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Numerical and experimental studies of coupling-induced phase shift in resonator and interferometric integrated optics devices


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Abstract: Coupling induced effects are higher order effects inherent in waveguide evanescent coupling that are known to spectrally distort optical performances of integrated optics devices formed by coupled resonators. We present both numerical and experimental studies of coupling-induced phase shift in various basic integrated optics devices. Rigorous finite difference time domain simulations and systematic experimental characterizations of different basic structures were conducted for more accurate parameter extraction, where it can be observed that coupling induced wave vector may change sign at the increasing gap separation. The devices characterized in this work were fabricated by CMOS-process 193nm Deep UV (DUV) lithography in silicon-on-insulator (SOI) technology.

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References and links

1. Introduction

Coupling-induced frequency shift (CIFS) [1–3] and coupling induced phase shift (CIPS) [4] have been shown to have deteriorating effects on optical filter devices that are based on coupled-resonator geometries [4–8]. Previous theoretical studies based on finite difference time domain (FDTD) [1], finite element method (FEM) [2], and quantum scattering formalism [3] have indicated that coupling induced effects are not the byproduct of fabrication error, but an intrinsic mechanism from higher order coupling effects. Although intuitively predicted as only having a negative sign, numerical simulations have confirmed that CIFS may have positive sign, suggesting the inadequacy of standard coupled mode theory (CMT) treatment. Such counterintuitive results have been theoretically explored from standing wave field patterns within the coupler [1] or from symmetry properties inherent in the coupler geometry [2]. However, to the best of our knowledge, this has not been explored experimentally.

Therefore, a detailed experimental characterization of coupling-induced effects would be necessary to further understand the nature of CIFS and CIPS. Experimentally, the characterization of coupling induced effects is challenging since it convolutes with the effects of fabrication imperfection such as optical proximity effects (OPE) in the coupling region or phase imbalance of 3dB coupler in the interferometric structure used for their characterization. The other reason is rather fundamental where the ideal resonance wavelength (uncoupled resonator) can never be obtained because coupling is always necessary for measurements.

The aim of this work is to present both numerical and experimental quantification of coupling induced effects in silicon-on-insulator (SOI) technology. The rest of this paper is organized as follows. In Section 2, the existence of CIPS and CIFS are first numerically demonstrated using FDTD simulation in Mach-Zehnder interferometer (MZI) and ring resonators. Then, in Section 3, rigorous and systematic experimental characterization of various basic structures (see Fig. 1) is carried out in order to obtain accurate extraction of coupling induced phase shift in ring-enhanced Mach-Zehnder interferometer (REMZI) for different coupler lengths and gap separations. Finally, the conclusion is given in Section 4.

![Fig. 1. The fabricated (a) Ring-enhanced Mach-Zehnder Interferometer (REMZI), (b) 3dB multimode interferometer (MMI), and (c) ring resonator side coupled to two bus waveguides (1R2B) devices.](image)

2. Numerical study of CIPS in Mach-Zehnder interferometers and resonators

The evanescent coupling of two closely spaced optical waveguides can be readily modeled using coupled-mode theory,
Here \( k_{pq} \) is the overlap field integral terms \( k_{pq} = (k_p^2 / 2\beta_p) \int E_p(r)\Delta \varepsilon q(r)E_q(r)dr \). where \( \beta_p (\beta_q) \) is the mode of waveguide \( p (q) \) which has index contrast profile \( \Delta \varepsilon q(\Delta \varepsilon_q) \). The eigenvalue problem in Eq. (1) can be readily solved, and the fields before and after coupling can be related by familiar coupling matrix \[9\]

\[
\begin{pmatrix}
A_1(L) \\
A_1(L)
\end{pmatrix} = \exp(-i\beta L)M
\begin{pmatrix}
A_i(0) \\
A_i(0)
\end{pmatrix},
\]

where \( L \) is the coupler length and the coupling matrix \( M \) is expressed as

\[
M = \begin{bmatrix}
\cos(sL) + i\frac{\Delta \beta}{2s} \sin(sL) & i\frac{k_{21}}{s} \sin(sL) \\
\frac{k_{21}}{s} \sin(sL) & \cos(sL) - i\frac{\Delta \beta}{2s} \sin(sL)
\end{bmatrix}.
\]

