Optimisation of Public Bus Transport Network in Singapore Using Spatial Parameters and Demographic Information

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SUMMARY

Transportation is an important aspect of our urban daily life. Unfortunately, the performance of urban transport systems is getting worse in many cities and traffic causes a lot of negative environmental effects. Developing better public transport and using non-polluting modes are believed to be the way towards sustainable transportation. Effective design of well-planned bus routes and services that are closely integrated with mass rapid transit can increase the attractiveness of public transportation in urban areas and improve its quality.

Public transport network design is always a challenge for transport planners and there is no simple approach to solve it. This project aims to develop a methodology for optimised bus route network design with the help of GIS. Firstly, the locations of bus stops are optimised with the objective of improving accessibility. Secondly, bus routes are optimised with the objective of minimising the total system cost which is the sum of travellers' cost and bus operating cost.

Two methods are proposed for the problem of improving the location of bus stops. The first one uses a randomised algorithm which moves the bus stops within their current road segments and can be applied on a large scale network. After hypothetical relocation of bus stops in Singapore, a small increase in the number of houses with good accessibility to public bus services was achieved. Good accessibility means that walking time to bus stop is less than 5 minutes. The second method is developed using a set-cover problem solution which requires no preliminary knowledge of the bus routes but needs a huge computer storage space. In the test case, 17% of bus stops could be removed without worsening accessibility level.

A hybrid simulated annealing (SA) and genetic algorithm (GA) approach is developed to design the bus routes. The proposed method is tested on four theoretical networks, a benchmark network and is also applied to the real transport network in a part of Singapore with real demographic data. Sensitivity analysis is performed on parameters of this SA-GA approach.

The proposed hybrid SA-GA was compared with methods previously reported in the literature and tested on the benchmark Mandl's network. In-vehicle travel time of SA-GA's solution is only 1% longer than theoretical minimum in-vehicle travel time, which is better than in previous solutions. SA-GA's solution is also better than previous solution in terms of percentage of direct trips, total travel time and transfer time. The SA-GA solution for the real network was evaluated and compared with the current situation. SA-GA's solution is better than current bus route network in terms of average number of transfers, percentage of direct trips, average total travel time, average in-vehicle time, average transfer time and average waiting time. The improved quality of bus route network is achieved with decreased operation cost. All the results show that the proposed SA-GA methodology generally outperforms methods used in previous studies in both the solution quality and the computing efficiency of the methodology itself.

Besides looking at transport network design, an evaluation of the current transportation system is performed in order to understand the status quo and be able to compare alternative improvement proposals. Three measures are used to evaluate the system performance. They are: pedestrian accessibility, connectivity among bus stops and travel time to employment. Evaluation shows that over 70% of buildings have good access (walking time less than 5 minutes) to bus stops. Over 99% bus stop to bus stop trips can be accomplished with two or fewer transfers.

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LIST OF ABBREVIATIONS

AI	Artificial intelligence
ATM	Average theoretical minimum travel time
AIVT	Average in-vehicle time
CBD	Central business district
DGP	Development Guide Plan
DSS	Decision Support System
FSLC	Fixed string length coded model
GA	Genetic algorithm
GIS	Geographic Information System
GIS-T	GIS in transportation
LRT	Light Rapid Transit
LTA	Land Transport Authority
MRT	Mass Rapid Transit
NEL	North East MRT Line
NDP	Network design problem
NP-hard	Nondeterministic polynomial-time hard
O-D	Origin-destination
SA	Simulated annealing
SA-GA	Hybrid simulated annealing and genetic algorithm methodology
S-P	Single-point crossover
SRFC	Simultaneous route and frequency coded model
TAZ	Transport analysis zones
T-P	Two-point crossover
URA	Urban Redevelopment Authority
VOT	Value of travel time
VSLC	Variable string length coded model

LIST OF COMMON SYMBOLS

$B = (1, 2 \ldots n)$	A set of bus stops
C_b	bus seating capacity (passengers/bus)
C_M	MRT seating capacity (passengers/train)
c_1	factor to convert user travel time to user travel cost ($/passenger·hour)
c_2	factor to convert bus running time to cost equivalent ($/bus·hour)
c_3	factor to convert MRT running time to cost equivalent ($/train·hour)
c_b	factor to convert bus-kilometres to cost equivalent ($/bus·kilometre)
c_p	factor to convert passenger travel time to passenger travel cost ($/passenger·hour)
E	the value of objective function ($/hour)
F	frequency (buses/hour)
f_k	frequency of *k*th bus route (buses/hour)
$f_{k,min}$	minimum frequency of buses operating on any route (buses/hour)
f_s	frequency of *s*th MRT route (trains/hour)
$f_{s,min}$	minimum frequency of MRT (trains/hour)
I	a parameter to control the number of iterations for each temperature
k	bus route number
L_k	round trip distance for *k*th bus route (kilometre)
l_k	load factor on bus route $k = Q_k^{\max} /(f_k \cdot C_b)$
$l_{k,max}$	maximum allowable load factor of bus service
l_s	load factor on MRT route $s = Q_s^{\max} /(f_s \cdot C_M)$
$l_{s,max}$	maximum allowable load factor of MRT
$N = (Rd, B)$	Road network

N_{fs}	fleet size (buses)
N_0	current solution which is a subset of candidate routes
N_1	new solution generated by GA
n	number of bus stops in the network
P	the probability of accepting the new solution
P_0	population size
P_c	crossover probability
P_m	mutation probability
$Q_k^{\max}$	maximum flow occurring on bus route k (passengers/hour)
$Q_s^{\max}$	maximum flow occurring on MRT route s (passengers/hour)
R	a random number (0<R<1)
R_1	set of bus routes in the solution
R_2	set of MRT routes in the solution
$R\ \{1, 2 \ldots k\}$	A set of candidate bus routes
$Rd = (1, 2 \ldots m)$	A set of roads
$R_{SR}\ \{1, 2 \ldots\}$	Solution
s	MRT route number
T	temperature
T_{ij}	travel demand from node i to j (passengers/hour)
T_{ij}^1	travel demand from bus stop i to j assigned to the first path (passengers/hour)
T_{ij}^2	travel demand from bus stop i to j assigned to the second path (passengers/hour)
t_c	cycle time (the sum of in-vehicle travel time and loading/unloading time) of a round trip for a bus service (hour)
t_{ij}	total travel time (the sum of in-vehicle travel time, waiting time and transfer time) between bus stop i and j (hour)
t_{ij}^1	total travel time for the first path between bus stop i and j (hour)

t_{ij}^2	total travel time for the second path found between bus stop i and j (hour)
t_k	round trip time for kth bus route (hour)
t_s	round trip time for sth MRT route (hour)
t_{wij}	average waiting time for passengers travelling from bus stop i to j (hour)

CHAPTER 1 INTRODUCTION

1.1 Background

With accelerated urbanisation, traffic congestion and environmental degradation caused by cars have become serious problems affecting the quality of life in major cities. These problems are increasingly severe with the rapid growth of car ownership in Asia. Faced with this urgent situation, transportation engineers and urban planners have to find various methods to improve transportation systems in cities. It is almost universally agreed that building more roads and expressways does not solve these problems (Makri and Folkesson, 1999). Public transport and using non-polluting modes of transport such as cycling are alternatives to unsustainable car-based transportation. Hence, transport planners in many countries advocate a policy for building mass rapid transit (MRT) systems and improving bus systems to ease the woes of congestion and to cater for the needs of the public. Effective design of well-planned bus routes and service frequencies that are closely integrated with mass rapid transit can increase the attractiveness of public transportation in urban areas.

Prior to the introduction of more sophisticated mass rapid transit systems (MRTs) to alleviate congestion, the humble bus service was the main mode of urban public transportation. The newly introduced MRT systems usually will have a great impact on existing bus transit system. As some demand shifts to the new rapid transit mode, there may be duplicated routes or redundant routes. The MRT system may also be underutilised if buses do not provide good feeder services to the stations. From the perspective of operators, they will lose money because of poorly integrated public transport routes. The high operation cost may be eventually transferred to travellers as higher fares. Therefore, there is a need for providing a comprehensive strategy to design the public bus transit system as a whole.

While it is impossible to satisfy all the people's requirements, the conflicting profit motivation between operators and passengers add another difficulty in public transport design. It is necessary to reach some kind of balance between operators' and passengers' interests in finding the best public transport solution.

Traditionally public bus systems are developed incrementally over time as the demographic distribution and road networks change and other services like mass rapid transport are introduced. However, with present advanced spatial management and analysis capabilities, it could be potentially more beneficial to re-engineer a bus network according to fundamental demographic pattern from scratch rather than to make incremental changes to the existing transport network. For example, the transportation system of Dordrecht in Holland was completely restructured to achieve high efficiency and coherence after thorough experiments revealed how inefficient the present system was (Cheung, 1998).

This approach is a radical change from the conventional incremental approach. Singapore's small geographical size and extensive spatial and demographic data offer ideal conditions for a case study using such an approach.

1.2 Problem statement

Bus service has been the conventional and main public transport service in most cities for a very long time. Major changes of land use and demographic pattern distribution may cause the whole network to be less efficient. New public transport services introduced incrementally into transport network in response to the expansion of the city or the increase of travel demand may result in local improvements of the transport system around the newly developed area. However, this incremental change may affect the whole public transport network negatively. Hence, a comprehensive approach must include all changes while considering the network as a whole to achieve a balanced and efficient transport system.

The network design problem (NDP) can be more formally presented in the following way. A road network $N = (Rd, B)$, comprises a set of roads $Rd = (1, 2 \ldots m)$ and a set of bus stops $B = (1, 2 \ldots n)$. A bus route is represented by a sequence of bus stop IDs. A set of candidate bus routes *R {1, 2 ... k}* is generated based on network *N*. Each bus route is given a unique number as its ID. The problem is to find the set of bus routes that will maximise an objective function value which represents the system efficiency while satisfying several constraints.

It is universally agreed that NDP is a combinatorial problem. There are plenty of combinations available due to the large number of bus stops and roads. How to select the best combination from so many candidates is a big challenge. In addition, complex constraints and conflicting objectives must be considered while solving this problem.

Constraints vary for different urban areas or countries. Most common constraints focus on fleet size, minimum and maximum service frequency etc. Some constraints may contradict each other. For example, high service frequency will definitely increase the fleet size.

The objectives of optimisation of bus route network primarily involve the following:

- Minimising the number of bus services and fleet size
- Minimising the operation cost
- Maximising the service quality (e.g. minimising travel time, waiting time and walking time)
- Maximising the service coverage.

The fact that maximising of service quality and coverage would result in increased operation cost is undeniable. Therefore, reaching a balance among these contradictory objectives is another challenge for NDP.

NDP involves not only planning bus routes based on a road network, but also transit demand assignment and frequency determination. The aim of bus route design is to plan sufficient bus routes to cover all the bus stops and accommodate all the travel demand while not violating any constraints.

NDP is an NP-hard (Nondeterministic polynomial-time hard) problem (Chan et al., 1990). It has been proven that it is computationally infeasible to find a globally optimal solution to this kind of NDP (Lin et al., 2003). Given the vast amount of data to process, it is crucial to balance between the computational resources (e.g. running time and space) and the algorithms for the network design.

The effectiveness of the proposed hybrid SA and GA approach will be tested in three ways. Firstly, it will be applied to four theoretical grid networks with different sizes. Secondly, it will be applied to a benchmark network which was used by other researchers. Lastly, it will be tested on a part of real network of Singapore.

1.3 Objectives and scope

This project aims to propose an advanced approach which is applicable to real and large scale road network to plan and design the public bus system with the help of GIS and spatial analysis. Specifically, the main objectives of this research are as follows:

(1) To develop methods of evaluating the current public transport system
Prior to re-designing the transport system, it is important to evaluate the existing system. The reason is two-fold: firstly, understanding the current network and providing a base for comparison with future designed system; secondly, evaluating the current system which reveals the strengths and weaknesses of the system and therefore provides valuable insights for proper optimisation.

(2) To propose and develop a method to optimise bus stop location
Two methods will be proposed. One is randomised algorithm which is applicable to area with existing bus stops. The other is solving the problem by transformation into set-cover problem. The second method is applicable when planning bus stop location from scratch. The objective of bus stop location design is to improve the pedestrian accessibility of bus stops.

(3) To propose and develop a method to solve NDP

The proposed method will involve a combination of simulated annealing (SA) and genetic algorithms (GA). With this method, an optimised bus route network based on the existing road infrastructure to meet the needs of the general population will be produced. This method should be applicable to large scale real network. Transit assignment needs to be considered in the evaluation. It is also necessary to take into account some real case issues like loop bus route, one-way street etc. to make this method practical.

(4) To demonstrate the application of the proposed method

The proposed method will be applied on both theoretical and real networks. One of the theoretical networks is the benchmark network used in many previous similar studies. The results from the proposed method and previously developed methods will be compared. The new bus route network designed by the proposed method will be compared with the current bus route network of the real network in Singapore. Extensive tests are necessary to analyse the optimised network, and more importantly, to verify that the proposed method can indeed result in an improvement of the overall efficiency.

Detailed bus stop to bus stop demand matrix is not available in Singapore. The data used in this study is estimated from zone to zone origin-destination travel survey. The estimation is based on the distribution of buildings and bus stops.

1.4 Organisation

This thesis consists of 11 chapters. They are organised in the following way:

Chapter 2 reviews some of the existing literature on public transport evaluation, application of GIS (Geographic Information System) in transportation and public transport network design. The advantages and disadvantages of existing approaches for NDP are summarised.

Chapter 3 evaluates the current public transport system of Singapore. Three measures are used. They are pedestrian accessibility, connectivity among bus stops and travel time between zones.

Chapter 4 discusses the proposed two approaches for design of bus stop locations. The first approach is a randomised algorithm. The second approach transforms the bus stop location problem to a set-cover problem. The application of these two approaches and their advantages and disadvantages are summarised.

Chapter 5 elaborates the framework of the proposed SA-GA methodology for bus route network design.

Chapter 6 discusses the application of the proposed methodology on four theoretical grid networks with different sizes.

Chapter 7 discusses the application of the proposed methodology on the benchmark network which has been used in many similar studies.

Chapter 8 explains the preparation process of the data needed for application of the proposed approach on real network in Tampines area which is a part of Singapore's real road network.

Chapter 9 discusses the application of the proposed methodology on Tampines network. The new designed bus routes network is compared with the existing network.

Chapter 10 provides an evaluation on the effectiveness and efficiency on the proposed methodology.

Chapter 11 finally presents the conclusions of this study and recommendations for further research.

CHAPTER 2 LITERATURE REVIEW

2.1 Introduction

To understand a transportation system and provide a base for comparison of different systems, it is important to select a suitable method for public transport performance evaluation. This is reviewed in Section 2.2.

Public transport network design is a perennial problem for planners. Various approaches to routing and scheduling have been used to solve aspects of the transportation problem. The traditional method employs analytical models on a simplified network to get the optimal values for key design variables such as bus stop spacing, route spacing, route length, operating headway, vehicle capacity etc. The relationships between those design variables and the quality of public transport system are analysed (Chien et al., 2001; Spasovic et al., 1993). A brief review of this approach is summarised in Section 2.4. Another broadly applied method is heuristic algorithms. In heuristic models, expert knowledge is introduced into the system for guiding the program to search for better solutions. Some studies in this field are reviewed in Section 2.5.

With advancing computer technology and artificial intelligence (AI), the ability to solve complex relationships and problems is now more readily available. Due to its powerful spatial representation and analysis capability, Geographic Information System (GIS) has been applied in transportation broadly. This is reviewed in Section 2.3. Expert system is one of AI tools, which has been used in solving transportation problems. Currently, genetic algorithm (GA) and simulated annealing (SA) algorithm have attracted much attention for their strong ability to solve complex optimisation problems. These three AI approaches are reviewed in Sections 2.9, 2.6 and 2.7, respectively. The newly developed combined algorithm of GA and SA is reviewed in Section 2.8.

2.2 Public transport performance evaluation

2.2.1 Introduction

In order to attract more people to use public transport instead of private cars, it is important to understand what they expect from a bus system. Kelly (1996) summarised people's requirements on bus systems as: reliability, frequency, stability, information, speed, access, value for money and impression created. Qualitative requirement alone is not sufficient to practically deliver a high quality bus service. It is necessary to develop some quantitative measures as the indicators of bus service quality.

Performance evaluation has gained increasing attention in transportation organisations and the reason is universal. People analyse the performance of a transport system from various perspectives since transportation is a very complex system which involves many factors with different characteristics such as land use, infrastructure and social-economic elements. Therefore, it is quite hard to construct a uniform evaluation method. Benn et al. (1995) conducted a research project on bus route evaluation standards in the United States and Canada. They found that the evaluation methodology had progressed considerably, for example 15 criteria were studied in 1984; but 44 criteria were used in 1994. Moreover, it was apparent that the focus of evaluation switched from "how much money a trip brought in", "was the trip profitable" to customer satisfaction and orientation. Although many criteria are being used by different agencies, they can be categorised into basically two groups: economic measures and technical measures.

2.2.2 Evaluation of public transport from economic perspective

The first group is measuring transport efficiency from an economic perspective. Efficiency indicators define the relationship between resource input and produced output (Fielding et al., 1985). Some generally used performance indicators are listed in Table 2-1. Traditionally, there are two kinds of approaches to measure the efficiency: econometric and mathematical programming (Costa and Markellos, 1997). Recently, some new techniques have been used in measuring the efficiency such as neural network models. Economic factors have a significant impact on transportation investment and construction and they may decide the feasibility of a transportation project eventually. Measurement of the performance of the current transport system in terms of economic efficiency is difficult because it requires accurate data.

Table 2-1 Inter-modal performance indicators

Categories	Indicators
Cost efficiency	Total cost per revenue vehicle capacity mile Total cost per revenue vehicle capacity hour
Cost effectiveness	Total cost per passenger trip Total cost per passenger mile Passenger revenue per total cost
Cost effectiveness	Passenger trip per revenue vehicle capacity mile Passenger trip per revenue vehicle capacity hour Passenger miles per revenue vehicle capacity mile Passenger miles per revenue vehicle capacity hour

Source: Li and Wachs (2000)

2.2.3 Evaluation of public transport from technical perspective

The second group of public transport evaluation is measuring its performance from a technical perspective. Because public transport is generally operated as a public service, profitability cannot be used as the sole indicator of success (Fielding et al., 1985). Therefore, it is necessary to measure its performance according to service utilisation, service quality and accessibility of service.

In practice, a large number of performance indicators are adopted by different transport organisations to evaluate the quality of transportation system. For example, there were 101 public transport performance measures in use in the US according to Ferreira's (2003) study. They were not quite suitable for measuring an integrated transport system (involving multi-mode). Ferreira summarised a list of performance indicators, either adopted or proposed to deal with the integration. All these indicators were defined in terms of connectivity or accessibility. Connectivity could be measured as the average number of transfers, percent trips needing transfers etc. These measures are adopted in this study. For measuring accessibility, travel distance and travel time were the most important components.

Accessibility is an important index for evaluating public transport and suitable for measuring degree of integration of transport (Ferreira, 2003). It is also the focus of public transport performance in this study. There are a number of definitions for accessibility in the literature (Makri and Folkesson, 1999; Kalsaas and Aase, 1998; Miller, 2000; Halden et al., 2000). Generally accessibility means the ease with which a person can reach and take part in, or use, an activity. In terms of transport system evaluation, measures of accessibility can be divided into two categories: place accessibility and individual accessibility. Individual accessibility measures are usually superior to place measures for they describe the individual's experiences on accessibility instead of assuming that all people from the same zone demonstrate the same behaviour and have the same level of accessibility. For example, retired people and working people like to visit different places and are subject to different spatial-temporal constraints, so the assumption of common accessibility cannot be absolutely accurate. However, the problem with using individual accessibility is that it requires processing individual information, which is virtually impossible to obtain for a large population. In contrast, place accessibility considers the overall accessibility of a certain area. In practice, place accessibility is often preferred for its relatively low requirement on data.

Makri and Folkesson (1999) discussed three most generally used models for place accessibility. The first one was distance measures, which simply used average distance or weighted area distance as the accessibility. The second one was cumulative-opportunity measures, which evaluated the accessibility in terms of the number or proportion of activities accessible within certain travel distance or time from a given location. The last one was gravity measures, which measured the accessibility as the sum of relative accessibility for all possible destinations divided by the total attraction of the urban area in interest. For example, the accessibility A_i for the zone i could be expressed as:

$$A_i = \frac{\sum_j a_j * f(d_{ij})}{F} \qquad \text{Equation 2-1}$$

where:

a_j: the attraction in zone j
d_{ij}: the travel time, distance or cost from zone i to zone j
$f(d_{ij})$: the impedance function
F: a standardisation factor

This formula is the basis of measuring average travel time to employment as one of the accessibility indices to evaluate the performance of public transport, which is discussed in Section 3.4.

Schoon et al. (1999) tested the application and effectiveness of accessibility indices in northeast Hampshire, England. They used the travel times or costs by car or bus between 15 pairs of selected origins and destinations to measure the accessibility indices. They defined the car travel time accessibility indices as the travel time by car divided by the average of travel time by car and bus, and the bus travel time accessibility indices as the travel time by bus divided by the average of travel time by car and bus. Accessibility indices for travel cost were calculated in the same way. Although their application of accessibility indices was very simple, the result was very encouraging. They found that the accessibility indices described above could effectively evaluate the existing transport system and could indicate which transport corridors had greatest potential for improvement. To make the application of accessibility indices more robust and efficient, they suggested measuring the accessibility in a weighting way, utilising computer techniques such as GIS, including public transport frequency etc.

Geurs and Wee (2003) conducted a comprehensive review of accessibility evaluation. They defined criteria for accessibility measures of land-use and transportation systems, which could be summarised as follows. Ideal accessibility measures should be:

- Sensitive to changes in transport system;
- Sensitive to changes in land-use pattern;

- Sensitive to temporal constraints of opportunities;
- Take individual needs, abilities and opportunities into account;
- Be practical, considering the availability of data, models and techniques, as well as time and budget;
- Easy to understand and interpret;
- Can be used as a social indicator;
- Can be used as an economic indicator.

The authors also indicated that it was not practical to apply the full set of criteria because it would imply a high cost and complexity in data collection and computation. As a result, they concluded that place-based accessibility could be considered as effective measure of accessibility.

From the review of the features and practice of public transport system evaluation, it is seen that:

- accessibility measures are suitable for evaluation of the performance of a public transport system;
- place accessibility measures are more practical then individual accessibility measures in terms of data collection and computation complexity.

2.3 Application of GIS in transportation

2.3.1 Introduction

Geographic Information System (GIS), which is essentially a database management system with the capability to manipulate and analyse spatial data, has particularly useful applications for the transportation industry. Some applications that are related with this project are reviewed in the following sections.

2.3.2 Application of GIS as a decision support tool

Since GIS has the ability to answer "what if" questions, it has been used as a decision support tool to evaluate different transportation plans such as transport infrastructure siting, route design etc. and observe the different impacts of these plans on operations in quantitative measures. For example, the number of cars that need to be diverted to the new highway so that the existing traffic congestion can be eased.

Sadek et al. (1999) developed a decision-aid tool within a GIS platform for multicriteria evaluation of highway alignment. This tool comprised three main components: GIS model, tool engine and evaluation framework. GIS model was the common component for all GIS projects. A geographically referenced database for the region of interest was constructed in this model. To implement multicriteria evaluation, the following layers were built in the database: political/administrative, existing roads, existing structures, land cover, land use, topography, rivers/streams, geology, soil, and depth of water table. Analysis was carried out in tool engine. In this module, the alignment of a new highway was analysed through some spatial analysis function such as intersecting of layers, cut and fill analysis, contour construction, buffer analysis etc. In the evaluation framework, the alignment of a new highway was evaluated based on the results derived from the tool engine. Various evaluation criteria were used and consisted of environmental criteria, community disruption criteria (number of structures hit), geometric evaluation criteria (number of curves, route length etc.) and geotechnical evaluation criteria (earthworks and type of material, slope stability). This approach was tested on a case study of the city of Beirut, Lebanon. The result demonstrated the feasibility and effectiveness of this approach.

Selection of the best route is a very importation problem in transport operation and has wide applications in real life problems such as goods delivery, emergency treatment etc. A method of choosing best route for freight transportation based on a multimodal transportation network was developed by Beuthe et al. (2001). The advantage of this work was that it organised all the modes (road, rail and waterway) of transportation and all the loading, unloading, transhipping and waiting operations in the same network. For those items that did not exist as real objects such as loading, transhipping and waiting, they were represented by "virtual links". The fictitious virtual network was generated from the basic geographical network automatically. After attaching the cost functions to each virtual link defined by a specific transport mode or operation, the best route was found through minimisation of the total cost with Dijkstra's algorithm. One shortcoming of the method was that it could not reach a solution consisting of more than one mode.

There are other examples of application of GIS as a decision support tool which are not reviewed in detail here. For example, a GIS in transportation (GIS-T) has been used as a decision support system (DSS) to facilitate transport planning and management in Portugal for ten years (Dias et al., 2000). They even proposed a model to evaluate the performance of GIS-T DSS according to their experiences.

2.3.3 Application of GIS in accessibility measurement

Accessibility measurement is another major application of GIS in transportation. Since this study is based on the city of Singapore, two works of accessibility measurement of Singapore are reviewed in detail.

Zhu and Liu (2003) used this technique to evaluate the impact of the MRT system on accessibility in Singapore. They measured this impact through comparing the accessibility to the central business district (CBD), to working population, and to industrial and commercial opportunities before and after the construction of the MRT system. Accessibilities to CBD, to working population and to industry were measured in terms of travel time during the peak hours. Accessibility to commercial opportunities was measured in terms of travel cost. They were all measured on a zone level. Based on the results, the authors concluded that the accessibility in almost all aspects decreased with increasing distance from East-West Line (one of the MRT lines). The construction of the North-East Line (the newly constructed MRT line) greatly improved the accessibility of northeastern areas, but had little impact on other areas.

Liu and Kam (2003) implemented accessibility measures using a gravity model in a GIS environment. They measured the accessibility on a zone level. Typically, planning areas or transport analysis zones (TAZ) are used in accessibility measurement. But hexagonal zones were used in this study because the authors did not have TAZ data and circle zones would result in overlapping and not continuous surface. The population was used as the attraction and the travel time was used as the impedance in the gravity model to calculate the accessibility for each zone. In order to compute the travel time between origins and destinations (centres of zones), they developed an algorithm, which estimated the travel time according to the actual situation. The general idea was that the total distance between the origin and destination consisted of three parts: the distance between the nearest point and the origin (multiplying the airline distance by a user-specified factor); the distance between the nearest point and the destination (multiplying the airline distance by a user-specified factor); and the distance between the two nearest points (multiplying the shortest path distance by a user-specified factor). Finally, they applied this approach to evaluate the accessibility conditions before and after the construction of a LRT line in Singapore. They concluded that general patterns of the "before" and "after" situations were quite similar but the improvement of accessibility due to the LRT line could be observed clearly. Therefore, users could easily understand the performance of a transport project from the accessibility perspective.

Besides the above two studies, a lot of publications can be found in this area. The basic concepts of all applications of accessibility measures used are quite similar. Therefore, only some improvements in accessibility measures are listed and discussed here.

Kalsaas and Aase (1998) proposed a method to measure the detailed accessibility at address-to-address level which was defined as travel time by bus. The difficulties in calculating travel time by bus based on point to point pattern are that: firstly, it involves transfers; secondly, several bus routes may share a common link (road); thirdly, walking network must be included if the origin and destination points are not on the road network. They tried to solve these problems through overlaying different layers of network, one for road network and one for walking network, or adding fictional routes which represented the links shared by more than one bus route. However, most GIS software only involved network calculation at one layer and adding fictional routes would enlarge the size of database considerably. So they did not implement their idea in reality in the time available.

Juliao (1999) extended the accessibility measurement in continuous surface instead of dividing an area into zones and considering the whole zone to have the same accessibility. This approach made it possible to examine the whole territory in detail.

In order to make the evaluation of accessibility more accurate, Thevenin (2001) added the time dimension in the accessibility analysis. Traditionally, travel time is measured using average speed and the average accessibility is taken as the performance indicator for the transport system, though accessibility can vary very much at different times of day. Most studies focus on the accessibility measures only during peak hours. The peak hour accessibility is not suitable for people who take transport outside peak hours. Therefore, the authors took the exact travel time into account in accessibility measurement through considering the service frequency.

Although GIS has strong capabilities in spatial analysis and its application in transportation area is quite beneficial, it is worth noting that the performance of GIS approach depends heavily on the nature of data. Moreover, the data structure of most GIS software does not support very detailed information for some particular analysis. It may be necessary to develop new data structures for special use.

2.4 Public transport network design with analytical methods

2.4.1 Introduction

A public transport system consists of a road network, a number of bus stops with fixed location, a demand table showing number of trips between any pair of bus stops and a series of operation parameters such as frequency etc. In a normal public bus system, a fixed number of buses run on fixed routes according to a fixed timetable and stop at regular bus stops. In theory, optimisation of bus routes for a city with a given road network, bus stop locations and passenger demand would involve deciding on the best number of routes as well as the best bus stop sequence for each route and the frequency, with the objective of minimising the sum of total passenger user costs and bus operating costs. To solve NDP analytically, it is necessary to transform the transport system including road network, bus stop, number of routes and operational parameters into mathematical description. Therefore, NDP is basically a combinatorial problem and the number of possible solutions grows exponentially with the network size.

Some early attempts to solve the problem analytically can be found in plenty of literature (Kuah and Perl, 1988; Spasovic et al., 1993; van Nes and Bovy, 2000; Chien and Spasovic, 2002 etc.) Most of them involved greatly simplified networks and abstract models. Usually, straight bus routes with rectangular or parallel pattern and uni-directional demand were assumed in these studies. The optimum values of key parameters like route length, route spacing, frequency and bus stop spacing could be determined by solving the objective function mathematically. However, real-life constraints on route length, route spacing, bus stop spacing and the many-to-many character of demand cannot be reflected in these analytical models. This makes the analytical approach not practical for real-life application and only suitable for theoretical research.

2.4.2 Importance of objectives in urban transit network

One of the major problems that make public transport design a difficult task is the conflict between interests of operators and passengers. Obviously passengers benefit most with a denser bus route network and a shorter operating headway. However a dense route network and short headway can be too expensive to operators. Divergent objectives result in different optimal value for key design variables. Therefore a suitable and balanced objective function should be formulated first.

The importance of objectives in urban transit network design was studied by van Nes and Bovy (2000). They postulated that six kinds of objective functions from three primary perspectives, the passenger, operator and social welfare perspectives, were widely used in past studies. Some of these objectives were:

- Minimise passenger travel time, which includes access time, waiting time and in-vehicle time;
- Minimise passenger travel time given a limited budget;
- Maximise cost effectiveness which is defined as the ratio of the total revenues to the operational cost;
- Maximise operator profit which is the total revenues minus the operational cost;
- Minimise the total cost which is the operational cost plus traveller cost (travel time multiplied by value of travel time (VOT));
- Maximise passenger load in public transport.

These six objectives were formulated into an analytical model based on an assumed urban area of one square kilometre. This area was served by one or more parallel transit lines having uniform line spacing. The stop spacing for all the transit lines was also uniform. The transit fares were fixed. The focus was on the influence of the objective on the resulting key design variables, i.e. stop spacing and line spacing. After comparing the different network attractiveness and performance characteristics resulting from different objectives, van Nes and Bovy concluded that minimising the total cost was the most suitable objective in urban transit network design.

2.4.3 Optimising bus stop spacing with a discrete approach

Furth and Rahbee (2000) proposed a discrete approach to optimise bus stop spacing with dynamic programming. First, a set of candidate bus stop locations along a route needed to be selected. All of the intersections along the route were used as candidate locations normally and a bus route was analyzed in a single direction. Second, a geographical model was used to distribute demand, which was assumed to be fixed, to the blocks in the route's service area. The demand was distributed in proportion to opportunities such as population, jobs, retail places etc. along each block. The objective of optimising bus stop spacing was to minimise the total cost which was the sum of cost of walking time, cost of riding delay and operating cost. The major constraint was a maximum-allowed stop spacing. Then, the optimisation progressed backward, starting from optimising the destination bus stop location and continued with the bus stop next to the destination until the origin bus stop. The effectiveness of this approach was tested and proved on one of Bonston's busiest bus routes. This model could be further improved by considering each travel direction, multiple periods and variable demand.

2.4.4 Optimising different design variables for simplified networks

Optimising design variables for feeder bus routes

Usually a number of variables including route spacing, route length, bus stop spacing, frequency and so on are necessary to define a bus transport system. A research on optimising feeder bus routes based on an assumed rectangular service area was done by Kuah and Perl (1988). In this service area, passengers took feeder buses to access a single transit line. The feeder bus routes were perpendicular to the rail line. They proposed an analytic model for designing the feeder bus routes for the peak hour with the assumption of inelastic demand. The advantage of this model is that it can optimise route spacing, operating headway and bus stop spacing simultaneously instead of optimising those variables separately. The authors found that the optimal route spacing and operating headway were cubic root functions of the system parameters (walking speed, value of riding time and average time lost at a bus stop) and were not sensitive to the changes in travel demand and other parameters. Three different conditions of stop spacing were considered: uniform spacing; constant spacing along routes; and varying spacing both between routes and along routes. It was shown that stop spacing was a square root function of the system parameters and it increased with walking speed, riding time, and average time lost at a bus stop, and decreased with the value of walking time.

Optimisation of parallel bus routes with elastic demand

Spasovic et al. (1993) extended the methodology for inelastic demand to the case of optimising route length, headway, route spacing and fare with elastic demand that was sensitive to the quality of public transport service and fare. The assumptions of the bus system serving a rectangular urban corridor were as follows: the total travel demand was uniformly distributed in the study area and over time; *n* parallel routes extended from the CBD outwards with the same route length and route spacing; operating headway was uniform among all parallel routes and as well as along routes; and there was no constraint on vehicle fleet size.

Two objective functions were used in this paper: maximisation of operator profit that was the difference between the revenue and operator cost; and maximisation of social welfare that was the sum of passenger surplus and operator surplus. The optimisation of social welfare was carried out under two different conditions: with unconstrained subsidy and break-even constraints. The algorithm to implement the optimisation could also incorporate vehicle capacity constraints.

The numerical results showed that profit and social welfare functions are relatively flat near optimum. In addition, the subsidy could be eliminated by providing a marginally worse service quality given a set of system parameters (route spacing, route length, headway, stop spacing) under the break-even constraints.

Design of bus service with demand equilibration

Kocur and Hendrickson (1982) used calculus and Lagrange multiplier to analyse optimal parameters in local feeder bus system design. For analytical tractability, the analysis was based on a model of infinitely fine street grid connecting to the rapid transit. In addition, demand functions were assumed to be linear for easy manipulation. The key design variables considered were bus route spacing, bus headway, and the fare. The authors selected three different objective functions: profit maximisation, profit maximisation plus some fraction of net user benefit, and net user benefit maximisation subject to deficit constraint. They found that optimal line spacing and headway are proportional to one another, independent from the objective function, whereas the optimum travel fare is very sensitive to the objective function.

Other applications of analytical approach

An approximate analytical model was proposed by Wirasinghe (1980) to find the nearly optimal value or bound for parameters such as rail station location, feeder bus route frequency of a rail/bus system. Peak-period many to one demand and rectangular highway grid with railway parallel to one axis was assumed. LeBlanc and Boyce (1986) proposed a bilevel programming algorithm to design road network. The objective was to minimise the sum of travel cost and improvement cost. Bilevel programming algorithm involved first finding the optimal values for minimising travel cost and then minimising the improvement cost based on the optimal values obtained in the first step. Theoretically their method could be used for network with no more than 200 nodes. However, no computational test was presented. Wirasinghe et al. (2002) presented an analytical optimisation method to determine transit network parameters. Specifically, they attempted the determination of the optimum terminal location of a cross-town rail line. However, they considered a dense rectangular grid local road network. There was only one rail line to be planned. As the result, the model is too idealised for most real-life scenarios.

2.4.5 Summary of the analytical approach

Generally, the analytical method can only be applied in a network with very simple structure. For example, designing feeder bus service routes that are perpendicular to a main transit route and the feeder bus routes that are parallel to each other. The travel pattern is simply one-to-many or many-to-one, such as from CBD to the surrounding area and vice versa. The service area is often assumed with a regular shape and most are assumed to be rectangular. All these assumptions make this approach impractical in a real case study. In addition, this method works well only with simple formulae with a few variables. The optimal values cannot be found if many variables are incorporated into the objective function. Basically, the model, which can describe a transport system well, is usually too complicated and has too many parameters for the analytical method to design a complete public transport network with irregular shape and many-to-many travel pattern.

2.5 Heuristic models for transit route network design

2.5.1 Introduction

Another approach for solving the bus network optimisation problem involves heuristic algorithms. Heuristic methods are approximate methods. Usually the route network is constructed in a step-by-step manner while using different rules. Relevant literature on heuristic models can be found in the works of Mandl (1980), Hasselstrom (1981), Ceder and Wilson (1986), Baaj and Mahmassani (1995) and Shih et al. (1998). Detailed reviews of these works are provided in the following sections.

2.5.2 Public transportation planning with interactive methodology

The problem of how to efficiently choose routes, vehicle-types and frequencies for a public transportation system with fixed routes was addressed in Hasselstrom's work (1981). It was solved by an interactive methodology based on the cooperation between local planners and computer. A set of candidate routes meeting certain restrictions were generated first. The routes included in the final solution would be selected from them. By assigning passengers to candidate routes, a set of promising routes were chosen and their frequencies were estimated with consideration of riding time, waiting time and number of transfers. Then an evaluation on the selected routes network was performed. Frequency and route connections were further optimised based on the evaluation result. An iteration might be needed to reach a nearly optimal solution. In each process, planners can provide their knowledge and experience to assist the planning by removing duplicate routes, re-combining route segments and so on. The methodology was tested for designing tramway routes in the Gothenburg public transportation system. A substantial improvement was achieved which suggests that the methodology can be efficiently used in a real world.

2.5.3 Two level bus network design

A comprehensive method proposed by Ceder and Wilson (1986) considered both passengers' and operators' viewpoints. In the first level, only passengers' viewpoints were considered. The objective function was formulated as minimisation of the sum of difference in passenger-hours between the route travel time and the shortest travel time and the transfer time. In the second level, both passengers' and operators' viewpoints were considered. The objective function was formulated as minimisation of the sum of four terms: difference in passenger-hours between the route travel time and the shortest travel time; the transfer time; the wait time and operation cost. All of them were converted into monetary value. A tree search algorithm was used in this study to find all feasible routes which gave all the possible route sets step by step. Although more choices may increase the probability of finding the optimum solution, it is impossible for planners to examine all the available solutions for a larger network. Therefore, it is more practical to use an algorithm with a screening ability so as to find a limited number of better solutions for planners to do the final evaluation.

2.5.4 Design of transit route network by improving or expanding initial skeleton

A heuristic model for design of transit route network through trial and error

Mandl (1980) proposed a heuristic approach which started with a feasible set of routes (for example the existing ones) and then searched for better solutions by trial and error. The improvement may be obtained through exchanging parts of routes; including a node that is close to a route; or excluding a node that is already served by another route. The advantage of this approach is that expert experience can be utilised to insure some important landmarks are covered by the public transport system and the searching space is therefore reduced. However, if wrong or inefficient public transport routes are selected into the feasible set of routes in the first place, it would be very hard to achieve an efficient public transport system. Passenger assignment and vehicle assignment (due to fixed number of available vehicles) were also discussed in this paper. The author proposed to assign passengers to public transport network proportionally to the arrival probability if the alternative path was not much longer than the shortest path. He also suggested that vehicle assignment should be performed before passenger assignment and the two assignments should be done in an iterative procedure. His method was tested on a theoretical network, called Mandl's network, which was used by many subsequent researchers. This network consists of 15 nodes (stops) and 21 links (roads). It is illustrated in Figure 2-1

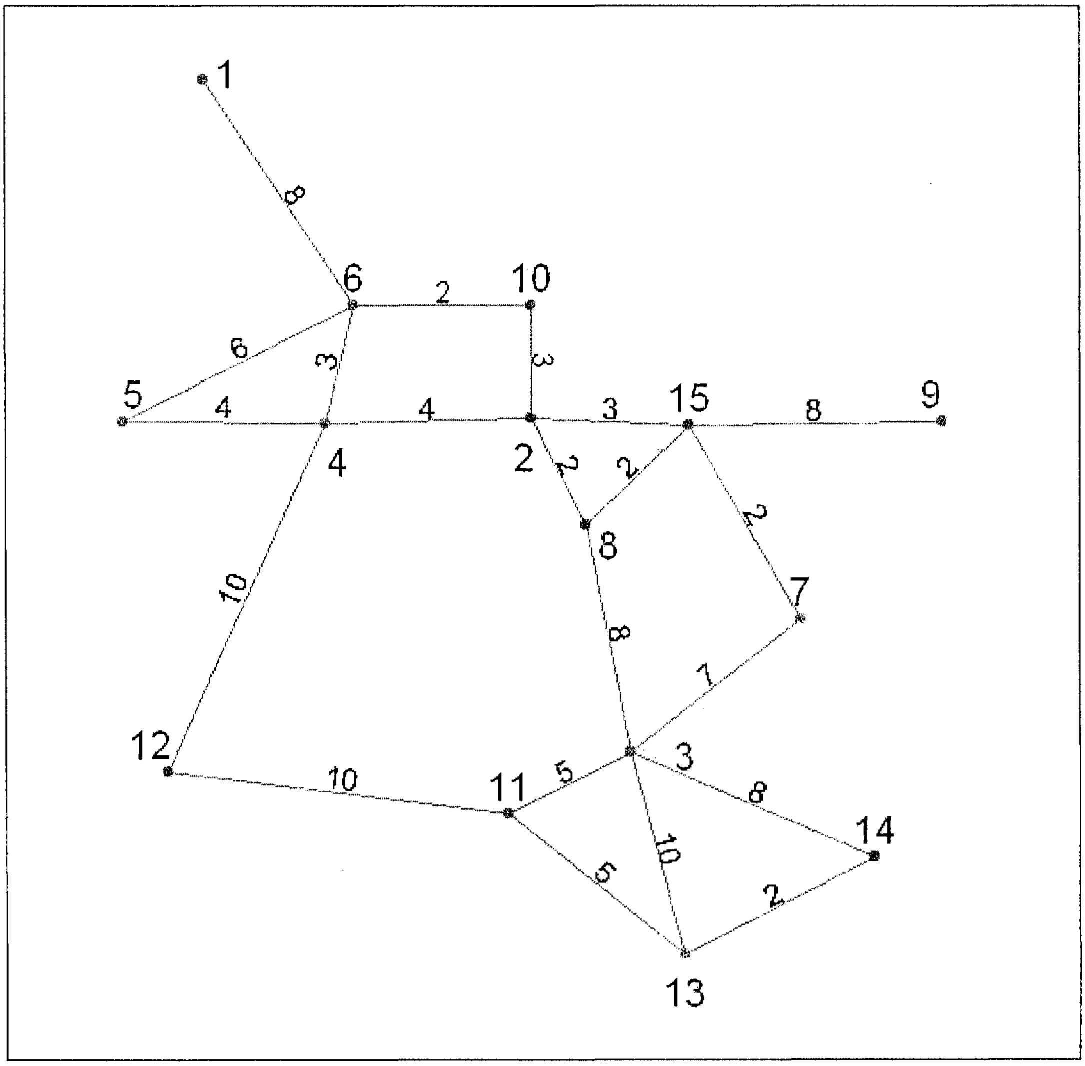


Figure 2-1 Mandl's Network

Design of transit route network through expanding an initial skeleton

Baaj and Mahmassani (1995) proposed a route generation algorithm. In their method a simple skeleton network was constructed first using the shortest or alternative paths between all pairs of high demand nodes. The decision on the size of the initial skeleton (number of routes included in the skeleton) was based on the planners' knowledge or experience. If no reliable estimation existed, only one route was selected. Then the skeleton was achieved by adding new routes until pre-defined size was reached. After that, the skeleton was extended to routes by inserting new nodes into existing ones until predefined demand satisfaction requirement was fulfilled. Three different rules were used for node insertion to generate three different sets of routes. They were:

- maximum increase in the network's total demand satisfied directly;
- maximum rate of the network's total demand satisfied directly over increase in route length;
- maximum rate of the network's total demand satisfied directly over increase in total in-vehicle travel time.

The expansion of a given route terminated when either a route-capacity or a route-length constraint, or both, were violated. In each step, designer's knowledge can be implemented so as to reduce the search space and it is clear that only the insertions which benefit the network the most are allowed. But it is a question whether the insertion added later may cause an earlier insertion to become a redundant one so that the efficiency of the whole network decreases.

A similar application of heuristic approach can be found in the study of Carrese and Gori (1998). They designed express lines for most distant O-D pairs or the ones with highest demand and integrated them with fixed supply such as railways and subways firstly. In subsequent phase, the main skeleton designed in previous stage was considered as a fixed supply. Main lines and feeder lines were designed based on demand matrix and all the fixed supplies.

Designing a bus transit network with coordinated operations

Shih et al. (1998) introduced a heuristic model to design a bus transit network with coordinated operations. This model comprised four sub-modules. The first module was route generation procedure. In this module, route network with a preset number of routes was generated among node pairs with high demand. Then the route network was expanded into a full route network through a node selection and insertion strategy, though this method was criticised by the authors in the early part of this paper. To reduce overlaps, the route whose nodes were subset of other routes was eliminated. This approach of reducing overlap was too arbitrary because some nodes had extremely high demand which could not be satisfied by one route only. The other modules were network analysis procedure, which measured the performance of the route network; public transport centre selection procedure, which identified some potential public transport centres to replace infeasible centres; and network improvement procedure, which did some adjustment to the resulting network in order to improve its performance. Finally, this model was tested in the city of Austin, Texas. The existing public transport network of Austin consisted of 40 bus routes. Using a given set of design parameters generated from the current public transport system, 27 new routes were designed which can satisfy 87.14 percent of the total demand. However, the authors did not compare the quality of current system and the designed system.

Design of highway system with heuristic approach

Solanki et al. (1998) studied the problem of selecting a subset of links from a base network to minimise inter-node travel distance for the specified travel demand subject to total link length constraint. Due to the intractable nature of this NP-hard problem, they used a combination of clustering method and heuristic-assisted branch-and-bound algorithm to find a good solution in a reasonable computation time. However, their model is used for highway design. It does not involve complex factors such as bus transfer, headway, and travel fare.

2.5.4 Summary of application of heuristic models in transit network design

The heuristic approaches have the advantage that expert knowledge can be included to reduce the search space. However, for newly developed urban areas past expert knowledge may not be sufficient and accurate to guide the algorithm which may result in excluding some good solutions. The incremental changes usually used by heuristic models might not always produce better results since every route network should be treated as a whole system because even small changes to one route might affect the overall network effectiveness.

2.6 Optimisation of public transport with genetic algorithm

2.6.1 Genetic algorithm

Genetic algorithm was first proposed by Holland (1975). It is a local search algorithm which simulates the evolution process of a living organism. It has since been broadly applied in many complex optimisation problems and brought better results than traditional methods.

The basis of genetic algorithm is the belief that the optimal solution potentially exists in the genetic pool of a given population. Its procedure can by simplified as follows:

1. Encode the problem into a binary string.
2. Generate a population of possible solutions randomly. A string denotes a solution.
3. Calculate the fitness value for each possible solution in the population.
4. Select solutions with better fitness value from the given population as the candidates for mating. This process is called reproduction.
5. Conduct operation of crossover and mutation to those selected solutions.
6. Iterate the above steps from step (3) until stopping criteria are met.

In the encoding process, a decimal number would be converted into a binary. For example, consider a road network with three bus stops (1, 2 and 3). All the possible candidate bus routes (assume that a bus route must have more than two stops) are:

1) 1-2-3
2) 1-3-2
3) 2-1-3
4) 2-3-1
5) 3-1-2
6) 3-2-1

The solution (bus route network) is a set of candidate routes. Suppose it includes route 1, 4 and 6. Instead of using decimal number string 1, 4, 6 to represent the solution, a binary string is used:

$$\underbrace{\boxed{0}\boxed{0}\boxed{1}}_{\text{(Route 1)}}\;\underbrace{\boxed{1}\boxed{0}\boxed{0}}_{\text{(Route 4)}}\;\underbrace{\boxed{1}\boxed{1}\boxed{0}}_{\text{(Route 6)}}$$

Reproduction, crossover and mutation are basic operators for a genetic algorithm. Reproduction makes copies of individual with better fitness values from the current population. The fitness function is derived from objective function value. The commonly adopted fitness function is as follows (Pattnaik et al., 1998):

$$F_i = V - \frac{O_i \cdot P_0}{\sum_{k=1}^{P} O_k} \qquad \text{Equation 2-2}$$

where:

F_i: fitness value of ith individual

O_i: objective function value of ith individual

P_0: population size

V: a large value to ensure non-negative fitness value

During crossover, each pair of strings in the population after reproduction exchanges their genes to produce two new strings according to some predefined rules. Figure 2-2 shows a simple crossover operator. The frequency of the crossover operation is controlled by crossover probability P_c (note that $0 < P_c < 1$). It is a fixed number usually considered as input genetic parameter.

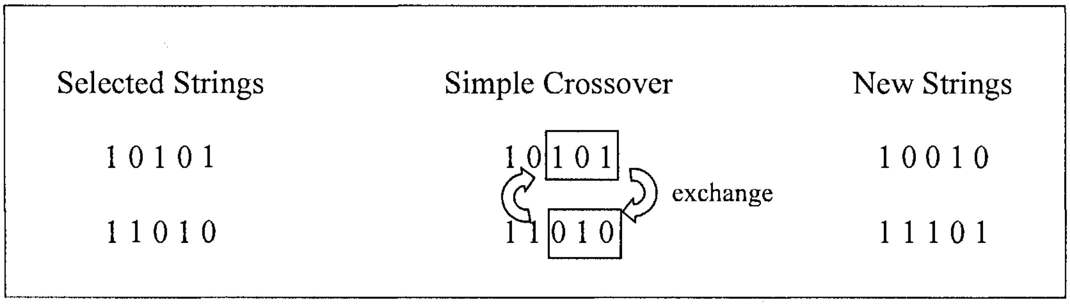


Figure 2-2 Simple crossover operator

Just as in nature, some chromosomes will have random mutations occurring in their genes. The mutation probability P_m (note that $0 < P_m < 1$) specifies the frequency that a given gene in a string will be mutated. Usually it is also a fixed number considered as input genetic parameter. If a gene is selected for mutation then its value will be changed. In the case of binary representation, the gene will simply be changed, that is, a one changed to a zero or a zero changed to a one.

Genetic algorithm exploits the potential of original population by reproduction, crossover and mutation. Although it cannot guarantee that the global optimum will be found, the result should be a good approximation of the optimal value. The crossover and mutation described above are the simplest and the most basic ones. Many other kinds of crossover and mutation can be applied. Generally, a reasonable and efficient objective function and a suitable representation of the problem will lead to an acceptable result.

2.6.2 General application of genetic algorithm

Two-phase design approach with genetic algorithm

A two- phase design process using genetic algorithm was proposed by Krishna Rao et al. (1998). In the first phase, optimal routes were worked out to minimise the in-vehicle travel time and the transfer time of passengers for the whole network. The starting point of routing model was to find the paramount corridors and nodes which generated most of trips. These nodes were the basis for developing routes. *K* shortest paths were developed for each pair of these nodes. The paths developed must satisfy some requirements, i.e. route length and route flow value. The possible route was selected from the *K* shortest paths on a random basis for each pair of nodes using genetic algorithm. The travel demand was then assigned to the route network. After that, the total in-vehicle time and the transfer time could be calculated. Genetic algorithm tried to find the optimal route choice with the minimum total travel time.

In the second phase, optimal schedule was obtained based on the set of routes developed in the first module. The objective of scheduling was to minimise the sum of passenger cost (which is defined as the sum of in-vehicle time, waiting time and transfer time) and operator cost. Genetic algorithm was used to get the optimal headway subject to the constraints of load factor and fleet size. The performance of this method was tested on a sample network with 15 nodes and 21 links. The result was better than that of Mandl (1980) and Baaj and Mahmassani (1990) in terms of in-vehicle travel time and the percentage of trips that could be achieved without transfer. Mandl and Baaj also proposed methods for optimising public transport system and tested their methods on the same sample network.

Although it is proven that genetic algorithm is a good way to solve routing problems in public transport, it has not been tested on a real network or a big and complicated network. To study the potential of applying this method on a bigger network and the effect of the parameters to its performance, a test based on a more realistic network needs to be done.

Combined optimisation of route network and headway

Another attempt of designing an urban bus route network with genetic algorithm was reported by Pattnaik et al. (1998). The difference was that they optimised the route network and bus frequency simultaneously and tested their model on a part of a real transportation network, the Madras Metropolitan City, India. First, a set of routes was developed as the candidates for the optimum solution. These candidate routes were ranked according to a performance index, i.e. demand satisfied. They were coded into the representation scheme which could be understood by the genetic algorithm. This was the key issue in application of genetic algorithm. The traditional coding method has fixed string length coding (FSLC). The size of the route set for the optimal solution is fixed in this coding method, i.e. the optimal network must consist of a given number of routes. Besides this method, Pattnaik et al. proposed a new coding method using variable string length coding (VSLC), in which the size of the route set for the optimal solution could vary, i.e. the number of routes in the optimal network was not constant but varied according to the optimisation process. Both methods were tried in this work. After coding, the genetic algorithm was used to find the optimal route network based on the objective of minimising overall cost, which was the sum of passenger cost and operator cost.

From comparison of the measures of performance values, Pattnaik et al. (1998) concluded that the fixed string length coding gave a better solution in terms of objective function value, demand satisfied, total travel time and fleet size. But its computation time was nearly ten times of that of the variable string length coding. The variable string length coding could provide a near-optimal solution within an acceptable computation time. However, application of both methods to a very large size network has yet to be studied.

Transit route design with variable transit demand

A method of optimising transit route network with GA based on variable transit demand was proposed by Fan and Machemehl (2006a). The main features of this work can be summarised as follows: first, the transit demand on public transport was not assumed to be fixed as most previous studies, only the total travelling demand was assumed as fixed. According to the travel time by auto and public transport, the total travel demand was split between the two modes. Second, long-walk and transfer were considered in trip demand assignment. The trip demand assigned to possible paths was inversely proportional to the total travel time. Third, comprehensive sensitivity study on the effect of route set size and GA parameters was conducted. Some recommendations were given on the selection of GA parameters like population size (60), crossover probability (0.8), mutation probability (0.1), number of generations (80), etc. The first two features made this method more practical and similar to the situations in real life compared with previous approaches. Whether this method can be applied in real case was not discussed in this paper. It was only tested on a theoretical network using two different assumptions, with demand aggregation and without demand aggregation. Under demand aggregation, total demand was aggregated to 28 zone centres and the computing time was 3 hours. Without demand aggregation, the zone demand was distributed to 95 centroid distribution nodes and the computing time was 72 hours. However, the optimised solution was not presented in the paper.

Comparison between genetic algorithm and exhaustive search algorithm

Chien et al. (2001) applied the genetic algorithm to design one feeder bus route from an irregular service area with 160 zones to the CBD. This service area had irregular grid street pattern and had unevenly distributed travel demand. In this paper, they formulated the calculation of access times to bus routes for 14 different zones with respect to the bus routes and incorporated them into their objective function, which was also the sum of passenger travel cost and operating cost.

To achieve the optimal bus route and headway, two methods were used here. One was the exhaustive search algorithm which found all the possible routes, compared their objective function values and searched the optimal solution with the minimum total cost. The other was the genetic algorithm. It worked in its most general way and with the most generally used operators, reproduction, crossover and mutation. Identical optimal solutions were found in both methods. That validated the correctness of the genetic algorithm. In addition, the computing time for genetic algorithm was only 1.1% of that for exhaustive search algorithm (e.g., 4.8 minutes and 7.5 hours, respectively), which showed the effectiveness of the genetic algorithm approach.

Although the result of this paper is encouraging, it should be noted that the travel pattern for this study is many-to-one and it only searches for one optimal route. Moreover, its grid street pattern is also a simplification of urban road network which usually is far more complicated. Therefore, the application of genetic algorithm for the design of a bus route network in an urban area with many-to-many travel pattern and complex road network still needs to be studied further.

2.6.3 Application of specialised genetic algorithms

Seven new operators especially for transport optimisation

The traditional operations applied in genetic algorithm are reproduction, crossover and mutation. They may be too simple compared with the complexity of transport routing problem. Ngamchai and Lovell (2003) proposed seven new operators according to the special features of bus transit routing problem in an attempt to improve the efficiency of reaching optimal solution. The seven operators were characterised as follows:

- Route-merge genetic operator tries to connect routes that share a common terminus and have high transfer demand.
- Route-break genetic operator tries to separate one route into two or three new routes if a great variation of demand exists in the original route.
- Route-sprout genetic operator tries to add an overlapping route between terminals with high demand.
- Add-link genetic operator tries to identify some missing links in the route network which can satisfy certain demand in a more direct way.
- Remove-link genetic operator tries to delete underutilised routes which serve a very small demand.
- Route-crossover genetic operator tries to change the current connectivity of routes to a new combination of routes with more consistent travel demand.
- Transfer-location genetic operator tries to change the transfer location between two routes in order to reduce the transfer demand.

The candidate routes to carry out these operations were selected at random. The probabilities for execution of the seven operations were functions of travel demand. The coordination of headway also mainly depended on the demand. This procedure was implemented after completing the configuration of bus route network. Their approach was tested on an assumed network with 19 nodes and 35 links. According to the sensitivity analysis of the seven operators, the suitable probabilities for them were determined respectively and the different levels of efficiency were also determined. The add-link genetic operator seemed to be the most efficient since some critical links may not be included in the initialisation of the route network. But there were still several user-specified parameters, such as the parameter for termination of a route, which could not be decided by a systematic approach. Moreover a test on a big network using this method is yet to be done.

Performing genetic algorithm in a parallel way

Lin et al. (2003) also suggested some improvements to the genetic algorithm in routing and assigning capacity for a computer network. Although the computer network is far simpler then a public transport network, the common characteristics in routing problem makes these improvements to the genetic algorithm also valid for public transport network design. First, a search for the optimal solution in a parallel way was done as follows: three independent populations were adopted in three different computers at the same time. In this way, the searching space was extended. Next, the three populations were mixed and migrated after each evolution. Therefore the diversity of populations was maintained and inbreeding was avoided. Third, genetic parameters, crossover probability P_c and mutation probability P_m were changed dynamically according to the performance of each evolution instead of using fixed values during the whole process.

The crossover probability P_c and mutation probability P_m are fixed numbers in the general form of genetic algorithm. But in this study they would be lowered to the initial value if a better result was obtained compared with the result of last generation; otherwise they would be increased by a predefined amount if the best result of successive generation had no improvement. So the process of optimisation was controlled by the performance of the genetic algorithm itself and not arbitrarily controlled by the analyst.

With these improvements, the genetic algorithm generated a much better solution than previous studies on a similar problem. But its effectiveness in designing a public transport network based on a real urban area has not been tested yet.

Agrawal and Mathew (2004) also presented a similar approach to solve NDP using parallel genetic algorithm. The difference is that the authors did not try to manage independent populations on different computers simultaneously. Different individual strings in a population were sent to different computers to conduct GA procedures such as evaluation, crossover, mutation etc.

A new method to compute fitness function values in genetic algorithms

Fitness function value is quite important in genetic algorithm application. What chromosome will be selected into the next generation and when to stop the iteration process are decided according to the fitness function value. Traditionally, the objective function value is considered as the fitness function value. But it cannot fully represent the performance of the route network because there are so many indicators such as number of vehicles, commuters per vehicle, number of stops etc. involved in determining the service quality of a transport system.

Bielli et al. (2002) proposed a multi-criteria analysis to synthesise 24 indicators into one fitness function value. They ranked the performance of networks according to the weighted sum of the indicators. The weights representing the importance of the indicators were predetermined before the computation started. They tested their method on a small city in Italy which has a bus network with 1,134 nodes, 3,016 arcs, 459 stops and 22 lines. The fitness function value was improved about 90% from the initial fitness function value. But the pre-decided weights could be too subjective. They need to be verified. The authors suggested keeping n-best solutions container for conducting a sensitivity analysis in order to find the most suitable weights for the network performance indicators.

Public transport route design with frequency coded GA

Tom and Mohan (2003) proposed a new coding approach, Simultaneous Route and Frequency Coded Model (SRFC), for solving transit route design problem with the genetic algorithm. Normally only the ranked candidate routes are considered as the variables for coding in previous studies. The authors of this study took the frequency of each route as a variable for coding too. In this manner, no initial frequency needs to be assumed for each candidate route in order to perform transit demand assignment. A comparison between SRFC and other two popular models used in public transport route design was conducted based on a sample network with 75 nodes and 125 links. The two popular models are Fixed String Length Coded Model (FSLC) and Variable String Length Coded Model (VSLC) (Pattnaik et al. 1998) as discussed in section 2.6.2. The results showed specific advantages of each model. FSLC gave solutions with the lowest waiting time and travel time. VSLC was more efficient in computation as it needed the lowest number of generations. The new model (SRFC) gave solutions with the least operation cost, lowest fleet size and highest direct demand allocation.

2.6.4 Summary of genetic algorithms

Since the invention of the genetic algorithm, it has been widely applied in optimisation problems due to its strong search capability. Its effectiveness and efficiency have been proved by many studies. However, it is generally known that GA may become stuck in a local optimum region due to the fact that GA only accepts better solutions in its reproduction process. Yet accepting a worse solution is sometimes beneficial as it may lead to searching over a wider space and finally reaching the global optimum.

In order to improve the GA performance, a lot of development has been made. But there are still limitations which need to be considered. An important problem of genetic algorithm is convergence. Reproduction from the same string with better fitness value will result in a fast convergence. It sometimes prevents finding of a better solution. To avoid this problem, a large population with high diversity is necessary. But a large population means a large search space which inevitably needs large amount of computation time. So an appropriate population size needs to be chosen to reach a balance between the diversity of the population and the computational time efficiency.

2.7 Designing transit network with simulated annealing approach

2.7.1 Simulated annealing algorithm

As its name implies, the simulated annealing (SA) exploits the analogy between the way in which a metal cools and freezes into a minimum energy crystalline structure (the annealing process) and the search for a minimum in a more general system. In the annealing process, a metal is first heated up to a very high temperature at which all particles of the metal arrange themselves randomly in the liquid phase. Then through lowering the temperature slowly, all the particles rearrange themselves into a stable state with low energy. If initial temperature is sufficiently high and the cooling process is sufficiently slow, the metal can reach a thermal equilibrium.

A system is said to be in thermal equilibrium at a temperature T, if the probability of being in state i with energy E_i is governed by a Boltzmann distribution (van Laarhoven and Aarts, 1987):

$$P(E_i) = \frac{\exp(-\frac{E_i}{T})}{Z(T)} \qquad \text{Equation 2-3}$$

where:

Z(T): a normalisation factor, known as the partition function depending on the temperature T

According to van Laarhoven and Aarts (1987), this algorithm is based on the 1953 work of Metropolis et al. (1953) who originally proposed a means of finding the equilibrium configuration of a collection of atoms at a given temperature. The general form of SA works in the following way. It begins with an initial configuration with energy E_0 and temperature T. At each step, the current configuration with energy E_i is given a small change so that it reaches another configuration with E_j. If the difference in energy is negative, i.e. $\Delta E = E_j - E_i < 0$, the process continues with the new configuration. If $\Delta E = E_j - E_i \geq 0$, the probability of acceptance of the new configuration is given by $\exp(-\frac{\Delta E}{T})$. This is known as the Boltzmann factor. A random number R between 0 and 1 is generated. If R is less then or equal to the Boltzmann factor, the new configuration is accepted as a starting point for the next step; otherwise the new configuration is rejected and the current configuration is still used as starting point. The process is repeated until the probability distribution of the configurations approaches the Boltzmann distribution at each temperature. With the decrease of temperature, the Boltzmann distribution concentrates on the state with the lowest energy.

The connection between this algorithm and mathematical minimisation was first noted by Pincus (1970). But it was Kirkpatrick et al. (1983) who proposed that it forms the basis of an optimisation technique for combinatorial (and other) problems. To apply SA in an optimisation problem, just consider a solution to the problem as a configuration while objective function C and the control parameter T take the roles of energy and temperature, respectively. The objective is minimisation of the value of the objective function C. Let $\Delta C_{ij} = C_j - C_i$, then the probability for solution j to be accepted as the starting point for next step is given by 1, if $\Delta C_{ij} < 0$; and by $\exp(-\frac{\Delta C_{ij}}{T})$, if $\Delta C_{ij} \geq 0$.

2.7.2 Equilibrium network design with simulated annealing approach

Friesz et al. (1992) applied SA to the equilibrium network design problem. The design problem was to find an optimal network design when the network flow pattern was constrained to be in equilibrium. Its objective was to minimise the total travel cost and construction cost with fixed demand. Since cooling scheme is paramount in SA, a self-regulating mechanism for the step size determination which was proposed by Vanderbilt and Louie (1984) was selected. At each step, an enhancement for design variables was decided by a matrix. This matrix was chosen at a given temperature so that the enhancement more or less explored the accessible search space. Given the enhancement of design variables, the objective function was recalculated with these new parameters. A decision to accept or reject the new solution was made according to the method described in the previous section. This process was repeated until the accepted solutions at each step approximated the Boltzman distribution.

2.7.3 Hybrid simulated annealing and case-based reasoning approach

A hybrid artificial intelligence approach based on combing case-based reasoning and simulated annealing was proposed by Sadek (2001). This approach intended to solve transportation problems which were non-convex or recurrent and had a very large search space or must be solved in real time. After a new problem was fed into the model, the system would check if there was a sufficiently similar case in the case base using the nearest neighbour algorithm. The system would output the solution to the similar case as the proposed solution to the new problem, if the search was successful. Otherwise the system would start to compute a new solution for the new problem with the simulated algorithm.

The difference between the SA applied here and the general application of SA was the selection of the temperature starting point. Basically, the initial temperature should be high enough to broaden search space and obtain a global optimum. But in this hybrid approach, the initial temperature could be fairly low because the system could find a similar case for the new problem from the case base, though this case was not close enough to treat its solution as the proposed solution to the new problem. Based on the assumption that the solution of the similar case was very near to the optimal solution of the new problem, it was reasonable to search only the space near the solution of the similar case. Thus computation time could be reduced dramatically.

