Initial Stiffness of Reinforced Concrete Columns with Moderate Aspect Ratios

Cao Thanh Ngoc Tran[1] and Bing Li[2,*]

[1]*Department of Civil Engineering, International University, Vietnam National University, Ho Chi Minh City, Vietnam*
[2]*School of Civil and Environment Engineering, Nanyang Technological University, Singapore 639798*
**Corresponding author. Email address: cbli@ntu.edu.sg; Tel: +65-6790-5292. Associate Editor: J.G. Dai.*

Abstract

The estimation of the initial stiffness of columns subjected to seismic loadings has long been a matter of considerable uncertainty. This paper reports a study that is devoted to addressing this uncertainty by developing a rational method to determine the initial stiffness of RC columns when subjected to seismic loads. A comprehensive parametric study based on a proposed method is initially carried out to investigate the influences of several critical parameters. A simple equation is then proposed to estimate the initial stiffness of RC columns. The applicability and accuracy of the proposed method and equation are then verified with the experimental data obtained from literature studies.

Keywords:reinforced concrete, column initial stiffness, stiffness ratio.

1. Introduction

In recent years, earthquake design philosophy has shifted from a traditional force-based approach toward a displacement-based ideology. The assumed initial stiffness of reinforced concrete (RC) columns could affect the estimation of the displacement and displacement ductility, which are crucial in displacement-based design. In addition, the assumed initial stiffness properties of columns also affect the estimation of the fundamental period and distribution of internal forces of structures. Therefore, an accurate evaluation of the initial stiffness of columns becomes an inevitable requirement.

Literature reviews show that there is a considerable amount of uncertainty regarding the estimation of the initial stiffness of columns when subjected to seismic loads. Current design codes often employ a stiffness reduction factor to deal with this uncertainty. In an attempt to address these uncertainties, the study presented within this paper is devoted to developing a rational method to determine the initial stiffness of RC columns when subjected to seismic loads. A comprehensive parametric study based on the proposedmethod was carried out to investigate the influences of several critical parameters. A simple equation to estimate the initial stiffness of RC columns is also proposed within this paper. The applicability and accuracy of the proposed method and equation are then verified with the experimental data obtained from the literature.

2. Defining Initial Stiffness of RC Columns

There are two methods as illustrated in Figure 1(a) that are commonly utilized to determine the initial stiffness of RC columns (K_i). In the first method, the initial stiffness of RC columns are estimated by using the secant of the shear force versus lateral displacement relationship passing through the point at which the applied force reaches 75% of the flexural strength (0.75 V_u). In the second method, the column is loaded until either the first yield occurs in the longitudinal reinforcement or the maximum compressive strain of concrete reaches 0.002 at a critical section of the column. This corresponds to point A in Figure 1(a). Generally, the two approaches give similar values. In this study, the later approach was adopted.

However, the above mentioned definition cannot be used for columns whose shear strengths do not substantially exceed its theoretical yield force. For these columns, defined as those whose maximum measured shear force was less than 107% of the theoretical yield force, the effective stiffness was defined based on a point on the measured force-displacement envelope with a shear force equal to 0.8 V_{max} as illustrated in Figure 1(b) (Elwood *et al.* 2009).

Assuming the column is fixed against rotation at both ends and has a linear variation in curvature over the height of the column, the measured effective moment of inertia can be determined as:

$$I_e = \frac{L^3 K_i}{12 E_c} \quad (1)$$

The stiffness ratio (κ) is defined as follows:

$$\kappa = \frac{I_e}{I_g} \times 100\% \quad (2)$$

where I_g is the moment of inertia of the gross section; K_i is the initial stiffness of columns and L is the height of columns and E_c is the elastic modulus of concrete.

3. Review of Existing Initial Stiffness Models

3.1.ACI 318-08 (2008)

ACI 318-08 (2008) recommends the following options for estimating member stiffness for the determination of lateral deflection of building systems subjected to factored lateral loads: (a) 0.35 EI_g for members with anaxial load ratio of less than 0.10 and 0.70 EI_g for members with an axial load ratio of more than or equal to 0.10; or (b) 0.50 EI_g for all members.

3.2. FEMA 356 (2000)

FEMA 356 (2000) suggests the variation of effective stiffness values with the applied axial load ratio. The effective stiffness is taken as 0.50 EI_g for members with an axial load ratio of less than 0.30, while a value of 0.7 EI_g is adopted for members with an axial load ratio of more than 0.50. This value varies linearly for intermediate axial load ratios as illustrated in Figure 2.

3.3. ASCE 41 (2007)

As shown in Figure 2, ASCE 41 (2007) recommends that the effective stiffness is taken as 0.30 EI_g for memberswith an axial load ratio of less than 0.10, as 0.7 EI_g for members with an axial load ratio of more than 0.50 and varies linearly for intermediate axial load ratios.

3.4. Paulay and Priestley (1992)

According to Paulay and Priestley's recommendation (1992), the effective stiffness is taken as 0.40 EI_g for members with an axial load ratio of less than −0.05, as 0.8 EI_g for members with an axial load ratio of more than 0.50 and varies linearly for intermediate axial load ratios as illustrated in Figure 2.

3.5. Elwood and Eberhard (2009)

Elwood and Eberhard (2009) recommend the following equation for estimating the initial stiffness of reinforced concrete columns subjected to seismic loading:

$$k = \frac{0.45 + 2.5P / A_g f_c'}{1 + 110\left(\frac{d_b}{h}\right)\left(\frac{h}{a}\right)} \leq 1 \quad \text{and} \quad \geq 0.2 \qquad (3)$$

where d_b is the diameter of longitudinal reinforcing bars; a is the shear span and h is the column depth; A_g is the gross sectional area of columns and f_c' is the compressive strength of concrete.

Figure 2 illustrates the variation of stiffness ratio based on Elwood and Eberhard's model (2009) versusthe axial load ratio for specimens with d_b and a equal to 25 mm and 850 mm

4. Experimental Investigation on Initial Stiffness of RC Columns

In this section, the experimental results obtained from testing of six RC columns conducted by Tran *et al.* (2009) are briefly discussed with respect to the initial stiffness of the test specimens. Four column axial loads of 0.05, 0.20, 0.35, 0.50 $f_c' A_g$ and two aspect ratios of 1.71 and 2.43 were investigated in this experimental program. Table 1 summarizes all the details of the test specimens. It is to be noted that only a brief summary of important test features that are relevant to this study are presented within this paper. Detailed information has been documented in another publication (Tran *et al.* 2009).

The relationships between initial stiffness and the column axial load ratio obtained from all the test specimens are tabulated in Table 2. The initial stiffness of SC-1.7 Series specimens enhanced by around 9.8%, 17.6%, and 40.4% as the column axial load was increased from 0.05 to 0.20, 0.35, and 0.50 $f_c^{'} A_g$, respectively. An analogous trend was observed in the specimens of RC-1.7 Series, whose initial stiffness experienced an enhancement of around 33.9%, 64.3% and 86.1% with an increase in the column axial load from 0.05 to 0.20, 0.35 and 0.50 $f_c^{'} A_g$, respectively. As compared to Specimen SC-2.4-0.20, Specimen SC-2.4-0.50 experienced anincrease in the initial stiffness of 20.2%. The aforementioned discussion clearly indicated that column axial load was beneficial to the initial stiffness of test specimens.

