<table>
<thead>
<tr>
<th><strong>Title</strong></th>
<th>Strength and behavior in shear of reinforced concrete deep beams under dynamic loading conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Author(s)</strong></td>
<td>Adhikary, Satadru Das; Li, Bing; Fujikake, Kazunori</td>
</tr>
<tr>
<td><strong>Date</strong></td>
<td>2013</td>
</tr>
<tr>
<td><strong>URL</strong></td>
<td><a href="http://hdl.handle.net/10220/9420">http://hdl.handle.net/10220/9420</a></td>
</tr>
<tr>
<td><strong>Rights</strong></td>
<td>© 2013 Elsevier B.V. This is the author created version of a work that has been peer reviewed and accepted for publication by Nuclear engineering and design, Elsevier B.V. It incorporates referee’s comments but changes resulting from the publishing process, such as copyediting, structural formatting, may not be reflected in this document. The published version is available at: DOI [<a href="http://dx.doi.org/10.1016/j.nucengdes.2013.02.016">http://dx.doi.org/10.1016/j.nucengdes.2013.02.016</a>].</td>
</tr>
</tbody>
</table>
Strength and Behavior in Shear of Reinforced Concrete Deep Beams under Dynamic Loading Conditions

Satadru Das Adhikary¹, Bing Li¹ and Kazunori Fujikake²

¹School of Civil and Environmental Engineering, Nanyang Technological University, Singapore 639798
²Department of Civil and Environmental Engineering, National Defense Academy, Yokosuka 239 8686, Japan

Abstract
Research on reinforced concrete (RC) deep beams has seen considerable headway over the past three decades; however, information on the dynamic shear strength and behavior of RC deep beams under varying rates of loads remains limited. This paper describes the experimental results of 24 RC deep beams with and without shear reinforcements under varying rates of concentrated loading. Results obtained serve as useful data on shear resistance, failure patterns and strain rates corresponding to varying loading rates. An analytical truss model approach proves its efficacy in predicting the dynamic shear resistance under varying loading rates. Furthermore, three-dimensional nonlinear finite element (FE) model is described and the simulation results are verified with the experimental results. A parametric study is then conducted to investigate the influence of longitudinal reinforcement ratio, transverse reinforcement ratio and shear span to effective depth ratio on shear behavior. Subsequently, two empirical equations were proposed by integrating the various parameters to assess the dynamic increase factor (DIF) of maximum resistance under varying rates of concentrated loading.
1. Introduction

During their service life, reinforced concrete structures in nuclear power plants (NPPs) may be subjected to various types of dynamic loading in terms of earthquakes, impact and blast. Nuclear power plants could be situated in earthquake-prone regions and hence may be subjected to dynamic loading due to ground oscillations. Moreover, impact loads may be imposed on RC structures in the NPPs from tornado-borne objects such as steel rods, steel pipes and wooden poles or from objects flown during an internal accident such as turbine failure or pipe breakage. Therefore, the global safety of existing or next generation NPPs depends on the resistance of internal and external reinforced concrete structures to the above-mentioned dynamic loadings. However, dynamic loading in terms of accidental and intentional blast loading is out of scope of this research. The main objective of the research is to improve the understanding of the response of RC deep beam under wide range of loading rates ($4 \times 10^{-4}$ to 2 m/s). Measurement of strain at the mid-span of longitudinal tensile reinforcements indicates that peak strain rate induced from these loading rates were in the range of $10^{-3}$ to 10/s. Strain rates induced from low ($4 \times 10^{-2}$ m/s) to medium ($4 \times 10^{-1}$ m/s) loading rates are analogous to the earthquake-range of strain rates whereas the same from high (2 m/s) loading rates are similar to impact-range of strain rate up to 10/s. Many researchers have investigated the behavior of RC slender beams under drop weight impact [1-4] and varying rates of concentrated loading [5-8]. However, limited literature exists pertaining to resistance and behavior of RC deep beams under varying rates of concentrated loading in displacement control. Displacement control is known to be indispensable for obtaining the entire post-peak response of RC structural elements. In varying loading rate testing, displacement control is even useful to reduce inertia effects during the pre-peak stage. With the aim of shedding some light in this area, an extensive experimental program was undertaken for RC deep beams subjected to a wide range of loading rates (Static: $4 \times 10^{-4}$, Low: $4 \times 10^{-2}$, Medium: $4 \times 10^{-1}$ and High: $2 \times 10^{0}$ m/s). This paper summarizes the details of the test program; including specimen & material properties, test setup, instrumentations and test procedures. The test results and major observations are presented and
critically discussed. Then, a simplified analytical method was developed using a truss model where
the beam was idealized by longitudinal, transverse and diagonal truss elements. This model
predicted the experimental dynamic shear resistance and the load-deformation response up to
ultimate resistance quite successfully. Moreover, three-dimensional nonlinear FE analysis is
performed using LS-DYNA software [9] for the numerical simulation of the RC deep beams under
varying rates of concentrated loading. Simulation results from FE model is verified with the
experimental results. FE models are deemed successful to predict the dynamic shear resistance and
shear failure mechanism of RC deep beams under the varying loading environments. This makes
finite element modeling an attractive alternative when high rates and/or impact testing facility is not
available or feasible. This is followed by a parametric study to investigate the influence of the
longitudinal reinforcement ratio, the transverse reinforcement ratio and the shear span to effective
depth ratio. Empirical equations are proposed in terms of various parameters to predict the dynamic
increase factor (DIF) of maximum resistance of RC deep beams (with and without transverse
reinforcements) under varying loading rates. DIF was calculated as a ratio of maximum resistance
at any loading rates (i.e. low, medium and high) to the corresponding static maximum resistance at
a loading rate of 4x10^{-4} m/s.