2.7.4 Using a simulated annealing algorithm to design public transport network

Fan and Machemehl (2006b) proposed a SA approach to solve public transport design problem. Their approach consisted of three major components. The first one was candidate route generation procedure which generated all feasible routes and stored then in the candidate route pool. The second one was a network analysis procedure which assigned transit demand, determined service frequencies and evaluated the solution. Both of the two components were the same as discussed in the authors' previous study (Fan and Machemehl, 2006a). The last component was SA procedure which was used as a search method to find an optimal subset of routes from the huge number of candidate routes. At the beginning of SA procedure, an initial solution network was generated which would be evaluated by the network analysis component later. After that, SA was called to search for a solution which is better than the current solution in the local neighbourhood. Local neighbourhood of a route was defined as the route right next to that route in the candidate route pool. If convergence or the number of iterations was satisfied, the SA search would stop for a specific solution size which was predefined. The whole procedure was repeated for each solution size that belonged to the planner defined solution size range.

2.7.5 Summary of simulated annealing algorithms

Generally, optimisation problems are solved with a method of steepest descent. It is very likely that the method of steepest descent can obtain a local minimum only because it does not accept a solution with worse objective value than the current solution like genetic algorithm. However, SA may accept a worse solution if some conditions are satisfied. This makes it possible to extend its searching space beyond the local optimum. Simulated annealing has been used in various combinatorial optimisation problems and has been particularly successful in circuit design problems (Kirkpatrick et al., 1983). Due to the similarity between network design problem and circuit design problem, it is believed that simulated annealing approach has the potential to work out a better solution than other approaches in solving bus route design problem. However, not much work in public transport network design with SA approach has been done so for.

2.8 Combination of simulated annealing and genetic algorithm

A combination of the two advanced search algorithms simulated annealing and genetic algorithm, was used recently to search for an optimal solution for public transport route network design problem in the work of Zhao and Zeng (2006a). They searched for the solution with the aim of minimising the average number of vehicle boardings that passengers have to make to complete their trips and with the constraints of maximum total route length and fixed number of routes in the solution. The search space was defined by a number of master paths and their surrounding local path space. A master path is a shortest path between a selected pair of origin and destination. Some nodes in the master path are selected as the key nodes. A local path space is derived by generating routes between the key nodes and the nodes which are connected to the key nodes by one road segment only. The search starts from an initial solution which consists of a fixed number of master paths. Then it picks one or more routes from the solution and exchanges it or them with the routes in the corresponding local path space until no improvement in the objective function value can be achieved. When route level search reaches the termination condition, a new search starts from network level with a new selection of fixed number of master paths and continue with route level search until a given maximum computation time is used. This approach was tested on Mandl's network (1979) and the real service area of Miami-Dade Transit agency. The results show that this approach generally outperforms other methods for Mandl's network and has potential to be an effective public transport route network design tool. The good thing about this approach is that it can be utilised on a large-scale real network and can produce a better solution than the existing public transport network within tolerable time. The way of selecting master paths and their local path spaces confines the search space to the area limited to the surroundings of the shortest path only. Fixing solution size is also arbitrary as the correct solution size would be usually unknown. A comprehensive objective like minimising total cost while considering waiting time and transfer penalties may make the application more practical and meaningful.

Later Zhao and Zeng (2006b) extended their method to optimise public transport network layout and headway with the objective of minimising users' cost which include waiting time, in-vehicle travel time and transfer penalty. Headway is determined by demand, bus sitting capacity, load factor, route length, and fleet size. The network layout and headway are optimised separately. Headway is obtained after the transit network layout is determined and it does not impact the following search process for network layout. This approach was also tested on Mandl's network and the real service area of Miami-Dade Transit agency. It also produces better results then previous solutions for Mandl's network and the existing transit network of Miami. It improves the zero-transfer trip dramatically for the Miami network, but only improves it by 3-4% in terms of total user cost. It is good to know that the combination of simulated annealing and genetic algorithm approach is applicable to a large-scale real network. Some real situations like uni-directional roads, loop services and so on are not solved in this paper. All of them should be considered to make the method better and applicable for practical use.

2.9 Application of other methods in public transit system analysis

Some researchers used expert system approach to solve NDP. A framework of a decision support system called PLANiTS (Planning and Analysis Integration for Intelligent Transportation Systems) was developed by Kanafani et al. (1994). This system was supported by four bases: data and knowledge base, method and tool base, strategy and action base and policy and goals base. Based on these, PLANiTS was designed to facilitate the whole process of planning, from problem identification, through method generation and evaluation, to programming. The performance of expert system depends heavily on the quality of its database. If its database is not comprehensive and there are too many data gaps, the system may fail to solve the problem. Other similar studies can be found in the works of Janarthanan and Schneider (1988) and Tung and Schneider (1987).

Simulation is also a useful approach for transportation study. Victor and Santhakumar (1986) presented a simulation model to evaluate the changes of some bus transit features on bus transit system. With the proposed model, the effect of removal of bus stop, changing bus running speed etc. were simulated and the results did not differ with the actual result more than 5%. The advantage of simulation approach is that the result of a plan can be presented before it is implemented so that some potential bad effects can be avoided. However, the big challenge of identifying the correct design of bus stops, bus routes or operation parameters such as headway etc. still exists and can only be solved by trial and error.

2.10 Summary

Transportation system can be evaluated using many different criteria. Technical measurement is selected in this study as it combines the operators' and users' point of view. Specifically, accessibility of public transport is the focus of the evaluation. The method of measurement is implemented in a GIS environment which makes the evaluation more efficient and easy to understand.

The review of the literature implies that the purely analytical methods can only be used to design bus routes for a regular service area with many-to-one or one-to-many travel patterns (van Oudheusden et al., 1987). Moreover, most transit networks studied in previous research work have a regular pattern e.g. parallel or perpendicular to a main transit line, radial pattern etc. Uniform route spacing and operating headway are assumed so that a simplified objective function can be formulated. However, whether an optimal value for each design variable can be obtained is uncertain because most transit route design problems are non-convex, non-linear and multi-objectives (Newell, 1979; Baaj and Mahmassani, 1991).

As network design problem (NDP) is complex, it is generally impossible to obtain an optimal solution with mathematical programming. That is because the accurate formulation of a practical problem results in a nonlinear problem. Therefore it is unlikely that the exact optimal value for design variables can be determined using mathematical calculations. Moreover, the real world transit network design problem inevitably involves a lot of uncertainty, constraints and complicated relationships over an extremely large search space. All of the above difficulties cannot be handled by mathematical approaches alone. The analytical method is not suitable for practical use and is more suitable for theoretical study.

Using the genetic algorithm, the whole transit network with varying route spacing and headways can be optimised. It is much more efficient than traditional exhaustive search algorithm in terms of computation time. The key issues in performing genetic algorithm optimisation are precise and efficient encoding of the transit network into strings and the construction of a simple but meaningful objective function which represents the quality of public transport.

Simulated annealing is based on the manner in which liquids freeze or metals recrystalize in the process of annealing. In an annealing process, a melt occurs initially at high temperature when the molecular system is disordered. Then it is slowly cooled so that the system at any time is approximately in thermodynamic equilibrium. This approach performs very well in some optimisation problems, such as circuit design which is quite similar with network design problem.

It is natural to think that the combination of two powerful search algorithms may achieve better results, especially when one algorithm complements the other. GA may become stuck in a local optimum region due to the fact that GA only accepts better solutions in its reproduction process. However, SA accepts some worse solutions when certain requirements are met so as to extend its searching space beyond the local optimum. Hence, a combined SA and GA approach is proposed in this study to solve public transport network design problem.

In this study, the bus service is treated as an independent service for transporting passengers directly from their origins to destinations. In addition, this study is based on a real urban area in Singapore with its road network, existing demography. The modelling function will be based on observed real values. It goes beyond theoretical problems and presents a challenging task in reorganising the actual transport network.

CHAPTER 3 EVALUATION OF CURRENT PUBLIC TRANSPORTATION IN SINGAPORE

People assess public transport systems from various perspectives. This gives rise to a number of different criteria for evaluation, like accessibility, travel time, bus punctuality, bus loading etc. (PTC, 2003). This chapter examines the bus-rail transport system in Singapore based on three generally adopted criteria: pedestrian accessibility (Section 3.2), connectivity among bus stops (Section 3.3) and travel time to employment (Section 3.4). Section 3.5 concludes the whole chapter.

3.1 Data description

Singapore is made up of several islands. Only the largest one which is the main island of Singapore (41 kilometres from east to west and 23 kilometres from north to south) (Gervero, 1998) is considered as the object of the evaluation.

To describe the current public transport system in Singapore, the following geographical information is assembled:

- Road network
- Bus stops
- MRT lines (Figure 3-1)
- MRT stations (Figure 3-1)
- LRT lines (treated the same as MRT lines)
- Buildings

The above data come from Agis Pte. Ltd (a local mapping company). They are all in ArcView format. In addition to their representation on digital maps, some attributes of the infrastructure elements are also included:

- Road network: 7,742 road links; road name; road type; one-way road or two-way road.
- Bus stops: 4,318 bus stops of which 3,555 have bus service numbers.
- MRT stations: 81 stations and their names.

- Buildings: 118,666 buildings represented by points; some of which are classified.

For the detailed information and examples, see Appendix A.

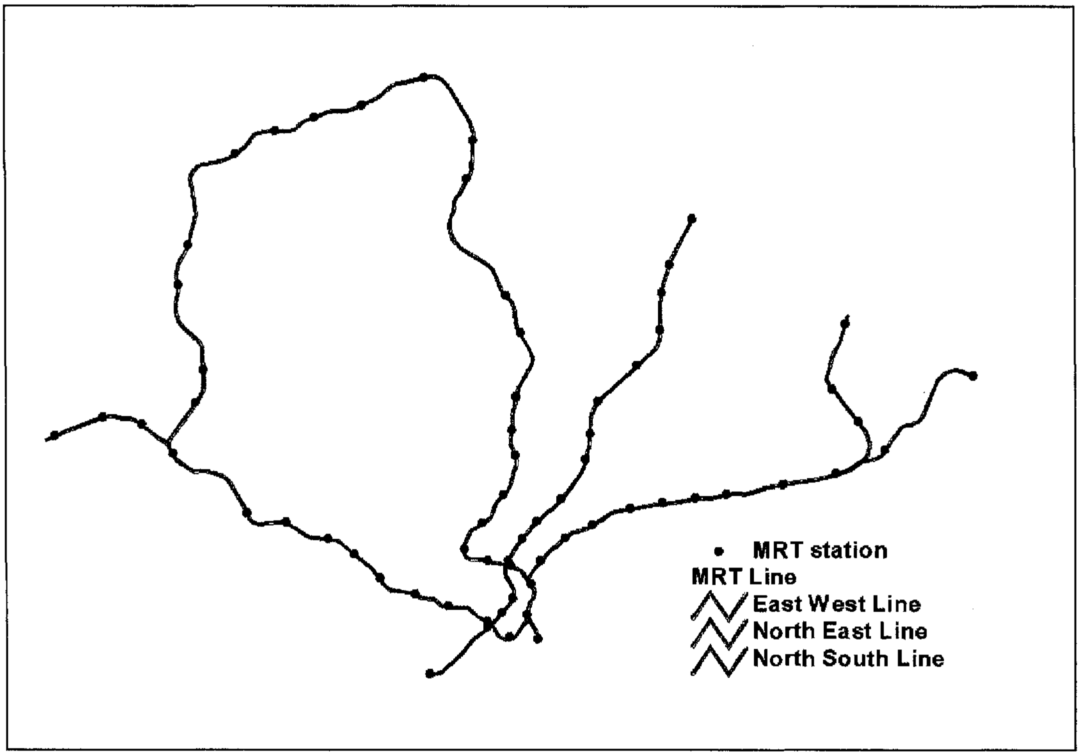


Figure 3-1 Map of MRT system in Singapore

In order to evaluate the travel time to employment, a zone system used by LTA was adopted. For transport planning purposes, Singapore is divided into 927 zones. Every zone is given a 5-digit ID. The first digit of all zone ID is 5. The next two digits represent the development guide plan (DGP) number. In 1991, Urban Redevelopment Authority (URA), Singapore's national planning and conservation authority, released a revised Concept Plan. This Plan maps out the vision for Singapore's long term physical development for a population of 4 million inhabitants. With the completion of the Concept Plan, URA prepared detailed plans (DGP) for gazetting as the new Master Plan. Since all the 927 zones are derived by subdividing DGP areas in 1995, the last two digits represent the zone number within a DGP area. For example, in case of Zone 50417, 5 is a zone ID, 04 means that this zone belongs to DGP area Boon Lay and 17 means that this zone is the

17th zone unit within Boon Lay. The map of zones is attached as Figure B2 in Appendix B.

The data on inter-zone travel time were obtained from LTA. The data consist of inter-zone travel time by bus, public transport (train and bus) and car (text file format); employment for every zone (Microsoft Excel format); and a digital zone map of Singapore and the map of the centres of zones (ArcView format). For the details, see Appendix B.

3.2 Evaluation of pedestrian accessibility

Bus stops should not be located too far from a passenger's origin and destination. American research has shown a strong inversely proportional relationship between public transport ridership and pedestrian access (Hsiao et al., 1997). When pedestrian access level decreases, a corresponding decline occurs in the demand for public transport. Therefore, good public transport pedestrian access encourages people to take public transport instead of using private cars, which may lead to relieving heavy traffic in an urban area and decreasing the level of air pollution. The evaluation of public transport pedestrian access is one of the key elements for understanding the quality of public transport service.

There are two approaches for pedestrian accessibility evaluation: aggregate approach and disaggregate approach. The former divides the area of interest into zones so that analysis of accessibility is performed on zonal level. The latter measures the access at the bus stop level. Here the disaggregate method is used. This microscopic method provides a detailed description of spatial data rather than at a more broad zonal level. Thus, it is possible to observe with more clarity the particular land use pattern and demographic characteristics that surround each individual bus stop and the relationship between pedestrian access and public transport ridership.

Fock (2003) conducted surveys among bus passengers and concluded that people generally think walking time to bus stop of less than 5 minutes is short and very convenient; 5-10 minutes is medium and comfortable; 10-15 minutes is long but still acceptable; over 15 minutes is too long. According to this survey, walking accessibility to bus stops and MRT station is divided into three levels: good, medium and bad. Good accessibility refers to walking time less than 5 minutes; medium accessibility refers to walking time between 5 to 10 minutes; and bad accessibility refers to walking time longer than 10 minutes. Normal human walking speed is 1.0 m/s (Lee, 2000) which takes into consideration the delay caused by crossing roads, and the average ratio of walking distance with respect to airline distance (Euclidean distance) is 1.3 (Olszewski and Wibowo, 2006). These findings and relationships are summarised in Table 3-1.

Table 3-1 Levels of pedestrian accessibility

	Good accessibility	Medium accessibility	Bad accessibility
Walking time	<5 minutes	5-10 minutes	>10 minutes
Walking distance	<300 metres	300-600 metres	>600 metres
Airline distance	<231 metres	231-461 metres	>461 metres

The existing arrangement of bus stops, MRT stations and buildings in Singapore is analysed using ArcView GIS software. The numbers of buildings with good, medium and bad walking accessibility to bus stops and MRT stations are calculated respectively. Figure 3-2 is a sample map that illustrates different levels of pedestrian accessibility to bus stops. The brown crosses in the figure represent bus stops. Those labelled with Bus Stop ID numbers are chosen as sample bus stops to which the pedestrian accessibility is measured. The green points (circles) that fall inside the border of good accessibility represent the buildings with good accessibility to these selected bus stops. Similarly, the pink points (triangles) which fall between the good accessibility boundary and medium accessibility boundary denote buildings with medium accessibility to the sample bus stops. The blue points (squares) outside of the border of medium accessibility are buildings with bad accessibility to the bus stops. Figure 3-3 shows the location of the sample bus stops and the general distribution of levels of pedestrian accessibility to bus stops in Singapore.

The accessibility distributions to bus stops and MRT stations are summarised in Tables 3-2 and 3-3, respectively.

Table 3-2 Distribution of buildings by pedestrian accessibility to bus stops

	Good accessibility	Medium accessibility	Bad accessibility	Total
Number of buildings	85,774	26,344	6,548	118,666
Percentage	72.3	22.2	5.5	100

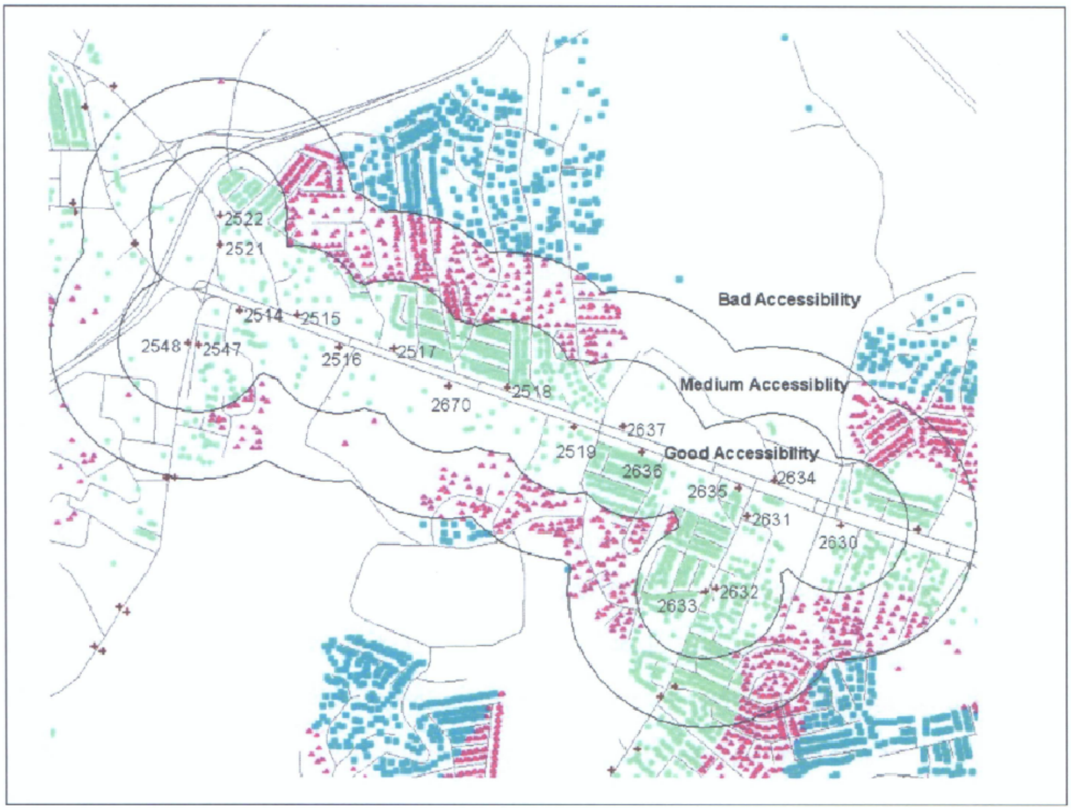


Figure 3-2 Example of pedestrian accessibility evaluation

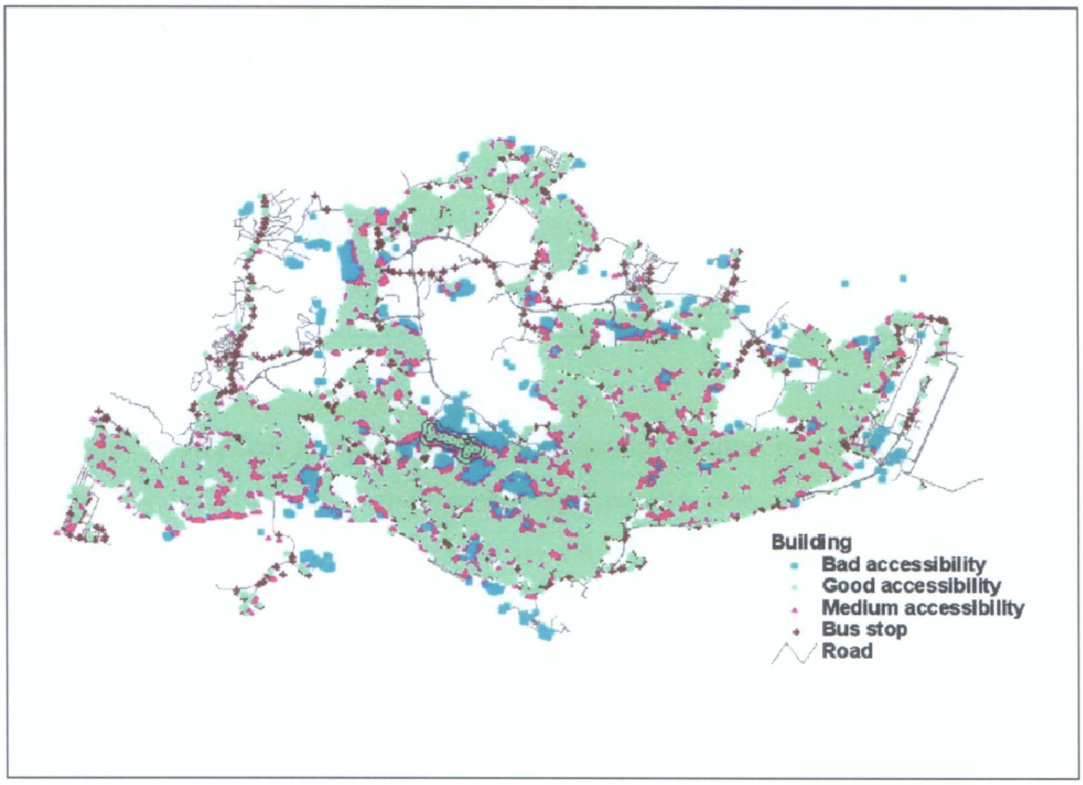


Figure 3-3 Map of pedestrian accessibility to bus stops in Singapore

Table 3-3 Distribution of pedestrian accessibility to MRT stations

	Good accessibility	Medium accessibility	Bad accessibility	Total
Number of buildings	3,474	12,570	102,622	118,666
Percentage	2.9	10.6	86.5	100

Obviously, it is more accurate to measure the pedestrian accessibility with walking distance. However, it is virtually impossible to collect data about walking distance or time for all bus stops and MRT stations due to the effort and cost required. Hence, airline distance has to be used as a proxy of walking distance to measure pedestrian accessibility. In addition, most bus routes are operated in two directions. Therefore, one bus stop located at each side of a road serves people headed in two opposite directions. It is better to differentiate the two bus stops serving people in different directions in the evaluation of pedestrian accessibility. Accessibility can not be considered good if passengers have good access to buses going in one direction only. However, whether passengers can walk across the road to take a bus running in the other direction is not known because such data would have to be collected in the field. Therefore, the differentiation of bus stops is practically not possible. Hence all the bus stops are treated as if passengers had walking access to them in this evaluation.

Figure 3-4 shows that almost 95% of buildings have good or medium pedestrian accessibility to bus stops, which means that people who live or work in these buildings can walk to a bus stop in no more than 10 minutes. The level of pedestrian accessibility to bus stops in Singapore is very good.

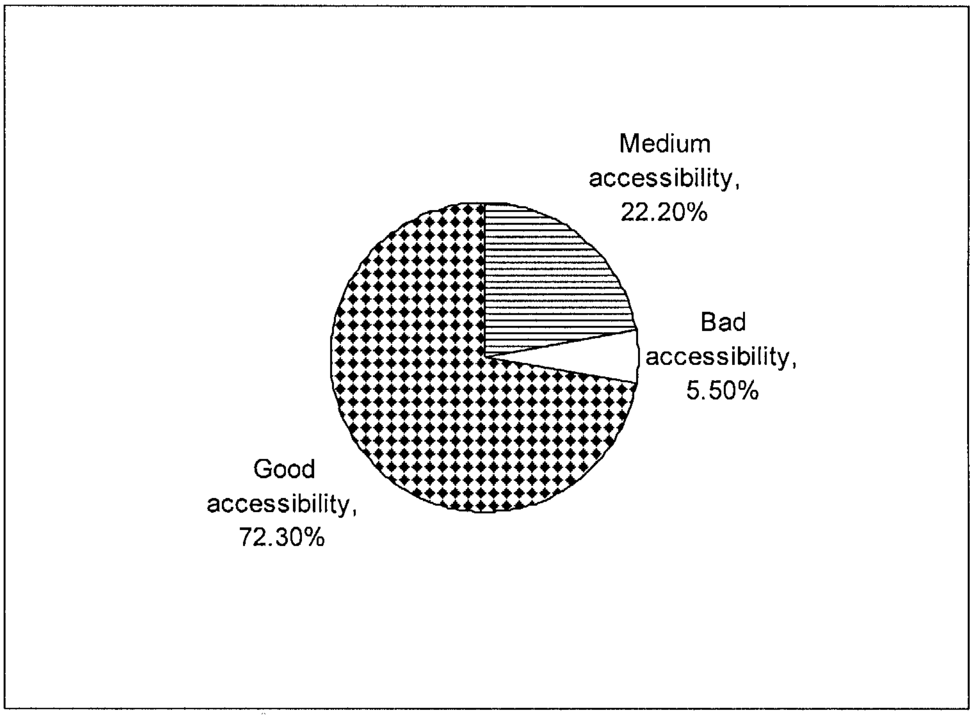


Figure 3-4 Distribution of buildings by pedestrian accessibility to bus stops

In contrast, MRT lines are only located in corridors where the travel demand is very high. The average MRT station spacing in Singapore is 1,613 m (calculated from our data). Only 13.5% (Figure 3-5) buildings have good or medium pedestrian accessibility to MRT stations, which is much lower than that of bus stops. Therefore, the problem of how to efficiently transport passengers to MRT stations using buses is important in transit network design.

3.3 Evaluation of connectivity among bus stops

This section evaluates connectivity among bus stops. The connectivity between two bus stops is measured by the minimum number of transfers one has to take to travel from one point to another. Although physical distance and travel time are not explicitly taken into account here, the connectivity is a good indicator of the minimum total travel time because every transfer implies longer travel time and inconvenience. The unavoidable transfer and waiting time would reduce bus use significantly when the direct route connection is not available (Hsu and Surti, 1975).

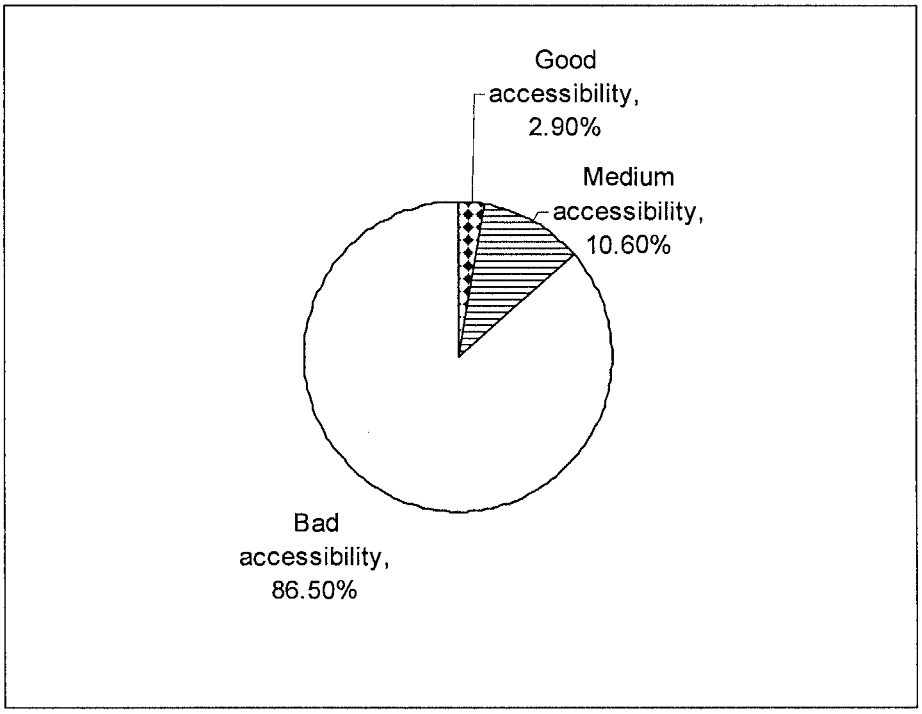


Figure 3-5 Distribution of pedestrian accessibility to MRT Stations

The concept of bus stop connectivity is extended from geometrical accessibility which was introduced by Jiang et al. (1999). In their work, Jiang et al. studied the geometrical accessibility as the number of paths (e.g. roads or streets) traversed from any node in a network to any other node (e.g. road crossings or some special places). However, the accessibility among streets and the nodes linked by streets is not directly related to connectivity of public transport network. Adaptation of the concept to the context of public transport network where each edge denotes a bus route transfer instead of a road or street is needed. The connectivity of a transit network can be viewed as a shortest-path problem where the length of a path is not measured by distance but the number of transfers.

Before giving the details of connectivity computation, one assumption needs to be made here. Passengers only have two options of transfer. The first one is that a passenger can transfer to other bus services at the same bus stop where he alights from the previous bus. The second one is that a passenger can also transfer to other bus services at the bus stops which are within half of the good accessibility walking distance (150 metres which is half of the average bus stop spacing in Singapore) from the bus stop where he alights. If a passenger can switch between bus routes A and B by taking only one transfer somewhere, the two bus routes are considered connected.

Shortest-path problem in a network may be solved using methods such as the breadth-first search algorithm, Dijkstra's algorithm or Floyd-Warshall algorithm (Cormen et al., 2001). However, none of these classical algorithms can be applied in the case where bus transfer is involved. This is because all these shortest-path algorithms require the following "intermediate node property":

If the length of the shortest path between any two nodes, $\boldsymbol{m}$ *and* $\boldsymbol{n}$ *is denoted by* $\boldsymbol{D_{m,n}}$, *then for any nodes* $\boldsymbol{i}$, $\boldsymbol{j}$ *and* $\boldsymbol{k}$ *such that the shortest path between* $\boldsymbol{i}$ *and* $\boldsymbol{j}$ *contains* $\boldsymbol{k}$ *as an intermediate node,* $\boldsymbol{D_{i,j} = D_{i,k} + D_{k,j}}$.

However, the "intermediate node property" no longer holds in a bus stop connectivity measurement problem. Consider a small example described below (Figure 3-6).

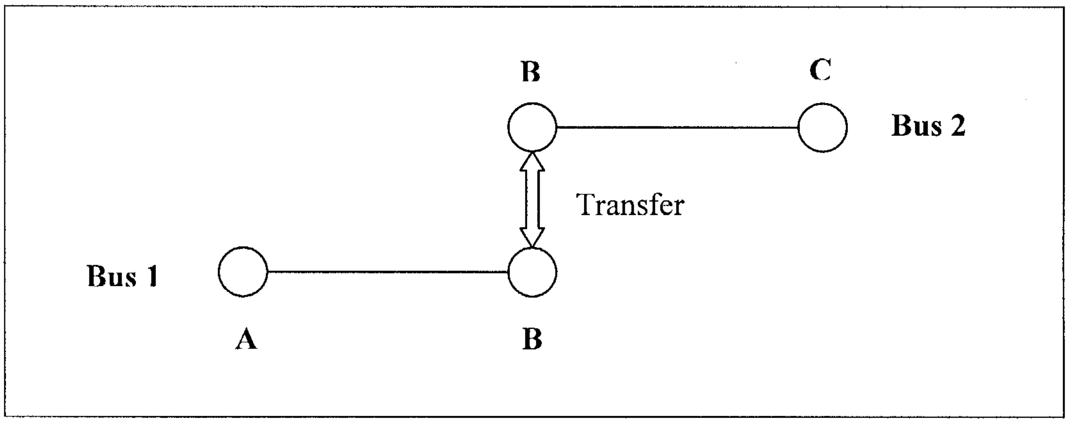


Figure 3-6 Illustration of the failure of "intermediate node property"

In the simple network shown in Figure 3-6, there are three bus stops, A, B and C. Bus 1 stops at A and B and Bus 2 stops at B and C. There is only one path from A to C, ABC. The length of this path, in the sense of connectivity, is 1, because one transfer is needed at B. However, both AB and BC have the same length 0, which violates the property stated above. The reason why the property fails is that the intermediate node itself introduces additional connectivity cost if a transfer is needed.

To adapt the standard shortest-path algorithm and overcome this problem, it is necessary to change the underlying network into one that satisfies the above property. For example, one way is to use virtual links. In this approach, each bus stop is virtually divided into a set of sub-nodes, the number of sub-nodes being equal to the number of bus routes that service this bus stop. Each sub-node belongs to one and only one bus route; and the set of sub-nodes are pairwise connected using virtual links, the weight of the virtual link being equal to the transfer cost. Two sub-nodes generated from different bus stops are connected if and only if they belong to the same bus route and their original bus stops are connected, and the weight of the edge between the two sub-nodes, if they are connected, is equal to the travel cost between their original bus stops. For illustration, consider the diagram in Figure 3-7.

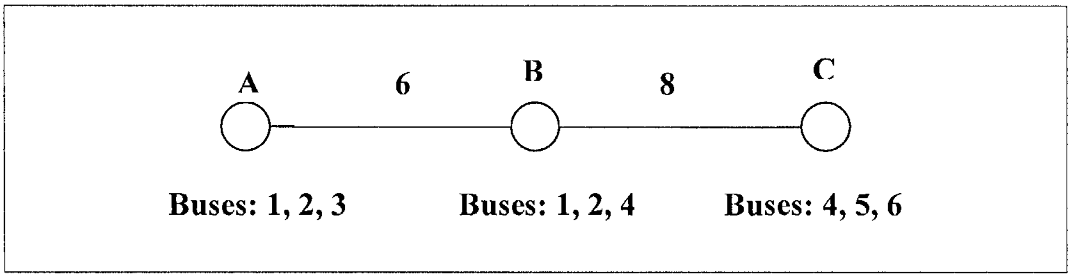


Figure 3-7 Underlying network

In this simple network, there are three bus stops, A, B and C. Each bus stop belongs to three bus routes. The travel cost of AB is 6, while that of BC is 8. We further assume that the transfer waiting cost at any bus stop is always 1. The network after expanding the nodes using virtual links is shown in Figure 3-8.

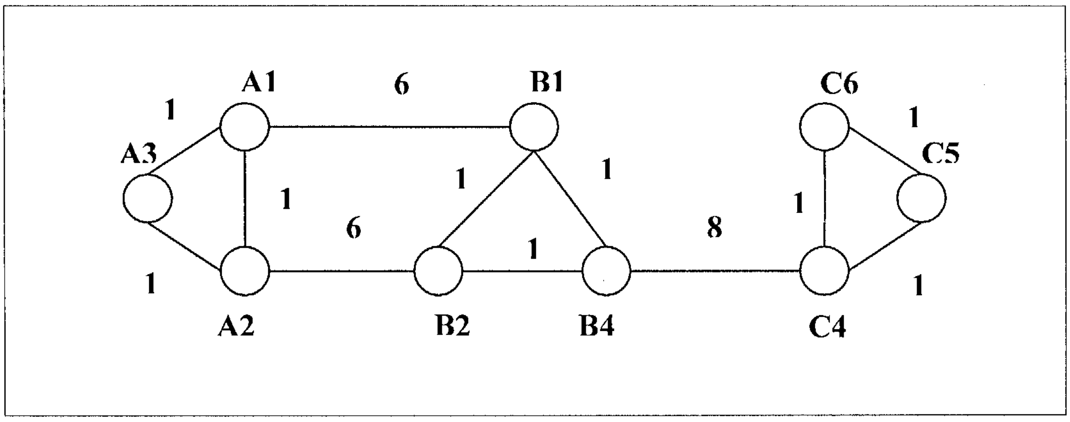


Figure 3-8 Extended network with virtual links

Each vertex of the network denotes a sub-node. For example, A2 represents the sub-node at A that belongs to bus route 2. The weight of the edge between B1 and B2 is 1 as the transfer waiting cost is assumed to be 1. A2 and B2 are connected because they belong to the same bus route and bus stops A and B are connected; and the weight of the edge between A2 and B2 is 6 for the travel cost between A and B is 6.

It is not hard to verify that the extended network satisfies the property that is necessary for the successful application of standard shortest-path algorithms like Dijkstra's Algorithm. For instance, the shortest path between A1 and C5 is A1-B1-

B4-C4-C5, the cost of which is exactly the minimum cost 16 to travel from Stop A taking Bus 1 to Stop C and taking Bus 5. To find the shortest path between two bus stops is just a little bit more complicated. Suppose Stop A is expanded to sub-nodes A_i and B to sub-nodes B_j. The shortest path between A and B is simply the minimum among the shortest paths between all pairs A_iB_j. In the above example, one of the shortest paths between A and C is A1-B1-B4-C4.

By adjusting the edge weight accordingly, the virtual link approach can answer all kinds of shortest path problems with bus transfer involved. For example, to minimise the number of transfers, simply set all the travel costs to be 0 and all transfer costs to be 1; to minimise total travel time, set all the travel costs to be the real in-vehicle travel time and the transfer costs to be the estimated transfer time. However, one major difficulty with the virtual link approach is the computational complexity. From the above example, one can see that a simple underlying network often produces a much more complicated extended network, which dramatically increases the complexity in terms of both time and space. It also makes the implementation much harder. Therefore, at this stage, this approach has not been implemented, but it can be used in the future work of this study. For connectivity problem alone, a much simpler algorithm was found and fully implemented. The algorithm is described below.

Define D (a, b) as the number of transfers between two bus routes, a and b. D(x, x), the number of transfers from one bus route to itself is naturally 0. Denote all the bus routes that pass bus stop S by S_i. Then a crucial observation is that for any pair of bus stops S and T, $D_{S,T} = \mathrm{Min}(D(S_i, T_j))$ which is the minimum number of transfers from bus stop S to bus stop T. The observation is true because if $D_{S,T} < \mathrm{Min}(D(S_i, T_j))$, then the sequence transfers for $D_{S,T}$ gives an even smaller $D(S_i, T_j)$ for some i and j. So instead of directly computing the number of transfers between any pair of bus stops, one can first compute a matrix of minimum numbers of transfers between any pair of bus routes, and then using this matrix to compute the number of transfers between any pair of bus stops.

An MRT route is simply treated as one of the bus routes. MRT stations are also treated in the same way as bus stops. However, a preparation work needs to be done before integrating MRT into the public transport network due to the lack of information on what buses stop at which MRT stations. The bus stops which are within good accessibility distance (231 metres in airline distance) to MRT stations are considered to be available for transfer to MRT. An MRT station is considered to have the available transfer option to the bus services which stop at the bus stops within good accessibility distance (231 metres in airline distance) to the station. Therefore, the public transport network prepared for evaluation of connectivity comprises 3,555 bus stops and 262 bus services and the MRT service. The relationship between bus stops and public transport services is listed in Table 3-4, which shows five bus stops as an example.

Define an undirected graph G based on Table 3-4 in the following way:

- Represent each bus service by a vertex,
- Connect any two vertices if and only if their corresponding bus services are connected.

Table 3-4 Public transport services available at each bus stop (Partial)

Bus Stop ID	SBS Service	TIBS Service	MRT
...	...	...	...
315	*s56,s105,s153,s232	**t167	y
316	s56,s105,s153	t167	y
317	s232		n
318	s232		n
319	s232,s235		y
...	...	...	...

**s denotes that bus service is offered by SBS Transit Ltd.*
*** t denotes that bus service is offered by TIBS Ltd.*

Computing the connectivity matrix among bus routes is equivalent to finding all-pair shortest paths of G, which can be easily solved using Floyd-Warshall algorithm (Cormen et al., 2001). Table 3-5 lists a few bus services together with each connecting services; and Figure 3-9 shows part of the connectivity matrix among bus services.

Table 3-5 Public transport service connectivity (Partial)

Service number	Services available for transfer
*s2	s10,s100,s103,s105,s106,s107,s111,s12,s123,s124,s13,s130 …
s3	s124,s125,s131,s186,s21,s7
s5	s100,s103,s105,s106,s107,s111,s12,s123,s124,s13,s130,s131 …
…	…
**t382	s51,s55,s61,s65,s66,s67,s7,s8,s854,s93,s94,t385,t61 …
…	…
MRT	s2,s5,s7,s9,s10,s12,s13,s14,s15,s16,s20,s21,s23,s24,s25,s26…

**s denotes that bus service is offered by SBS Transit Ltd.*
*** t denotes that bus service is offered by TIBS Ltd.*

$$
\begin{array}{c|ccccccccc}
 & s2 & s3 & s5 & s6 & s7 & s8 & s9 & s10 & \cdots \\
s2 & 0 & 2 & 1 & 2 & 1 & 2 & 1 & 1 & \cdots \\
s3 & 2 & 0 & 2 & 2 & 1 & 2 & 2 & 2 & \cdots \\
s5 & 1 & 2 & 0 & 2 & 1 & 2 & 2 & 2 & \cdots \\
s6 & 2 & 2 & 2 & 0 & 2 & 2 & 2 & 2 & \cdots \\
s7 & 1 & 1 & 1 & 2 & 0 & 2 & 2 & 2 & \cdots \\
s8 & 2 & 2 & 2 & 2 & 2 & 0 & 1 & 2 & \cdots \\
s9 & 1 & 2 & 2 & 2 & 2 & 1 & 0 & 1 & \cdots \\
s10 & 1 & 2 & 2 & 2 & 2 & 2 & 1 & 0 & \cdots \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots
\end{array}
$$

Figure 3-9 Connectivity matrix for bus service

The bus stops connectivity is evaluated twice. The first evaluation takes MRT into consideration so that an integrated public transport network is assessed. In the second evaluation, only buses are considered, so transfers to MRT are assumed impossible. Therefore, a comparison between public transport network with and without MRT can be done. To facilitate the two evaluations, two connectivity matrices among bus services are actually constructed, one with access to MRT, the other one without.

It is also assumed that the trip from bus stop A to bus stop B is the same as the trip from bus stop B to stop A. A total of 6,317,235 trips are considered in this evaluation since there are 3,555 bus stops that have information about what bus services stop there. The number of possible interchange is: (3,555*(3,555 - 1) / 2 = 6,317,235). For each bus stop to bus stop pair (A_i, B_j), the number of transfers needed to change bus from A_i to B_j can be found by looking up the connectivity matrix. The minimum of all those entries is obviously the minimum number of transfers we need to take to travel from stop A to stop B. Thus, the connectivity of public transport network with MRT and without MRT is obtained. The procedure is also illustrated in Figure 3-10 and the result is shown in Table 3-6 and Figure 3-11.

The depth of a transit network is defined as the maximum number of transfers among all pairs of bus stops. From the tables and figures, the depth of the current public transport network is three without MRT; or two if transfers to MRT are considered. That is to say, without MRT sometimes it is necessary to transfer three times when going from one place to another, but only two transfers are necessary if MRT is taken into account. Generally, two or fewer transfers are considered acceptable. There are only less than 1% pairs of bus stops which need three transfers when travelling between them if not considering MRT. With MRT, all the trips between any pair of bus stops can be finished with no more than two transfers. However, only less than 14% trips can be made without transfer, which is less than satisfactory.

Table 3-6 Connectivity among bus stops

	without MRT		with MRT	
Number of Transfers	***Trips***	***Percentage***	***Trips***	***Percentage***
0	856,624	13.56	870,936	13.79
1	4,279,258	67.74	4,358,499	68.99
2	1,165,921	18.46	1,087,800	17.22
3	15,432	0.24	0	0
Total	6,317,235	100	6,317,235	100

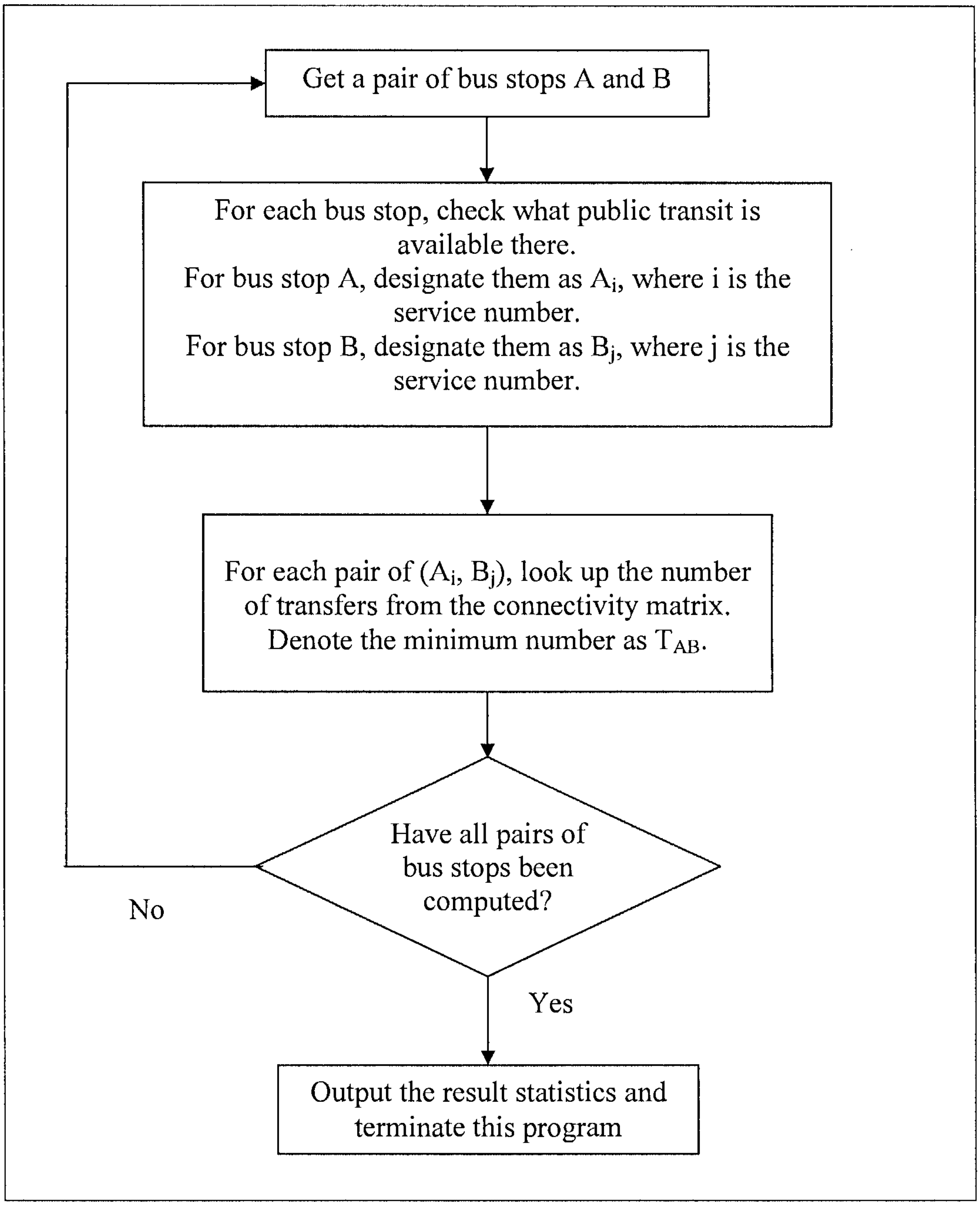


Figure 3-10 Procedure of connectivity measurement

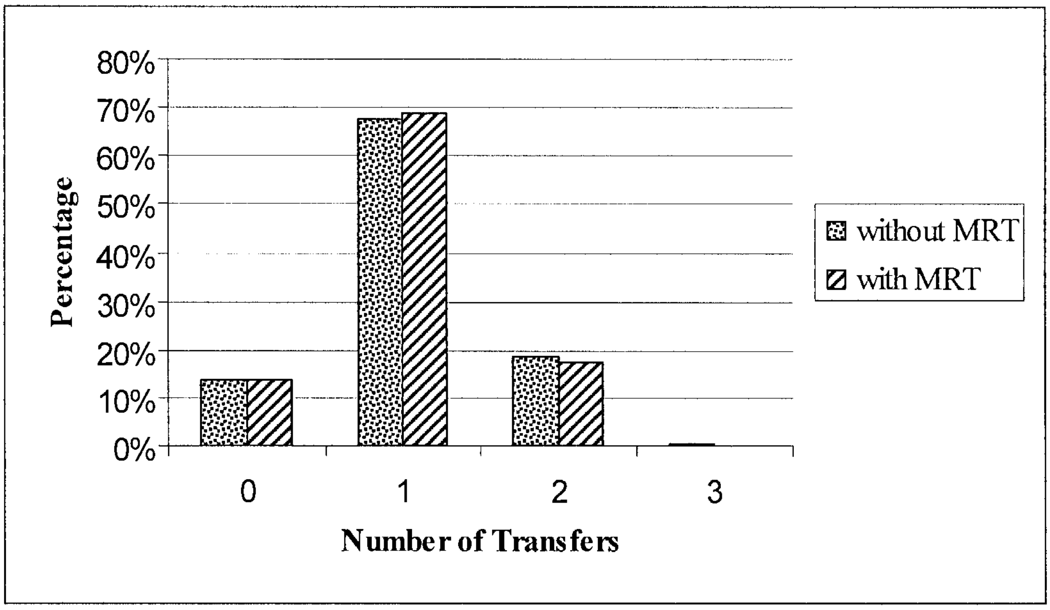


Figure 3-11 Connectivity of public transport network of Singapore

3.4 Travel time analysis

Travel time is also a very important indicator of the quality of a transport system. In particular, this section studies travel time of work trips. There are two main reasons for selecting travel time to place of employment as the destination. Firstly, work trips constitute a large proportion of the overall travel demand. Study of travel time to employment reveals the overall picture of transportation quality. Secondly, performance of the transport system during the peak period has most economic significance. Work trips are the major component of the total trips during peak period. Work trips usually have fixed origins and destinations, as people do not change jobs often. All the necessary demographic data are available from employment departments. Travel time is only measured up to inter-zone level, since the point-to-point measurement would require too detailed input.

3.4.1 Verification of LTA travel time data

Verification of LTA's travel time data was first made. A sample of zones was selected. The travel times among the sample zones were calculated as the sum of walking time, waiting time and in-vehicle travel time according to the transport network (Thevenin, 2001). Lastly, the results were compared with the data provided by LTA. The procedure is explained below.

Verifying the inter-zone travel time by bus

Ten zones were selected randomly for comparison. They are Zones: 50113, 50232, 50602, 50817, 51824, 54413, 54922, 55115, 55605 and 55715, illustrated in Figure 3-12.

Inter-zone travel time by bus is calculated as follows (taking travel time by bus between Zone 50113 and Zone 50232 as an example):

(i) Identify the bus stops in each zone. These are: bus stops 2085 and 2101 in Zone 50113; bus stops 2060 and 2761 in Zone 50232.

(ii) Compute the shortest in-vehicle travel time by bus from any bus stop in Zone 50113 to any bus stop in Zone 50232 with the shortest path function of ArcView GIS 3.2a. The average speed for a bus service is taken as 18 km/h, the same input speed used by LTA. The result is shown in Table 3-7.

Table 3-7 Travel time (min) by bus between bus stops

Zone	50113		
	Bus stop	***2085***	***2101***
50232	***2060***	37.36	39.32
	2761	38.43	40.39

(iii) Select all possible travel routes for each pair of zones. For example, four routes are available for travelling from Zone 50113 to Zone 50232. They are travelling from bus stop 2085 to bus stop 2060 or 2761 and travelling from bus stop 2101 to bus stop 2060 or 2761. Therefore the average in-vehicle time between this pair of zones is calculated as follows:

$$\begin{aligned} Zivt_{50113,50232} &= (Bivt_{2085,2060} + Bivt_{2085,2761} + Bivt_{2101,2060} + Bivt_{2101,2761}) / 4 \\ &= (37.36 + 38.43 + 39.32 + 40.39) / 4 \\ &= 38.875 \end{aligned}$$

where:

$Zivt_{i,j}$: in-vehicle travel time between zone i and j

$Bivt_{i,j}$: in-vehicle travel time between bus stop i and j

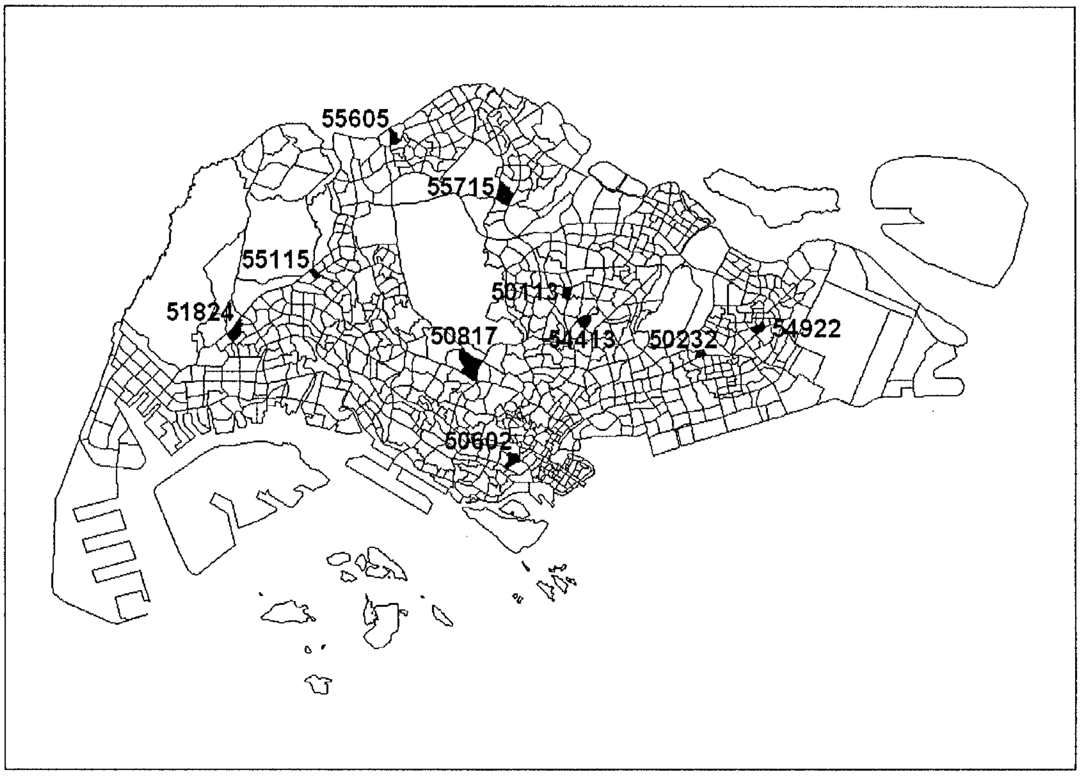


Figure 3-12 Location of zones selected for comparison of travel time by bus

(iv) Calculate average walking time from the centre of each zone to all the bus stops within the zone. Walking speed is taken as 4.5 km/h which is also the value used by LTA. The average walking time from the centre to a bus stop in Zone i is denoted by wal_{i}.

(v) Assume the waiting time for bus (W) is constant. Since the average frequency of a bus service in the morning peak hours is 10 minutes (SBS, 2003), the waiting time W is assigned as 5 minutes for all zones.

(vi) Take the travel time between centres of zones as the inter-zone travel time. Hence

$$T_{500113,\,50232} = Zivt_{50113,\,50232} + wal_{50113} + wal_{50232} + W$$

(vii) Iterate the above steps until travel times for all pairs of zones are computed.

Figure 3-13 shows the flow diagram of the above procedure. The result is shown in Appendix C. Since the assumption that travel time from Zone i to Zone j is equal to travel time from Zone j to Zone i, the result consists of only 45 unique travel times ((10*10-10)/2=45). The comparison is depicted in Figure 3-14.

We can see from Figure 3-14 that generally LTA data are in agreement with the calculated data. Therefore, LTA data are deemed reliable and acceptable for accessibility evaluation.

Verifying the inter-zone travel time by public transport

In order to calculate travel time by public transport, 5 zones that have MRT stations were randomly selected for comparison. They are Zones: 50203, 51307, 51407, 55202 and 55607, illustrated in Figure 3-15. The calculation is similar to the calculation of travel time by bus. The algorithm is as follows:

(i) Select the MRT station within each zone.

(ii) Find the in-vehicle travel time between stations from the travel time table (see Appendix D) available at the website of SMRT (2003).

(iii) Assume that all passengers only walk to the MRT stations (this is done in order to simplify the calculation because the purpose here is only to verify the reliability of travel time data obtained from LTA). Calculate the average walking time to a MRT station as the walking time needed to walk from the centre of each zone to its corresponding MRT station. The walking speed is also taken as 4.5 km/h which is the value used by LTA.

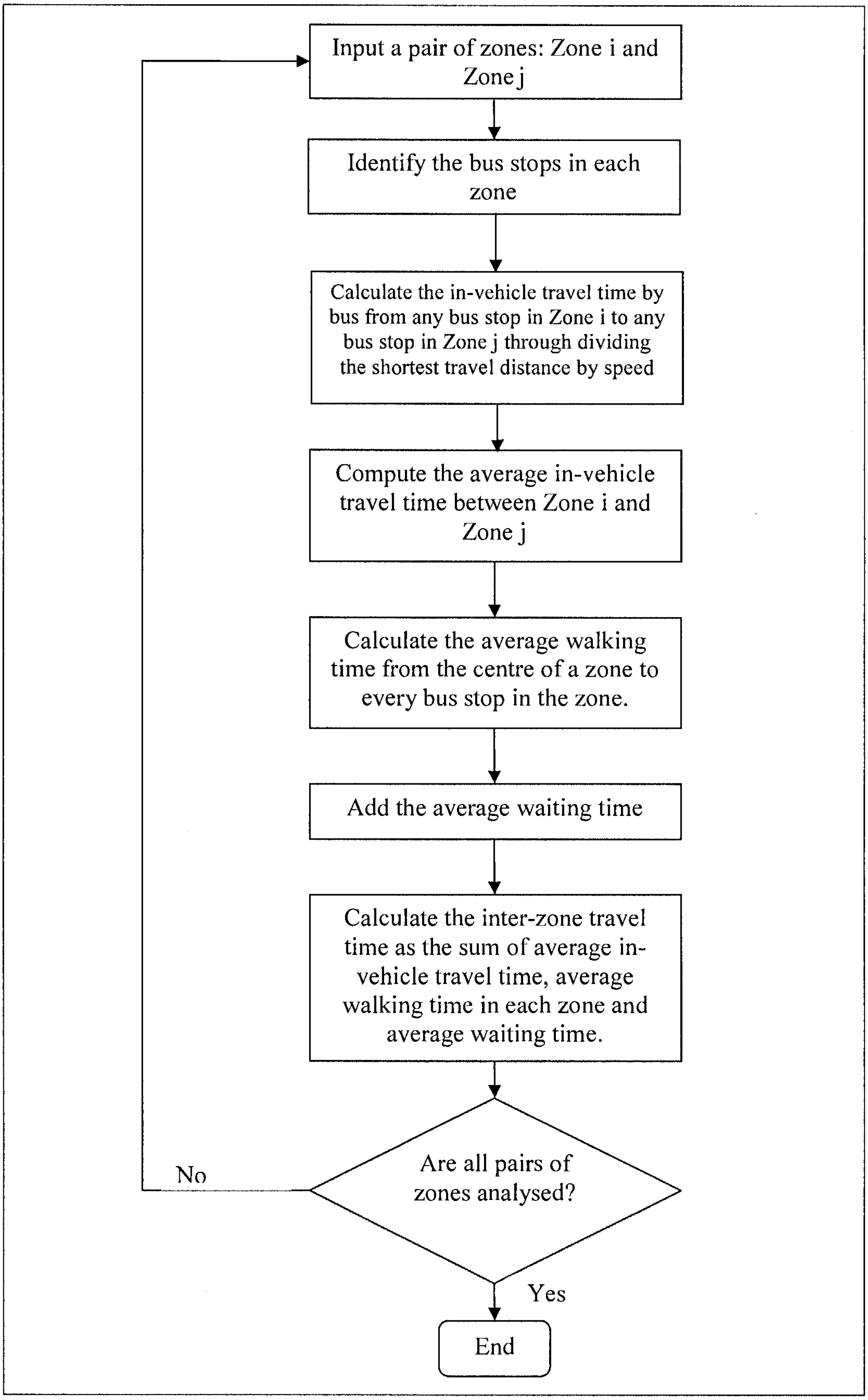


Figure 3-13 Procedure of calculation of inter-zone travel time by bus

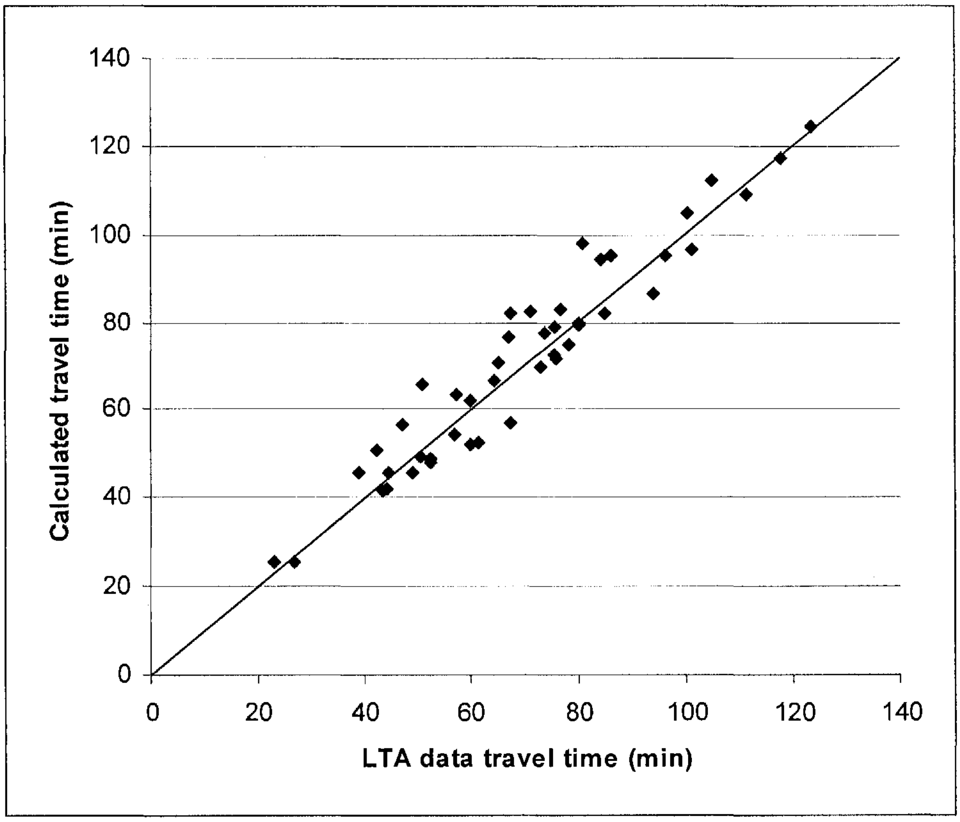


Figure 3-14 Comparison of travel time by bus between LTA data and calculated using the bus network

(iv) Assume that average waiting time in the morning peak hour is 2 minutes according to the information at the website of SMRT (2003).

(v) The total travel time is the sum of in-vehicle (MRT) travel time, walking time in each zone and the average waiting time.

(vi) Iterate the above steps until the travel times for all pairs of zones are computed.

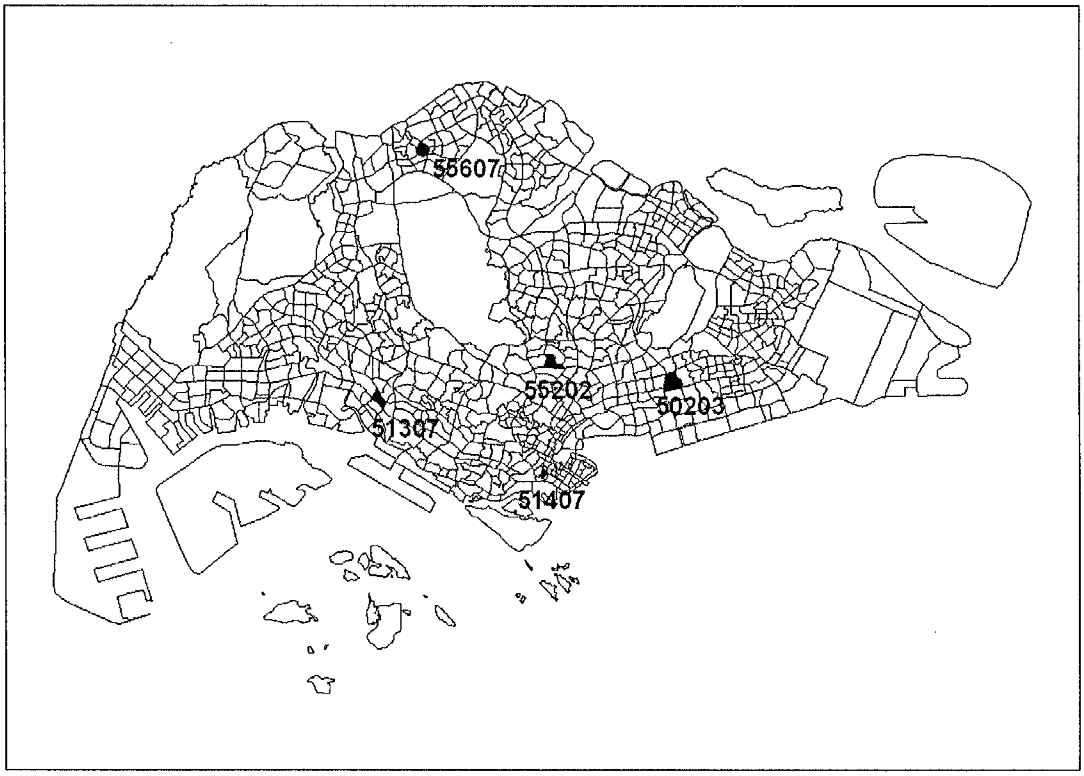


Figure 3-15 Location of zones selected for comparison of travel time by public transport

Figure 3-16 is the flow diagram of the above procedure. The result is shown in Table C2 in Appendix C. Due to the assumption that travel time from Zone i to Zone j is equal to travel time from Zone j to Zone i, the result consists only 10 unique travel times ((5*5-5)/2=10). The comparison is presented in Figure 3-17.

Figure 3-17 shows that generally the LTA data are in agreement with the calculated data and can therefore be adopted for accessibility evaluation.