The initial stiffness of Specimens SC-2.4-0.20, SC-1.7- 0.20, SC-2.4-0.50 and SC-1.7-0.50 obtained from the tests were 12.9 kN/mm, 26.9 kN/mm, 15.5 kN/mm and 34.4 kN/mm respectively. The increase in the initial stiffness when comparing between Specimens SC-1.7-0.20 and SC-2.4-0.20 was 108.5%. Similarly, an enhancement in the initial stiffenss of 121.9% was observed in Specimen SC-1.7-0.50 as compared to Specimen SC-2.4-0.50.

The initial stiffness of test columns calculated based on ACI 318-2008 (2008), FEMA 356 (2000), ASCE 41 (2007), Paulay and Priestley (1992), and Elwood and Eberhard (2009) are also all tabulated in Table 2. All these models tend to overestimate the initial stiffness of the test columns. Amongst all of these existing models, Elwood and Eberhard (2009) provides the best mean ratio of the experimental to predicted initial stiffness. However none of these models are accurate.

5. Proposed Method

5.1.Yield Force (V_y)

The initial stiffness of columns is determined by applying the second method as described in the previous section. The yield force (V_y) corresponding to point A in Figure 1(a) is obtained from the yield moment (M_y) when the reinforcing bar closest to the tension edge of columns has reached its yield strain. Moment-curvature analysis is adopted to determine this moment.

5.2. Displacement at Yield Force ($\Delta' y$)

The displacement of a column at yield force (V_y) can be considered as the sum of the displacement due to flexure, bar slip and shear.

$$\Delta'_y = \Delta'_{flex} + \Delta'_{shear} \quad (4)$$

where Δ_y' is the displacement of a column at yield force; Δ_{flex}' is the displacement due to flexure and bar slip at yield force; and Δ_{shear}' is the displacement due to shear at yield force

5.2.1. Flexure deformations (Δ_{flex}')

In this proposed method, the simplified concept of an effective length of the member suggested by Priestley *et al.* (1996) was used to account for the displacement due to bar slip in flexure deformations. Assuming a linear variation in curvature over the height of the column, the contribution of flexural deformations and bar slips to the displacement at the yield force for RC columns with a fixed condition at both ends can be estimated as follows:

$$\Delta'_{flex} = \frac{\phi_y'\left(L + 2L_{sp}\right)^2}{6} \tag{5}$$

where ϕ_y' is the curvature at the yield force determined by using moment-curvature analysis and L is the clear height of columns.

The strain penetration length (L_{sp}) is given by:

$$L_{sp} = 0.022 f_{yl} d_b \tag{6}$$

where f_{yl} is the yield strength of longitudinal reinforcing bars; and d_b is the diameter of longitudinal reinforcing bars.

5.2.2. Shear deformations (Δ_{shear}')

The idea of utilizing the truss analogy to model cracked RC elements has been around for many years. The truss analogy is a discrete modeling of actual stress fields within RC members. The complex stress fields within structural components resulting from applied external forces are simplified into discrete compressive and tensile load paths. The analogy utilizes the general idea of concrete in compression and steel reinforcement in tension. The longitudinal reinforcement in a beam or column represents the tensile chord of a truss while the concrete in the flexural compression zone is considered as part of the longitudinal compressive chord. The transverse reinforcement serves as ties holding the longitudinal chords together. The diagonal concrete compression struts, which discretely simulate the concrete compressive stress field, are connected to the ties and longitudinal chords at rigid nodes to attain static equilibrium within the truss. The truss analogy is a very promising way to treat shear because it provides a visible representation of how forces are transferred in a RC members under an applied shear force.

Park and Paulay (1975) derived a method to determine the shear stiffness by applying the truss analogy for short or deep rectangular beams of unit length. The shear stiffness is the magnitude of the shear force, when applied to a beam of unit length that will cause unit shear displacement at one end of the beam relative to the other. This model is reliable in estimating shear deformations of short or deep beams in which the influences of flexure are negligible. The behaviors of RC columns under seismic loading are much more complex because of the interaction between shear and flexure. The influences of axial strain due to flexure in estimating shear deformations of RC columns should be considered to accurately predict the initial stiffness of RC columns. By applying a method that is similar to Park and Paulay's analogous truss model (1975), theshear stiffness of RC columns is derived in this part of the paper. The effects of flexure in shear deformations are incorporated in the proposed model through the axial strains at the center of columns ($\varepsilon_{y,CL}$).

Assuming that transverse reinforcing bars start resisting the applied shear force when the shear cracking starts occurring, the stress in transverse reinforcing bars at the yield force is calculated as:

$$f_{sy} = \frac{\left(V_y - V_{cr}\right)s}{A_{st} d \tan\theta} \tag{7}$$

where d is the distance from the extreme compression fiber to centroid of tension reinforcement; s is the spacing of transverse reinforcement; A_{st} is the total transverse steel area within spacing s; and θ is the angle of diagonal compression strut. Hence the strain in transverse reinforcing bars is:

$$\varepsilon_x = \frac{f_{sy}}{E_s} \le \varepsilon_{yt} \tag{8}$$

Where ε_{yt} is the yield strain of transverse reinforcing bars; E_s is the elastic modulus of steel.

Similar to Park and Paulay' s model (1975), the concrete compression stress at the yield force is given as:

$$f_2 = \frac{V_y}{bL_{cs}\cos\theta} \tag{9}$$

where b is the width of columns; $L_{cs} = d\, sin\theta$ is the effective depth of the diagonal strut as shown in Figure 3.

Hence the strain in the concrete compression strut is given as:

$$\varepsilon_2 = \frac{f_2}{E_c} \tag{10}$$

where E_c is the elastic modulus of concrete given as:

$$E_c = 5000\sqrt{f_c} \tag{11}$$

Based on Vecchio and Collins's model (1986), the effective compressive strength of concrete is calculated as follows:

$$f_{ce} = \frac{f_c^{'}}{0.8 + 170\varepsilon_1} \le f_c^{'} \tag{12}$$

By applying Mohr's circle transformation for the mean strains at the center of Section C-C as shown in Figure 4, it gives:

$$\varepsilon_1 = \frac{\varepsilon_x + \varepsilon_{y,CL}}{2} + \sqrt{\left(\frac{\varepsilon_x - \varepsilon_{y,CL}}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} \tag{13}$$

$$\varepsilon_2 = \frac{\varepsilon_x + \varepsilon_{y,CL}}{2} - \sqrt{\left(\frac{\varepsilon_x - \varepsilon_{y,CL}}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} \tag{14}$$

$$\tan 2\theta = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_{y,CL}} \tag{15}$$

For the axial mean strains, compatibility requires that the plain sections remain plane. Hence the mean strain at the center of section C-C is given as:

$$\varepsilon_{y,CL} = \frac{\varepsilon_{y,top} + \varepsilon_{y,bot}}{2} \tag{16}$$

where $\varepsilon_{y,top}$,$\varepsilon_{y,bot}$ are the axial strains at the extreme tension and compression fibers, respectively as shown in Figure 4(b).