\( \tilde{\beta}_p = \beta_p + k_{mm} \) is the modified waveguide mode propagation constant in the presence of self coupling term \( k_{mm} \). \( \beta = (\tilde{\beta}_1 + \tilde{\beta}_2)/2 = (\beta_1 + \beta_2)/2 + (k_{11} + k_{22})/2 \) is the average mode propagation constant, \( s = \sqrt{(\Delta \beta/2)^2 + k_{21}^2} \) is the coupling constant arising from mode beating, and \( \Delta \beta = \tilde{\beta}_1 - \tilde{\beta}_2 \) is the phase mismatch between two waveguide modes.

Note that Eq. (3) is derived from simplistic CMT treatment which does not take into account the full-vectorial nature of the mode evanescent coupling \[1\], which renders Eq. (3) inadequate to fully describe coupling induced effects. However, both Eq. (2) and Eq. (3) are sufficient to be used for obtaining general insight into the origins of coupling induced phase shift, which come from a modified pre-factor term \( \exp(-i\beta L) \) and complex conjugated diagonal elements of the coupling matrix. The phase relation among the coupling matrix elements in Eq. (3) is actually a general consequence of reciprocity along the horizontal direction, which remains true with or without the existence of coupling induced phase shift. Hence, the general form of the coupling matrix can be written as

\[
M = \begin{bmatrix}
\exp(-i\phi_r) & it \\
it & \exp(i\phi_t)
\end{bmatrix},
\]

where the self-coupling (r) and cross-coupling (t) are modeled to have a phase of \( \phi_r \) and \( \pi/2 \), respectively. Note that the phase relations in Eq. (4) are consistent with those in Ref. 1 (see Eq. (7) and the related discussions therein), where the cross-coupling terms are purely imaginary (consequence of reciprocity) and the self-coupling are complex conjugates ( \( \pm \phi_r \)). It is important to note that the phase relation in Eq. (4) holds true for a coupler with or without vertical symmetry \[1\].

In the case of a ring resonator side-coupled with a waveguide, by incorporating the additional phase both in prefactor and in self-coupling coefficients, it is possible to deduce the phase modification within the resonator and in the waveguide bus. In the ring resonator, the roundtrip phase (\( \delta' \)) becomes \( \delta' = \delta + (\beta - \beta)L_c + \phi \), where \( \delta \) is the unperturbed roundtrip phase, \( (\beta - \beta)L_c \) is the phase originated from the prefactor, and \( \phi \) is the intrinsic phase of the self-coupling coefficient. On the other hand, the perturbed phase in the waveguide bus (\( \phi' \)) is \( \phi' = (\beta - \beta)L_c - \phi \). It is interesting to note that the phase shift inside and outside the resonator are different, and this is caused by the complex conjugate terms in Eq. (4), where
the light within the resonator and the light in the waveguide bus experiences opposite phase contribution of \( \phi \).

The coupling-induced phase within the resonator, i.e., \( \phi_{\text{res}} = (\delta' - \delta) \), then translates to the shift of resonance wavelength, \( \Delta \lambda_{\text{res}} = (\phi_{\text{res}}/2\pi)\Delta \lambda_{\text{FSR}} \), where the free-spectral range \( \Delta \lambda_{\text{FSR}} = \lambda_0^2/(n_g L_{\text{cav}}) \) is expressed in terms of resonance wavelength \( \lambda_0 \), group index \( n_g \) for a given wavelength band \( \Delta \lambda \) and the cavity circumference \( L_{\text{cav}} \). The coupling induced effects have been numerically demonstrated in isolated rings side-coupled with optical waveguides, or in a directional coupler, where the phase response is typically extracted from a suitable reference plane before and after coupling [1]. In our numerical simulation, however, we use interferometric structure instead of individual resonator, where phase difference between the two arms is deduced from amplitude responses of both output arms. Thus, there is no further need to find suitable reference plane and the simulation can be done much faster compared to simulations based on resonant structures. The structure to be simulated is shown in Fig. 2, which consists of a Mach-Zehnder interferometer with the light splitter/combiner elements modified to directional coupler of variable coupler length.

