying, because $\tilde{x}$ is positively related to the spacing between the two sources.

It is interesting to investigate what happens if we extend one layer of the stratified medium at position $P$ from infinitesimally thin to a finite thickness $QP$, as shown in Fig. 1(c). Due to the extension of thickness at $P$, the left half part of the original stratified medium is pushed to the left by a distance of $QP$. Therefore, by setting the coordinates of $Q$ and $P$ to be $\ln a$ and $\ln b$, respectively ($a < b$), the refractive index profile for the left half part now is $n(\tilde{x}) = e^{\tilde{x}+(\ln b-\ln a)} = \frac{b}{a} e^{\tilde{x}}$. Assuming $CP=BQ$, a propagating plane wave with $k_0 = m$ excited at point $B$ in Fig. 1(c) looks to an observer in the far field similar to a plane wave excited at point $C$ in Fig. 1(b), except there is a phase shift due to traveling through the additional layer of $QP$. In the paraxial approximation, all propagating waves in the newly extended layer of $QP$ are almost parallel. Hence, they experience nearly the same phase shift after traveling though $QP$. In other words, a virtual image will appear at point $C$ of the true source at $B$ for the observer in the far field. Accordingly, in the physical space of cylindrical coordinates, the spacing between the two point sources is magnified. The magnification can also be clearly seen in Fig. 1(d), where the new profile of refractive index is plotted as the red dotted line. For the two point sources at $B$ in the extended-layer medium at coordinate of $e^{\tilde{x}} = r'$, the virtual images will appear as if they are located at point $C$ with coordinate $e^{\tilde{x}} = \frac{b}{a} r'$ in the unextended medium.

Now we go from the transformed space $(\tilde{x}, \tilde{y}, \tilde{z})$ back to the physical space $(r, \phi, z)$. The extended and shifted stratified medium then becomes a concentric cylindrical structure with inner radius $a$ and outer radius $b$ as shown in Fig. 1(a) and described by Eq. (1). The magnification
that this magnifier can provide is \( \frac{b}{a} \). We can understand this magnifier’s subwavelength imaging capability from an intuitive viewpoint. The high refractive index of \( \frac{b}{a} \) in the core converts some originally evanescent waves into propagating waves. The concentric shell with the GRIN profile \( \frac{b}{r} \) is able to deliver all propagating waves to the far field because this concentric shell is transformed from a homogeneous slab which guarantees that a propagating wave on one side will still be propagating on the other side after it traverses this slab. The GRIN profile inside the shell can be achieved by constructing a multi-layer structure of several different materials or by drilling air holes in a high refractive index material according to some location-dependent ratio that can be calculated in a straightforward fashion [16]. We will choose the latter for ease of numerical demonstration.

3. Numerical demonstration of subwavelength resolution

We set \( b = 4a = 2 \mu \text{m} \) and the free space wavelength \( \lambda_0 = 500 \text{nm} \), so the refractive index in the core \( r < a \) is 4, close to the refractive index of silicon, and the magnification is 4. In Fig. 2(a), two coherent and in-phase point sources are located along the \( x \) axis and separated by \( \lambda_0/4 \). We can see that they are difficult to distinguish. In Fig. 2(b), these two point sources are now placed inside the magnifier, and they generate a recognizable pattern in the far field. Fig. 2(c) shows the \( E \) field with these two sources in free space but with the separation increased to \( 2\lambda_0 \). The far field in Fig. 2(c) is almost the same as that in Fig. 2(b), except there is an evident phase shift which is caused by the insertion of a finite layer in Fig. 1(c). It also confirms the magnification of 4 of this magnifier. In practice, a complete circular structure is difficult to use. Here we keep a half of the magnifier and study its performance by finite-element-analysis (FEA) simulation as in Fig. 2(d). The two sources now are attached to the bottom interface at \( y = 0 \) with separation of \( \lambda_0/4 \). The radiation pattern has been largely preserved in the upper half space.

![Image intensity of two equally bright incoherent point sources with different separations at free-space wavelengths of (a) 506 nm, (b) 605 nm and (c) 709 nm, respectively.](image)

Now we examine the resolution that the half-cylinder magnifier of Fig. 2(d) can provide in a real fluorescence imaging system. We choose the \( y \) axis as the optical axis and suppose a well-corrected microscope with numerical aperture of 0.95 (receiving angle of 144° subtended by the origin in the object plane). To simulate the imaging function of the microscope, we set a finite plane above the magnifier where the \( E \) field is recorded with 144° receiving angle, collect the propagating plane wave components with \( k_z \) smaller than \( k_0 \), and finally propagate back these same plane wave components to reconstruct an image incoherently, i.e. superimposing the point image intensities. To emulate the GRIN profile dictated by Eq. (1) in the realistic case of silicon operating at broadband, we set up the following procedure: we assume a distribution of air-filled holes of negligible diameter compared to the shortest wavelength and of gradient density \( f \) according to Eq. (1) with the ideal values \( b = 4a \). We then scale the effective complex
refractive index in the region $a < r < b$ according to $n_{\text{eff}}^2 = (1 - f)n_{\text{si}}^2 + fn_{\text{air}}^2$, using $n_{\text{si}}(506\text{nm}) = 4.26 + 0.0439i$, $n_{\text{si}}(605\text{nm}) = 3.93 + 0.0191i$, and $n_{\text{si}}(709\text{nm}) = 3.76 + 0.00992i$ for the green, orange and red colors, respectively [17]. Figure 3 provides the intensities of the reconstructed images of two incoherent sources with different separations. We can see that resolution below 100 nm can be achieved for our entire chosen wavelength range.

Since our design works over broadband, we can image color objects. We consider a 1D fluorescent object emitting a mix of green, orange and red spectral lines at the respective wavelengths mentioned above, as shown in Fig. 4(a). We assume the object to have uniform brightness. Figures 4(b-d) show the images obtained at wavelengths of 506 nm, 605 nm and 709 nm, respectively. These images can be obtained simultaneously by splitting the radiation for three imaging systems. We can also observe the object directly using a single broadband microscope, which will produce an image as in Fig. 4(e). Clearly, the color information is preserved. An alternative way is to combine Fig. 4(b-d) to reconstruct an image where the color at every point is determined by the largest intensity among different color components at corresponding points in Fig. 4(b-d). We are able to get an image as in Fig. 4(f). Setting a minimum intensity threshold is similar to shortening the imaging exposure time. Here we set 97 percent of the highest intensity in Fig. 4(f) as the imaging threshold and get the result shown in Fig. 4(g). A color image is obtained with high fidelity compared to the original object. More sophisticated inversion techniques which also take into account the noise present in the image can be used in a realistic situation, but their complete analysis is beyond the scope of this paper.

4. Conclusion

In conclusion, we have proposed an optical subwavelength imaging technique using a dielectric metamaterial magnifier with gradient refractive index that can be implemented by current metamaterial technologies. This magnifier creates a virtual color image with sub-100 nm resolution over broadband that can be captured directly by a conventional microscope in the far field. Our approach only uses low-loss dielectrics, which is appealing for photon-starved applications such as real-time molecular imaging.
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