There are six variables, namely ε_x, $\varepsilon_{y,CL}$, γ_{xy}, ε_1, ε_2 and θ; and six independent Eqns 8, 10, 13, 14, 15 and 16. By solving these six independent equations, the shear strain (γ_{xy}) at the center of section C-C could be determined.

The column is divided into several segments along its height of the column to determine the total shear deformation at the top of the column. The mean axial strain at the center of the section is determined based onthe moment-curvature analysis. The shear strains at the lower and upper section of the segment are calculated using the above equations. Hence, the total shear displacement caused by the yield force can be calculated as follows:

$$\Delta'_{shear} = \sum_{i=1}^{n} \left(\frac{\gamma_{xy}^{i} + \gamma_{xy}^{i+1}}{2}\right) h_i \tag{17}$$

where γ_{xy}^{i} and γ_{xy}^{i+1} are the shear strains at the lower and upper section of the segment *i*; h_i is the height of segment *i* and *n* is the number of segments.

5.3. Initial Stiffness

Once the flexural and shear deformations at the top of columns under yield force are obtained, the initial stiffness of columns can be determined as:

$$K_i = \frac{V_y}{\Delta'_{flex} + \Delta'_{shear}} \tag{18}$$

6. Validation of the Proposed Method

The proposed method is validated by comparing its results to the initial stiffness of six columns obtained from the experimental study previously conducted by Tran *et al.* (2009).

It was found that the average ratio of experimental to predicted initial stiffness by the proposed methodwas 0.735 as tabulated in Table 2. It shows a relatively good correlation between the analytical and experimental results. The initial stiffness of the tested columns calculated based on ACI 318-2008 (2008), FEMA 356 (2000), ASCE 41 (2007), Paulay and Priestley (1992), and Elwood and Eberhard (2009) are also tabulated in Table 2. The mean ratio of the experimental to predicted initial stiffness and its coefficient of variation were 0.242 and 0.060, 0.301 and 0.076, 0.262 and 0.054, 0.312 and 0.084, 0.232 and 0.046, and 0.588 and 0.104 for ACI 318-2008 (2008a), ACI 318-2008 (2008b), FEMA 356 (2000), ASCE 41 (2007), Paulay and Priestley (1992), and Elwood and Eberhard (2009) respectively. Comparison of available models with experimental data indicated that the proposed method produced a better mean ratio of the experimental to predicted initial stiffness than other models. The proposed method may be suitable as an assessment tool to calculate the initial stiffness of RC columns.

7. Parametric Studies

A parametric study conducted to improve the understanding of the effects of various parameters on the initial stiffness of RC columns is presented within this section. The parameters investigated are transverse reinforcement ratios (ρ_v), longitudinal reinforcement ratios (ρ_l), yield strength of longitudinal reinforcing bars (f_{yl}), concrete compressive strength (f_c'), aspect ratio (*a/d*) and axial load ratio ($P/f_c' A_g$). In the parametric study, the effects of the parameters that were investigated on the initial stiffness of RC columns are presented by the dimensionless stiffness ratio (*k*).

Specimen SC-2.4-0.20 with an aspect ratio of 2.4 isconsidered as the reference specimen in the parametricstudy. An axial load of 0.2 was applied to the specimen. The concrete compressive strength of the specimen (f_c') at 28 days was 25.0 MPa. The longitudinal reinforcement consisted of 8-T20 (20 mm diameter). This resulted in the ratio of longitudinal steel area to the gross area of column to be 2.05%. The transverse reinforcement consisted of R6 bars (6 mm diameter) with 135° bent spaced at 125 mm, corresponding to a transverse reinforcement ratio of 0.129%.

7.1.Influence of Transverse Reinforcement Ratio

The analyses as illustrated in Figure 5 were conducted to assess the influence of transverse reinforcement on effective moment of inertia. Two column axial loads of 0.05 $f_c^{'} A_g$ and 0.20 $f_c^{'} A_g$ were considered. Five types of transverse reinforcement, R6-125 mm, R8-125 mm, R8- 100 mm, R10-125 mm and R10-100 which correspond to five transverse reinforcement ratios ρ_v of 0.129%, 0.230%, 0.287%, 0.359% and 0.449% respectively, were investigated.

Figure 5 shows that with an increase in transverse reinforcement content from 0.129% to 0.230%, 0.287%, 0.359% and 0.449%, stiffness ratios rose slightly by approximately 3.4%, 4.5%, 5.5%, 6.4%, respectively for columns under an axial load of 0.20 $f_c^{'} A_g$. The stiffness ratios increased by approximately 2.3%, 3.6%, 4.9%, 6.1% for columns under an axial load of 0.05 $f_c^{'} A_g$ with an increase in transverse reinforcement content from 0.129% to 0.230%, 0.287%, 0.359% and 0.449%, respectively. This suggested that the effect of transverse reinforcement ratios on stiffness ratios is insignificant. In addition, Figure 5 shows a clear indication that stiffness ratio increases with an increase in column axial load.

7.2.Influence of Longitudinal Reinforcement Ratio

The influence of longitudinal reinforcement ratios on stiffness ratios is presented in Figure 6 for two different column axial loads of 0.05 $f_c^{'} A_g$ and 0.20 $f_c^{'} A_g$. Four types of longitudinal reinforcement, 8T16, 8T20, 8T22 and 8T25 corresponding to longitudinal reinforcement ratios ρ_l of 1.66%, 2.05%, 2.48% and 3.21% respectively, were considered.

As shown in Figure 6, the stiffness ratios for columns under an axial load of 0.05 $f_c^{'} A_g$ were observed to rise slightly with an increase in longitudinal reinforcement ratio; while for columns under an axial load of 0.20 $f_c^{'} A_g$ the stiffness ratios almost remained the same. This suggested that for simplicity the influence of longitudinal reinforcement ratio on the initial stiffness of RC columns could be ignored.

7.3.Influence of Yield Strength of Longitudinal Reinforcing Bars

Four yield strengths of longitudinal reinforcing bars, 362 MPa, 412 MPa, 462 MPa and 512 MPa were chosen to investigate the influences of this variable on stiffness ratios. As shown in Figure 7, with a decrease in yield strength of longitudinal reinforcing bars from 512 MPa to 462 MPa, 412 MPa and 362 MPa; the stiffness ratios increased slightly by approximately 3.1%, 4.3%, and 5.0%, respectively for columns under an axial load of 0.05 $f_c^{'} A_g$; whereas stiffness ratios almost remains the same for column under an axial load of 0.20 $f_c^{'} A_g$. Theanalytical results suggested that the influences of yield strength of longitudinal reinforcing bars on stiffness ratios are negligible.

