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Voigt Airy surface magneto plasmons

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Abstract: We present a basic theory on Airy surface magneto plasmons (SMPs) at the interface between a dielectric layer and a metal layer (or a doped semiconductor layer) under an external static magnetic field in the Voigt configuration. It is shown that, in the paraxial approximation, the Airy SMPs can propagate along the surface without violating the nondiffracting characteristics, while the ballistic trajectory of the Airy SMPs can be tuned by the applied magnetic field. In addition, the self-deflection-tuning property of the Airy SMPs depends on the direction of the external magnetic field applied, owing to the nonreciprocal effect.

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References and links

1. Introduction

Airy wave packets, which were first proposed as nonspreading beams by Berry and Balaz [1], have attracted a surge of interest since their experimental observation in free space [2, 3]. Airy beams are well known by the intriguing properties of nondiffracting, asymmetric field profile, self-bending, and self-healing [3]. This kind of novel light beam has been widely studied in various materials, such as nonlinear mediums [4, 5] and uniaxial crystals [6], and broadly applied in particle cleaning [7] and light bullet generation [8] applications.

Compared with other diffraction-free wave packets, e.g. Bessel [9] and Mathieu [10] beams, Airy beams have a unique property such that they are the only nonspreading solution to the one-dimensional potential-free Schrödinger equation. This suggests that only Airy surface plasmon (SP) beams can propagate on a metal surface without diffraction. It is found in both theoretical studies [11, 12] and experimental observations [13, 14] that Airy SPs remain the properties of both SPs and free space-propagating Airy beams, including the energy confinement at a subwavelength scale and self-bending property of the Airy beams.

On the other hand, it is known that when an external static magnetic field is applied on a metal or a semiconductor, propagation of the SP wave (also called surface magneto plasmons (SMPs) [15]) can be changed, due to the existence of the Lorentz force which alters the response of free carriers. In this situation, the resonant oscillation of free carriers (which causes SP waves) is not only characterized by the plasma frequency \( \omega_p \) and the incident frequency \( \omega_c \), but also by the cyclotron frequency \( \omega_c \), which is a function of the external magnetic field. In consequence, the medium becomes highly anisotropic (the permittivity of the conductor becomes a tensor) under an external magnetic field – even though the medium is isotropic. Therefore, SMPs have some unique and intriguing features, compared with general SP waves [15, 16]. For example, in the Voigt configuration (the applied magnetic field is parallel to the surface and perpendicular to the propagating direction of SMPs), the nonreciprocal effect can be observed, i.e. SMP waves propagating in two opposite directions have different propagating constants and cutoff frequencies [15, 17, 18].

In this paper, we analytically investigate TM-polarized paraxial Airy SMPs in the Voigt configuration. It is found that unlike the Airy SPs on a metal surface, the electromagnetic field components of the Airy SMPs are coupled in the wave equation due to the anisotropy of the metal or semiconductor when a magnetic field is applied. Thus, it is difficult to obtain analytical derivations. While, in the paraxial approximation, a relatively simple expression on the electromagnetic field components of Airy SMPs can be derived. The analytical and simulation results show that the external magnetic field can manipulate the self-deflection property of the Airy SMPs by tuning the wave vector of SMPs. Furthermore, due to the nonreciprocal effect, the magnetic field applied in one direction can significantly change the tilting angle of the Airy SMPs, while has little effect when applied in the opposite direction.

2. Theory of Airy surface magneto plasmons

The schematic structure of Airy SMPs propagating at the interface of a metal (or semiconductor) (region I) and a dielectric (region II) is depicted in Fig. 1. The Airy SMP wave is excited at the plane \( z = 0 \), and propagates along the \( z \)-axis. An external static magnetic field \( B \) is applied uniformly on the whole structure along the \( y \)-axis, forming the so called Voigt configuration.
Fig. 1. Schematic structure of Airy SMPs on the interface of a metal/semiconductor and a dielectric. The external magnetic field is applied along the y-axis.

