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Analytical Model on the Bond Stress-Slip Relationship between Steel Reinforcement and Concrete for RC Beam-Column Joints

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Abstract. There are no conventionally accepted failure criteria for progressive collapse, and often times, deflection of affected beams over the "missing column" are often used as performance criteria. However, when simulating the deformation behaviour and the strength of reinforced concrete (RC) framed structures for progressive collapse analysis, besides the flexural deformations, the so-called "fixed end" rotation induced by the longitudinal bar slip at the beam-column ends connected to the joints can be significant and result in additional lateral deformations not accounted for in the initial analysis. Hence, it is important to quantify the deformations arising from fixed end rotations. Several bond stress-slip relationships between steel reinforcement and concrete were previously proposed in the literature. In the present work, their merits and demerits are discussed in terms of application limitation. To address the limitations of previous bond-slip models, a new analytical model based on the bond stress integration along the bar stress propagation length is proposed to predict the bar-slip behaviour in the RC beam-column joints. The proposed analytical model on the bond stress-slip relationship is validated against experimental studies from the literature and is shown to be simple and reliable.

1 Introduction

Previous experimental studies [1-4] under generalized excitations showed that besides the flexural deformation, significant additional deformation (Fig. 1) was caused by the fixed end rotations due to slippage of longitudinal steel reinforcement at the beam-column ends. Other than the total deformations, validated numerical simulations by Shima et al. [4] showed that the behaviour of RC members with and without bond action is quite different from each other in terms of the predicted structural ductility. Besides, the bar slip behaviour also caused significant stiffness degradation in the load-deformation relationships of moment-resisting frames [1]. In some extreme situations, brittle failure due to sudden loss of bond action between reinforcing bars and concrete in anchorage zones may cause severe local damage or even collapse of the whole structure [1]. Therefore, bar slip behaviour should be incorporated if accurate predictions are needed when analyzing RC beam-column structures for progressive collapse.

In the previous studies, several local bond stress-slip relationships between steel reinforcement and concrete subjected to axial pullout have been proposed and can be generally classified in terms of bond stress distribution. One of them is based on piecewise non-uniform distributions [1, 2, 4-11] found in the literature. A common demerit of this type of idealization is that the embedment length has to be divided into many segments, upon which iterative calculations have to be performed to satisfy the steel stress-strain relationship, the local equilibrium between bond force and bar force, and boundary conditions for different segments of embedment lengths. Consequently, the analytical model based on piecewise non-uniform bond stress distributions is far too complex and requires too much computational time when analyzing large-scale RC framed structures, even though some of them are capable of predicting well the test results. The other one is based on a piecewise uniform distribution [3, 12, 13], that is, bond stress distribution is idealized as two segments of uniform bond stress along the whole embedment length to represent mechanical bond and frictional bond, denoted as τ_E and τ_Y, respectively, as shown in Fig. 2

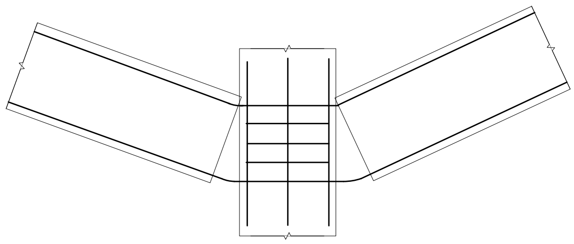

Fig. 1 Additional deformation resulting from local bar slip at the "fixed end condition"

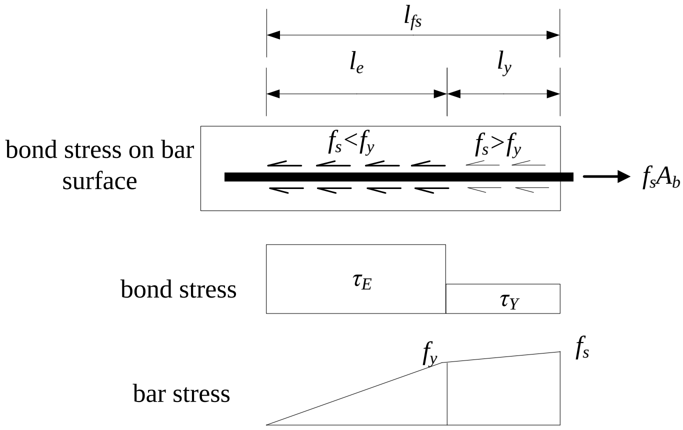

Fig. 2 Assumed bond stress and bar stress distribution for a reinforcing bar anchored in a joint