7.4.Influence of Concrete Compressive Strength

Figure 8 illustrates the influence of concrete compressive strength on stiffness ratios for two different axial loads of 0.05 $f_c^{'} A_g$ and 0.20 $f_c^{'} A_g$. The concrete compressive strengths investigated were 25 MPa, 35 MPa, 45 MPa, and 55 MPa. For both axial loads, with an increase in concrete compressive strength, no significant changes on stiffness ratios were observed.

7.5.Influence of Aspect Ratio

Figure 9 and Table 3 show the influence of aspect ratioon stiffness ratios of RC columns. Six aspect ratios of1.50, 1.80, 2.10, 2.43, 2.70, and 3.00 were investigated. In general, the stiffness ratio increased with an increase in aspect ratio.

Figure 9 shows that with an increase in aspect ratio from 1.50 to 1.80, 2.10, 2.43, 2.70, and 3.00; the stiffness ratios of columns without axial loads rose by approximately 18.5%, 39.8%, 62.8%, 83.6%, 109.4%, respectively. Similar trends were observed for the columns with an axial load ratio of 0.20. The stiffness ratios increased by approximately 15.6%, 27.4%, 37.8%, 45.2% and 52.3% for columns under an axial load of 0.60 $f_c^{'} A_g$ with an increase in aspect ratio from 1.50 to 1.80, 2.10, 2.43, 2.70, and 3.00, respectively. This suggested that the aspect ratio significantly influences the stiffness ratio.

7.6.Influence of Axial Load

It is generally recognized that the presence of column axial load can effectively increase the flexural strength of columns and thus lead to larger initial flexural stiffness, which results in a higher stiffness ratio. The analyses as illustrated in Figure 10 and tabulated in Table 3 were carried out to assess the influence of axial load ratio on stiffness ratio The axial load ratio was varied from 0 to 0.60.

In general, the stiffness ratio increased with an increase in axial load ratio. Figure 10 showed that with an increase in axial load ratio from 0 to 0.20, 0.40, and 0.60; the stiffness ratios for specimens with an aspect ratio of 1.5 rose by approximately 35.2%, 98.7% and 167.9%, respectively. Similar trends were observed for other aspect ratios. It can thus be concluded that the axial load ratio significantly affects the stiffness ratio.

8. Proposed Equation for Effective Moment of Inertia of RC Columns

It is observed that the stiffness ratio apparently increased with an increase in aspect ratios (R_a) and axial load ratio (R_n). The transverse and longitudinal reinforcement ratios, yield strength of longitudinal bars and concrete compressive strength insignificantly influenced the stiffness ratio of RC columns. For simplicity, the influences of these factors were ignored. Based on the results of the parametric study, the stiffness ratio (κ) is given by the following equation:

$$\kappa = \left(2.043R_n^2 + 2.961R_n + 1.739\right)\left(3.023R_a + 2.573\right) \quad (19)$$

Berry *et al.* (2004) collected a database of 400 tests of RC columns, which contained the hysteretic response, geometry, column axial load and material properties of test specimens. This database provided the data needed to evaluate the accuracy of the proposed equation for the stiffness ratio. The verification was limited to the range of the parametric study. The axial load was limited from 0 to 0.60 $f_c^{'}A_g$, and the aspect ratio was limited from 1.5 to 3.0. Only rectangular columns tested in the double-curvature configuration under unidirectional quasi-static cyclic lateral loading were chosen. Details of the chosen RC columns are tabulated in Table 4.

It was found that the average ratio of the experimental to predicted stiffness ratio by the proposed equation is 0.945 as shown in Figure 11 and Table 4, showing a good correlation between the proposed equation and experimental data. Therefore, the proposed equation may be suitable as an assessment tool to calculate the stiffness ratio of RC columns within the range of the parametric study.

The stiffness ratio of columns calculated based on ACI 318-2008 (2008), FEMA 356 (2000), ASCE 41 (2007), Paulay and Priestley (1992), and Elwood and Eberhard (2009) are also shown in Table 4. The mean ratio of the experimental to predicted stiffness ratio and its coefficient of variation were 0.406 and 0.136, 0.409 and 0.095, 0.399 and 0.097, 0.571 and 0.151, 0.380 and 0.096, and 0.855 and 0.202 for ACI 318-2008 (2008a), ACI 318-2008 (2008b), FEMA 356 (2000), ASCE 41 (2007), Paulay and Priestley (1992), and Elwood and Eberhard (2009) respectively. Comparison of available models with experimental data indicated that the proposed equation produced a better mean ratio of the experimental to predicted stiffness ratio than other models. It is to be noted that the proposed equation gives slightly conservative estimation of stiffness ratio in some cases and acceptable small underestimation in other cases.

9. Conclusions

This paper presents an analytical method to estimate the initial stiffness of RC columns. A comprehensive parametric study is carried out based on the proposed method to investigate the influences of several critical parameters. A simple equation to estimate the initial stiffness of RC columns is also proposed. The following provides specific findings of the paper:

Comparisons made between the analytical results and the experimental results of the six specimens tested in Tran *et al.*'s study (2009) show relatively good agreement. This shows the applicability and accuracy of the proposed method to estimate initial stiffness of RC columns.

The parametric study based on the proposed method shows that the stiffness ratio (κ) increases along with aspect ratios (R_a) and axial load ratio (R_n). The transverse and longitudinal reinforcement ratios, yield strength of longitudinal bars and concrete compressive strength showed a negligible impact on the stiffness ratio.

It was found that by the proposed equation, the average ratio of the experimental to predicted stiffness ratio is 0.945, showing a good correlation between the proposed equation and the experimental data. The proposed equation may be suitable as an assessment tool to calculate the stiffness ratio of RC columnswithin the range of the parametric study, where the axial load was limited from 0 to 0.60 $f_c' A_g$, and the aspect ratio limited from 1.5 to 3.0. Only rectangular columns tested in the double-curvature configuration under unidirectional quasi-static cyclic lateral loading were chosen.

References

ACI 318 (2008).*Building Code Requirements for Structural Concrete (ACI 318-08) and Commentary*, American Concrete Institute, Farmington Hills, Michigan, USA.

ASCE 41 (2007).*Seismic Rehabilitation of Existing Buildings*, American Society of Civil Engineers, Reston, VA, USA.

Arakawa, T., Arai, Y., Mizoguchi, M. and Yoshida, M. (1989).“Shear resisting behavior of short reinforced concrete columns under biaxial bending-shear”, *Transactions of the Japan Concrete Institute*, Vol. 11, pp. 317–324.

Bett, B.J., Klingner, R.E. and Jirsa J.O. (1985).*Behavior of Strengthened and Repaired Reinforced Concrete Columns Under Cyclic Deformations*, PMFSEL Report No. 85-3, Department of Civil Engineering, University of Texas at Austin, Austin, Texas, USA.