One problem with LTA data is that they do not account for intra-zone travel time. To make the measurement of accessibility complete, the intra-zone travel time is estimated in the following way. Consider Zone i. Let T_{ij} be the average travel time from Zone i to any other Zone j. Select three minimum values from among all T_{ij} and denote them as T_a, T_b and T_c. The intra-zone travel time for Zone i, T_{ii} is computed with Equation 3-1 and this value will be used for Equation 3-2.

$$T_{ii} = (T_a + T_b + T_c) / 3 * (2 / 3) \qquad \text{Equation 3-1}$$

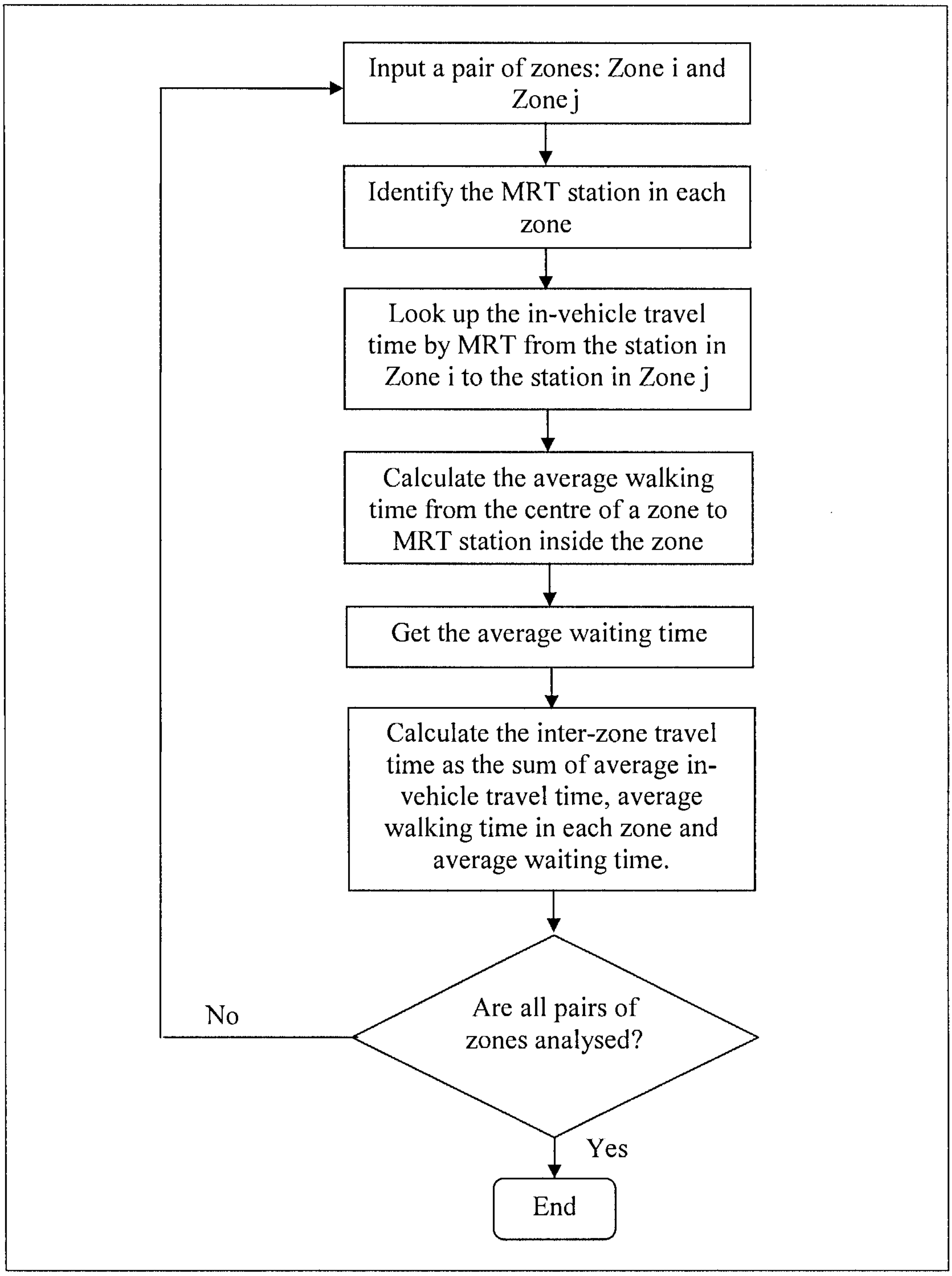


Figure 3-16 Procedure of calculation of inter-zone travel time by MRT

It can be shown numerically (and theoretically by integration) that for a square the average distance between its internal points is equal to 0.5 of its side length. Therefore, for a regular square zoning grid the equation should have a factor of ½.

However, 2/3 is used as a conservative assumption to account for the fact that the zones are all irregular in shape.

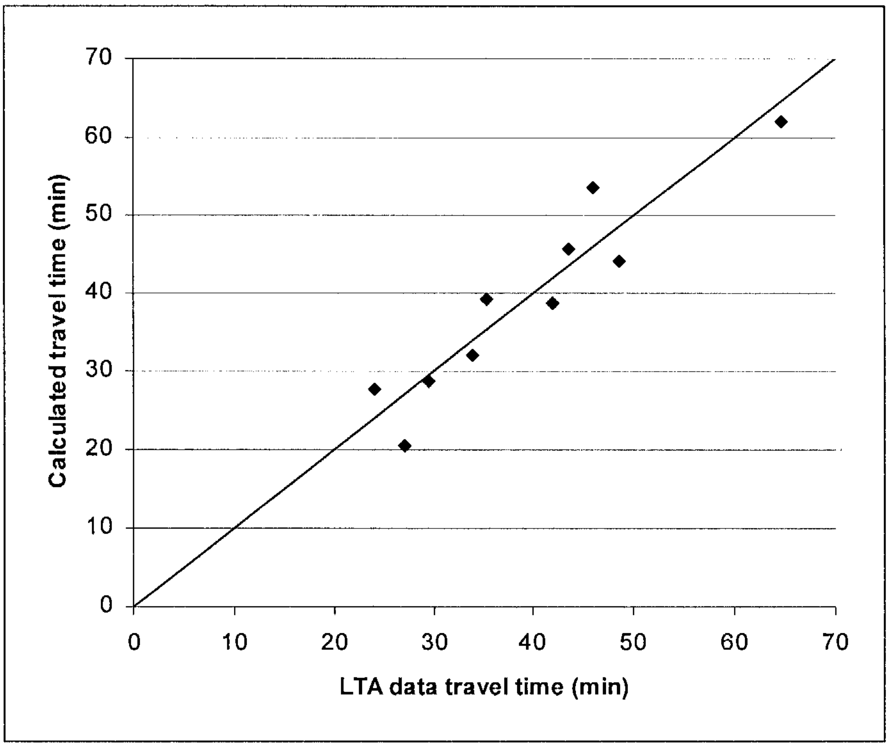


Figure 3-17 Comparison of travel time by public transport between LTA data and calculated using the MRT network

3.4.2 Computation of average travel time to employment by bus

The average travel time to all employment zones by bus, ATE_i^{bus} is used as the measure of accessibility of bus services for Zone i. The following equation gives the average travel time to employment zones:

$$ATE_i^{bus} = \frac{\sum_j E_j t_{ij}^{bus}}{\sum_j E_j} \qquad \text{Equation 3-2}$$

where:

ATE_i^{bus} : average travel time to employment from zone i by bus

E_j : employment in zone j

t_{ij}^{bus} : travel time from zone i to zone j by bus

The map of ATE^{bus} for the whole of Singapore is depicted in Figure 3-18.

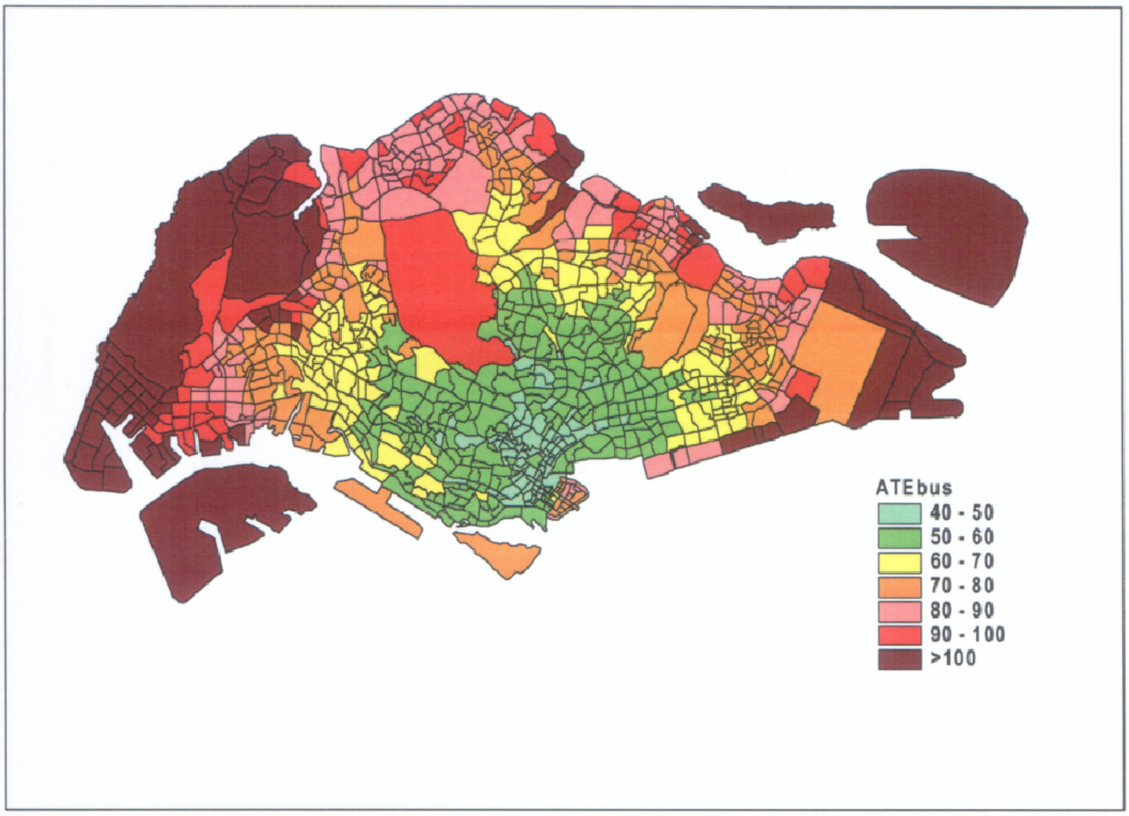


Figure 3-18 Map of ATE for bus service (minute)

3.4.3 Computation of average travel time to employment by public transport

Similarly the accessibility by public transport for Zone i is measured using the average travel time to place of employment, ATE_i^{pub}. It is calculated using the following equation:

$$ATE_i^{pub} = \frac{\sum_j E_j t_{ij}^{pub}}{\sum_j E_j} \qquad \text{Equation 3-3}$$

where:

ATE_i^{pub}: average travel time to employment zone from zone i by public transport (bus and MRT)

t_{ij}^{pub}: travel time from zone i to zone j by public transport (bus and

MRT)

The current situation of ATE^{pub} for whole Singapore is shown in Figure 3-19. It is apparent that the zones near MRT lines have relatively lower average travel times to employment, which is in part attributed to shorter travel time by MRT. However, the difference between using and not using MRT is not so dramatic.

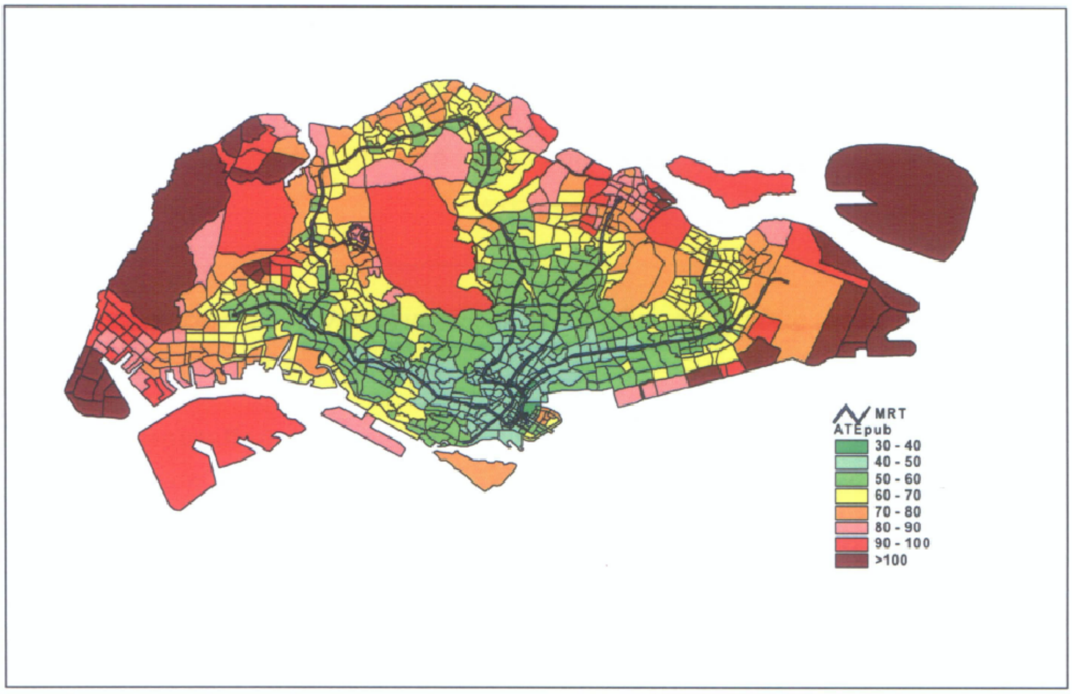


Figure 3-19 Map of ATE for public transport (minute)

A comparison between ATE by bus and by public transport is illustrated in Figure 3-20. The travel time by bus to employment averaged for all origin zones is 71.75 minutes. Half of the 927 zones have average travel times lower than 70 minutes. The travel time by public transport to employment averaged for all origin zones is 64.18 minutes. Even with MRT, only half of zones can travel to other zones within one hour on average. This implies that passengers do not have good access to MRT. This is a significant problem and should be studied further.

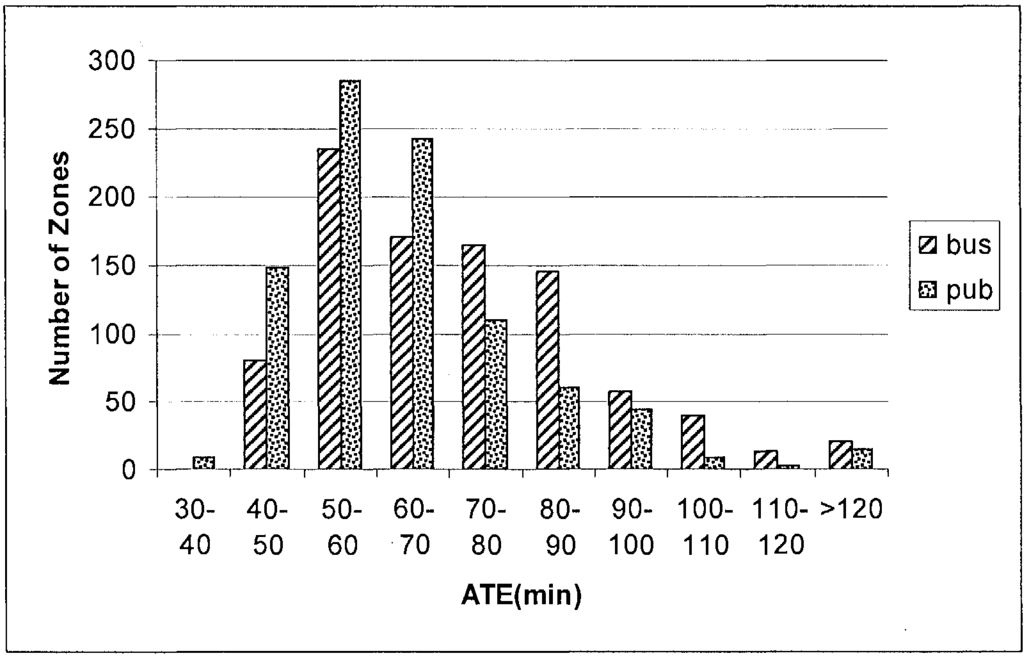


Figure 3-20 Distributions of ATE by bus and public transport

3.4.4 Computation of average travel time to employment by private car

For comparison, the accessibility by private car for Zone i is calculated using the average travel time to employment by car, ATE_i^{car}. The corresponding equation is:

$$ATE_i^{car} = \frac{\sum_j \mathrm{E_j t_{ij}^{car}}}{\sum_j \mathrm{E_j}} \qquad \text{Equation 3-4}$$

where:

ATE_i^{car} : average travel time to employment from zone i by car

t_{ij}^{car} : travel time from zone i to zone j by car

The current situation of $\mathrm{ATE}^{\mathrm{car}}$ for the whole of Singapore is depicted in Figure 3-21. Average travel times to employment for bus, public transport and car are compared in Figure 3-22. The advantage of car is very apparent. 90% of zones have the ATE by car less than 40 minutes. But almost no zone has ATE by bus or public transport less than 40 minutes. There is a long way to go in order to improve the attractiveness of public transport. When comparing ATE by bus and public transport, the difference is not very much. Half of the zones are with ATE by bus less than 70 minutes. Half of the zones have the ATE by public transport less than 60 minutes. 90% of zones' ATEs by bus are less than 100 minutes. 90% of zones' ATEs by public transport are less than 90 minutes. Therefore, how to take advantage of rail transit should be studied further.

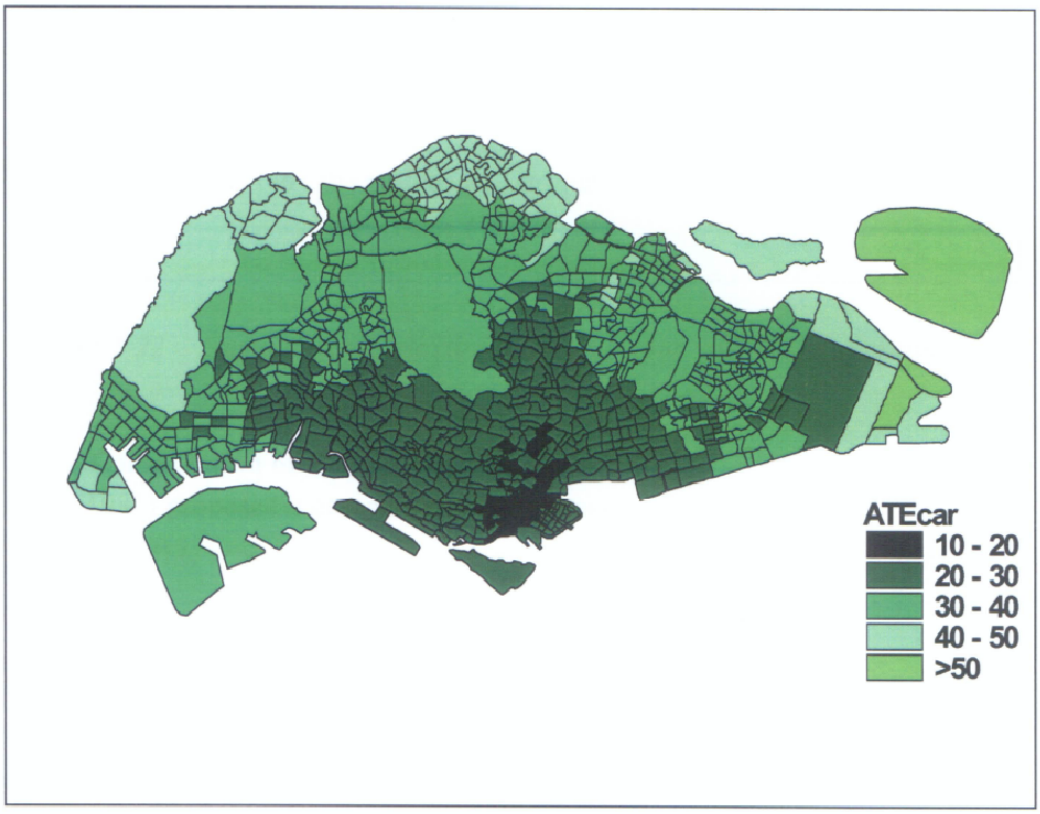


Figure 3-21   Map of ATE for car (minute)

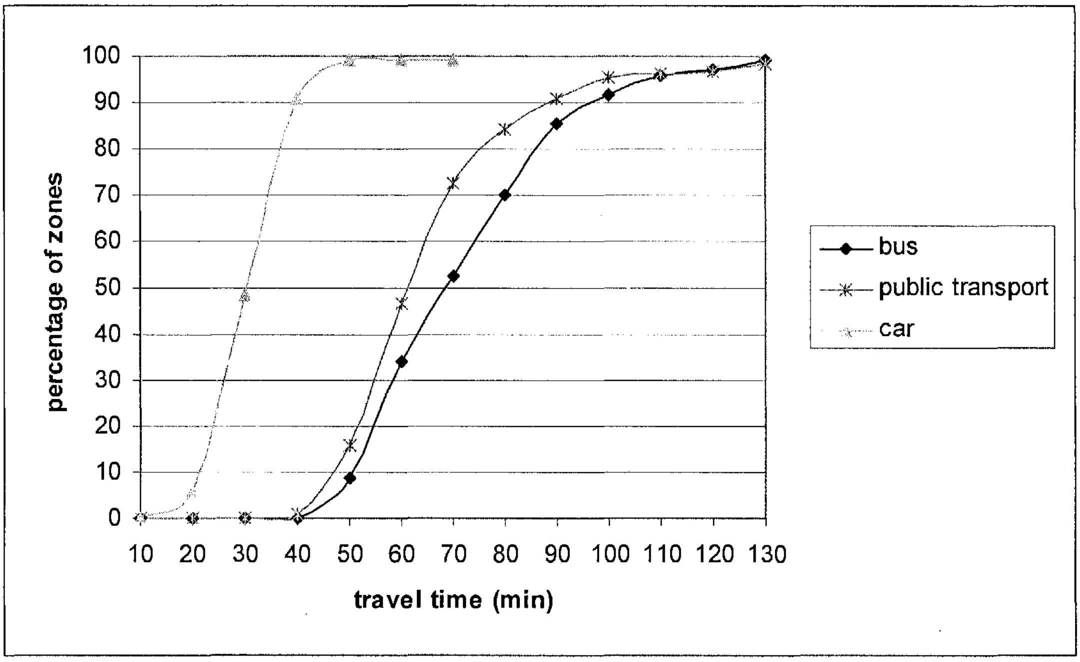


Figure 3-22 Cumulative distributions of ATE by bus, public transport and car

3.4.5 Computation of average travel time ratio

Jones (1981) proposed a comparison between accessibility of public transport and private car. This is applied here for measuring travel time ratio for each zone. This ratio is:

$$TTRB_i = \frac{ATE_i^{bus}}{ATE_i^{car}} \qquad \text{Equation 3-5}$$

where:

$TTRB_i$: travel time ratio of bus service for zone i

$$TTRP_i = \frac{ATE_i^{pub}}{ATE_i^{car}} \qquad \text{Equation 3-6}$$

where:

$TTRP_i$: travel time ratio of public transport for zone i

The results are shown in Figure 3-23 and Figure 3-24. It is interesting to note that zones with the lowest level of ATE by bus and public transport do not have the lowest levels of TTR. The reason is that the zones with the lowest level of ATE by bus and public transport also have the lowest level of ATE by car. Therefore, the ratio (TTRB and TTRP) calculated as the quotient of these values may not be the lowest.

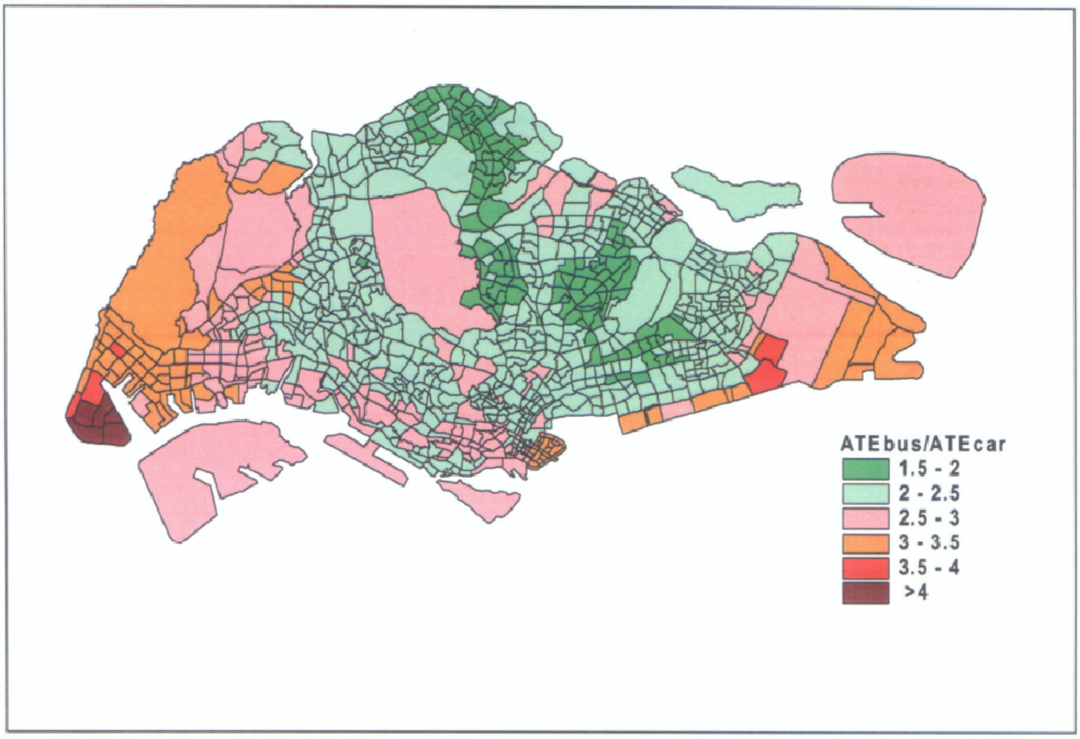


Figure 3-23 Map of TTRB

TTRB and TTRP are compared in Figure 3-25. The range for TTRB is 1.71 to 4.52. The mean TTRB is 2.40. The range for TTRP is 1.24 to 4.05. The mean TTRP is 2.16. The mean TTRP is not much less than the mean TTRB. However, with the MRT, 170 more zones have the average travel time less than two times that of ATE by car, compared with the situation of travelling only by bus services.

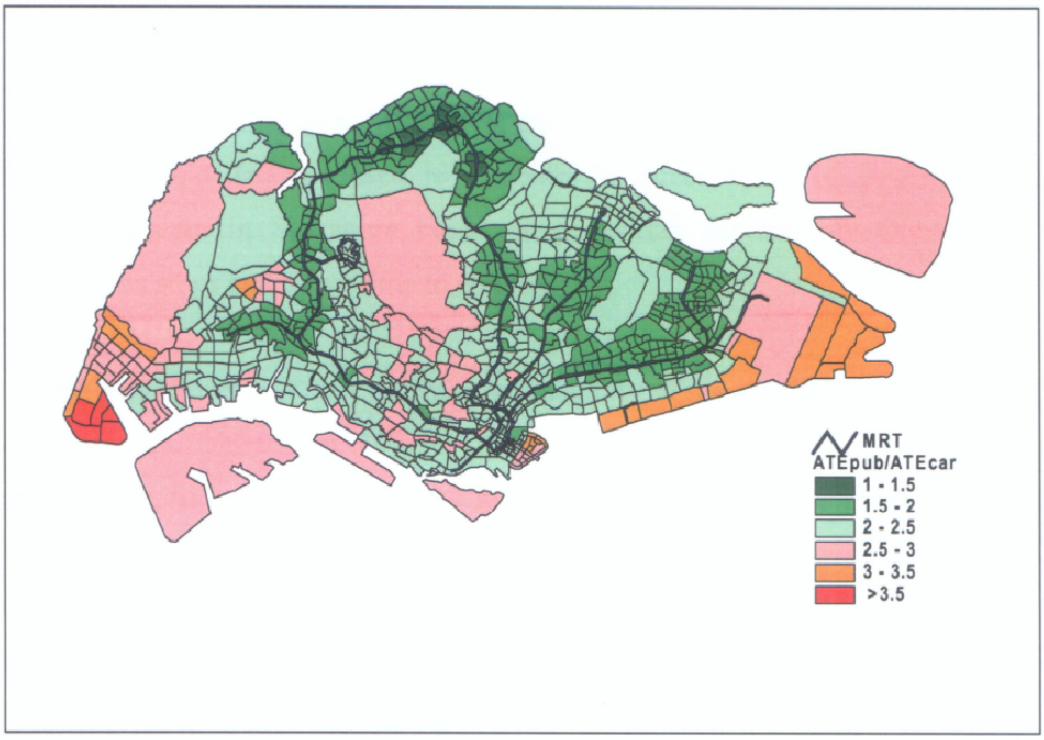


Figure 3-24 Map of TTRP

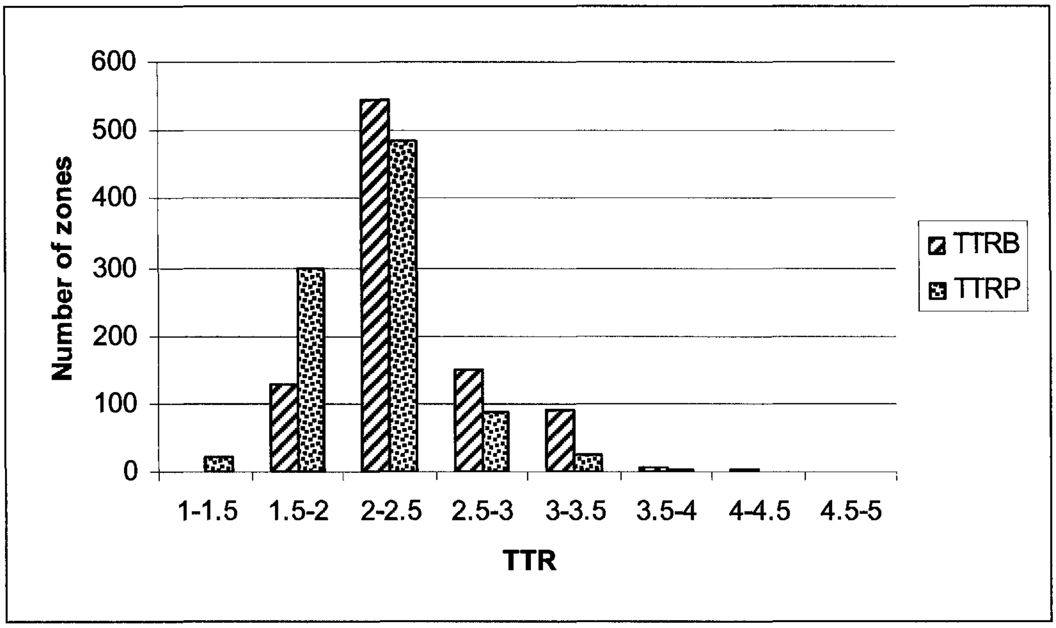


Figure 3-25 Distribution of TTRB and TTRP

3.5 Summary

The basic data used in this study are introduced and described in this chapter. They are: road network, bus stops, MRT lines, MRT stations, LRT lines, LRT locations, buildings. These data are used to evaluate the accessibility of current public transport system in Singapore based on pedestrian accessibility, connectivity between bus stops and travel time to employment.

The analysis shows that the pedestrian accessibility of the current transport system is quite good. More than 70% of buildings have good access to bus stops. Nevertheless, indications are that the number of bus stops can be greatly reduced without worsening the pedestrian accessibility, which is very desirable. This is examined in the next chapter. However, only less than 3% of buildings have good access to MRT stations. This is mainly due to the larger MRT station spacing.

The current connectivity among bus stops is overall satisfactory because more than 99% of bus stop to bus stop trips can be accomplished with no more than 2 transfers if MRT is taken into consideration. Even without considering MRT, the number of transfers needed would only increase by one.

Based on the analysis of inter-zone travel times, it is apparent that zones near MRT lines have relatively low average travel time to employment. Average travel times increase with the increase of the distance to MRT lines. Hence improving the public accessibility to MRT and integrating bus services with it can hopefully enhance the quality of public transportation in Singapore.

CHAPTER 4 OPTIMISATION OF BUS STOP LOCATION

Due to the large size of the bus system, optimisation cannot be accomplished all at once. Instead, the whole project is divided into two phases. Phase I focuses on optimising the location of bus stops in terms of accessibility without considering bus routes, Phase II optimises bus routes based on the set of bus stops with fixed locations. This chapter focuses on optimisation of bus stop location and is organised as follows. Section 4.1 states the basis of this study; Section 4.2 discusses the location of bus stops in detail; Section 4.3 concludes this chapter. Method for optimisation of bus routes will be discussed in Chapter 5.

4.1 Basis of project study

This case study considers the whole area of Singapore. To design an improved transit network, comprehensive infrastructure and demographic data are assembled. These data are all in ArcView format and include:

- Road network of Singapore
- Bus stops with information on what bus services stop there
- MRT lines
- MRT stations
- LRT lines (treated the same as MRT lines)
- Buildings with classification

This project evaluates the bus network and tries to redesign bus stop locations to improve its efficiency. The MRT system is considered a fixed infrastructure. Travel demand for public transport is assumed not to change over a short time. Without detailed data regarding travel demand, the data based on buildings are used as a proxy. All buildings are categorised into several groups, i.e. residential (HDB, condominium and private house); commercial; industry; education etc. Average population can be estimated in individual buildings for different categories.

4.2 Location of bus stops

The arrangement of bus stops certainly affects the pedestrian accessibility to public transport (Tyler, 1999). Moreover, the ridership of public transport increases as the accessibility improves; and growing ridership generates revenue which provides the incentive to the operator to improve the quality of public transport further. It is important to carefully design the locations of bus stops in relation to convenient access. Therefore, it is separated from bus route optimisation and performed before network design.

While MRT stations are considered fixed, placement of bus stop locations are much more flexible. This section concentrates on how to optimise the accessibility of bus stops by finding the best location for them.

A randomised algorithm is proposed to search for good placement by randomly moving bus stops from their current locations. It is described in Section 4.2.1.

4.2.1 Randomised algorithm

In brief, the algorithm works by randomly selecting and moving bus stops until the objective function cannot be further improved. All the bus stops are treated in the same way, regardless of their locations on which side of a road it is situated.

Previous studies show that transportation facilities such as bus stops, etc. should be located within acceptable walking distance from major generators such as homes and offices (Seneviratne, 1985). Therefore, improving pedestrian accessibility of bus stops is a good objective for optimisation of bus stop location. Two different objectives are applied here. One is to maximise the number of houses in good accessibility range. The other is to minimise the number of houses in bad accessibility range. The randomised algorithm is applied twice, each time with a different objective.

When a bus stop gets selected, the algorithm looks for a better position for the stop and moves it to that position. However, this search for better position must be limited to a certain range. In this randomised algorithm, the bus stops are only allowed to move along the road and cannot be moved too far away from their initial position because it is definitely undesirable to have situations where a few bus stops cluster together while there are long distances without a single bus stop along the bus route. Given the average bus stop spacing being 300 metres in Singapore, the maximum moving distance is defined to be 120 metres in each direction. The unit step size is defined to be 30 metres, which means that a bus stop can be moved by 0, 30, 60, 90 or 120 metres up or down the road. In order to space the stops evenly, it is also required that distance between any two neighbouring bus stops is at least 200 metres. In addition, bus stops should not be located nearer than 30 metres from cross roads for traffic concerns. Finally, bus stops are not allowed to be moved out of the road segment.

In order to describe the algorithm clearly, some terms are defined first.

Target bus stop: the bus stop to be moved

Area of interest: a circle centred at the target bus stop. Its radius depends on the objective. With the objective of maximising good accessibility, the radius used is 582 m, which is twice the airline distance defining good accessibility plus the maximum bus stop movement distance (2*231+120=582 m). Under the objective of minimising bad accessibility, the radius is 1,042 m, which is twice the airline distance of the medium accessibility plus the maximum movement distance (2*461+120=1,042 m).

Neighbouring bus stops: bus stops which fall within the area of interest. They are called neighbouring bus stops because they are the only bus stops that can potentially service the same house as the target bus stop.

Neighbouring houses: houses which fall within the area of interest. They are called neighbouring houses because they are the only houses whose accessibility can be possibly affected by moving the target bus stop.

Coverage: houses which are within good/medium distances of the target bus stop, depending on objective, from the target bus stop. Coverage is denoted by C.

Unaffected houses: the houses which fall into the area of interest and are also serviced by some of neighbouring bus stops. Again, the service radius depends on objective. The set of unaffected houses is denoted by U.

Net coverage: the set of houses serviced only by target bus stop, which is denoted as N. The selection of service distance is the same as for unaffected house. It can be calculated as:

$$N = C - (C \cap U) \qquad \text{Equation 4-1}$$

The implementation of the algorithm includes the following four subroutines:

Subroutine **Main**:

1. Generate a permutation sequence of bus stops;
2. Get a new target bus stop according to the order of the sequence or exit if all the bus stops have been exhausted;
3. Call Subroutine **BusStopReposition** to find the best new position for target bus stop;
4. If the new position is not the same as the current position of target bus stop, move it to that position and update the map and database;
5. Go back to step 3.

There are 3 types of permutation sequence considered in Step 1. They will be discussed later.

Subroutine **BusStopReposition:**

1. Call Subroutine **UnaffectedHouses** to compute the unaffected houses of the target bus stop;
2. Find the road along which the target bus stop is placed;

3. Generate a list of candidate positions where the target bus stop can be placed (30, 60, 90, and 120 metres up and down the road from the original position unless the constraints are violated, e.g. too close to cross road or other bus stops);
4. For each candidate position, call Subroutine **NetCoverage** to compute its corresponding net coverage;
5. Return the position with highest net coverage.

Subroutine **UnaffectedHouses:**

1. Find the neighbouring bus stops for the target bus stop;
2. Compute the set of unaffected houses U, and return it to the Function **BusStopPosition**.

Subroutine **NetCoverage:**

1. Compute the coverage C;
2. Calculate net coverage $N = C - (C \cap U)$

The flow diagram of the program is illustrated in Figure 4-1.

Since varying one bus stop may affect the net coverage of the neighbouring bus stops, the final outcome depends on the order in which the bus stops are moved. Three kinds of sequences of moving bus stops are experimented with. The first one simply follows the original ordering of the database. The second one is a random permutation sequence. The third one is the best-first sequence, best in terms of net coverage. To perform it, first run the above procedure one time and record the increase of net coverage for each bus stop; then sort the bus stops according to the recorded net coverage increase in descending order. The resulting ordering is the best-first sequence.

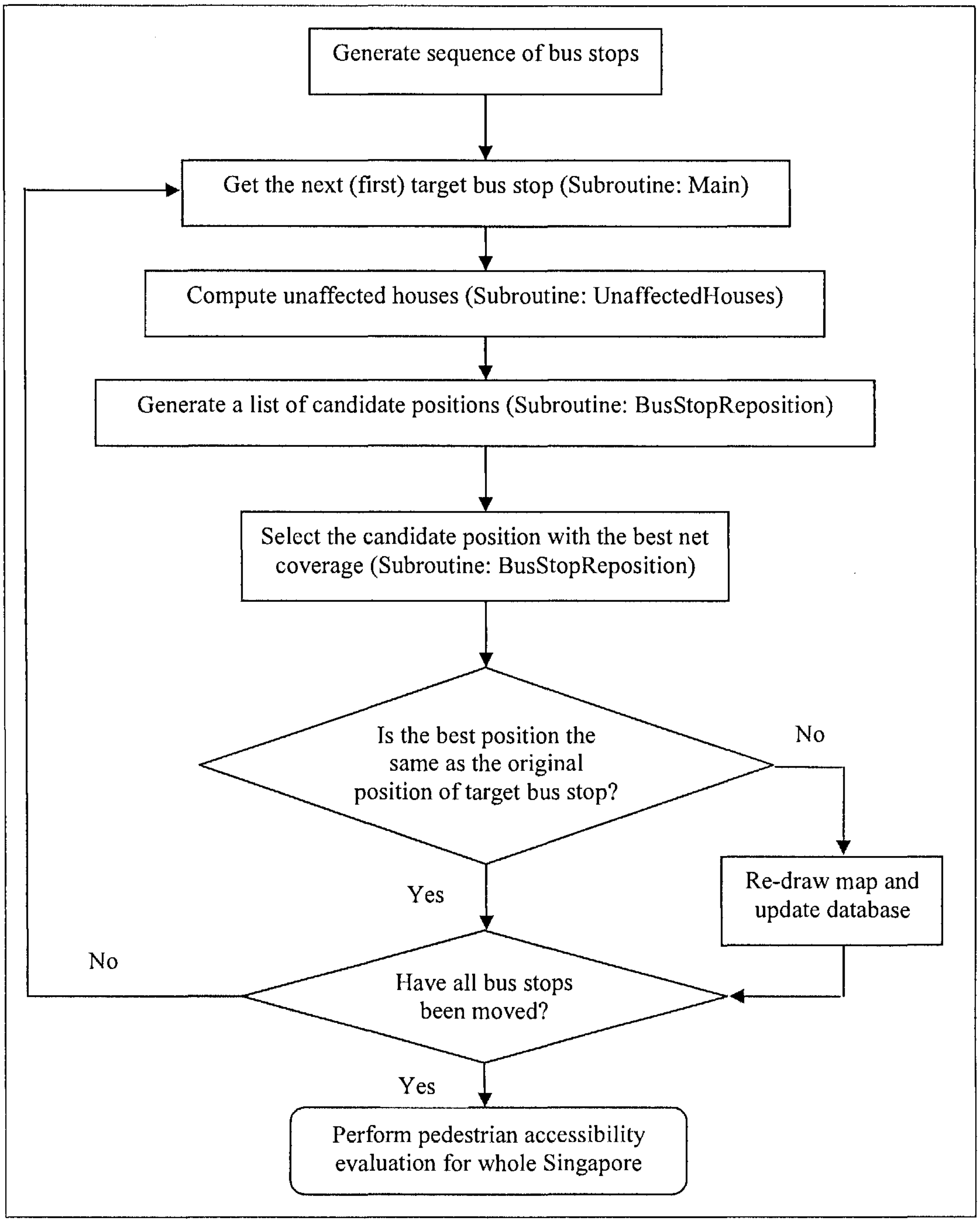


Figure 4-1 *Procedure of moving bus stops*

To help explain the algorithm, one iteration of the computation (under the objective of maximising the number of bus stops with good accessibility) is described in detail below.

a) In subroutine **Main** a new target bus stop is selected. Assume it is Bus Stop 174 (174 is not a bus No. but the bus stop ID). The original location of the target bus stop and its record number are passed to subroutine **BusStopReposition**. Bus Stop 174 is marked with a red star in Figure 4-3.

b) Subroutine **BusStopReposition** firstly passes the position of target bus stop to subroutine **UnaffectedHouses**. In subroutine **UnaffectedHouses**, the neighbouring bus stops (within 582 m) of the target bus stop are selected. To Bus Stop 174, there is only one neighbouring bus stop, which is marked with a blue flag in Figure 4-3.

c) Subroutine **BusStopReposition** calls subroutine **UnaffectedHouses**. **UnaffectedHousess** computes the set of unaffected houses to Bus Stop 174, denoted as U. In Figure 4-3, U includes all the houses that fall within the intersection between the large circle (radius = 582 m) centred at the target bus stop (red star) and the small circle (radius = 231 m) centred at the neighbouring bus stop (blue flag). Those houses are represented as bigger orange points. After U is computed, the control is passed back from **UnaffectedHouses** to **BusStopReposition**.

d) In subroutine **BusStopReposition**, the road to which the target bus stop is attached is found. Here the road ID is 1856. Then **BusStopReposition** generates the candidate positions of the target bus stop. Besides the original position, the candidate positions include 30, 60, 90 and 120 m up and down the road. They are represented by green triangles in Figure 4-2.

e) Each candidate position is further tested to see if it meets all the constraints (e.g. not too close to crossroads or other bus stops). Any candidate position that violates any of the constraints is removed from the candidate list. In this example, all the candidate positions pass the test.

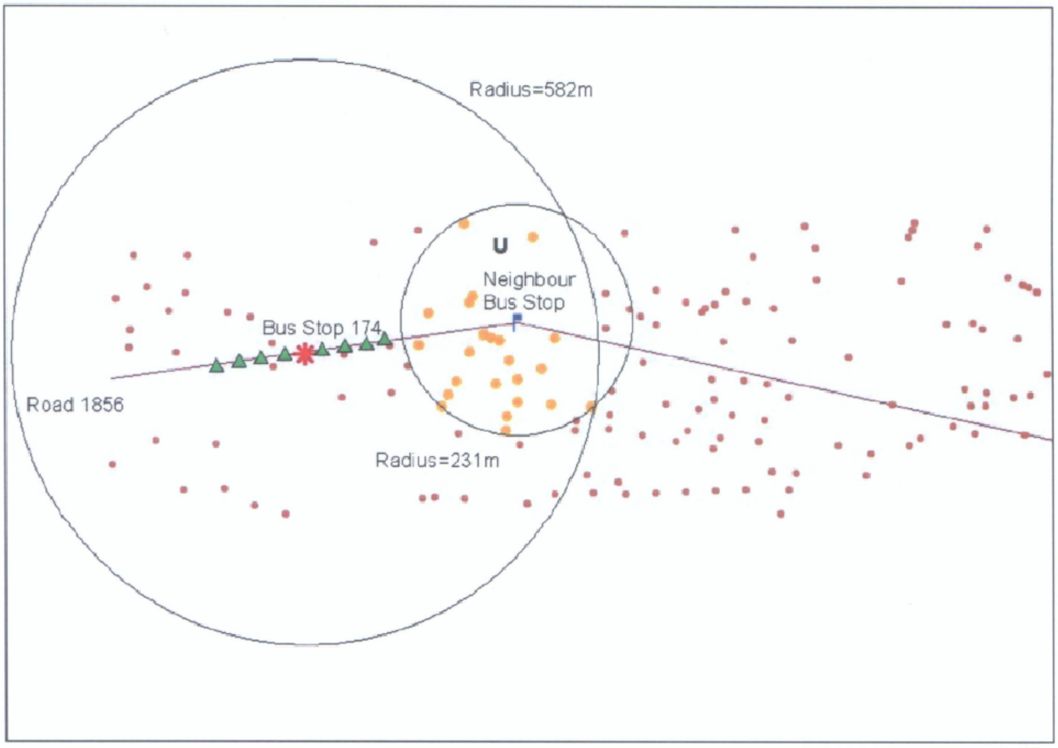


Figure 4-2 Calculation of unaffected houses

f) **BusStopReposition** calls **NetCoverage** to calculate the net coverage, N of each candidate position according to Equation 4.1. The results are $N_{original} = 9$; $N_{R1} = 9$; $N_{R2} = 9$; $N_{R3} = 9$; $N_{R4} = 9$; $N_{L1} = 11$; $N_{L2} = 10$; $N_{L3} = 15$ and $N_{L4} = 13$ (R1~4 denote the four candidate positions to the right of the original position while L1~4 denote the four to the left). To illustrate this, only the calculation for $N_{original}$ is shown in Figure 4-3. There are 10 houses in the coverage of the target bus stop at its original position. They are the nine purple crosses plus the pink square. $C \cap U$ is represented as the pink square. Therefore, $N_{original} = 10 - 1 = 9$. The calculations of N for other candidate positions are similar.

g) The candidate position with the highest net coverage is considered the best position and returned to subroutine **Main**. In the example of Bus Stop 174, the best position is L3 which is represented as pink triangle in Figure 4-3.

h) Since the best position is not the original position, subroutine **Main** repositions Bus Stop 174 at L3 and updates the map and database.

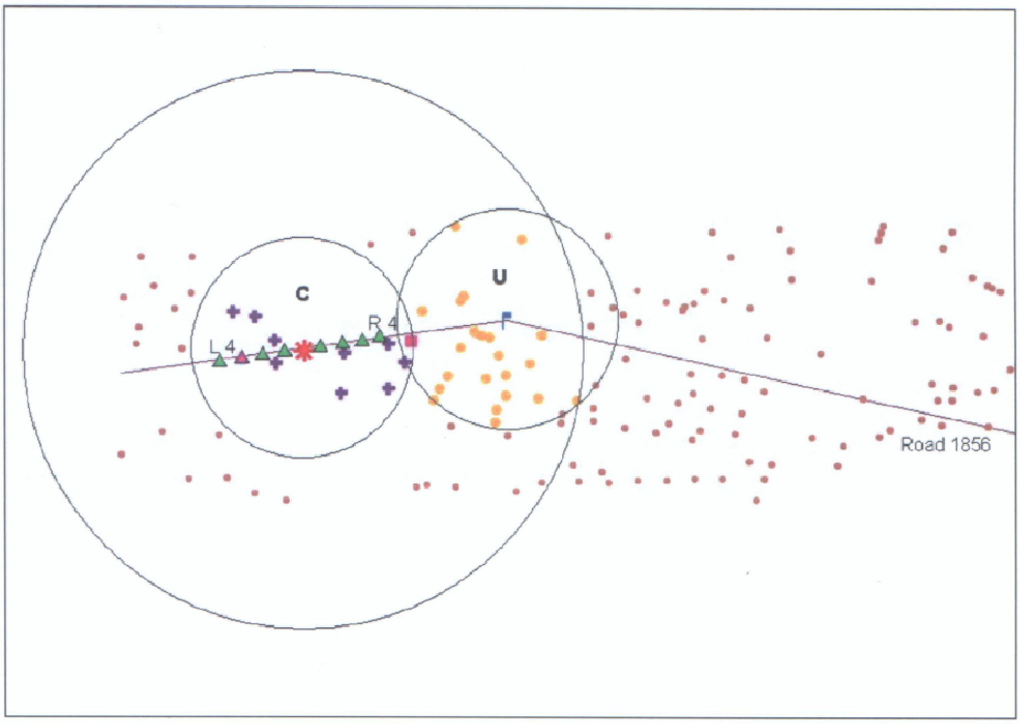


Figure 4-3 *Calculation of net coverage for target bus stop at its original position*

The above process illustrates one iteration of the computation. The program iterates over all the bus stops in the sequence to improve the accessibility.

Although in each iteration, the local optimal position of the target bus stop is found, it does not mean that the position will remain being locally optimal. This is because all bus stops are interrelated. The reposition of one bus stop affects the local optimality of the neighbouring bus stops. In another word, one run of the program cannot ensure the convergence to a locally optimal placement of bus stops. Therefore the optimisation procedure is run many times until no further improvement can be achieved. Based on trials, four runs are enough to reach the locally optimal bus stop placement to achieve the objective of minimising bad accessibility; while six runs are enough to achieve the objective of maximising good accessibility regardless of what sequence is used.

Combination of different objectives and different sequences produces different experimental results. The outputs from the computation with the objective of minimising bad accessibility under three different sequences are shown in Tables 4-1, 4-2 and 4-3. The performance of the three different sequences is compared in Figure 4-4. The outcomes from the three sequences are similar. Locating bus stops with best-first sequence only changes the positions of 73 bus stops. The other two approaches both change the positions of more than 80 bus stops.

Table 4-1 Minimising bad accessibility with original sequence

No. of Runs	GOOD	GOOD%	MEDIUM	MEDIUM%	BAD	BAD%	CHANGED
0	85,774	72.28	26,344	22.20	6,548	5.52	0
1	86,119	72.57	26,143	22.03	6,404	5.40	82
2	86,169	72.61	26,136	22.02	6,361	5.36	26
3	86,192	72.63	26,121	22.01	6,353	5.35	13
4	86,193	72.63	26,124	22.01	6,349	5.35	1
5	86,193	72.63	26,124	22.01	6,349	5.35	0

Table 4-2 Minimising bad accessibility with random sequence

No. of Runs	GOOD	GOOD%	MEDIUM	MEDIUM%	BAD	BAD%	CHANGED
0	85,774	72.28	26,344	22.20	6,548	5.52	0
1	86,131	72.58	26,131	22.02	6,404	5.40	80
2	86,176	72.62	26,129	22.02	6,361	5.36	23
3	86,199	72.64	26,114	22.01	6,353	5.35	11
4	86,200	72.64	26,117	22.01	6,349	5.35	1
5	86,200	72.64	26,117	22.01	6,349	5.35	0

Table 4-3 Minimising bad accessibility with best-first sequence

No. of Runs	GOOD	GOOD%	MEDIUM	MEDIUM%	BAD	BAD%	CHANGED
0	85,774	72.28	26,344	22.20	6,548	5.52	0
1	86,142	72.59	26,121	22.01	6,403	5.40	73
2	86,191	72.63	26,115	22.01	6,360	5.36	20
3	86,214	72.65	26,100	21.99	6,352	5.35	10
4	86,215	72.65	26,103	22.00	6,348	5.35	1
5	86,215	72.65	26,103	22.00	6,348	5.35	0

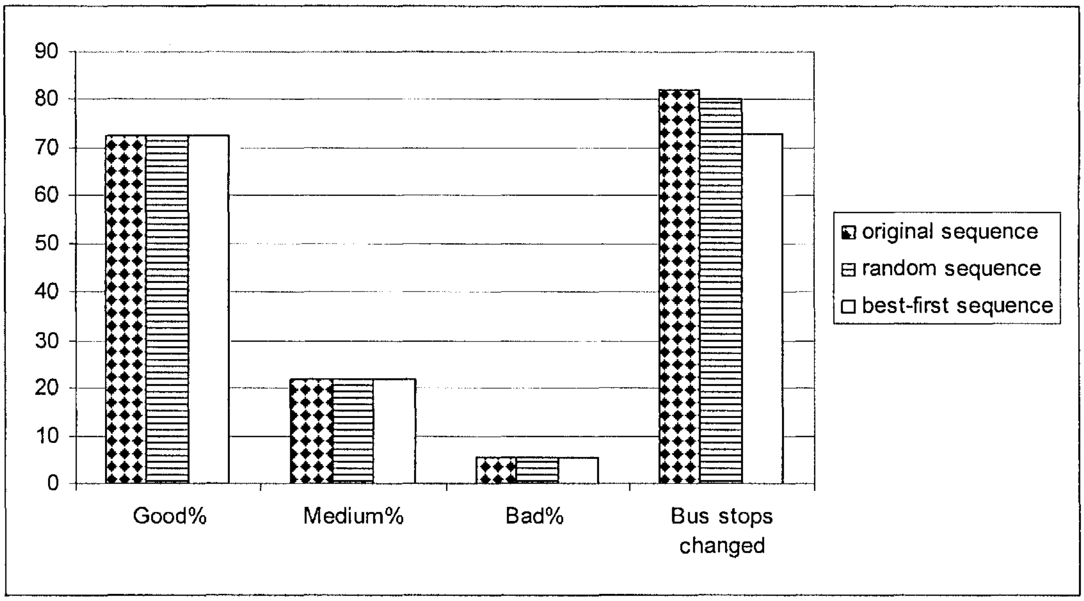


Figure 4-4 Comparison of the performances for the three sequences with the objective of minimising bad accessibility

The best result of minimising bad accessibility is shown in Figure 4-5. Compared with the original situation, 0.37% (441) more houses are within good accessibility and houses within bad accessibility are reduced by 0.17% (200).

The outputs for the computation with the objective of maximising good accessibility under three different sequences are shown in Tables 4-4, 4-5 and 4-6. The performance of the three different sequences is compared in Figure 4-6. The outcomes from the three sequences are the same. However, procedure with best-first sequence only changes the positions of 262 bus stops. The other two both change the positions of more than 270 bus stops.

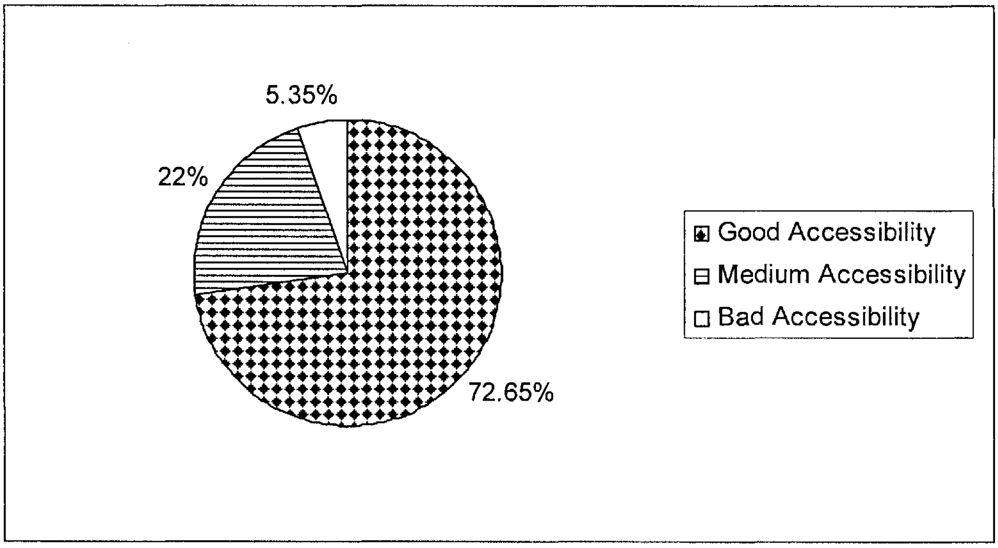


Figure 4-5 Best result of minimising bad accessibility

The best result of maximising good accessibility is shown in Figure 4-7. Compared with the original situation, 1.37% more houses (1515) are within good accessibility and 0.13% fewer houses (153) are within bad accessibility.

Table 4-4 Maximising good accessibility with original sequence

No. of Runs	GOOD	GOOD%	MEDIUM	MEDIUM%	BAD	BAD%	CHANGED
0	85,774	72.28	26,344	22.20	6,548	5.52	0
1	86,914	73.24	25,315	21.33	6,437	5.42	275
2	87,153	73.44	25,106	21.16	6,407	5.40	83
3	87,243	73.52	25,019	21.08	6,404	5.40	36
4	87,284	73.55	24,982	21.05	6,400	5.39	13
5	87,286	73.56	24,980	21.05	6,400	5.39	2
6	87,287	73.56	24,979	21.05	6,400	5.39	1
7	87,287	73.56	24,979	21.05	6,400	5.39	0

Table 4-5 Maximising good accessibility with random sequence

No. of Runs	GOOD	GOOD%	MEDIUM	MEDIUM%	BAD	BAD%	CHANGED
0	85,774	72.28	26,344	22.20	6,548	5.52	0
1	86,918	73.25	25,311	21.33	6,437	5.42	273
2	87,156	73.45	25,103	21.15	6,407	5.40	83
3	87,247	73.52	25,015	21.08	6,404	5.40	36
4	87,287	73.56	24,979	21.05	6,400	5.39	14
5	87,289	73.56	24,977	21.05	6,400	5.39	2
6	87,290	73.56	24,976	21.05	6,400	5.39	1
7	87,290	73.56	24,976	21.05	6,400	5.39	0

Table 4-6 Maximising good accessibility with best-first sequence

No. of Runs	GOOD	GOOD%	MEDIUM	MEDIUM%	BAD	BAD%	CHANGED
0	85,774	72.28	26,344	22.20	6,548	5.52	0
1	86,937	73.26	25,297	21.32	6,432	5.42	262
2	87,168	73.46	25,096	21.15	6,402	5.39	76
3	87,256	73.53	25,011	21.08	6,399	5.39	35
4	87,287	73.56	24,984	21.05	6,395	5.39	13
5	87,289	73.56	24,982	21.05	6,395	5.39	2
6	87,289	73.56	24,982	21.05	6,395	5.39	0

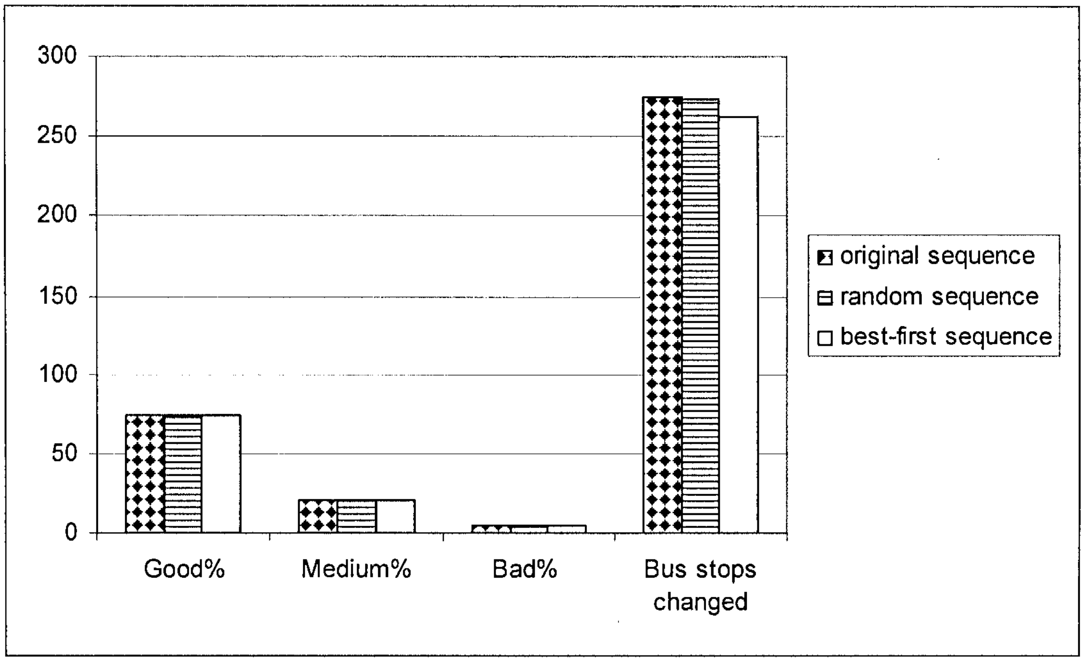


Figure 4-6 Comparison of performance of the three sequences with the objective of maximising good accessibility

Although some improvements in pedestrian accessibility are achieved, the improvement is not very significant and perhaps cannot justify the cost for relocating the bus stops. Moreover, the randomised algorithm cannot lower the number of bus stops. Is it necessary to have so many bus stops? It will be of great value if the number of bus stops can be reduced for they are not only a source of delay but also incur infrastructure cost and maintenance cost to the operator of the bus system. The next approach of optimisation seems more promising.

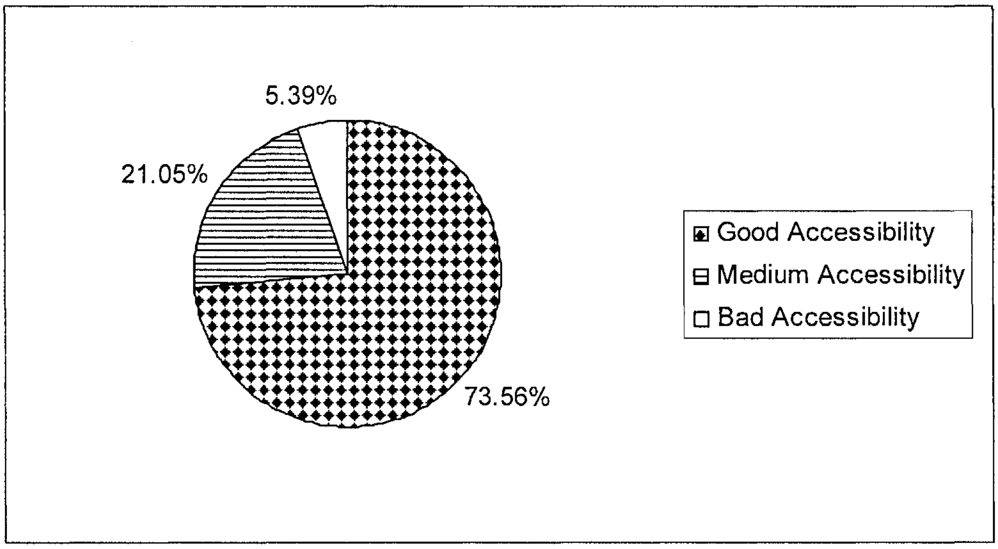


Figure 4-7 Best result of maximising good accessibility

4.2.2 Transformation into set-cover problem

The set-cover problem is a well-studied subject in computer science. The definition of set-cover problem is as follows:

> Given a collection C of subsets S_i of a set S, a sub-collection C′ of C covers S if and only if for any member m in S, some subset S_i can be found such that m is also a member of S_i. Supposing that C can cover S, what is the minimum sub-collection C_{min}?

For example, let S = {1, 2, 3, 4} and C = {{1}, {2, 3, 4}, {1, 3}, {2}, {1, 4}}. Obviously, C can cover S, as for any member m of S, there is a subset S_i in C such that m is in S_i (e.g. 2 is in subset {2, 3, 4}). However, C is more than enough to cover S. For instance, Sub-collection C′ = {{1}, {2, 3, 4}} can already cover S. In fact, it is not hard to verify that C′ is the minimal sub-collection of C that can cover S, C_{min} = C′.

It is interesting that the bus stops placement problem can actually be cast as a set cover problem. Imagine that the whole road network is cut into thousands of little segments. If a bus stop is placed along some segment, it can cover a number of houses. Let S be the set of all houses in Singapore. Then each segment represents a subset of S. And the collection of all these segments is C. Minimising the number of bus stops is equivalent to finding the minimum sub-collection of segments that can cover S.

Set-cover is a very complex problem, as no known algorithm can solve it in polynomial time. (Polynomial time algorithms are those algorithms whose running time grows polynomially fast with respect to the problem size as opposed to exponential time algorithms. The running time of exponential algorithms grows so fast that an exponential time algorithm often takes thousands of years to solve a medium sized problem. Therefore, in practice, only polynomial time algorithm can be applied.) However, efficient algorithms have been developed that can solve special cases of set-cover problem approximately well. In particular, greedy algorithm (Cormen et al., 2001) and ε-net algorithm perform very well on collections whose VC-dimensions are constant (Bronnimann and Goodrich, 1994). Roughly speaking, the VC-dimension is a measure of the complexity of a collection of subsets. It turns out that the VC-dimension of the bus stops placement problem is constant. Therefore, many results for efficiently solving set cover problem can be applied here. The algorithm below is derived from the set-cover problem:

1. Draw a circle of good/medium accessibility around the first house h_1. This circle may cut through a few roads. Segment these roads within and intersected by the circle. Delete the original roads from the database of roads and add the segments back. Label those segments that fall inside the circle with the house's ID, 1.