![Fig. 2. The schematic of MZI for phase extraction.](image)

The light-splitting and light-combining section in Fig. 2 can be described by Eq. (4), where \( \phi \) is assumed to be the same since the directional coupler is symmetrical with respect to both vertical and horizontal axis [1]. Note that the \( \exp(-i\beta L_c) \) prefactor from two coupling junctions do not contribute to the phase shift between two arm lengths, as they cancel each other. The bar \( (T_B) \) and cross \( (T_C) \) transmissions can be written as \( T_B = (t^2 - r^2) + 4r^2t^2 \sin^2 \phi \) and \( T_C = 4r^2t^2 \cos^2 \phi \), respectively. The phase \( \phi \) can then be deduced from the ratio of bar and cross transmissions,

\[
\phi = \sin^{-1} \sqrt{\frac{(T_B/T_C) - [(t^2 - r^2)/(2rt)]^2}{1 + (T_B/T_C)}},
\]

where coupling coefficients \( (r,t) \) are independently obtained from FDTD simulations of directional couplers for a given coupler length \( L_c \).

The simulations of directional coupler (with 5\( \mu \)m bending radius assumed in each simulation) are carried out with the variation of the coupler length \( (L_c) \) and gap separation \( (g) \) from 2\( \mu \)m to 14\( \mu \)m and 200nm to 400nm, respectively. In order to save computation time, the FDTD simulations are limited only to two dimensions, with the waveguide core index deduced by effective index method of the actual silicon-on-insulator material. The core index is calculated to be \( \approx 2.84 \) and the cladding is SiO\(_2\) \( (n = 1.444) \). In general, disagreement between 3D and 2D simulation is to be expected, since the 2D simulations are limited to Semi-Vectorial wave propagation in contrast to the Full-Vectorial propagation in the 3D counterparts. As a result, the group index and the field confinement factor are expected to be lower, as is evident from a rather large coupling strength, compared with experimental values in the Section 3.
Fig. 3. The simulation of coupling coefficients as a function of wavelength for different coupler lengths and gap separations.

The waveguide mode is launched at the input port in the form of Gaussian pulse. The time domain fields at the output ports are Fourier transformed and normalized with respect to the input spectrum in order to obtain the coupling coefficient spectrum. Figure 3 shows the coupling coefficients for coupler lengths of 6μm, 8μm, and 10μm for 4 different gap separations as a function of wavelength in the L-band (1500nm to 1600nm). These coupling coefficients are then inserted into Eq. (5) in order to get the induced phase shift. Figure 4 shows the plot of induced phase shift for different coupler lengths and gap separation. Each point in Fig. 4 represents the average phase values taken within a particular wavelength band of interest (within 1560nm-1580nm wavelength band), together with their standard deviation represented in the error bar.

Fig. 4. (a) The induced phase shift as a function of coupler length and gap separation. The wavelength at which the phase is extracted is ~1564.9nm (for 200nm), ~1576.2nm (for 300nm), and ~1576.5nm (for 400nm). (b) The coupling induced phase wavevector as a function of gap separation.

As shown in Fig. 4(a), the induced phase follows linear relationship with the coupler length, which seems to confirm theoretical aspect of CIPS. However, in the varying gap separation, a somewhat different trend can be observed where the phase slope changes sign as the gap separation increases. This is rather different from conventional CMT treatment which states that the phase slope depends on the overlap integral between the square of electric field
of one waveguide with the index contrast of the other waveguide, which thus leads to a positive-definite values of the integral. This rather counter-intuitive aspect of CIPS will be verified later in our experiment in the Section 3. In addition, by using standard CMT treatment, the induced wavevector $\Delta K_r$ can be calculated from the phase slope ($\Delta K_r = d\phi/dL_C$), which is shown in Fig. 4(b) as a function of gap separation. It can be seen that Fig. 4(b) roughly follows the exponential trend because of the exponentially decreasing coupling strength in the increasing of gap separation.