Berry, M.P., Parrish, M. and Eberhard, M.O. (2004).*PEER Structural Performance Database User’s Manual*, Pacific Earthquake Engineering Research Center, University of California, Berkeley, USA.

FEMA 356 (2000).*Prestandard and Commentary for the Seismic Rehabilitation of Buildings*, Federal Emergency Management Agency, Washington DC, USA.

Elwood, K. and Eberhard, M.O. (2009).“Effective stiffness of reinforced concrete columns”, *ACI Structural Journal*, Vol. 106, No. 4, pp. 476–484.

Esaki, F. (1996) “Reinforcing effect of steel plate hoops on ductility of R/C column”, *Proceedings of the 11th World Conference on Earthquake Engineering*, Acapulco, Mexico, June.

Ohno, T. and Nishioka, T. (1984). “An experimental study on energy absorption capacity of columns in reinforced concrete structures”, *Proceedings of the JSCE: Structural Engineering/Earthquake Engineering*, Vol. 1, No. 2, pp. 137–147.

Ohue, M., Morimoto, H.F.S. and Morita, S. (1985). “The behavior of RC short columns failing in splitting bond-shear under dynamic lateral loading”, *Transactions of the Japan Concrete Institute*, Vol. 7, pp. 293–300.

Park, R. and Paulay, T. (1975).*Reinforced Concrete Structures*, John Willey & Sons, New York, USA.

Paulay, T. and Priestley, M.J.N. (1992).*Seismic Design of Reinforced Concrete Masonry Buildings*, John Willey & Sons, New York, USA.

Priestley, M.J.N., Seible, F., Xiao, Y. and Verma, R. (1994). “Steel jacket retrofitting of reinforced concrete bridge columns for enhanced shear strength - Part II: test results and comparison with theory”, *ACI Structural Journal*, Vol. 91, No. 5, pp. 537–551.

Priestley, M.J.N., Seible, F. and Calvi, G.M. (1996).*Seismic Design and Retrofit of Bridge Structures*, John Willey & Sons, New York, USA.

Pujol, S. (2002).*Drift Capacity of Reinforced Concrete Columns Subjected to Displacement Reversals*, PhD Thesis, Purdue University, USA.

Tran, C.T.N. (2010). *Experimental and Analytical Studies on the Seismic Behavior of RC Columns with Light Transverse Reinforcement*, PhD Thesis, Nanyang Technological University, Singapore.

Umehara, H. and Jirsa, J.O. (1982).*Shear Strength and Deterioration of Short Reinforced Concrete Columns Under Cyclic Deformations*, PMFSEL Report No. 82-3, Department of Civil Engineering, University of Texas at Austin, Austin, Texas, USA.

Vecchio, F.J. and Collins, M.P. (1986).“The modified compression-field theory for reinforced concrete elements subjected to shear”,*ACI Journal Proceedings*, Vol. 83, No. 2, pp. 219–231.

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Specimen	Longitudinal reinforcement	Transverse reinforcement	f_c' (MPa)	$b \times h$ (mm × mm)	L (mm)	$\frac{P}{f_c' A_g}$
SC-2.4-0.20						0.20
SC-2.4-0.50					1700	0.50
SC-1.7-0.05	8-T20	2-R6 @ 125				0.05
SC-1.7-0.20	$\rho_l = 2.05\%$	$\rho_v = 0.13\%$	25.0	350 × 350		0.20
SC-1.7-0.35					1200	0.35
SC-1.7-0.50						0.50

Table 1

Specimen	$K_{i-\exp}$ (kN/mm)	$\frac{K_{i-\exp}}{K_{i-p}}$	$\frac{K_{i-\exp}}{K_{i-ACI(a)}}$	$\frac{K_{i-\exp}}{K_{i-ACI(b)}}$	$\frac{K_{i-\exp}}{K_{i-FEMA}}$	$\frac{K_{i-\exp}}{K_{i-ASCE}}$	$\frac{K_{i-\exp}}{K_{i-PP}}$	$\frac{K_{i-\exp}}{K_{i-EE}}$
SC-2.4-0.20	12.9	0.782	0.254	0.355	0.355	0.444	0.305	0.793
SC-2.4-0.50	15.5	0.572	0.301	0.421	0.301	0.301	0.263	0.525
SC-1.7-0.05	24.5	0.918	0.319	0.223	0.223	0.372	0.236	0.560
SC-1.7-0.20	26.9	0.865	0.169	0.236	0.236	0.295	0.203	0.590
SC-1.7-0.35	28.8	0.653	0.188	0.263	0.239	0.239	0.190	0.553
SC-1.7-0.50	34.4	0.620	0.220	0.308	0.220	0.220	0.193	0.507
Mean		0.735	0.242	0.301	0.262	0.312	0.232	0.588
Coefficient of Variation		0.141	0.060	0.076	0.054	0.084	0.046	0.104

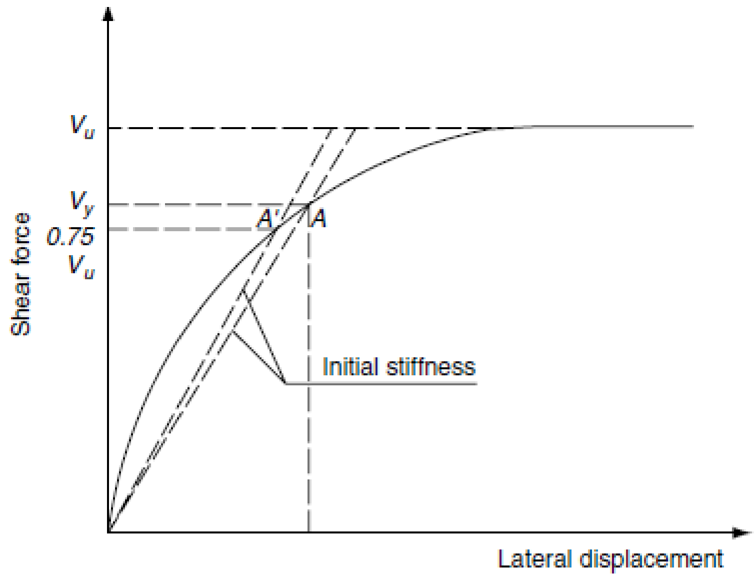
Table 2

$P / f_c' A_g$	a / h					
	1.50	**1.80**	**2.10**	**2.43**	**2.70**	**3.00**
0.00	11.22	13.30	15.69	18.27	20.60	23.50
0.05	12.27	14.24	16.64	19.24	21.13	23.90
0.10	13.32	15.45	17.78	20.23	22.21	24.20
0.15	14.23	16.54	18.85	21.46	23.37	25.27
0.20	15.17	17.66	20.13	22.83	24.80	26.70
0.25	16.43	19.23	22.56	25.61	27.75	29.76
0.30	17.90	21.83	25.70	29.06	31.30	33.22
0.35	19.78	24.85	28.77	31.91	33.85	35.50
0.40	22.30	27.57	31.27	34.22	36.05	37.73
0.45	24.74	29.70	33.27	36.12	38.01	39.81
0.50	26.82	31.73	35.28	38.14	40.16	42.08
0.55	28.56	33.37	36.82	39.86	41.94	43.95
0.60	30.06	34.74	38.30	41.42	43.66	45.77