Nevertheless, some shortcomings of these models with piecewise uniform bond stresses [3, 12] have to be addressed as follows. Firstly, the boundary conditions considered are limited to a few cases. Consequently, these models are only capable of predicting some of the failure modes, such as fracturing of steel reinforcement with a sufficient embedment length, or bond slip failure of steel reinforcement with an insufficient embedment length. Other possible failure modes such as fracturing of steel reinforcement with an insufficient embedment length are not considered. Secondly, these analytical models were proposed for seismic loading. Since the focus in the present study is on progressive collapse analysis where only monotonic loading condition is considered, column removals are taken at the beginning of progressive collapse and the original reaction forces at the removed columns are assumed to be the applied loading. Therefore, the proposed bond stress distributions in these models have to be validated against detailed bar-slip experiments subjected to monotonic loading.

Based on the above discussions on the merits and demerits of the previous analytical models for axial pullout tests, the ideal bond stress-slip relationship for large-scale structures should be simple and reliable, incorporating important factors associated with nonlinearity of the steel constitutive model and different embedment lengths. Therefore, as the main novelty of the present study, such an analytical model is proposed based on a piecewise uniform bond stress distribution. Almost all the important factors, including bond deterioration, pullout failure, steel post-yielding range, steel fracturing, various bar embedment lengths and boundary conditions which have not been systematically incorporated in the previous analytical models, will be considered in the current derivations based on the concept of *stress propagation length* in Section 2. In the derivation, steel constitutive law, equilibrium of bond and bar forces, and compatibility of deformation along the steel reinforcement are accounted for. In Section 3, the proposed analytical model is validated against experimental studies considering axial pullout action. Finally, some brief conclusions are summarized for the proposed analytical model.

2 Analytical model on the bond stress-slip relationship under axial pullout action

In this paper, the term *slip* is defined as the relative displacement between the main steel reinforcement and the surrounding concrete. Only the relative deformation along the longitudinal direction of the steel reinforcement is considered, while the contact of steel reinforcement with concrete in the transverse direction is assumed to be perfect. For the surrounding concrete, it is assumed to be well confined by sufficient steel reinforcement or with sufficient cover (concrete cover $\geqq 5d_b$ and clear spacing between bars $\geqq 10d_b$ as stipulated in [14], where d_b is the bar diameter). Thus, no splitting failure is considered in the proposed analytical model. In fact, Alsiwat and Saatcioglu [13] reported that pullout cone failure does not occur at the beam-column joints with transverse reinforcement under monotonic loading. Moreover, compared with concrete, the area of steel reinforcement is small and the steel strain is sufficiently large, so that it is commonly assumed that there is negligible influence of concrete deformation on longitudinal bar slip.

In the present study, only the bond-slip behaviour under tension is of interest. For sufficient embedment in which bar fracturing failure occurs, the value of $1.4\sqrt{f'_c}$ (the average of suggested values by Lowes and Altoontash [12] and Sezen and Moehle [3] and the shortcomings of these two models have been clarified in Section 1) is taken for τ_E, and the value of τ_Y is conservatively taken as $0.4\sqrt{f'_c}$. For insufficient embedment in which pullout failure probably occurs, the value of $2.5\sqrt{f'_c}$ is selected for τ_E, which is the maximum bond stress proposed by Eligehausen et al. [1] based on their experimental study and has been adopted by the CEB-FIP Model Code 2010 [14]. As for τ_Y, a relatively larger value of $0.8\sqrt{f'_c}$ is taken to reflect an increase of embedment length due to penetration at the unloaded end under a large strain.

However, it should be noted that the effective embedment length of a steel reinforcement is not necessarily taken as the actual embedment length of steel reinforcement to resist slip. In reality, the effective length of steel reinforcement to resist a bar slip should depend on the magnitude of the applied pullout load and the bond condition. Therefore, a more realistic concept called *stress propagation length* is proposed here to describe the propagation of bar stress along the steel reinforcement with variations of applied load and bond deterioration. By using the proposed concept of stress propagation length, the bar-slip resistance can be obtained by integrating the bond stress over the circumferential area and also along the effective length of steel reinforcement. Such a calculation approach is able to overcome the disadvantages of previous analytical models [3, 12], such as the predictions of bar-slip behaviour with an insufficient embedment length of steel reinforcement.

