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Long term surface deformation of Soufrière Hills Volcano, Montserrat from GPS geodesy: Inferences from simple elastic inverse models

Glen S. Mattioli, Richard A. Herd, Michael H. Strutt, Graham Ryan, Christina Widiwijayanti, Barry Voight

1Department of Geosciences, University of Arkansas, Fayetteville, Arkansas, USA.
2Montserrat Volcano Observatory, Fleming, Montserrat.
3School of Environmental Sciences, University of East Anglia, Norwich, UK.
4British Geological Survey, Keyworth, UK.
5Institute of Earth Science and Engineering, University of Auckland, Auckland, New Zealand.
6Earth and Mineral Sciences, Pennsylvania State University, University Park, Pennsylvania, USA.

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We examine a number of elastic models for episodes from 1995 to 2009, evaluated with descriptive statistics, to test basic ideas about the geometry and temporal evolution of the SHV magmatic system. Our goal is to assess whether we can uniquely define the geometry of the SHV plumbing system using GPS geodesy alone.

2. Previous Models of the SHV Plumbing System

[3] Mattioli et al. [1998] reported on campaign and semi-continuous GPS observations for the period 1995 to 1997. They used forward elastic half-space models to suggest that the early surface deformation field at SHV was dominated by a shallow, vertical, NW-trending dike coupled with a point source [Mogi, 1958; hereafter Mogi source] at approximately 6 km depth, consistent with petrological constraints [Barclay et al., 1998] and maximum depths of volcano-tectonic earthquakes [Aspinall et al., 1998]. Mattioli et al. [1998] further showed that the inferred source volume change was smaller than the volume of erupted lava during that same time period [Sparks et al., 1998].

[4] Mattioli and Herd [2003] analyzed GPS data from 1998 to 2001, including newly installed cGPS sites. They demonstrated that there was a strong correlation between the state of eruptive activity at SHV and the surface deformation field: episodes of lava efflux/dome growth correlated with subsidence, and episodes of little or no eruption, the so called repose periods [Norton et al., 2002], correlated with uplift. They also inferred Mogi-source depths at ~12 km, deeper than initially estimated for 1995–1997. These conclusions were corroborated by cGPS data through 2007 [Mattioli et al., 2008].

[5] Elsworth et al. [2008] combined GPS data with SHV efflux data and assumed a magmatic system with Mogi sources fixed at 6 and 12 km. The additional constraint from lava efflux allowed estimation of fluxes at the base of the crustal magma system, and volume changes and fluxes for the two crustal chambers. Elsworth et al. [2008] concluded that the deeper chamber dominated the overall geodetic signal, and that a valve mechanism controlled magma transit between the chambers. Voight et al. [2010] have suggested that the vertically–stacked, two-source geometry may not be realistic. They propose that a vertical, prolate ellipsoid better fits the 2003–2005 uplift and 2005–2007 subsidence episodes and conclude that up to ~40% of observed misfit between the calculated volume change for the crustal source and the surface lava efflux may be explained by volatile-saturated, compressible magma. In addition to models that focus on the geometry of the mid-crustal magmatic plumbing system at...
SHV, Foroozan et al. [2010] and Hautmann et al. [2010] examine effects of non-uniform crustal elastic moduli, conditioned on seismic velocities from the SEA-CALIPSO experiment [Shalev et al., 2008]. If the elastic modulus increases with depth (~60 GPa at 4 km to ~120 GPa at 20 km [Foroozan et al., 2010]), then inferred source depths increase by ~1.5 km.

3. GPS Network Topology, Observations, and Processing

[6] Since the early phase of the eruption at SHV in 1995, considerable effort has been expended to expand, harden, and improve the GPS network [Mattioli et al., 2004]. Given the brevity of this report, much of the data and analysis is presented in the auxiliary material.1 The first dual-frequency, code–phase GPS observations were made in autumn 1995 (Figure S1 and Table S1). While these campaign data are included here for completeness, the primary focus of this report is the analysis of the cGPS data acquired since 1998.

