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Neural Adaptive Back Stepping Flight Controller for a Ducted Fan UAV

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Abstract—In this paper, we present a neural adaptive back-stepping flight controller for a ducted fan UAV whose dynamics is characterized by uncertainties and highly coupled nonlinearities. The proposed neural adaptive back-stepping controller can handle unknown nonlinearities, unmodeled dynamics and external wind disturbances. A single layer radial basis function network is used to approximate the virtual control law derived using back stepping approach, which provides necessary stability and tracking performances. The neural controller parameters are adapted online using Lyapunov based update laws. The proposed controller is evaluated using nonlinear simulation model of a typical ducted fan UAV performing bop-up maneuver. Three neural adaptive controllers are implemented to handle attitude command altitude hold system, one in each body axis. A separate neural controller is implemented to track the height command for autonomous takeoff and landing. The results indicate that the proposed controller can stabilize the ducted fan UAV and provide necessary tracking performance.


I. INTRODUCTION

Unmanned aerial vehicles (UAV) have greatly reduced human effort and have enormous potential applications with a large operating environment. UAVs can fly to places where it is potentially dangerous for a human being to reach in emergency situations therefore reducing human loss. Further fixed wing UAVs which provide hovering, vertical take-off and landing (VTOL) capabilities will increase the mission envelope and the potential applications of the UAVs in the military and civil environment to a great extent. Ducted fan UAV is one such example with hover, VTOL and forward flight capabilities which has both advantages of the fixed and rotary wing UAVs in a single design. These features make the ducted fan UAV as a prominent research topic among the aerospace community, and currently many researchers are working to exploit the transition and VTOL capabilities of the ducted fan vehicles[1-4].

Ducted fan UAVs can perform both the operations of aircraft and helicopter opening a wide area of applications. The unstable and highly coupled dynamics of the ducted fan UAV present challenges in stability and control of the vehicle in transition and forward flight. Uncertain aerodynamics of the duct and nonlinear, coupled nature of the rotor restricts the operating envelope of the UAV further which has been highlighted in many research works[2, 4].

Uncertain and nonlinear coupled aerodynamics poses great challenges in designing an autonomous flight control system and these problems should be addressed while choosing an appropriate control strategy. High lift and poor drag characteristics of the duct poses challenges in the translational flight reducing the tracking performance of the vehicle. Hence, the need for a suitable control strategy to provide accurate tracking performance and stability to the ducted fan created lot of interest among researchers.

Many established linear and nonlinear control strategies have been developed in the past decade to handle different kinds of control problems in aircraft. Some of them are linear control[4], gain scheduling[5], Eigen structure assignment[6], dynamic inversion[7], Back stepping[8, 9], direct and indirect adaptive control schemes[10-14], adaptive critic control[15], robust control[16-18] and intelligent control[11-14] used widely in aircraft. Gain scheduling[5] uses a number of linear controllers for different trim points and flight stability is achieved by migrating between different trim points. Eigen structure assignment[6] involves selecting a desired Eigen structure which is then projected into a different subspace like linear quadratic regulator to achieve global boundedness and stability. These linear control approaches cannot handle nonlinearities and have very poor robustness characteristics. Dynamic inversion[7] and back stepping methods[9] decouple the nonlinearities by means of virtual stabilizing functions but the nonlinearities have to be known in advance. Adaptive control schemes[10-15] provide a dynamic control law which can adapt to the nonlinearities and uncertainties and flight stability is guaranteed by Lyapunov synthesis. Intelligent control schemes[11-14] uses evolutionary algorithm like genetic algorithm and neural networks to estimate the nonlinearities and can be combined with any of the above control strategies to provide asymptotic flight stability.

Among such methods, back stepping and adaptive control schemes has gained prominent importance these days because of their ability to handle the nonlinear dynamics.
Since the control problem in ducted fan involves unknown nonlinearities, a combination of back stepping and neural networks[19] is selected as a suitable control approach to handle the problems posed by uncertainties, coupled nonlinearities, and unmodeled dynamics. Asymptotic stability is guaranteed by Lyapunov synthesis and a dynamic control law can be designed to provide the required tracking performance.