Firstly, the bond force and the bar force for an infinitesimal element dx with respect to the unloaded end are in an equilibrium state, that is, $f_s\, A_b = \tau_E\, \pi\, d_b dx$. In addition, with an assumed bi-linear stress-strain relationship, the steel reinforcement strain is given in Eq. (1).

$$\varepsilon_s = \begin{cases} f_s\ / E_s & when\ f_s \le f_y \\ f_y/E_s + (f_s - f_y)/E_h & when\ f_s > f_y \end{cases} \tag{1}$$

where f_s is the bar stress at the point of interest, f_y is the steel yield strength, E_s is the steel Young's modulus, E_h is the hardening modulus, A_b and d_b are the cross-sectional area and diameter of steel reinforcement, respectively.

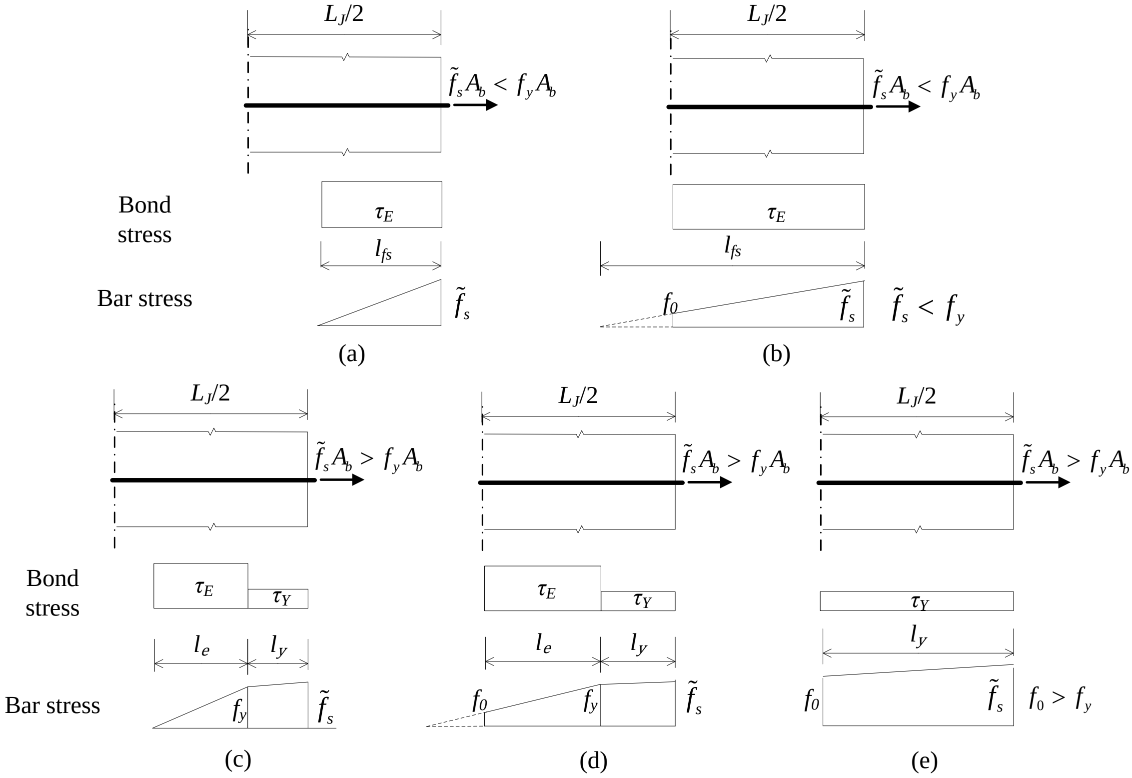

Fig. 3 Stress propagation of the steel reinforcement and the corresponding bond stress

Based on the equilibrium condition and steel bilinear constitutive model, the global relationship of bar slip and bond stress can be obtained for any magnitude of applied load. For simplicity of derivation, assuming that the load transfer along each steel reinforcing bar throughout the interior joint region is symmetric, the point with zero slip is assumed at the middle point of the steel embedment length within the joint region. Thus, the anchorage length for the bond-slip behaviour is limited to one-half of the joint width L_J and the bar stress f_0 at the joint centre is taken as the boundary condition to balance the applied pullout force. Since the bar stress propagates from the loaded end of the steel reinforcement with increasing applied load, the distributions of the bar stress and the associated bond stress are shown in Fig. 3 for all possibilities of the boundary conditions.