We first solve the wave equation of the electric field in the paraxial approximation in the region (I) \((x<0)\). It can be written, according to the Maxwell equations, as

\[
\nabla \times (\nabla \times \bar{E}) - k_0^2 \epsilon_m \bar{E} = 0
\]

where \(k_0\) is the wave vector in free space. When \(B\) is applied, the permittivity of the metal/semiconductor \(\epsilon_m\) becomes a tensor caused by the Lorentz force on the free electrons, which is expressed by [17, 19]:

\[
\epsilon_m = \begin{bmatrix}
\epsilon_{xx} & 0 & \epsilon_{xz} \\
0 & \epsilon_{yy} & 0 \\
-\epsilon_{zx} & 0 & \epsilon_{ss}
\end{bmatrix}
\]

where, in the lossless case, \(\epsilon_{xx} = \epsilon_{\infty} \left[1 - \alpha_0^2 \left(\alpha^2 - \omega_c^2\right)\right]\), \(\epsilon_{zy} = -i \epsilon_{\infty} \frac{\omega_p^2}{\omega^2 - \omega_c^2}\), in which \(\omega\) is the angular frequency of the incident wave, \(\omega_p\) is the plasma frequency of the metal/semiconductor, \(\epsilon_{\infty}\) is the high-frequency permittivity, and \(\omega_c = eB/m^*\) is the cyclotron frequency. \(e\) and \(m^*\) are the charge and the effective mass of electrons, respectively. \(B\) is the applied external magnetic field. Here, it is noted that we use the Drude model to calculate the elements in Eq. (2) [19]. Considering the exponential decay of the SP waves in the metal/semiconductor material, we can express the electric field components as

\[
E^{(i)}(x,y,z) = A_{x,y,z}(y,z) e^{\alpha x}
\]

where \(\alpha_1\) is the decay factor in \(x\)-direction. For a paraxial Airy SMP, \(\alpha_1\) does not change much with that of a plane SMP wave [11], and consequently, it can be calculated by \(\alpha_1^2 = k_{\text{mp}}^2 - k_0^2 \epsilon_v\), where \(\epsilon_v\) is the Voigt dielectric constant, defined by \(\epsilon_v = \epsilon_{xx} + \epsilon_{ss} / \epsilon_{xx}\) [15]. \(k_{\text{mp}}\) is the propagation constant of the SMPs, calculated by a transcendental equation:

\[
\epsilon_v \sqrt{k_{\text{mp}}^2 - \epsilon_v k_0^2} + \epsilon_v \sqrt{k_{\text{mp}}^2 - \epsilon_v k_0^2} + i \frac{E_\text{d} E_{\text{ss}}}{\epsilon_{xx}} k_{\text{mp}} = 0
\]

in which \(\epsilon_d\) is the permittivity of the dielectric. Substitute Eqs. (2) and (3) into Eq. (1), and conduct the Fourier transform on the equations with respect to \(y\), we obtain

\[
\frac{\partial^2 \tilde{A}_x}{\partial z^2} - \alpha_1 \frac{\partial \tilde{A}_x}{\partial z} + (\epsilon_{ss} k_0^2 - k_v^2) \tilde{A}_x - i k_v \alpha_1 \tilde{A}_y + \epsilon_{ss} k_0^2 \tilde{A}_z = 0
\]
where \( \tilde{A}_{x,y,z}(k_y,z) \) are the Fourier transforms of \( A_{x,y,z}(y,z) \), respectively. From Eq. (5) it is observed that the electric field components are coupled due to the permittivity tensor, and cannot be solved by separated scalar wave equations like Airy SPs [11]. To solve Eq. (5), we use \( \tilde{A}_x \) and \( \tilde{A}_y \) to express \( \tilde{A}_z \) by Eq. (5c) and substitute it into Eqs. (5a) and (5b). The differential equations with respect to \( \tilde{A}_x \) and \( \tilde{A}_y \) are obtained as