In this paper, we present a back stepping control design to decouple the nonlinear and coupled dynamics of the ducted fan by designing a virtual stabilizing function. The unknown nonlinearities in this stabilizing function are approximated by a radial basis function neural networks (RBFNN) whose weights are updated using Lyapunov based update law.

For autonomous take-off and landing, a neural adaptive controller is designed in the vertical axis to track the altitude command with attitude hold. To provide stability in translational forward flight, three neural adaptive controllers have been proposed one in each axis (roll, pitch and yaw) holding altitude. The proposed control law is then tested on a six degree of freedom model of the ducted fan implemented in MATLAB. This controller is then analyzed under a staircase height command, and its results are presented.

This paper is structured as follows: Section II describes the mathematical model of the ducted fan UAV briefly, followed by section III describing the neural adaptive controller design with Lyapunov stability analysis. Section IV presents the simulation results for autonomous take-off under a staircase height command. Finally, section V summarizes the conclusion from this study.

II. MATHEMATICAL MODEL OF THE UAV

A. Problem formulation

To facilitate a more intuitive understanding of the type of control problem that will be encountered in the ducted fan UAV, lets first look at the problem formulation before presenting the mathematical model of the UAV dynamics. Consider a class of nonlinear systems described by

\begin{align}
\dot{x} &= f(x, u, t) \\
y &= g(x)
\end{align}

where \(x \in \mathbb{R}^n\) is the state vector with \(n\)-dimensional state vector, the control input \(u \in \mathbb{R}^m\) with \(m\)-dimensional inputs, \(y^p\) is the \(p\)-dimensional output vector and \(f\) is a nonlinear function of the state vector, control input and time \(t\).

The control objective is to find out the bounded control law \(u\) such that

- The plant output \(y\) follows the reference command \(y_d\) asymptotically.
- All the signals in the closed loop remain uniformly bounded over time.

\[
\lim_{t \to \infty} e(t) = y_d - y = 0, \forall y \in \mathbb{R}^p, u \in \mathbb{R}^m, x \in \mathbb{R}^n \tag{3}
\]

where \(y_d\) is generated as shown by

\[
\dot{x} = A_d x_d + B_d r \\
y_d = C_d x_d
\]

where \(A_d, B_d, C_d\) are the desired system matrices and \(r\) is the reference input. To meet the control objective given in Eq. (3), the system should satisfy the following assumption which is well documented in the literature[20-22].

Assumption 1: Let \(x_d \in \mathbb{R}^m\) be the set of states that can be tracked accurately and the remaining states \(x_r \in \mathbb{R}^{n-m}\) approach asymptotically to the equilibrium point.

\[
\dot{x}_r = f_r(x, u), x \in \mathbb{R}^n, u \in \mathbb{R}^m \tag{6}
\]

\[
\dot{r} = f_r(x, u), x \in \mathbb{R}^n, u \in \mathbb{R}^m \tag{7}
\]

where \(f_r(x, u)\) is a smooth function. With this assumption a desired control function \(u_d(t)\) can be expressed as

\[
u_d(t) = \tilde{f}(x, \dot{x}_t) \tag{8}
\]

Assumption 2: If the nonlinear function \(f_r(x, u)\) has a continuous bounded partial derivative in a certain neighbourhood of all the points along the desired trajectory \(x_d\) and the matrix \(\frac{\partial f_r(x, u)}{\partial u}\) is nonsingular, then by Implicit Function Theorem the desired control function \(u_d(t)\) can be expressed as

\[
u_d(t) = \tilde{f}(x, \dot{x}_t) \tag{8}
\]
The dynamical model of the UAV with reference to inertial frame of reference is
\[
\begin{align*}
\dot{p} &= v \\
\dot{v} &= F/m \\
\dot{\omega} &= I^{-1}(M - \omega \times I\omega)
\end{align*}
\]
where \(p \in \mathbb{R}^3\) is the position vector, \(\omega \in \mathbb{R}^3\) is the angular velocity vector, \(v \in \mathbb{R}^3\) is the velocity vector, \(I \in \mathbb{R}^3\) is the moment of inertia of the vehicle and \(m\) is the mass of the vehicle. Let \(F\) and \(M\) represents the sum of the aerodynamics forces and moments acting on the vehicle respectively.