2. Outline of experiment

2.1 Specimen details, material characteristics and instrumentation

The test specimens consisted of twenty-four RC deep beams (total span length 1100 mm), having
rectangular cross-sections of 150 mm width and 250 mm depth. Shear-span to effective-depth ratio
of all the beams was 1.9. The layout of the longitudinal reinforcements, spacing of transverse
reinforcements and the measuring points (acceleration and strain for steel reinforcing bars) are
shown in Fig. 1. Longitudinal reinforcements consisted of four deformed steel bars of D22 mm
while D6 deformed bars were used for shear reinforcements. All the specimens had the same
degree (2.4%) of longitudinal reinforcements. 40 mm concrete cover was provided in all the
specimens. There was no transverse reinforcement in RC1.9_S0 whereas 0.42% and 0.84%
transverse reinforcements were provided in RC1.9_S42 and RC1.9_S84 respectively. Ready mix concrete was used to cast the specimens. The specified concrete compressive strength at 28 days was 40 MPa with maximum aggregate size of 20 mm for all specimens. 150 mm diameter and 300 mm high concrete cylinders were used to determine the compressive strength of concrete. It is to be noted that the concrete cylinders were cast together with the specimens using the same batch of concrete mix and were cured under identical conditions. Standard coupon tests were carried out to determine the mechanical properties of steel reinforcements. Yield strength of longitudinal and shear reinforcements were determined to be 371 MPa and 342 MPa respectively. All specimens were well instrumented to capture the load, displacements, accelerations and steel reinforcement strains. The instrumentation for this test program included a load cell, strain gauges and accelerometers. A load cell of capacity of 980kN and measuring frequency of 5 kHz was attached to the actuator to measure the load. 2mm strain gauges were installed in the mid-span of the longitudinal tensile reinforcements and in the mid-point of the two legs of the transverse reinforcements. Five accelerometers (capacity of 1000 times gravity and resonance frequency more than 70 kHz) were mounted for each test (except static loading) on the specimens to measure the accelerations for low, medium and high rates of concentrated loading. The midspan deflection was measured by laser-type variable displacement transducers (LVDTs) which have a measuring range of 80 mm and sampling rate of 50 kHz. Data from the sensors were collected by a digital data acquisition system which has a sampling rate of 100 Hz, 10 kHz, 100 kHz, and 200 kHz for static, low, medium and high loading rates respectively. The specific locations of the strain gauges and accelerometers are shown in Fig.1. A steel plate (40 mm thickness) was placed on the top of the beam at loading point to transfer well-distributed force to the specimens. The use of digital photography and high-speed video recording proved to be valuable in providing insights into the cracking patterns and failure modes of the beams, particularly the spalling of concrete, and the bending of the longitudinal reinforcing bars.
2.2 Shear resistance of deep beams in code provisions

Empirical formulae have been given in ACI 318-99 [10] and CIRIA Guide 2 [11] to calculate the shear capacities of RC deep beams. The formula from CIRIA Guide 2 could not be used herein as the effective span-to-depth ratio of beam is not less than 2. The shear resistance is calculated to be 101.5 kN, 136.4 kN and 171.2 kN for RC1.9_S0, RC1.9_S42 and RC1.9_S84 respectively as per ACI 318-99. Furthermore, ACI 318-08 [12] specifies strut-and-tie models for estimating the load carrying capacity of beams having shear span-to-overall depth ratio less than 2. According to this code, in simply supported deep beams, load transfer from loading point to supports through concrete struts. Thus, the shear resistance of RC1.9_S0 (without shear reinforcements) owing to failure of concrete strut would be around 95.7 kN. However, no guidelines are provided on the load-carrying mechanism in strut-and-tie models of ACI 318-08 for beam having shear reinforcements. Specimens satisfying the requirements of orthogonal shear reinforcement ratios as per equation A-4 in ACI 318-08, exhibit enhanced shear resistance by 25%. Therefore, for RC1_S42 and RC1_S84, shear resistance would be around 119.6 kN.

2.3 Testing plan

Two variables such as amount of shear reinforcement ratios (0%, 0.42%, and 0.84%) and rate of loading under displacement control [static (S), low (L), medium (M) and high (H)] were taken into consideration in designing the scheme of testing. There were three types of RC deep beams, distinguished in terms of shear reinforcement ratios (0%, 0.42% and 0.84%). For each type there were four pairs of specimens subjected to four different loading rates. A schematic diagram of the test setup is shown in Fig. 2. A digitally controlled servo-hydraulic test system was used in this study. The test frame had been designed to be stiff enough. The static test was completed in a few minutes while the duration of low, medium and high rate tests were 1 second, 100 milliseconds and 20 milliseconds respectively.
3. Experimental results and discussions

3.1 Load-midspan deflection curves

The load acting on specimens was measured by a load cell attached to the piston of the servo-hydraulic machine and the accelerations were measured by the accelerometers placed along the specimens. Inertia force was calculated by considering the linear variation of the accelerations between two adjacent accelerometers. Then, inertia force was eliminated from the measured load by load cell under high loading rate only to evaluate the true resistance of the RC deep beams. However, for low and medium loading rates the inertia effects are generally considered insignificant. Deflection was measured at the mid-span of the reinforced concrete beams. From Figs. 3-5, it is obvious that with an increasing loading rate, the ultimate shear resistance increases. The load carrying capacity of RC1.9_S0 increased by 50%, 100% and 130% respectively for low, medium and high rates of loading compared to the static load carrying capacity. Moreover, the residual strength for all other loading rates was more than that under static loading. Under medium and high rates, the load vs. mid-span deflection curve showed some indentation before reaching the peak load, indicating the development of cracking. For RC1.9_S4, the peak load increased by 11%, 33% and 53% respectively for low, medium and high rates of loading in comparison to the static peak load. However, RC1.9_S42 had sufficient residual strength under all loading rates as compare to RC1.9_S0. Similarly, for RC1.9_S84 load resistance increased by 14%, 27% and 35% respectively for low, medium and high rates of loading compared to the static case. Due to the presence of high amount of transverse reinforcement ratios (0.84%), RC1.9_S84 exhibited ductile nature for all loading rates which is clearly visible from load-midspan deformation curves. Fig. 6(a) shows the variation of the dynamic shear resistance of each specimen under four different loading rates. Furthermore, the ultimate shear strength of the RC deep beams increased with the increment of the shear reinforcement ratios. Moreover, it was observed that the slope of post-peak branch (i.e., descending branch) of the load-midspan deformation curve increased for all loading rates due to the increment of shear reinforcement ratio. DIF of maximum resistance under different loading rates is
shown in Fig. 6(b). From the above-mentioned figure, it could be enunciated that the influence of loading rate on the DIF of maximum resistance is more significant for specimens without transverse reinforcements as compared to those having shear reinforcements.