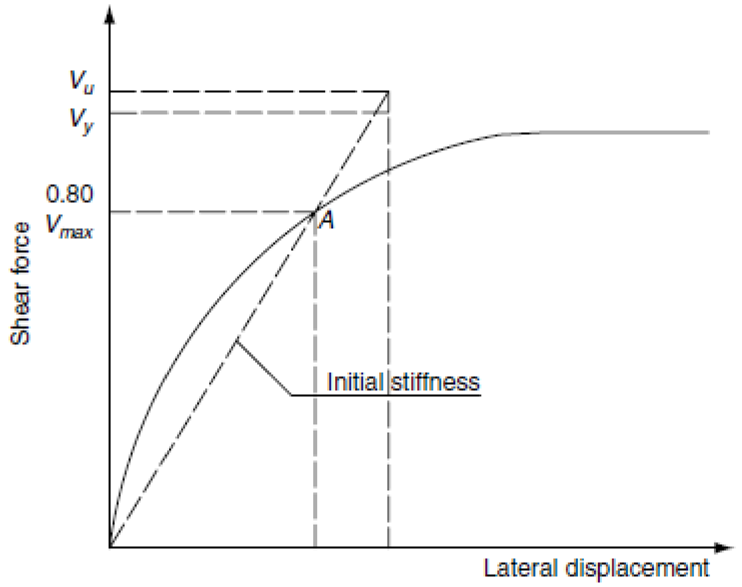
Table 3

Table 4

	Specimen	R_a	R_u	κ_p	κ_{exp}	$\frac{\kappa_{ixp}}{\kappa_p}$	$\frac{\kappa_{ixp}}{ACI(a)}$	$\frac{\kappa_{ixp}}{\kappa_{ACI(b)}}$	$\frac{\kappa_{ixp}}{\kappa_{FEMA}}$	$\frac{\kappa_{ixp}}{\kappa_{ASCE}}$	$\frac{\kappa_{ixp}}{\kappa_{PP}}$	$\frac{\kappa_{ixp}}{\kappa_{EE}}$
Tran *et al.* (2009)	SC-2.4-0.20	2.43	0.200	23.9	17.8	0.745	0.254	0.355	0.355	0.444	0.305	0.793
	SC-2.4-0.50	2.43	0.500	37.0	21.1	0.570	0.301	0.421	0.301	0.301	0.263	0.525
	SC-1.7-0.05	1.71	0.050	14.6	11.2	0.767	0.319	0.223	0.223	0.372	0.236	0.560
	SC-1.7-0.20	1.71	0.200	18.7	11.8	0.631	0.169	0.236	0.236	0.295	0.203	0.590
	SC-1.7-0.35	1.71	0.350	23.4	13.1	0.560	0.188	0.263	0.239	0.239	0.19	0.553
	SC-1.7-0.50	1.71	0.500	28.9	15.4	0.533	0.220	0.308	0.220	0.220	0.193	0.507
Arakawa *et al.* (1989)	No. 102	1.50	0.333	20.9	16.7	0.799	0.426	0.596	0.559	0.559	0.441	0.493
Ohue *et al.* (1985)	2D16RS	2.00	0.143	19.0	14.5	0.763	0.349	0.488	0.488	0.713	0.569	0.725
	4D13RS	2.00	0.153	19.3	15.2	0.788	0.389	0.544	0.544	0.795	0.634	0.76
Ohno *et al.* (1984)	CA025C	1.50	0.257	18.7	14.4	0.770	0.394	0.552	0.552	0.604	0.443	0.591
Umehara *et al.* (1982)	CUW	1.96	0.162	19.3	16.2	0.839	0.374	0.524	0.524	0.724	0.473	0.81
Bett *et al.* (1985)	No. 1-1	1.50	0.104	14.7	11.2	0.762	0.16	0.224	0.224	0.368	0.257	0.56
Pujol *et al.* (2002)	No. 10-2-3N	2.25	0.085	18.8	17.9	0.952	0.511	0.358	0.358	0.597	0.359	0.895
	No. 10-2-3S	2.25	0.085	18.8	19.6	1.043	0.56	0.392	0.392	0.653	0.394	0.98
	No. 10-3-1.5N	2.25	0.089	18.9	18.6	0.984	0.531	0.372	0.372	0.62	0.371	0.93
	No. 10-3-1.5S	2.25	0.089	18.9	21.2	1.122	0.606	0.424	0.424	0.707	0.423	1.06
	No. 10-3-3N	2.25	0.096	19.1	19.4	1.016	0.554	0.388	0.388	0.647	0.383	0.97
	No. 10-3-3S	2.25	0.096	19.1	20.4	1.068	0.583	0.408	0.408	0.680	0.403	1.02
	No. 10-3-2.25N	2.25	0.105	19.4	21.4	1.103	0.306	0.428	0.428	0.713	0.417	1.07
	No. 10-3-2.25S	2.25	0.105	19.4	20.6	1.062	0.294	0.412	0.412	0.687	0.402	1.03
	No. 20-3-3N	2.25	0.158	21.2	22.7	1.071	0.324	0.454	0.454	0.634	0.412	1.087
	No. 20-3-3S	2.25	0.158	21.2	25.0	1.179	0.357	0.500	0.500	0.698	0.454	1.197
	No. 10-2-2.25N	2.25	0.082	18.7	18.8	1.005	0.537	0.376	0.376	0.627	0.379	0.94
	No. 10-2-2.25S	2.25	0.082	18.7	20.2	1.080	0.577	0.404	0.404	0.673	0.407	1.01
	No. 10-1-2.25N	2.25	0.078	18.6	18.8	1.011	0.537	0.376	0.376	0.627	0.381	0.94
	No. 10-1-2.25S	2.25	0.078	18.6	19.5	1.048	0.557	0.390	0.390	0.650	0.396	0.975
Priestley *et al.* (1994)	R1A	2.00	0.054	16.4	20.0	1.22	0.571	0.400	0.400	0.667	0.42	0.928
	R3A	2.00	0.059	16.6	20.3	1.223	0.580	0.406	0.406	0.677	0.424	0.922
	R5A	1.50	0.063	13.7	17.1	1.248	0.489	0.342	0.342	0.570	0.355	0.855
Esaki *et al.* (1985)	H-2-1/5	2.00	0.200	20.8	23.6	1.135	0.337	0.472	0.472	0.590	0.405	1.116
	HT-2-1/5	2.00	0.200	20.8	19.6	0.942	0.280	0.392	0.392	0.490	0.337	0.922
	H-2-1/3	2.00	0.334	25.5	28.1	1.102	0.401	0.562	0.526	0.526	0.414	0.982
	HT-2-1/3	2.00	0.333	25.4	26.1	1.028	0.373	0.522	0.489	0.489	0.384	0.914
	Mean				0.945	0.406	0.409	0.399	0.571	0.380	0.855	
	Coefficient of Variation				0.202	0.136	0.095	0.097	0.151	0.096	0.202	