(a) Elastic state. As shown in Fig. 3 (a), the applied pullout load is not so large and there is a sufficient length for the propagation of bar stress, that is, $\tilde{f}_s \le f_y$ and $l_{fs} \le L_J/2$, where L_J is the width of the joint and $l_{fs} = \frac{\tilde{f}_s}{\tau_E}\frac{A_b}{\pi\, d_b}$ is the propagation length of the bar stress as indicated in Fig. 2. The slip at the loaded end can be obtained from Eq. (2).

$$Slip = \int_0^{l_{fs}} \varepsilon_E\, x\, dx = \int_0^{l_{fs}} \frac{f_s}{E_s}\, x\, dx = \int_0^{l_{fs}} \frac{\tau_E\, \pi\, d_b}{E_s\, A_b}\, x\, dx = \frac{2\, \tau_E}{E_s\, d_b}\left(l_{fs}\right)^2 \tag{2}$$

(b) Elastic state with non-zero stress boundary. With increasing applied bar stress $\tilde{f}_s$ at the joint perimeter, the bar stress will propagate towards the joint centre along the steel reinforcement. If the joint width is insufficient for embedment length and the yield strength is relatively large, then the distributions of bond stress and bar stress as shown in Fig. 3 (b) are mobilized for the scenario with $\tilde{f}_s \le f_y$ and $l_{fs} > L_J/2$. The integrated slip at the loaded end can be obtained from Eq. (3).

$$Slip = \int_0^{\frac{L_J}{2}} \varepsilon_E\, x\, dx = \int_0^{\frac{L_J}{2}} \frac{f_0 + \frac{2x(f_s - f_0)}{L_J}}{E_s}\, dx = \int_0^{\frac{L_J}{2}} \frac{f_0 + \frac{\tau_E\, \pi\, d_b}{A_b} x}{E_s}\, dx = \frac{f_0}{E_s}\frac{L_J}{2} + \frac{2\tau_E\, \pi\, d_b}{E_s\, d_b}\left(\frac{L_J}{2}\right)^2 \tag{3}$$

with $f_0 = \tilde{f}_s - \frac{\tau_E\, \pi\, d_b}{A_b}\frac{L_J}{2}$.

It is evident in Fig. 3 (b) that the middle point along the steel reinforcement acts as a force boundary to provide the anchorage force f_0 to resist the applied load $\tilde{f}_s$ at the joint perimeter.

(c) Elasto-plastic state with zero stress boundary. Besides the scenario in Fig. 3 (b), the other possibility is that the joint width is sufficient for embedment but the yield strength is relatively small. Then, the distributions of bond stress and bar stress are mobilized as shown in Fig. 3 (c) for the scenario with $\tilde{f}_s > f_y$ and $l_{fs} = l_y + l_e < L_J/2$, where $l_e = \frac{f_y}{\tau_E}\frac{A_b}{\pi\, d_b}$ and $l_y = \frac{\tilde{f}_s - f_y}{\tau_Y}\frac{A_b}{\pi\, d_b}$ are the lengths of elastic and plastic steel reinforcement, respectively, as indicated in Fig. 2. In such a situation, the corresponding slip at the loaded end is given in Eq. (4).

$$Slip = \frac{2\, \tau_E}{E_s\, d_b}(l_e)^2 + \frac{f_y l_y}{E_s} + \frac{2\, \tau_Y}{E_h\, d_b}\left(l_y\right)^2 \tag{4}$$

(d) Elasto-plastic state with non-zero stress boundary. No matter what the scenario is in Fig. 3 (b) or Fig. 3 (c), with increasing load at the end, the following stage given in Fig. 3 (d) will occur for $\tilde{f}_s > f_y$, $l_{fs} = l_y + l_e > L_J/2$ and $l_y < L_J/2$. Yielding occurs for a certain range of steel reinforcement near the loaded end of steel reinforcement. Since the local strain of the yielded steel is greater than that of the elastic steel, the corresponding bond stress for yielded steel is activated as shown in Fig. 3 (d). The integrated slip at the loaded end is given in Eq. (5) with $f_0 = f_y - \frac{\tau_E\, \pi\, d_b}{A_b}\left(\frac{L_J}{2} - l_y\right)$.