3.2 Stiffness

Stiffness is calculated by using the secant of the load vs. midspan deflection curve passing through the points of ultimate shear resistance of deep beam under different loading rates. Fig. 7(a) shows the variation in stiffness for all specimens under static, low, medium and high loading rates. Stiffness of RC1.9_S0 enhanced by around 7.4%, 17.3% and 36.7% when the loading rate was shifted from static to low, medium and high respectively. Moreover, for RC1.9_S42 the increment in stiffness was observed to be 22.2%, 24.5% and 25.1% respectively and for RC1.9_S84 the stiffness was augmented by 35.9%, 39.7% and 40% respectively. Shear reinforcements had some beneficial effect to increase the stiffness of deep beam under higher loading rates as compared to static. However, for specimens having shear reinforcements, the stiffness remains almost same for low, medium and high loading rates.

3.3 Strain at the midspan of longitudinal tensile reinforcements and corresponding strain rates

After analyzing the strain history data of longitudinal tensile reinforcements and converting it to stress history by multiplying it by the elastic modulus of steel, it was recognized that the yield stress of tensile reinforcing bars was increased as compared to the static yield stress (371 MPa) for medium and high loading rates. For RC1.9_S0, 25% and 46% (Fig. 7b) enhancement in yield stress in the tensile reinforcing bars was observed in the case of medium and high loading rates respectively. Similarly, it was calculated to be 32% and 39% more than the static yield stress of the tensile reinforcing bars of RC1.9_S42 for medium and high loading rates respectively. Again for RC1.9_S84, 40% and 62% increment in yield stress in the tensile reinforcing bars were observed for medium and high loading rates respectively. The peak strain rates (at the midspan of longitudinal tensile reinforcements) were computed to be 0.0059/s, 0.067/s, 0.59/s and 3.5/s for
static, low, medium and high rates of loading respectively for RC1.9_S0. Similarly, the peak strain rates were calculated to be 0.0068/s, 0.041/s, 0.48/s and 3.1/s for static, low, medium and high rates of loading respectively for RC1.9_S42. Again for RC1.9_S84, peak strain rates were computed to be 0.0027/s, 0.029/s, 0.46/s and 3.7/s for static, low, medium and high rates of loading respectively. Approximately, one order increment (approximately, 10 times) in the peak strain rate was observed as the loading rates progressed from low to high.

3.4 Crack profiles

The high-resolution digital photography produced images of the side surface of specimens that yielded detailed deflection and a visual record of the crack profiles under four different loading rates are illustrated in Fig. 8. RC1.9_S0 exhibited diagonal splitting and/or crushing strut failure under static loading. Moreover, arch-rib cracks were developed from the compressive zone and extended to the side of the beam. For low rates of loading, shear compression and crushing strut failure were noticed and few arch-rib cracks were formed at the top face of the specimen. Formation of diagonal strut and crushing of compression concrete was observed in medium loading rate, moreover bearing failure occurred in one side and arch-rib cracks were developed in the top face. For high loading rates, crushing strut failure including bearing failure at loading and support region was observed. Massive spalling of concrete in top, bottom and diagonal region and exposure of tensile and compressive reinforcements were also perceived for this loading case. For RC1.9_S42, diagonal splitting failure was observed under static loading, subsequently arch-rib cracks were started to develop from the top face of the beam. Under low loading rate, crushing strut and diagonal splitting failure were noticed. Bearing and crushing strut failure were perceived for medium loading rate. Crushing strut failure with massive spalling of concrete in strut region was observed in high rates with the exposure of top and bottom reinforcements. For RC1.9_S84, one-sided diagonal cracks and flexural cracks were formed in the initial stage of static loading. Spalling of concrete below the loading plate lead to bearing failure and subsequently crushing strut failure was observed. Moreover, few arch-rib cracks were also appeared in the top face of the beam.
Diagonal splitting and bearing failure under loading point was observed for low rate. For medium loading rate, diagonal splitting failure with crushing of compression concrete was observed. Crushing strut failure and spalling of concrete from compression and diagonal region were noticed under high loading rate.

4. Analytical truss model approach

Typically, reinforced concrete members are designed to resist the actions based on the assumption of linear strain distribution at a section referred to as Bernoulli hypothesis or beam theory. The region of structures where the Bernoulli hypothesis is valid referred to as a B region. Whereas, deep beam must be considered differently from a section design because the linear strain distribution assumption is no longer valid. ACI 318-08 [12] recommends using strut-and-tie models for designing the discontinuity regions of RC structures under static loading. However, it does not provide specific guidance on suitable strut-and-tie models for different structural elements with non-linear strain distribution at a section. Truss model has been used for analysis of RC beams, columns and their subassemblies subjected to monotonic or cyclic loading [13, 14]. However, truss model to analyze RC members under varying loading rates is scarce in literature. Therefore, efforts have been devoted herein to analyze the RC deep beam under varying loading rates by using well-known strut-and-tie or truss model approach. In this truss model, the RC beam is idealized by longitudinal, transverse and diagonal truss elements which duly consider the strain rate effects of concrete and reinforcing steel.