2. For each subsequent house h_i, repeat step 1. The only difference is that besides roads, the circle may also cut through segments of roads. In this case, the segments will be further segmented; and the new segments will inherit the label of the old segment. For those segments that fall inside the circle, add the house's ID, i to their labels. For example, suppose that in iteration 7, house No. 7 is processed and a segment with label {1, 4, 5} falls

inside the circle, the new label for this segment will be {1, 4, 5, 7}. An interpretation of the label of a segment is that if a bus stop is placed along this segment, all the houses whose IDs are included in the label will be covered by the bus stop.

3. After all the houses have been processed, select the segment s_{max} whose label contains the maximum number of house IDs and place a bus stop along this segment. Then for each segment s_i, delete the IDs of those houses that have already been covered by s_{max} from the label of s_i. Delete s_{max} from the list of segments.

4. Repeat Step 3 until the label of every segment becomes empty.

The above algorithm has been tested on a sample area with 1,014 houses (points). This area is serviced by 12 bus stops (triangles). There are 954 houses within the good accessibility range. It is shown in Figure 4-8.

Figure 4-8 Current situation of the sample area

After relocation using the above algorithm, only 5 bus stops (triangles) are needed to cover the whole area up to the same accessibility level (see Figure 4-9) and 12 bus stops would be enough to cover all the 1,014 houses within good accessibility range. The test shows that the improvement in terms of both accessibility and number of bus stops is quite dramatic. However, it should be noted that the five bus stops are virtual bus stops which are located at the centre of a road. In practice, 10 bus stops are needed to service this area supposing all bus routes run on two-way roads. In addition, pedestrian accessibility to bus stops on one-way roads and two-way roads are different. It is more accurate to identify one-way road and its direction beforehand and treat it differently from a two-way road. The relocation process does not impose any constraints to the siting of the relocated bus stop. In practice, there are many possible constraints such as the road width (the location may be too narrow to site a bus stop) and other mandatory requirements such as having a bus stop within a certain distance of a school. As no usage data is available in the building data, a single unit weight is assigned to all buildings. This will distort the final location of the bus stop as the demand from a public housing building can be grossly different from a detached bungalow. The considerations made in the relocation algorithm could not accommodate these constraints largely because data for such constraints are not easily available. Nonetheless, the principles embodied in the algorithm are generic in nature and could be improved if relevant data are made available.

The problem with the above algorithm is that it does not scale very well with respect to the problem size. It is easy to see that the algorithm makes heavy use of set operations. As the problem size grows, the set operation becomes very costly. However, these difficulties can be circumvented by applying techniques like the clustering method.

Figure 4-9 New bus stops after relocation

4.3 Summary

The optimisation of bus stop location was discussed in this chapter. It addressed the bus stop location problem in its implementation stage. The main data used in this task are: road network of Singapore, bus stop locations and building locations. Improving pedestrian accessibility to bus stops is chosen as the objective for optimisation of bus stop locations.

Optimisation based on moving bus stops has been completed for the whole Singapore and some preliminary results have been presented. The proposed randomised algorithm can improve the pedestrian accessibility by 1%. This is a marginal improvement because the current pedestrian accessibility to bus stops of Singapore is quite good. Experiments show that moving bus stops with best-first sequence can reach the best accessibility with minimum re-location of bus stops. In this case, only 2% (73 / 3,555) of bus stops need to be re-located.

Optimisation based on set-cover algorithm performs well in the tested area. Some 17% reduction of bus stops is possible while maintaining the same accessibility level. However, the problem of this algorithm is that the roads need to be divided into small segments. Although clustering may solve this problem, the objective of this study is to propose a method of designing bus network in a disaggregate manner. This set-cover algorithm is not used on the whole network as the size is beyond the computational capacity.

CHAPTER 5 METHODOLOGY - OPTIMISATION OF BUS ROUTES

Numerous papers and reviews on transit route design suggest that analytical method and mathematical programming cannot solve large-scale network design problems. Artificial intelligence (AI) methods (e.g. simulated annealing, genetic algorithm and expert system) are usually adopted. Each of these methods has its own strengths and weaknesses. Traditionally they are applied independently to solve much smaller and simpler problems. Combined AI approaches could help solve complex problems (Sadek, 2001). Garcia et al. (1998) compared a series of hybrid algorithms (including hybrid genetic /simulated annealing) with pure genetic or local search algorithms based on a problem in telecommunications planning, the multiperiod network expansion problem, and concluded that the hybrid algorithm outperforms both pure genetic and pure local search algorithm. In this research a hybrid simulated annealing (SA) and genetic algorithm (GA) is proposed to solve the network design problem.

5.1 Objective formulation

In re-designing the public transport system, it is important that a precise measure can be adopted to approximately quantify the optimality of a system. Otherwise, different designs cannot be compared. In terms of optimisation theory, this measure is also called the objective function. The objective function should reflect the true performance of the system, yet it must be simple and concise. A transport system, however small, is characterised by many features and variables. For example, the travel time between any pair of locations is one variable that describes, to a certain extent, the quality of the whole system. Some of the features like travel time, cost, transfer time etc. are more related to the optimality compared to others such as comfort of travel etc. Moreover, many features are dependent on each other. An objective function that takes into account all the features will be overly complex. It not only complicates calculations but may also be inaccurate due to data dependency.

Previous studies show that the most appropriate objective for transport network planning is minimising the total cost, which is the sum of operation cost and passenger cost (van Nes and Bovy, 2000). This function is also adopted in this study. The objective function can be formulated as follows (adopted from Pattnaik et al., 1998):

$$Min: E = c_1 \sum_{i=1}^{n} \sum_{j}^{n} T_{ij} \cdot t_{ij} + c_2 \sum_{k \in R_1} f_k \cdot t_k + c_3 \sum_{s \in R_2} f_s \cdot t_s \qquad \text{Equation 5-1}$$

subject to the following constraints:

$f_k \geq f_{k,\min} \forall k \in R_1$ (frequency feasibility)

$f_s \geq f_{s,\min} \forall s \in R_2$ (frequency feasibility)

$l_k \leq l_{k,\max} \forall k \in R_1$ (load factor constraint)

$l_s \leq l_{s,\max} \forall s \in R_2$ (load factor constraint)

where T_{ij} = travel demand from node i to j (passengers/hour) which is the number of people aiming to travel from i to j in a hour; t_{ij} = total travel time (the sum of in-vehicle travel time, waiting time and transfer time) between node i and j (hour); f_k = frequency of kth bus route (buses/hour); t_k = round trip time for kth bus route (hour); f_s = frequency of sth MRT route (trains/hour); t_s = round trip time for sth MRT route (hour); c_1 = factor to convert user travel time to user travel cost (\$/passenger·hour); c_2 = factor to convert bus running time to cost equivalent (\$/bus·hour); c_3 = factor to convert MRT running time to cost equivalent (\$/train·hour). The time unit for all parameters is one hour.

The first term of this model represents the total travel time cost of passengers per hour. The second term represents the operation cost of bus service per hour. The third term represents the operation cost of MRT per hour. The sum of the last two terms is the total operation cost of the transit network per hour. Hence, the sum of the three terms is the overall travel cost per hour which is represented by E. This objective function will be solved using the hybrid simulated annealing and genetic algorithm approach.

5.2 The basic framework of network design with hybrid SA and GA

The hybrid SA and GA is used to plan bus routes. MRT stations are treated the same way as the terminal nodes. Terminal nodes are where bus routes start and end. MRT lines are considered as fixed routes which must be included in both candidate solutions and the optimal solution. This framework uses SA as the main algorithm to search for the optimal solution. In every application of SA, the current solution needs to be given a small change. GA is used to give a random disturbance to the current solution. Since every solution is actually a set of bus routes, it is necessary to generate enough candidate routes for each pair of terminals in a road network before running SA-GA. This is discussed in Section 5.2.1. Then the main structure of the hybrid SA and GA is discussed in the following Section 5.2.2. The procedure of applying GA is specified last.

5.2.1 Candidate route generation module

In this module, a set of candidate routes is generated. Ideally candidate routes should start from a terminal node and end at another terminal node. However, candidate routes may also be generated between bus stops or bus stop and terminals. If all possible routes are considered as candidate routes, the size of the candidate route set will be enormous. Hence, the selection of candidate routes might be guided by demand, constraints of routes and planners' knowledge. This module consists of the following steps:

1) Identify terminal nodes according to demand.
2) Generate k shortest paths between every pair of terminal nodes. k is a number specified by the planner.
3) A route length constraint could be enforced so that all routes which are too long or too short are excluded from candidate route set.

All MRT stations and bus stops are considered as terminal nodes in step 1. Ideally, in step 2 the shortest path for each pair of terminal nodes should be generated by the k-shortest path algorithm (Hershberger et al., 2003) because no closed loop is allowed in real life. However, this algorithm has a high computational complexity and its computation time would be extremely large. Therefore, a modified Dijkstra algorithm (Cormen et al., 2001) is applied to find a number of candidate routes between all pairs of bus terminals in order to save computation time. An amendment is done to this algorithm to exclude all paths with closed loops from candidate routes (see Appendix E). Under these circumstances, it is not practical to ensure the candidate routes generated are the k shortest paths within an acceptable computation time limit. This method guarantees that the shortest path is included as one of the candidate routes. The shortest path is defined as the one that involves the minimum travel time among all the alternatives.

5.2.2 Main structure of the hybrid SA and GA

The procedure of the hybrid SA and GA approach is summarised in Figure 5-1.

The general form of simulated annealing algorithm starts from a random point. This framework is very flexible as it can be adapted to both incremental and macro approaches in the network design. To incrementally improve the transport system from its current configuration, it is necessary to take the current bus stops and MRT stations as the terminal nodes for designing the transit network. Then the current transit route network can be used as the initial solution N_0. To re-design the system from scratch, the relocated bus stops (described in Section 4.2) and MRT stations may be used as the terminal nodes and a random-generated network used as the initial network N_0.

The initial network N_0 is generated by a GA sub-function called *initialisation* and its objective function value, E_0, is calculated based on Equation 5-1. Since every network is actually a set of bus routes, it is necessary to generate enough candidate routes for each pair of bus stops in a road network before running SA-GA. All candidate routes are generated by candidate route generation module which will be discussed later.

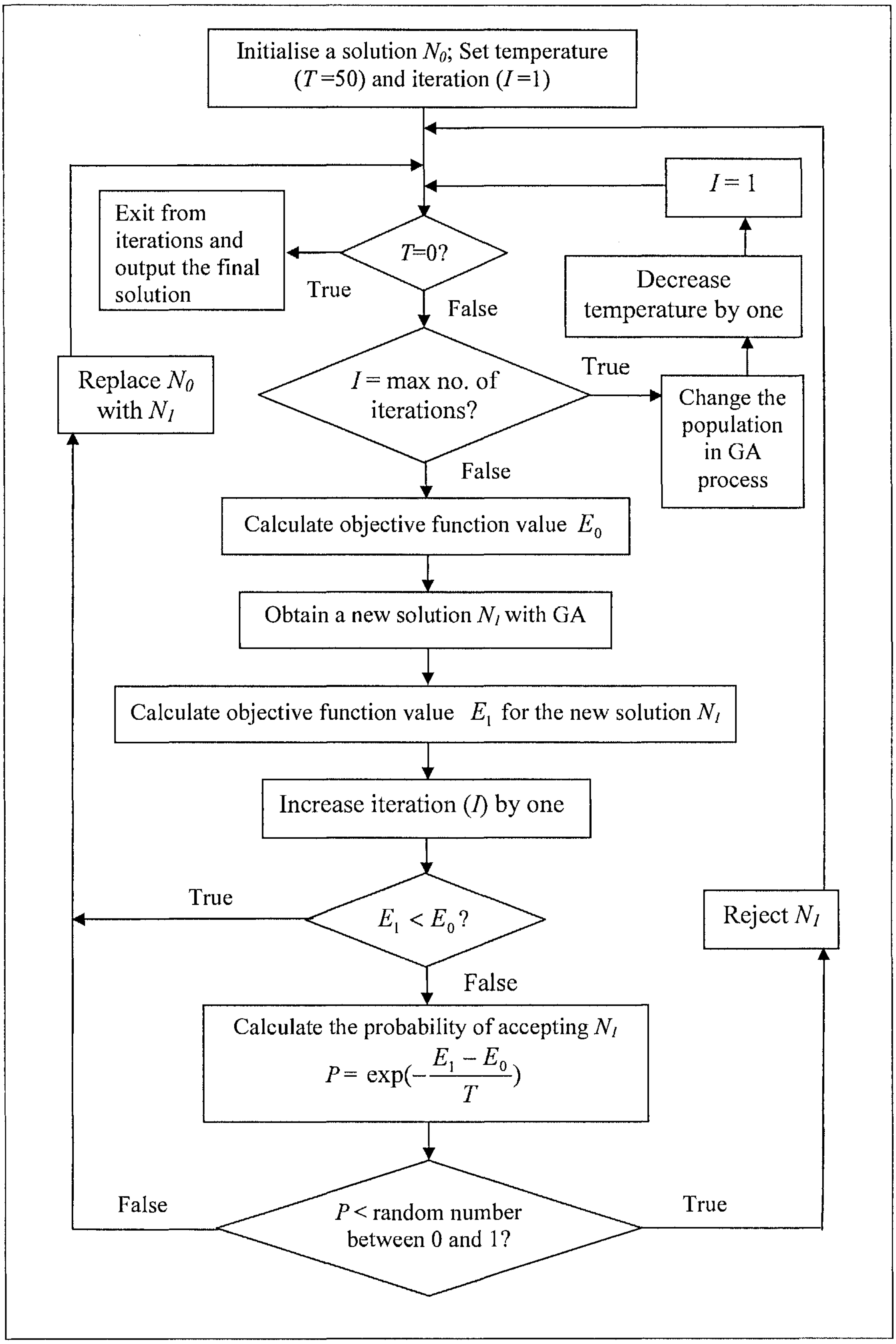


Figure 5-1 Hybrid SA and GA algorithm for network design

A new solution N_1 is obtained from the GA process and evaluated to obtain E_1 also based on Equation 5-1. If the new solution is better than the old one (i.e. $E_1 < E_0$), it is accepted and N_1 replaces N_0. Even if the new solution is worse than N_0 (i.e. $E_1 - E_0 > 0$), it can still be accepted based on acceptance probability which is a function of the two objective function values and the current temperature (Figure 5-1). If the acceptance probability is greater than a random number between 0 and 1 generated by the computer, the new solution N_1 is accepted, otherwise it is rejected.

5.2.3 Application of GA

GA is used to generate the new potential solutions among which the one with the best fitness value is selected and subsequently used by the SA process for further network selection. In the case of cost minimisation, the fitness value is usually assumed to be the reciprocal of the objective function value. The general structure of GA is presented in Figure 5-2.

Representation of route network

In order to solve network design problem using the genetic algorithm, it must be first represented as a GA problem on which GA operators can work. This is an important step in applying GA. An appropriate representation can not only link the real-word problem very well with a GA problem but can also improve the efficiency of this algorithm.

Every candidate route is considered as a variable in the route network. Usually each variable is encoded into a binary string in traditional GA approaches. However, according to the nature of this problem, it is natural to use real number coding method to encode networks. That means each candidate route is identified by its ID and using the combination of route IDs to represent a bus route network. For example, a route network consists of 5 bus routes which are routes 4, 27, 34, 9, and 12. It will be simply represented as a string with the length of 5:

4	27	34	9	12

It should be noted that the sequence of the IDs is not relevant which means that string 4, 27, 34, 9, 12 and string 4, 34, 9, 12, 27 represent the same route network.

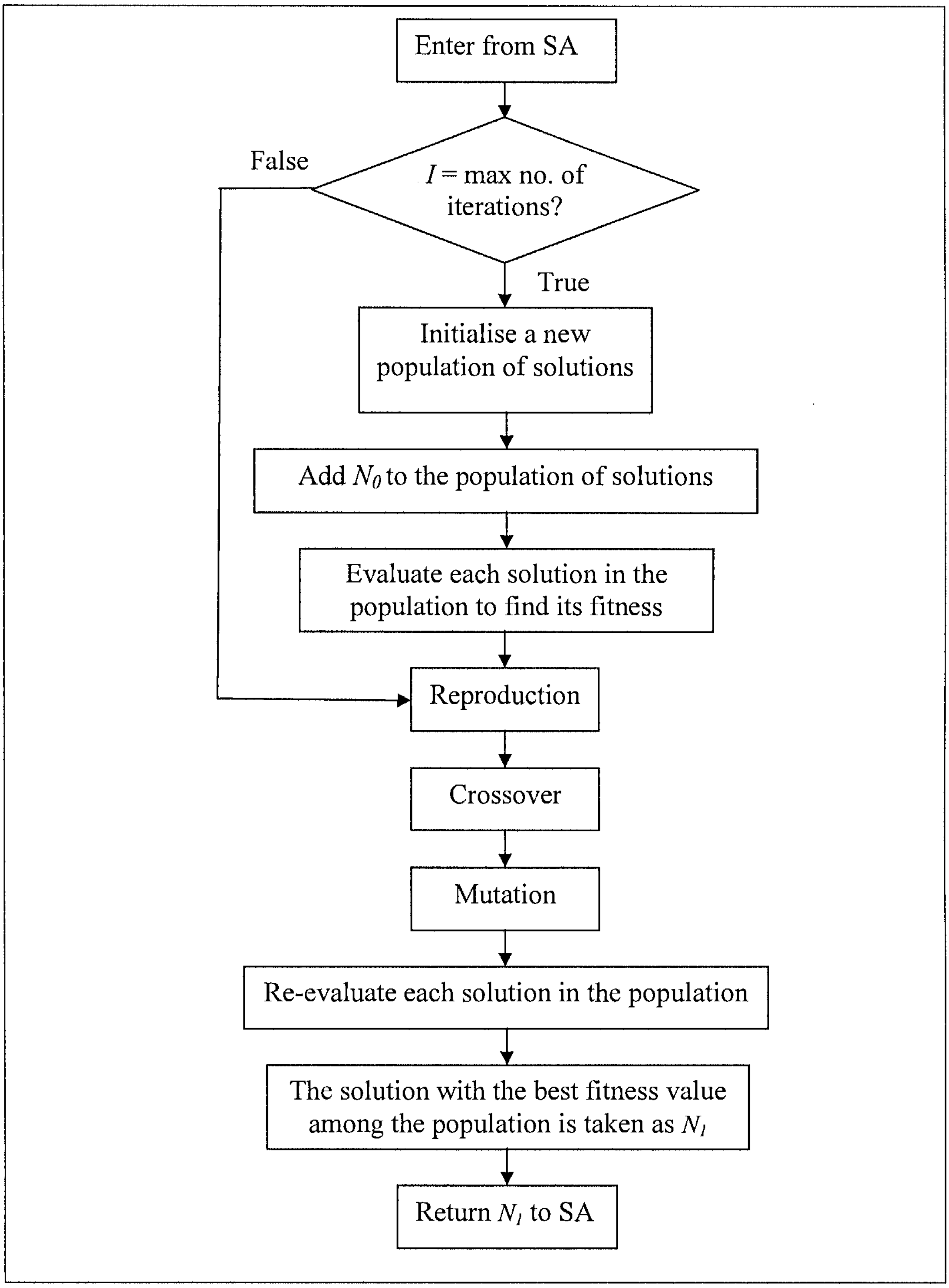


Figure 5-2 General structure of GA

Application of GA with variable string length coding

Traditionally the candidate network consists of a fixed number of candidate routes. However, it is not known how many routes constitute the optimal or near-optimal network. Therefore many trials have to be done since different numbers of routes in the optimal solution have to be tested. This needs tremendous computation time. Hence a variable string length coding (Pattnaik et al., 1998) is used here. In this coding method, the length of the string which represents a candidate network varies. The size of the solution s is selected at random within a predefined range. Once the value of s is set, solution N_0 is generated in *initialisation*.

Initialisation

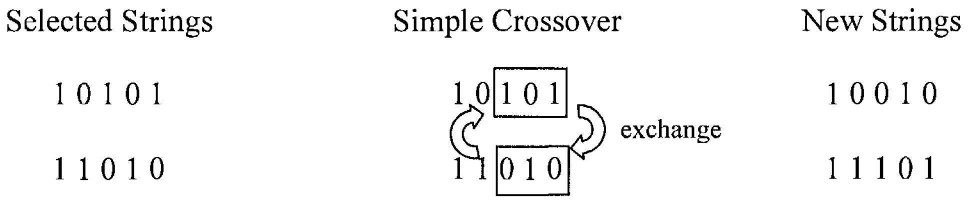
During the *initialisation* phase, two tasks are accomplished. The first task is to generate a solution known as N_0. This will be used by SA as the initial solution. A solution is a string containing some bus route IDs. Since the size of the optimum solution (i.e. how many routes should be included) is not known beforehand, variable string length coding is applied. Solution N_0 is generated by selecting s candidate routes at random. The second task is to generate a population of solutions. A commonly used population size is 50. The bigger the population size, the better the diversity of the population. Therefore, the chance of finding better solutions increases. However, a bigger population increases the computation time. Using the same method as for initialising N_0, the other solutions in the population are generated. Each individual solution is evaluated based on the objective function formula and its fitness value is determined for reproduction purpose in the GA process. How a solution is evaluated is discussed in Section 5.2.3.

Reproduction

Reproduction is a process in which some strings with better fitness values are selected from the population for further GA operations. The popular roulette wheel selection method (Goldberg, 1989) is used in this study. In this way, strings with a higher fitness value have a higher probability of being selected.

Crossover

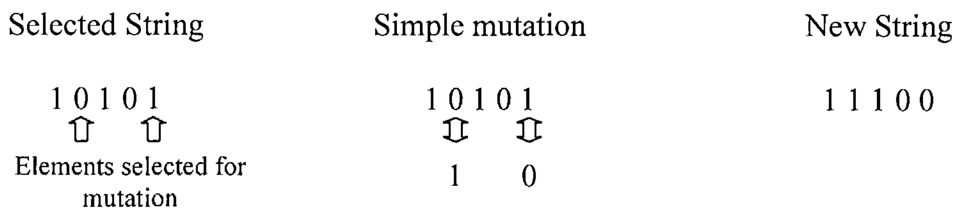
Crossover is a process to generate new strings from a given pair of strings in the process of reproduction. Generally crossover is performed on a population of strings between the first and second strings, third and fourth strings, and so on. In addition, performing the crossover on a pair of strings is controlled by crossover probability P_c, which has a predefined fixed value. For each pair of strings ready for crossover, a random number between 0 and 1 is generated. Crossover is performed only if the random number is smaller then P_c. Usually, P_c should be relatively large so that it is possible for the program to introduce improvements to the current solution. There are a number of crossover methods available. Each has its special characteristics and is potentially more efficient for certain types of problems. The common crossover approach applied in most GA applications is single-point crossover (Goldberg, 1989). It works in the following way:



To test the effectiveness of different crossover methods, three other kinds of crossover: two-point, uniform and convex (Goldberg, 1989) are also implemented in addition to single-point crossover.

Mutation

Mutation is the occasional random alteration of the value of an element of a string. It is necessary because sometimes useful information may be lost during reproduction and crossover. The mutation operator helps to introduce some potentially useful components (i.e. good bus stop sequences). Generally, it works in the following way:



As in crossover, this process is also controlled by a probability parameter P_m, (mutation probability). For each element of a string, a random number between 0 and 1 is generated. If it is smaller than P_m, mutation is performed; otherwise no mutation is performed for this element. The frequency of mutation is usually very small in most GA applications. For example, one mutation per thousand elements is a typical value and produced best results in most applications (Goldberg, 1989). The value of $P_m = 0.001$ is also used in this study. In total, two types of mutations are used: uniform mutation and dynamic mutation. It should be noted that crossover and mutation may result in duplicate elements in child strings. Since in the bus route network design problem no duplication is allowed, an additional procedure to remove duplicates is performed after mutation. In this process, each duplicate is replaced by a route ID chosen randomly from candidate routes. This replacement must not be a duplicate of any existing element in the string.

Termination

Ideally the SA-GA process should terminate when the optimal solution is found or the program converges. But due to the size and complexity of the bus route design problem, it is not likely that an optimal solution or a convergence would happen in an acceptable time period. In practice, only a fixed number of iterations are conducted. This can be controlled by temperature T and I in this study.

5.2.3 Evaluation

Evaluating the solution is probably the most important and complicated part of the whole process. The adopted procedure is illustrated in Figure 5-3. Since the solution is randomly generated and always changing, two sets of values need to be found before calculation of the objective function value and fitness based on

Equation 5-1. These are the in-vehicle travel times between all pairs of bus stops and frequencies for all bus routes in the solution. Frequency is usually obtained by dividing demand for a route by bus capacity. Since bus capacity is fixed at 80 passengers per bus and all buses are assumed as travelling with full load in this study, calculation of frequency for each route in the solution is equivalent to calculation of the maximum demand for each route. All buses are assumed travelling with full load in this study because the morning peak period demand matrix will be used for bus route design and almost every bus running in morning peak period is fully loaded. The demand for each route will keep changing while different routes are selected into the solution network. Therefore, it is necessary to do transit assignment each time a solution is formed or changed. The method adopted is to assign demand to two selected paths. The rules for selecting the two paths are:

- the first path must be the shortest path between the pair of bus stops; and
- the travel time of the second path cannot be more than 20% longer than the travel time of the shortest path.

This method guarantees that the shortest path is included as one of the two paths. There is no guarantee that the second path found is the second shortest path due to the complexity and computation time constraint of this problem when no closed loop is allowed as discussed in the section on candidate route generation. If the second path found for a particular pair of bus stops is more than 20% longer than the in-vehicle travel time of the shortest path, all the demand is assigned to the shortest path.

Once a solution is generated and passed to the evaluation module, the first thing to do is to assign demand based on the given solution (route network). Transit assignment is done in two iterations. In the first iteration the demand between each pair of bus stops (T_{ij}) is assigned equally to the two selected routes.

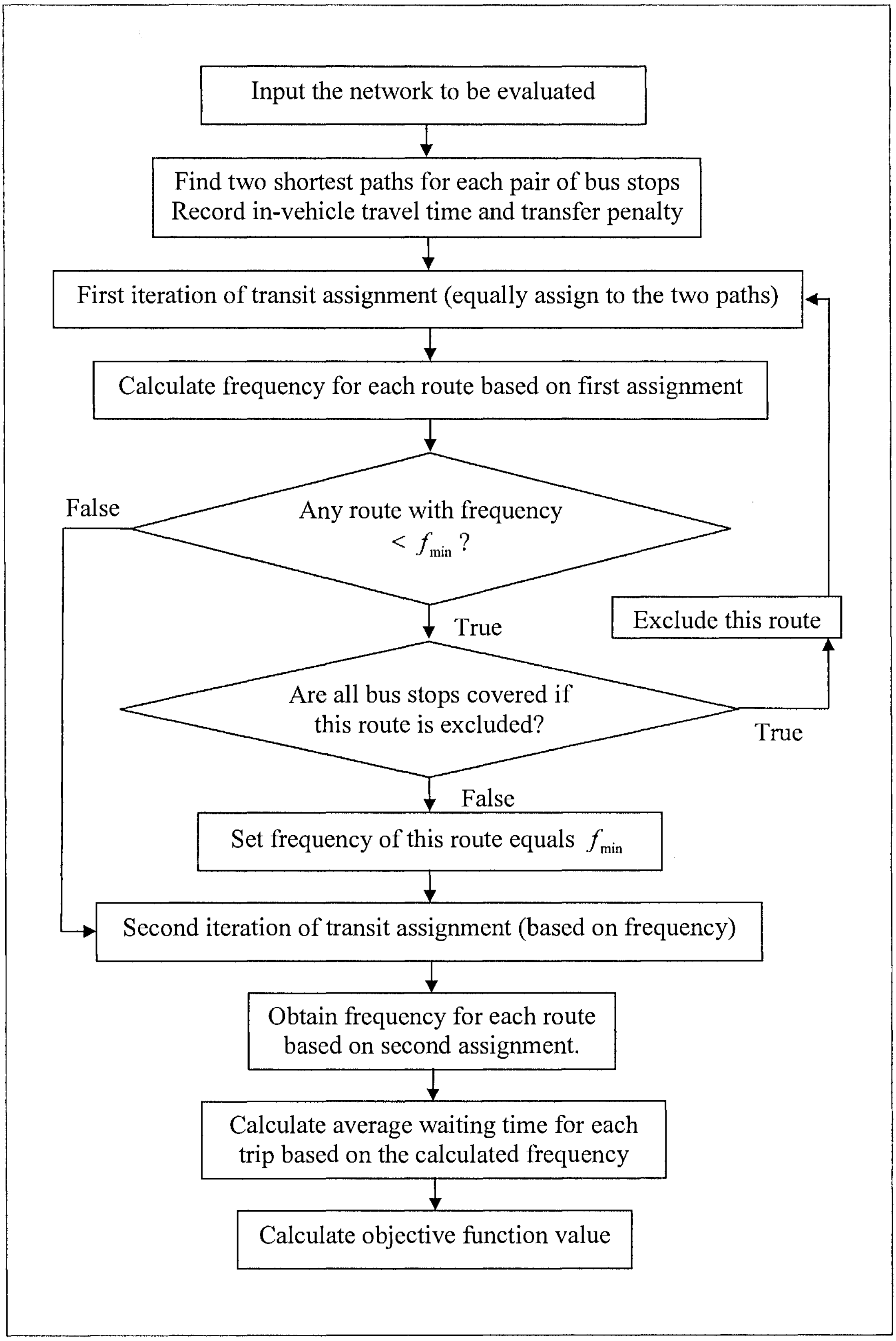


Figure 5-3 Procedure of network evaluation

Next, the frequencies for all routes are calculated. If the frequencies of all bus routes are larger than the minimum frequency f_{min} (3 buses per hour is used in this study), the program directly goes to the second iteration. If some bus routes with frequencies less than f_{min} are found, the program will do the followings:

i. Find the bus route with the minimum frequency among all the bus routes with frequencies less than f_{min};
ii. If all bus stops are covered without this bus route, it is deleted from the solution and the program goes back to first iteration. Otherwise its frequency is set to f_{min} and the program goes to second iteration;

In the second iteration, the demand T_{ij} to the two routes is re-assigned according to their calculated frequencies. If transfers are involved in a path, demand is assigned according to the lowest frequency among the routes involved in the path.

Two options are considered when transfers have to be involved. One is direct transfer at the alighting bus stop. The other is an inter-connecting transfer by walking from the alighting bus stop to another nearby bus stop to transfer to another service. The inter-connecting limit is set to 230 metres in airline distance. This assumes that passengers are willing to walk 300 metres to transfer to another bus, since the average ratio of walking distance with respect to airline distance (Euclidean distance) is approximately 1.3 (Olszewski and Wibowo, 2006). The transfer penalty and walking time are added to the total path time.

The final frequency of each route can be calculated based on the resulting demand from the second transit assignment. The average waiting time for each trip can also be calculated. Now all the data needed for evaluation based on Equation 5-1 are available.

5.4 Summary

In this chapter, the main proposed framework of bus route design is explained. First of all, a suitable objective which is minimisation of total cost is selected for optimisation of bus routes. This objective is formulated into mathematical formula so that it can be used as the objective function to evaluate every solution generated by the proposed method.

In the proposed methodology, a candidate route generation module is first performed to find sufficient possible routes between any pair of origin and destination. The routes in the final solution are all selected from these candidate routes.

SA is used as the main algorithm to select the optimal candidate route combination. GA is used as a subroutine to create a new solution for SA. In solution evaluation, two paths including the shortest one are considered for transit demand assignment. Two iterations of assignment considering minimum frequency constraint are also utilised. In the first iteration, the demand is assigned equally to the two routes; in the second iteration, the demand is assigned based on frequencies calculated in the first iteration. In order to implement the proposed method, some initial assumptions are made. They are:

- Maximum number of iterations for each temperature (I): 20
- Crossover probability (P_c): 1
- Mutation probability (P_m): 0.001
- Population size: 50

These initial assumptions will be tested later by performing a sensitivity analysis.

CHAPTER 6 INPLEMENTATION OF SA-GA ALGORITHM ON THEORETICAL NETWORKS

In this chapter, the framework described in Chapter 5 is fully and successfully implemented on four theoretical grid networks with different numbers of terminals, bus stops, road links and demand. The planning results of the four networks are also illustrated and compared in this chapter.

6.1 Construction of theoretical networks

The smallest network N1 which consists of six squares with side length of 600 metres is illustrated in Figure 6-1. Each 600 metre-long side represents a road. There are 14 bus stops (large dots) distributed on this network. Location of these bus stops is based on the distribution of household buildings (small dots). The method described in Section 4.2.1 was used to maximise the number of households within good accessibility to the bus stops. Household buildings are mapped onto the network according a demand matrix between any pair of bus stops which is presented in Table 6-1. The demand between all pairs of bus stops is fixed which means that the demand does not depend on the quality of bus service. There are three terminals among the 14 bus stops. They are bus stops No. 1, 8 and 14 (crosses). All bus stops, bus terminals and road intersections are called nodes. Links are the edges between two neighboring nodes. Links between any pair of neighboring road intersection nodes are called road links. Links between any pair of any nodes are called bus links.

The other three networks are constructed based on N1 network. Their demand matrices are also extended from the demand matrix of the first network. Network N2 consists of nine grids. As shown in Figure 6-2, there are 20 bus stops (dots) on the network among which 4 are terminals (bus stops No. 1, 8, 14 and 20 illustrated by crosses). From Figure 6-2 it is easy to see that Bus Stops 3, 4, 5, 6, 7, 8 and bus stops No. 15, 16, 17, 18, 19, 20 are symmetric replication with respect to bus stops No. 1 and 2. The demand matrix between pairs of bus stops of network N2 is presented in Table F1 in Appendix F.

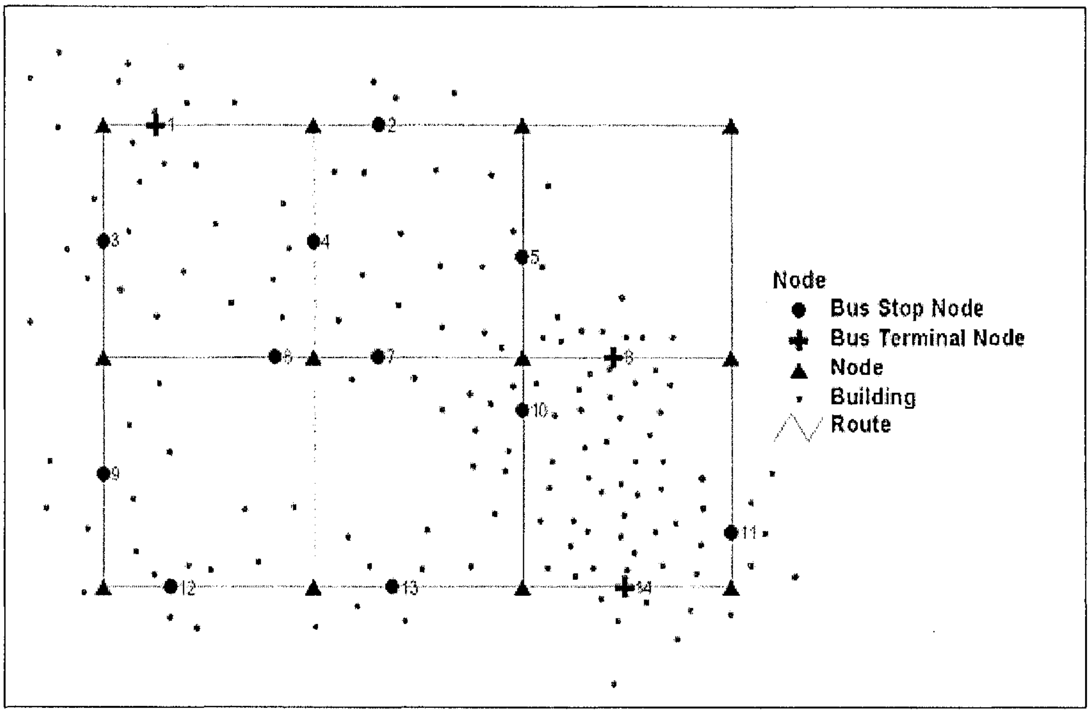


Figure 6-1 Network N1

Table 6-1 Demand matrix for Network N1

Bus Stop From\to	1	2	3	4	5	6	7	8	9	10	11	12	13	14	Total
1	0	32	26	39	45	51	71	116	64	96	51	39	45	122	797
2	40	0	10	15	16	20	28	45	25	38	20	15	18	48	338
3	46	14	0	17	20	23	31	51	29	43	23	17	20	54	388
4	51	16	13	0	23	26	35	58	32	48	26	19	23	61	431
5	57	18	14	21	0	29	39	64	36	54	29	21	25	68	475
6	51	16	13	19	23	0	35	58	32	48	26	19	23	61	424
7	57	18	14	21	25	29	0	64	36	54	29	21	25	68	461
8	80	25	20	30	35	40	55	0	50	75	40	30	35	95	610
9	29	9	7	11	12	13	20	32	0	27	14	11	13	34	232
10	63	20	16	24	28	31	43	71	38	0	31	24	28	75	492
11	46	14	11	17	20	23	31	52	29	43	0	17	20	54	377
12	46	14	11	18	20	23	32	51	29	43	22	0	20	54	383
13	40	13	10	15	18	20	28	45	25	38	20	16	0	47	335
14	91	29	23	34	40	46	63	103	57	85	46	34	40	0	691
Total	697	238	188	281	325	374	511	810	482	692	377	283	335	841	6434

Using a similar method networks N3 and N4 are obtained. Network N3 consists of twelve squares. There are 5 terminals (bus stops No. 1, 8, 14, 20 and 26 illustrated by crosses) out of 26 bus stops (dots) in this network which is illustrated in Figure 6-3. The demand matrix of network N3 is shown in Table F2 in Appendix F.

Network N4 is made up of 15 grids. In total 32 bus stops (also dots) are distributed on this network. There are 6 terminals (crosses) among all the bus stops (Figure 6-4). The demand matrix of network N4 is also shown in Table F3 in Appendix F.

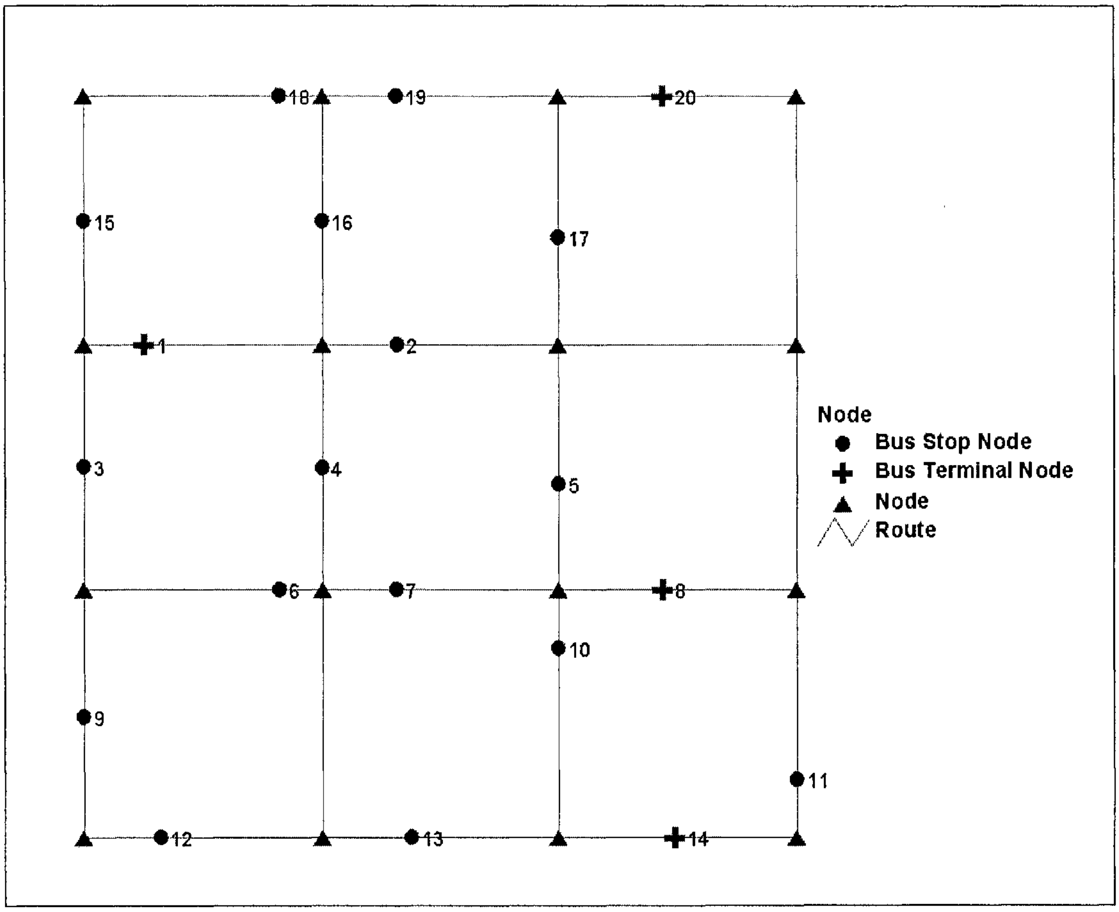


Figure 6-2 Network N2

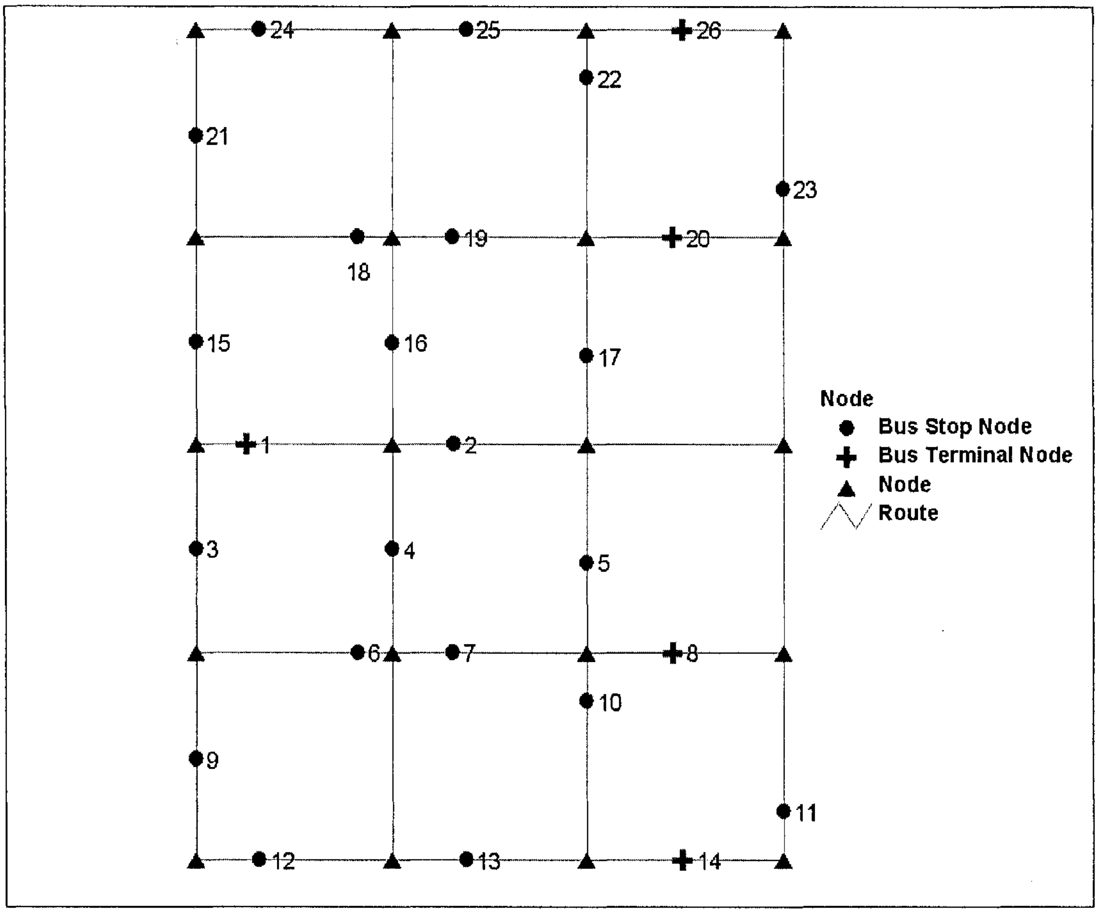


Figure 6-3 Network N3

6.2 Generation of candidate routes

Before generating candidate routes, some assumptions are made: all bus routes should begin from a terminal and end at another terminal; all bus routes are two way routes which means that the route from bus stop A to bus stop B is the same as the route from bus stop B to bus stop A for the same bus service.

First preparation work needs to be done. It is necessary to break all roads into links according to nodes. There are three kinds of nodes. They are bus stop nodes, bus terminal nodes and road intersection nodes which are simply called nodes as shown in Figures 6-1, 6-2, 6-3 and 6-4. Links are the edges between two neighbouring nodes. The resulting attributes of the four networks are summarised in Table 6-2.

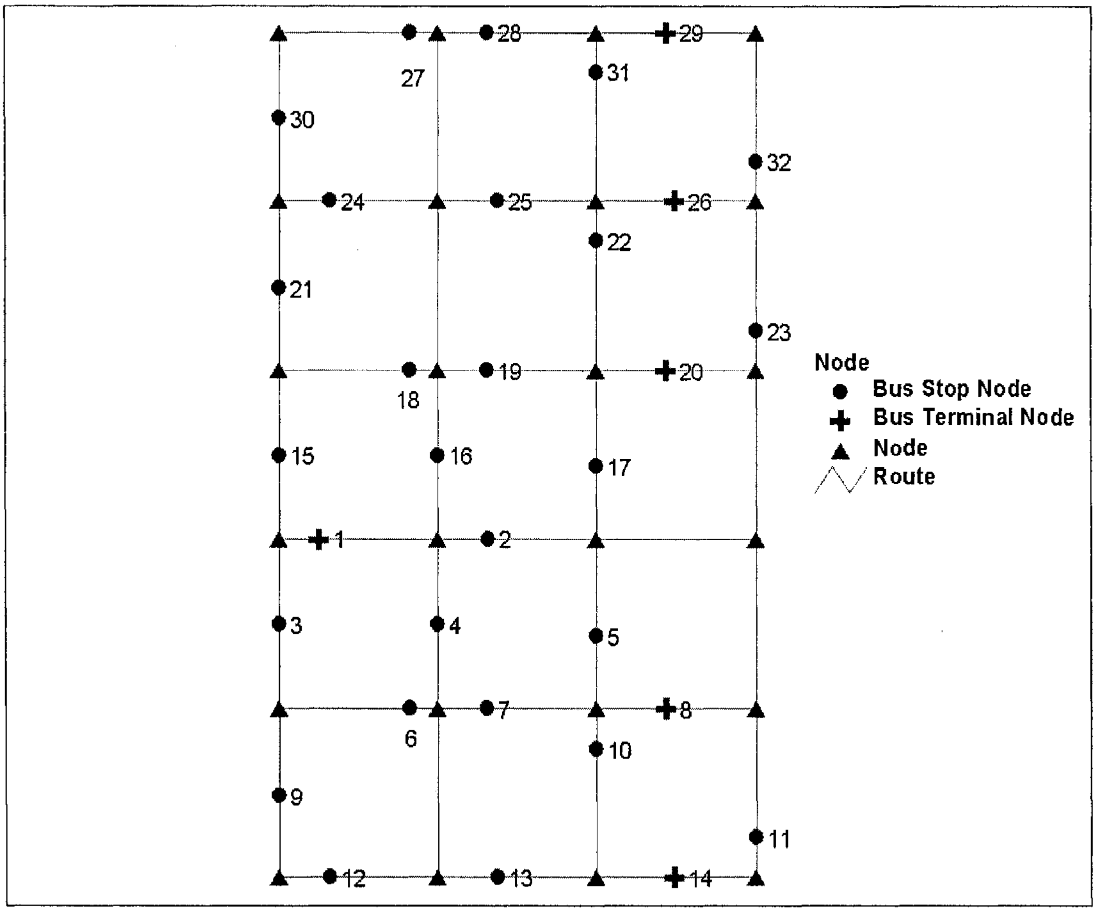


Figure 6-4 Network N4

It should be noted that some road links which have no bus stops and do not form essential connections are excluded in the generation of candidate routes. For example, the links used for candidate routes generation in network N1 are plotted in Figure 6-5. For the other three networks, please see Appendix G.

Table 6-2 Attributes of networks

Network	No. of Bus Stops (including terminals)	No. of Terminals	No. of Road Intersection Nodes	No. of Road Links	No. of Bus Links
N1	14	3	12	17	31
N2	20	4	16	24	44
N3	26	5	20	31	57
N4	32	6	24	38	70

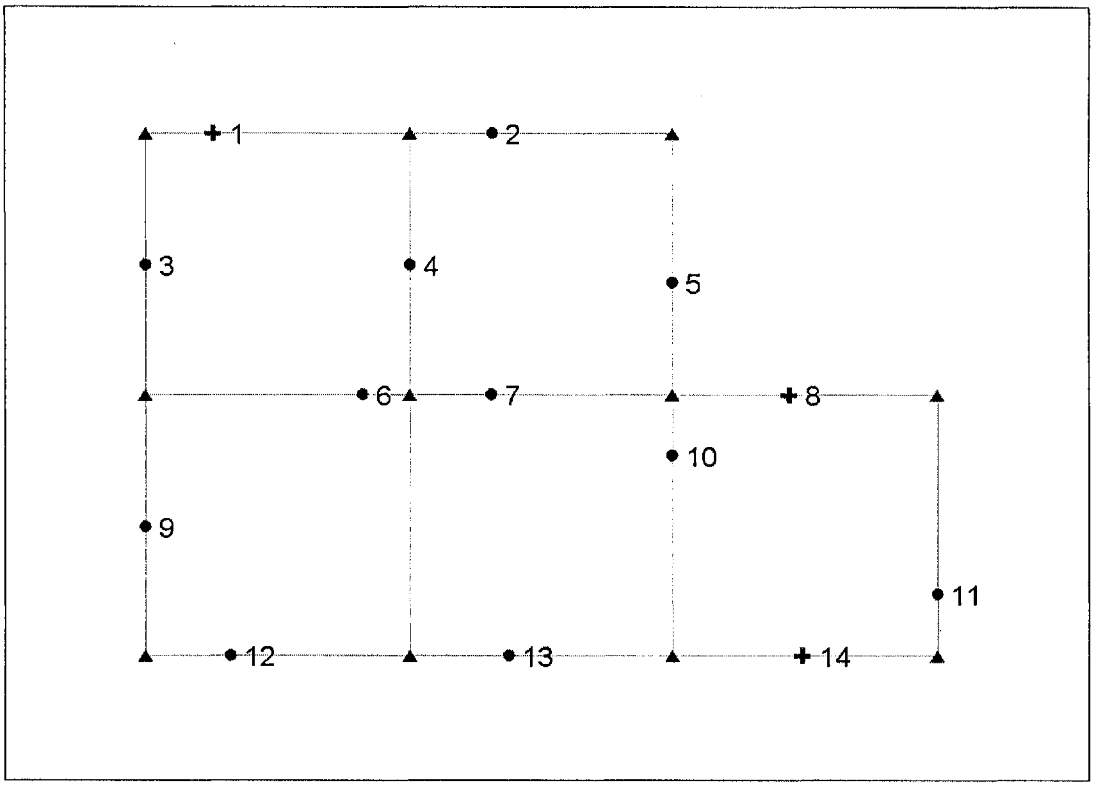


Figure 6-5 Links of network N1 for candidate route generation

The route generation module is implemented in Visual C++. In candidate route generation, it is assumed that all roads are two-directional. The modified Dijkstra's algorithm (Appendix E) is used to generate up to 50 routes for each pair of terminals. Not all pairs of terminals can get 50 routes. The results are summarised in Table 6-3. The computation times needed for all four networks are all less than 30 seconds. The attributes of candidate routes for network N1 are listed in Table 6-4. The others are presented in Appendix H. These candidate routes are used to do the network design experiments for all the four networks.

Table 6-3 Result of candidate route generation

Network	No. of Bus Stops	No. of Terminals	No. of Road Link used	No. of Bus Link used	No. of Candidate routes
N1	14	3	15	29	21
N2	20	4	20	46	171
N3	26	5	28	54	454
N4	32	6	35	67	720

Table 6-4 Candidate routes for network N1

Route ID	Bus stops travelled	Single trip distance (km)
1	1 2 5 8	1.91
2	1 4 7 8	1.91
3	1 4 13 10 8	3.11
4	1 3 9 12 13 10 8	3.42
5	1 4 13 14 11 8	3.79
6	1 3 9 12 13 14 11 8	4.09
7	1 4 13 14	2.54
8	1 4 7 10 14	2.54
9	1 3 9 12 13 14	2.85
10	1 4 7 8 11 14	3.15
11	1 2 5 8 11 14	3.15
12	1 2 5 7 13 14	3.74
13	1 4 13 10 8 11 14	4.35
14	1 3 9 12 13 10 8 11 14	4.66
15	8 10 14	1.16
16	8 11 14	1.24
17	8 7 13 14	2.36
18	8 5 2 4 13 14	3.56
19	8 7 6 9 12 13 14	3.56
20	8 5 2 4 6 9 12 13 14	4.76
21	8 7 4 1 3 9 12 13 14	4.76

6.3 Planning bus routes for the four simplified networks with SA-GA approach

6.3.1 Objective function and parameter settings

Since only bus service is considered in the theoretical test and in order to make the application more realistic, two-path assignment is used in this study. Two paths are considered between any two bus stops and used in demand assignment. Therefore, the objective function (Equation 5-1) is modified as follows:

$$Min: E = c_p \sum_{i=1}^{n} \sum_{j=1}^{n} (T_{ij}^1 \cdot t_{ij}^1 + T_{ij}^2 \cdot t_{ij}^2 + T_{ij} \cdot t_{wij}) + c_b \sum_{k \in R_{SR}} f_k \cdot L_k \qquad \text{Equation 6-1}$$

subject to the following constraints:

i. all bus stops are covered

ii. $f_k \geq f_{k,\min}$

The superscripts 1 and 2 represent the two paths used in the assignment – the first path is always the shortest and the second an alternative path between bus stops i and j. The average waiting time for passengers travelling from bus stop i to j (t_{wij}), is calculated in the following way:

$t_{wij} = 0.5 * \frac{1}{f_k}$ if the two paths start with the same bus route k or one of the path is by walking only;

$t_{wij} = 0.5 * \frac{1}{f_1 + f_2}$ if the two paths start with different bus routes with different frequencies, f_1 and f_2, respectively.

Usually, a frequency constraint is also imposed for the objective function. For a bus route with a frequency lower than the minimum frequency, the system will check whether the bus stop coverage constraint will be violated if this route was excluded from the solution. If the constraint is not violated, this route will be excluded and the first iteration of demand assignment is performed again. Otherwise this route will be kept and frequency for this route is set to the predefined minimum frequency. However, a maximum frequency constraint was not used as it may exclude some actually very good routes which carry a lot of travellers. In real life, this problem can be solved manually by splitting one heavily loaded route into two similar routes.

Before performing the SA-GA, some parameter values need to be specified. These are assumed as in Table 6-5. The assumed value for bus operating cost c_b is 5.24. It is calculated as follows: $\frac{\text{Total Expenses}}{\text{Vehicle Revenue Miles} * 1.6} = \frac{15{,}613}{1{,}864 * 1.6}$. These data are obtained from 2002 bus transit data in USA (APTA, 2003). The value of travellers' time is assumed as \$8.00 per hour based on American data (VTPI, 2005).

Table 6-5 Assumed values for parameters used in SA-GA

Parameter	***Unit***	***Assumed value***
Bus capacity	passengers / bus	80
Travellers' value of time (c_p)	$ / hour	8.00
Bus operating cost (c_b)	$ / bus-km	5.24
Transfer penalty	seconds / transfer	400
Minimum bus frequency ($f_{k,\ min}$)	buses / hour	3

6.3.2 Simulated annealing process

The main (outer) process is the simulated annealing algorithm (SA). Genetic algorithm (GA) is used as its sub-process to give a disturbance to the current solution kept in SA in order to obtain a better solution. The details are explained in Section 5.2.2. It is implemented in Visual C++ environment. The maximum temperature (T) is set to 50. Parameter I is used to control the iterations of the GA process. The maximum number of iterations is set to 20.

6.3.3 Genetic algorithm process

Genetic algorithm (GA) is used as the approach to generate new potential solutions among which the one with the best fitness value is picked and kept in simulated annealing (SA) process for further selection. The details are presented in Section 5.2.3. In the case of minimisation, the fitness value is usually assumed to be the reciprocal of the objective function value.

Initialisation

The ranges selected for the four theoretical networks in order to do variable string length coding are given in Table 6-6. The population size is selected as 99 which is chosen after the initial experiments showed that this did not increase the computation time dramatically.

Table 6-6 String length ranges for the four theoretical networks

	Network 1	Network 2	Network 3	Network 4
String length range	1---15	1---20	1---25	1---30

Evaluation

Evaluating the solution is probably the most important and complicated part of the whole process. Every solution is evaluated based on the objective function (Equation 6-1) and given a fitness value which is the reciprocal of the objective function value.

To explain the transit assignment process more clearly, a simple example is shown here. Suppose that only two routes are included in the network. They are Route 2 and Route 8 in Figure 6-6. Numbers along roads represent the bus services travelling along that road. For example, there are in total 71 people (see Table 6-1) that need to travel from Bus Stop 1 to Bus Stop 7. Obviously, the two paths are taking Bus Service No. 2 or taking Bus Service No. 8. They have the same in-vehicle travel time. In this case, the demand of 71 people is assigned equally to Bus Routes 2 and 8, which is 35.5 for each route. Actually the demand is assigned to each segment of the route. In this case, segments 1-4 and 4-7 of Route 2; segments 1-4 and 4-7 of Route 8. Using this method, the demands between 17 pairs of bus stops are assigned to the routes included in the network. They are listed in Table 6-7. For some trips, like travelling from Bus Stop 1 to Bus Stop 8, there is only one path available. All the 116 demand is assigned to the only path which is Route 8. It should be noted that an assumption is made that all routes are two-way. It is necessary to distinguish the travel directions. After assigning the demands of all trips, a sum of assigned demands for each segment of each route is calculated. The largest sum for each route is then taken as the highest demand of this route and used to calculate frequency. For Route 2, 412 is the highest demand. Therefore, its frequency is 412/80 = 5.2 buses/hour. For Route 8, the highest demand is 619. Then its frequency is 619/80 = 7.7 buses/hour.

Next, second transit assignment would be conducted. Since two paths are selected based on in-vehicle travel time and transfer penalty only, the two paths found for each pair of bus stops would be the same as the two paths found for first transit assignment. The only difference is the amount of demand assigned to the routes. Once again, consider trips from Bus Stop 1 to Bus Stop 7 as the example. As the demand is assigned according to frequency this time, so the demand assigned to the first path would be 71 * 5.2 / (5.2+7.7) = 28.6; and the demand assigned to the second path would be 71 * 7.7 / (5.2+7.7) = 42.4. With this approach, the demands of all trips are assigned and recorded in Table 6-8.

Table 6-7 First transit assignment

Trips	Path	Demand	Route 2			Route 8			
			1---4	4---7	7---8	1---4	4---7	7---10	10---14
1→4	1: (1)==[2]==(4)	39/2=19.5	19.5						
	2: (1)==[8]==(4)					19.5			
1→7	1: (1)==[2]==(4) ==[2]==(7)	71/2=35.5	35.5	35.5					
	2: (1)==[8]==(4) ==[8]==(7)					35.5	35.5		
1→8	1: (1)==[2]==(4) ==[2]==(7) ==[2]==(8)	116	116	116	116				
1→10	1: (1)==[8]==(4) ==[8]==(7) ==[8]==(10)	96				96	96	96	
1→14	1: (1)==[8]==(4) ==[8]==(7)==[8] ==(10)==[8]==(14)	122				122	122	122	122
4→7	1: (4)==[2]==(7)	35/2=17.5		17.5					
	2: (4)==[8]==(7)						17.5		
4→8	1: (4)==[2]==(7) ==[2]==(8)	58		58	58				
4→10	1: (4)==[8]==(7) ==[8]==(10)	48					48	48	
4→14	1: (4)==[8]==(7) ==[8]==(10) ==[8]==(14)	61					61	61	61
7→8	1: (7)==[2]==(8)	64			64				
7→10	1: (7)==[8]==(10)	54						54	
7→14	1: (7)==[8]==(10) ==[8]==(14)	68						68	68
8→10	1: (8)==[2]==(7) ==[8]==(10)	75						75	
8→14	1: (8)==[2]==(7) ==[8]==(10) ==[8]==(14)	95						95	95
10→8	1: (10)==[8]==(7) ==[2]==(8)	71			71				
10→14	1: (10)==[8]==(14)	75							75
14→8	1: (14)==[8]==(10) ==[8]==(7) ==[2]==(8)	103			103				
		Sum	171	227	**412**	273	380	**619**	421

Notes:
Trip 1→4: trip starts from Bus Stop 1 and ends at Bus Stop 4;
(1): Bus Stop 1;
[2]: bus Route 2;
Path (1)==[2]==(4)==[2]==(7): take bus Route 2 at Bus Stop 1, then pass Bus Stop 4 and finally get off at Bus Stop 7;
1—4: route segment starts from Bus Stop 1 and ends at Bus Stop 4.

Table 6-8 Second transit assignment

Trips	Path	Demand	Route 2			Route 8			
			1---4	4---7	7---8	1---4	4---7	7---10	10---14
1-->4	1: (1)==[2]==(4)	39*5.2/12.9	15.7						
	2: (1)==[8]==(4)	39*7.7/12.9				23.3			
1-->7	1: (1)==[2]==(4) ==[2]==(7)	71*5.2/12.9	28.6	28.6					
	2: (1)==[8]==(4) ==[8]==(7)	71*7.7/12.9				42.4	42.4		
1-->8	1: (1)==[2]==(4) ==[2]==(7) ==[2]==(8)	116	116	116	116				
1→10	1: (1)==[8]==(4) ==[8]==(7) ==[8]==(10)	96				96	96	96	
1→14	1: (1)==[8]==(4) ==[8]==(7)==[8] ==(10)==[8]==(14)	122				122	122	122	122
4-->7	1: (4)==[2]==(7)	35*5.2/12.9		14.1					
	2: (4)==[8]==(7)	35*7.7/12.9					20.9		
4-->8	1: (4)==[2]==(7) ==[2]==(8)	58		58	58				
4→10	1: (4)==[8]==(7) ==[8]==(10)	48					48	48	
4→14	1: (4)==[8]==(7) ==[8]==(10) ==[8]==(14)	61					61	61	61
7-->8	1: (7)==[2]==(8)	64			64				
7→10	1: (7)==[8]==(10)	54						54	
7→14	1: (7)==[8]==(10) ==[8]==(14)	68						68	68
8→10	1: (8)==[2]==(7) ==[8]==(10)	75						75	
8→14	1: (8)==[2]==(7) ==[8]==(10) ==[8]==(14)	95						95	95
10→8	1: (10)==[8]==(7) ==[2]==(8)	71			71				
10→14	1: (10)==[8]==(14)	75							75
14→8	1: (14)==[8]==(10) ==[8]==(7) ==[2]==(8)	103			103				
		Sum	160.3	216.7	**412**	283.7	390.3	**619**	421

Notes:
Trip 1→4: trip starts from Bus Stop 1 and ends at Bus Stop 4;
(1): Bus Stop 1;
[2]: bus Route 2;
Path (1)==[2]==(4)==[2]==(7): take bus Route 2 at Bus Stop 1, then pass Bus Stop 4 and finally get off at Bus Stop 7;
1—4: route segment starts from Bus Stop 1 and ends at Bus Stop 4.

With the information given in Table 6-8, the highest demand of each route may also be calculated in the same way as in the first iteration. The result does not change as marked by bold characters in the table. Besides that, the total passengers who take route 2 and route 8 can be calculated too. That would be done by adding the sums together for each route. For example for direction 1 of Route 2, the total number of passengers is 160.3 + 216.7 + 412 = 789. For direction 2 of Route 2, the total number is 335 + 141.5 +123.6 = 600.1.

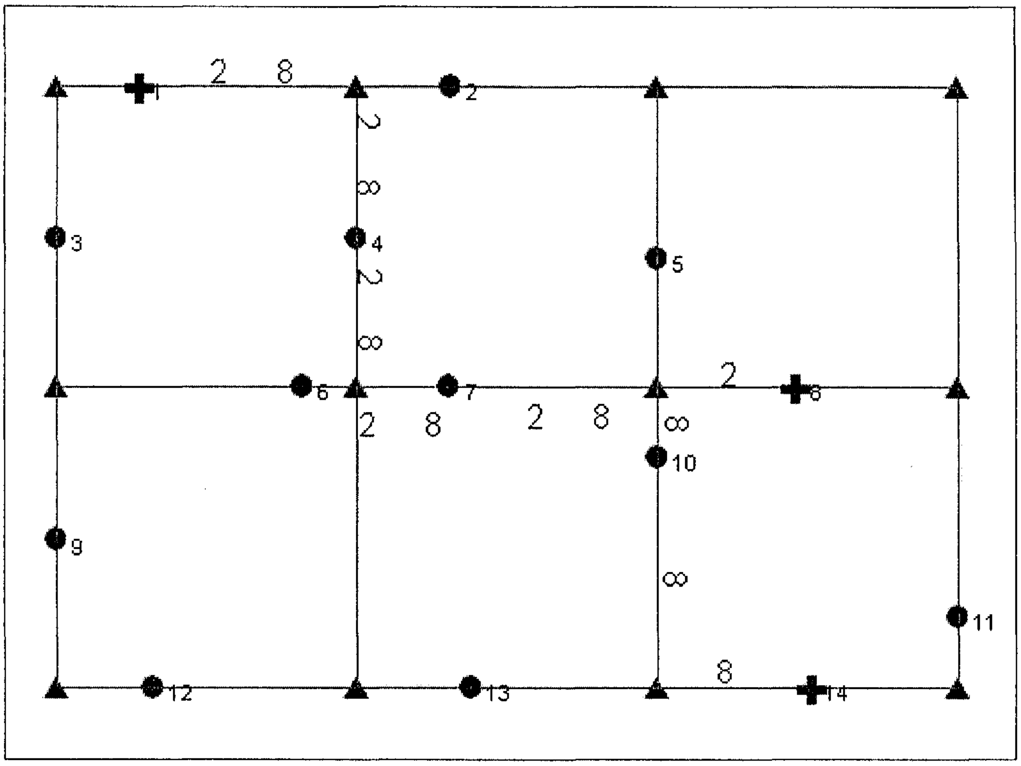


Figure 6-6 Map for transit assignment example

Reproduction

Reproduction is a process in which some strings with better fitness values are selected from the population for further GA operations. Before doing reproduction, N_0 is combined with the 99 strings to form a population with 100 strings. The popular roulette wheel selection method (Goldberg, 1989) is used in this study. In this method, each string is assigned a probability of being selected in proportion to its fitness value. Then several random numbers between 0 and 1 are generated to decide how many copies of each string should be selected into the mating pool. A simple example shown in Table 6-9 is used to demonstrate this process. Suppose the population of four strings have the objective function values and fitness values as shown in the table. The fitness proportions for each string are calculated and also listed in the table. A probability range is assigned to each string according to its fitness proportion. Then four random numbers (the number of random numbers is equal to the population size) are generated. For example, they are: 0.038, 0.476, 0.530 and 0.987. Since the first number falls in the range for string 1, the second and third numbers fall in range for string 3 and the fourth number falls in the range for string 4, one copy of string 1, no copy of string 2, two copies of string 3 and one copy of string 4 are taken into the mating pool, as shown in the table. The population of the mating pool is then:

2 13 6 3 2 19 4 16 2 19 4 16 1 8 6 15

Finally, the strings in the mating pool are randomly sorted.