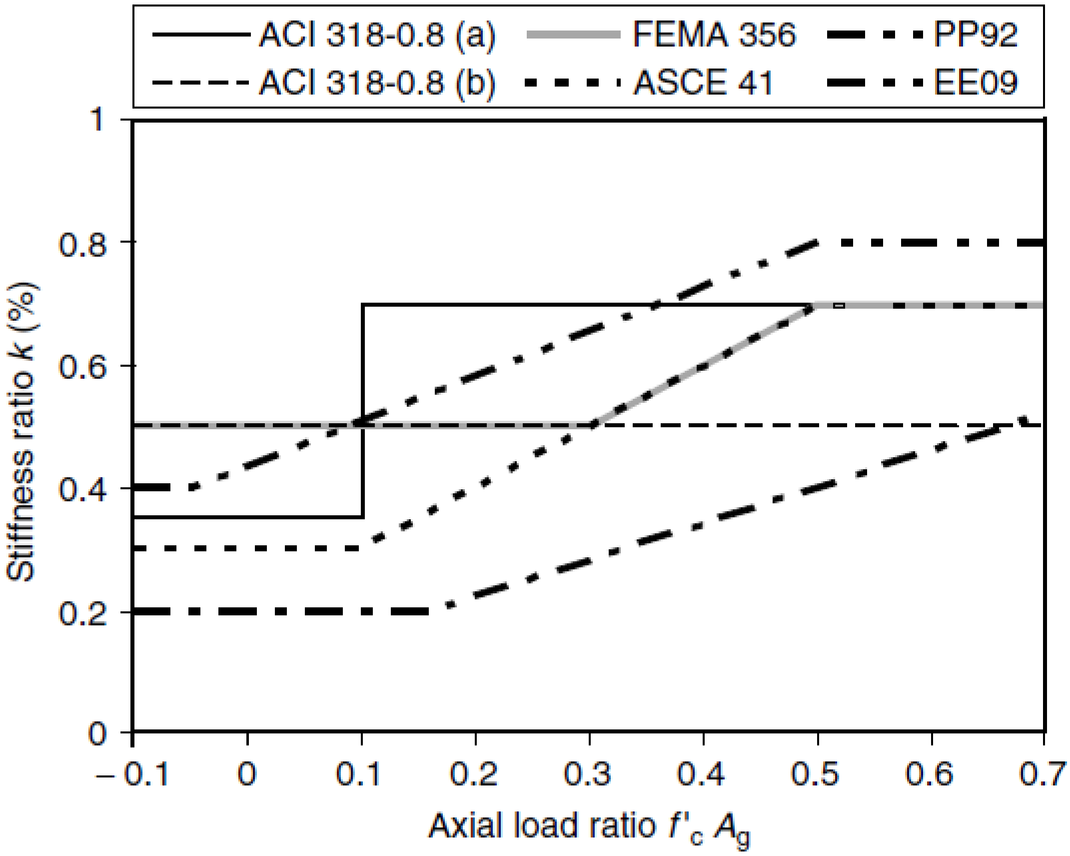


(a)



(b) (Elwood *et al.* 2009)

Fig. 1



ACI 318-0.8 (a)
ACI 318-0.8 (b)
FEMA 356
ASCE 41
PP92
EE09
Stiffness ratio k (%)
Axial load ratio f'c Ag
1
0.8
0.6
0.4
0.2
0
– 0.1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7

Fig. 2

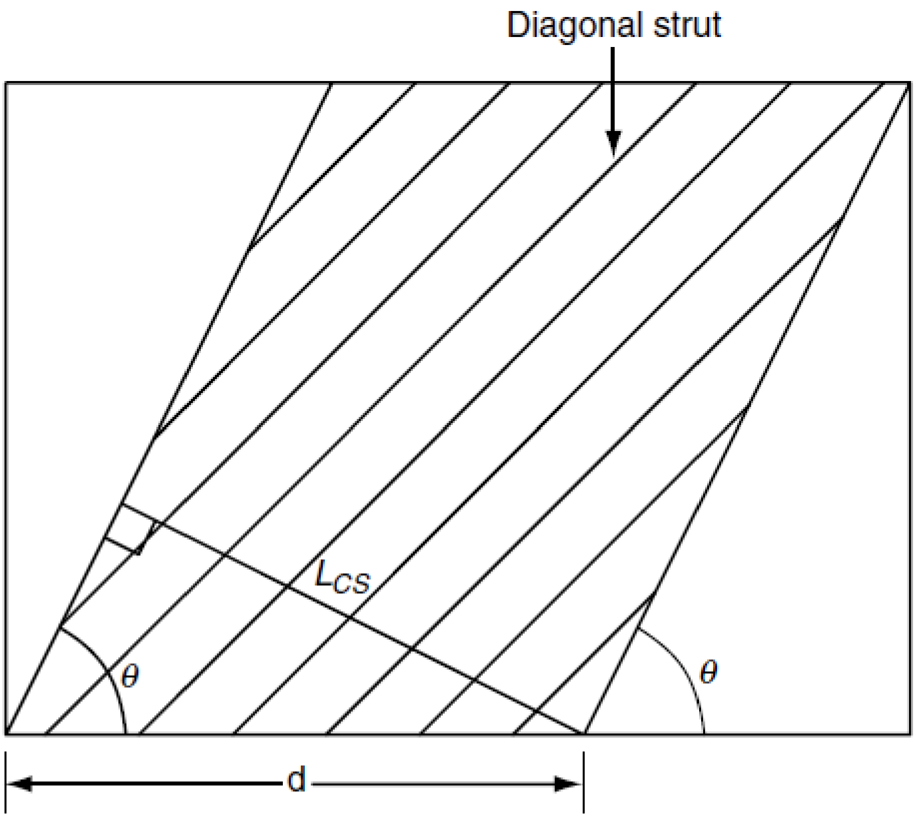
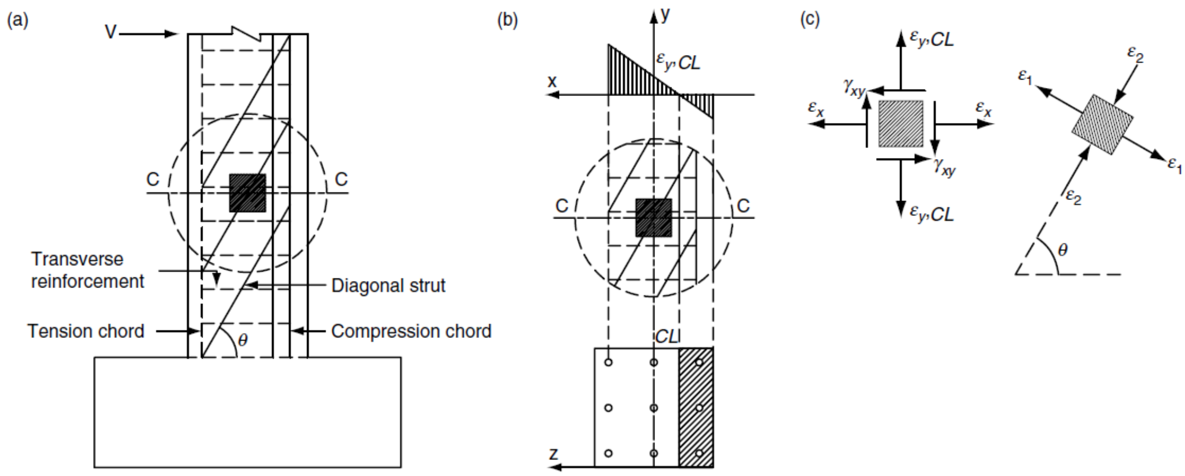