$$Slip = \frac{f_0}{E_s}\left(\frac{L_J}{2} - l_y\right) + \frac{2\tau_E}{E_s\, d_b}\left(\frac{L_J}{2} - l_y\right)^2 + \frac{f_y l_y}{E_s} + \frac{2\tau_Y}{E_h\, d_b}\left(l_y\right)^2 \tag{5}$$

(e) Plastic state. The ultimate stage of the bond stress-slip behaviour is shown in Fig. 3 (e), in which the whole steel embedment within the joint region has yielded with $\tilde{f}_s > f_y$, $l_{fs} = l_y + l_e > L_J/2$ and $l_y > L_J/2$. The slip at the loaded end can be obtained from Eq. (6) with $f_0 = \tilde{f}_s - \frac{\tau_Y \pi d_b}{A_b} \frac{L_J}{2}$.

$$Slip = \int_0^{\frac{L_J}{2}} \left(\frac{f_y}{E_s} + \frac{f_0 - f_y + (f_s - f_0)x/(\frac{L_J}{2})}{E_h} \right) dx = \frac{f_y}{E_s}\frac{L_J}{2} + \frac{f_0 - f_y}{E_h}\frac{L_J}{2} + \frac{2\tau_Y}{E_h d_b}\left(\frac{L_J}{2}\right)^2 \quad (6)$$

3 Validations

In the present study, in order to validate the prediction accuracy of the proposed analytical model in the axial pullout loading scenario, the experimental studies by Ueda et al. [8] and Shima et al. [4] are employed due to their comprehensive descriptions about the test details and well interpreted test results. Only the predicted results of three specimens in [4] are discussed in detail due to page limit.

On comparison of experimental results, the proposed analytical model is capable of predicting the general trend of the bond stress distribution and the critical point between elastic and plastic ranges of steel reinforcement. Even though there is a strain plateau at the yield strength and the strain variation is difficult to be accurately determined, the predictions of steel stress and strain distributions are reasonably acceptable. Additionally, the predicted slip by the proposed analytical model agrees well with the measured slip, which means that the assumed bilinear bond stress distribution can be quite accurate in an average sense for both the elastic and the plastic ranges along the steel reinforcement. The relation between the slip at the loaded end and applied bar stress is demonstrated in Fig. 4, which shows good agreement between the predictions by the proposed analytical model and experimental results. It is found that even though the slip is calculated based on strain integration for each discretized segment along the stress propagation length, the accumulated error is not so significant and, thus, the proposed analytical model is considerably reliable in terms of accuracy. Besides, the predicted failure mode for all the three specimens herein is fracturing of bars, which agrees well with experimental observations.

It should be clarified that the ultimate tensile strengths of the steel bars in the three specimens [4], are 540 MPa for Specimen SD30, 800 MPa for Specimen SD50 and 910 MPa for Specimen SD70, respectively. As shown in Fig. 4, the predicted failure points are not coincident with those in reported experimental results, because the experiments had not been conducted until the nominal failure points with ultimate tensile strengths of the steel bars. However, the ultimate tensile strengths are taken as the fracturing criterion in the predictions. Thus, the maximum slips and ultimate applied bar stresses predicted by the proposed analytical model are slightly greater than those obtained from the experimental studies. In general, the proposed analytical model is capable of predicting the bond-slip behaviour with the failure modes of pullout failure and bar fracturing due to axial pullout action.

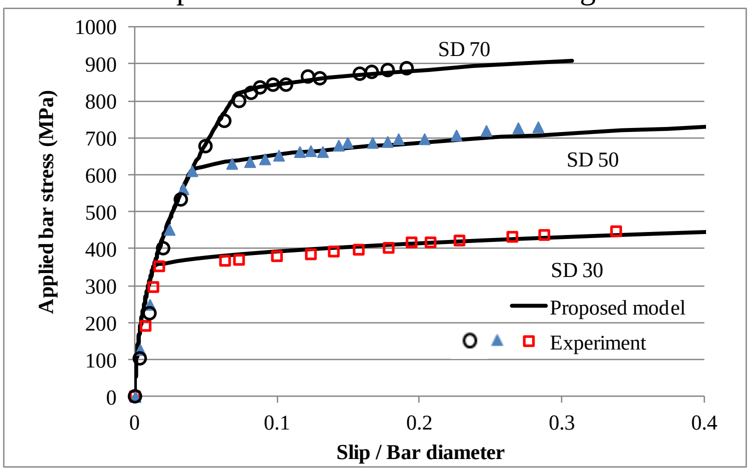

Fig. 4 Relations between the slip at the loaded end and applied bar stress in Shima's tests [4]

4 Conclusions

In the present paper, a simple and reliable analytical model based on a bi-uniform bond stress distribution is proposed to predict the relationship between the slip at the loaded end and the vertical applied load in RC joints.