\[
\left[ \frac{k_0^2 \varepsilon_{xx} - k_y^2}{\kappa_x^2} D^2 + \left( \frac{k_0^2 \varepsilon_{yy}}{\kappa_y^2} \right)^2 + k_0^2 \varepsilon_{xx} - k_y^2 \right] \frac{i k_0 \alpha}{\kappa_x^2} D^2 + \frac{k_0^2 \varepsilon_{xx}}{\kappa_y^2} D - ik_0 \alpha \left[ \frac{k_0^2 \varepsilon_{xx} + \alpha_i^2}{\kappa_x^2} D^2 + \alpha_i^2 + k_0^2 \varepsilon_{yy} \right] \tilde{A}_y = 0
\]

(6)

where \( \kappa_x^2 = k_0^2 \varepsilon_{xx} + \alpha_i^2 - k_y^2 \). \( D \) is the partial differential with respect to \( z \). The eigen values can be calculated by nontrivial solutions of Eq. (6) as

\[
\Lambda = \frac{1}{2} \left[ (\kappa_x^2 + \kappa_y^2) \pm \sqrt{(\kappa_x^2 - \kappa_y^2)^2 - 4 \left( \frac{\varepsilon_{yy}}{\varepsilon_{xx}} k_y^2 \right) \kappa_x^2 \kappa_y^2} \right]
\]

(7)

in which \( \kappa_x^2 = k_0^2 \varepsilon_{xx} + \alpha_i^2 - k_y^2 \) and \( \kappa_y^2 = k_0^2 \varepsilon_{yy} + \alpha_i^2 - k_y^2 \). Using the paraxial approximation expression \( k_y << k_0 \) and \( k_y << k_{\text{amp}} \), we expand Eq. (7) up to the second and the zeroth order of \( k_y / k_0 \) and \( k_y / k_{\text{amp}} \), respectively. The simplified eigen values are calculated by

\[
\Lambda_1 = ik \parallel
\]

(8a)

\[
\Lambda_2 = i \left( k_{\text{amp}} - \frac{\varepsilon_{ss} + \varepsilon_{yy}}{4 \varepsilon_{ss}} \frac{k_y^2}{k_{\text{amp}}} \right)
\]

(8b)

where \( k_0^2 \varepsilon_{ss} + \alpha_i^2 \). It can be found that Eqs. (8a) and (8b) correspond to the TE (\( E_x = 0 \)) and TM modes (\( E_y = 0 \)) of a plane wave propagating in Voigt-magnetized plasma [18], respectively, except that in Ref [18], \( \Lambda_2 = ik_{\text{amp}} \). Therefore, we choose Eq. (8b) as the eigen value in our calculations to ensure \( E_x \) is predominant in the electric field to excite plasmons on the surface. Consequently, the solution of Eq. (6) is

\[
\tilde{A}_z = C_1 \exp \left[ i \left( k_{\text{amp}} - \frac{\varepsilon_{ss} + \varepsilon_{yy}}{4 \varepsilon_{ss}} \frac{k_y^2}{k_{\text{amp}}} \right) z \right]
\]

(9)

where \( C_1 \) is a function of \( k_y \) needing to be determined. Like other works of Airy beams [1–6], we assume that the \( E_x \) component of the Airy SMPs in the metal/semiconductor layer at the input plane \( z = 0 \) takes a form of [11]

\[
E_x^{(1)}(x,y,0) = Ai \left( \frac{y}{w_0} \right) e^{ay^2 + \alpha_i x}
\]

(10)
where $w_0$ is the characteristic width of the first Airy beam lobe, and $a$ is the decay factor [3]. It can be inferred from the Fourier transform of Eq. (10) that

$$C_1 = w_0 \exp \left[ \frac{(a-iw_0k_x)^3}{3} \right]$$

(11)