[7] While the GPS network at SHV has changed considerably from 1995 to 2009, our processing procedures have not. We have used NASA’s GIPSY–OASISII (GOAII, v. 4) and a non-fiducial, absolute point positioning strategy. Details may be found in the auxiliary material; methods were similar to those of Jansma and Mattioli [2005], with coordinate time series and site velocities referenced to the fixed-Caribbean [DeMets et al., 2007]. Our processing relies on JPL GOAII data products exclusively. JPL has changed the number of global tracking stations and the length of arc for orbit estimation among other processing changes over the course of our observations at SHV (see auxiliary material for detailed discussion and Figures S4a–S4e). A study of the time series data from HARR and MV01 (Table S2 and Figures S2 and S3), however, shows the time series at our cGPS sites are unaffected by the changes to the JPL products, and therefore any apparent changes in component variance or velocity may be attributed to either equipment changes (which were accounted for in our processing) or volcanological processes at SHV.

[8] The time series for each cGPS site shows distinctly non-linear motion (Figures 1 and S4a–S4e), with amplitudes that are well outside any standard noise sources that are often considered to affect cGPS time series [Mao et al., 1999]. In order to simplify the analysis of the SHV surface deformation field, we divided the entire non-linear coordinate time series at each site into piecewise linear segments, whose boundaries were determined by the observed changes from subsidence to uplift in the vertical component at HARR and MV01, the two longest running cGPS sites. We hereafter refer to these as dome growth and repose episodes starting with D1, R1, etc. The bounds are shown on the vertical cGPS time series in Figure 1. Note that episode duration is not uniform and that these bounds are closely, although not exactly, aligned in time with initiation and cessation of lava efflux at SHV [Sparks et al., 1998; Wadge et al., 2010]. We used the piecewise linear segments for each dome growth and repose episode to obtain a site velocity for every GPS site for which there was data available (Table S3). Figure S5 illustrates our method for two sequential epochs for repose (R2) and dome growth (D3) at HARR and MV01. Component site velocities relative to ITRF05 as well as fixed CAR, and their 1–σ errors, are presented in Table S3.

4. Inversion of Elastic Half-Space Models

[9] The surface velocity field for each episode was used as input to constrain uniform half-space elastic inversion models (Table S3). The justifications for use of elastic models at SHV are discussed thoroughly by Voight et al. [2010]. Our inverse modeling code is based on a simplex method combined with a simulated annealing algorithm to help avoid local minima during inversion [Press et al., 1992]. Simplex (v. 4) was modified to include Mogi sources within an elastic half-space. The model solution was obtained by chi-squared minimization, and the best-fit model parameters along with their 1–σ errors, and the correlation and covariance matrices were output [Beverington, 1969]. In almost all cases, the algorithm converged rapidly without simulated annealing. All models were fit using the 1–σ component velocity uncertainties from each cGPS site (Table S3) and final reduced chi-squared statistics are weighted by reciprocal variance (Table S4).

[10] Because the GPS network changed substantially from 1995 to 2009, we wanted to know if these changes could have biased our inversions. We examined the possible effect of this on the derived Mogi parameters by eliminating one or more sites from the surface deformation field for a particular episode. The source representation is simple but its use in all inversions provides consistency and facilitates the tracking of changing conditions. We computed over 50 different combinations of data, model geometry (see below), and fixed versus adjustable parameters for any given geometry.