The sum of the force and moment vectors is as follows.
\[
\begin{align*}
F &= F_a + F_r + F_d + F_{cs} + F_{fin} + F_g \\
M &= M_a + M_r + M_d + M_{cs} + M_{fin} + M_{gy}
\end{align*}
\]
where \(F_a\) is the force vector due to fuselage, \(F_r\) is the force vector due to rotor, \(F_d\) is the force vector due to the duct, \(F_{cs}\) is the force vector due to the control surfaces and \(F_{fin}\) is the force vector due to the fin. Similarly \(M_a\) is the moment vector due to fuselage, \(M_r\) is the moment vector due to rotor, \(M_d\) is the moment vector due to the duct, \(M_{fin}\) is the moment vector due to the fin and \(M_{cs}\) is the moment vector due to the control surfaces.

Rotor dynamics is modeled using classical blade momentum theory[24], duct dynamics is modeled using independent sectional integration of annular airfoils stacked in the radial direction. Force due to the control surfaces, fin and fuselage can be calculated directly from the basic aerodynamic equations of lift and moment. Gyroscopic moments can be calculated from the body rates and gravitational contribution is also added with a transformation matrix to transform the vectors from the local frame to the body frame. For a detailed explanation of the dynamic equations and the mathematics behind each component can be found in [23].

The total summed up forces and moments can be used in the aircraft equations of motion to form the six degree of freedom model by assuming the UAV as a point mass and rigid body. This six degree of freedom model is implemented in MATLAB which is used to design the neural adaptive controller in the subsequent sections.

III. NEURO-ADAPTIVE BACKSTEPPING CONTROLLER

The neural adaptive control strategy employed in this paper is shown in Fig 2. This control strategy combines the back stepping control and neural networks to design a dynamic control law as shown in the Fig 2. This control law can stabilize the UAV dynamics by compensating the unmodeled dynamics and nonlinearities present in the ducted fan system. By means of two back stepping steps, a virtual stabilizing function and a control law is designed to ensure stability of the UAV dynamics. Since the modeled UAV dynamics has nonlinearities and uncertainties, neural networks are employed to approximate these nonlinear functions. RBFNN is chosen in this paper as a neural network compensator because they can approximate any nonlinear function to arbitrary accuracy when projected onto a higher dimensional space[25]. This neural network controller can be trained offline first and can be adapted online during flight. The weights of the neural network can be updated by using a Lyapunov based update law.

The control objective of this paper is to design a neural adaptive back stepping controller to track a desired height command \(h_d\) and to provide global boundedness and stability to the closed loop system.

The vertical dynamics of the system is given by
\[
\begin{align*}
\dot{V}_z &= -V_y p - V_z q + \frac{F_z}{m} + g \cos \theta \cos \phi \\
\dot{h} &= -V_x \sin \theta + V_y \sin \phi \cos \theta + V_z \cos \phi \cos \theta
\end{align*}
\]
where \(V_x, V_y, V_z\) are the velocities in the x, y and z directions respectively, \(p, q\) are the roll and pitch rate respectively, and \(\theta, \phi\) are the pitch and roll angle respectively. \(F_z\) is the total force produced by the ducted fan in the z direction and \(m\) is the mass of the vehicle. The force vector \(F_z\) is a combination of forces produced by rotor(\(r\)), fuselage(\(a\)), duct(\(d\)), fin(\(fin\)) and control surfaces(\(cs\)).

Let us define the first error variable
\[
z_1 = h - h_d
\]
where \(z_1\) is the first error function, \(h\) is the measured height and \(h_d\) is the desired height command from the reference system.