4.1 Stress-strain relationships of concrete and reinforcing steel

The constitutive relationships of concrete and reinforcing steel [7] for the truss model are shown in Fig. 9. The dynamic compressive strength of concrete is defined by the following equation as the function of strain rate:

\[ f'_{cd} = f_c \left( \frac{\varepsilon}{\varepsilon_s} \right)^{0.056[\log_{10}(\varepsilon/\varepsilon_s)]^{1.03}} \]  

(1)
where, \( f_{cd}' \) = dynamic compressive strength under strain rate \( \dot{\varepsilon} \), \( f_{c}' \) = compressive strength under static loading (MPa), \( \dot{\varepsilon}_s = 1.2 \times 10^{-5} \) (1/s).

And the dynamic elastic modulus of concrete is specified as the function of strain rate by the following equation:

\[
E_{0d} = E_0 \left( \frac{\varepsilon}{\varepsilon_s} \right)^{0.002 \log_{10}(\dot{\varepsilon}/\dot{\varepsilon}_s)}^{1.12}
\]  

(2)

where, \( E_0 = \) elastic modulus of concrete under static loading; \( E_0 = 3320 \sqrt{f_c} + 6900 \) (MPa); \( \dot{\varepsilon}_s = 1.2 \times 10^{-5} \) (1/s).

The relationship between the dynamic yield strength \( (f_{syd}) \) of reinforcing steel and the strain rate \( (\dot{\varepsilon}) \) is given as

\[
f_{syd} = f_{sys} \left( 1.202 + 0.040 \times \log_{10}(\dot{\varepsilon}) \right) \geq f_{sys}
\]  

(3)

where, \( f_{sys} = \) yield strength under static loading.

4.2 Modeling of truss elements

Strut-and-tie models are discrete representations of actual stress fields resulting from applied load and support conditions. These models represent the load-carrying mechanisms of structural members by approximating the flow of internal forces by means of struts representing the flow of compressive stresses and ties representing the flow of tensile stresses. A graphical presentation of the overall configuration of the truss model is illustrated in Fig. 10. Compressive members are shown in dotted lines whereas tensile members are shown in solid lines. For RC1.9_S0 (without shear reinforcements), two support points are connected by tension tie which represents the longitudinal reinforcing bars whereas the loading point to support point is connected by diagonal compression strut that represents the cracked concrete in compression. However, in the case of RC1.9_S42 and RC1.9_S84 (with shear reinforcements), the shear span is divided into two parts by placing one tension tie in the transverse direction at the mid of shear span. Although the same model has been used for RC1.9_S42 and RC1.9_S84, the cross sectional area of transverse tie was
being varied in accordance to the amount of transverse reinforcement ratio. Arch mechanism is duly considered in this truss model by introducing compression strut which directly connects the loading points with the supports [15].

Many researchers have opined that the truss model analogy tends to overestimate the shear resistance and stiffness of RC beams when the failure stress of the concrete strut is assumed to be uniaxial concrete compressive strength. Thus, the stress in each concrete element in truss model must be considered carefully. Here the suggestion from Schlaich et al. [16] is adopted for the effective compressive stress of concrete. Therefore, $0.85f_c'$ is taken for the top longitudinal concrete element. A compressive strength of $0.68f_c'$ is chosen for the diagonal compressive struts which accounts for the detrimental effects of transverse tensile stresses to the compressive strength. Moreover, $0.35E_0$ is specified for the elastic modulus of diagonal struts for taking into account of cracked concrete. For the longitudinal bottom chord elements, the properties of longitudinal bottom reinforcements are defined and successively for transverse tension ties, the properties of shear reinforcements are specified.

Although the compression struts are idealized as a uniform cross sectional member, they generally vary in cross section along their length by the spreading of compression stress acting on them. The spreading of compression stress results in transverse tension stress in the compression struts, which may cause cracking. The transverse tension stress, therefore, may play a very important role under high loading rate, because the strain rate effect on concrete strength is more significant in tension than in compression. Thus, it may appear that the increase in the transverse tension strength of concrete with increasing the strain rate makes the thickness of the concrete compression strut bigger.

The relationship between the thickness of the compression strut and the strain rate is looked into over the experimental data and the following equation is obtained.

$$t_{cd} = t_c \left( \frac{\dot{\varepsilon}}{\dot{\varepsilon}_t} \right)^{0.006[\log_{10}(\dot{\varepsilon}/\dot{\varepsilon}_t)]^{1.05}}$$

(4)
where, $t_{cd}$ = thickness of compression strut at any strain rate $\dot{\varepsilon}$, $t_c$ = thickness of compression strut at $\dot{\varepsilon}_a = 1.2 \times 10^{-5}$ (1/s).

### 4.3 Verification of truss model approach

To verify the proposed truss model approach, a comparison of dynamic shear resistance and load-midspan deflection response under varying loading rates acquired from test results is demonstrated. Using the methodology as pioneered previously, the truss model approach with established member properties was analyzed by the LS-DYNA solver. Fig. 11 shows that the proposed truss model predicts the experimental dynamic shear resistance of RC1.9_S0 and RC1.9_S84 with a reasonable accuracy. Moreover, the truss model is also validated by plotting the analytical results in terms of load versus midspan deflection response and comparing it to the experimental results of RC1.9_S42 under static, low, medium and high loading rates as shown in Fig. 12. Hence, the proposed truss model demonstrates good reliability in reasonable matching of test and analysis curves in terms of ultimate resistance and stiffness. However, to predict the post-peak descending branch of load versus midspan deflection responses, more rigorous model can be developed.