Table 6-9 Example of reproduction

String number	String	Objective function value	Fitness	% of total fitness	Probability range	No. selected
1	2 13 6 3	5	0.2	18.6	0-0.186	1
2	7 14 5 12	8	0.125	11.6	0.186-0.302	0
3	2 19 4 16	2	0.5	46.5	0.302-0.767	2
4	1 8 6 15	4	0.25	23.3	0.767-1	1
Total			1.075	100		4

Crossover

Crossover is a process to generate new strings from a given pair of strings in the process of reproduction. Whether crossover is performed on a pair of string is controlled by crossover probability P_c, which has a predefined fixed value. To maximise the exploration space, P_c is set to 1. This means crossover is performed on every pair of strings. There are a number of crossover methods available. Each has its special characteristics and is potentially more efficient for certain types of problems. To test the effectiveness of different crossover methods, four kinds of crossover: single-point, two-point, uniform and convex are implemented.

Single-point crossover is the most basic one. Each pair of strings undergoes crossover as follows: an integer position p is selected at random where $1 \le p \le l-1$, with l being the length of the string. Two new strings are created by swapping all elements between position p+1 and l. For example, suppose the two strings:

Parent 1	2	13	6	3
Parent 2	2	19	4	16

are selected for crossover and the value p, where $1 \le p \le 3$, is chosen at random to be 2. Then the third and fourth elements of the two strings are exchanged to obtain two new strings:

Child 1	2	13	4	16
Child 2	2	19	6	3

Now the operation of single-point crossover is complete.

Two-point crossover is very similar to single-point crossover. Two integer positions, instead of one, are selected at random and the segments between them swapped. For example, the two positions chosen are 1 and 3. Then the resulting new strings are:

Child 1	2	19	4	3
Child 2	2	13	6	16

Uniform crossover is different from the previous two crossovers. It has the ability to change every element in a string. It works as follows: a template with only 0 and 1 is constructed first. A 1 in the template indicates that the first parent string will contribute its element to the first child string and the second parent string will contribute its element to the second child string, while a 0 in the template indicates the opposite. Here is an example:

Parent 1	2	13	6	3
Parent 2	2	19	4	16
Template	1	1	1	0
Child 1	2	13	6	16
Child 2	2	19	4	3

Convex crossover is a type of arithmetical operator. Parent x is replaced by $x' = a_1 x + a_2 y$ and parent y is replaced by $y' = a_1 y + a_2 x$, where $a_1 + a_2 = 1$ and $a_1, a_2 > 0$. For example:

Parent 1	2	13	6	3
Parent 2	2	19	4	16

a_1 is selected at random as 0.3. Then a_2 is 0.7. The first element of child 1 should be [0.3*2+0.7*2] = 2. The second element of child 2 should be [0.3*19+0.7*13] = [14.8] =14. Therefore, the new pair of strings are:

Child 1	2	17	4	12
Child 2	2	14	5	6

It should be noted that the length of the two strings for crossover may be different when using variable string length coding. Therefore the shorter length between the two strings is taken as l. For uniform crossover, the length of the template is the same as the length of the shorter string. For convex crossover, it is also only done on the part of the two parents whose length is the same as the length of the shorter string. All the above rules are made to avoid creating child strings which are longer than their parents and destroying the consistency of the process.

Mutation

Mutation is the occasional random alteration of the value of an element of a string. It is necessary because sometimes useful information (e.g. bus stop sequence) may be lost during reproduction and crossover. The mutation operator helps to introduce some potentially useful components (i.e. good bus stop sequences). As in crossover, this process is also controlled by a probability parameter P_m, (mutation probability). For each element of a string, a random number between 0 and 1 is generated. If it is smaller than P_m, mutation is performed; otherwise no mutation is performed for this element. The frequency of mutation is usually very small in most GA applications. For example, one mutation per thousand elements is a typical value which produces best results in most applications (Goldberg, 1989). The value of $P_m = 0.001$ is also used in this study.

As in the case of crossover, two types of mutation methods are implemented to test the effectiveness of different methods on this bus route design problem. One is uniform mutation. Once an element of a string is confirmed for mutation, it will be replaced by a route ID which is chosen at random from the candidate route IDs. The other type is dynamic mutation. If the element x_k is selected for mutation, then the new x_k' is chosen with equal probability from the two choices:

$$x_k' = x_k + r(b_k - x_k)(1 - \frac{t}{T})^b \qquad \text{or} \qquad x_k' = x_k - r(x_k - a_k)(1 - \frac{t}{T})^b$$

where r is a random number chosen uniformly from [0, 1], t is the current generation number, T is the maximum number of generations and b is a parameter determining the degree of nonuniformity. Here it is a random number chosen uniformly from interval (0, 1]. (a_k, b_k) is the range of possible values that x_k can take. So a_k is 1 and b_k is the maximum number of candidate routes for each theoretical network.

Termination

The whole GA process is controlled by the parameter I. Under each temperature, GA process is performed for I_{max} times. After I_{max} iterations are completed, a new population of 99 solutions is generated to maintain the diversity of the overall population.

6.3.4 Results of computations

Since four types of crossover and two types of mutation are used, in total eight different combinations are applied for each theoretical network. Optimal values obtained in sensitivity analysis (Section 6.4) are used for the experiments on the four theoretical networks. The computation times for one combination of crossover / mutation on each network are recorded in Table 6-10 and plotted in Figure 6-7. The figure shows that the computation time goes up with the increasing network size and its rate of increase is growing.

Table 6-10 Computation time for four theoretical networks

Network	No. of Bus Stops	No. of Terminals	Maximum size of route network	Average time for SA-GA (min)
N1	14	3	15	7
N2	20	4	20	37
N3	26	5	25	81
N4	32	6	30	133

The best solutions found for each combination of reproduction methods for network N1 are listed in Table 6-11. The overall best solution for network N1 is marked with a star. It is plotted on Figure 6-8 and its attributes are summarised in Table 6-12. Demand is the total number of passengers travelling with the route. Numbers along roads represent the bus services travelling along that road. With this solution, only 38 O-D pairs out of 182 O-D pairs need one transfer in the shortest travel path. The others are all direct paths without transfer. The transfer rate is 7,778 / 6,434 = 1.21.

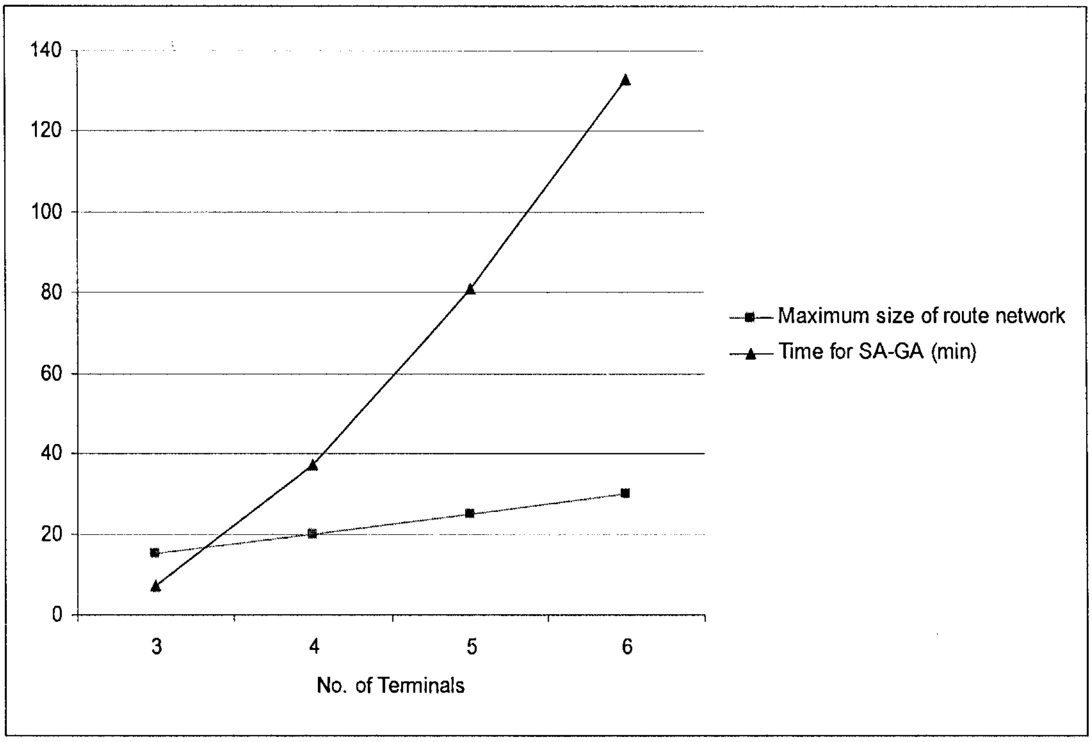


Figure 6-7 Computation time for four theoretical networks

Table 6-11 Result for network N1

Crossover	Mutation	Objective function value	Travellers' cost	Operation cost	Solution size	Solution
Single-point	Uniform	9,509	8,267	1,242	5	11,14,16,19,21
Single-point	Dynamic	9,525	8,422	1,103	5	1,8,11,14,20
Two-point	Uniform	9,523	8,459	1,064	5	8,9,11,14,19
Two-point	Dynamic	9,486	8,283	1,203	4	*11,14,19,21
Uniform	Uniform	9,523	8,459	1,064	5	8,9,11,14,19
Uniform	Dynamic	9,491	8,275	1,216	5	4,10,11,14,20
Convex	Uniform	9,537	8,374	1,163	4	10,11,14,19
Convex	Dynamic	9,541	8,485	1,056	5	8,9,11,14,19

*overall best solution

Table 6-12 Features of the best solution for network N1

Route	Frequency	Length (km)	Max demand along the route	Demand
11	9.5	6.30	757	1,967
14	8.1	9.32	649	2,376
19	5.6	7.12	445	1,528
21	5.8	9.52	461	1,907
Total				7,778

The best solutions found for each combination of crossover / mutation for network N2 are listed in Table 6-13. The overall best solution for network 2 is marked with a star and its attributes are summarised in Table 6-14. This best solution is plotted on Figure 6-9. Numbers along roads represent the bus services travelling along that road. With this solution, only 52 O-D trips (bus stop to bus stop) out of 380 pairs need one transfer in the shortest travel path. The others are all direct paths without transfer. The transfer rate is 14,632/13,079 = 1.12. This means that 89% of trips were accomplished without any transfer.

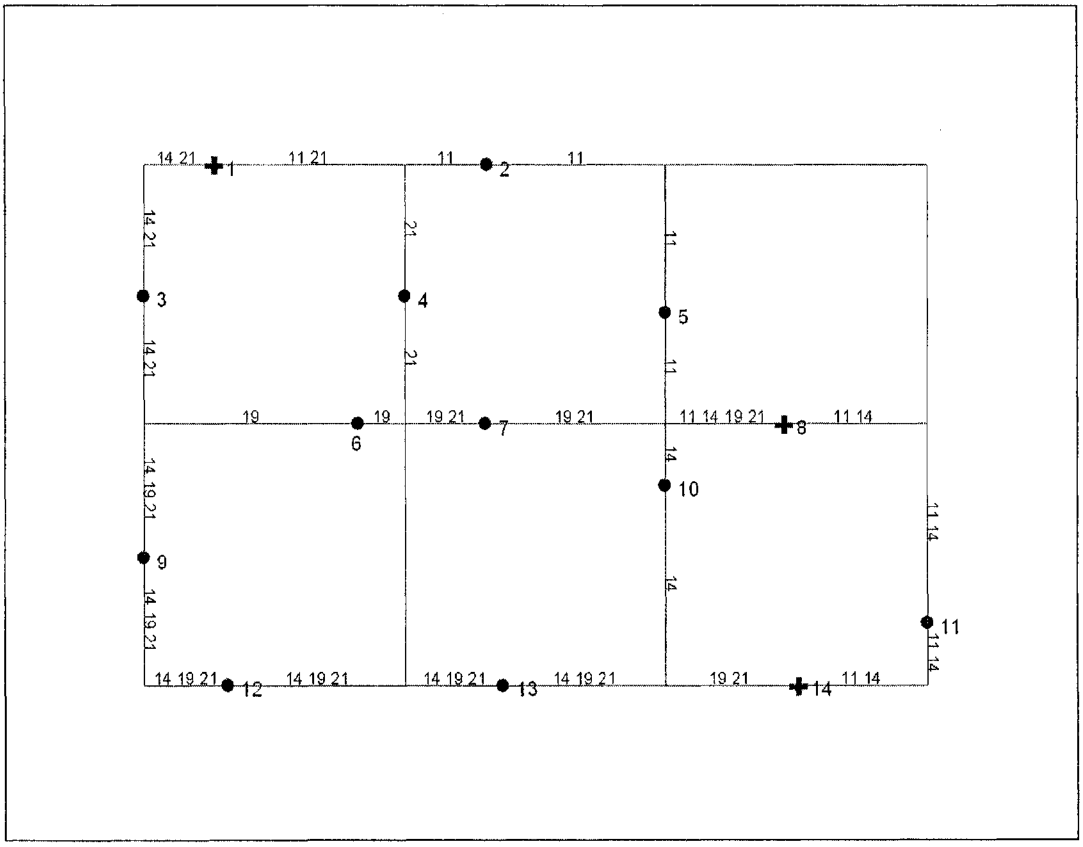


Figure 6-8 Solution for network N1

The best solutions found for each combination of crossover / mutation for networks N3 and N4 are listed in Tables I1 and I3 in Appendix I. The overall best solutions for networks N3 and N4 are marked with a star and their attributes are summarised in Tables I2 and I4 in Appendix I. The star solutions are plotted in Figures I1 and I2 in Appendix I. Numbers along roads represent the bus services travelling along that road. With the best solution, only 12% O-D pairs of network N3 need one transfer in the shortest travel path. The others are all direct paths without transfer. The transfer rate of network N3 is 25,244 / 22,570 = 1.12. This means that 89% of trips can be accomplished without any transfer. For network N4, 88% of the O-D pairs can be travelled without any transfer in the shortest travel path. The transfer rate is 42,315 / 35,914 = 1.18. This means that 85% of trips were accomplished without any transfer. A summary of the best result for all four networks is in Table 6-15.

Table 6-13 Results for network N2

Crossover	Mutation	Objective function value	Travellers' cost	Operation cost	Solution size	Solution
Single-point	Uniform	20,133	17,310	2,823	5	106,58,145,52,62
Single-point	Dynamic	20,031	17,205	2,826	6	152,40,79,87,111,151
Two-point	Uniform	20,201	17,453	2,749	7	152,54,148,134,111,24,36
Two-point	Dynamic	19,986	17,032	2,954	6	*151,39,91,42,62,58
Uniform	Uniform	20,098	16,987	3,111	5	153,131,62,116,58
Uniform	Dynamic	20,011	16,932	3,079	6	67,152,54,107,20,116
Convex	Uniform	20,222	17,189	3,033	5	13,58,116,102,62
Convex	Dynamic	20,633	17,783	2,850	7	131,153,151,93,133,57,103

*overall best solution

Table 6-14 Features of the best solution for network N2

Route	Frequency	Length (km)	Max demand along the route	Demand
151	8.0	9.52	641	2,573
39	5.2	13.50	412	1,538
91	5.9	8.26	471	1,727
42	7.9	14.68	634	2,404
62	11.5	14.04	921	4,270
58	6.8	13.42	544	2,121
Total				14,632

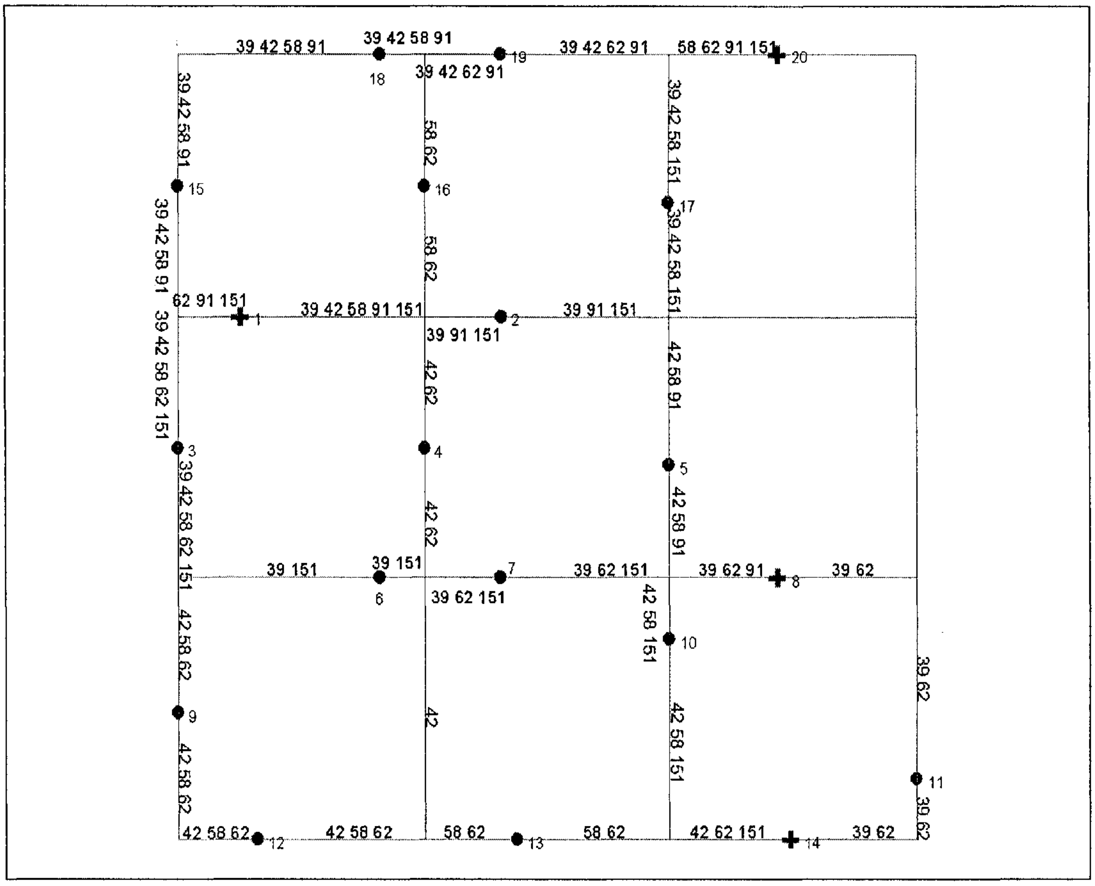


Figure 6-9 Solution for network N2

Table 6-15 Summary of computation results of four theoretical networks

Network	*Max. solution size allowed*	*Crossover used by the best result*	*Mutation used by the best result*	*Objective function value*	*Travellers' cost*	*Operation cost*	*Solution size*	*Average time for SA-GA (min)*
N1	15	T-P	Dynamic	9,486	8,283	1,203	4	7
N2	20	T-P	Dynamic	19,986	17,032	2,954	6	37
N3	25	S-P	Dynamic	36,599	31,419	5,180	10	81
N4	30	T-P	Dynamic	61,698	52,573	9,125	19	133

6.4 Sensitivity analysis and discussion

The computation times of all the networks are indicated in Table 6-15. Obviously, the computation time increases very quickly with the increase of network size. This is not surprising because it is generally accepted that bus network design problem is a highly computationally-intensive problem (NP-hard).

When comparing the features of the best results for all four networks, it can be seen that all the best solutions came from dynamic mutation and three best solutions came from the two-point crossover. This suggests that dynamic mutation and two-point crossover are more suitable than the other mutation and crossover techniques for solving the bus network design problems with GA.

The performance of SA-GA may depend upon the selection of parameters like probability of crossover (*Pc*), probability of mutation (*Pm*), population size, and the maximum number of iterations (*I*). The following sections present the sensitivity analysis of these parameters for network N1.

6.4.1 Effect of crossover probability

The effect of crossover probability is investigated by choosing different numbers ranging from 0.1 to 1 for probability of conducting crossover and the result is provided in Figure 6-10. As can be seen, the lowest objective function value is achieved with 1 as the crossover probability. Therefore, 1 is recommended as the crossover probability.

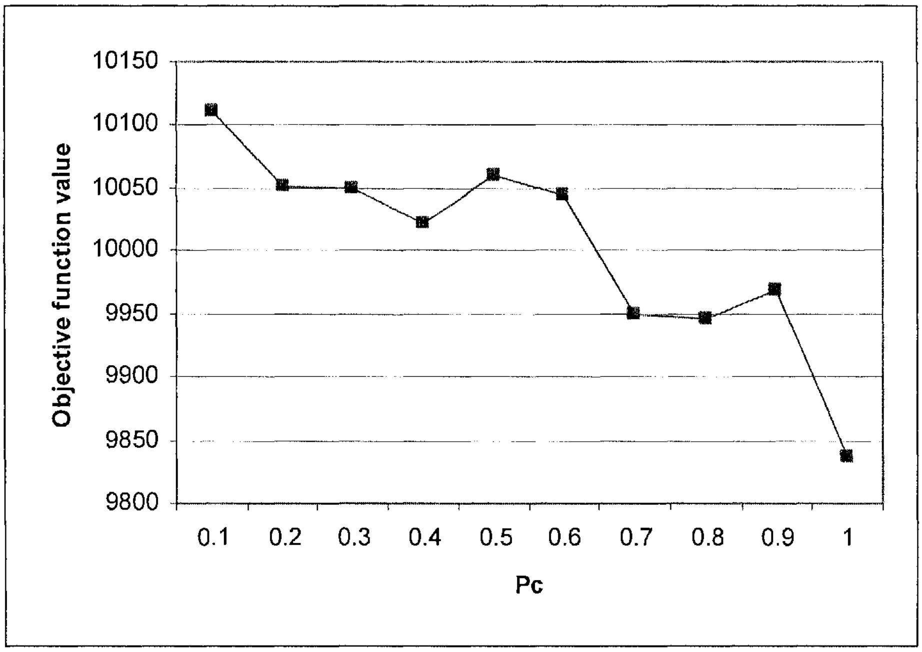


Figure 6-10 Effect of crossover probability

6.4.2 Effect of mutation probability

Similarly the effect of mutation probability is examined by varying this value from 0.00001 to 1 and the result is presented in Figure 6-11. The lowest objective function value is achieved at 0.001. This suggests that the optimal mutation probability might be 0.001.

6.4.3 Effect of population size

The effect of population size is investigated by varying this value from 50 to 300 and the result is shown in Figure 6-12. It can be seen that the objective function value tends to decrease with the increase of the population size. It is also observed that the larger the population size, the longer the computation time. When population size is 200, the lowest objective function value is achieved. This suggests that 200 should be used as the optimal population size for network N1.

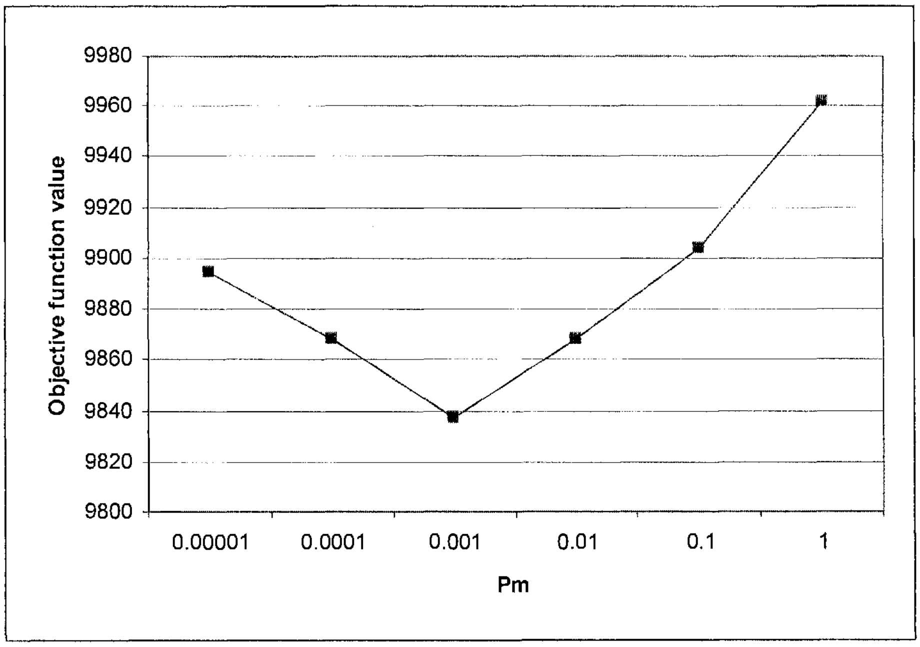


Figure 6-11 Effect of mutation probability

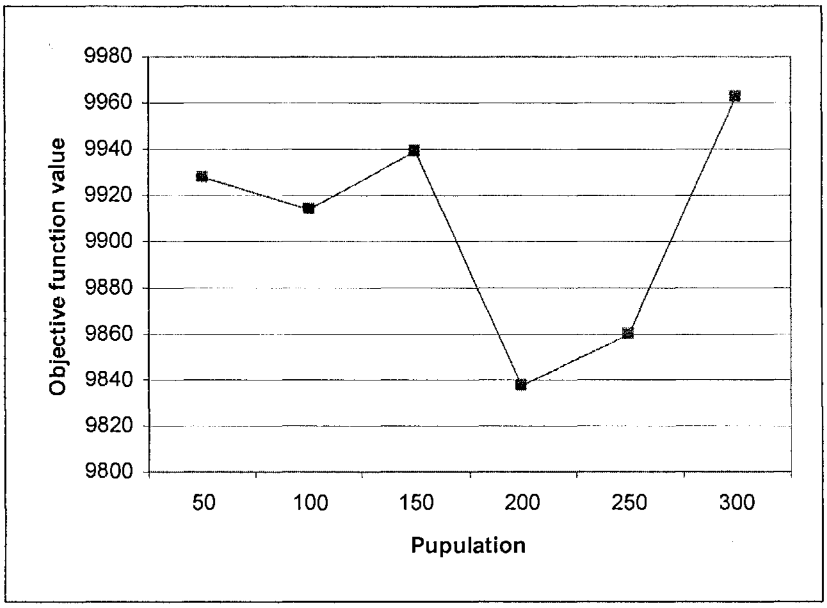


Figure 6-12 Effect of population size

6.4.4 Effect of the number of iterations

Parameter I is the number of iterations performed with GA and using the same population. A new population will be generated to increase the diversity after the Ith iteration of GA is completed. The effect of parameter (I) is investigated by varying its value from 5 to 30 and the result is shown in Figure 6-13. It can be seen that the best objective function value is achieved when I is 20. Therefore, 20 is assumed for this parameter.

6.4.5 Effect of the scaling constant

As mentioned before, the acceptance of new solution in simulated annealing is decided by a probability function $P = \exp(-\frac{E_1 - E_0}{T})$ when the objective value of a new solution is greater than the objective value of current solution kept in SA. Obviously the probability is affected by the objective function value and this value is affected by the size of the network. Since the same T value is used for all networks, it would be useful to introduce a scaling factor to normalise the numerator of the probability function. Therefore, a scaling constant is introduced so that the probability function is changed to:

$$P = \exp\left[-\left(\frac{E_1 - E_0}{E_0}\right)\frac{C}{T}\right] \quad \text{Equation 6-2}$$

where C is the scaling constant.

The usefulness of scaling constant is investigated by varying its value from 1,000 to 12,000 and the result is plotted in Figure 6-14. Unfortunately no clear trend can be observed from the figure. However, it seems that a bigger scaling constant (10,000 to 12,000) would result in a better objective value.

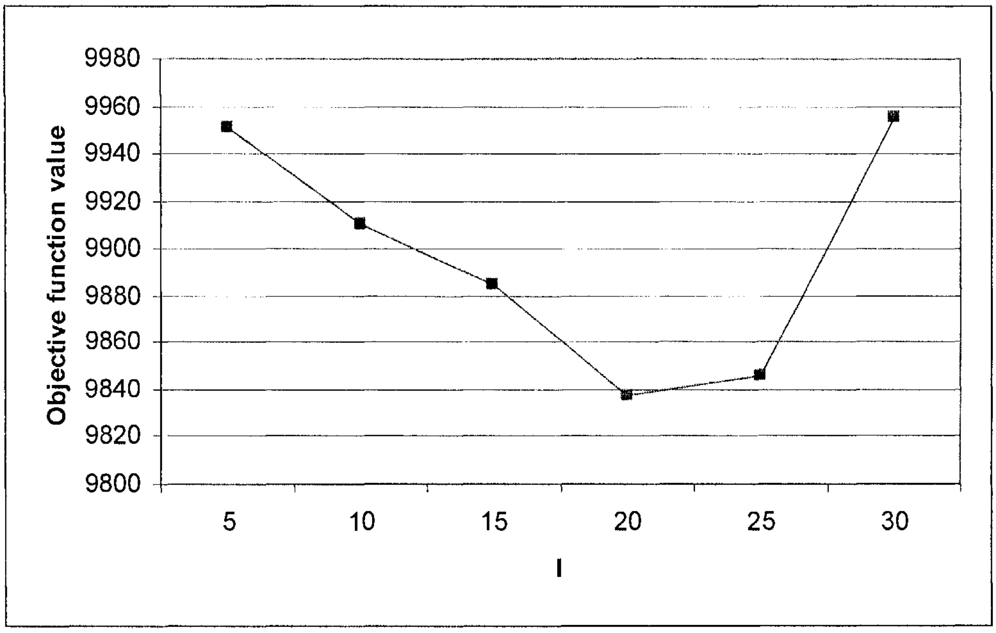


Figure 6-13 Effect of the number of iterations

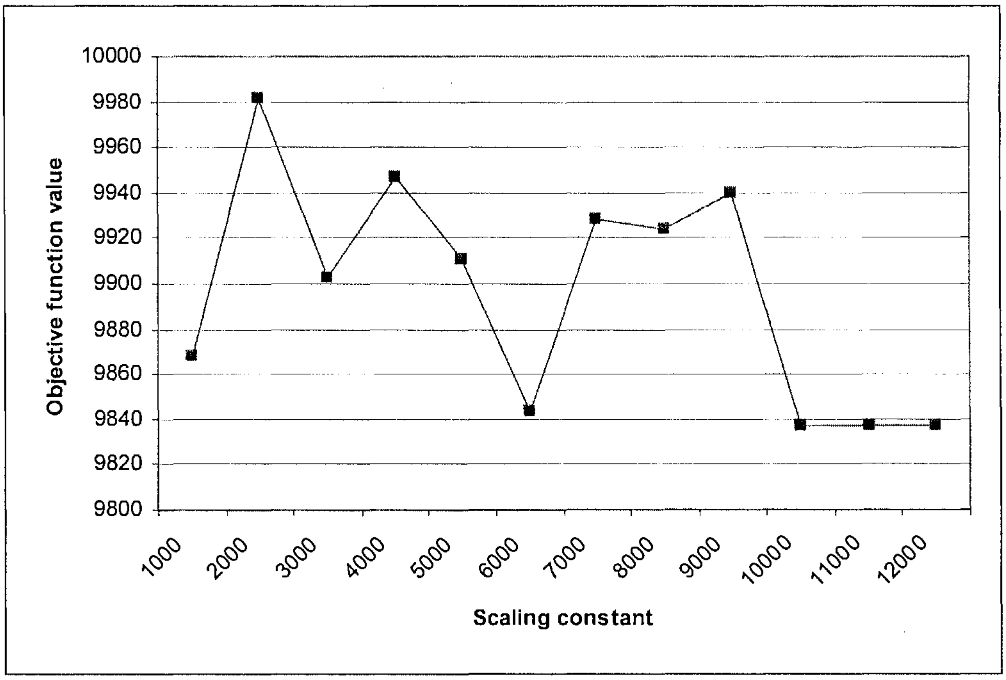


Figure 6-14 Effect of scaling constant

6.4.6 Discussion

The computation times of all the networks are indicated in Table 6-15. Obviously, the computation time goes up very quickly with the increase of network size. This is not surprising because it is widely accepted that bus network design problem is a highly computationally-intensive problem (NP-hard).

It can be noticed that all the eight solutions for network N1 (Table 6-11) have the similar objective function values which are within 0.6% range. This may indicate that there are some factors other than those included in the objective function which may also govern the solution to be selected. These factors may include delay in each bus stop caused by queuing, boarding and alighting when demand is close to capacity; variable transfer penalty for different situations such as transfer with walking and transfer without walking; fleet size constraint and so on. A more detailed objective function would give a more precise representation of the bus transport service quality so that a better optimal solution may be produced. However, the tradeoff would be in increased computation complexity and computation time. The calibration of a balanced objective function is worth of further study.

When comparing the features of the best results for all four networks (Table 6-15), it can be seen that the best solutions for all networks resulted from dynamic mutation and three best solutions resulted from two-point crossover. This suggests that dynamic mutation and two-point crossover are more suitable than the other mutation and crossover techniques for solving the bus route network design problems with GA.

Sensitivity analysis shows that the best results are obtained with the following parameter values for a theoretical network with 14 bus stops (3 are terminals), 12 intersection nodes, 17 road links and 31 bus links:

- Crossover probability = 1
- Mutation probability = 0.001
- Population size = 200
- Number of iterations = 20

Sensitivity analysis was also done on parameters c_b and c_p for the range of 1-10 for both parameters. Results show that improvement on objective function value is not sensitive to the change of c_b and $c_{p.}$ The percentage decreases in objective function value are comparable with different values of c_b and $c_{p.}$ Therefore, the values shown in Table 6-5 were not changed and used in subsequent analysis.

CHAPTER 7 IMPLEMENTATION OF SA-GA ALGORITHM ON BENCHMARK NETWORK

In this chapter, the proposed framework is tested on an existing problem, which can be considered as a benchmark reported in the literature. This problem is known as Mandl's network as it was first presented by Mandl (1979). Later this network was used as the testing case by other researchers. The solutions obtained in previous studies are compared with the solution generated by the framework proposed in this study.

7.1 Mandl's network

Figure 7-1 shows the Mandl's network. There are 15 nodes (bus stops) and 21 links (roads). The link length is defined with in-vehicle travel time in minutes (as shown on the links). The demand matrix is presented in Table 7-1.

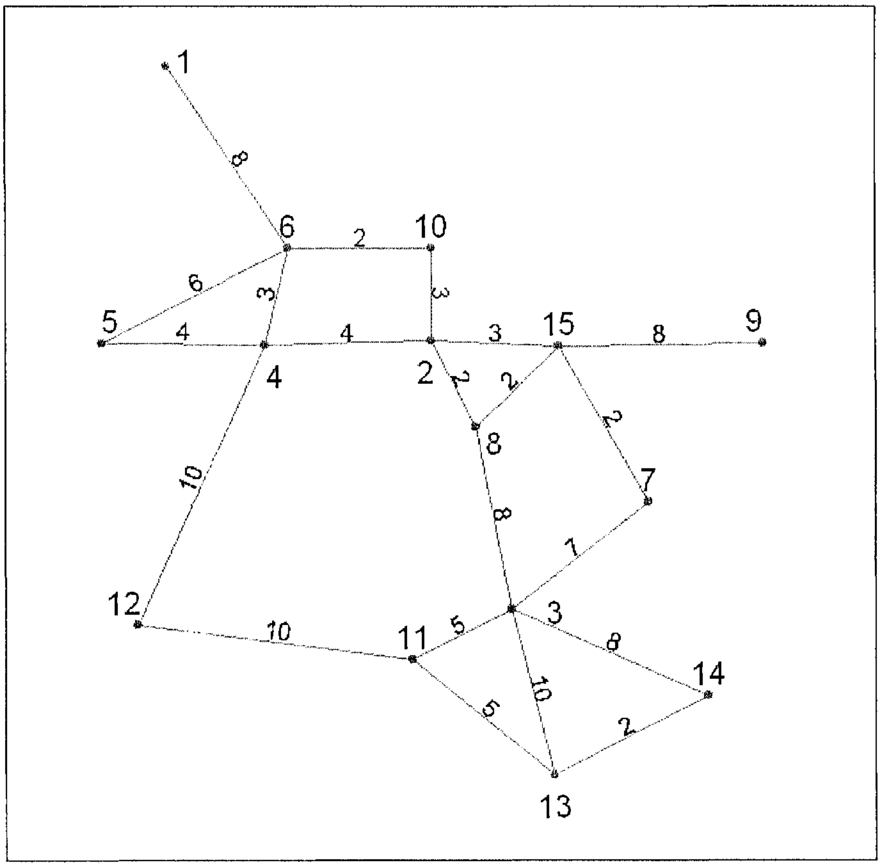


Figure 7-1 Mandl's network

Table 7-1 Demand matrix of Mandl's network

Node	***1***	***2***	***3***	***4***	***5***	***6***	***7***	***8***	***9***	***10***	***11***	***12***	***13***	***14***	***15***
1	0	150	160	60	80	400	75	75	30	200	30	25	35	0	0
2	150	0	880	100	50	180	100	100	30	180	60	15	15	10	0
3	160	880	0	240	120	130	440	440	140	45	600	250	500	200	0
4	60	100	240	0	50	120	50	50	15	40	40	25	10	5	0
5	80	50	120	50	0	20	25	25	10	60	20	15	5	0	0
6	400	180	130	120	20	0	90	90	15	50	20	10	10	5	0
7	75	100	440	50	25	90	0	50	15	90	35	10	10	5	0
8	75	100	440	50	25	90	50	0	15	90	35	10	10	5	0
9	30	30	140	15	10	15	15	15	0	15	20	5	0	0	0
10	200	180	45	40	60	50	90	90	15	0	20	10	10	5	0
11	30	60	600	40	20	20	35	35	20	20	0	75	95	15	0
12	25	15	250	25	15	10	10	10	5	10	75	0	70	0	0
13	35	15	500	10	5	10	10	10	0	10	95	70	0	45	0
14	0	10	200	5	0	5	5	5	0	5	15	0	45	0	0
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

7.2 Generation of candidate routes

Since no terminal was used in previous studies on Mandl's network, no terminal is set in this testing either in order to make the solution comparable. Candidate routes are generated for each pair of nodes (bus stops). In total, 1183 candidate routes are generated (see Appendix J).

7.3 Objective and parameter setting

Equation 6-1 is used as the objective function here. However, minimising total travel time was used as the objective in previous studies. To make the result comparable, two objectives are used here. One is minimising total travel time. The other is minimising the total cost which is the same as the objective used in testing on theoretical networks. To implement the first objective, we only need to set the operating cost to zero and set c_p (the factor to convert passenger travel time to passenger travel cost) to 1.

Parameter settings are the same as those used in the previous studies:

- Bus capacity: 50 passengers per bus;
- Transfer penalty: 5 minutes.

No constraint is set on frequency.

7.4 SA-GA numerical results and comparison

Testing on Mandl's network is done using the same procedure as used for theoretical networks. The results for SA-GA with two different objectives and the results of other methods are listed in Table 7-2. Theoretical minimum is the minimum total in-vehicle travel time, assuming that the shortest path is available for all pairs of bus stops. Calculated for Mandl's network it is equal to 155,856 minutes.

Table 7-2 Comparison of results for Mandl's network

	Total travel time	*In-vehicle time*	*Waiting time*	*Transfer time*	*Direct trip %*	*One transfer %*	*Two transfer %*	*No. of routes*
Mandl's final solution	219,044 (141%)	177,400 (114%)	18,144	23,500	69.94	29.93	0.13	4
Baaj & Mahmassani's best solution	205,656 (132%)	168,076 (108%)	20,930	16,650	78.61	21.39	0	6
Zhao & Zeng's best solution	186,545 (120%)	NA	NA	NA	95.95	4.05	0	8
SA-GA (minimise total cost)	186,018 (119%)	157,428 (101%)	19,708	8,882	94.53	5.47	0	6
SA-GA (minimise travel time)	181,043 (116%)	157,678 (101%)	20,203	3,162	96.35	3.65	0	10

* Percentage in the brackets is the percentage over the theoretical minimum.

Some previous studies using Mandl's network are selected for comparison purpose. They are:

- Mandl (1979) used heuristic method to optimise the urban public transport network with the objective of minimising average travel time (discussed in Section 2.5.3).
- Baaj and Mahmassani (1995) used heuristic method to design transit network with multiple objectives (discussed in Section 2.5.3).
- Zhao and Zeng (2006b) used hybrid SA and GA method to solve transit route design problem with the objective of minimising total travel time (discussed in Section 2.8).

SA-GA is performed on Mandl's network with the optimal parameter values obtained in sensitivity analysis (Section 7.5). The solution from SA-GA with minimisation of travel time as the objective has result closest to the theoretical minimum. It is just 1% more. This solution is also 37.8% better than Mandl's final solution, 22.6% better than Baaj and Mahmassani's best solution and 0.4% better than Zhao and Zeng's best solution in terms of direct trip. The result obtained with objective of minimisation of travel time is 17.7% better than Mandl's solution; 12.1% better than Baaj and Mahmassani's solution and 3.3% better than Zhao and Zeng's solution in terms of total travel time. The solution is shown in Figure 7-2 and summarised in Tables 7-3 and 7-4. The solution obtained from SA-GA with minimisation of total cost as the objective (Figure 7-3) is slightly worse from the user's point of view because a balance between operating cost and user cost is being achieved. However, it is still 35.2% better than Mandl's final solution and 20.3% better than Baaj and Manmassani's best solution in terms of percentage of direct trips and 0.8% better than Zhao and Zeng's best solution in terms of closeness to theoretical minimum in the total in-vehicle travel time. This solution is summarised in Tables 7-3 and 7-5.

Table 7-3 Summary of solutions

Objective	*Total cost*	*Traveller's cost*	*Operation cost*	*Average No. of transfer*	*Solution size*	*Solution (route ID)*
Min. travel time	26,765	21,621	5,144	1.04	10	21,37,92,126, 198,734,802, 837,1020,1037
Min. total cost	24,465	22,132	2,333	1.12	6	59,104,504, 553,601,1014

7.5 Sensitivity analysis and discussion

As mentioned before, the performance of SAGA might depend upon the selection of parameters like probability of crossover (*Pc*), probability of mutation (*Pm*), population size, number of iterations and scaling constant. The following sections present a sensitivity analysis of these parameters.

Table 7-4 Features of solution for minimisation of travel time

Route	Frequency	Length (km)	Max demand along the route	Demand
21	13	17.4	1,041	3,032
37	9.2	19.5	740	2,774
92	8.2	12.9	656	2,845
126	14.3	13.2	1,144	3,342
198	0.8	15.3	63	186
734	5.1	11.1	410	1,138
802	3.7	9.9	298	840
837	2.6	11.7	211	366
1020	8.6	16.8	688	1,562
1037	0.5	2.4	38	52
Total				16,137

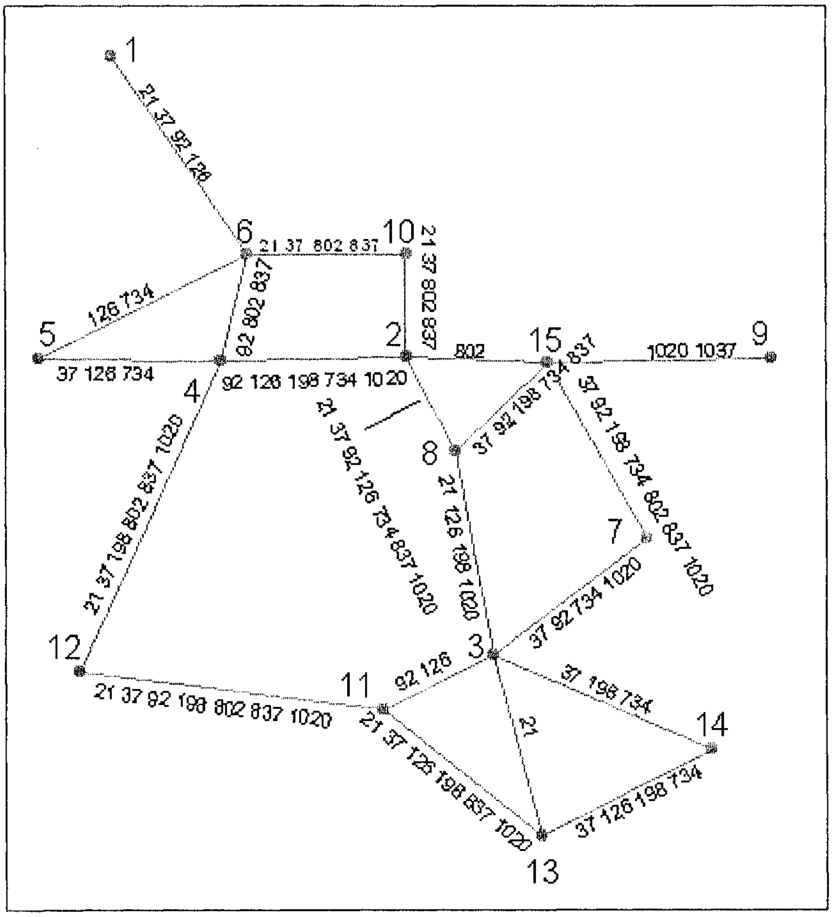


Figure 7-2 Mandl's network - solution for minimisation of travel time

Table 7-5 Features of solution for minimisation of total cost

Route	Frequency	Length(km)	Max demand along the route	Demand
59	5.6	7.5	447	1,816
104	14.3	9.9	1,142	6,049
504	8.8	10.5	703	3,959
553	15.3	7.5	1,227	3,827
601	1.4	3.0	115	332
1014	5.5	8.4	444	1,392
Total				17,375

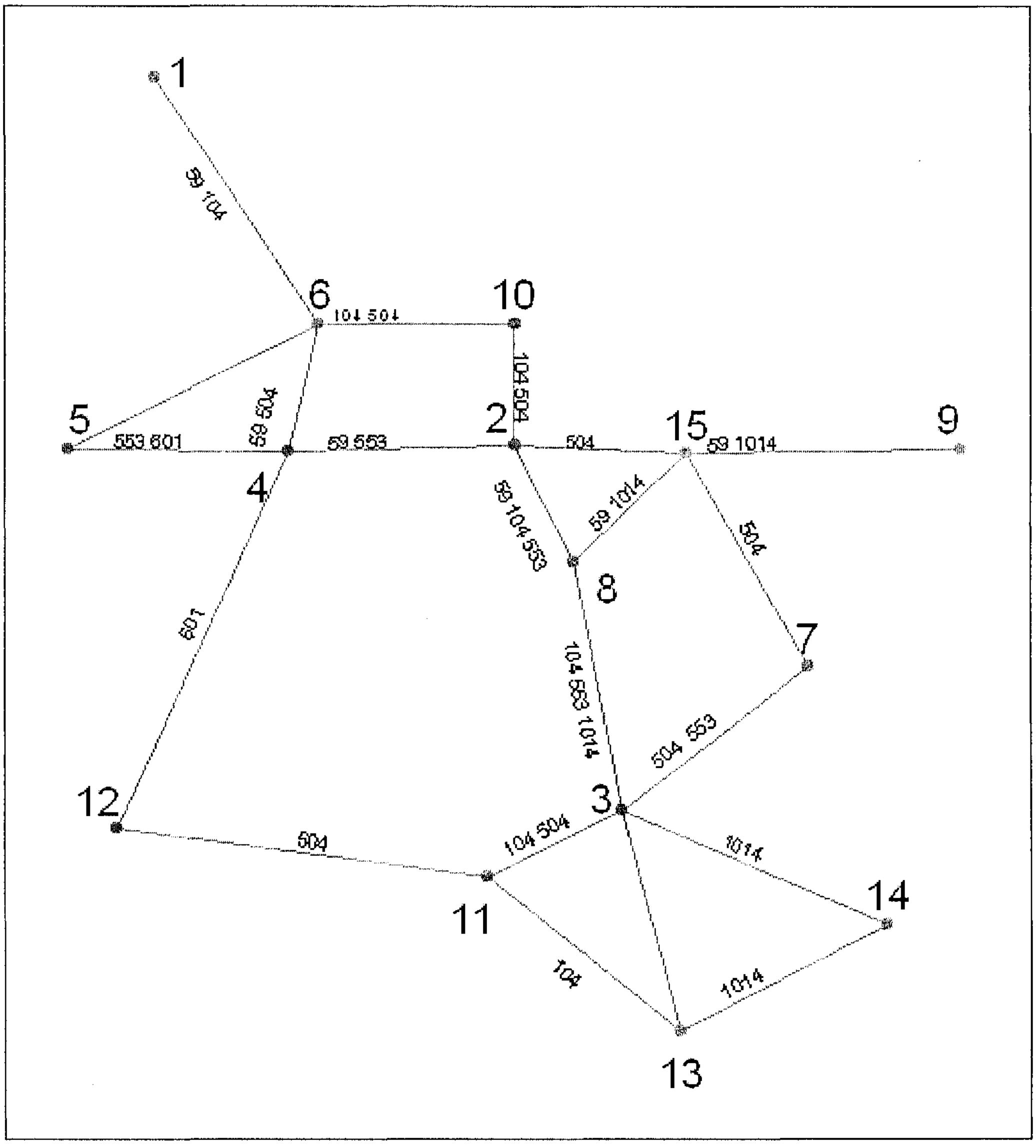


Figure 7-3 Mandl's network - Solution for minimisation of total cost

7.5.1 Effect of crossover probability

The effect of crossover probability is investigated by choosing different numbers ranging from 0.1 to 1 for probability of conducting crossover and the result is shown in Figure 7-4. It can be seen that the lowest objective value is achieved with 1 as the crossover probability. This is consistent with the result from the sensitivity analysis on network N1. Therefore, 1 is recommended as the crossover probability.

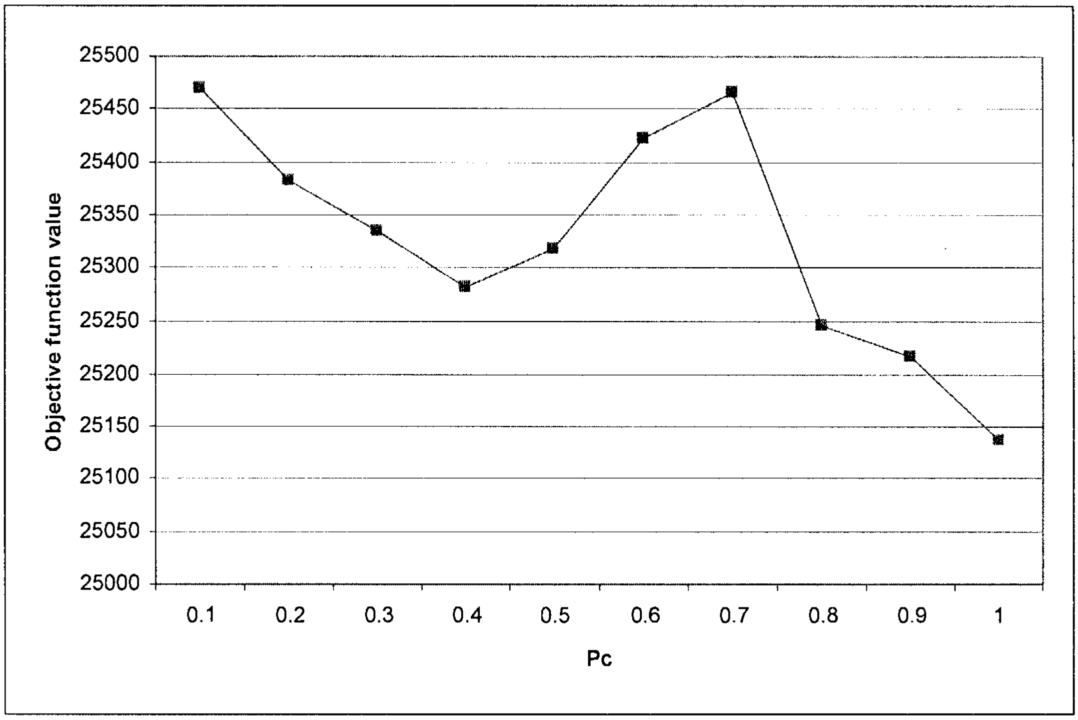


Figure 7-4 Effect of crossover probability – Mandl's network

7.5.2 Effect of mutation probability

Similarly the effect of mutation probability is examined by varying this value from 0.00001 to 1 and the result is presented in Figure 7-5. The lowest objective function value is achieved at 0.001 which is also consistent with analysis result on network N1. This suggests that the optimal mutation probability is 0.001.

7.5.3 Effect of population size

The effect of population size is investigated by varying this value from 50 to 300 and the result is given in Figure 7-6. It can be seen that the objective value tends to decrease with the increase of the population size. It is also observed that the larger the population size, the longer the computation time. When population size is 200, the optimal objective value is achieved. This suggests that 200 should be used as the optimal population size for this network and this result is again consistent with that obtained from analysis on network N1.

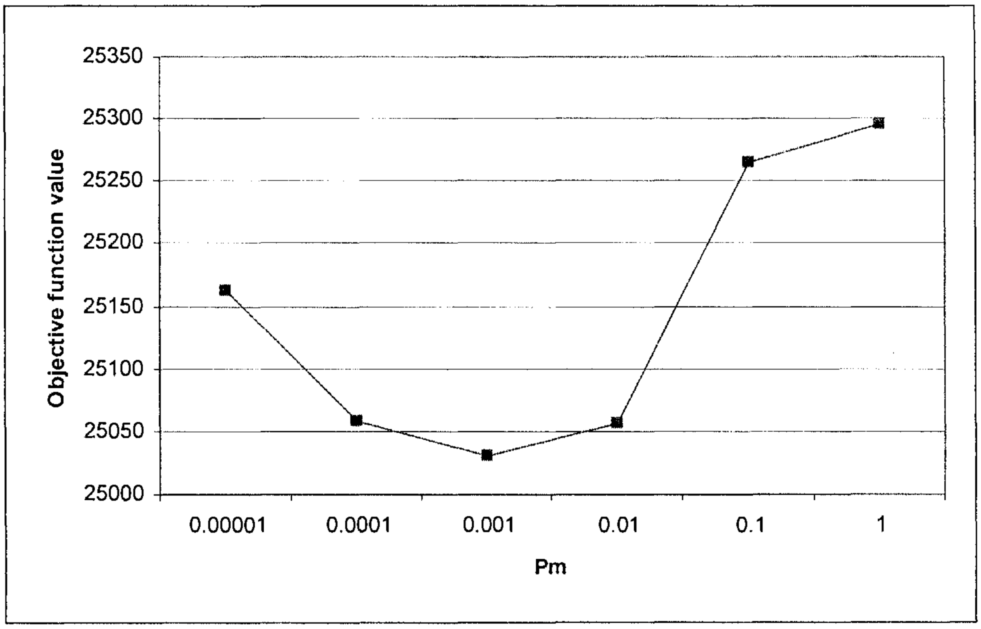


Figure 7-5 Effect of mutation probability – Mandl's network

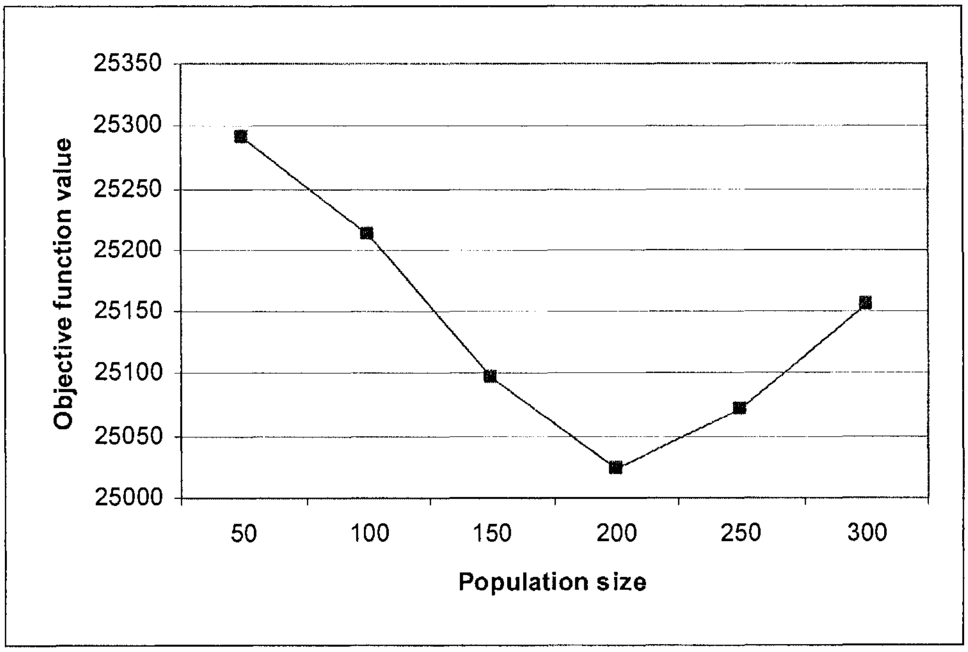


Figure 7-6 Effect of population size- Mandl's network

7.5.4 Effect of the number of iterations

The effect of number of iteration (*I*) is investigated by varying its value from 5 to 30 and the result is shown in Figure 7-7. It can be seen that the optimal objective value is achieved when *I* is 20 which is the same as the result obtained for network N1. Therefore, 20 is suggested to be used as parameter.

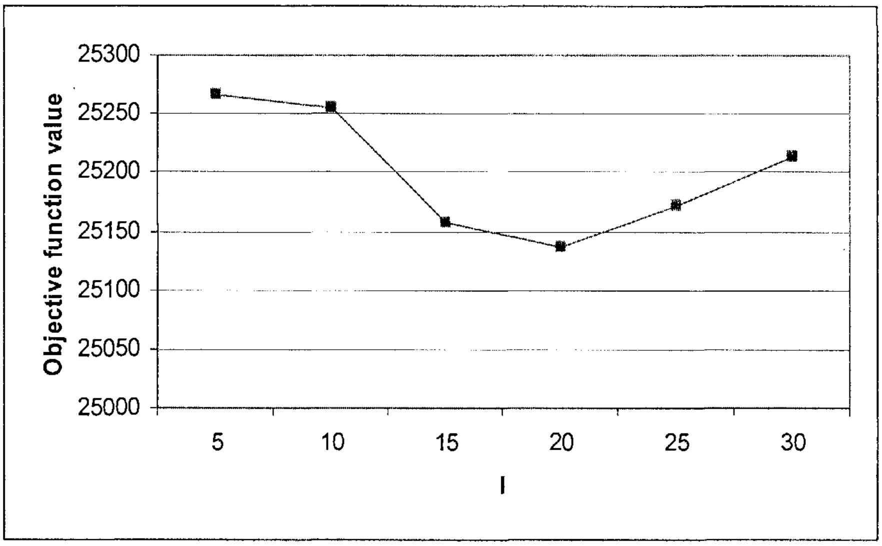


Figure 7-7 Effect of number of iterations – Mandl's network

7.5.5 Effect of the scaling constant

The usefulness of scaling constant is investigated by varying its value from 1,000 to 12,000 and the result is plotted in Figure 7-8. Unfortunately still no clear trend can be observed from the figure. However, it seems that the middle range of the scaling constant (7,000) is more suitable for Mandl's network.

7.6 Summary

The proposed SA-GA approach was performed on Mandl's network with two different objectives, minimising total cost (both operating cost and user cost) and minimising travel time. Results show that the proposed approach can produce better solution than previous studies of Mandl, Baaj and Mahmassani, and Zhao and Zeng. The in-vehicle travel time of SA-GA solutions from both objectives are

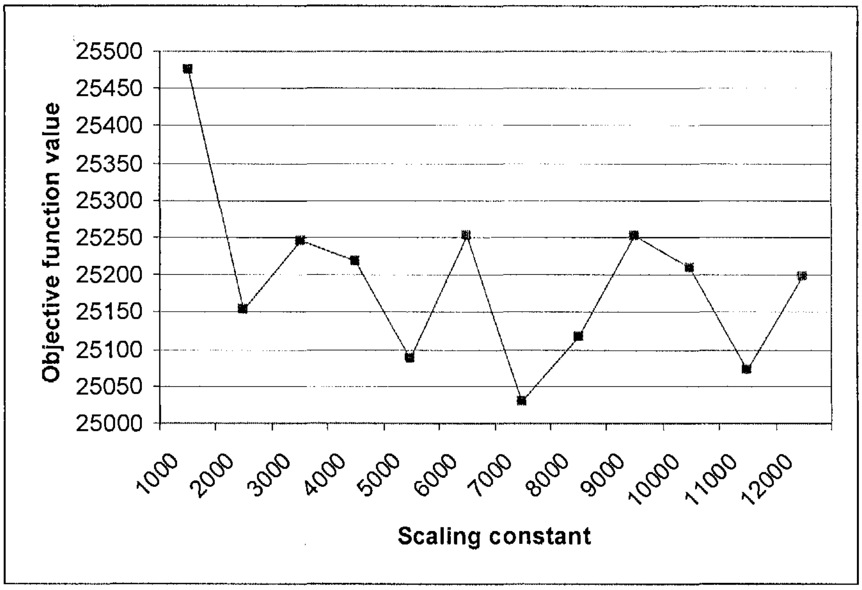


Figure 7-8 Effect of the scaling constant – Mandl's network

the closest (1% higher) to the theoretical minimum. The best solution from previous studies is 8% higher than the theoretical minimum. In terms of total travel time, the SA-GA solution with objective of minimising travel time is the best. It is 17.7% better than Mandl's solution; 12.1% better than Baaj and Mahmassani's solution and 3.3% better than Zhao and Zeng's solution. Although the solution obtained from SA-GA with minimisation of total cost as the objective is slightly worse from the user's point of view, it is still 35.2% better than Mandl's final solution and 20.3% better than Baaj and Manmassani's best solution in terms of percentage of direct trips.

Sensitivity analysis of probability of crossover, probability of mutation, population size, number of iterations and scaling constant has been performed on Mandl's network as well. Consistent results with the sensitivity analysis performed on Network 1 are obtained for probability of crossover, probability of mutation, population size, and the number of iterations.

CHAPTER 8 PREPARATION OF TAMPINES NETWORK

The experience obtained in the method implementation on theoretical networks shows that the computational time goes up quickly with the increase in the network size. It is not practical to implement the SA-GA approach on the entire network of Singapore which has 4,318 bus stops. Therefore, a residential area of Singapore called Tampines is selected as the testing network.

8.1 Characteristics of Tampines area

Tampines is a high-density residential area located in the eastern part of Singapore. Its total area is 10.37 km^2 (calculated from our data). Some 77% of all the buildings in this area are for residential purpose and 95% (calculated from our data) of the residential buildings are high-density housing (URA, 2006). The total population of Tampines is 212,913 in 2000 (SDOS, 2006). The Tampines area currently offers a wide range of job opportunities. There are two industrial parks. The Temasek Polytechnic and Eastern General Hospital are also located in the planning area. The total employment in this area is 68,430. The area is served by only one MRT station as shown in Figure 8-1. It has one bus interchange which is located beside the MRT station. Currently, 32 bus services are running through this area and 6 of them are loop services. The total number of bus stops is 71.

8.2 Preparation of road network

The main difference between theoretical network and real network is that the assumption of two-way road is not valid in the latter. One-way roads have to be considered in the real case application. To meet this requirement, the real road network is prepared in the following way: all bus stops, bus terminals and road intersections are called nodes and numbered sequentially. Links are the edges between two neighboring nodes. If a road or a part of a road is two-way which means that buses can run on it in two directions, two links are created for this part. Each link is one-directional and has its own unique ID. If a road or a part of a road is one-way (buses can run on it in one direction only), one link in that direction is created for that part of the road. Therefore, a directional network is constructed according to the real situation in Tampines as the base network for application of SA-GA. Its attributes are presented in Table 8-1.

There is only one bus terminal in Tampines which is the bus interchange. It is necessary to create some virtual bus terminals in order to generate bus routes. Seven virtual bus terminals are created on major roads which connect Tampines to other places of Singapore. They are assigned numbers 1 to 7 as the bus stop IDs.