So far the coupling induced phase shift has been numerically demonstrated in horizontally symmetric directional coupler. For completeness, it is instructive to also simulate the coupling induced phase shift when the horizontal symmetry is broken. This can be found in the case of resonator side-coupled with two optical bus waveguides (termed as 1R2B) shown in Fig. 5. In this case, we deduce the coupling induced phase shift inside the resonator, which manifests in the shift of resonance wavelength. The induced phase shift in 1R2B is calculated from the resonance wavelength shift, $\phi_{\text{res}} = 2\pi(\lambda_1 - \lambda_0)/\Delta \lambda_{\text{fsr}}$, where $\lambda_1$ ($\lambda_0$) is the resonance wavelength for the coupled (uncoupled) case. The resonance wavelength for coupled case [Fig. 5(a)], is deduced from normalization of the transmission spectrum (drop or through port) with the input spectrum, while the resonance wavelength for the uncoupled case [Fig. 5(b)] is deduced from normalization of intra-cavity field with respect to the input spectrum.

![Fig. 5. The simulation strategy for 1R2B when it is (a) coupled and (b) uncoupled. For the coupled case, the drop transmission is normalized with respect to the input spectrum. For the uncoupled case, the intra-cavity field is normalized with respect to the input spectrum. (c) Induced phase shift in 1R2B as a function of coupler length for three different gaps. The resonance wavelength shift is measured from three resonance orders. Each point represents the average resonance shift and its standard deviation.](image)

The induced phase shifts for 1R2B structures is shown in Fig. 5(c), where slight bowing effects can be observed for different gap separations. The phase slope is generally positive, which is very different than those in Fig. 4(a) where the slope changes sign in the increasing gap separation. This is a result of different symmetry conditions in both structures, which leads to different phase conditions. Furthermore, we note that the induced phase in Fig. 4 is much smaller than those in Fig. 5, which in an indicative of opposite phase contribution of $\phi_r$ in the resonator as compared to that in the waveguide.

3. Experimental study of CIPS in resonator-enhanced Mach-Zehnder interferometers

The devices were fabricated in silicon-on-insulator (SOI) technology using 193nm Deep-UV Lithography [10, 11]. The REMZI, as shown in Fig. 1, consists of Mach-Zehnder-Interferometer (MZI) whose one of the arms is side-coupled with a ring resonator. In order to vary the coupling coefficients and to isolate optical proximity effects, both the coupler length ($L_C$) and gap separation ($g$) are varied while the ring radius is fixed to 5μm. The waveguide
dimensions are of 450nm width and 220nm thickness. The sample is clad by SiO$_2$ by default, unless specified otherwise. To facilitate input/output coupling, a curved second order gratings are fabricated at both ends of the devices [see Fig. 6(a)]. The grating coupler is much more compact than those in our previous work due to the curvature introduced in the grating, which functions to focus the light onto the waveguide and reduce the possibility for higher order mode conversion. The length of the grating is less than 100μm, which gives the device density about three times than those in our previous fabrication batches.

In order to have a reasonable coupling coefficient at large gap, the coupler length is varied from 6μm to 14μm (at 2μm increment) with gap separation varied from 200nm to 400nm (at 100nm increment), totaling to 15 REMZI devices. In the REMZI, we used 3dB 2x2 MMI coupler for light division and combination. Here, 2x2MMI is employed instead of 1x2MMI for isolating the contribution of MMI phase imbalance from extracted phase shift, thereby giving a more accurate measurement. It should be noted that ring resonator side-coupled to two optical waveguides (1R2B) are also fabricated on the same chip with the same variation in coupler lengths and gap separations (15 1R2B devices), as shown in Fig. 1. From these structures the coupling coefficients as a function of gap and coupler length are independently obtained, leaving REMZI fitting reduced to only one fitting parameter, which is the coupling-induced phase shift $\phi_{	ext{CIPS}}$. In the following subsections, waveguide loss, coupling coefficients, and MMI loss and phase imbalances are briefly characterized.

3.1. Characterization of waveguide loss

The most accurate deduction of waveguide propagation loss is by the measurement of transmission spectrum of spiral waveguides. In the sample, spiral waveguides of lengths spanning from 1mm to 4cm was also fabricated. Here, for the purpose of having three sets of coupling coefficients, we consider three kinds of cladding, namely the SiO$_2$, i-Line resist, and the air cladding (bare silicon). The SiO$_2$ film is of 600nm nominal thickness and was deposited at 300°C temperature. The other two claddings are used to create less field confinement (for i-Line resist) to increase the coupling, and more field confinement (for air cladding) to decrease the coupling.