Substitute Eq. (11) into Eq. (9), and conduct the inverse Fourier transform on the equation, we can obtain the electric field $E_z$ of the Airy SMPs in the metal/semiconductor as

$$E_z^{(r)} = E_0^{(r)} Ai \left[ f_1(y,p) \right] \exp \left[ f_2(y,p) \right] e^{i\alpha_y} e^{ik_{zmp}z}$$

(12)

in which

$$f_1(y,p) = \frac{y}{w_0} - p^2 + i2ap$$

(13a)

$$f_2(y,p) = a \frac{y}{w_0} - 2ap^2 + i \left( -\frac{2}{3}p^3 + a^2p + \frac{y}{w_0}p \right)$$

(13b)

$E_{z0}^{(r)}$ is the amplitude. $p = z \left( \epsilon_{xx} + \epsilon_{yy} \right) / \left( 4\epsilon_{xx} k_{zmp} w_0^2 \right)$. For an Airy surface plasmon, because the electric field components are not invariant in the $y$-direction, the definition of TE and TM modes is not the same as that of an Airy beam in free space (for TE mode: $H_y = 0$, TM mode: $E_x = 0$). However, in the paraxial approximation, they can be divided by $E_z = 0$ for TE mode and $H_z = 0$ for TM mode [11]. For simplicity, we only consider TM mode. Therefore, according to the Maxwell equations, other electric field components in regions (I) are derived as

$$E_x^{(r)} = \frac{k_{zmp}^2 - \epsilon_{xx} k_0^2}{\epsilon_{xx} k_0^2 - ik_{zmp} \alpha_y} \left[ E_0^{(r)} Ai \left[ f_1(y,z) \right] \exp \left[ f_2(y,z) \right] e^{i\alpha_y} e^{ik_{zmp}z} \right]$$

(14)

$$E_y^{(r)} = -j \frac{k_{zmp}}{w_0} \left( \frac{k_{zmp}^2 - \epsilon_{xx} k_0^2}{\epsilon_{xx} k_0^2 - ik_{zmp} \alpha_y} \right) E_0^{(r)} \exp \left[ f_2(y,z) \right] e^{i\alpha_y} e^{ik_{zmp}z}$$

(15)

$$\times \left[ f_1(y,z) K_{2/3} \left( 2^{3/2} f_1^{3/2}(y,z) \right) + (ip + a) Ai \left[ f_1(y,z) \right] \right]$$

In the dielectric, the electric field components can be calculated by the same procedure, by replacing $\epsilon_{xx}$ with $\epsilon_d$, $\alpha_l$ with $-\alpha_d$, and $\epsilon_{xx} = 0$, where $\alpha_d^2 = k_{zmp}^2 - k_0^2 \epsilon_d$. Together with the consideration of the boundary conditions, we obtain:

$$E_x^{(r)} = E_0^{(r)} Ai \left[ f_1(y,z) \right] \exp \left[ f_2(y,z) \right] e^{-i\alpha_y} e^{i\beta_{zmp}z}$$

(16)

$$E_y^{(r)} = \frac{k_{zmp}^2 - \epsilon_d k_0^2}{\epsilon_d k_0^2 - ik_{zmp} \alpha_l} \left[ E_0^{(r)} e^{i\alpha_l} e^{ik_{zmp}z} \right]$$

(17)
\[
E_y^{(II)} = -\frac{1}{w_0 \epsilon_0} \frac{\epsilon_{zz}}{\epsilon_{xx}} k_{\text{zmp}}^2 - i \epsilon_{zz} k_{\text{zmp}}^2 E_0^{(I)} \exp \left[ f_2(y, z) \right] e^{-a z} e^{i k_{\text{zmp}} z} \\
\times \left[ -\frac{f_1(y, z)}{\pi^{\frac{1}{3}}} K^{\frac{2}{3}} + \frac{2}{3} f_1^{1/2} (y, z) \right] + (ip + a) A i \left[ f_1(y, z) \right] \right]
\] (18)

in which

\[
E_0^{(I)} = \frac{\epsilon_{zz} k_0^2 - i k_{\text{zmp}} \alpha_i}{\epsilon_{xx} k_{\text{zmp}}^2 - i \epsilon_{zz} k_{\text{zmp}} \alpha_i}
\] (19a)

\[
E_0^{(II)} = \frac{1}{\epsilon_d}
\] (19b)