### 5. Finite element model

In the present study, finite element code LS-DYNA was employed because of its proven effectiveness in geometric modeling and analysis capability under high loading rates. The adoption of 3D analysis arose from the need to account for the effects of inertia and the nonlinear behavior of concrete and steel. The description of modeling includes structural geometry, boundary conditions, application of loads, and relevant material models.

#### 5.1 Structural modeling

Fig. 13 shows the three-dimensional FE model of tested RC deep beams. Eight node solid hexahedron elements with a single integration point were used to represent concrete while beam elements (2-node Hughes-Liu beam element formulation with 2x2 Gauss quadrature integration) were used to model steel reinforcing bars. Comparing the pre-analysis results with the experimental ones, the mesh geometry was chosen as follows considering the fact that the mesh aspect ratio is
smaller than 1.5. A mesh size of 25 mm was used to create the solid element in the span direction of RC beam whereas the mesh configuration in sectional directional comprised: (1) in the depth direction of RC beam, 8 elements for concrete between the top and bottom longitudinal reinforcements and 2 elements for concrete cover; (2) in the width direction, 4 elements between two adjoining longitudinal reinforcements and 2 elements for concrete cover. In total, approximately, 6900 nodal points and 4220 elements were used to model the whole structures. Numerical convergence study showed that further decrease in mesh size had insignificant effect on the accuracy of results, while increasing the duration of analysis. The mesh discretization was executed in such a way that the reinforcement nodes coincided with the concrete nodes. The steel reinforcing bars were modeled explicitly using beam elements connected to the concrete mesh nodes. The nodes that linked the concrete and reinforcement mesh were shared and therefore unable to slip. Due to this assumption of complete compatibility of strains between the concrete and steel nodes, they formed a perfect bond.

5.2 Boundary conditions, application of load and contact algorithm

To simulate the actual experimental conditions, the beams were supported on two rigid cylinders made of solid elements. Constraints were defined to the support cylinder, so that it could rotate about its own longitudinal axis but would not be able to translate. Displacement was prescribed in the rigid loading plates located at the mid-span of the RC beam. The rigid loading plate was allowed to move only in the vertical direction. The prescribed displacement was linear, going from zero displacement to 40 mm displacement under certain time duration, depending on the desired rate of loading. The corresponding applied load due to the prescribed displacement was then determined by monitoring the vertical reaction forces at the concrete nodes in contact with the support solid cylinders. The algorithm CONTACT_AUTOMATIC_SINGLE_SURFACE in LS-DYNA was used to model the contact between the support cylinder, loading plate and RC beam. This algorithm automatically generates slave and master surfaces and uses a penalty method where nominal interface springs are used to interpenetration between element and surfaces. The interface
stiffness is computed as a function of the bulk modulus, volume and face area of the elements on the contact surface.

5.3 Material characteristics

Concrete
Material type 072 R3 (MAT_CONCRETE_DAMAGE_REL3), was the third release of Karagozian and Case (K&C) concrete model and was utilized in this study. It includes implementation of a third, independent yield failure surface, removal of tension cutoff and extension of the plasticity model in tension; shift of the pressure cutoff; implementation of three invariant formulation for the failure surfaces; determination of triaxial extension to triaxial compression ratio as a function of pressure; shear modulus correction and implementation of a radial path strain rate enhancement [17]. The model has a default parameter generation function based on the unconfined compressive strength of the concrete [18]. A detailed description of the concrete material model was provided by Wu et al. [19].

Steel
The steel reinforcement bars (longitudinal and shear reinforcements) within the beam were modeled as a strain sensitive uniaxial elastic-plastic material to account for its strain rate sensitivity as well as stress-strain history dependence. Material model PIECEWISE_LINEAR_PLASTICITY (MAT_024) from LS-DYNA was employed in this study to incorporate the strain rate effect. The expressions proposed by Malvar [20] on strain rate effect were utilized in this study. Wu et al. vividly depicted the details of the steel material modeling.

Loading plate, support roller and plate
MAT_RIGID (MAT_020) was used from the LS-DYNA material library to model the loading plates, support rollers and plates. Realistic values of Young’s modulus and Poisson’s ratio of the rigid material need to be defined to avoid numerical problems in contact. Young’s modulus and Poisson’s ratio of steel material were used for the rigid material in the numerical simulation.
6. Verification of finite element model

Numerical simulation results of reinforced concrete beams subjected to four different loading rates were calibrated with the experimental results.

6.1 Load vs. midspan deflection

To simulate the same experimental conditions, the beams were analyzed in displacement control in LS-DYNA. Table 1 presents the comparisons of the numerical simulations results with experimental dynamic shear resistance of RC deep beams under four different rates of loading conditions. It was found that the mean, and coefficient of variation, of the predicted to experimental shear resistance was 1.04 and 0.04 respectively, showing a good correlation between the FE simulation and experimental results.

6.2 Crack profiles on side surface of RC deep beams

The damage of the beams obtained from numerical simulation is shown by plotting the fringes of effective plastic strain. These effective plastic strain contours reveal the strain localization where failure propagates. Fig. 14 shows the crack pattern of the beams of the present study compared with the damage plot of numerical simulation results. From these comparisons, it can be seen that the damage plot of numerical simulation results can capture the experimental crack profiles under varying loading quite satisfactorily.