![Image of waveguide loss characterization](image)

Fig. 6. (a) The input/output curved gratings. (b) The waveguide loss characterization in spiral structures in three different SOI samples: (left) the PECVD SiO$_2$ coated, (middle) the i-line resist coated, and (right) bare silicon.

Figure 6(b) shows the loss characterization of the three samples. Note that the loss of SiO$_2$ cladded sample ($\alpha_{\text{wg}} = 2.84$dB/cm) is higher than that for the air cladded samples ($\alpha_{\text{wg}} = 2.34$dB/cm) because of the non-ideal PECVD condition, which is supposed to be done at much higher temperature (~700°C) for more conformal and stoichiometric film deposition.
The non-stoichiometric SiO$_2$ film may generate silicon nanocrystals which then behave as absorption centers. However, higher loss in $i$-Line resist cladding sample is to be expected because of the absorbing nature of $i$-Line photoresist ($\alpha_{wg} = 5.74$dB/cm). It is also interesting to note that the fiber-waveguide coupling efficiency is higher for $i$-Line resists clad samples ($\alpha_c = 3.59$dB/facet) compared with the other two ($\alpha_c = 5$dB/facet). This is because of lower index contrast in $i$-Line resist and much better phase matching condition with the gratings.

3.2. Characterization of coupling coefficients in ring resonator structures

The coupling coefficients can be extracted by using the drop transmission formula of 1R2B structure [12]

$$D = \frac{a(1 - r^2)}{1 + a^2 r^4 - 2a r^2 \cos \delta},$$

where $a = \exp(-\alpha L_{cav}/2)$ is the roundtrip loss for resonator with cavity length $L_{cav}$, $r$ is the self-coupling coefficients, and $\delta = 2\pi n_g L_{cav}/\lambda$ is the round trip phase for a given group index $n_g$. The drop amplitude is strongly dependent on loss, which may be influenced by other loss not originated from the resonator, such as grating loss. Therefore, loss extraction from 1R2B structure is less accurate compared to those from waveguides. Furthermore, since resonance linewidth depends on both roundtrip loss and coupling coefficients, it is rather difficult to isolate one from another. However the above problem can be avoided by linearizing Eq. (6) into the following form,

$$\frac{1}{D} = \frac{(1 - ar^2)^2 + 4ar^2 \cos \delta}{a(1 - r^2)^2} = \frac{(1 - ar^2)^2}{a(1 - r^2)^2} + \frac{4r^2}{(1 - r^2)^2} \cos \delta,$$

so that the first term depends on the coupling coefficients and round trip loss, while the second term solely dependent on the coupling coefficients. Thus, from the slope of Eq. (7) (with $\cos \delta$ as the abscissa), it is possible to extract coupling coefficient accurately without the information of round trip loss. The fitting methodology consists of two steps: (1) determination of group index, and (2) slope extraction of the linearized drop transmission [see Eq. (7)]. Figure 7(a) shows the offset drop transmissions of 1R2B structures for 300nm gap separation in SiO$_2$ PECVD cladding sample. The coupler length is varied from 6μm to 14μm. Prior to coupling coefficient extraction, an accurate group index needs to be determined. The initial condition of the group index is obtained from free-spectral range by $n_g = \lambda_0/(\Delta\lambda_{FSR} L_{cav})$, which is then readjusted to match $\delta = 2\pi n_g /\lambda_0$ [or $\delta = (2m + 1)\pi$] for the resonance (or anti-resonance) wavelength condition. The group index is found to be ~4.25 for PECVD and $i$-Line resist coated samples, in agreement with those in [13,14], and about ~4.5 for bare silicon sample. The higher group index in bare silicon is to be expected due to the higher field confinement in higher index contrast waveguide.