[11] A summary of the best-fit solutions for two different single Mogi source models using somewhat different data sets for each of the three dome growth and repose episodes is presented in Table S4. The average of each best-fit Mogi model for all possible data subsets reported (two different models for each of the three dome growth and repose episodes, for a total of 12) yields horizontal position estimates of X = 0.3 ± 0.5 km, Y = 0.8 ± 0.4 km, and a depth of Z = 10.4 ± 2.1 km (1–σ, with Z positive down), relative to the center of the model domain, which is close to the long-term location of the vent on SHV. If we weight the derived parameters by their respective reciprocal variance [Beverington, 1969], then we obtain the following parameter estimates: X = 0.4 ± 0.1 km, Y = 0.8 ± 0.1 km, and a depth of Z = 10.2 ± 2 km, which is not significantly different from the global average, although the standard deviations are substantially reduced, as expected. While the inferred horizontal position estimates are invariant within error for all single Mogi source models over the entire eruption sequence, the inferred mean Mogi depth for effusive (deflation) versus repose (inflation) for all possible data subsets is different, yielding Z = 11.4 ± 2.0 km and Z = 9.3 ± 1.6 km, respectively. Based on a simple t-test, we reject the null hypothesis and conclude that the difference in average Mogi source depth for effusive (deflation) and repose (inflation) is significant at better than 90% confidence.

[12] Given that the cGPS network has been relatively constant since the massive dome collapse of July 2003

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1Auxiliary materials are available in the HTML. doi:10.1029/2009GL042268.
[Herd et al., 2005], we focus additional scrutiny on the uplift of 13 July 2003 to 01 November 2005 and the subsidence during 02 November 2005 to 01 April 2007. First, we examined the effect of individual cGPS sites on a single Mogi source model. We did this by eliminating, in sequence, the site with the highest summed velocity uncertainty, and fitting the remaining nine sites using Simplex inversion. We then compared the 10-site model with the ensemble of the nine individual, 9-site models as well as the best-fitting model from that group (see auxiliary material). We conclude that there is no statistical motivation to eliminate any cGPS site from the post-2003 inversion models.

Next we examine several different proposed geometries for the SHV magmatic plumbing system. We set up seven “straw-man” models, one of which is the basic model of a single Mogi source with X, Y, Z, and volume change, ΔV, free to vary (Table S4), and we examined the 2003–2005 inflation (R2) and the 2005–2007 deflation (D3) episodes. There were three basic models: 1) a vertically-stacked geometry with Mogi sources at 6 and 12 km depth; 2) a single Mogi source; and 3) a vertical, prolate ellipsoid. The predicted model velocities and their residuals for all components are presented in Tables S5 and S6 for the 2003–2005 and 2005–2007 episodes, respectively, with the derived model parameters, 1-σ uncertainties, and the chi-squared statistics. Observed and predicted surface deformation vectors for three of the seven models for each epoch are shown in Figure 2. The component residuals for these models are shown versus radial distance from the dome or vent in Figure 3. Prolate ellipsoid models were calculated with a grid search algorithm based on the analytical solution of Yang et al. [1988].

5. Discussion

[14] The following should be clear from inspection of Tables S5 and S6 and Figures 2 and 3: 1) all the models examined for the 2003–2005 uplift fit the observations better than those from the 2005–2007 deflation as evidenced by their lower reduced chi-squared statistics; 2) consistent with the findings of Elsworth et al. [2008], the deeper Mogi source in the vertically-stacked, two-source models requires a ΔV that is more than an order of magnitude larger than the shallower Mogi source; 3) the ΔV for the inflation episode is less than half that required for the deflation episode despite the longer duration of the inflation versus deflation (Figure 1 and Table S3); and 4) there is a systematic misfit in the vertical component for both the two-source and single Mogi source models relative to the prolate ellipsoid model for the 2003–2005 epoch (Figure 3). Now we apply the F-test to determine whether the increasing complexity of the models is merited [Bevington, 1969].