7. Parametric study

7.1 Numerical simulation case studies

After verification of the FE model against the experimental results, this section presents a parametric investigation to capture more information about the behavior of RC deep beams under varying loading rates. The response was studied by varying key parameters such as shear span to effective depth ratios (a/d: 1.4, 1.9 and 2.26), longitudinal reinforcement ratios (ρ: 2.4% and 1.27%), transverse reinforcement ratios (ρ: 0%, 0.42%, and 0.84%) and loading rates (δ: 4x10⁻⁴, 4x10⁻², 4x10⁻¹ and 2x10⁰ m/s). Fig. 15 illustrates the general schematic diagram of the beams and Table 2 summarizes the specimen characteristics of the simulation matrix.
7.2 Effect of longitudinal reinforcement ratio

Fig. 16 illustrates the effect of the longitudinal reinforcement ratio on DIF of maximum resistance under varying loading rates. There were three types of beams in terms of their slenderness ratio (a/d - 1.4, 1.9 & 2.26). Fig. 16 shows that for specimens having a/d - 2.26 and without transverse reinforcements, there is no significant difference in DIF whereas for deep beam having a/d - 1.4 and 1.9, DIF increases with the increment of longitudinal reinforcement ratio. However, specimens having transverse reinforcements, DIF was observed to be on the higher side with low longitudinal reinforcement ratio (1.27%) of the RC beams at low, medium and high loading rates. This means that although the ultimate shear resistance is low in RC deep beams with low longitudinal reinforcement ratio for all loading rates, the rate of increment of ultimate load carrying capacity was higher as compared to beams having high amount of longitudinal reinforcements.

7.3 Effect of transverse reinforcement ratio

Fig. 17 shows the influence of transverse reinforcement on DIF of maximum resistance under varying loading rates. Specimens without transverse reinforcements but with a shear span to effective depth ratio between 1 and 2 show a higher DIF as compared to specimens having transverse reinforcements for loading rates. However, for specimens of shear span to effective depth ratio of 2.26, this difference is not so significant. Nevertheless, large transverse reinforcement ratios (0.42% and 0.84%) in the RC deep beams resisted the catastrophic failure mode (i.e., huge cracking and spalling of concrete, and exposure and bending of longitudinal reinforcements) under loading rates greater than static loading by providing additional confinement to the core concrete and supplementing lateral restraint capacity against buckling of the longitudinal reinforcements. So, transverse reinforcement does not have important influence on DIF, but significantly affects the deformation ductility, failure mode and ultimate shear resistance of RC deep beam.

7.4 Effect of shear span to effective depth ratio
In this study, the cross-section was kept constant for all the deep beam specimens. An increase or decrease in beam shear span to effective depth ratio meant a corresponding increase or decrease in span length. The effect of shear span to effective depth ratio on the DIF of maximum resistance of the RC deep beam is shown in Fig. 18. For specimens without transverse reinforcements, it is tough to comment on the influence of shear span to effective depth ratio on DIF. However, for specimens with transverse reinforcement, DIF increases with the increment of shear span to effective depth ratio for all loading rates, except for a few cases of lower amount of longitudinal reinforcement ratio.

7.5 Proposed equations for estimating DIF of maximum resistance of RC deep beams at a wide range of loading rates

Parametric study through numerical simulation revealed the significance of parameters that affect the DIF of RC deep beams under varying loading rates. Two empirical equations were proposed through multivariable regression analysis in terms of various parameters to predict the DIF. The empirical equation for beams without transverse reinforcements is expressed as follows:

\[
DIF = \left(0.45 + 0.09 \rho_g + 0.48 \left(\frac{a_f}{d}\right)\right) e^{0.30 - 0.05 \rho_g - 0.05 \left(\frac{a_f}{d}\right)}
\]  

The other empirical equation for the case of beams having transverse reinforcements is depicted as follows:

\[
DIF = \left[1.25 - 0.04 \rho_g - 0.13 \rho_v + 0.05 \left(\frac{a_f}{d}\right)\right] e^{0.22 - 0.03 \rho_g - 0.17 \rho_v + 0.03 \left(\frac{a_f}{d}\right)}
\]

Examples presenting the comparison of proposed equations with the numerical analysis results are presented in Fig. 19. The solid and dotted lines in each plot denote the proposed equations, while the scatter data points represent the numerical analysis results. After inspecting the plots, it can be addressed that for most of the cases, the proposed equations matched the numerical analysis results within an acceptable accuracy.
8. Conclusions

Based on the results presented in this study, the following conclusions were drawn:

1. The dynamic shear resistance of RC deep beams was found to increase as the loading rates were increased. The ultimate shear resistance also increased with the increment of shear reinforcement ratios. Shear reinforcements had some beneficial effect in increasing the stiffness of deep beams under higher loading rates as compared to static loading. However, for specimens with shear reinforcements, the stiffness remains almost same for loading rates higher than static. Moreover, it was observed that the slope of the post-peak branch (i.e., descending branch) of the load-midspan deformation curve increased with increasing shear reinforcement ratio. The influence of loading rate on the DIF of maximum resistance is more significant for beams without transverse reinforcements as compared to those having shear reinforcements. Peak strain rate was amplified in one order of magnitude (approximately, 10 times) as the loading rates progressed from low to high.

2. Shear compression and/or diagonal splitting type failure was observed in all specimens irrespective of applied concentrated loading rates at their midspan, however much ductile behavior was perceived due to the increment of shear reinforcement ratios under any loading rate.

3. The simplified truss model analysis results demonstrated that it could predict the experimental dynamic shear resistance and the load-deformation response up to ultimate resistance under varying loading rates quite satisfactorily.

4. Transverse reinforcement does not have important influence on DIF of maximum resistance, but significantly affect the deformation ductility, failure mode and ultimate shear resistance of RC deep beam. Moreover, high amount of transverse reinforcement ratio resisted the catastrophic failure mode under loading rate greater than static loading by providing additional confinement to the core concrete.