The coupling coefficients extracted from Eq. (7) for different coupler lengths and gap separations are presented in Fig. 7(b), where the coupling coefficients are linearized with respect to $L_C$, i.e., $(2/\pi)\cos^{-1}(r)$, for the purpose of finding the beating length ($L_b$) and phase offset ($\phi_0$). The beating length $L_b$ is the length at which all the light power is transferred to the other waveguide, while the phase offset is the lumped phase parameter which takes into account the coupling from non-straight parts of the directional coupler. By means of linear regression, it is then straightforward to obtain the beating length and phase offset from the slope and offset, respectively. Figure 8 presents the beating length and phase offset for different gap separations in the PECVD coated sample (here we choose only one sample to represent the whole curve fitting of 1R2B devices from the three different claddings). Note that the beating length increases exponentially with the gap separation, while the phase offset decrease exponentially. This is to be expected from field overlap integral which has exponential dependence on gap separation. The empirically deduced expression of coupling coefficient can then be written as
where the beating length \( L_x = p_0 \exp(p_1 g) \) and phase offset \( \phi_0 = q_0 \exp(-q_1 g) \) are empirically modeled as the exponential function of gap separation \( g \). This is in agreement with theoretical formulation of 2D directional coupler [15], in which the coupling constant \( \kappa \) is an exponentially dependent on gap separation.

\[
    r = \cos \left( \frac{\pi L_x}{2L_x(g)} + \phi_0(g) \right), \quad t = \sin \left( \frac{\pi L_x}{2L_x(g)} + \phi_0(g) \right),
\]

Fig. 7. (a) The measured drop transmissions of 1R2B structures for 300nm gap separation. Each measurement is offset by 10dB from each other. (b) Linearized coupling coefficients in different gap separations.

Fig. 8. Beating length and phase offset for different gap separations.

3.3. Characterization of MMI phase imbalance

Next is the characterization of phase imbalance in MMI-based 3dB splitter. For 3dB splitter application, MMI is often used in interferometer structures due to its broadband wavelength operations, in contrast to directional coupler. However, the required MMI length for 3dB splitting in device design is often different from the actual required MMI length in fabrication. Furthermore, the splitting ratio of MMI cannot be arbitrarily designed because of the self-
imaging principle inherent in MMI, in contrast to directional coupler where the splitting ratio can be easily controlled by adjusting the coupler length. Therefore, a mismatch of MMI length may give moderate insertion loss (about 1-2dB). However, the broadband nature of MMI renders the insertion loss almost independent of wavelength. Hence, the almost flat spectrum of insertion loss does not affect the interference condition in interferometric structures. Apart from additional insertion loss, a mismatch in MMI length may also give phase imbalance between bar and cross transmissions, which may significantly affect the interference condition between two arms in MZI structure, due to their wavelength dependence.

The MMI phase imbalance can be measured by integrating the MMI in question into the MZI configuration, as shown in Fig. 9(a). The ideal phase-balanced MMI is when the MZI has the cross transmission of almost unity and the bar transmission of almost zero. This is because of purely destructive (constructive) interference in the bar (cross) port. Figure 9(b) shows the bar and cross transmission of MMI with different MMI lengths. For the purpose of illustration, only PECVD coated sample is shown here. The MMI in our case was designed using RSOFT commercial software [16], which has MMI dimensions of 45μm MMI length, 3.5μm MMI width, 1.78μm port separation, 5μm taper length, and 1.3μm port width. However, as we can see from Fig. 9(b), the most ideal MMI seems to be when \( L_{\text{MMI}} \) is 41μm.

The phase imbalances \( \phi_{\text{MMI}} \) can be deduced by the ratio between the Bar \( T_B = \sin^2(\phi_{\text{MMI}}/2) \) and Cross \( T_C = \cos^2(\phi_{\text{MMI}}/2) \) transmission

\[
\phi_{\text{MMI}} = 2\tan^{-1}\sqrt{T_B/T_C}. \tag{9}
\]

Figure 9(c) shows the MMI phase imbalances when the MMI length is varied from 41μm to 51μm. Clearly, the phase imbalance is higher when the MMI length is more mismatched from the ideal one. It can also be observed that the phase imbalance is generally wavelength dependent, unlike the insertion loss and splitting ratio. This significantly affects the interference condition in REMZI devices which will be discussed in the next subsection.