The second error variable is defined as,
\[
z_2 = V_z - \mu
\]
where \(z_2\) is the second error function, \(V_z\) is the measured velocity in the z direction and \(\mu\) is the stabilizing function. By defining these two error variables the tracking problem in (6) and (7) can be transferred into a regulation problem of stabilizing the two error functions in two back stepping steps. By choosing the virtual control law as
\[
\begin{align*}
\dot{V}_z &= \mu = -c_1 z_1 + f(x) \\
\dot{z}_2 &= g(x) + \frac{F_z(r)}{m}
\end{align*}
\]
where \(f(x) = -V_y p + V_x q + g \cos \theta \cos \phi + \frac{F_{cs,a,d,fin}}{m}\) Equation (7) can be expressed as
\[
\begin{align*}
\dot{V}_z &= g(x) + \frac{F_z(r)}{m}
\end{align*}
\]
where \(g(x) = V_x \sin \theta - V_y \sin \phi \cos \theta + h_d\) is a function of roll and
pitch rates and their respective angles, and also a function of velocities in the x and y direction. Nominal values of \( g(x) \) can be represented as \( g^*(x) \). Since neural networks have the property of approximating any nonlinear function to arbitrary accuracy when projected onto a higher dimensional space, a RBFNN is adopted to approximate \( g^*(x) \)

\[
V_z = W^* \varphi(x) + \varepsilon + \frac{F_z(x)}{m} \tag{20}
\]

where \( W^* \) is the optimal weighting matrix of RBFNN

\[
W^* = [w_1, w_2, \ldots, w_{c_n}], x = \begin{pmatrix} V_x \\ V_y \\ p \\ q \\ \phi \\ \theta \end{pmatrix} \tag{21}
\]

\( \varphi(x) = e^{-\frac{(x-c)^2}{2\sigma^2}} \) is the Gaussian basis function, \( c \) is the centre of the gaussian function, \( \sigma \) is the width of the gaussian function, \( \varepsilon \) is the approximation error of the neural networks and \( c_n \) is the node number of the hidden layer. The ideal weight vector \( W^* \) is defined as the value of \( W \) that minimizes \( |\varepsilon| \) defined by

\[
W^* = \arg \min_{W \in \mathbb{R}^{c_n}} \sup_{x \in \mathbb{R}^6} |g^*(x) - g(x)| \tag{22}
\]

\( W^* \) is bounded, \( \|W^*\|_F \leq \zeta \) where \( \zeta \) is a positive real number and \( \|\cdot\|_F \) denotes Frobenius norm. Choosing the control law as

\[
F_z(x) = m(-c_2z_2 + c_1z_1 + f(x) - \hat{W} \varphi) \tag{23}
\]

where \( \hat{W} \varphi \) is the estimate of the unknown function \( g(x) \) from neural networks. Since the control input \( F_z(x) \) and the throttle can be linked directly by an expression, throttle input can be calculated directly. Next we construct a Lyapunov function and derive the parameter update rule,

\[
\dot{V} = \frac{1}{2} z_1^2 z_1 + \frac{1}{2} z_2^2 z_2 + \frac{1}{2} \Gamma tr(\hat{W} T \hat{W}^T) \tag{24}
\]

where \( \gamma \in \mathbb{R}^+ \) and \( \Gamma \) is a positive definite matrix.

\[
\dot{V} = z_1^2 z_1 + z_2^2 z_2 + \frac{1}{2} \Gamma tr(\hat{W} T \hat{W}^T) \tag{25}
\]

\[
\dot{V} = -c_1\|z_1\|^2 \cos \phi \cos \theta + z_2 \left( -c_2 z_2 - \hat{W} \varphi + W^* \varphi + \varepsilon \right) + \frac{1}{2} \Gamma tr(\hat{W} T \hat{W}^T) \tag{26}
\]

Letting \( \hat{W}_t = W^* - \hat{W} \)

\[
\dot{V} = -c_1\|z_1\|^2 \cos \phi \cos \theta - c_2\|z_2\|^2 + z_2 \left( \hat{W} \varphi + \varepsilon \right) + \frac{1}{2} \Gamma tr(\hat{W} T \hat{W}^T) \tag{27}
\]

By using the property of trace,

\[
\dot{V} = -c_1\|z_1\|^2 \cos \phi \cos \theta - c_2\|z_2\|^2 + z_2 \left( \hat{W} \varphi + \varepsilon \right) + \frac{1}{2} \Gamma tr(\hat{W} T \hat{W}^T) \tag{28}
\]