Based on experimental results obtained from the literature, the bi-uniform bond stress distribution is suggested. Due to insufficient embedment length of steel reinforcement in some cases, different equations according to the proposed stress propagation length are derived to satisfy the equilibrium and compatibility conditions in the axial pullout loading scenario.

The proposed bond-slip analytical model is validated against experimental results subjected to axial pullout. The validations for axial pullout predictions include not only the comparisons of the relationship between the slip at the loaded end and the vertical applied load, but also the comparisons of the detailed distributions of bond stress, bar stress and bar strain along the steel reinforcement. It is shown that the proposed analytical model is considerably reliable in terms of accuracy, even though the slip is calculated based on strain integration along the stress propagation length.

In conclusion, the proposed simple and reliable analytical model on the bond stress-slip relationship is capable of effectively predicting the bar-slip behaviour subjected to axial pullout in the RC beam-column joints.

References

[1] Eligehausen, R., E.P. Popov, and V.V. Bertero: *Local bond stress-slip relationships of deformed bars under generalized excitations* in *Report UCB/EERC-83/23, Earthquake Engineering Research Center*: University of California, Berkeley, California, U.S (1983).

[2] Russo, G., G. Zingone, and F. Romano: *Analytical solution for bond-slip of reinforcing bars in R.C. joints*. Journal of Structural Engineering-ASCE Vol. 116 (1990), p. 336-355.

[3] Sezen, H. and J.P. Moehle: *Bond-slip behavior of reinforced concrete members*. in *fib-Symposium: Concrete Structures in Seismic Regions*. CEB-FIP, Athens, Greece (2003).

[4] Shima, H., L.L. Chou, and H. Okamura: *Micro and macro models for bond in reinforced concrete*. Journal of the faculty of engineering, the university of Tokyo Vol XXXIX (1987), p. 133-194.

[5] Kwak, H.G. and F.C. Filippou: *Finite element analysis of reinforced concrete structures under monotonic loads*. Department of Civil Engineering, University of California, Berkeley, California, U.S., Rep. UCB/SESM-90/14 (1990).

[6] Kwak, H.G. and F.C. Filippou: *Nonlinear FE analysis of R/C structures under monotonic loads.* Computers & Structures Vol. 65 (1997), p. 1-16.

[7] Khalfallah, S. and M. Ouchenane: *Prediction of bond between steel and concrete by numerical analysis.* The Open Civil Engineering Journal Vol. 2 (2008), p. 1-8.

[8] Ueda, T., I. Lin, and N.M. Hawkins: *Beam bar anchorage in exterior column-beam connections.* Journal of the American Concrete Institute Vol. 83 (1986), p. 412-422.

[9] Yankelevsky, D.Z.: *Analytical model for bond-slip behavior under monotonic loading.* Building and Environment Vol. 20 (1985), p. 163-168.

[10] Noh, S.Y.: *Tension Stiffening Model for Numerical Analysis of RC Structures by Using Bond-Slip Relationship.* Journal of Advanced Concrete Technology Vol. 7 (2009), p. 61-78.

[11] Menegotto, M. and P.E. Pinto: *Method of analysis for cyclically loaded R.C. plane frames including changes in geometry and non-elastic behavior of elements under combined normal force and bending*. in *Proceedings of the International Association for Bridge and Structural Engineering on the resistance and ultimate deformability of structures acted on by well defined repeated loads*. Lisbon, Portugal (1973).

[12] Lowes, L.N. and A. Altoontash: *Modeling reinforced-concrete beam-column joints subjected to cyclic loading.* Journal of Structural Engineering-ASCE Vol. 129 (2003), p. 1686-1697.

[13] Alsiwat, J.M. and M. Saatcioglu: *Reinforcement anchorage slip under monotonic loading.* Journal of Structural Engineering-ASCE Vol. 118 (1992), p. 2421-2438.

[14] CEB: *CEB-FIP Model Code 2010*, Lausanne, Switzerland (2010).