![Fig. 9. (a) The schematic of MZI based on different MMI geometries. (b) Experimental measurements of Bar and Cross MMI transmissions for different MMI lengths. (c) Measured MMI phase imbalances for different MMI lengths.](image)

### 3.4. Characterization of CIPS in REMZI structures

The presence of induced phase shift in REMZI gives asymmetrical spectrum near the resonance. Thus, the induced phase shift \( \phi_{\text{CIPS}} \) can be obtained by fitting the spectrum asymmetry by using the MZI formula

\[
T_{\text{BAR}} = \frac{1}{2} \left| t_r \exp(-i\phi_{\text{CIPS}}) - \exp(-i\phi_{\text{MMI}}) \right|^2, \quad T_{\text{CROSS}} = \frac{1}{2} \left| t_r \exp(-i\phi_{\text{CIPS}}) + \exp(-i\phi_{\text{MMI}}) \right|^2, \tag{10}
\]

where the MMI phase imbalance \( \phi_{\text{MMI}} \) has been taken into account, and \( t_r = [r + a\exp(-i\delta)]/[1 - ar\exp(-i\delta)] \) is the transmittance of ring resonator side-coupled to one optical waveguide whose values is nearly unity since it is very far from critical coupling
condition \((r = a)\) [17]. The sharpness of the asymmetrical resonance in REMZI is mainly determined by the finesse of the resonator and the phase bias (the phase shift in the waveguide bus) introduced by the coupling induced phase shift, and it is independent of whether the roundtrip phase of the ring has been modified as a result of phase shift within the resonator. The finesse of the resonator is controlled by the coupling coefficients \((r)\) and cavity roundtrip loss \((a)\), which have been independently obtained. Therefore, since the MMI phase imbalance \(\phi_{\text{MMI}}\) has been obtained the previous section, we only have the \(\phi_{\text{CIPS}}\) to be curve-fitted in REMZI structure.

Although the MMI power imbalance is present in the MMI, we do not take that into account in our fitting. This is because the lineshape asymmetry is independent of the MMI power imbalance. The cross to bar transmission ratio in unloaded MZIs have been measured to be more than 10dB for our chosen MMI structure, suggesting MMI splitting ratio of almost unity. The extracted coupling induced phase values for all the three samples are respectively summarized in Table 1, Table 2, and Table 3 in the Appendix. Some of the fittings are shown in the Fig. 10, where the chosen sample is again the SiO\(_2\) PECVD coated sample. Note that the transmission contrast decreases when the gap separation is increased. This is evidently caused by resonant enhancement of cavity loss originated from critical coupling condition, which occurs when the coupling strength is equal to the cavity roundtrip loss \((r = a)\) [17]. The coupling condition has progressively moved from over-coupling (gap = 200nm) to near critical coupling (gap = 400nm), where the coupling strength is small enough to match the roundtrip loss. This also explains the apparent discontinuity in the resonance lineshape for 400nm gap separation, which originates from the almost discontinuous phase response of the ring transmittance near critical coupling condition [17].

Fig. 10. The measured Bar transmissions and their best fittings for 6μm coupler length for different gap separations.

The deduced coupling-induced phase shift \(\phi_{\text{CIPS}}\) for different coupler lengths and gap separations is shown in Fig. 11(a). It can be observed that \(\phi_{\text{CIPS}}\) is almost linear with coupler length \((L_c)\), as expected from simulation results. However, it should be noted that only one MZI arm length is side-coupled with the ring, which makes the term \(\exp(-i\beta L_c)\) cannot be neglected in the calculation. As previously discussed in Section 2, the modified coupling induced phase in this case is \(\phi_{\text{CIPS}} = (\beta - \beta)L_c - \phi\), where \(\phi\) originates from the self-coupling coefficient. It should be noted that, due to the double coupling nature in 1R2B in Fig. 5(c), the induced phase shift should be about twice the induced phase shift in REMZI (which is singly coupled). By comparing the induced phase shift in Fig. 5(c) with that in Fig. 11(a), we can see that the induced phases deduced from 1R2B and REMZI are of the same order.