Choosing the update law for the weights as

\[
\dot{\hat{W}} = \gamma \Gamma^{-1} \varphi \hat{z}_2^T - \gamma \Gamma^{-1} n_z \|z_2\| \hat{W} \tag{29}
\]

By using another property of trace,

\[
\dot{\hat{W}} = \gamma \Gamma^{-1} \varphi \hat{z}_2^T - \gamma \Gamma^{-1} n_z \|z_2\| \hat{W} \tag{30}
\]

\[
\dot{\hat{W}} = -c_1\|z_1\|^2 \cos \phi \cos \theta - c_2\|z_2\|^2 + z_2 \left( \hat{W} \varphi + \varepsilon \right) + \frac{1}{2} \Gamma tr(\hat{W} T \hat{W}^T) \tag{31}
\]

By using another property of trace,

\[
\dot{\hat{W}} = \gamma \Gamma^{-1} \varphi \hat{z}_2^T - \gamma \Gamma^{-1} n_z \|z_2\| \hat{W} \tag{32}
\]

\[
\dot{\hat{W}} = \gamma \Gamma^{-1} \varphi \hat{z}_2^T - \gamma \Gamma^{-1} n_z \|z_2\| \hat{W} \tag{33}
\]
\[
\dot{V} \leq -c_1 \|z_1\|^2 \cos \phi \cos \theta - c_2 \|z_2\|^2 + z_2^T \varepsilon \\
+ n.\|z_2\| \cdot \left((||W|| F - \frac{1}{2} \zeta)^2 - \frac{1}{4} \zeta^2 \right)
\] (35)

If \( \|z_2\| \geq \frac{n.\zeta^2 + 4.||\varepsilon||}{4.c_2} \), then \( \dot{V} < 0 \) is negative definite and the errors \( z_1 \) and \( z_2 \) asymptotically converges to zero. By proper selection of \( n, c_1, c_2, \gamma \) and \( \Gamma \) the closed loop dynamics can be proved asymptotically stable and is bounded inside the compact set defined by \( \Omega = [\|z_2\| \geq \frac{n.\zeta^2 + 4.||\varepsilon||}{4.c_2}] \). Similarly neural adaptive back stepping controllers can be designed in all the three axes, one in each axis to track roll, pitch and yaw commands respectively.

IV. SIMULATION RESULTS

The mathematical model of the ducted fan UAV along with the neural adaptive back stepping controller is implemented in the MATLAB/SIMULINK interface. In this study we present an autonomous take-off simulation of the ducted fan in presence of a staircase height command. This simulation uses 100 hidden layer neurons in the RBFNN. A staircase command with six steps with step of 3m height as shown in Fig 3 is given to the controller and its performance is evaluated. Each step is timed in such a way that the velocity response from the system will not be oscillatory. From

Fig 5, we can observe the velocity response of the ducted fan UAV under staircase command. It can be seen that the velocity response is smooth and settles down quickly without any oscillations. The height response of the controller is presented in Fig 3 and it can be clearly seen that the proposed neural adaptive controller tracks the desired height command accurately. A second order filter of frequency 1.4Hz & damping 0.7 and a proper timing of the staircase command based on the rise time facilitates smooth velocity response from the controller. From Fig 5, the angular rates and the corresponding euler angle responses can be seen, and it is shown to be asymptotically stable during autonomous take-off. Fig 6 shows the three dimensional trajectory of the UAV under staircase command and it clearly shows perfect vertical takeoff with no displacements in the x and y direction. Similar results can be obtained for the proposed controller in pitch, roll and yaw axis with the help of three neural adaptive controllers one in each axis respectively.

Fig. 3. Height response for the staircase command

Fig. 4. Velocity response of the ducted fan UAV in the x, y and z direction

Fig. 5. Angular rates and euler angle response in the x, y and z direction

Fig. 6. Trajectory of the ducted fan UAV
V. CONCLUSION

In this paper, we presented the nonlinearities and uncertainties posed by the ducted fan UAV and proposed a neural adaptive back stepping controller to overcome it ensuring global tracking performance and stability. An autonomous take-off simulation is performed and the performance of the neural adaptive controller is evaluated. The results clearly indicate that the proposed controller provides good tracking performance and ensures global asymptotic stability of the ducted fan UAV.

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