[15] For the 2003–2005 (R2) episode, the single Mogi with only Z and ΔV free to vary (n = 2; ν = 28) versus a model with
all 4 parameters free ($n = 4; \nu = 26$), yields $F = 0.270$. Similarly, the single Mogi with $X, Y$ fixed compared with the two-source, stacked with $X, Y$ free for the upper source, yields $F = 0.415$. In both cases, the improvement to the fit with the addition of more adjustable parameters is not statistically significant at even 90% confidence. In contrast, the single Mogi source versus the vertical, prolate ellipsoid (both models have $X, Y$ fixed at 0, 0) yields a $F = 3.553$, which is significant at >95% confidence. Examining the models from 2005–2007 (D3) yields a similar result. While the additional parameters for the single Mogi models and the single versus two-source Mogi models come closer to being statistically significant, with $F = 2.239$ and 2.348, respectively, both fall short of 90% confidence. The single Mogi source versus the vertical, prolate ellipsoid, however, yields $F = 9.709$, which is significant at 99% confidence. In summary, rigorous statistical comparison of our “straw-man” models demonstrates that only the vertical, prolate ellipsoid with its two additional adjustable parameters is merited over either a single Mogi source or two-source Mogi geometry.

Models for 2003–2005 fit better than any of the models for 2005–2007, which might be explained by the inflationary, repose episode being longer than the subsequent deflationary dome-growth episode. In general, component site velocity uncertainties should decrease with increasing number of observations or time, depending on the type of noise [Mao et al., 1999]. To examine this quantitatively, we set the uncertainties for all the cGPS sites for both episodes to equal weights of 1 mm/yr for a single Mogi source model with 4 free parameters. The ratio of the actual GPS error-weighted chi-squared statistics is 2.082. When equal weighting is used, the reduced chi-squared values increase to 60.706 and 241.959, respectively, and the ratio increases to 3.986. We infer that our measured uncertainties are a reasonable approximation to the true uncertainties and that the worse fit of the deflation of 2005–2007 is not solely related to the shorter duration of the cGPS time series. We suggest that the difference is related to the magmatic plumbing system at SHV. We speculate that the effective stress transmitted to the surface during the inflation epoch

Figure 2. Observed site velocities and 1-σ errors (black arrows and ellipses) and elastic inversion models (red arrows). Model residuals are shown in Figure 3. Upper maps show three models for 13 Jul 2003 to 01 Nov 2005 (no lava effusion). Lower maps show three models for 02 Nov 2005 to 01 Apr 2007 (lava effusion and dome growth).
may be more uniform, while during the deflation epoch it is less so, perhaps related to magma extraction or movement over a larger spatial zone. The cGPS data and our static models would tend to "average" out any temporally varying magma extraction depths and thus likely result in the poorer fit.

6. Conclusions

We have examined the most extensive surface deformation data set available for any actively erupting, andesitic stratovolcano. Based on our inversion of the piecewise linear geodetic site motions and derived model statistics, we conclude that the SHV magmatic plumbing system cannot be uniquely defined based solely on surface deformation data. Complex geometries that include deep, multiple Mogi sources (or by extension of our analysis, other types of deformation sources, for example dikes or sills), are not justified statistically over simpler models. Any number of plausible geometries fit the data equally well when only GPS data are used to condition the model. A vertical, prolate ellipsoid model improves the fit over other models at better than 95% confidence for the 2003–2005 inflationary and 2005–2007 deflationary episodes. The assumption of an elastic crustal model is justified based on the observations and models and this implies that if a visco-elastic zone is present in the mid-crust at SHV, it must either be relatively small (e.g., a very thin shell) or its viscosity must not be very different from the incompressible magma and surrounding crust.

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References


R. A. Herd, School of Environmental Sciences, University of East Anglia, Norwich NR4 7TJ, UK.

G. S. Mattioli, Department of Geosciences, University of Arkansas, 113 Ozark Hall, Fayetteville, AR 72701, USA.

G. Ryan, Institute of Earth Science and Engineering, University of Auckland, PB 92019, Auckland 1142, NZ.

M. H. Strutt, British Geological Survey, Keyworth NG12 5GG, UK.

B. Voight and C. Widiwijayanti, Earth and Mineral Sciences, Pennsylvania State University, 503 Deike Bldg., University Park, PA 16802, USA.