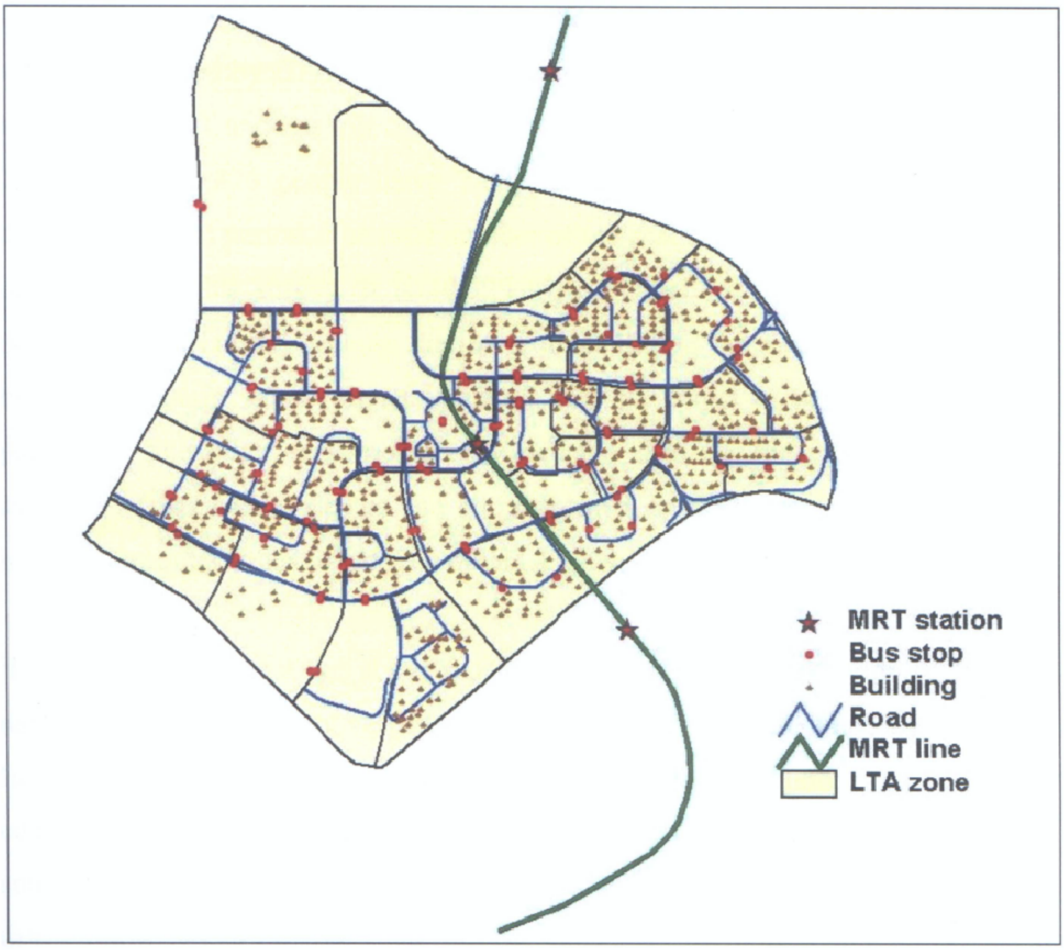


Figure 8-1 Map of Tampines

Table 8-1 Attributes of Tampines real network

No. of Bus Stops (including terminals)	No. of Terminals	No. of Nodes	No. of Links
79	8	159	383

8.3 Preparation of demand matrix

The demand matrix for Tampines network is also extracted and constructed from the real data. First of all, all the trips in Tampines area are divided into three types:

- Internal trip with both origin and destination in Tampines;
- Outbound trip the origin of which is in Tampines;
- Inbound trip the destination of which is in Tampines.

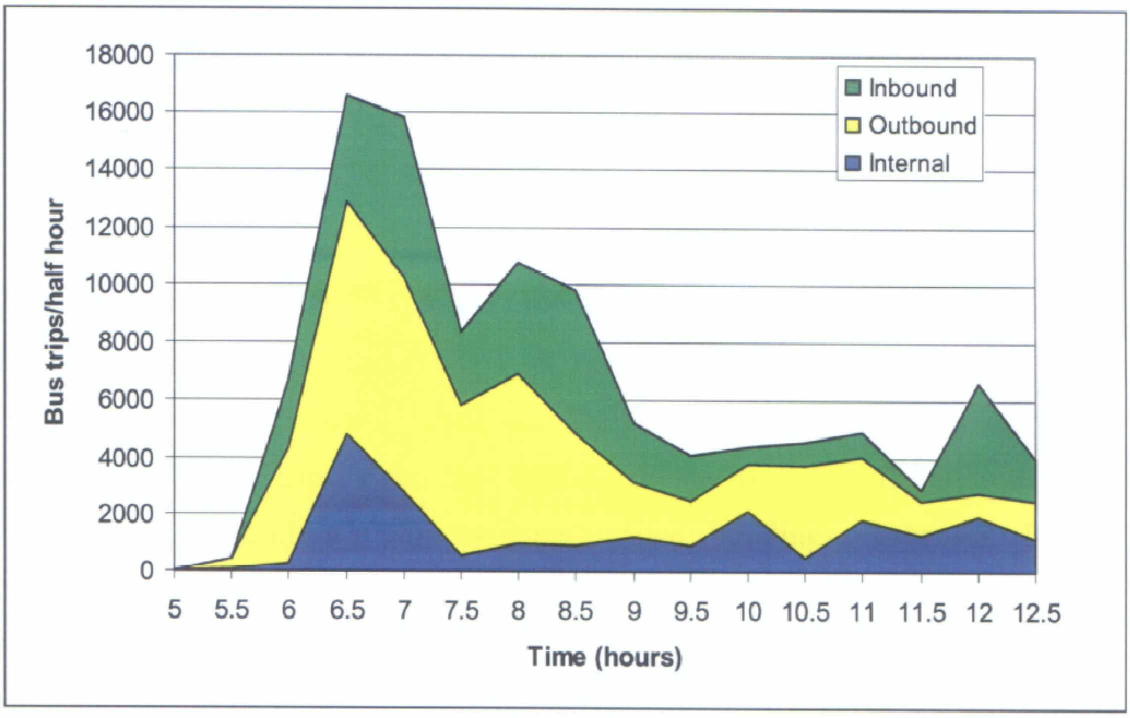
The whole Singapore is divided into 58 DGP zones. Zone to zone public transport travel demand matrix for the morning peak period (6-9am) is obtained from a survey conducted by LTA as shown in Table 8-2. Besides trips by bus only, there are trips by MRT and bus too. According to the survey, 766 people go to Tampines by MRT and 9,876 people leave Tampines for other areas by MRT everyday during the 6-9am period. The total number of internal trips within Tampines is also obtained from this survey. It is 10,329 trips in total for the 6-9am period. AM demand matrix is used because the travel times during the morning peak have the highest economic value (higher value of time for travel to work than for other purposes). The other reason was only demand data for the am peak period is available. Tests and evaluates can be performed on the final solution using OD matrices for other times of a day to obtain a network that is valid for the entire day.

It should be noted that the above data obtained from LTA survey are three-hour data. It is necessary to convert them into one hour data to calculate the objective function value. It was found that trips in the morning peak hour (6:30-7:30am) constitute approximately 47.6% of the trips in the three-hour period based on the summary of daily trip distribution in Tampines (Figure 8-2). Therefore, all the three-hour data can be converted into one-hour data by multiplying by 47.6%, as shown in Table 8-3. The peak hour numbers are:

- Inbound trips: 22,220 * 47.6% = 10,578
- Outbound trips: 34,172 * 47.6% = 16,268
- Internal trips: 10,329 * 47.6% = 4,917

Table 8-2 Zone demand (6-9am) extracted from LTA survey data

Macro zone No.	*Consolidated Zones*	*To Tampines*	*From Tampines*
1	ANG MO KIO	1,037	532
2	BEDOK	5,289	6,359
3	BISHAN+TOA PAYOH+NOVENA	1,750	1,195
4	CENTRAL	423	1,136
5	BT PANJANG+BT TIMAH+CHOA CHU KANG	157	162
6	CHANGI	311	3,325
7	GEYLANG+KALLANG+MARINE PAR	2,121	3,985
8	HOUGANG+SERANGOON	1,396	1,203
9	JURONG	445	126
10	NORTH+MALAYSIA	2,237	1,078
11	PASIR RIS+ISLANDS	2,315	3,180
12	PAYA LEBAR	0	249
13	PUNGGOL+SENGKANG	1,866	344
14	TANAH MERAH	2,107	1,422
15	ALL OTHER ZONES (BY MRT)	766	9,876
	TOTAL	22,220	34,172



The next step is to distribute the trips to all the bus stops in the Tampines area. There are 79 bus stops including 8 bus terminals in this area as shown in Figure 8-3. Bus terminals are indicated by squares. Bus stops are represented by dots. Only

Terminal 8 is the actual bus terminal existing in the real life which is located next to the Tampines MRT station. The other seven are all artificial terminals representing the "boundary nodes" and created for the purpose of generating bus routes for this area.

Table 8-3 One hour inter-zonal demand (6:30 ~7:30 am)

Macro zone No.	*CONSOLIDATED ZONES*	*To Tampines*	*From Tampines*
1	ANG MO KIO	494	253
2	BEDOK	2,518	3,027
3	BISHAN+TOA PAYOH+NOVENA	833	569
4	CENTRAL	201	541
5	BT PANJANG+BT TIMAH+CHOA CHU KANG	75	77
6	CHANGI	148	1,583
7	GEYLANG+KALLANG+MARINE PAR	1,010	1,897
8	HOUGANG+SERANGOON	664	573
9	JURONG	212	60
10	NORTH+MALAYSIA	1,065	513
11	PASIR RIS+ISLANDS	1,102	1,514
12	PAYA LEBAR	0	119
13	PUNGGOL+SENGKANG	888	164
14	TANAH MERAH	1,003	677
15	ALL OTHER ZONES (BY MRT)	365	4,701
Total		10,578	16,268

Trips are distributed according to the production and attraction factors around each bus stop. Production factors include all residential buildings like HDB blocks, condominiums and private housing etc. They are assumed to be the origins of morning peak hour trips. Attraction factors include basically all buildings other than residential buildings like schools, office buildings, industrial blocks, commercial buildings etc. They are assumed to be the destinations of morning peak hour trips. All the buildings need to be assigned to bus stops as production or attraction factors. This is done with the data described in Section 3.1 in ArcView GIS 3.2a environment. First, all the buildings are categorised into two groups: production buildings and attraction buildings. Then the following steps for production buildings and attraction building are done separately:

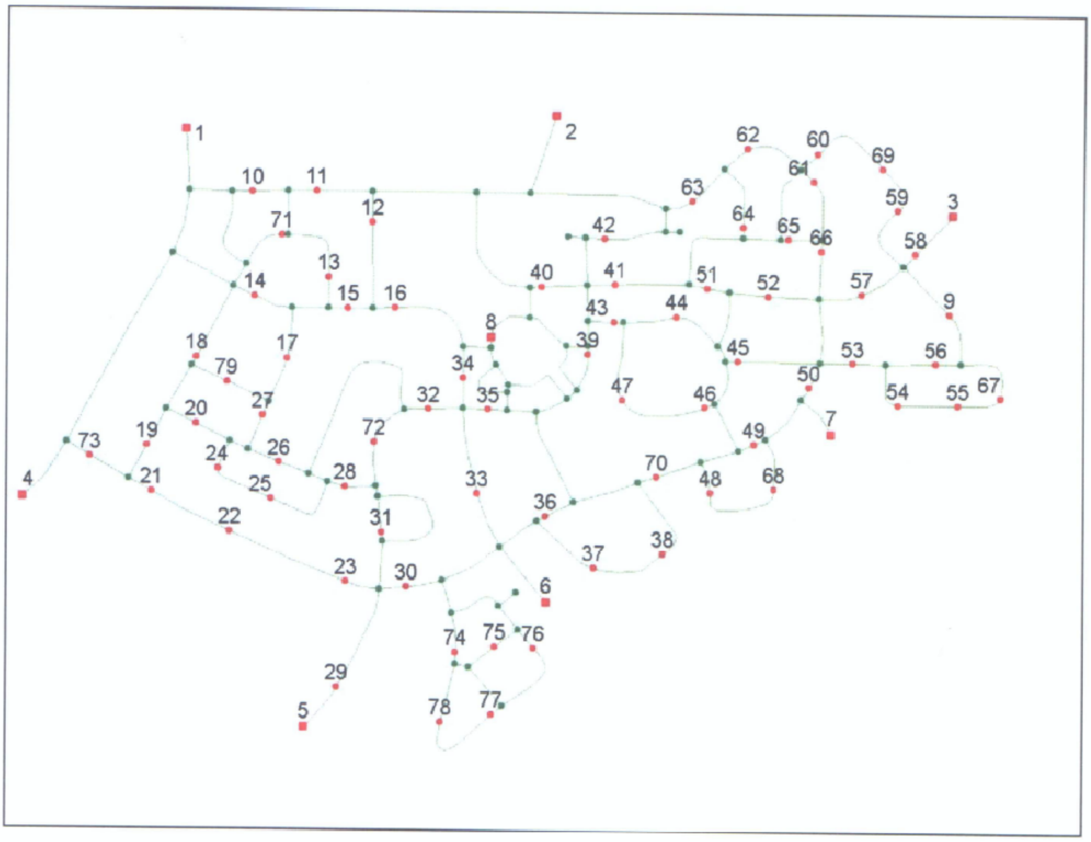


Figure 8-3 Map of Tampines with bus stops

- For each building, all the nearby bus stops are found (excluding artificial terminals) within 231m (see Section 3.2) in airline distance from the building;
- The airline distances from the building to all nearby bus stops are measured and added.
- Each building is assigned as a production or attraction factor to each nearby bus stop according to the distance. For example, suppose there are three stops (Stops 1, 2 and 3) within 231m from Building A. The airline distance between the building and the three bus stops are: s_1, s_2 and s_3 respectively. The percentage of the building assigned to Stop1 is $\frac{1/s_1}{1/s_1 + 1/s_2 + 1/s_3}$, thus assuming that passengers prefer to take a bus at the nearer bus stop.

Similarly, the percentages for Stop2 and Stop3 are: $\dfrac{1/s_2}{1/s_1+1/s_2+1/s_3}$ and $\dfrac{1/s_3}{1/s_1+1/s_2+1/s_3}$.

Finally, the total number of production buildings and attraction buildings assigned to each bus stop is calculated. The result is summarised in Table 8-4.

In distributing the internal trips, the following two steps are performed:

- Assign 4,917 internal trips to each bus stop in proportion to the number of production factors assigned to it. The production factors are taken as the origins of these internal trips. For example, 4,917*(1.71/707.73) = 11.88 origination trips are assigned to Bus Stop 8 which is also Terminal 8.

- Distribute the trips originating from each bus stop (assigned in the previous step) to all bus stops other than itself in proportion to the number of attraction factors attached to those bus stops so that an internal trip demand matrix can be obtained. The attraction factors are taken as the destinations of these internal trips. For example, 11.88 trips were assigned to Bus Stop 8 in last step. Since no trip among these trips should be assigned to Bus Stop 8, the total attraction factor is: 153.06 - 4.65 = 148.38. Therefore, 11.88*(6.27/148.38) = 0.50 trip should be assigned to Bus Stop 74. The 0.5 trip originating form Bus Stop 8 ends at Bus Stop 74.

Two decimal places are kept in calculating trips in the above two steps. A sum of total assigned internal trips is calculated for each bus stop after completion of the two steps. The total number of trips for each bus stop is rounded to a whole number. This method is also used for external trips.

Table 8-4 Production and attraction factors attached to each bus stop

ID	*PRODUCTION*	*ATTRACTION*	*ID*	*PRODUCTION*	*ATTRACTION*
1-7	0.00	0.00	44	10.79	3.31
8	1.71	4.65	45	13.60	0.62
9	13.12	1.53	46	9.77	1.56
10	7.30	0.08	47	11.28	1.46
11	7.77	0.11	48	7.87	1.47
12	5.62	0.10	49	10.42	0.66
13	12.92	0.26	50	16.42	0.67
14	10.23	0.20	51	12.28	1.06
15	13.84	0.44	52	12.12	0.19
16	6.95	1.03	53	15.41	0.98
17	15.37	0.30	54	14.66	0.56
18	4.71	0.11	55	12.06	0.64
19	2.87	1.52	56	14.58	1.86
20	7.65	0.51	57	9.79	0.92
21	3.83	3.32	58	12.94	1.00
22	4.42	6.05	59	13.87	0.37
23	7.81	1.95	60	15.10	0.68
24	7.77	1.14	61	13.82	1.26
25	9.52	3.15	62	24.02	1.12
26	11.33	2.65	63	18.25	2.18
27	12.65	1.37	64	14.68	0.84
28	11.23	5.25	65	11.93	0.57
29	0.00	0.00	66	9.27	0.46
30	7.69	1.05	67	10.31	0.73
31	12.07	2.46	68	8.74	0.26
32	8.69	2.17	69	12.88	0.18
33	12.81	2.12	70	9.04	2.08
34	3.12	5.01	71	12.40	0.21
35	5.26	6.68	72	13.00	2.75
36	13.82	2.39	73	0.41	1.13
37	16.18	1.09	74	0.29	6.27
38	12.26	1.38	75	0.76	11.64
39	9.51	2.47	76	2.84	11.34
40	9.09	1.40	77	0.00	12.70
41	11.78	1.60	78	0.00	9.94
42	12.98	1.87	79	8.25	0.32
43	10.00	1.66	**Total**	**707.73**	**153.06**

For external trips (inbound and outbound trips), it is necessary to assign the zone demand in Table 8-3 to the terminals before distributing them to bus stops. Obviously, the demand by MRT (No.15) should be assigned to Terminal 8 since that is where the MRT station is located (see Table 8-5 and 8-6). Other zone demand (No.1-14) is assigned to other seven boundary terminals according to the routing of current bus services. Take zone No. 2 (Bedok) as an example. Currently there are 13 bus services going between Tampines and Bedok. One bus service passes Terminal 4, six services pass Terminal 5, three services pass Termial 6 and three services pass Terminal 7. Therefore, one thirteenth of the inbound demand (from Bedok to Tampines) and outbound demand (from Tampines to Bedok) is assigned to Terminal 4; six thirteenths are assigned to Terminal 5; three thirteenths are assigned to Terminals 6 and 7. In this way, the inter-zonal demand in Table 8-3 is assigned to all terminals as shown in Table 8-5 and 8-6.

It is assumed that all inbound trips originate from the boundary terminals and terminate at all the other bus stops which have attraction factors around them. Based on this assumption, inbound trips are distributed to bus stops in proportion to the number of attraction factors attached to each bus stop. For example, 3,325*(0.08/153.06) = 1.74 inbound trips from Terminal 1 will be assigned to Bus Stop 10. The 1.74 trips originate from Terminal 1 and terminate at Bus Stop 10. In this way, the full demand matrix for inbound trips can be obtained.

Table 8-5 Inbound trips (6:30~7:30 am) assigned to each boundary terminal

Macro	***Terminal***							
Zone	***1***	***2***	***3***	***4***	***5***	***6***	***7***	***8***
1	247				247			
2				193	1,162	581	582	
3				333	333	167		
4				50	50	50	51	
5					75			
6						21	127	
7					252	505	253	
8	221		111	221		111		
9				106	106			
10	532		533					
11	138	138	826					
12								
13	444		444					
14						669	334	
15								365
Total	1,582	138	1914	903	2,225	2,104	1,347	365

Similarly, all outbound trips are assumed to originate from bus stops which have production factors around them and terminate at one of the eight terminals. Outbound trips are distributed to bus stops in proportion to the number of production factors attached to each bus stop. For example, when distributing the trips ending at Terminal 8, no trip should be assigned to Terminal 8 itself. So the total production factor is 707.73-1.71 = 706.02. Then 4,701*(13.12/706.02) = 87.36 trips should be assigned to Bus Stop 9 as originating trips. These trips start from Bus Stop 9 and end at Terminal 8. In this way, a demand matrix for outbound trips can be constructed.

Table 8-6 Outbound trips (6:30~7:30 am) assigned to each boundary terminal

Macro Zone	***Terminal***							
	1	***2***	***3***	***4***	***5***	***6***	***7***	***8***
1	127				126			
2				232	1,397	699	699	
3				228	228	113		
4				135	135	135	136	
5					77			
6			1,357			226		
7					474	949	474	
8	191		96	191		95		
9				30	30			
10	256		257					
11	189	189	1,136					
12				30	30	30	29	
13	82		82					
14						451	226	
15								4,701
Total	845	189	2,928	846	2,497	2,698	1,564	4,701

A complete demand matrix can be obtained by combining internal trip demand matrix, inbound trip demand matrix and outbound trip demand matrix. The demand matrix is shown in Appendix K.

8.3 Current real bus routes serving Tampines

The advantage of application of the proposed SA-GA approach on real network is that the planning result can be compared with the current real bus route set serving the same area. Thus, the real bus routes in Tampines are extracted from the 'Singapore Public Transport Guide 2006' (SCIP, 2006). A description of the routing of bus service No. 3 is shown in Table 8-7 as an example. For more information on all the 32 bus routes currently serving Tampines, see Appendix L. The numbers labelled besides each bus stop and terminals in Figure L1 are bus service numbers.

In order to make the real bus routes available for evaluation, it is necessary to represent them in the following way:

- Loop service: preparing the list of bus stops as the real sequence and preparing the list of links also according to the real sequence;
- Non-loop service: splitting into two parts for two directions first and then preparing the list of bus stops and links in the same way as for the loop services.

Table 8-7 Description of routing of bus No. 3 in Tampines

Direction	***Bus Stop Sequence***	***Link Sequence***
1	8 34 35 39 43 47 46 49 50 57 58 3	176 198 289 102 292 107 228 46 47 302 117 39 312 125 306 308 121 13 300 115 56 321 136 224 339 154 343 366
2	3 58 57 45 46 47 43 39 35 34 8	180 157 340 153 38 320 133 212 307 122 120 311 126 225 303 116 233 232 42 293 106 288 103 12 362

8.4 Optimisation of bus stop location for Tampines area

The two proposed optimisation algorithms in Chapter 4 are applied in Tampines area. Randomised algorithm is performed first. No new location can be found for any of the current bus stops to improve the accessibility because currently 94.9% of buildings are already within good accessibility range.

Set-cover algorithm is applied as the second step. It is first tried in ArcView GIS 3.2a environment. However, after cutting roads into segments, the segments turned out to be too small to be accepted by the software. Therefore, it has to be implemented in C environment. The result is that 54 new bus stops (Figure 8-4) are found and they can cover 99% of buildings within good accessibility. In total the new solution has 62 stops when combined with the 7 virtual terminals and 1 MRT station. Currently there are 79 stops located in Tampines (Figure 8-3).

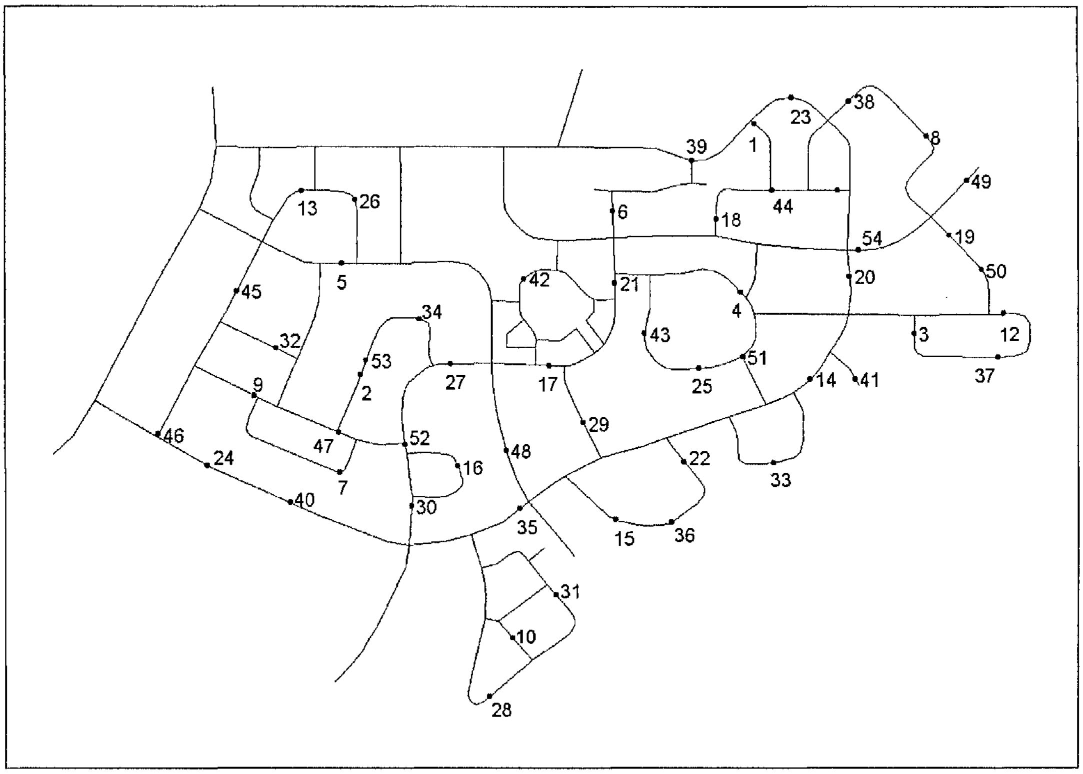


Figure 8-4 New bus stops designed by Set-cover algorithm

There is a big reduction in terms of the number of bus stops. However, these new bus stops will not be used in the implementation of SA-GA approach for new bus routes planning for the following reasons:

- To validate the effectiveness of the proposed SA-GA approach on network design problem, it is more reasonable and meaningful to compare the SA-GA solution with the current bus routes while the two are based on the same set of bus stops.
- In real life, minimum number of bus stops does not mean good design. Fewer bus stops may result in too many people going to the same bus stop to take a bus. This makes the bus stop too crowded and increases the

boarding time. Fewer bus stops may also cause too many buses stopping at the same stop. This would cause traffic congestion. The two situations definitely imply worse service quality.

- With the set-cover algorithm, there is a tendency to locate the bus stop at an intersection or very close to crossing to cover more buildings. This situation happens to new bus stops 1, 9, 39, 44, 46, 47, 51 and 52.
- It is not practical and economic to remove all the current bus stops and build new ones if no strong evidence is available to show that a dramatic improvement of public bus service quality will result after re-location of all the current bus stops. A similar situation called "NEL issue" happened in Singapore (Goh, Sep. 15, 2003) after opening a new NEL MRT line. The public made a lot of complaints after bus services duplicating with NEL were changed.

However, the planning result from set-cover algorithm provides planners a new angle and starting point for designing bus stops. Planners may use the result as a base plan and make some adjustments by moving or adding bus stops according to the real situation in the area to reach a more realistic and convenient design.

8.5 Summary

In this chapter, background information of Tampines area is presented. To make the selected real bus network suitable for testing, the following preparations are made:

- Creating virtual bus terminals to generate bus routes;
- Sequential numbering of nodes including bus stops, bus terminals and road intersections;
- Creating a directional network which recognise both two-way road and one-way roads;
- Creating bus stop to bus stop demand matrix;
- Gathering information about the current real bus routes serving Tampines area.

Bus stop location optimisation algorithms were tested on Tampines area. The randomised algorithm can not produce improved locations for current bus stops because the accessibility of the current bus stops is quite good (94.9% of buildings within good accessibility). Set-cover algorithm can produce a result which has fewer bus stops to cover more buildings within good accessibility (99%). However, the new planned bus stops will not be used for bus routes planning to make the validation of the effectiveness of the proposed SA-GA approach more convincing. Still, the set-cover result is a good indication for a more practical and convenient planning of bus stops.

CHAPTER 9 IMPLEMENTATION OF SA-GA ALGORITHM ON THE TAMPINES NETWORK

In this chapter, the framework described in Chapter 5 and tested on theoretical networks (Chapter 6) as well as on Mandl's network (Chapter 7) is implemented on the real network of Tampines. All the necessary data are prepared as described in Chapter 8. The planning results obtained with different parameter values are presented and compared in this chapter.

9.1 Generation of candidate routes

The method described in Section 5.2.1 is also applied here to generate the candidate routes. These candidate routes start from a terminal and end at a bus stop instead of having both ends in terminals. In real life, buses originate and terminate at terminals so that it is easy to manage and schedule the buses. However, this would limit the performance of the designed bus route network. Moreover, seven out of the eight terminals are artificial terminals. In order to make the application on real network comparable with previous studies, this compromise method is used. In addition, there are two other differences from the candidate route generation for theoretical networks:

- The first difference is due to the one-way roads existing in the real network. In this situation, bus routes in different directions between a pair of stops might not be the same. Thus, it is necessary to generate bus routes with different directions separately for each pair of bus terminals.

- The second difference is that loop services are considered for certain bus terminals according to the real life situation. In reality, Terminal 8 is located next to the MRT station. A number of loop services are provided from Terminal 8 in order to provide convenient feeder services for MRT passengers. Therefore, a number of loop routes are generated for Terminal 8 only in the candidate route generation module. Attributes of candidate routes are listed in Appendix M. In total, 4,139 candidate routes are generated for Tampines.

9.2 Coverage enforcement module for the real case study

In preliminary running of SA-GA, it was noticed that it was very hard for the program to find a solution which could cover all the bus stops. This was because of the large size of the real network. To fix this problem, a Coverage Enforcement Module was inserted into Evaluation Module as the first step in evaluation for the real case application. Thus, the structure of the evaluation algorithm for real case application (Figure 9-1) would be a little different from the theoretical application.

The Coverage Enforcement Module works in the following way:

1. Find what bus stops are not covered by the current solution (a route network);
2. Select one candidate route meeting the selection criterion (defined below) from the candidate routes which are not in the current solution and include this route in the current solution.
3. Repeat step 1 until all the bus stops are covered.

Two selection criteria were tested. The first criterion was to select the candidate route which can cover most not-covered bus stops. For example, there are 10 bus stops (1 to 10) in total and not-covered bus stops are: 1, 2, 3, 4 and 5. There are two candidate routes available. They are: Route 1 which passes bus stops 1, 2, 3, 6, 7, 8, 9 and 10; Route 2 which passes bus stops 4, 5, 7 and 9. Since Route 1 covers 3 not-covered bus stops and Route 2 covers only 2 not-covered bus stops, Route 1 is selected. The second criterion was to select the candidate route with the highest "not-covered to covered ratio". Using the above example for illustration, the "not-covered to covered ratio" for Route 1 is 3:5 because it has three not-covered bus stops (1, 2 and 3) and 5 covered bus stops (6 to 10). Route 2 has 2 not covered and 2 covered bus stops. Thus, the "not-covered to covered ratio" for Route 2 is 2:2. Route 2 has the higher ratio and therefore it will be selected into the current solution.

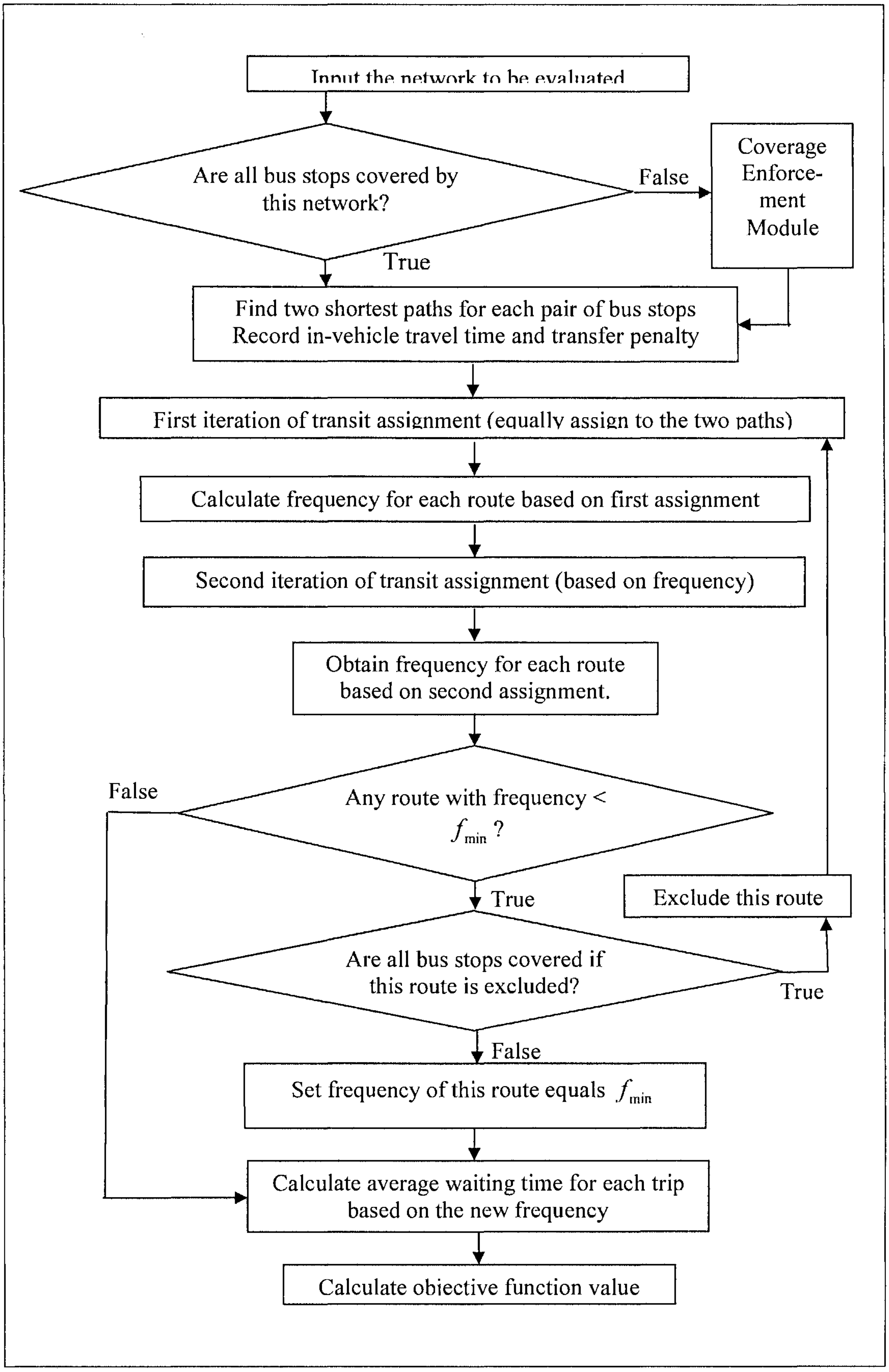


Figure 9-1 Structure of evaluation module for the real case application

After running preliminary tests, it turned out that criterion two gave better solution than criterion one in terms of total cost probably because criterion two can reduce the probability of having redundant routes. Therefore, criterion two is used in the following experiments.

9.3 Results of computation and sensitivity studies

Since a series of experiments have already been performed on theoretical networks and Mandl's network and consistent results were obtained, the same experiment would not be repeated on Tampines network. The values of other parameters such as crossover type, crossover probability, population size etc. are based on recommended values obtained from experiments on theoretical networks and Mandl's network (Table 9-1). The best result for Tampines is presented in Table 9-2. Its features are summarised in Table 9-3.

Table 9-1 Values of parameters used in application on Tampines network

Parameter	*Value*
Crossover	Two-point crossover
Crossover probability	1.0
Mutation	Dynamic mutation
Mutation probability	0.001
Maximum Temperature	50
No. of iterations	20
Population size	200

For the purpose of comparison, the existing real bus routes are evaluated using the current frequency (obtained from www.sbstransit.com.sg) and the same evaluation approach as used in SA-GA. The results are summarised in Table 9-4. The theoretical minimum average travel time (theoretical min) calculated for Tampines is 7.22 minutes per trip. The average in-vehicle travel time with the current bus routes is 9.7 minutes which is 34% more than the theoretical minimum. The average in-vehicle travel time for SA-GA solution is only 16% more than the theoretical minimum. In terms of directness, 19% more trips can be accomplished without transfer with the new bus routes planned by the SA-GA approach.

Table 9-2 *Bus routes in the best solution for Tampines*

Route	***Bus Stop***
28	1 14 15 16 34 33 6
284	1 10 13 15 16 8 42
399	1 10 11 63 62 60 69 59
479	1 10 13 15 16 34 35 70
523	1 14 17 27 26 28 31 30 74 75 76
810	2 42 39 40
833	2 40 43 44
835	2 42 43 44
1036	2 11 71 18 19 73
4000	3 58 59 69 60
4033	3 58 9 67 55 54 53 66 65 64
4043	3 58 9 56 53 66
4110	3 58 57 52 44 43 39 36 74 75
1761	4 18 20 24 25
1870	4 73 19 20 26 28 72 32 35 39 41
2034	4 73 19 20 27 17 15 16 8 41 64
2102	4 73 21 22 23 30 76 77 78 74
2243	5 29 23 22 21
2442	5 29 30 36 39 43 44 45 50
2475	5 29 30 36 70 49 50 53 56 67 55
2520	5 29 31 72 32 34 8 40 42 63 62 61
2679	6 36 16 12 11
2750	6 33 32 72 28 26 27 79 19 21
2772	6 36 34 32 72 28 26 24
2862	6 37 38
2949	6 36 39 43 44 45 50
3078	6 36 39 43 44 49 68
3144	6 76 77 78
3145	6 75 77 78
3169	7 50 66 65 64 62 60 69 59 9
3296	7 49 70 36 30 31 28
3325	7 50 45 44 43 39 35 32
3341	7 50 45 44 43 40 8 34
3429	7 49 46 47
3482	7 50 53 54 55
3566	7 50 53 54 55 67
3573	7 49 48 68
3577	7 50 52 48 68
3642	7 49 70 36 74 78
1105	8 35 32 72 31 30 6
1319	8 38 37
1328	8 36 37 38
1397	8 43 47 46 48
1419	8 41 51
1604	8 35 33 74 78

Table 9-3 Features of the best solution for Tampines

Route	*Frequency*	*Length(km)*	*Max demand*	*Demand*	*Fleet size*
28	10.3	3.17	828	2,006	6
284	5.5	3.25	442	739	3
399	3	3.75	201	508	2
479	11	3.41	880	1,175	7
523	15.8	3.88	1,265	2,422	8
810	3	2.68	61	133	2
833	3	1.93	95	170	2
835	3	2.14	23	43	2
1036	3	1.87	51	54	2
1105	3.5	2.51	277	717	3
1319	3	1.90	162	240	2
1328	5.5	1.96	438	682	3
1397	4.5	2.10	361	676	2
1419	3	1.73	34	71	2
1604	13.1	2.08	1,045	1,339	6
1761	3	2.87	44	64	2
1870	6.0	3.70	477	1,129	4
2034	9.6	4.70	766	2,102	7
2102	12.2	3.83	977	2,505	8
2243	5.1	1.76	412	641	2
2442	7.7	3.79	612	1,191	5
2475	14.2	4.04	1,137	2,399	9
2520	25.2	4.28	2,016	4,756	17
2679	5.2	2.89	418	562	3
2750	6.3	3.44	504	1,329	4
2772	6.6	3.39	531	1,205	4
2862	3	1.15	57	64	1
2949	7.5	2.84	597	919	4
3078	4.5	3.10	358	639	3
3144	3	2.04	111	124	2
3145	3	2.07	84	106	2
3169	6.4	3.30	512	809	4
3296	5.6	2.84	446	951	3
3325	3.6	2.46	285	624	2
3341	7.9	2.53	630	1,231	4
3429	3	1.23	75	145	1
3482	3	1.13	136	158	1
3566	3	1.33	147	177	1
3573	3	1.25	20	40	1
3577	3	2.49	46	78	2
3642	9.3	2.61	742	947	5
4000	8.4	1.22	670	775	3
4033	10.6	3.00	845	1,206	6
4043	11.4	1.92	915	1,331	5
4110	13.7	4.13	1,100	1,893	9
Total				41,075	176

According to the 'Bus Passenger Satisfaction Survey' conducted by Public Transport Council of Singapore, the most dissatisfying attributes of bus service were:

- Waiting time, walking time and travel time in survey of 2003-2004 (PTC, 2004);
- Waiting time, travel time, need for transfer and value-for-money in survey of 2004-2005 (PTC, 2005).

Among these five factors, three of them (waiting time, travel time and need-for-transfer) are improved in the SA-GA solution as compared with the current situation (Table 9-4). The improved quality of bus service has not resulted in increased operation cost. On the contrary, the operating cost is reduced. For example, the average bus route length is 19% shorter than current bus routes; the average frequency is 29% lower.

Table 9-4 Comparison between the current route set and SA-GA solution

	Current	*SA-GA result*	*Change*
Objective function value	96,159	86,593	-10%
Travellers' cost	85,839	76,776	-11%
Operation cost	10,320	9,817	-5%
Average no. of transfers	1.43	1.29	-10%
Average round route length (km)	6.58	5.32	-19%
Average frequency (buses/hour)	9.47	6.76	-29%
No. of routes	32	45	+41%
Direct trips	55.23%	65.46%	+19%
One- transfer trips	41.64%	31.73%	-24%
Two- transfer trips	0.81%	0.49%	-40%
Average total travel time (min)	16.3	14.2	-13%
Average in-vehicle time (min)	9.7	8.4	-13%
Average transfer time (min)	4	3.3	-18%
Average waiting time (min)	2.6	2.5	-4%
Average load factor	0.78	0.77	-1%
Fleet size	240	176	-27%

9.4 Sensitivity analysis and discussion

As mentioned before, the performance of SA-GA might depend upon the selection of parameters like probability of crossover (*Pc*), probability of mutation (*Pm*), population size, parameter (*I*) and scaling constant. Since consistent results were obtained from sensitivity studies of probability of crossover (*Pc*), probability of mutation (*Pm*), population size, and parameter (*I*) on theoretical network N1 and Mandl's network, experiments of these parameters were not done on Tampines network. However, the following section presents a sensitivity analysis of the scaling constant.

The usefulness of scaling constant is investigated by varying its value from 1,000 to 12,000 and the result is plotted in Figure 9-2. It seems that a small scaling constant (3000) is more suitable for the Tampines network. Comparison of the analysis results obtained for network N1, Mandl's network and Tampines network might suggest that a bigger scaling constant works well for smaller networks and a smaller scaling constant works well for bigger networks.

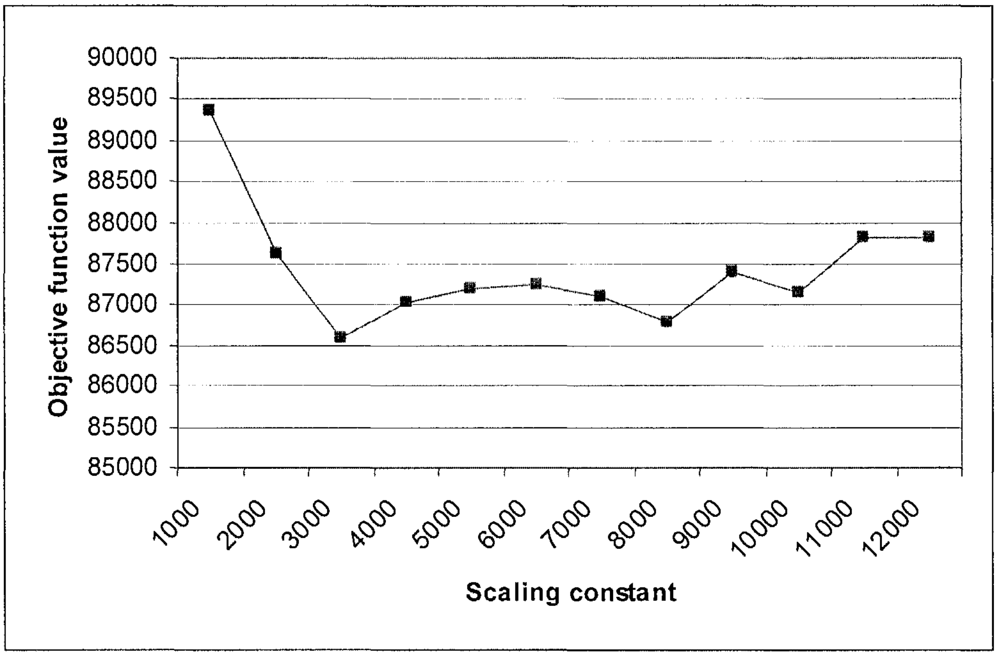


Figure 9-2 Effect of the scaling constant for Tampines network

CHAPTER 10 EVALUATION AND DISCUSSION

In this chapter, the proposed SA-GA solution is evaluated with the methods presented in Chapter 3 and in other ways. The evaluation results show that SA-GA is an effective approach to design bus routes because it can produce better quality bus service than the current service and achieve result very close to the theoretically minimum travel time. After evaluation, a discussion is presented on the proposed SA-GA approach.

10.1 Evaluation of connectivity

The connectivity between two bus stops is measured by the minimum number of transfers that one has to take to travel from one point to another. The whole network of Singapore is evaluated in Chapter 3 and the result is summarised in Table 3-6. The depth of Singapore network is 3 because the largest number of transfers needed for some trips between bus stops is 3. Table 9-4 shows that Tampines network also needs at most 3 transfers with both the current bus services and the planned new bus routes. However, the connectivity is improved greatly. Currently, 13.56% (Table 3-6) of trips for the whole Singapore and 55.23% (Table 9-4) of trips related with Tampines can be accomplished without any transfer. With SA-GA solution, 19% more trips within Tampines can be accomplished without transfer.

10.2 Evaluation of average travel time to employment

Average travel time to employment (ATE) is defined by Equation 3-2 in Chapter 3. Travel time between a pair of zones A and B, t_{AB}, is calculated as:

$$t_{AB} = \frac{\sum_{i \in A} \sum_{j \in B} t_{ij}}{N_A N_B}$$

where: N_A : number of bus stops in zone A

N_B : number of bus stops in zone B

ATE is calculated for each zone in Tampines area under both the current bus services and the planned bus routes. The travel time is the average travel time to other zones in Tampines calculated based on current bus services and the planned bus routes. The employment data are the same as used for calculation of ATE for the whole Singapore in Chapter 3.

Total travel time is the sum of in-vehicle travel time, waiting time, transfer time and transfer penalty. ATE calculated based on the current bus service is depicted in Figure 10-1. ATE calculated based on the new solution is depicted in Figure 10-2. With the current bus services, no zone has average travel time to employment under 5 minutes. However, three zones have average travel time to employment lower than 5 minutes with the planned bus services. The average ATE for all zones with the planned bus services is 9.64 min which is a 7% improvement from the average ATE with the current services.

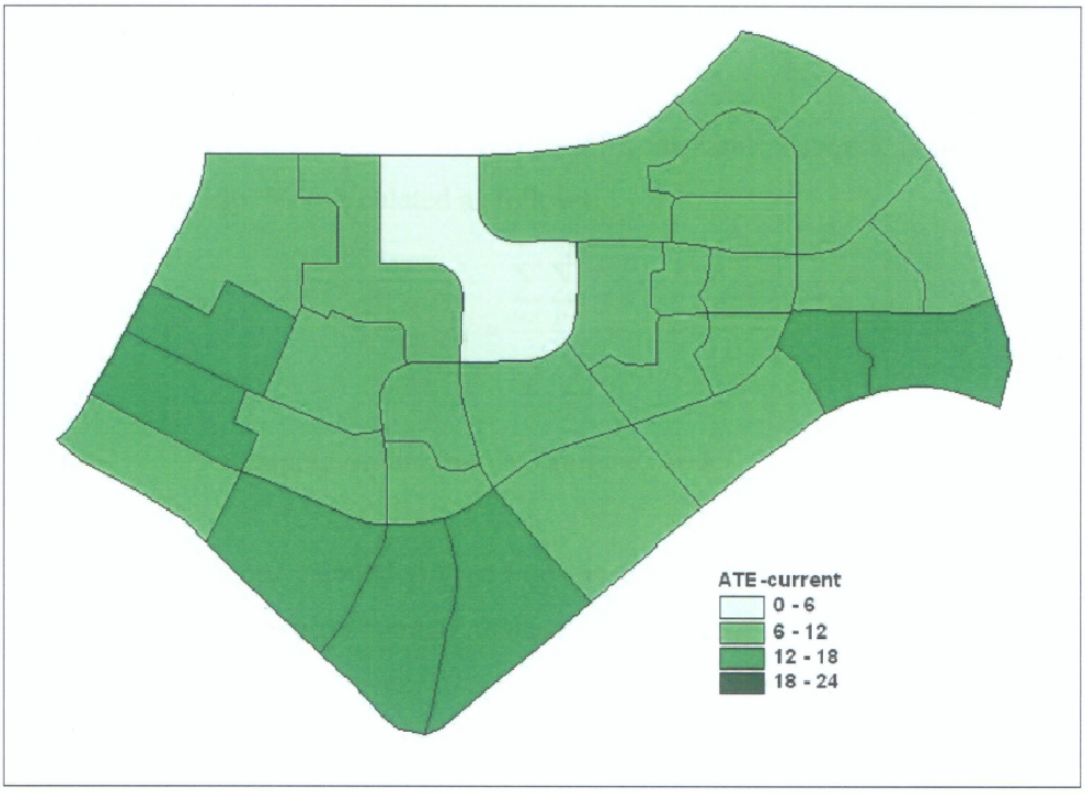


Figure 10-1 Average travel time to employment - current situation

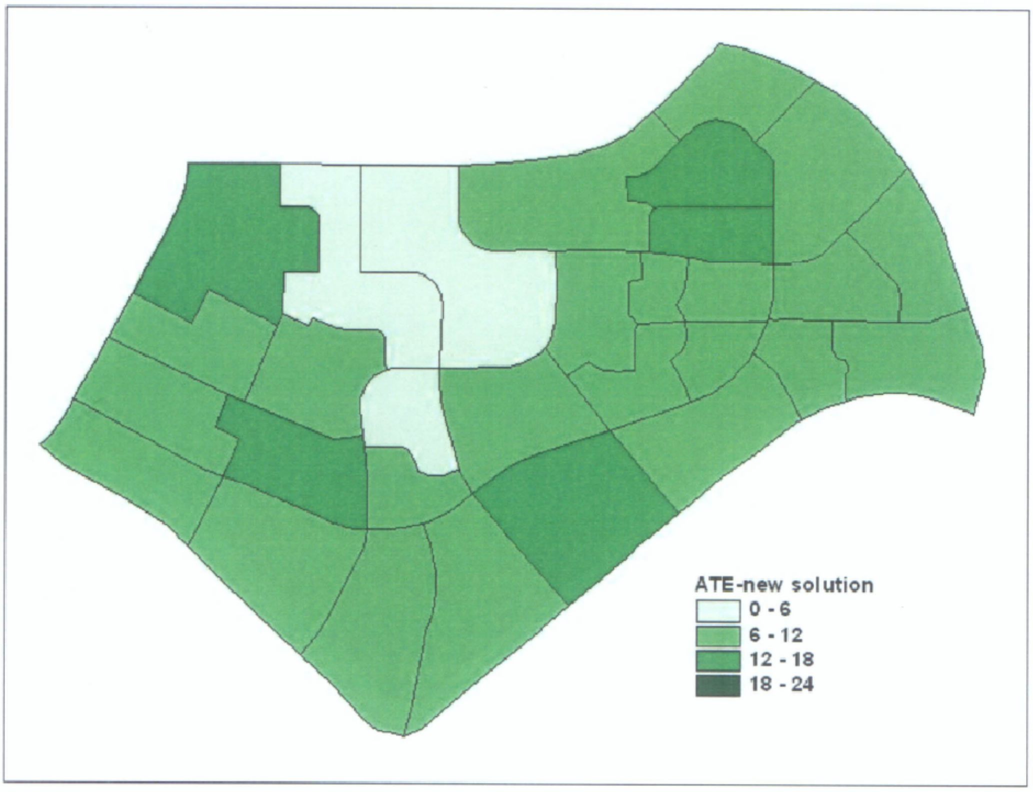


Figure 10-2 Average travel time to employment – new solution

10.3 Comparison of the average travel time

Another important indicator of the quality of a bus route network is the average travel time per trip. It is calculated as follows:

$$\bar{t}_A = \frac{\sum_{i \in A} \sum_{j=1}^{N} T_{ij} t_{ij}}{\sum_{i \in A} \sum_{j=1}^{N} T_{ij}}$$

where: N: number of bus stop in Tampines area

Average travel times calculated based on the current bus service are depicted in Figure 10-3. Average travel times calculated based on SA-GA are shown in Figure 10-4. With current bus services, 16 zones (53%) have average travel time above 12 minutes. The longest average travel time is 17.5 minutes. With the SA-GA solution, 14 zones (47%) have the average travel time above 12 minutes and the longest average travel time is 16.3 minutes which is 7% better than the current situation.

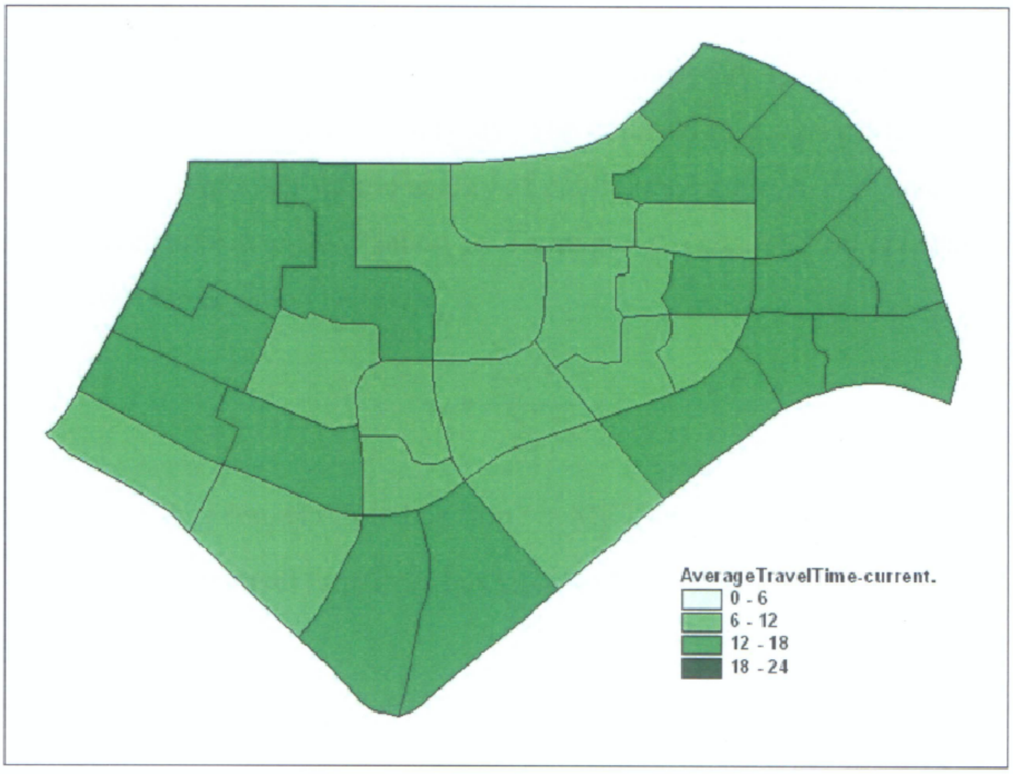


Figure 10-3   Average travel time – current situation

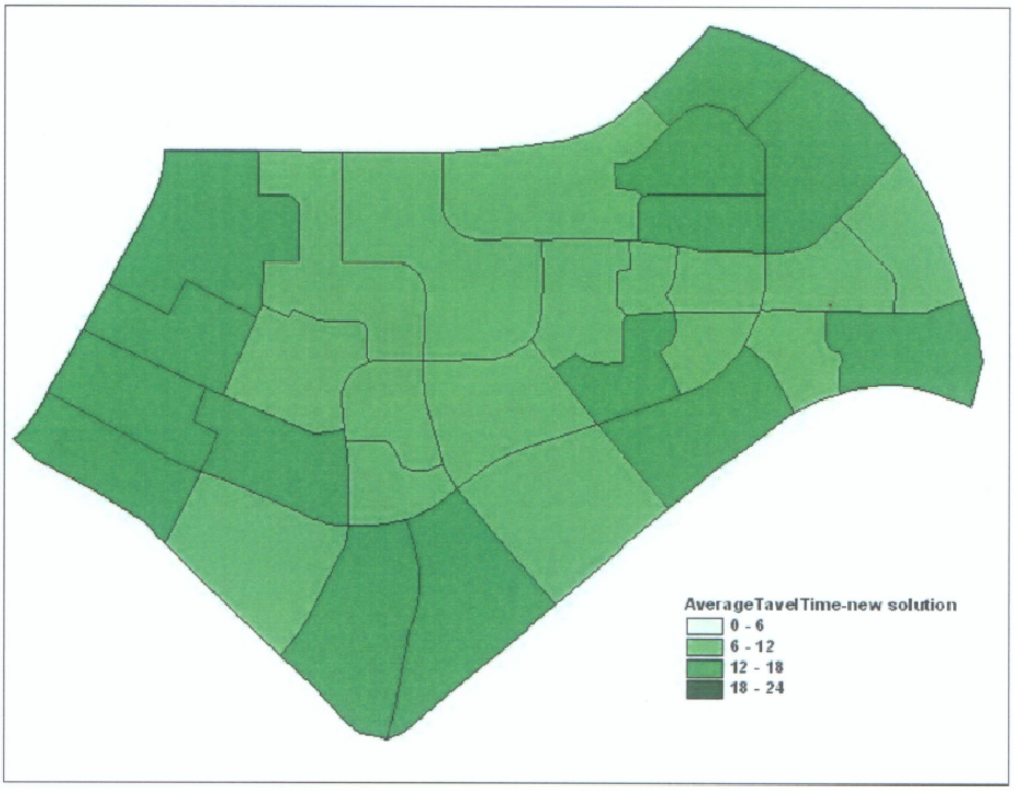


Figure 10-4   Average travel time – new solution

10.4 Comparison of the average travel time to MRT station

MRT is the main mode of public transport in Singapore. It carries millions of passengers to work and school everyday. The connection between MRT and bus services is very important for the quality of public transport system. Therefore, the average travel time to MRT station in Tampines is calculated as an evaluation indicator with the equation below:

$$\bar{t}_{Am} = \frac{\sum_{i \in A} T_{im} t_{im}}{\sum_{i \in A} T_{im}}$$

where: T_{im} : demand from bus stop *i* to MRT station

t_{im} : travel time from bus stop *i* to MRT station

Average travel times to MRT station calculated based on the current bus service network are depicted in Figure 10-5. Average travel times to MRT station calculated based on SA-GA solution are shown in Figure 10-6.

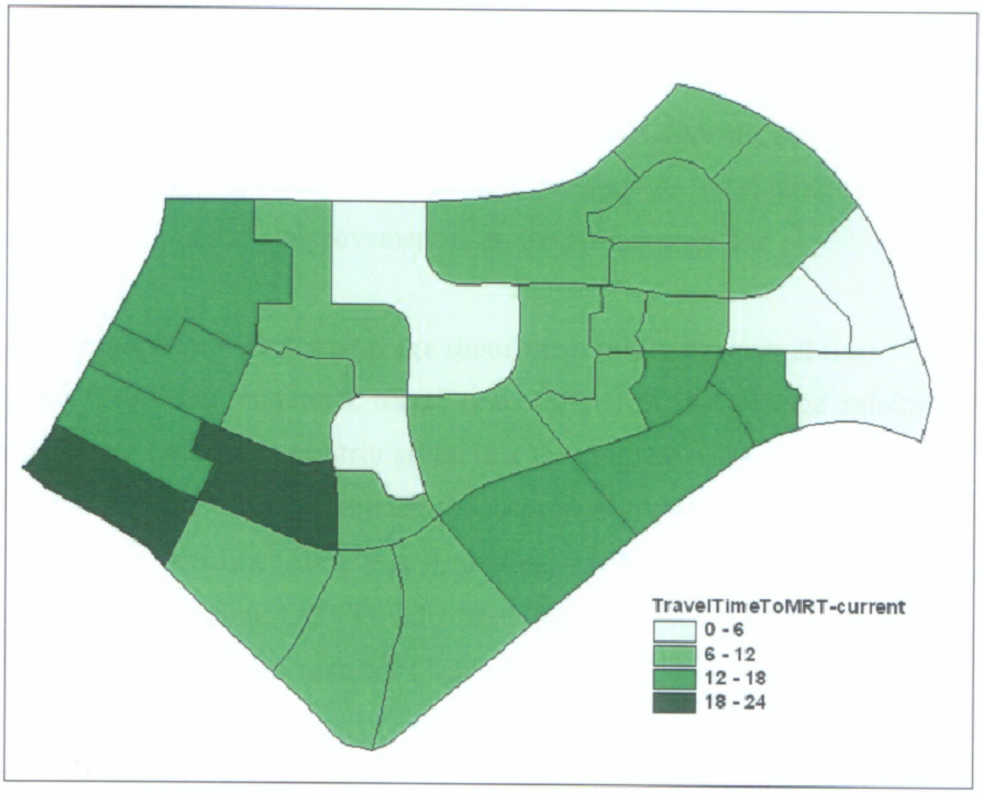


Figure 10-5 Average travel time to MRT station – current situation

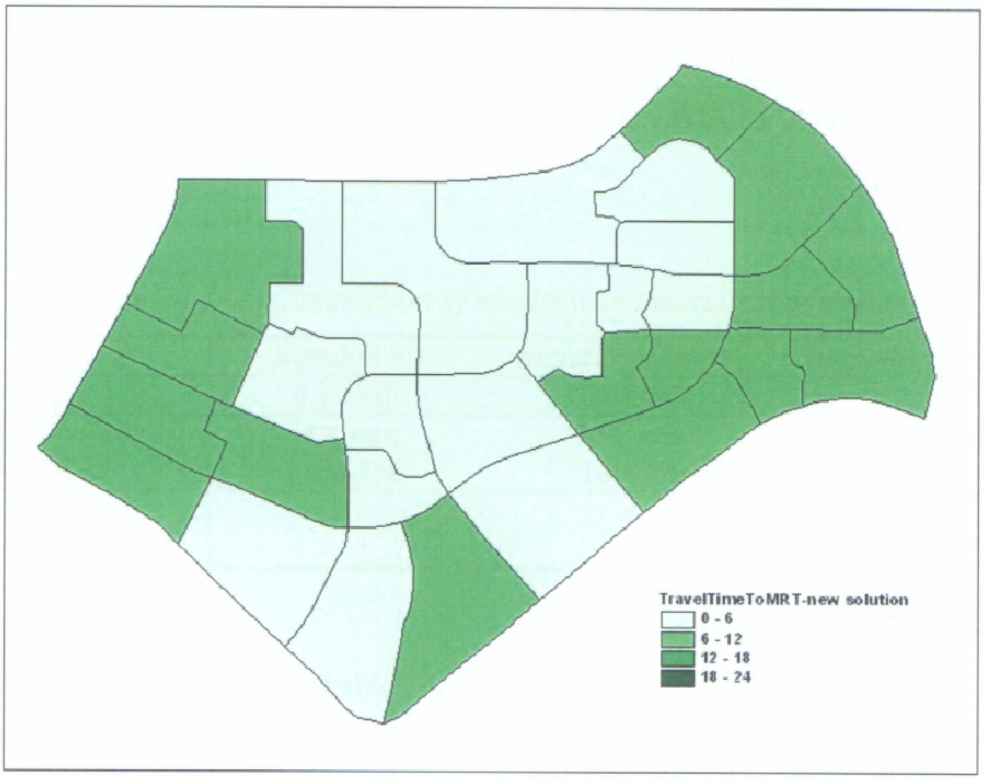


Figure 10-6 Average travel time to MRT station – new solution

With the current bus services, 2 zones have average travel time above 18 minutes and 7 zones have average travel time between 12-18 minutes. The longest average travel time is 23.06 minutes. With the SA-GA solution, no zone have average travel time above 12 minutes and the longest average travel time is only 11.54 minutes which is a 50% improvement from the current situation.

10.5 Comparison with the average theoretical minimum travel time

Average theoretical minimum travel time (ATM) is the average minimum in-vehicle travel time for every trip given that the shortest paths are available for all pairs of bus stops. It was calculated for theoretical network N1, Mandl's network and Tampines network. Then this theoretical minimum was compared with the average in-vehicle time (AIVT) calculated under the solution obtained from the SA-GA approach for each network (Table 10-1).

Table 10-1 shows that SA-GA can almost reach the average theoretical minimum for Mandl's network. For a small network like network N1, SA-GA can accomplish the average in-vehicle time which is less than 5% more than the theoretical minimum. Though SA-GA's result is 16% over the theoretical minimum for a large network like the Tampines network, it constitutes an 18% improvement from the current bus network.

Table 10-1 Comparison of results with theoretical minimum

	Network N1	*Mandl's Network*	*Tampines Network*
ATM	4.22 min	10.01 min	7.22 min
AIVT (SA-GA)	4.4 min	10.1 min	8.4 min
% over ATM	104.3%	100.9%	116.3%
AIVT (current)			9.7 min
% over ATM			134.3%

10.6 Comparison with fleet size

Fleet size is another important performance indicator of public transport system. It is calculated as the sum of the fleet size for all individual bus services in the transport system. The fleet size for individual bus service can be calculated as:

$$N_{fs} = [F * t_c]^{+}$$

where F is the frequency (buses/h) and t_c is the cycle time in hours. The brackets indicate integer value equal or greater than the computed value. This formula is adopted from Vuchic (2004). Cycle time is calculated as the round trip distance divided by average speed which is 18 km/h for both current bus service and SA-GA solution.

After calculation of the fleet size for each individual bus service, the sum of the fleet sizes for all bus service is calculated for both current transport system and SA-GA solution. It is noted that 240 buses (Table L1 in Appendix L) are required to satisfy all the demand based on the current public transport system. However, only 176 (27% less) buses (Table 9-3) can handle all the demand based on the SA-GA solution. SA-GA solution for Tampines is not only an improved solution from travellers' perspective (improved travel time, waiting time, transfer time, direct trip, number of transfers), it also an improved solution for the operators (decreased fleet size and operation cost).

10.7 Comparison with load factor

Load factor is an indicator for service quality of public transport system. Smaller loading factor means fewer people are loaded in a bus, and therefore, a higher service quality. Load factor is calculated as:

$$l_k = Q_k^{\max} /(f_k \cdot C_b)$$

This formula is adopted from Pattnaik et al. (1998). C_b is taken as 80 (passengers/bus) for both the current bus routes and SA-GA solution. Average load factor for current bus routes is 0.78. Average load factor for SA-GA solution is 0.77 which is slightly lower. Therefore, it is seen that the reduced travel time and operation cost is not obtained at the cost of decreased service quality at least from capacity point of view.

10.8 Discussion

SA-GA approach is an effective method for solving the bus route design problem according to the evaluation and comparison of planning result with previous studies or existing situation. However, there are some issues need to be carefully considered when apply this method. For example, the selection of parameter values in SA-GA approach is important for application of this method. The optimal values for parameters are selected through sensitivity analysis performed on network N1 and Mandl's network. Consistent results are obtained for both networks. The optimal value for crossover probability is 1.0. The optimal value for mutation probability is 0.001. The optimal value for population size is 200. The optimal value for the parameter which controls the number of iterations performed at each temperature is 20.

When comparing the features of the best results for all four theoretical networks, it can be seen that all the best solutions came from dynamic mutation and three best solutions came from the two-point crossover. This suggests that dynamic mutation and two-point crossover are more suitable than the other mutation and crossover techniques for solving the bus network design problems with GA.

However, values of some other parameters still need to be investigated by running sensitivity tests. In transit demand assignment, two paths are selected for assignment. It was assumed that the two paths must not have in-vehicle travel time longer than 20% of the shortest in-vehicle travel time. Subsequently, transit assignment is done in two iterations. In the first iteration, demand is assigned equally to the two paths and in the second assignment it is based on frequency. The number of paths considered, the acceptable time range and the number of iterations are assumed in this way in order to control the computation time. With three or more paths, the effects of setting larger acceptable time range or more assignment iterations are subject to further investigation. A further sensitivity analysis on the factors to convert passenger travel time and bus operation distance to cost is also necessary, although sensitivity analysis in this study shows that improvement in objective function value is not sensitive to them. A broader range of values may be selected to perform the sensitivity analysis.