It is also interesting to note in Fig. 11(a) that the sign of the induced phase changes as the gap separation is increased from 200nm to 400nm. Such a change of sign may be attributed to
the following reason. Figure 4 (a) indicates that $\phi_r$ is more sensitive to coupler length than to gap separation. Thus, at increasing gap separation, the $\phi_r$ starts to dominate the whole induced phase contribution because the term $(\beta - \bar{\beta})L_c$ exponentially decreases with increasing gap separation. Thus, this may give negative net induced phase in the REMZI devices. The demonstration of negative coupling induced phase confirms that the sign of CIPS is not necessarily positive-definite. We further observe that the corresponding coupling-induced wavevector, as deduced from the phase slope, also changes sign at the increasing gap separation. In principle, using conventional CMT treatment, the phase slope depends on the overlap integral between the square of electric field of one waveguide with the index contrast of the other waveguide, which suggests the integration should be positive-definite. Clearly, this is not the case in experiments. The phase slope, as deduced from linear regression (see solid lines in Fig. 11(a)), is plotted as a function of gap separation in Fig. 11(b), which seems to resemble a negatively offset exponential-like curve. Although the exponential tail is rather expected from exponential dependence of the overlap integral, the origin of negative offset in the coupling-induced wave vector still needs further investigation. A possible explanation for such a negative offset may be attributed to $-d\phi_r / dL_c$ term in $d\phi_{CIPS} / dL_c$.

![Graph](image)

Fig. 11. (a) The measurements of coupling induced phase shift for different gap separations and upper claddings. The PECVD was done at 300C with 600nm nominal thickness. The 2 points in 200nm gap in the first two panels are excluded because the measured coupling is too strong, which renders the curve-fitting no longer accurate. This is also the case for the rightmost panel for 400nm gap when the coupling is far too weak. (b) The phase slope as a function of gap separation for the three samples.

In 193nm DUV CMOS process used in this work, the OPE start to predominate in closely spaced waveguides when the gap is smaller than 150nm. At 200nm gap (circle markers) the positive values of the extracted phase shows that it could not have been caused by optical proximity effect. This is because optical proximity effect, in the technology used here, slightly decreases the waveguide width while slightly increase the gap separation, which in turn gives smaller effective index of the guided mode and hence results in negative phase. Thus, it can be deduced that the positive phase found here is attributed to coupling-induced effects, not OPE. Furthermore, at 300nm gap (square markers), the slope decreases to almost flat line, and finally at 400nm gap (triangular markers), the slope increases again but with negative sign. This is particularly interesting because the optical proximity effect is completely gone at 400nm gap, and yet the phase is negative. The same measurements and curve-fittings are conducted on the other two samples of different upper claddings (i-Line resist and air cladding), from which the same trend is found. The consistent trend in the measurements of three different samples suggests that our observation cannot have been caused by measurement noise or fabrication error.
4. Conclusions

We have characterized both numerically and experimentally coupling induced phase shifts in various optical structures for different coupler lengths and gap separations. Using FDTD simulations, coupling-induced phase shift (CIPS) from MZI structures and coupling-induced frequency shift (CIFS) from 1R2B simulations have been thoroughly calculated. While the shift of resonance wavelength in 1R2B is rather expected, the trend arising from MZI structure however remains interesting because of the change in sign of the coupling-induced wavevector. Experimentally, the characterization of CIPS was conducted in REMZI structures. In order to prevent inaccuracies and ambiguity in the curve-fitting, independent extractions of waveguide propagation loss, coupling coefficients, and MMI phase imbalances are done from separate structures, leaving the coupling-induced phase shift $\phi_{\text{CIPS}}$ the only parameter to be fitted in REMZI structures. The effect of fabrication errors manifested in optical proximity effects (OPE) and MMI phase imbalance have been taken into account in the fitting, and it is observed that the induced phase and the phase slope can be negative when the gap separation is increased. Based on 193nm DUV lithography process, the optical proximity effect only predominates when the gap is smaller than 150nm, suggesting that the induced phase shift could not have been caused by the OPE. This is also confirmed by the fact that the induced phase shift is positive, instead of negative (if OPE is assumed to exist).

Appendix: Best fit parameters of SiO$_2$, i-Line resist, and air cladded samples

Table 1. SiO$_2$ (PECVD Coated)

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<th>$g = 400\text{nm}$</th>
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<td>$\phi_0/\pi$</td>
<td>$n_1$</td>
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Table 2. i-Line Resist Coated

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Table 3. Bare Silicon

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Acknowledgment

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