In Fig. 2, the electric field intensity \( |E|^2 \) distributions of Airy SMPs on both the \( x\)-\( y \) plane and \( y\)-\( z \) plane without the magnetic field are plotted. In order to ensure \( \epsilon_y < 0 \) (under this condition, SMPs can exist), the incident frequency and the magnetic field are set as \( \omega = 0.8 \omega_p \) and \( \omega = 0.1 \omega_p \), respectively. Without loss of generality, the characteristic width and the decay factor are chosen as \( w_0 = 2 \lambda \) and \( a = 0.1 \), respectively, where \( \lambda \) is the incident wavelength. We choose InSb as the semiconductor material, which is often used in the SMPs experiments [21–23]. The corresponding parameters of InSb are \( m^* = 0.014 m_0 \) (\( m_0 \) is the free electron mass in vacuum), \( \omega_p = 12.6 \text{THz} \), and \( \epsilon_\infty = 15.68 \) [23]. It can be seen from Figs. 2(a) to 2(c) that the Airy plasmons are confined on the surface of the InSb material, keeping the diffraction-free characteristic even after propagation distance of 80\( \lambda \). With the increase of the propagating length, the diffraction effect becomes obvious due to the decay factor \( a \) in the \( y \)-direction, which is the same as that of a free space Airy beam [2, 3]. When they propagate along the \( z \)-direction, the main lobe moves toward the \( + y \)-axis. Figure 2(d) shows the self-bending property of Airy SMPs.

![Electric field distributions of the Airy SMPs on the planes perpendicular and parallel to the surface when \( \omega = 0.1 \omega_p \) and \( \omega = 0.8 \omega_p \). (a)-(c) Electric field distributions in the \( x\)-\( y \) plane when \( z = 0, 50 \lambda \), and \( 100 \lambda \), respectively. (d) Electric field distribution in the \( y\)-\( z \) plane when \( x = 0 \).](image)

3. Ballistic trajectory tuning by the external magnetic field

In this section, we will study the influence of the external magnetic field on the Airy SMPs. It is well known that the Airy beams and Airy plasmons perform ballistic dynamics similar to those of projectiles moving under gravity [11, 20]. From Eqs. (12) and (16), it is found that like Airy surface plasmons, the Airy SMPs also follow a ballistic trajectory in the \( y\)-\( z \) plane, which is described by
It can be clearly seen that if the transverse scale \( w_0 \) is set, the ballistic dynamics of the Airy SMPs is mainly determined by \( k_{\text{smpp}} \). In Fig. 3, we plot the dispersion curve of SMPs under different external magnetic field intensities with \( \omega < \omega_p \). It is found that when the magnetic field is applied along the \(+y\)-axis (denoted by \( B > 0 \) in the inset), the \( k_{\text{smpp}} - \omega \) curve nearly keeps unchanged. While, when the magnetic field is applied along the \(-y\)-axis (denoted by \( B < 0 \)), the dispersion curve moves toward lower frequencies side. This intriguing nonreciprocal effect can be understood by Eq. (4). When the magnetic field is applied in the opposite direction, \( \varepsilon_{xz} \) in the last term changes its sign. Thus the dispersion equations become different, having different cutoff frequencies [24].

![Dispersion relation of SMPs under different external magnetic field intensities.](image)

Fig. 3. Dispersion relation of SMPs under different external magnetic field intensities. If the external magnetic field is along the \(-y\)-axis (denoted by \( B = 0 \)), the dispersion curve is red-shifted with the increase of the magnetic field. However, if the external magnetic field is along the \(+y\)-axis (denoted by \( B = 0 \) in the inset), the dispersion curve is nearly unchanged.