5. Two empirical equations were proposed in terms of various parameters by multivariable
regression analysis to predict the DIF of maximum resistance of the RC deep beams (with and without transverse reinforcements) under varying loading rates. Comparison of the proposed equations with numerical analysis results showed that the proposed curves can delineate the inclination of the DIF under varying loading rates within a reasonable accuracy. Therefore, these two equations can be useful in estimating the DIF of RC deep beam at a wide range of loading rates during preliminary investigation. However, future experimental investigation of RC deep beams under varying loading rates is indeed necessary and should consider other parameters such as various grades of concrete and steel reinforcing bars for further incorporation in the proposed equations.
References


5. Mutsuyoshi, H., and Machida, A. “Properties and Failure of Reinforced Concrete Members Subjected to Dynamic Loading,” Transactions of the Japan Concrete Institute, V. 6, 1984, pp. 521-528.


10. ACI Committee 318, “Building Code Requirements for Structural Concrete (ACI 318-99) and Commentary,” American Concrete Institute, Farmington Hills, Mich., 1999.


12. ACI Committee 318, “Building Code Requirements for Structural Concrete (ACI 318-08) and Commentary,” American Concrete Institute, Farmington Hills, Mich., 2008.


List of Tables

Table 1  Comparison of dynamic shear resistance of RC deep beam by FE simulation and experimental results
Table 2  Specimen characteristics of the simulation matrix
## List of Figures

<table>
<thead>
<tr>
<th>Fig.</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Dimensions of RC deep beams, layout of reinforcements, location of strain gauges and accelerometers</td>
</tr>
<tr>
<td>2</td>
<td>Test setup</td>
</tr>
<tr>
<td>3</td>
<td>Load-midspan deflection diagram for RC1.9_S0 under four different loading rates</td>
</tr>
<tr>
<td>4</td>
<td>Load-midspan deflection diagram for RC1.9_S42 under four different loading rates</td>
</tr>
<tr>
<td>5</td>
<td>Load-midspan deflection diagram for RC1.9_S84 under four different loading rates</td>
</tr>
<tr>
<td>6</td>
<td>(a) Variation of dynamic shear resistance; (b) DIF of maximum resistance for different loading rates</td>
</tr>
<tr>
<td>7</td>
<td>(a) Stiffness of RC deep beams under static, low, medium and high loading rate; (b) Strain histories of RC1.9_S0 under medium and high loading rate</td>
</tr>
<tr>
<td>8</td>
<td>Cracking patterns of RC deep beam under varying loading rates</td>
</tr>
<tr>
<td>9</td>
<td>Constitutive relationships of concrete and reinforcing steel</td>
</tr>
<tr>
<td>10</td>
<td>Truss model</td>
</tr>
<tr>
<td>11</td>
<td>Comparison of predicted (truss model) and experimental dynamic shear resistance of RC1.9_S0 &amp; RC1.9_S84 under varying loading rates</td>
</tr>
<tr>
<td>12</td>
<td>Comparison of predicted (truss model) and experimental load-midspan deflection responses of RC1.9_S42 under varying loading rates</td>
</tr>
<tr>
<td>13</td>
<td>Finite element model of RC deep beam</td>
</tr>
<tr>
<td>14</td>
<td>Comparison of cracking pattern of RC deep beams under: (a) Static; (b) Low; (c) Medium; and (d) High loading rates</td>
</tr>
<tr>
<td>15</td>
<td>General schematic diagram of RC deep beam</td>
</tr>
<tr>
<td>16</td>
<td>Effect of longitudinal reinforcement ratios on DIF of maximum resistance of RC deep beams: (a) a/d-1.4; (b) a/d-1.9; (c) a/d-2.26</td>
</tr>
<tr>
<td>17</td>
<td>Effect of transverse reinforcement ratios on DIF of maximum resistance of RC deep beams: (a) a/d-1.4; (b) a/d-1.9; (c) 2.26</td>
</tr>
<tr>
<td>18</td>
<td>Effect of shear span to effective depth ratios on DIF of maximum resistance of RC deep beams: (a) ρ_s-2.4%; (b) ρ_s-1.27%</td>
</tr>
<tr>
<td>19</td>
<td>Comparisons of numerical results with proposed equations: (a) without transverse reinforcements; (b) with transverse reinforcements</td>
</tr>
</tbody>
</table>
Table 1 Comparison of dynamic shear resistance of RC deep beam by FE simulation and experimental results

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Loading rate (m/s)</th>
<th>Dynamic shear resistance (kN)</th>
<th>LS-DYNA</th>
<th>LS-DYNA/Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>RC1_S0</td>
<td>0.0004</td>
<td>170.1</td>
<td>175.6</td>
<td>1.03</td>
</tr>
<tr>
<td></td>
<td>0.04</td>
<td>246.5</td>
<td>239.2</td>
<td>0.97</td>
</tr>
<tr>
<td></td>
<td>0.4</td>
<td>290.5</td>
<td>293.4</td>
<td>1.01</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>353.6</td>
<td>366</td>
<td>1.04</td>
</tr>
<tr>
<td>RC1_S42</td>
<td>0.0004</td>
<td>271.9</td>
<td>300.2</td>
<td>1.10</td>
</tr>
<tr>
<td></td>
<td>0.04</td>
<td>300.6</td>
<td>324</td>
<td>1.08</td>
</tr>
<tr>
<td></td>
<td>0.4</td>
<td>361.3</td>
<td>375.2</td>
<td>1.04</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>417</td>
<td>421.5</td>
<td>1.01</td>
</tr>
<tr>
<td>RC1_S84</td>
<td>0.0004</td>
<td>331.8</td>
<td>338</td>
<td>1.02</td>
</tr>
<tr>
<td></td>
<td>0.04</td>
<td>378.2</td>
<td>381.7</td>
<td>1.01</td>
</tr>
<tr>
<td></td>
<td>0.4</td>
<td>420.75</td>
<td>422.7</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>446.7</td>
<td>459.3</td>
<td>1.03</td>
</tr>
</tbody>
</table>