The computation times of all the networks analysed are indicated in Table 10-2. Obviously, the computation time goes up very quickly with the increase of network size. This is not surprising because the bus network design problem is a highly computationally-intensive optimisation problem (NP-hard).

Table 10-2 Computation time for all networks

	No. of stops	*No. of links*	*Computation time (min)*
Network1	14	31	7
Network2	20	44	37
Network3	26	57	81
Network4	32	70	133
Mandl	15	21	35
Tampines	79	197	786

The problem of size of area applicable for optimisation should not only be considered a matter of computational complexity and computation time. A normalised presentation is required for an area which needs to be optimised. As the area size increases, more region specific characteristics need to be eliminated or compromised. The optimal solution obtained based on the normalised presentation may not remain the optimum with the real situations of specific sub-regions. Therefore, the meaning of optimisation exists only with limited problem size.

The proposed approach is applicable to other networks or cities as long as the following data are available:

- Network with roads, bus stops and/or road intersections. Each bus stop or road intersection is a node with unique ID. Every node should have X Y coordinates. Every road consists of several links. Each end of a link is a node. A link is represented by two node IDs. The sequence of the IDs shows the direction of the link.
- Bus stop to bus stop demand table.

When applying the proposed approach on a large network such as the whole Singapore road network, a good decomposition is essential to reduce the computational complexity. A possible solution is to decompose the problem into three levels. The whole Singapore can be divided into macro-zones such as Tampines. Each zone would have one or more bus interchanges and several virtual terminals which are located on the main roads connecting different zones with each other. Only bus interchanges can be origins or destinations of bus routs. Bus stops need to be located in every zone to let the public have good access to bus services. Demand information regarding both intra-zone travel and inter-zone travel must be available.

The first level is intra-zone level. Intra-zone travel demand need to be assigned to each bus stop or bus interchange in every zone. Bus routes are designed to satisfy all intra-zone travel demand. These bus routes must have at least one end at bus interchange. The other end can be bus interchange or virtual terminal. The frequency associated with each inter-zone route is called intra-zone frequency.

The second level is inter-zone level. Inter-zone travel demand needs to be assigned to bus interchanges or virtual terminals. Bus routes are designed to satisfy all inter-zone travel demand. Unlike intra-zone routes, inter-zone bus routes can have virtual terminals as both origins and destinations. The frequency associated with each inter-zone route is called inter-zone frequency.

The third level is combination. Intra-zone and inter-zone bus routes are combined on this level. Requirements for combination are:

- Routes selected for combination must have the common virtual terminals as origins or destinations.
- Select the routes with closest direction and frequency to combine.
- The resulting bus route must have both ends at bus interchanges.
- If there is one route left with one or both ends are virtual terminals, a duplicate route is made to combine with it so that it has both ends at bus interchanges. Suggest making a duplicate of the route with the highest frequency among candidates.

An evaluation should be performed to calibrate the parameters such as frequency etc. The calculated intra-zone and inter-zone frequencies can be used as the boundaries of frequency selection. Different combinations of intra-zone and inter-zone routes can also be evaluated and compared to find a better solution.

CHAPTER 11 CONCLUSIONS

11.1 Conclusions

Network design is crucial for transit planning. It has wide applications in other areas besides transportation. Researchers have been attempting to resolve this extremely complex problem for decades.

This study starts from evaluation of the current public transport network of Singapore from three different perspectives: pedestrian accessibility, connectivity among bus stops and travel time to employment zones. The evaluation shows that the existing bus system does not integrate very well with the MRT system.

Then a new approach for transit network design is introduced. Firstly, methods for optimisation of bus stop location are proposed. A randomised algorithm with the aim of optimisation of bus stop placement with respect to pedestrian accessibility is introduced. The algorithm repeatedly picks bus stops according to a certain ordering and moves the stop to its local optimal position. The required inputs of this approach are the roads where bus stops are sited or the original locations of bus stops and the location of trip generators like housing blocks and other buildings. The advantages of this approach are: easy to implement; low requirement for computation resources. The disadvantages of the approach are that some preliminary knowledge of the target area is necessary and selection of movement distance is arbitrary.

To overcome these shortcomings, another bus stop location method which transforms this problem to set-cover problem is introduced. Small scale test shows that this method can efficiently utilise bus stops to maximise accessibility. However, this method has heavy requirements of computational storage and time. Therefore, the application of this method on large scale networks still needs more exploration and improvements. There is a tendency to locate the bus stop at an intersection or very close to an intersection in the set-cover solutions. It happens due to the program trying to cover as many as buildings as possible. Moreover, the minimum number of bus stops does not mean good design in real life. Fewer buses stopping may result in too many people going to the same bus stop to take bus and too many buses stopping at the same bus stop. These concentrations make the bus stop too crowded, increase the boarding time and cause traffic congestion. It may not be practical and economical to remove all the current bus stops and build new ones if no strong evidence is available to show that a dramatic improvement of public bus service quality will result from after re-location of all the current bus stops.

Secondly, a framework of designing bus route network with hybrid SA and GA algorithm is proposed. This new combined SA-GA approach is implemented in two phases: a set of candidate routes is first developed and then an optimal subset of routes is selected. With the help of advanced algorithms: simulated annealing (SA) and genetic algorithm (GA), the design problem is solved in minutes for theoretical networks and the Mandl's network and in hours for the real case network with 79 bus stops.

Results from the applications on four theoretical networks suggest that different genetic operators have different levels of efficiency. Since all the best solutions for the four networks come from dynamic mutation and three best solutions come from two-point crossover, the indication is that dynamic mutation and two-point crossover are more suitable for solving the bus network design problem with GA than the other mutation and crossover methods.

A directional network is used in real case application of the proposed approach so that one-way roads are taken into consideration in the application and it is possible for the program to deal with the situation when the routes in two directions between a pair of bus stops are different which does happen in real life. Moreover, loop services are also used in the application. All of the situations discussed above are quite common in real life but seldom mentioned in previous studies on bus route design. The successful implementation of these cases makes the proposed approach more suitable for practical use than the other methods.

Transit demand assignment is an important part in the evaluation module. In this part, two paths are considered in two iterations of assignment. Actually, the program is capable of dealing with n path assignment. In practice, it is not realistic to expect that travellers have the knowledge about all the available paths between any pair of origin and destination. Therefore, two path assignment is acceptable to save computational time. An advantage of the assignment utilised in this study is that low demand route can be eliminated in the assignment module. In the first iteration, the demand is assigned equally to the two paths and then the frequency is calculated based on the demand assigned. If any route has the frequency lower than a predefined minimum frequency, this route would be eliminated from the solution as long as all the bus stops can be covered without this particular route. In this way, any redundant or less efficient route can be excluded from the solution thus making the final solution more efficient.

Theoretically, the general network design problem is proven to be NP-hard, i.e. it is almost impossible to find an efficient algorithm that can solve this kind of problem in polynomial time (Karp, 1975). There is simply no way to find the absolute optimal solution for any medium-sized network design problem in tolerable time. Moreover, not many real case studies have been completed. This project does not aim to reduce the time complexity or achieve any theoretical breakthrough. Instead, the goal is to adapt and combine the existing techniques to develop a feasible model for transit network design suitable for a real case application.

There are very few publications on large scale real case applications of bus network design. The relatively recent large scale application can be found in Fan and Machemehl (2006a) who used GA approach. It was implemented in two ways: with and without demand aggregation. With demand aggregation, all zone demand was loaded on one node (bus stop). In this way the network size was reduced to 28 stops. Without demand aggregation, each zone could have more than one exit (bus stop) so that the network size was increased dramatically to 95 stops. The computation time for demand aggregation approach was 3 hours while without demand aggregation it was 72 hours. Both computations were performed with 80 generations. In this study, the real case application is performed on a network with 79 bus stops with 1,000 generations and the computation finished within 13.1 hours. This indicates that the proposed approach outperforms the method of Fan and Machemehl (2006a) in terms of computation time.

The proposed approach produced better result in total travel time, in-vehicle travel time, transfer time and direct trip percentage than most previous studies on Mandl's network. Particularly, the average in-vehicle travel time for Mandl's network under SA-GA solution is only 0.9% more than the theoretical minimum, which indicates the effectiveness of the proposed approach.

In the real case application on Tampines network, SA-GA solution is also better than the current bus network. With SA-GA solution, average travel time is 13% less than the current situation; 19% more trips can be completed without transfer; 50% improvement is achieved on average travel time to MRT station. In addition to these improvements, total cost is decreased by 10% and operation cost by 5%.

Based on these evaluation and comparison, it is confirmed that SA-GA can produce a nearly optimal solution for a small network and a much improved solution compared to the current bus network for a large real network within a reasonable computation time given all the assumptions made. Some of the important assumptions are:

- Fixed demand: this is a simplification of the problem which is due to practical reasons. Planners can only optimise the system assuming a fixed demand pattern. If planners knew the demand elasticity with respect to travel cost, they could use an iterative procedure as follows: assume fixed demand → optimise the network → find new travel costs → adjust the demand matrix using the elasticity function → optimise the network again → and so on.
- Two path assignment: this is also a simplification of the problem for practical reasons. Ideally all available paths should be considered. However,

travellers may not have enough information to know all the available paths and some alternative paths may not be considered by travellers in real life if they require long travel times.

- Traveller's choice when faced with two best paths: it is assumed that travellers will board the first bus which arrives, belonging to one of the two best paths. This assumption is used based on previous studies such as Chriqui and Robillard (1975) and Mandl's(1980) works and proved to be valid when the alternative paths are not much different in length.
- Assignment of unit weight to waiting time, in-vehicle travel time and transfer time: this assumption is made due to the lack of a survey of Singapore travellers to indicate different weights. The calculation of travellers' perceived cost would be more accurate if different weights were used for these components of total travel time and the designed bus routes would be more acceptable for travellers. A weighted solution would generate more acceptable routes for travellers.

11.2 Limitation of this research

Demand matrix used in real case application is estimated from survey data. This survey was done by LTA on a sampling basis. Estimation of the demand matrix was based on current bus service routing and virtual terminals. The demand matrix may not be a true indication of real demand distribution and might affect the efficiency of the SA-GA solution.

Although all components of travel time i.e. waiting time, in-vehicle travel time and transfer time are taken into account in the calculation of total travel cost, equal weight is given to all of them. However, travellers generally attach different importance to these components when they are facing the choice of different travel paths (Bovy and Hoogendoorn-Lanser, 2005). Therefore, the measurement of traveller's cost and total cost would be more accurate through assigning different weights to waiting time, in-vehicle travel time and transfer time based on their average importance in travellers' opinion. Accordingly, the designed bus route network would be more acceptable by travellers. This is not done in this research because of the lack of such a survey for Singapore travellers.

11.3 Recommendations for future research

The Network Design Problem has long been recognised to be one of the most difficult problems in transport studies (Yang and Bell, 1998). This is a nonlinear optimisation problem with many variables. Previous studies show that computational complexity increases exponentially as the number of nodes and arcs grows. Solving large scale (more than 50 nodes) network design problems is one of the most challenging problems in modern computer science (Magnanti and Wong, 1984).

The larger the search space, the better the SA-GA algorithm performs. However, larger search space also means longer computation time. Currently, the selection of the most suitable size of search space is done through testing. No systematic way of selection of search space size for networks with different sizes is available. If a scientific method for reaching the balance between search space size and computation time can be discovered, the utilisation of SA-GA can probably be further improved.

In optimisation of bus stop locations, improving accessibility is chosen as the objective. Discrete classification of accessibility into good, medium and bad ranges is used. The boundaries between classes are defined based on a survey among bus passengers done in Singapore without further study into the sensitivity of these boundaries. To be more precise, a continuous representation of accessibility could be a better basis for evaluation of accessibility and optimisation of bus stop location. The optimisation algorithm used in this thesis is a simplified model without constraints. Further improvements could be made with more realistic constraints such as demand based on weighted building classification and road constraints such as the number of lanes in a given stretch of road. Unfortunately, many of these practical constraints are not readily available and therefore are omitted in the study.

Shortest path or k-shortest paths are used for the candidate routes generation in this study and almost all other applications of genetic algorithms and simulated annealing algorithm according to available literature. It is quite natural to think that short or shortest path may save travel time and then produce smaller cost. However, there is no proof that the combination of all shortest paths would produce a better solution for all pairs of origins and destinations. Therefore, it is worth to try other methods to produce candidate routes.

In this study, fixed travel demand is assumed. This means that the quality of the transit system would not affect people's choice between public transport and private car. In fact, high quality (fast, comfortable) and inexpensive public transport attracts more people to use buses instead of cars; low quality public transport forces people to use private transport. Therefore, incorporation of correct function of travel demand with respect to public transport quality would make this design approach more accurate and realistic. In addition, planning and designing of transit route in this study is based on the morning peak hour demand. This may produce a biased design. It would be fair to design the transit network while considering both peak and off-peak period demand.

Good integration of bus service with mass rapid transit (MRT) is one of the possible solutions to ease the traffic congestion in big cities. It would be valuable to include future MRT lines into the bus transit network design process. It is also interesting to compare the result of "planning from scratch" with new bus stops and new bus routes with the current system and examine how much improvement can be achieved.

It should also be noted that solving transportation problems transcends scientific and engineering knowledge. It has also a lot to do with human factors, including political, administrative, social and economic issues (Vuchic, 2004). Thus, it is possible that the computational planning does not adequately reflect the real situation in the real world. Take the example of NEL issue that occurred in Singapore recently. Before the change of the bus services duplicating NEL took place, the planners did predict the possible effects using a computer program named “Transport Improvement Planning System”. However, the real effects were quite different from what the program predicted (Goh, Sep. 15, 2003).

Human factors are too complex to model and virtually impossible to predict accurately. If more accurate statistics on travellers’ behaviour can be obtained, more accurate or desirable design can be expected. These statistics may include: penalty for transfer; travellers’ cost for in-vehicle travel time, waiting time, and transfer time; travellers’ choice when facing different routes with the same total travel time etc. All of these findings may produce a more suitable and precise objective function. The sensitivity analysis of these factors and different effects of divergent objective functions on optimisation result is also worth doing. Thus, there remains a considerable scope for further improvements of the proposed SA-GA methodology.

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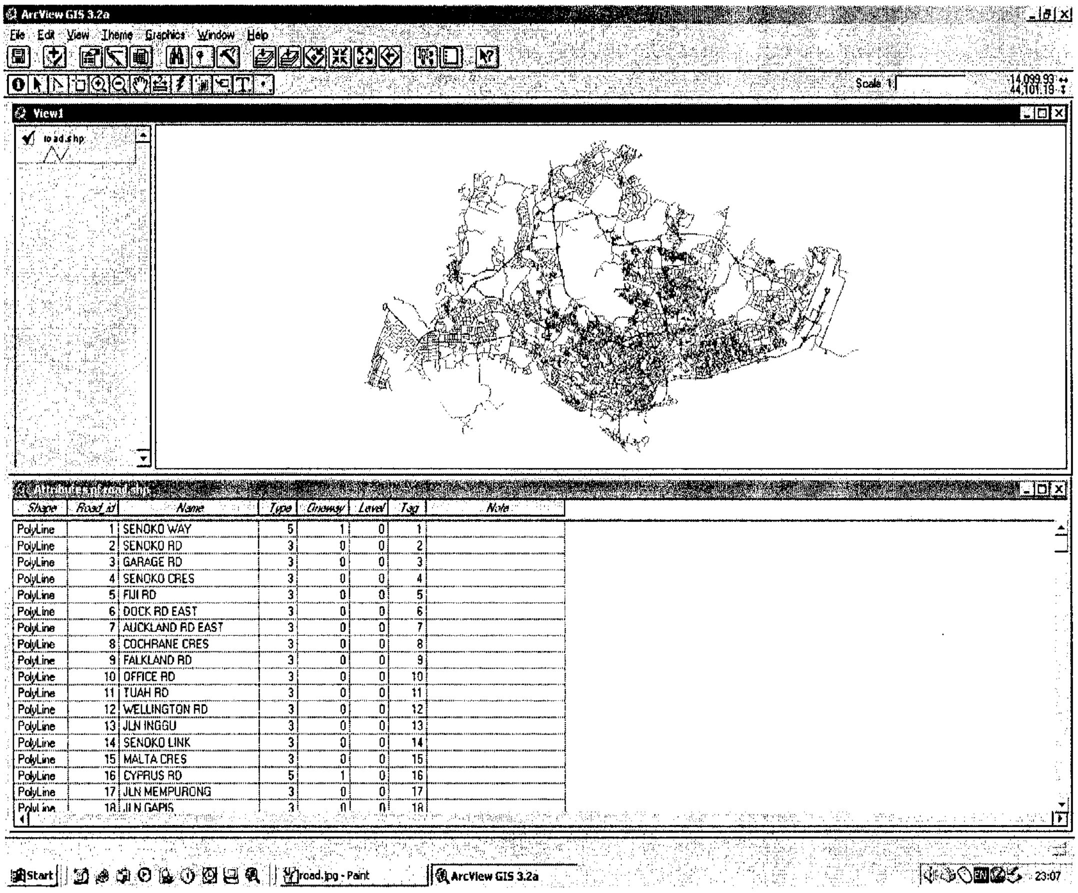
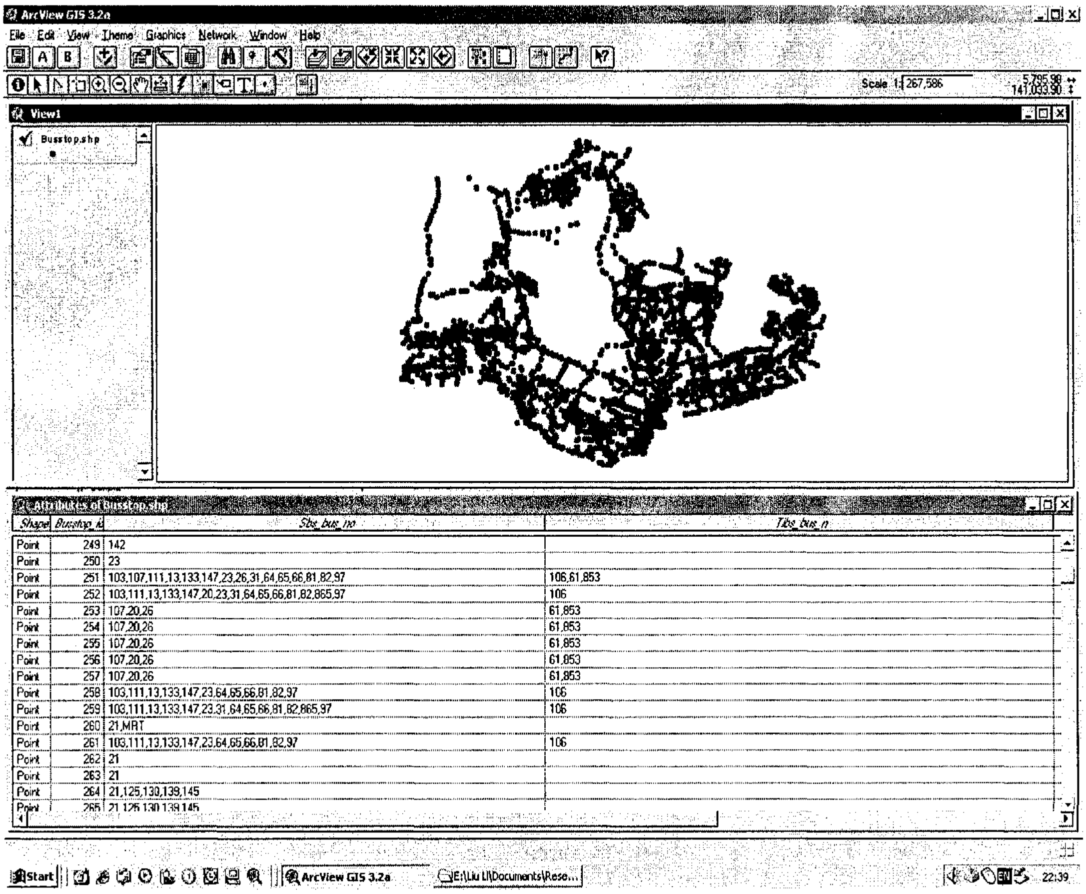
APPENDIX A

INFRASTRUCTURE DATA



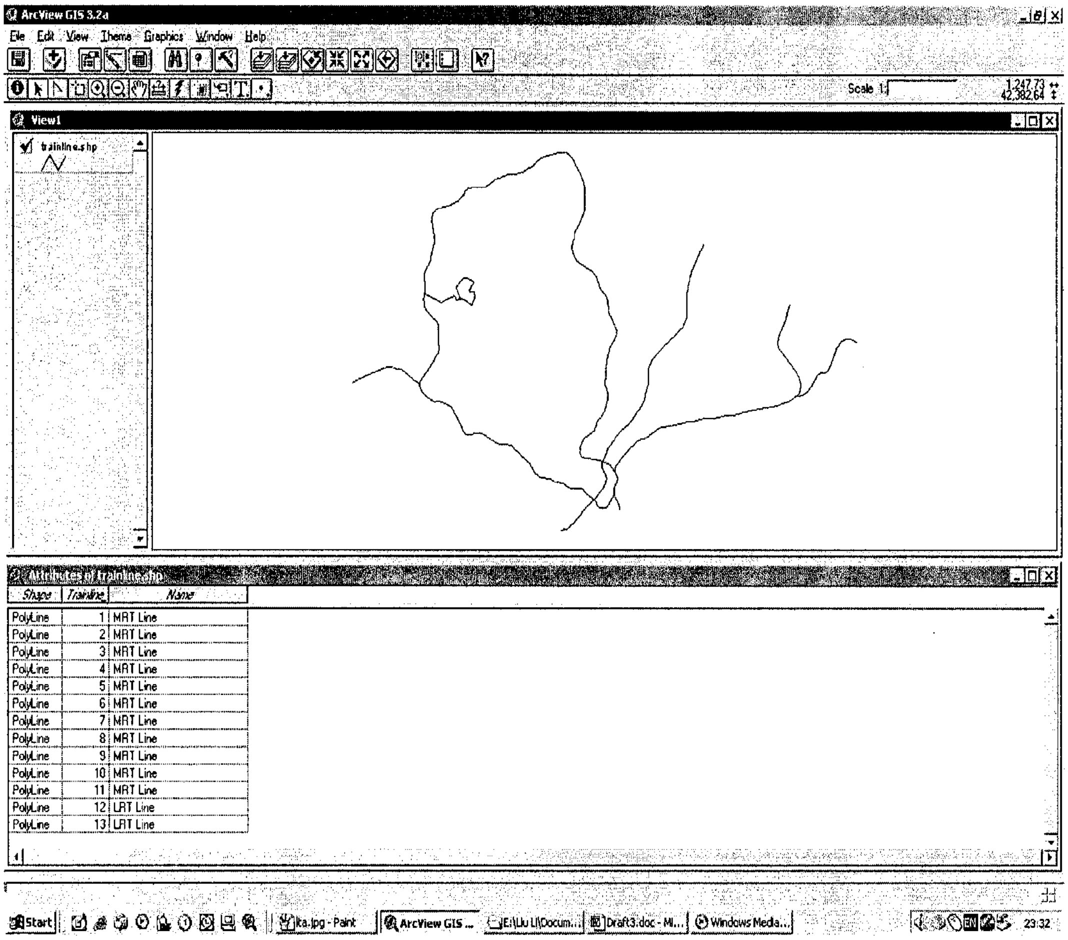
Shape	Road_id	Name	Type	Oneway	Level	Tag	Note
PolyLine	1	SENOKO WAY	5	1	0	1	
PolyLine	2	SENOKO RD	3	0	0	2	
PolyLine	3	GARAGE RD	3	0	0	3	
PolyLine	4	SENOKO CRES	3	0	0	4	
PolyLine	5	FIJI RD	3	0	0	5	
PolyLine	6	DOCK RD EAST	3	0	0	6	
PolyLine	7	AUCKLAND RD EAST	3	0	0	7	
PolyLine	8	COCHRANE CRES	3	0	0	8	
PolyLine	9	FALKLAND RD	3	0	0	9	
PolyLine	10	OFFICE RD	3	0	0	10	
PolyLine	11	TUAH RD	3	0	0	11	
PolyLine	12	WELLINGTON RD	3	0	0	12	
PolyLine	13	JLN INGGU	3	0	0	13	
PolyLine	14	SENOKO LINK	3	0	0	14	
PolyLine	15	MALTA CRES	3	0	0	15	
PolyLine	16	CYPRUS RD	5	1	0	16	
PolyLine	17	JLN MEMPURONG	3	0	0	17	
PolyLine	18	JLN GAPIS	3	0	0	18	

Figure A1 Road network



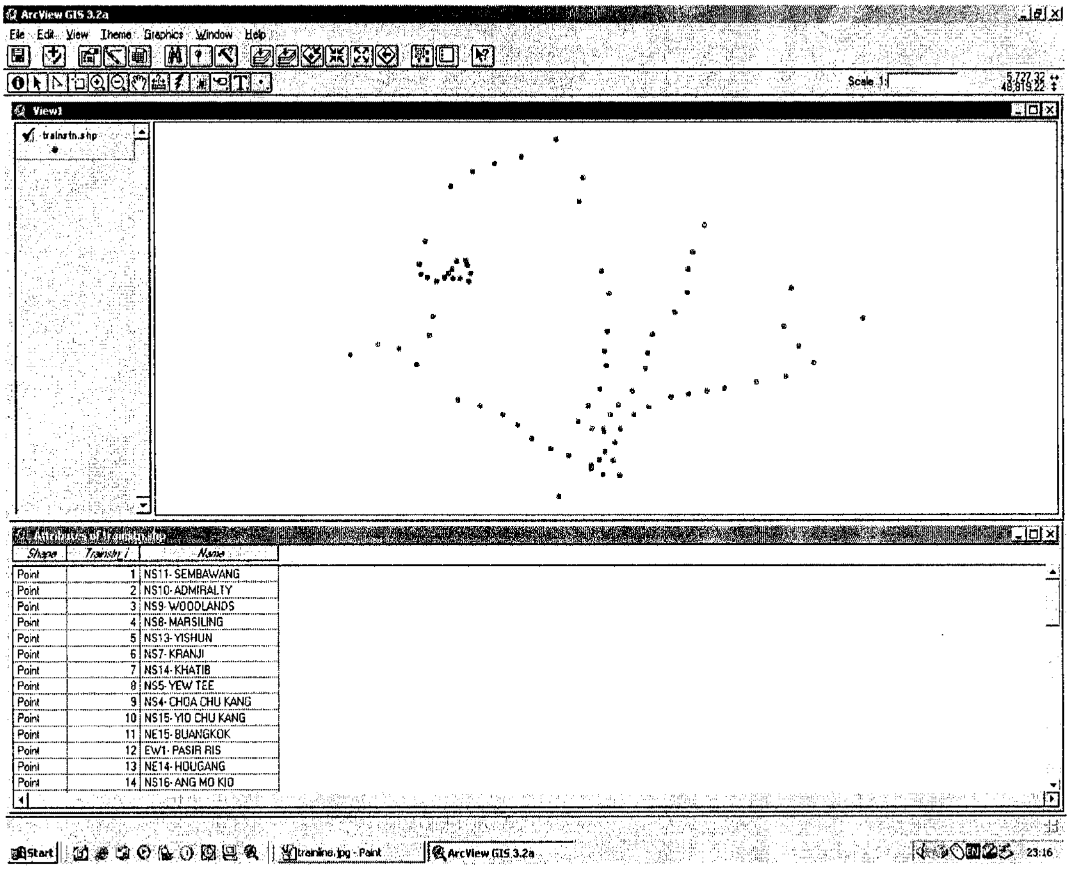
Shape	Busstop_id	Sbs_bus_no	Tibs_bus_n
Point	249	142	
Point	250	23	
Point	251	103,107,111,13,133,147,23,26,31,64,65,66,81,82,97	106,61,853
Point	252	103,111,13,133,147,20,23,31,64,65,66,81,82,865,97	106
Point	253	107,20,26	61,853
Point	254	107,20,26	61,853
Point	255	107,20,26	61,853
Point	256	107,20,26	61,853
Point	257	107,20,26	61,853
Point	258	103,111,13,133,147,23,64,65,66,81,82,97	106
Point	259	103,111,13,133,147,23,31,64,65,66,81,82,865,97	106
Point	260	21,MRT	
Point	261	103,111,13,133,147,23,64,65,66,81,82,97	106
Point	262	21	
Point	263	21	
Point	264	21,125,130,139,145	
Point	265	21,125,130,139,145	

Figure A2 Bus stops



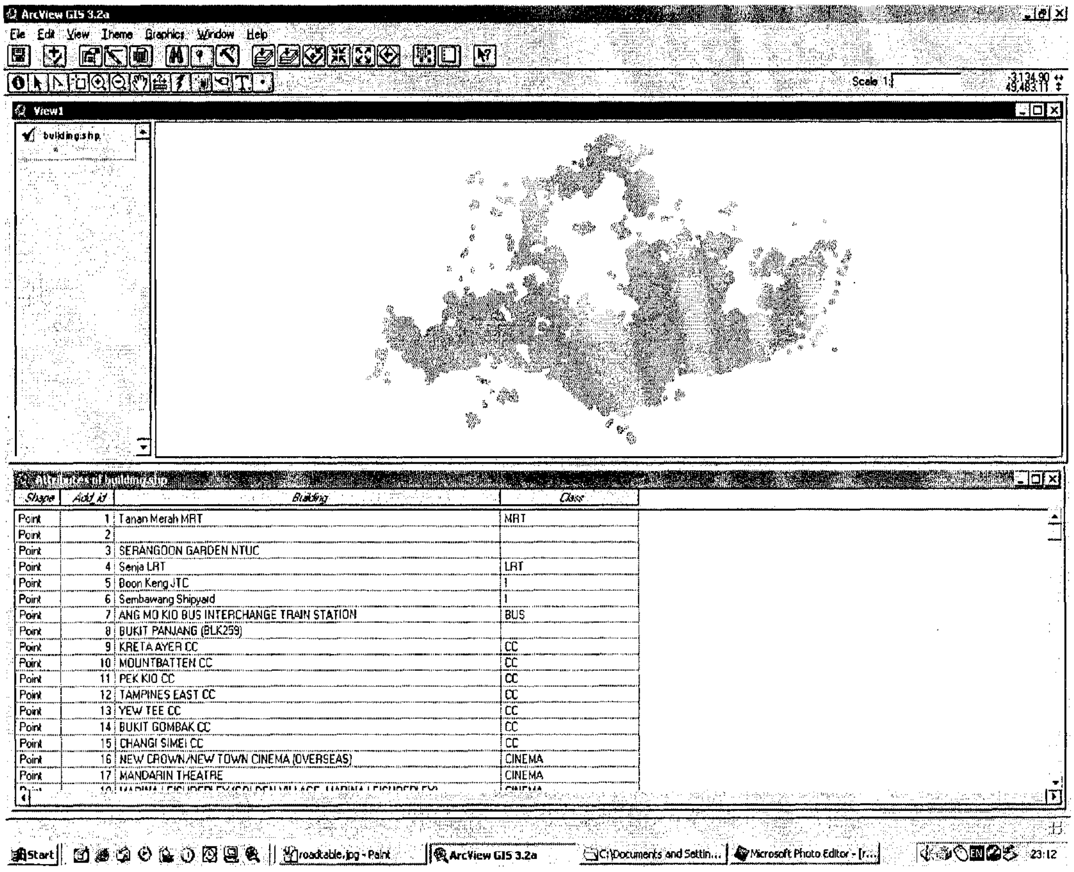
Shape	Trainline	Name
PolyLine	1	MRT Line
PolyLine	2	MRT Line
PolyLine	3	MRT Line
PolyLine	4	MRT Line
PolyLine	5	MRT Line
PolyLine	6	MRT Line
PolyLine	7	MRT Line
PolyLine	8	MRT Line
PolyLine	9	MRT Line
PolyLine	10	MRT Line
PolyLine	11	MRT Line
PolyLine	12	LRT Line
PolyLine	13	LRT Line

Figure A3   Train lines (MRT lines and LRT lines)



Shape	Transtn_i	Name
Point	1	NS11- SEMBAWANG
Point	2	NS10- ADMIRALTY
Point	3	NS9- WOODLANDS
Point	4	NS8- MARSILING
Point	5	NS13- YISHUN
Point	6	NS7- KRANJI
Point	7	NS14- KHATIB
Point	8	NS5- YEW TEE
Point	9	NS4- CHOA CHU KANG
Point	10	NS15- YIO CHU KANG
Point	11	NE15- BUANGKOK
Point	12	EW1- PASIR RIS
Point	13	NE14- HOUGANG
Point	14	NS16- ANG MO KIO

Figure A4 MRT stations



Shape	Add_id	Building	Class
Point	1	Tanan Merah MRT	MRT
Point	2		
Point	3	SERANGOON GARDEN NTUC	
Point	4	Senja LRT	LRT
Point	5	Boon Keng JTC	1
Point	6	Sembawang Shipyard	1
Point	7	ANG MO KIO BUS INTERCHANGE TRAIN STATION	BUS
Point	8	BUKIT PANJANG (BLK259)	
Point	9	KRETA AYER CC	CC
Point	10	MOUNTBATTEN CC	CC
Point	11	PEK KIO CC	CC
Point	12	TAMPINES EAST CC	CC
Point	13	YEW TEE CC	CC
Point	14	BUKIT GOMBAK CC	CC
Point	15	CHANGI SIMEI CC	CC
Point	16	NEW CROWN/NEW TOWN CINEMA (OVERSEAS)	CINEMA
Point	17	MANDARIN THEATRE	CINEMA

Figure A5 Buildings

APPENDIX B

LTA TRAVEL TIME AND EMPLOYMENT DATA

Table B1 Inter-zone travel by bus

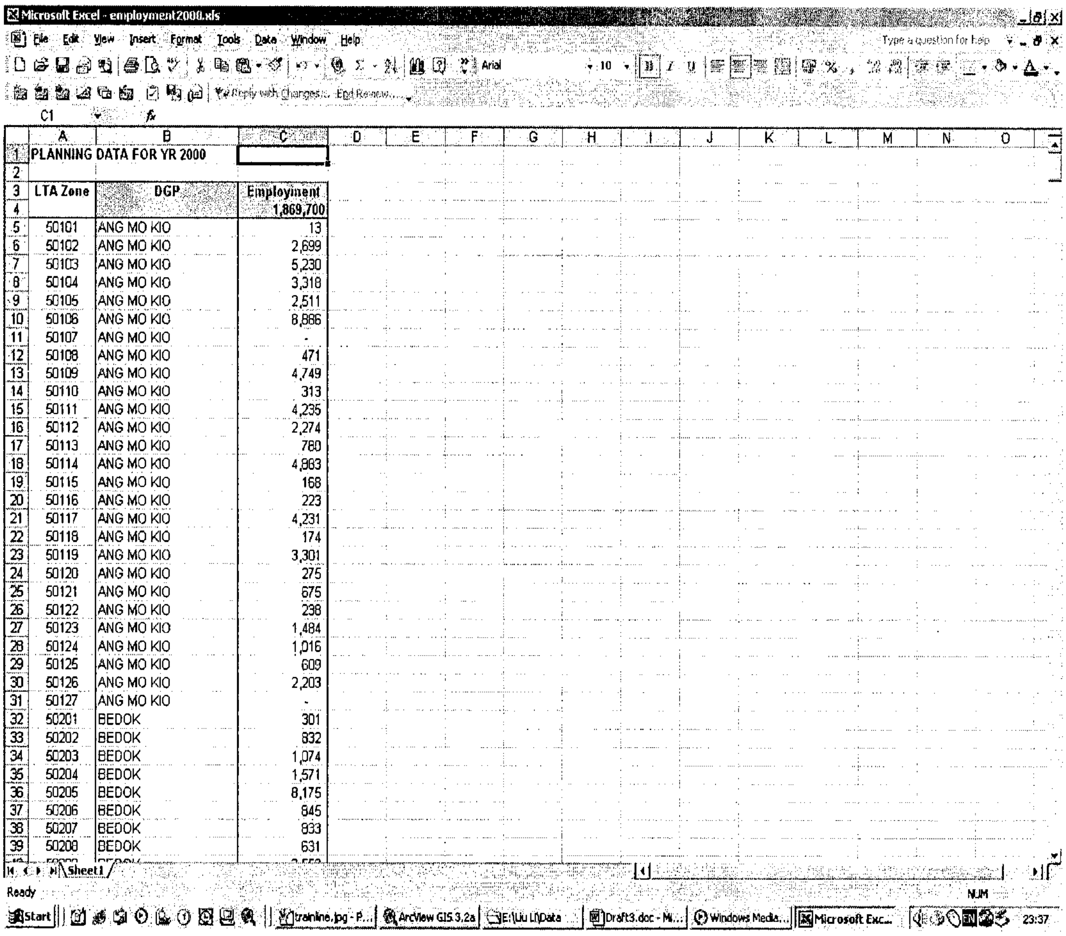
```
c EMME/2 Module:  3.14(v9.02)  Date: 03-04-10 16:27   User:
E508/LTA.......psl
c Project:        2000 Model
t matrices
a matrix=mf11 bustt      0 bus travel time (ivt+wait+walk)
  50101  50102:   19.78  50103:   16.92  50104:   24.35  50105:
21.53
  50101  50106:   17.97  50107:   33.27  50108:   26.76  50109:
29.47
  50101  50110:   20.51  50111:   27.64  50112:   25.82  50113:
28.74
  50101  50114:   19.51  50115:   18.89  50116:   19.56  50117:
24.64
  50101  50118:   19.85  50119:   21.87  50120:   22.04  50121:
14.12
  50101  50122:   13.57  50123:   18.27  50124:   17.72  50125:
20.79
  50101  50126:   28.47  50127:   14.10  50201:   70.17  50202:
61.53
  50101  50203:   55.26  50204:   50.33  50205:   50.53  50206:
58.42
  50101  50207:   59.46  50208:   62.90  50209:   58.56  50210:
64.06
  50101  50211:   61.43  50212:   63.89  50213:   62.53  50214:
73.43
  50101  50215:   74.23  50216:   94.68  50217:  104.12  50218:
79.83
  50101  50219:   69.48  50220:   67.28  50221:   65.57  50222:
66.23
  50101  50223:   67.42  50224:   93.07  50225:   55.40  50226:
59.60
  50101  50227:   65.08  50228:   70.04  50229:   50.33  50230:
53.88
  50101  50231:   58.79  50232:   57.49  50233:   65.27  50234:
66.06
  50101  50235:   64.92  50236:   63.80  50237:   58.89  50238:
69.60
  50101  50239:   62.40  50240:   67.17  50241:   68.75  50242:
69.38
  50101  50243:   73.32  50244:   65.40  50245:   67.32  50246:
70.13
  50101  50247:   70.06  50248:   75.30  50249:   82.13  50250:
74.63
  50101  50251:   72.21  50252:   77.73  50253:  111.35  50254:
57.30
  50101  50255:   93.56  50301:   28.51  50302:   20.87  50303:
17.63
  50101  50304:   33.09  50305:   38.04  50306:   20.65  50307:
20.38
  ...
```

Table B2 Inter-zone travel by public transport

```
c EMME/2 Module:   3.14(v9.02)  Date: 03-04-10 16:28   User:
E508/LTA.......psl
c Project:        2000 Model
t matrices
a matrix=mf11 MRTtt       0 MRT travel time (ivt+wait+walk)
  50101  50102:   19.77  50103:   16.93  50104:   26.12  50105:
22.65
  50101  50106:   20.20  50107:   34.10  50108:   26.70  50109:
31.22
  50101  50110:   22.74  50111:   28.64  50112:   28.79  50113:
30.20
  50101  50114:   19.28  50115:   21.10  50116:   21.77  50117:
24.58
  50101  50118:   19.87  50119:   21.82  50120:   24.22  50121:
14.13
  50101  50122:   17.60  50123:   21.87  50124:   17.73  50125:
21.58
  50101  50126:   32.67  50127:   18.13  50201:   73.95  50202:
68.81
  50101  50203:   63.31  50204:   67.74  50205:   73.49  50206:
78.57
  50101  50207:   64.64  50208:   73.18  50209:   69.64  50210:
70.09
  50101  50211:   66.92  50212:   66.52  50213:   67.72  50214:
79.18
  50101  50215:   79.98  50216:   97.68  50217:  111.20  50218:
79.49
  50101  50219:   67.89  50220:   73.48  50221:   70.69  50222:
72.05
  50101  50223:   66.69  50224:   97.68  50225:   65.04  50226:
68.92
  50101  50227:   70.29  50228:   70.29  50229:   68.57  50230:
71.24
  50101  50231:   75.04  50232:   77.64  50233:   70.51  50234:
71.77
  50101  50235:   71.92  50236:   69.72  50237:   62.78  50238:
74.39
  50101  50239:   66.12  50240:   73.25  50241:   74.95  50242:
79.62
  50101  50243:   78.93  50244:   70.22  50245:   72.62  50246:
74.23
  50101  50247:   78.52  50248:   78.78  50249:   81.76  50250:
78.12
  50101  50251:   76.02  50252:   77.36  50253:  114.35  50254:
76.09
  50101  50255:  100.80  50301:   28.23  50302:   20.81  50303:
17.59
  50101  50304:   31.34  50305:   37.92  50306:   20.60  50307:
20.33
...
```

Table B3   Inter-zone travel by car

```
c EMME/2 Module:  3.14(v9.02)  Date: 03-04-10 16:28   User:
E508/LTA.......psl
c Project:        2000 Model
t matrices
a matrix=mf68 cartt      0 car travel time
  50101  50102:    3.42  50103:    3.33  50104:    7.57  50105:
5.25
  50101  50106:    4.63  50107:    6.98  50108:    7.13  50109:
9.18
  50101  50110:    5.55  50111:    6.38  50112:    8.21  50113:
8.38
  50101  50114:    4.57  50115:    5.41  50116:    5.60  50117:
6.95
  50101  50118:    3.88  50119:    5.68  50120:    5.76  50121:
2.45
  50101  50122:    3.00  50123:    5.19  50124:    4.93  50125:
3.54
  50101  50126:    6.56  50127:    3.15  50201:   40.23  50202:
36.88
  50101  50203:   33.48  50204:   29.84  50205:   29.22  50206:
34.24
  50101  50207:   36.17  50208:   36.92  50209:   37.19  50210:
34.72
  50101  50211:   34.74  50212:   38.39  50213:   39.42  50214:
42.54
  50101  50215:   42.77  50216:   45.75  50217:   27.59  50218:
29.28
  50101  50219:   28.51  50220:   39.45  50221:   37.13  50222:
37.61
  50101  50223:   30.59  50224:   44.24  50225:   35.89  50226:
37.73
  50101  50227:   33.32  50228:   38.32  50229:   31.21  50230:
32.46
  50101  50231:   34.08  50232:   34.38  50233:   34.66  50234:
35.37
  50101  50235:   37.94  50236:   36.29  50237:   39.03  50238:
38.20
  50101  50239:   38.93  50240:   39.81  50241:   40.20  50242:
40.17
  50101  50243:   38.50  50244:   37.04  50245:   37.72  50246:
38.83
  50101  50247:   39.60  50248:   40.93  50249:   29.76  50250:
40.74
  50101  50251:   30.45  50252:   29.06  50253:   48.01  50254:
40.23
  50101  50255:   42.37  50301:    9.37  50302:    8.35  50303:
5.77
  50101  50304:   11.89  50305:   12.72  50306:    7.33  50307:
7.69
...
```



PLANNING DATA FOR YR 2000

LTA Zone	DGP	Employment
		1,869,700
50101	ANG MO KIO	13
50102	ANG MO KIO	2,699
50103	ANG MO KIO	5,230
50104	ANG MO KIO	3,318
50105	ANG MO KIO	2,511
50106	ANG MO KIO	8,886
50107	ANG MO KIO	-
50108	ANG MO KIO	471
50109	ANG MO KIO	4,749
50110	ANG MO KIO	313
50111	ANG MO KIO	4,235
50112	ANG MO KIO	2,274
50113	ANG MO KIO	780
50114	ANG MO KIO	4,883
50115	ANG MO KIO	168
50116	ANG MO KIO	223
50117	ANG MO KIO	4,231
50118	ANG MO KIO	174
50119	ANG MO KIO	3,301
50120	ANG MO KIO	275
50121	ANG MO KIO	675
50122	ANG MO KIO	238
50123	ANG MO KIO	1,484
50124	ANG MO KIO	1,016
50125	ANG MO KIO	609
50126	ANG MO KIO	2,203
50127	ANG MO KIO	-
50201	BEDOK	301
50202	BEDOK	832
50203	BEDOK	1,074
50204	BEDOK	1,571
50205	BEDOK	8,175
50206	BEDOK	845
50207	BEDOK	833
50208	BEDOK	631

Figure B1 Employment data of zones

Figure B2 Zone map and zone centres

APPENDIX C

TRAVEL TIME DATA VERIFICATION

Table C1 Verification results of inter-zone travel time by bus

ZoneID i	50602			50817		
ZoneID j	LTA	Calculated	Difference	LTA	Calculated	Difference
50113	61.8	52.6	9.1	40.5	45.6	-5.2
50232	64.7	57.2	7.5	61.6	66.6	-5.0
50602	0.0	0.0	0.0	49.6	41.9	7.8
50817	44.0	41.9	2.2	0.0	0.0	0.0
51824	73.7	77.7	-4.0	66.9	76.5	-9.6
54413	52.5	47.9	4.6	44.3	45.6	-1.4
54922	73.1	69.6	3.5	75.7	79.0	-3.3
55115	78.3	74.9	3.4	57.3	63.5	-6.2
55605	94.2	86.8	7.4	71.2	82.7	-11.5
55715	75.7	72.7	3.0	50.9	65.5	-14.6
ZoneID i	51824			54413		
ZoneID j	LTA	Calculated	Difference	LTA	Calculated	Difference
50113	85.1	95.2	-10.1	29.2	25.6	3.6
50232	108.1	112.3	-4.2	42.3	41.6	0.6
50602	81.4	77.7	3.7	50.4	47.9	2.5
50817	64.0	76.5	-12.5	51.5	45.6	5.8
51824	0.0	0.0	0.0	92.5	95.2	-2.7
54413	96.5	95.2	1.3	0.0	0.0	0.0
54922	123.4	124.7	-1.2	56.9	54.0	2.8
55115	38.9	45.5	-6.6	85.0	82.0	3.0
55605	80.2	79.8	0.3	80.2	79.4	0.8
55715	80.9	98.2	-17.3	46.9	56.3	-9.5
ZoneID i	54922			55115		
ZoneID j	LTA	Calculated	Difference	LTA	Calculated	Difference
50113	64.1	61.9	2.2	84.2	83.1	1.1
50232	23.1	25.6	-2.5	102.2	96.6	5.6
50602	75.2	69.6	5.6	87.6	74.9	12.7
50817	81.2	79.0	2.2	64.0	63.5	0.4
51824	118.9	124.7	-5.7	38.2	45.5	-7.3
54413	55.9	54.0	1.8	92.1	82.0	10.2
54922	0.0	0.0	0.0	124.0	109.0	15.0
55115	111.5	109.0	2.5	0.0	0.0	0.0
55605	117.8	117.3	0.5	60.0	52.2	7.8
55715	84.2	94.3	-10.2	65.1	70.5	-5.4
ZoneID i	55605			55715		
ZoneID j	LTA	Calculated	Difference	LTA	Calculated	Difference
50113	82.2	71.7	10.6	42.6	50.6	-8.0
50232	106.8	104.9	1.9	70.8	81.9	-11.1
50602	99.8	86.8	12.9	72.2	72.7	-0.5
50817	74.8	82.7	-8.0	65.0	65.5	-0.5
51824	81.9	79.8	2.0	82.9	98.2	-15.3
54413	83.2	79.4	3.8	48.6	56.3	-7.7
54922	118.0	117.3	0.7	81.1	94.3	-13.2
55115	59.5	52.2	7.3	66.0	70.5	-4.5
55605	0.0	0.0	0.0	48.2	48.8	-0.6
55715	52.2	48.8	3.4	0.0	0.0	0.0

Table C2 *Verification results of inter-zone travel time by public transit*

Zone i	50203			51307		
Zone j	LTA	Calculated	Difference	LTA	Calculated	Difference
50203	0	0	0	46	53.7	-7.7
51307	46.01	53.7	-7.69	0	0	0
51407	29.49	28.64	0.85	24.15	27.64	-3.49
55202	41.95	38.64	3.31	43.6	45.64	-2.04
55607	64.53	62.12	2.41	35.34	39.12	-3.78
Zone i	51407			55202		
Zone j	LTA	Calculated	Difference	LTA	Calculated	Difference
50203	29.47	28.64	0.83	41.85	38.64	3.21
51307	24.15	27.64	-3.49	43.57	45.64	-2.07
51407	0	0	0	27.05	20.58	6.47
55202	27.07	20.58	6.49	0	0	0
55607	48.55	44.06	4.49	33.92	32.06	1.86
Zone i	55607					
Zone j	LTA	Calculated	Difference			
50203	64.43	62.12	2.31			
51307	35.34	39.12	-3.78			
51407	47.9	44.06	3.84			
55202	33.92	32.06	1.86			
55607	0	0	0			

APPENDIX D

AVERAGE TRAVEL TIME ON MRT

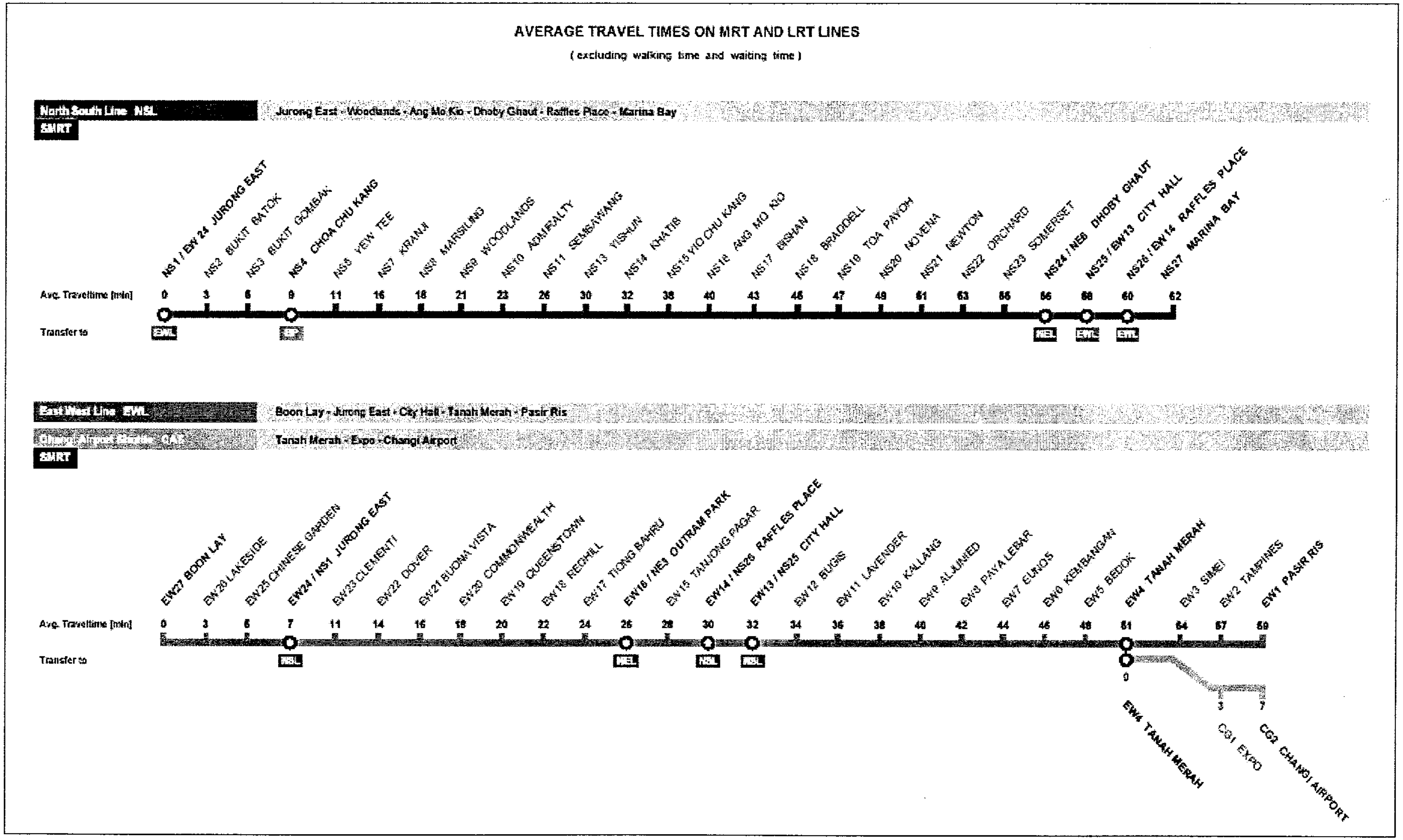


Figure D1 Average travel time on MRT

APPENDIX E

A MODIFIED VERSION OF DIJKSTRA'S ALGORITHM

Section 5.2.1 mentions that a modified version of Dijkstra's Algorithm to compute the k-shortest loopless paths. The algorithm is described below.

The original Dijkstra's Algorithm maintains a priority queue of active nodes based on their current path length to the source. The priority queue is initialised with only the source node **s** with path length 0. In each iteration, the node **n** with the least path length is taken out from the queue. Assume the path length to **n** be l_n. Then all the neighbouring nodes of **n** are examined. For a neighboring node **m**, let path length to **m** be l_m (infinity if the node has not been added to the queue) and the distance between **m** and **n** be d. There can be 3 cases for **m**: 1) **m** has already been de-queued. In this case l_m is less than l_n, so nothing is done; 2) **m** has not entered the queue. In this case, **m** is added to the queue with an initial path length $l_m = l_n + d$; and 3) **m** is already in the queue. In this case, if the current path length of **m** is larger than $l_n + d$, update the path length $l_m = l_n + d$, otherwise nothing is done. The algorithm terminates when the queue becomes empty. It is well known that the Dijkstra's Algorithm finds the shortest path to all the nodes that are connected to the source.

The modified version of Dijkstra's Algorithm works by assigning to each node **n** k dummies $\mathbf{n}_1,\ldots,\mathbf{n}_k$. Instead of the nodes themselves, the dummies are added to and deleted from the queue in the modified algorithm. As to be shown, the *i*th dummy is used to compute the *i*th shortest path. The priority queue is initialised with only the first dummy of the source node $\mathbf{s}_1$ with path length 0. In each iteration, a dummy $\mathbf{n}_i$ with the least path length l_{ni} is taken out from the queue. Then all the neighbouring nodes of **n** (the node $\mathbf{n}_i$ represents) are examined. For a neighbouring node **m**, let the path length to any dummy $\mathbf{m}_j$ be l_{mj} (infinity if the dummy has not been added to the queue) and the distance between **m** and **n** be d. First the path to **n** represented by $\mathbf{n}_i$ is scanned to make sure **m** has not already appeared in the path, otherwise the extended path to **m** contains a loop and should be discarded. Then there are 3 cases: 1) the number of dummies in queue or de-queued for node **m** is less than **k**. In this case, a new dummy $\mathbf{m}_j$ is added to the queue with path length $l_{ni} + d$; 2) the number of dummies in queue or de-queued is equal to **k** and $l_{ni} + d > l_{mj}$ for all dummies $\mathbf{m}_j$ that currently reside in the queue. In this case, nothing needs to be done; and 3) $l_{ni} + d < l_{mh}$ for some h, then a new dummy $\mathbf{m}_j$ is added to the queue with path length $l_{ni} + d$. At the same time, the dummy of **m** with the longest path length is discarded from the queue. The algorithm terminates when the queue becomes empty.

APPENDIX F

DEMAND MATRIX FOR NETWORKS N2, N3 AND N4

Table F1 *Demand matrix for network N2*

Bus stop																					
From\to	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**Total**
1	0	32	26	39	45	51	71	116	64	96	51	39	45	122	26	39	45	51	71	116	1145
2	40	0	10	15	16	20	28	45	25	38	20	15	18	48	10	15	16	20	28	45	472
3	46	14	0	17	20	23	31	51	29	43	23	17	20	54	12	17	20	23	31	51	542
4	51	16	13	0	23	26	35	58	32	48	26	19	23	61	13	19	23	26	35	58	605
5	57	18	14	21	0	29	39	64	36	54	29	21	25	68	14	21	25	29	39	64	667
6	51	16	13	19	23	0	35	58	32	48	26	19	23	61	13	19	23	26	35	58	598
7	57	18	14	21	25	29	0	64	36	54	29	21	25	68	14	21	25	29	39	64	653
8	80	25	20	30	35	40	55	0	50	75	40	30	35	95	20	30	35	40	55	90	880
9	29	9	7	11	12	13	20	32	0	27	14	11	13	34	7	11	12	13	20	32	327
10	63	20	16	24	28	31	43	71	38	0	31	24	28	75	16	24	28	31	43	71	705
11	46	14	11	17	20	23	31	52	29	43	0	17	20	54	11	17	20	23	31	52	531
12	46	14	11	18	20	23	32	51	29	43	22	0	20	54	11	18	20	23	32	51	538
13	40	13	10	15	18	20	28	45	25	38	20	16	0	47	10	15	18	20	28	45	471
14	91	29	23	34	40	46	63	103	57	85	46	34	40	0	23	34	40	46	63	103	1000
15	46	14	12	17	20	23	31	51	29	43	23	17	20	54	0	17	20	23	31	51	542
16	51	16	13	19	23	26	35	58	32	48	26	19	23	61	13	0	23	26	35	58	605
17	57	18	14	21	25	29	39	64	36	54	29	21	25	68	14	21	0	29	39	64	667
18	51	16	13	19	23	26	35	58	32	48	26	19	23	61	13	19	23	0	35	58	598
19	57	18	14	21	25	29	39	64	36	54	29	21	25	68	14	21	25	29	0	64	653
20	80	25	20	30	35	40	55	90	50	75	40	30	35	95	20	30	35	40	55	0	880
Total	1039	345	274	408	476	547	745	1195	697	1014	550	410	486	1248	274	408	476	547	745	1195	13079

Table F2 *Demand matrix for network N3*

Bus Stop **From\to**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**Total**
1	0	32	26	39	45	51	71	116	64	96	51	39	45	122	26	39	45	51	71	116	64	96	51	39	45	122	1562
2	40	0	10	15	16	20	28	45	25	38	20	15	18	48	10	15	16	20	28	45	25	38	20	15	18	48	636
3	46	14	0	17	20	23	31	51	29	43	23	17	20	54	12	17	20	23	31	51	29	43	23	17	20	54	728
4	51	16	13	0	23	26	35	58	32	48	26	19	23	61	13	19	23	26	35	58	32	48	26	19	23	61	814
5	57	18	14	21	0	29	39	64	36	54	29	21	25	68	14	21	25	29	39	64	36	54	29	21	25	68	900
6	51	16	13	19	23	0	35	58	32	48	26	19	23	61	13	19	23	26	35	58	32	48	26	19	23	61	807
7	57	18	14	21	25	29	0	64	36	54	29	21	25	68	14	21	25	29	39	64	36	54	29	21	25	68	886
8	80	25	20	30	35	40	55	0	50	75	40	30	35	95	20	30	35	40	55	90	50	75	40	30	35	95	1205
9	29	9	7	11	12	13	20	32	0	27	14	11	13	34	7	11	12	13	20	32	18	27	14	11	13	34	444
10	63	20	16	24	28	31	43	71	38	0	31	24	28	75	16	24	28	31	43	71	38	58	31	24	28	75	959
11	46	14	11	17	20	23	31	52	29	43	0	17	20	54	11	17	20	23	31	52	29	43	23	17	20	54	717
12	46	14	11	18	20	23	32	51	29	43	22	0	20	54	11	18	20	23	32	51	29	43	22	17	20	54	723
13	40	13	10	15	18	20	28	45	25	38	20	16	0	47	10	15	18	20	28	45	25	38	20	16	15	47	632
14	91	29	23	34	40	46	63	103	57	85	46	34	40	0	23	34	40	46	63	103	57	85	46	34	40	109	1371
15	46	14	12	17	20	23	31	51	29	43	23	17	20	54	0	17	20	23	31	51	29	43	23	17	20	54	728
16	51	16	13	19	23	26	35	58	32	48	26	19	23	61	13	0	23	26	35	58	32	48	26	19	23	61	814
17	57	18	14	21	25	29	39	64	36	54	29	21	25	68	14	21	0	29	39	64	36	54	29	21	25	68	900
18	51	16	13	19	23	26	35	58	32	48	26	19	23	61	13	19	23	0	35	58	32	48	26	19	23	61	807
19	57	18	14	21	25	29	39	64	36	54	29	21	25	68	14	21	25	29	0	64	36	54	29	21	25	68	886
20	80	25	20	30	35	40	55	90	50	75	40	30	35	95	20	30	35	40	55	0	50	75	40	30	35	95	1205
21	29	9	7	11	12	13	20	32	18	27	14	11	13	34	7	11	12	13	20	32	0	27	14	11	13	34	444
22	63	20	16	24	28	31	43	71	38	58	31	24	28	75	16	24	28	31	43	71	38	0	31	24	28	75	959
23	46	14	11	17	20	23	31	52	29	43	23	17	20	54	11	17	20	23	31	52	29	43	0	17	20	54	717
24	46	14	11	18	20	23	32	51	29	43	22	17	20	54	11	18	20	23	32	51	29	43	22	0	20	54	723
25	40	13	10	15	18	20	28	45	25	38	20	16	15	47	10	15	18	20	28	45	25	38	20	16	0	47	632
26	91	29	23	34	40	46	63	103	57	85	46	34	40	109	23	34	40	46	63	103	57	85	46	34	40	0	1371
Total	1354	444	352	527	614	703	962	1549	893	1308	706	529	622	1621	352	527	614	703	962	1549	893	1308	706	529	622	1621	22570

Table F3 *Demand matrix for network N4*

Bus Stop																				
From\to	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
1	0	32	26	39	45	51	71	116	64	96	51	39	45	122	26	39	45	51	71	116
2	40	0	10	15	16	20	28	45	25	38	20	15	18	48	10	15	16	20	28	45
3	46	14	0	17	20	23	31	51	29	43	23	17	20	54	12	17	20	23	31	51
4	51	16	13	0	23	26	35	58	32	48	26	19	23	61	13	19	23	26	35	58
5	57	18	14	21	0	29	39	64	36	54	29	21	25	68	14	21	25	29	39	64
6	51	16	13	19	23	0	35	58	32	48	26	19	23	61	13	19	23	26	35	58
7	57	18	14	21	25	29	0	64	36	54	29	21	25	68	14	21	25	29	39	64
8	80	25	20	30	35	40	55	0	50	75	40	30	35	95	20	30	35	40	55	90
9	29	9	7	11	12	13	20	32	0	27	14	11	13	34	7	11	12	13	20	32
10	63	20	16	24	28	31	43	71	38	0	31	24	28	75	16	24	28	31	43	71
11	46	14	11	17	20	23	31	52	29	43	0	17	20	54	11	17	20	23	31	52
12	46	14	11	18	20	23	32	51	29	43	22	0	20	54	11	18	20	23	32	51
13	40	13	10	15	18	20	28	45	25	38	20	16	0	47	10	15	18	20	28	45
14	91	29	23	34	40	46	63	103	57	85	46	34	40	0	23	34	40	46	63	103
15	46	14	12	17	20	23	31	51	29	43	23	17	20	54	0	17	20	23	31	51
16	51	16	13	19	23	26	35	58	32	48	26	19	23	61	13	0	23	26	35	58
17	57	18	14	21	25	29	39	64	36	54	29	21	25	68	14	21	0	29	39	64
18	51	16	13	19	23	26	35	58	32	48	26	19	23	61	13	19	23	0	35	58
19	57	18	14	21	25	29	39	64	36	54	29	21	25	68	14	21	25	29	0	64
20	80	25	20	30	35	40	55	90	50	75	40	30	35	95	20	30	35	40	55	0
21	29	9	7	11	12	13	20	32	18	27	14	11	13	34	7	11	12	13	20	32
22	63	20	16	24	28	31	43	71	38	58	31	24	28	75	16	24	28	31	43	71
23	46	14	11	17	20	23	31	52	29	43	23	17	20	54	11	17	20	23	31	52
24	46	14	11	18	20	23	32	51	29	43	22	17	20	54	11	18	20	23	32	51
25	40	13	10	15	18	20	28	45	25	38	20	16	15	47	10	15	18	20	28	45
26	91	29	23	34	40	46	63	103	57	85	46	34	40	109	23	34	40	46	63	103
27	51	16	13	19	23	26	35	58	32	48	26	19	23	61	13	19	23	26	35	58
28	57	18	14	21	25	29	39	64	36	54	29	21	25	68	14	21	25	29	39	64
29	80	25	20	30	35	40	55	90	50	75	40	30	35	95	20	30	35	40	55	90
30	29	9	7	11	12	13	20	32	18	27	14	11	13	34	7	11	12	13	20	32
31	63	20	16	24	28	31	43	71	38	58	31	24	28	75	16	24	28	31	43	71
32	46	14	11	17	20	23	31	52	29	43	23	17	20	54	11	17	20	23	31	52
Total	1680	546	433	649	757	865	1185	1916	1096	1613	869	651	766	2008	433	649	757	865	1185	1916