![Ballistic trajectory curves of the Airy SMPs](image)

Fig. 4. Ballistic trajectory curves of the Airy SMPs when the magnetic field is applied along the \(-y\)-axis. The cyclotron frequencies are \( \omega = 0, 0.1\omega_p, 0.2\omega_p \), and \( 0.25\omega_p \), respectively. The incident frequency is \( \omega = 0.85\omega_p \). The inset shows the “gravity” \( g \) as a function of the external magnetic field when the magnetic field is applied in the \(+y\)-axis and \(-y\)-axis, respectively.

From Fig. 3, it can be inferred that if the magnetic field increases in the region \( B < 0 \), \( k_{\text{smpp}} \) will increase for a monochromatic wave. Consequently, according to Eq. (20), the self-bending effect of the Airy SMPs will be alleviated. In Fig. 4, the ballistic curve of Airy SMPs
varying with respect to the magnetic field is plotted for an electromagnetic wave at $\omega = 0.85 \omega_p$. It can be seen that with the increase of the magnetic field, the tilting angle of the Airy SMPs decreases. However, this phenomenon cannot be observed for $B>0$, which is clearly shown in the inset of Fig. 4. In this situation, the “gravity” in the Newtonian equation of Eq. (21) changes very little (red dashed line), compared with that in the situation $B<0$ (blue line). In Fig. 5, the electric field distributions of Airy SMPs are plotted without any magnetic field, and with magnetic fields such that $\omega_c = 0.25 \omega_p$ along the +y-axis and −y-axis, respectively. It can be clearly seen that, when $B < 0$, the Airy SMPs can be tuned by the magnetic field. It should also be noted that the nonreciprocal effect can be observed by changing not only the direction of the external magnetic field, but also the propagation direction of the Airy SMPs.

In order to verify our theoretical derivations, 3D finite-difference time-domain (FDTD) simulations are carried out to simulate Airy SMP waves propagating on a semiconductor surface. The parameters used in the simulation are the same as those in Fig. 5. The field distribution of the Airy SMP source in the semiconductor is calculated by Eq. (10), and that in the dielectric is calculated by Eq. (7) of Ref [11]. The source is launched along the +z direction at $z = 0$ plane. Due to the limitation on our PC memories, the FDTD simulation is conducted in the region ($-2\lambda < x < 5\lambda, -30\lambda < y < 30\lambda, 0 < z < 50\lambda$). The analytical and FDTD-simulation results are compared in Fig. 6. It shows that the theoretical model gives good predictions of the main lobe and the side lobes of the Airy SMPs except for a slight shift. We
believe this shift is caused by the insufficient grid density. When a magnetic field is applied \( (\omega = 0.25 \omega_p) \) along the \(-y\)-axis, the main lobe moves about \(1.5\lambda\) toward the \(-y\) direction, calculated by the theoretical model, while the FDTD simulation gives a shift of about \(1.55\lambda\).

![Fig. 6. Comparison of theory (red lines) and FDTD simulation (blue lines) results of normalized electric field distributions of the Airy SMPs on the \(x=0\) plane at \(z=50\lambda\). The incident frequency is \(\omega = 0.85 \omega_p\). Solid lines: field distributions when \(\omega_c = 0 \omega_p\). Dashed lines: field distributions when \(\omega_c = 0.25 \omega_p\) for \(B<0\).](image)

4. Conclusions

In this paper, we study the Airy surface plasmon under an external magnetic field. When a magnetic field is applied perpendicular to the propagating direction of the Airy SMPs and perpendicular to the surface, the ballistic trajectory of the Airy SMPs can be tuned. When the applied magnetic field increases, the tilting angle of the Airy ballistic waves decreases. The FDTD full vectorial method demonstrates the accuracy of our analytical model. The proposed tuning mechanism, we believe, can be applied to design different types of Airy plasmonic devices.

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