Mean 1.03
C.O.V 0.03

Table 2 Specimen characteristics of the simulation matrix

<table>
<thead>
<tr>
<th>Beam mark</th>
<th>Beam Specifications</th>
<th>Shear span to effective depth ratio</th>
<th>Longitudinal reinforcement</th>
<th>Shear reinforcement</th>
<th>Loading rates (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>f′ (MPa) h (mm) b (mm) d (mm) a (mm) L (mm) a/d</td>
<td>Top Diameter (mm) Ratio (%) fy (MPa)</td>
<td>Bottom Diameter (mm) Ratio (%) fy (MPa)</td>
<td>Diameter (mm) Ratio (%) fy (MPa)</td>
<td></td>
</tr>
<tr>
<td>RC1.4_50_1</td>
<td>272 2.4 272 2.4</td>
<td>371 1.27 371 R6 4.2</td>
<td>342 4×10⁴</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RC1.4_542_1</td>
<td>272 2.4 272 2.4</td>
<td>371 1.27 371 R6 4.2</td>
<td>342 4×10⁴</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RC1.4_543_1</td>
<td>272 2.4 272 2.4</td>
<td>371 1.27 371 R6 4.2</td>
<td>342 4×10⁴</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RC1.4_50_2</td>
<td>272 2.4 272 2.4</td>
<td>371 1.27 371 R6 4.2</td>
<td>342 4×10⁴</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RC1.4_542_2</td>
<td>272 2.4 272 2.4</td>
<td>371 1.27 371 R6 4.2</td>
<td>342 4×10⁴</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RC1.4_543_2</td>
<td>272 2.4 272 2.4</td>
<td>371 1.27 371 R6 4.2</td>
<td>342 4×10⁴</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RC1.9_50_1</td>
<td>272 2.4 272 2.4</td>
<td>371 1.27 371 R6 4.2</td>
<td>342 4×10⁴</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RC1.9_542_1</td>
<td>272 2.4 272 2.4</td>
<td>371 1.27 371 R6 4.2</td>
<td>342 4×10⁴</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RC1.9_543_1</td>
<td>272 2.4 272 2.4</td>
<td>371 1.27 371 R6 4.2</td>
<td>342 4×10⁴</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RC1.9_50_2</td>
<td>272 2.4 272 2.4</td>
<td>371 1.27 371 R6 4.2</td>
<td>342 4×10⁴</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RC1.9_542_2</td>
<td>272 2.4 272 2.4</td>
<td>371 1.27 371 R6 4.2</td>
<td>342 4×10⁴</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RC1.9_543_2</td>
<td>272 2.4 272 2.4</td>
<td>371 1.27 371 R6 4.2</td>
<td>342 4×10⁴</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: T-Deformed bar; R-Plain bar; fy YIELD strength of reinforcements; f′-Compressive strength of concrete; *Experimental specimens
Fig. 1. Dimensions of RC deep beams, layout of reinforcements, location of strain gauges and accelerometers

Fig. 2. Test setup
Fig. 3. Load-midspan deflection diagram for RC1.9_S0 under four different loading rates

Fig. 4. Load-midspan deflection diagram for RC1.9_S42 under four different loading rates

Fig. 5. Load-midspan deflection diagram for RC1.9_S84 under four different loading rates
Fig. 6. (a) Variation of dynamic shear resistance; (b) DIF of maximum resistance for different loading rates

Fig. 7. (a) Stiffness of RC deep beams under static, low, medium and high loading rate; (b) Strain histories of RC1.9_S0 under medium and high loading rate
Fig. 8. Cracking patterns of RC deep beam under varying loading rates

Fig. 9. Constitutive relationships of concrete and reinforcing steel

Concrete
Reinforcing steel

Fig. 10. Truss model
Fig. 11. Comparison of predicted (truss model) and experimental dynamic shear resistance of RC1.9_S0 & RC1.9_S84 under varying loading rates

Fig. 12. Comparison of predicted (truss model) and experimental load-midspan deflection responses of RC1.9_S42 under varying loading rates
Fig. 13. Finite element model of RC deep beam

Fig. 14. Comparison of cracking pattern of RC deep beams under: (a) Static; (b) Low; (c) Medium; and (d) High loading rates
Fig. 15. General schematic diagram of RC deep beam

(a) \( \rho_v = 0\% \), \( a/d = 1.4 \)

(b) \( \rho_v = 0\% \), \( a/d = 1.9 \)

\( \rho_g = 2.4\% \), \( \rho_g = 1.27\% \)
Fig. 16. Effect of longitudinal reinforcement ratios on DIF of maximum resistance of RC deep beams: (a) a/d-1.4; (b) a/d-1.9; (c) a/d-2.26
Fig. 17. Effect of transverse reinforcement ratios on DIF of maximum resistance of RC deep beams: (a) a/d=1.4; (b) a/d=1.9; (c) 2.26
DIF of maximum resistance

Loading rate, \( \delta \) (m/s)

\( \rho_g = 2.4\% , \rho_v = 0\% , \)
\( a/d = 1.4 \)
\( a/d = 1.9 \)
\( a/d = 2.26 \)

\( \rho_g = 1.27\% , \rho_v = 0\% , \)
\( a/d = 1.4 \)
\( a/d = 1.9 \)
\( a/d = 2.26 \)

\( \rho_g = 2.4\% , \rho_v = 0.84\% , \)
\( a/d = 1.9 \)
\( a/d = 2.26 \)

\( \rho_g = 1.27\% , \rho_v = 0.42\% , \)
\( a/d = 1.9 \)
\( a/d = 2.26 \)
Fig. 18. Effect of shear span to effective depth ratios on DIF of maximum resistance of RC deep beams: (a) $\rho_g = 2.4\%$; (b) $\rho_g = 1.27\%$

Fig. 19. Comparisons of numerical results with proposed equations: (a) without transverse reinforcements; (b) with transverse reinforcements