Table F3 *Demand matrix for network N4 (continued)*

Bus Stop													
From\to	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**Total**
1	64	96	51	39	45	122	51	71	116	64	96	51	2012
2	25	38	20	15	18	48	20	28	45	25	38	20	814
3	29	43	23	17	20	54	23	31	51	29	43	23	931
4	32	48	26	19	23	61	26	35	58	32	48	26	1043
5	36	54	29	21	25	68	29	39	64	36	54	29	1156
6	32	48	26	19	23	61	26	35	58	32	48	26	1038
7	36	54	29	21	25	68	29	39	64	36	54	29	1144
8	50	75	40	30	35	95	40	55	90	50	75	40	1563
9	18	27	14	11	13	34	13	20	32	18	27	14	577
10	38	58	31	24	28	75	31	43	71	38	58	31	1241
11	29	43	23	17	20	54	23	31	52	29	43	23	929
12	29	43	22	17	20	54	23	32	51	29	43	22	935
13	25	38	20	16	15	47	20	28	45	25	38	20	821
14	57	85	46	34	40	109	46	63	103	57	85	46	1785
15	29	43	23	17	20	54	23	31	51	29	43	23	943
16	32	48	26	19	23	61	26	35	58	32	48	26	1055
17	36	54	29	21	25	68	29	39	64	36	54	29	1168
18	32	48	26	19	23	61	26	35	58	32	48	26	1050
19	36	54	29	21	25	68	29	39	64	36	54	29	1156
20	50	75	40	30	35	95	40	55	90	50	75	40	1575
21	0	27	14	11	13	34	13	20	32	18	27	14	589
22	38	0	31	24	28	75	31	43	71	38	58	31	1253
23	29	43	0	17	20	54	23	31	52	29	43	23	941
24	29	43	22	0	20	54	23	32	51	29	43	22	947
25	25	38	20	16	0	47	20	28	45	25	38	20	833
26	57	85	46	34	40	0	46	63	103	57	85	46	1797
27	32	48	26	19	23	61	0	35	58	32	48	26	1059
28	36	54	29	21	25	68	29	0	64	36	54	29	1165
29	50	75	40	30	35	95	40	55	0	50	75	40	1584
30	18	27	14	11	13	34	13	20	32	0	27	14	598
31	38	58	31	24	28	75	31	43	71	38	0	31	1262
32	29	43	23	17	20	54	23	31	52	29	43	0	950
Total	1096	1613	869	651	766	2008	865	1185	1916	1096	1613	869	35914

APPENDIX G

LINKS OF NETWORKS N2, N3 AND N4 USED FOR CANDIDATE ROUTE GENERATION

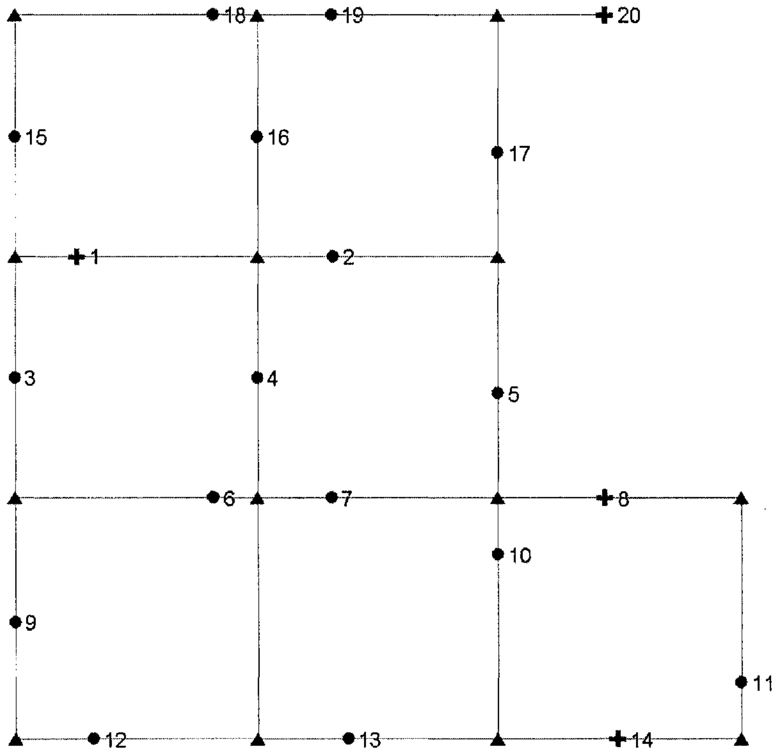


Figure G1 Links of network N2 for candidate route generation

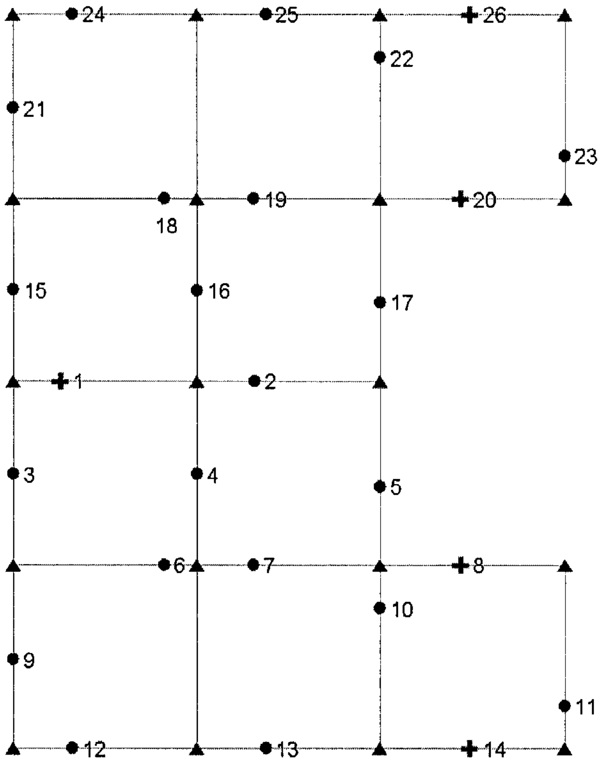


Figure G2   Links of network N3 for candidate route generation

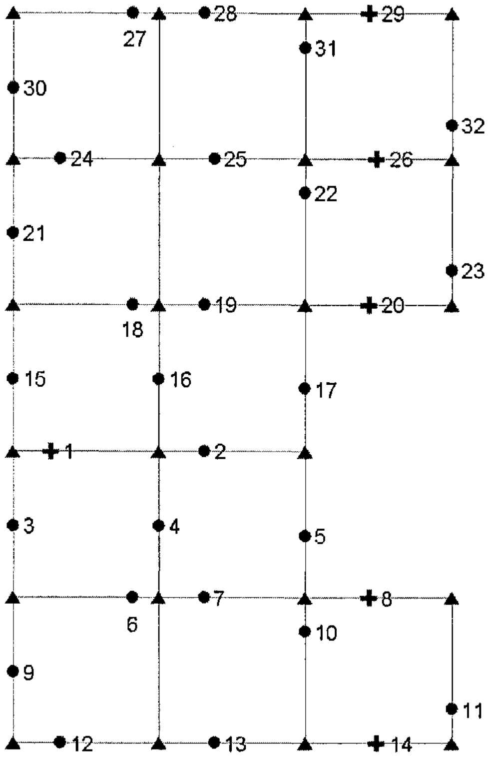


Figure G3 Links of network N4 for candidate route generation

APPENDIX H

ATTRIBUTES OF CANDIDATE ROUTES OF NETWROKS N2, N3 AND N4

Table H1 Candidate routes for network N2

ID	Bus stops travelled	Single trip distance (km)
1	1 2 5 8	1.91
2	1 4 7 8	1.91
3	1 4 13 10 8	3.11
4	1 3 9 12 13 10 8	3.42
5	1 4 13 14 11 8	3.78
6	1 3 9 12 13 14 11 8	4.09
7	1 16 18 15 3 6 13 10 8	5.51
8	1 2 17 19 18 15 3 6 7 8	5.51
9	1 16 18 15 3 9 12 7 8	5.51
10	1 16 18 15 3 9 12 13 10 8	5.51
11	1 16 18 15 3 6 13 14 11 8	6.18
12	1 16 18 15 3 9 12 13 14 11 8	6.18
13	1 4 12 9 3 15 18 19 17 5 8	6.71
14	1 2 17 19 18 15 3 6 13 10 8	6.71
15	1 2 17 19 18 15 3 9 12 13 10 8	6.71
16	1 2 17 19 18 15 3 9 12 7 8	6.71
17	1 2 17 19 18 15 3 9 12 13 14 11 8	7.38
18	1 16 18 15 3 9 12 7 10 14 11 8	7.38
19	1 2 17 19 18 15 3 6 13 14 11 8	7.38
20	1 2 17 19 18 15 3 6 7 10 14 11 8	7.38
21	1 4 12 9 3 15 18 19 17 5 10 14 11 8	8.58
22	1 2 17 19 18 15 3 9 12 7 10 14 11 8	8.58
23	1 4 13 14	2.54
24	1 4 7 10 14	2.54
25	1 3 9 12 13 14	2.85
26	1 4 7 8 11 14	3.15
27	1 2 5 8 11 14	3.15
28	1 2 5 7 13 14	3.74
29	1 4 13 10 8 11 14	4.35
30	1 3 9 12 13 10 8 11 14	4.66
31	1 16 18 15 3 9 12 13 14	4.94
32	1 16 18 15 3 6 13 14	4.94
33	1 2 17 19 18 15 3 6 13 14	6.14
34	1 16 18 15 3 9 12 7 10 14	6.14
35	1 2 17 19 18 15 3 9 12 13 14	6.14
36	1 2 17 19 18 15 3 6 7 10 14	6.14
37	1 16 18 15 3 9 12 7 8 11 14	6.75
38	1 16 18 15 3 6 13 10 8 11 14	6.75
39	1 2 17 19 18 15 3 6 7 8 11 14	6.75
40	1 16 18 15 3 9 12 13 10 8 11 14	6.75
41	1 2 17 19 18 15 3 9 12 7 10 14	7.34
…	…	…

Table H2 Candidate routes for network N3

ID	Bus stop travelled	Single trip distance (km)
1	1 2 5 8	1.91
2	1 4 7 8	1.91
3	1 4 13 10 8	3.11
4	1 3 9 12 13 10 8	3.42
5	1 4 13 14 11 8	3.78
6	1 3 9 12 13 14 11 8	4.09
7	1 16 25 22 17 5 8	4.31
8	1 15 21 24 25 22 17 5 8	4.62
9	1 16 25 26 23 20 17 5 8	5.51
10	1 16 18 15 3 9 12 13 10 8	5.51
11	1 16 24 21 15 3 6 7 8	5.51
12	1 16 18 15 3 6 13 10 8	5.51
13	1 15 21 24 25 22 19 16 4 7 8	5.82
14	1 15 21 24 25 22 19 16 2 5 8	5.82
15	1 15 21 24 25 22 17 2 4 7 8	5.82
16	1 16 25 22 17 5 10 14 11 8	6.18
17	1 16 24 21 15 3 9 12 7 8	6.71
18	1 2 17 22 25 24 21 15 3 6 7 8	6.71
19	1 2 17 19 24 21 15 3 6 7 8	6.71
20	1 2 17 22 25 18 15 3 6 7 8	6.71
21	1 2 17 19 18 15 3 9 12 13 10 8	6.71
22	1 16 24 21 15 3 6 13 10 8	6.71
23	1 16 24 21 15 3 9 12 13 10 8	6.71
24	1 4 12 9 3 15 18 19 17 5 8	6.71
25	1 16 25 26 23 20 17 5 10 14 11 8	7.38
26	1 16 24 21 15 3 9 12 13 14 11 8	7.38
27	1 16 24 21 15 3 6 13 14 11 8	7.38
28	1 16 25 22 17 5 7 13 14 11 8	7.38
29	1 15 21 24 25 22 19 16 4 7 10 14 11 8	7.69
30	1 15 21 24 25 22 19 16 4 13 14 11 8	7.69
31	1 15 21 24 25 22 17 2 4 13 14 11 8	7.69
32	1 15 21 24 25 22 17 5 7 13 14 11 8	7.69
33	1 15 21 24 25 22 19 16 2 5 10 14 11 8	7.69
34	1 15 21 24 25 22 17 2 4 7 10 14 11 8	7.69
35	1 4 12 9 3 15 18 25 22 17 5 8	7.91
36	1 2 17 19 24 21 15 3 9 12 13 10 8	7.91
37	1 4 12 9 3 15 21 24 25 22 17 5 8	7.91
38	1 2 17 20 23 26 25 18 15 3 6 7 8	7.91
39	1 2 17 22 25 24 21 15 3 9 12 7 8	7.91
40	1 2 17 20 23 26 25 24 21 15 3 6 7 8	7.91
41	1 2 17 22 25 24 21 15 3 9 12 13 10 8	7.91
…	…	…

Table H3 Candidate routes for network N4

ID	Bus stop travelled	Single trip distance (km)
1	1 2 5 8	1.91
2	1 4 7 8	1.91
3	1 4 13 10 8	3.11
4	1 3 9 12 13 10 8	3.42
5	1 4 13 14 11 8	3.78
6	1 3 9 12 13 14 11 8	4.09
7	1 16 25 22 17 5 8	4.31
8	1 15 21 24 25 22 17 5 8	4.62
9	1 16 28 31 22 17 5 8	5.51
10	1 16 25 26 23 20 17 5 8	5.51
11	1 16 18 15 3 6 13 10 8	5.51
12	1 16 18 15 3 9 12 13 10 8	5.51
13	1 15 21 24 25 22 19 16 4 7 8	5.82
14	1 15 21 24 25 22 19 16 2 5 8	5.82
15	1 15 21 24 25 22 17 2 4 7 8	5.82
16	1 16 25 22 17 5 10 14 11 8	6.18
17	1 16 28 29 32 23 20 17 5 8	6.71
18	1 2 17 22 25 18 15 3 6 7 8	6.71
19	1 2 17 22 25 24 21 15 3 6 7 8	6.71
20	1 16 25 31 29 32 23 20 17 5 8	6.71
21	1 16 28 31 26 23 20 17 5 8	6.71
22	1 4 12 9 3 15 18 19 17 5 8	6.71
23	1 16 24 21 15 3 9 12 13 10 8	6.71
24	1 16 28 29 32 26 22 17 5 8	6.71
25	1 2 17 19 24 21 15 3 6 7 8	6.71
26	1 16 27 30 21 15 3 6 7 8	6.71
27	1 16 24 30 27 28 31 22 17 5 8	6.71
28	1 2 17 19 18 15 3 9 12 13 10 8	6.71
29	1 2 17 19 18 15 3 6 13 10 8	6.71
30	1 16 25 26 23 20 17 5 10 14 11 8	7.38
31	1 16 28 31 22 17 5 10 14 11 8	7.38
32	1 16 24 21 15 3 9 12 13 14 11 8	7.38
33	1 15 21 24 25 22 19 16 4 7 10 14 11 8	7.69
34	1 15 21 24 25 22 17 2 4 13 14 11 8	7.69
35	1 15 21 24 25 22 19 16 4 13 14 11 8	7.69
36	1 15 21 24 25 22 17 2 4 7 10 14 11 8	7.69
37	1 15 21 24 25 22 17 5 7 13 14 11 8	7.69
38	1 15 21 24 25 22 19 16 2 5 10 14 11 8	7.69
39	1 16 24 30 27 28 31 26 23 20 17 5 8	7.91
...	....	...

APPENDIX I

SOLUTIONS FOR NETWORKS N3 AND N4

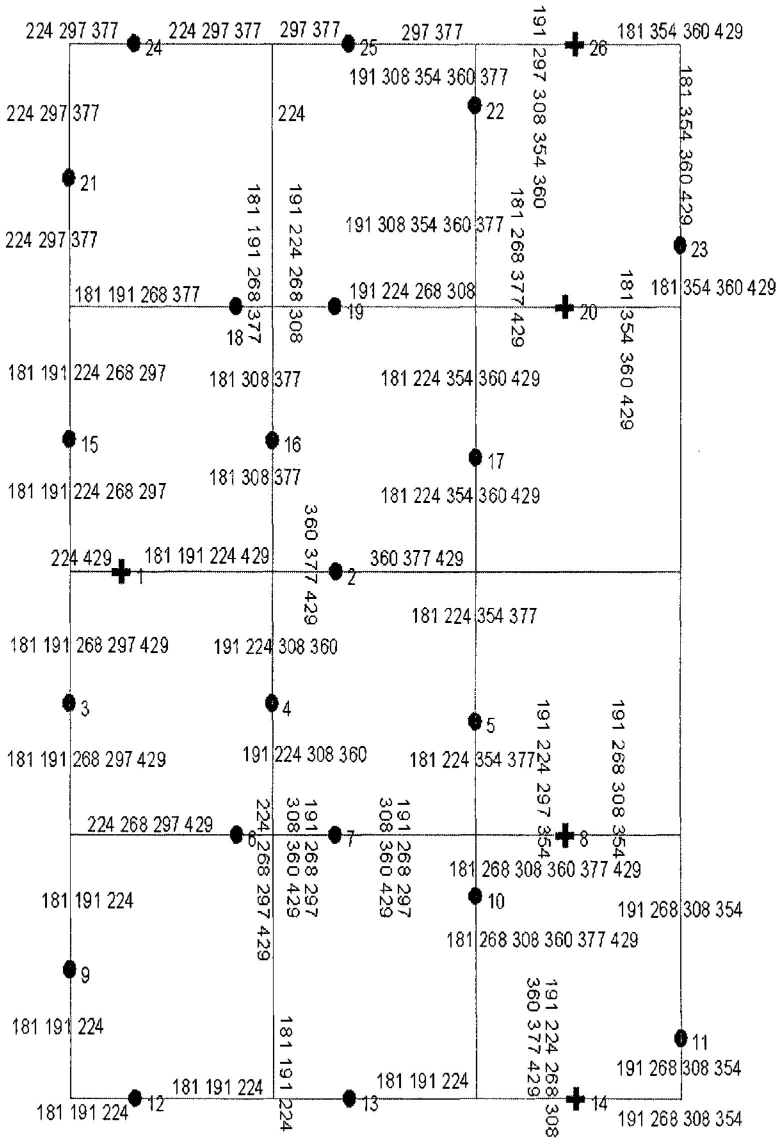


Figure I1 *Solution for network N3*

TableI1 Results for network N3

Crossover	Mutation	Objective function value	Travellers' cost	Operation cost	Solution size	Solution
Single-point	Uniform	37036	32336	4700	10	322, 234, 73, 197, 124, 299, 384, 429, 377,332
Single-point	Dynamic	36599	31419	5180	10	*308, 268, 377, 360, 297, 181, 224, 191, 429, 354
Two-point	Uniform	36799	31796	5003	9	442, 262, 14, 417, 116, 302, 354, 380, 25
Two-point	Dynamic	36834	31646	5189	10	373, 410, 136, 97, 348, 299, 409, 75, 362, 161
Uniform	Uniform	36627	30930	5696	8	426, 47, 93, 96, 133, 165, 377, 402
Uniform	Dynamic	36664	31445	5219	10	302, 328, 258, 357, 198, 413, 263, 170, 426, 418
Convex	Uniform	37070	31880	5190	9	289, 276, 171, 64, 176, 212, 147, 297, 211
Convex	Dynamic	37342	31623	5719	10	95, 81, 187, 96, 197, 373, 306, 285, 294, 258

Table I2 Features of the best solution for network N3

Route	Frequency	Length (km)	Max demand along the route	Demand
308	10.0	10.68	798	2704
268	5.3	12.00	422	1791
377	8.8	11.92	705	3169
360	5.7	10.86	455	1356
297	5.9	9.52	469	1662
181	10.3	15.90	827	3438
224	5.6	16.72	448	2655
191	7.2	17.08	580	3118
429	9.2	12.00	739	2753
354	10.5	9.68	840	2598
Total				25244

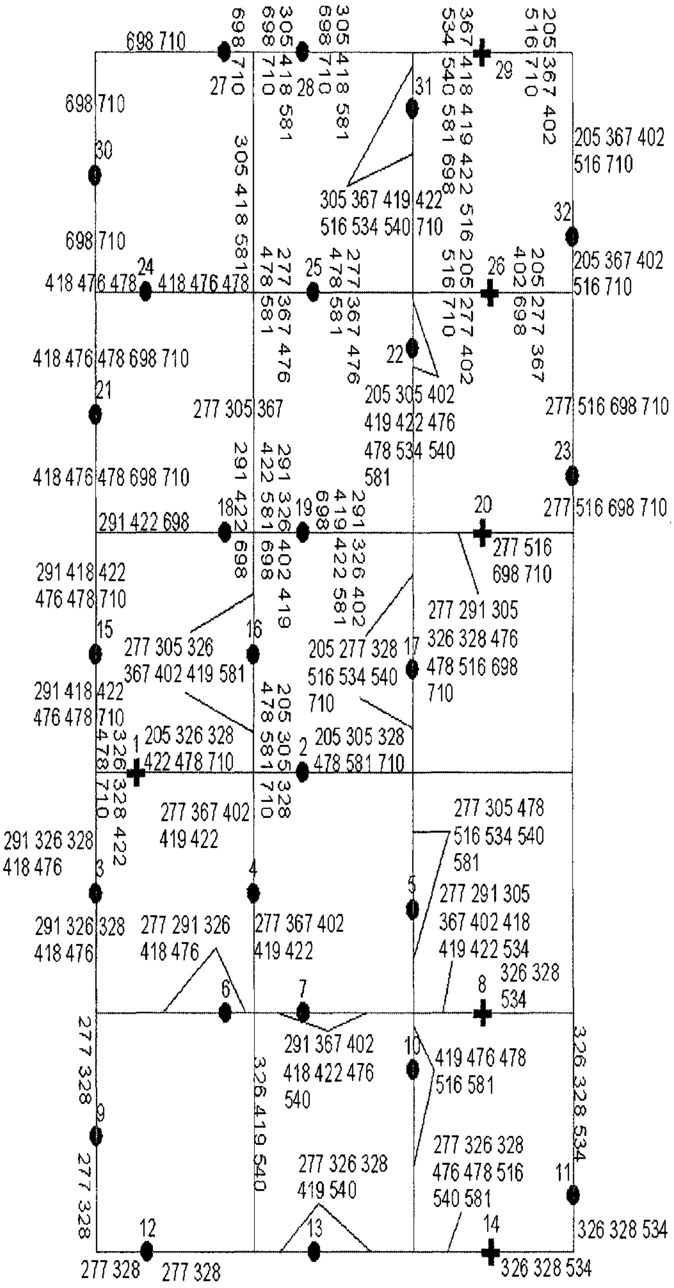


Figure I2 Solution for network N4

Table I3 Results for network N4

Crossover	Mutation	Objective function value	Travellers' cost	Operation cost	Solution size	Solution
Single-point	Uniform	61975	53211	8764	14	565, 271, 346, 138, 220, 111, 85, 297, 124, 472, 235, 655, 654, 421
Single-point	Dynamic	61886	52747	9138	16	575,365,117,485,328,30,40,118, 367,368,13,303,187,473,512,181
Two-point	Uniform	62058	52705	9353	14	409, 500, 526 ,241, 466, 249, 648, 686, 146, 314, 96, 712, 221, 231
Two-point	Dynamic	61698	52573	9125	19	*328, 540, 698, 305, 581, 367, 291, 402, 478, 326, 419, 422, 277, 418, 534, 476, 710, 516, 205
Uniform	Uniform	62089	52456	9632	13	500, 526, 241, 466, 249, 648, 686, 146, 314, 96, 712, 221, 231
Uniform	Dynamic	61834	53144	8690	15	702, 329, 372, 494, 79, 146, 623, 325, 558, 629, 85, 481, 522 ,427, 554
Convex	Uniform	62103	53447	8656	16	340, 376, 476, 164, 242, 474, 2, 553, 511, 718, 218, 655, 93, 654, 533, 713
Convex	Dynamic	63312	53820	9492	18	463, 198, 317, 543, 439, 614, 404, 453, 324, 240, 566, 576, 231, 448, 506, 12, 334, 125

Table I4 Features of the best solution for network N4

Route	Frequency	Length (km)	Max demand along the route	Demand
328	13.8	12.00	1108	3373
540	7.3	9.52	584	1733
698	8.4	10.74	669	2651
305	4.6	10.66	367	1573
581	8.4	11.92	671	2164
367	3.9	10.74	310	907
291	4.6	8.26	365	1123
402	8.6	9.60	689	2041
478	7.2	11.92	577	1877
326	5.0	12.00	396	1434
419	6.8	10.66	546	1851
422	7.2	10.66	575	2255
277	10.8	16.72	861	3813
418	8.1	10.66	650	1932
534	12.2	8.34	976	2704
476	7.1	11.92	564	2391
710	11.4	15.66	910	4187
516	11.8	10.78	948	2894
205	6.9	7.56	552	1413
Total				42315

APPENDIX J

CANDIDATE ROUTES OF MANDL'S NETWORK

Table J1 *Attributes of candidate routes for Mandl's network*

Route ID	Bus stops travelled	Single trip distance (m)
1	1 6 10 2	3900
2	1 6 4 2	4500
3	1 6 5 4 2	6600
4	1 6 10 2 8 3	6900
5	1 6 4 2 8 3	7500
6	1 6 10 2 15 7 3	7500
7	1 6 10 2 15 8 3	7800
8	1 6 10 2 8 15 7 3	7800
9	1 6 4 2 15 7 3	8100
10	1 6 4 2 8 15 7 3	8400
11	1 6 5 4 2 8 3	9600
12	1 6 5 4 2 15 7 3	10200
13	1 6 5 4 2 8 15 7 3	10500
14	1 6 4	3300
15	1 6 10 2 4	5100
16	1 6 5 4	5400
17	1 6 10 2 8 3 11 12 4	14400
18	1 6 10 2 15 7 3 11 12 4	15000
19	1 6 10 2 15 8 3 11 12 4	15300
20	1 6 10 2 8 15 7 3 11 12 4	15300
21	1 6 10 2 8 3 13 11 12 4	17400
22	1 6 10 2 8 3 14 13 11 12 4	17400
23	1 6 10 2 15 7 3 14 13 11 12 4	18000
24	1 6 10 2 8 15 7 3 14 13 11 12 4	18300
25	1 6 10 2 15 8 3 14 13 11 12 4	18300
26	1 6 5	4200
27	1 6 4 5	4500
28	1 6 10 2 4 5	6300
29	1 6 10 2 8 3 11 12 4 5	15600
30	1 6 10 2 15 7 3 11 12 4 5	16200
31	1 6 10 2 8 15 7 3 11 12 4 5	16500
32	1 6 10 2 15 8 3 11 12 4 5	16500
33	1 6 10 2 8 3 14 13 11 12 4 5	18600
34	1 6 10 2 8 3 13 11 12 4 5	18600
35	1 6 10 2 15 7 3 14 13 11 12 4 5	19200
36	1 6 10 2 15 8 3 14 13 11 12 4 5	19500
37	1 6 10 2 8 15 7 3 14 13 11 12 4 5	19500
38	1 6	2400
39	1 6 10 2 15 7	5400
40	1 6 10 2 8 15 7	5700
41	1 6 4 2 15 7	6000
...	......	...

APPENDIX K

DEMAND MATRIX FOR TAMPINES NETWORK

Table K1 Demand matrix of Tampines network

Bus Stop	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	0	0	0	0	0	0	0	48	16	1	1	1	3	2	5	11	3	1	16	5	34	63	20	12
2	0	0	0	0	0	0	0	4	1	0	0	0	0	0	0	1	0	0	1	0	3	5	2	1
3	0	0	0	0	0	0	0	58	19	1	1	1	3	3	6	13	4	1	19	6	42	76	24	14
4	0	0	0	0	0	0	0	27	9	0	1	1	2	1	3	6	2	1	9	3	20	36	12	7
5	0	0	0	0	0	0	0	68	22	1	2	1	4	3	6	15	4	2	22	7	48	88	28	17
6	0	0	0	0	0	0	0	64	21	1	2	1	4	3	6	14	4	2	21	7	46	83	27	16
7	0	0	0	0	0	0	0	41	13	1	1	1	2	2	4	9	3	1	13	4	29	53	17	10
8	2	0	7	2	6	7	4	0	4	0	0	0	1	1	1	3	1	0	4	1	8	15	5	3
9	16	4	54	16	46	50	29	90	0	0	0	0	0	0	0	1	0	0	1	0	2	4	1	1
10	9	2	30	9	26	28	16	50	1	0	0	0	0	0	0	0	0	0	1	0	1	2	1	0
11	9	2	32	9	27	30	17	53	1	0	0	0	0	0	0	0	0	0	1	0	1	2	1	0
12	7	2	23	7	20	21	12	39	0	0	0	0	0	0	0	0	0	0	0	0	1	2	0	0
13	15	3	53	15	46	49	29	89	1	0	0	0	0	0	0	1	0	0	1	0	2	4	1	1
14	12	3	42	12	36	39	23	70	1	0	0	0	0	0	0	0	0	0	1	0	2	3	1	1
15	17	4	57	17	49	53	31	95	1	0	0	0	0	0	0	1	0	0	1	0	2	4	1	1
16	8	2	29	8	25	26	15	48	0	0	0	0	0	0	0	0	0	0	0	0	1	2	1	0
17	18	4	64	18	54	59	34	106	1	0	0	0	0	0	0	1	0	0	1	0	2	4	1	1
18	6	1	19	6	17	18	10	32	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0
19	3	1	12	3	10	11	6	20	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
20	9	2	32	9	27	29	17	53	1	0	0	0	0	0	0	0	0	0	1	0	1	2	1	0
21	5	1	16	5	14	15	8	26	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
22	5	1	18	5	16	17	10	30	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
23	9	2	32	9	28	30	17	54	1	0	0	0	0	0	0	0	0	0	1	0	1	2	0	0
24	9	2	32	9	27	30	17	53	1	0	0	0	0	0	0	0	0	0	1	0	1	2	1	0
25	11	3	39	11	34	36	21	65	1	0	0	0	0	0	0	0	0	0	1	0	1	3	1	1
26	14	3	47	14	40	43	25	78	1	0	0	0	0	0	0	1	0	0	1	0	2	3	1	1
27	15	3	52	15	45	48	28	87	1	0	0	0	0	0	0	1	0	0	1	0	2	4	1	1
28	13	3	46	13	40	43	25	77	1	0	0	0	0	0	0	1	0	0	1	0	2	3	1	1

Table K1 Demand matrix of Tampines network (continued)

Bus Stop	***1***	***2***	***3***	***4***	***5***	***6***	***7***	***8***	***9***	***10***	***11***	***12***	***13***	***14***	***15***	***16***	***17***	***18***	***19***	***20***	***21***	***22***	***23***	***24***
29	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
30	9	2	32	9	27	29	17	53	1	0	0	0	0	0	0	0	0	0	1	0	1	2	1	0
31	14	3	50	14	43	46	27	83	1	0	0	0	0	0	0	1	0	0	1	0	2	3	1	1
32	10	2	36	10	31	33	19	60	1	0	0	0	0	0	0	0	0	0	1	0	1	2	1	0
33	15	3	53	15	45	49	28	88	1	0	0	0	0	0	0	1	0	0	1	0	2	4	1	1
34	4	1	13	4	11	12	7	21	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
35	6	1	22	6	19	20	12	36	0	0	0	0	0	0	0	0	0	0	0	0	1	2	0	0
36	17	4	57	17	49	53	31	95	1	0	0	0	0	0	0	1	0	0	1	0	2	4	1	1
37	19	4	67	19	57	62	36	111	1	0	0	0	0	0	0	1	0	0	1	0	2	4	1	1
38	15	3	51	15	43	47	27	84	1	0	0	0	0	0	0	1	0	0	1	0	2	3	1	1
39	11	3	39	11	34	36	21	65	1	0	0	0	0	0	0	0	0	0	1	0	1	3	1	1
40	11	2	38	11	32	35	20	62	1	0	0	0	0	0	0	0	0	0	1	0	1	3	1	0
41	14	3	49	14	42	45	26	81	1	0	0	0	0	0	0	1	0	0	1	0	2	3	1	1
42	16	3	54	16	46	49	29	89	1	0	0	0	0	0	0	1	0	0	1	0	2	4	1	1
43	12	3	41	12	35	38	22	69	1	0	0	0	0	0	0	0	0	0	1	0	2	3	1	1
44	13	3	45	13	38	41	24	74	1	0	0	0	0	0	0	1	0	0	1	0	2	3	1	1
45	16	4	56	16	48	52	30	93	1	0	0	0	0	0	0	1	0	0	1	0	2	4	1	1
46	12	3	40	12	34	37	22	67	1	0	0	0	0	0	0	0	0	0	1	0	1	3	1	1
47	13	3	47	13	40	43	25	78	1	0	0	0	0	0	0	1	0	0	1	0	2	3	1	1
48	9	2	33	9	28	30	17	54	1	0	0	0	0	0	0	0	0	0	1	0	1	2	1	0
49	12	3	43	12	37	40	23	72	1	0	0	0	0	0	0	0	0	0	1	0	2	3	1	1
50	20	4	68	20	58	63	36	113	1	0	0	0	0	0	0	1	0	0	1	0	2	5	1	1
51	15	3	51	15	43	47	27	84	1	0	0	0	0	0	0	1	0	0	1	0	2	3	1	1
52	14	3	50	14	43	46	27	83	1	0	0	0	0	0	0	1	0	0	1	0	2	3	1	1
53	18	4	64	18	54	59	34	106	1	0	0	0	0	0	0	1	0	0	1	0	2	4	1	1
54	18	4	61	18	52	56	32	101	1	0	0	0	0	0	0	1	0	0	1	0	2	4	1	1
55	14	3	50	14	43	46	27	83	1	0	0	0	0	0	0	1	0	0	1	0	2	3	1	1
56	17	4	60	17	51	56	32	100	1	0	0	0	0	0	0	1	0	0	1	0	2	4	1	1

Table K1 *Demand matrix of Tampines network (continued)*

Bus Stop	***1***	***2***	***3***	***4***	***5***	***6***	***7***	***8***	***9***	***10***	***11***	***12***	***13***	***14***	***15***	***16***	***17***	***18***	***19***	***20***	***21***	***22***	***23***	***24***
57	12	3	40	12	35	37	22	67	1	0	0	0	0	0	0	0	0	0	1	0	1	3	1	1
58	15	3	53	15	46	49	29	89	1	0	0	0	0	0	0	1	0	0	1	0	2	4	1	1
59	17	4	57	17	49	53	31	95	1	0	0	0	0	0	0	1	0	0	1	0	2	4	1	1
60	18	4	62	18	53	58	33	104	1	0	0	0	0	0	0	1	0	0	1	0	2	4	1	1
61	17	4	57	17	49	53	31	95	1	0	0	0	0	0	0	1	0	0	1	0	2	4	1	1
62	29	6	99	29	85	92	53	165	2	0	0	0	0	0	0	1	0	0	2	1	4	7	2	1
63	22	5	75	22	64	70	40	125	1	0	0	0	0	0	0	1	0	0	1	0	3	5	2	1
64	18	4	61	18	52	56	32	101	1	0	0	0	0	0	0	1	0	0	1	0	2	4	1	1
65	14	3	49	14	42	45	26	82	1	0	0	0	0	0	0	1	0	0	1	0	2	3	1	1
66	11	2	38	11	33	35	20	64	1	0	0	0	0	0	0	0	0	0	1	0	1	3	1	0
67	12	3	43	12	36	39	23	71	1	0	0	0	0	0	0	0	0	0	1	0	2	3	1	1
68	10	2	36	10	31	33	19	60	1	0	0	0	0	0	0	0	0	0	1	0	1	2	1	0
69	15	3	53	15	45	49	28	88	1	0	0	0	0	0	0	1	0	0	1	0	2	4	1	1
70	11	2	37	11	32	34	20	62	1	0	0	0	0	0	0	0	0	0	1	0	1	3	1	0
71	15	3	51	15	44	47	27	85	1	0	0	0	0	0	0	1	0	0	1	0	2	3	1	1
72	16	3	54	16	46	50	29	89	1	0	0	0	0	0	0	1	0	0	1	0	2	4	1	1
73	0	0	2	0	1	2	1	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
74	0	0	1	0	1	1	1	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
75	1	0	3	1	3	3	2	5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
76	3	1	12	3	10	11	6	20	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
77	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
78	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
79	10	2	34	10	29	31	18	57	1	0	0	0	0	0	0	0	0	0	1	0	1	2	1	0

Table K1 *Demand matrix of Tampines network (continued)*

Bus Stop	***25***	***26***	***27***	***28***	***29***	***30***	***31***	***32***	***33***	***34***	***35***	***36***	***37***	***38***	***39***	***40***	***41***	***42***	***43***	***44***	***45***	***46***	***47***	***48***
1	33	27	14	54	0	11	25	22	22	52	69	25	11	14	26	14	17	19	17	34	6	16	15	15
2	3	2	1	5	0	1	2	2	2	5	6	2	1	1	2	1	1	2	1	3	1	1	1	1
3	39	33	17	66	0	13	31	27	27	63	84	30	14	17	31	18	20	23	21	41	8	20	18	18
4	19	16	8	31	0	6	15	13	13	30	39	14	6	8	15	8	9	11	10	20	4	9	9	9
5	46	39	20	76	0	15	36	32	31	73	97	35	16	20	36	20	23	27	24	48	9	23	21	21
6	43	36	19	72	0	14	34	30	29	69	92	33	15	19	34	19	22	26	23	45	9	21	20	20
7	28	23	12	46	0	9	22	19	19	44	59	21	10	12	22	12	14	16	15	29	5	14	13	13
8	8	7	3	13	0	3	6	6	5	13	17	6	3	4	6	4	4	5	4	8	2	4	4	4
9	2	2	1	3	0	1	1	1	1	3	4	1	1	1	1	1	1	1	1	2	0	1	1	1
10	1	1	0	2	0	0	1	1	1	2	2	1	0	0	1	0	1	1	1	1	0	1	0	0
11	1	1	0	2	0	0	1	1	1	2	2	1	0	0	1	0	1	1	1	1	0	1	1	1
12	1	1	0	1	0	0	1	1	1	1	2	1	0	0	1	0	0	0	0	1	0	0	0	0
13	2	2	1	3	0	1	1	1	1	3	4	1	1	1	1	1	1	1	1	2	0	1	1	1
14	1	1	1	2	0	0	1	1	1	2	3	1	1	1	1	1	1	1	1	2	0	1	1	1
15	2	2	1	3	0	1	2	1	1	3	4	2	1	1	2	1	1	1	1	2	0	1	1	1
16	1	1	0	2	0	0	1	1	1	2	2	1	0	0	1	0	1	1	1	1	0	0	0	0
17	2	2	1	4	0	1	2	2	1	4	5	2	1	1	2	1	1	1	1	2	0	1	1	1
18	1	1	0	1	0	0	1	0	0	1	1	1	0	0	1	0	0	0	0	1	0	0	0	0
19	0	0	0	1	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
20	1	1	0	2	0	0	1	1	1	2	2	1	0	0	1	0	1	1	1	1	0	1	1	1
21	1	0	0	1	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	1	0	0	0	0
22	1	1	0	1	0	0	1	0	0	1	1	0	0	0	1	0	0	0	0	1	0	0	0	0
23	1	1	0	2	0	0	1	1	1	2	2	1	0	0	1	1	1	1	1	1	0	1	1	1
24	1	1	0	2	0	0	1	1	1	2	2	1	0	0	1	0	1	1	1	1	0	1	1	1
25	0	1	1	2	0	0	1	1	1	2	3	1	0	1	1	1	1	1	1	1	0	1	1	1
26	2	0	1	3	0	1	1	1	1	3	3	1	1	1	1	1	1	1	1	2	0	1	1	1
27	2	2	0	3	0	1	1	1	1	3	4	1	1	1	1	1	1	1	1	2	0	1	1	1
28	2	1	1	0	0	1	1	1	1	3	4	1	1	1	1	1	1	1	1	2	0	1	1	1

Table K1 Demand matrix of Tampines network (continued)

Bus Stop	*25*	*26*	*27*	*28*	*29*	*30*	*31*	*32*	*33*	*34*	*35*	*36*	*37*	*38*	*39*	*40*	*41*	*42*	*43*	*44*	*45*	*46*	*47*	*48*
29	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
30	1	1	0	2	0	0	1	1	1	2	2	1	0	0	1	0	1	1	1	1	0	1	1	1
31	2	1	1	3	0	1	0	1	1	3	4	1	1	1	1	1	1	1	1	2	0	1	1	1
32	1	1	1	2	0	0	1	0	1	2	3	1	0	1	1	1	1	1	1	1	0	1	1	1
33	2	2	1	3	0	1	1	1	0	3	4	1	1	1	1	1	1	1	1	2	0	1	1	1
34	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
35	1	1	0	1	0	0	1	1	1	1	0	1	0	0	1	0	0	0	0	1	0	0	0	0
36	2	2	1	3	0	1	2	1	1	3	4	0	1	1	2	1	1	1	1	2	0	1	1	1
37	2	2	1	4	0	1	2	2	2	4	5	2	0	1	2	1	1	1	1	2	0	1	1	1
38	2	1	1	3	0	1	1	1	1	3	4	1	1	0	1	1	1	1	1	2	0	1	1	1
39	1	1	1	2	0	0	1	1	1	2	3	1	0	1	0	1	1	1	1	1	0	1	1	1
40	1	1	1	2	0	0	1	1	1	2	3	1	0	1	1	0	1	1	1	1	0	1	1	1
41	2	1	1	3	0	1	1	1	1	3	4	1	1	1	1	1	0	1	1	2	0	1	1	1
42	2	2	1	3	0	1	1	1	1	3	4	1	1	1	1	1	1	0	1	2	0	1	1	1
43	1	1	1	2	0	0	1	1	1	2	3	1	1	1	1	1	1	1	0	2	0	1	1	1
44	2	1	1	3	0	1	1	1	1	3	3	1	1	1	1	1	1	1	1	0	0	1	1	1
45	2	2	1	3	0	1	2	1	1	3	4	1	1	1	2	1	1	1	1	2	0	1	1	1
46	1	1	1	2	0	0	1	1	1	2	3	1	0	1	1	1	1	1	1	1	0	0	1	1
47	2	1	1	3	0	1	1	1	1	3	3	1	1	1	1	1	1	1	1	2	0	1	0	1
48	1	1	0	2	0	0	1	1	1	2	2	1	0	0	1	1	1	1	1	1	0	1	1	0
49	1	1	1	2	0	0	1	1	1	2	3	1	1	1	1	1	1	1	1	2	0	1	1	1
50	2	2	1	4	0	1	2	2	2	4	5	2	1	1	2	1	1	1	1	2	0	1	1	1
51	2	1	1	3	0	1	1	1	1	3	4	1	1	1	1	1	1	1	1	2	0	1	1	1
52	2	1	1	3	0	1	1	1	1	3	4	1	1	1	1	1	1	1	1	2	0	1	1	1
53	2	2	1	4	0	1	2	2	1	4	5	2	1	1	2	1	1	1	1	2	0	1	1	1
54	2	2	1	4	0	1	2	1	1	3	4	2	1	1	2	1	1	1	1	2	0	1	1	1
55	2	1	1	3	0	1	1	1	1	3	4	1	1	1	1	1	1	1	1	2	0	1	1	1
56	2	2	1	4	0	1	2	1	1	3	4	2	1	1	2	1	1	1	1	2	0	1	1	1

Table K1 *Demand matrix of Tampines network (continued)*

Bus Stop	***25***	***26***	***27***	***28***	***29***	***30***	***31***	***32***	***33***	***34***	***35***	***36***	***37***	***38***	***39***	***40***	***41***	***42***	***43***	***44***	***45***	***46***	***47***	***48***
57	1	1	1	2	0	0	1	1	1	2	3	1	0	1	1	1	1	1	1	1	0	1	1	1
58	2	2	1	3	0	1	1	1	1	3	4	1	1	1	1	1	1	1	1	2	0	1	1	1
59	2	2	1	3	0	1	2	1	1	3	4	2	1	1	2	1	1	1	1	2	0	1	1	1
60	2	2	1	4	0	1	2	1	1	3	5	2	1	1	2	1	1	1	1	2	0	1	1	1
61	2	2	1	3	0	1	2	1	1	3	4	2	1	1	2	1	1	1	1	2	0	1	1	1
62	3	3	2	6	0	1	3	2	2	6	7	3	1	2	3	2	2	2	2	4	1	2	2	2
63	3	2	1	4	0	1	2	2	2	4	6	2	1	1	2	1	1	2	1	3	1	1	1	1
64	2	2	1	4	0	1	2	1	1	3	4	2	1	1	2	1	1	1	1	2	0	1	1	1
65	2	1	1	3	0	1	1	1	1	3	4	1	1	1	1	1	1	1	1	2	0	1	1	1
66	1	1	1	2	0	0	1	1	1	2	3	1	0	1	1	1	1	1	1	1	0	1	1	1
67	1	1	1	2	0	0	1	1	1	2	3	1	1	1	1	1	1	1	1	2	0	1	1	1
68	1	1	1	2	0	0	1	1	1	2	3	1	0	1	1	1	1	1	1	1	0	1	1	1
69	2	2	1	3	0	1	1	1	1	3	4	1	1	1	1	1	1	1	1	2	0	1	1	1
70	1	1	1	2	0	0	1	1	1	2	3	1	0	1	1	1	1	1	1	1	0	1	1	1
71	2	1	1	3	0	1	1	1	1	3	4	1	1	1	1	1	1	1	1	2	0	1	1	1
72	2	2	1	3	0	1	1	1	1	3	4	1	1	1	1	1	1	1	1	2	0	1	1	1
73	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
74	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
75	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
76	0	0	0	1	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
77	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
78	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
79	1	1	1	2	0	0	1	1	1	2	3	1	0	1	1	1	1	1	1	1	0	1	1	1

Table K1 *Demand matrix of Tampines network (continued)*

Bus Stop	***49***	***50***	***51***	***52***	***53***	***54***	***55***	***56***	***57***	***58***	***59***	***60***	***61***	***62***	***63***	***64***	***65***	***66***	***67***	***68***	***69***	***70***	***71***	***72***
1	7	7	11	2	10	6	7	19	10	10	4	7	13	12	23	9	6	5	8	3	2	22	2	28
2	1	1	1	0	1	1	1	2	1	1	0	1	1	1	2	1	1	0	1	0	0	2	0	2
3	8	8	13	2	12	7	8	23	12	13	5	9	16	14	27	11	7	6	9	3	2	26	3	34
4	4	4	6	1	6	3	4	11	5	6	2	4	7	7	13	5	3	3	4	2	1	12	1	16
5	10	10	15	3	14	8	9	27	13	15	5	10	18	16	32	12	8	7	11	4	3	30	3	40
6	9	9	15	3	13	8	9	26	13	14	5	9	17	15	30	12	8	6	10	4	2	29	3	38
7	6	6	9	2	9	5	6	16	8	9	3	6	11	10	19	7	5	4	6	2	2	18	2	24
8	2	2	3	0	2	1	2	5	2	3	1	2	3	3	6	2	1	1	2	1	0	5	1	7
9	0	0	1	0	1	0	0	1	1	1	0	0	1	1	1	1	0	0	0	0	0	1	0	2
10	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	1
11	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	1
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	1
13	0	0	1	0	1	0	0	1	1	1	0	0	1	1	1	0	0	0	0	0	0	1	0	2
14	0	0	0	0	0	0	0	1	0	0	0	0	1	1	1	0	0	0	0	0	0	1	0	1
15	0	0	1	0	1	0	0	1	1	1	0	0	1	1	1	1	0	0	0	0	0	1	0	2
16	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	1
17	0	0	1	0	1	0	0	1	1	1	0	0	1	1	2	1	0	0	1	0	0	1	0	2
18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
20	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	1
21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
23	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	1
24	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	1
25	0	0	0	0	0	0	0	1	0	0	0	0	1	0	1	0	0	0	0	0	0	1	0	1
26	0	0	1	0	1	0	0	1	0	1	0	0	1	1	1	0	0	0	0	0	0	1	0	1
27	0	0	1	0	1	0	0	1	1	1	0	0	1	1	1	0	0	0	0	0	0	1	0	2
28	0	0	1	0	1	0	0	1	0	1	0	0	1	1	1	0	0	0	0	0	0	1	0	1

Table K1 Demand matrix of Tampines network (continued)

Bus Stop	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72
29	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
30	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	1
31	0	0	1	0	1	0	0	1	1	1	0	0	1	1	1	0	0	0	0	0	0	1	0	2
32	0	0	0	0	0	0	0	1	0	0	0	0	1	0	1	0	0	0	0	0	0	1	0	1
33	0	0	1	0	1	0	0	1	1	1	0	0	1	1	1	0	0	0	0	0	0	1	0	2
34	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
35	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	1
36	0	0	1	0	1	0	0	1	1	1	0	0	1	1	1	1	0	0	0	0	0	1	0	2
37	0	0	1	0	1	0	0	1	1	1	0	1	1	1	2	1	0	0	1	0	0	2	0	2
38	0	0	1	0	1	0	0	1	1	1	0	0	1	1	1	0	0	0	0	0	0	1	0	2
39	0	0	0	0	0	0	0	1	0	0	0	0	1	0	1	0	0	0	0	0	0	1	0	1
40	0	0	0	0	0	0	0	1	0	0	0	0	1	0	1	0	0	0	0	0	0	1	0	1
41	0	0	1	0	1	0	0	1	0	1	0	0	1	1	1	0	0	0	0	0	0	1	0	1
42	0	0	1	0	1	0	0	1	1	1	0	0	1	1	1	1	0	0	0	0	0	1	0	2
43	0	0	0	0	0	0	0	1	0	0	0	0	1	1	1	0	0	0	0	0	0	1	0	1
44	0	0	1	0	0	0	0	1	0	1	0	0	1	1	1	0	0	0	0	0	0	1	0	1
45	0	0	1	0	1	0	0	1	1	1	0	0	1	1	1	1	0	0	0	0	0	1	0	2
46	0	0	0	0	0	0	0	1	0	0	0	0	1	1	1	0	0	0	0	0	0	1	0	1
47	0	0	1	0	1	0	0	1	0	1	0	0	1	1	1	0	0	0	0	0	0	1	0	1
48	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	1
49	0	0	1	0	0	0	0	1	0	0	0	0	1	1	1	0	0	0	0	0	0	1	0	1
50	0	0	1	0	1	0	0	1	1	1	0	1	1	1	2	1	0	0	1	0	0	2	0	2
51	0	0	0	0	1	0	0	1	1	1	0	0	1	1	1	0	0	0	0	0	0	1	0	2
52	0	0	1	0	1	0	0	1	1	1	0	0	1	1	1	0	0	0	0	0	0	1	0	2
53	0	0	1	0	0	0	0	1	1	1	0	0	1	1	2	1	0	0	1	0	0	1	0	2
54	0	0	1	0	1	0	0	1	1	1	0	0	1	1	1	1	0	0	0	0	0	1	0	2
55	0	0	1	0	1	0	0	1	1	1	0	0	1	1	1	0	0	0	0	0	0	1	0	2
56	0	0	1	0	1	0	0	0	1	1	0	0	1	1	1	1	0	0	0	0	0	1	0	2

Table K1 *Demand matrix of Tampines network (continued)*

Bus Stop	***49***	***50***	***51***	***52***	***53***	***54***	***55***	***56***	***57***	***58***	***59***	***60***	***61***	***62***	***63***	***64***	***65***	***66***	***67***	***68***	***69***	***70***	***71***	***72***
57	0	0	0	0	0	0	0	1	0	0	0	0	1	1	1	0	0	0	0	0	0	1	0	1
58	0	0	1	0	1	0	0	1	1	0	0	0	1	1	1	0	0	0	0	0	0	1	0	2
59	0	0	1	0	1	0	0	1	1	1	0	0	1	1	1	1	0	0	0	0	0	1	0	2
60	0	0	1	0	1	0	0	1	1	1	0	0	1	1	2	1	0	0	1	0	0	1	0	2
61	0	0	1	0	1	0	0	1	1	1	0	0	0	1	1	1	0	0	0	0	0	1	0	2
62	1	1	1	0	1	1	1	2	1	1	0	1	1	0	2	1	1	1	1	0	0	2	0	3
63	1	1	1	0	1	0	1	2	1	1	0	1	1	1	0	1	0	0	1	0	0	2	0	2
64	0	0	1	0	1	0	0	1	1	1	0	0	1	1	1	0	0	0	0	0	0	1	0	2
65	0	0	1	0	1	0	0	1	1	1	0	0	1	1	1	0	0	0	0	0	0	1	0	1
66	0	0	0	0	0	0	0	1	0	0	0	0	1	0	1	0	0	0	0	0	0	1	0	1
67	0	0	0	0	0	0	0	1	0	0	0	0	1	1	1	0	0	0	0	0	0	1	0	1
68	0	0	0	0	0	0	0	1	0	0	0	0	1	0	1	0	0	0	0	0	0	1	0	1
69	0	0	1	0	1	0	0	1	1	1	0	0	1	1	1	0	0	0	0	0	0	1	0	2
70	0	0	0	0	0	0	0	1	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	1
71	0	0	1	0	1	0	0	1	1	1	0	0	1	1	1	0	0	0	0	0	0	1	0	2
72	0	0	1	0	1	0	0	1	1	1	0	0	1	1	1	1	0	0	0	0	0	1	0	0
73	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
74	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
75	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
76	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
77	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
78	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
79	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	1

Table K1 *Demand matrix of Tampines network (continued)*

Bus Stop	***73***	***74***	***75***	***76***	***77***	***78***	***79***
1	12	65	120	117	131	103	3
2	1	6	10	10	11	9	0
3	14	78	146	142	159	124	4
4	7	37	69	67	75	59	2
5	16	91	169	165	185	145	5
6	16	86	160	156	175	137	4
7	10	55	102	100	112	87	3
8	3	16	30	29	32	25	1
9	1	4	7	7	8	6	0
10	0	2	4	4	4	3	0
11	0	2	4	4	4	4	0
12	0	2	3	3	3	3	0
13	1	4	7	7	7	6	0
14	1	3	5	5	6	5	0
15	1	4	7	7	8	6	0
16	0	2	4	4	4	3	0
17	1	4	8	8	9	7	0
18	0	1	2	2	3	2	0
19	0	1	2	1	2	1	0
20	0	2	4	4	4	3	0
21	0	1	2	2	2	2	0
22	0	1	2	2	3	2	0
23	0	2	4	4	5	4	0
24	0	2	4	4	5	4	0
25	0	3	5	5	6	4	0
26	1	3	6	6	7	5	0
27	1	4	7	7	7	6	0
28	1	3	6	6	7	5	0

Table K1 Demand matrix of Tampines network (continued)

Bus Stop	73	74	75	76	77	78	79
29	0	0	0	0	0	0	0
30	0	2	4	4	4	3	0
31	1	3	6	6	7	6	0
32	0	3	5	5	5	4	0
33	1	4	7	7	7	6	0
34	0	1	2	2	2	1	0
35	0	2	3	3	3	2	0
36	1	4	7	7	8	6	0
37	1	5	9	8	9	7	0
38	1	4	7	6	7	6	0
39	0	3	5	5	6	4	0
40	0	3	5	5	5	4	0
41	1	3	6	6	7	5	0
42	1	4	7	7	8	6	0
43	1	3	5	5	6	5	0
44	1	3	6	6	6	5	0
45	1	4	7	7	8	6	0
46	1	3	5	5	6	4	0
47	1	3	6	6	7	5	0
48	0	2	4	4	5	4	0
49	1	3	6	5	6	5	0
50	1	5	9	8	10	7	0
51	1	4	7	6	7	6	0
52	1	3	6	6	7	5	0
53	1	4	8	8	9	7	0
54	1	4	8	8	8	7	0
55	1	3	6	6	7	5	0
56	1	4	8	8	9	7	0

Table K1 *Demand matrix of Tampines network (continued)*

Bus Stop	*73*	*74*	*75*	*76*	*77*	*78*	*79*
57	1	3	5	5	6	4	0
58	1	4	7	7	8	6	0
59	1	4	7	7	8	6	0
60	1	4	8	8	9	7	0
61	1	4	7	7	8	6	0
62	1	7	13	12	14	11	0
63	1	5	10	10	11	8	0
64	1	4	8	8	9	7	0
65	1	3	6	6	7	5	0
66	0	3	5	5	5	4	0
67	1	3	5	5	6	5	0
68	0	2	5	5	5	4	0
69	1	4	7	7	7	6	0
70	0	3	5	5	5	4	0
71	1	4	7	6	7	6	0
72	1	4	7	7	8	6	0
73	0	0	0	0	0	0	0
74	0	0	0	0	0	0	0
75	0	0	0	0	0	0	0
76	0	1	2	0	2	1	0
77	0	0	0	0	0	0	0
78	0	0	0	0	0	0	0
79	0	2	4	4	5	4	0

APPENDIX L

EXISTING BUS ROUTES IN TAMPINES AREA

Table L1 Existing bus routes in Tampines area

No.	*Distance*	*No. of stop*	*Bus stop sequence*	*Fleet Size*
3	3973.19	12	8 34 35 39 43 47 46 49 50 57 58 3	3
3	3810.56	11	3 58 57 45 46 47 43 39 35 34 8	3
8	5921.81	15	8 40 41 51 52 50 49 70 36 30 23 22 21 73 4	6
8	5921.81	15	4 73 21 22 23 30 36 70 49 50 52 51 41 40 8	6
10	1377.11	4	8 34 33 6	2
10	1377.11	4	6 33 34 8	2
18	5921.81	15	8 40 41 51 52 50 49 70 36 30 23 22 21 73 4	5
18	5921.81	15	4 73 21 22 23 30 36 70 49 50 52 51 41 40 8	5
19	2309.28	8	8 40 41 51 52 57 58 3	3
19	2309.28	8	3 58 57 52 51 41 40 8	3
22	2228.53	6	8 32 72 31 29 5	3
22	2228.53	6	5 29 31 72 32 8	3
23	3440.22	9	8 34 33 30 23 22 21 73 4	6
23	3440.22	9	4 73 21 22 23 30 33 34 8	6
28	7683.08	18	8 40 42 63 62 60 69 59 9 67 53 50 49 70 36 30 29 5	5
28	7036.55	18	5 29 30 36 70 49 50 53 56 9 59 69 60 62 63 42 40 8	4
29	3275.05	10	8 40 42 63 62 61 66 57 58 3	3
29	3275.05	10	3 58 57 66 61 62 63 42 40 8	3
31	1377.11	4	8 34 33 6	4
31	1377.11	4	6 33 34 8	2
37	2309.28	8	8 40 41 51 52 57 58 3	3
37	2309.28	8	3 58 57 52 51 41 40 8	3
38	2259.05	7	8 40 41 51 52 50 7	2
38	2259.05	7	7 50 52 51 41 40 8	2
39	3725.17	10	8 34 33 36 70 49 50 57 58 3	2
39	3725.17	10	3 58 57 50 49 70 36 33 34 8	2
65	2324.06	6	8 34 33 30 29 5	3
65	2324.06	6	5 29 30 33 34 8	3
67	2324.06	6	8 34 33 30 29 5	3
67	2324.06	6	5 29 30 33 34 8	3
68	5879.52	14	8 32 72 31 74 75 76 77 78 74 31 72 32 8	1
69	4109.05	11	8 16 15 14 18 19 21 22 23 29 5	4
69	4109.05	11	5 29 23 22 21 19 18 14 15 16 8	4
72	1954.97	6	8 16 12 11 10 1	5
72	1954.97	6	1 10 11 12 16 8	3
81	2309.28	8	8 40 41 51 52 57 58 3	3
81	2309.28	8	3 58 57 52 51 41 40 8	2
291	5123.72	14	8 16 15 17 27 26 25 24 20 79 17 15 16 8	7
291A	5551.47	16	8 40 43 44 45 53 56 67 55 54 53 45 44 43 40 8	8
292	5638.41	14	8 34 33 36 70 48 68 49 70 38 37 33 34 8	4
293	6595.12	19	8 34 32 72 28 26 20 18 14 13 71 18 20 26 28 72 32 34 8	8
293A	5890.44	16	8 40 42 63 62 60 69 59 57 66 65 64 63 42 40 8	8
168	4918.03	10	5 29 30 33 35 39 40 11 10 1	3
168	4918.03	10	1 10 11 40 39 35 33 30 29 5	3

518	5032.91	13	4 73 21 22 23 30 36 70 49 50 57 58 3	4
518	5032.91	13	3 58 57 50 49 70 36 30 23 22 21 73 4	4
27	3971.08	8	6 33 35 39 40 11 10 1	6
27	3971.08	8	1 10 11 40 39 35 33 6	8
9	1370.15	5	7 50 57 58 3	15
9	1370.15	5	3 58 57 50 7	15
12	1370.15	5	7 50 57 58 3	2
12	1370.15	5	3 58 57 50 7	2
17	2969.80	8	6 36 70 49 50 57 58 3	2
17	2969.80	8	3 58 57 50 49 70 36 6	2
21	4591.18	12	5 29 30 33 35 39 41 51 52 57 58 3	3
21	4591.18	12	3 58 57 52 51 41 39 35 33 30 29 5	3
15	5375.49	11	2 40 39 35 33 30 23 22 21 73 4	4
15	5375.49	11	4 73 21 22 23 30 33 35 39 40 2	4
Total				240

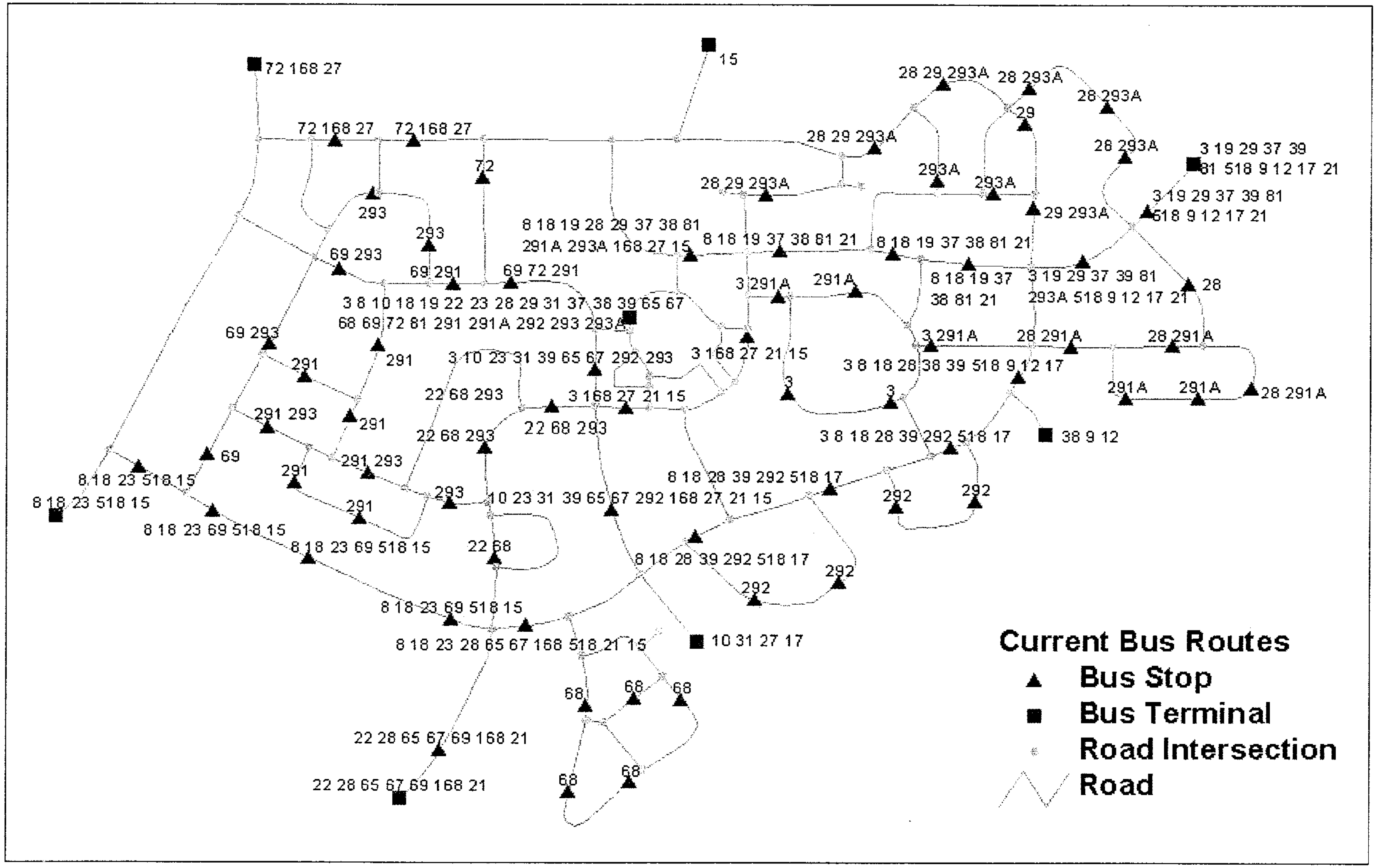


Figure L1 *Map of Current Bus Routes in Tampines*

APPENDIX M

CANDIDATE ROUTES OF TAMPINES NETWORK

Table M1 *Candidate routes of Tampines network*

Route ID	Bus stops travelled	Single trip distance (m)
1	1 10 11 2	2094.9
2	1 71 11 2	2789.3
3	1 10 11 2	2973.8
4	1 14 15 12 2	3011.1
5	1 14 13 11 2	3335.8
6	1 14 15 16 3 2	3611.3
7	1 10 13 15 16 3 2	3669.6
8	1 14 15 16 3	2124.1
9	1 10 13 15 16 3	2182.5
10	1 10 11 3	2408.3
11	1 14 15 16 34 35 3	2759.1
12	1 10 13 15 16 34 35 3	2817.5
13	1 18 79 17 15 16 3	3043.1
14	1 10 13 15 16 3	3061.4
15	1 18 19 73 4	2415.7
16	1 18 19 73 4	2498.7
17	1 10 71 18 19 73 4	2793.1
18	1 14 17 79 19 73 4	3093.2
19	1 14 17 27 20 19 73 4	3125.7
20	1 18 79 27 20 19 73 4	3185.4
21	1 14 17 27 26 28 31 29 5	3492.3
22	1 18 79 27 26 28 31 29 5	3552.0
23	1 18 20 26 28 31 29 5	3573.7
24	1 18 79 27 26 28 31 29 5	3635.0
25	1 18 20 26 28 31 29 5	3656.7
26	1 18 19 21 22 23 29 5	3677.6
27	1 18 19 21 22 23 29 5	3760.6
28	1 14 15 16 34 33 6	3168.2
29	1 10 13 15 16 34 33 6	3226.6
30	1 14 15 16 35 33 6	3503.2
31	1 10 71 14 15 16 34 33 6	3545.6
32	1 10 13 15 16 35 33 6	3561.6
33	1 14 15 16 36 6	3616.1
34	1 10 13 15 16 36 6	3674.5
35	1 10 11 40 43 44 45 50 7	3943.6
36	1 10 11 40 41 51 52 50 7	3976.7
37	1 10 11 43 44 45 50 7	4078.3
38	1 10 11 40 43 47 46 49 7	4197.2
39	1 14 15 16 3 43 44 45 50 7	4228.6
40	1 10 11 63 64 65 66 50 7	4249.8
41	1 10 13 15 16 70 49 7	4306.0
	......	...