Fig. 3



(a)
V
C
C
Transverse reinforcement
Diagonal strut
Tension chord
Compression chord
θ
(b)
y
x
ε_y, CL
C
C
CL
z
(c)
ε_y, CL
γ_xy
ε_x
ε_x
γ_xy
ε_y, CL
ε_1
ε_2
ε_1
ε_2
θ

Fig. 4

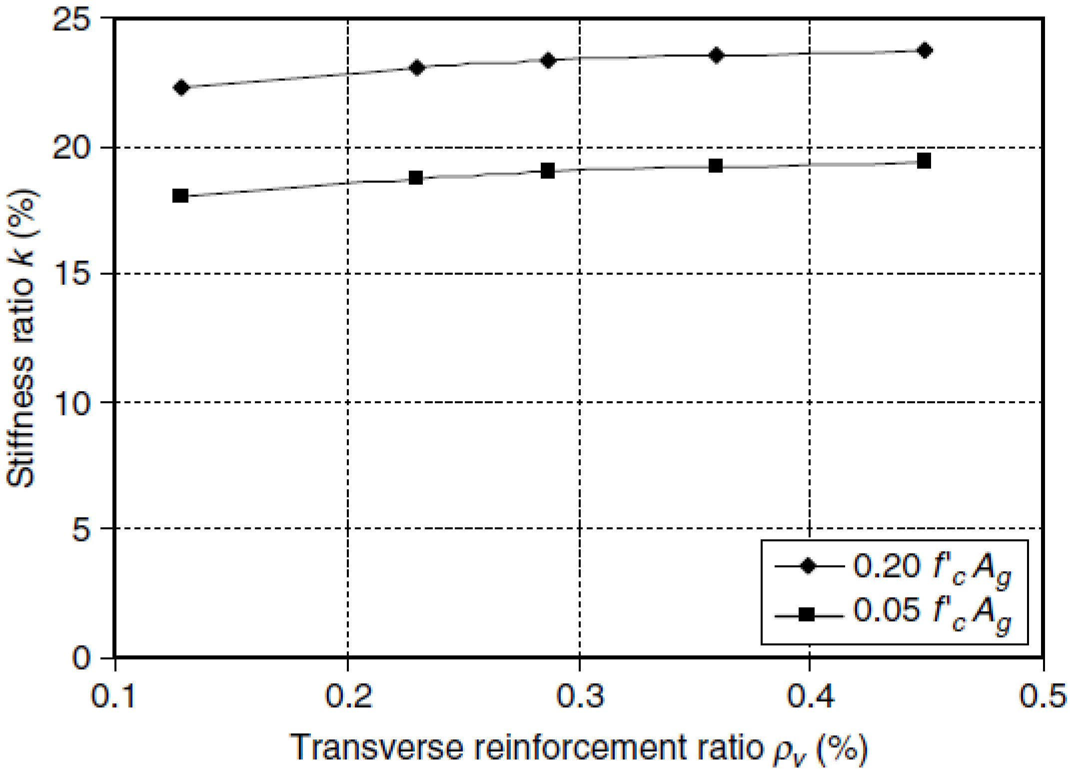
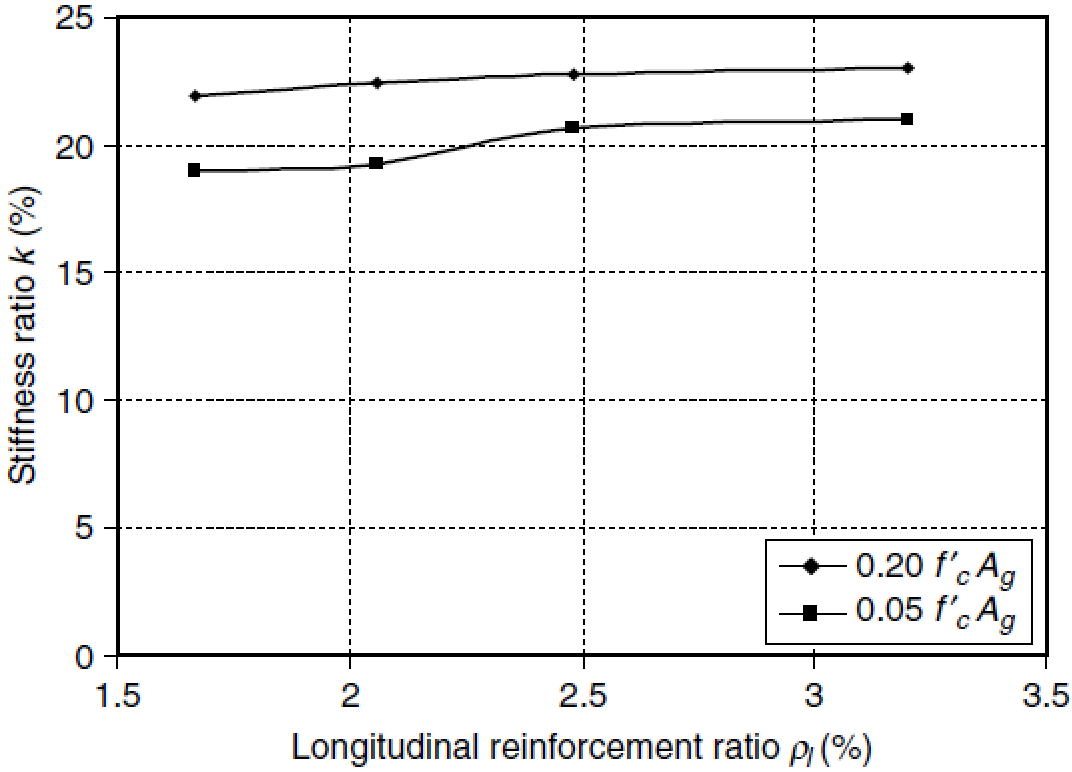
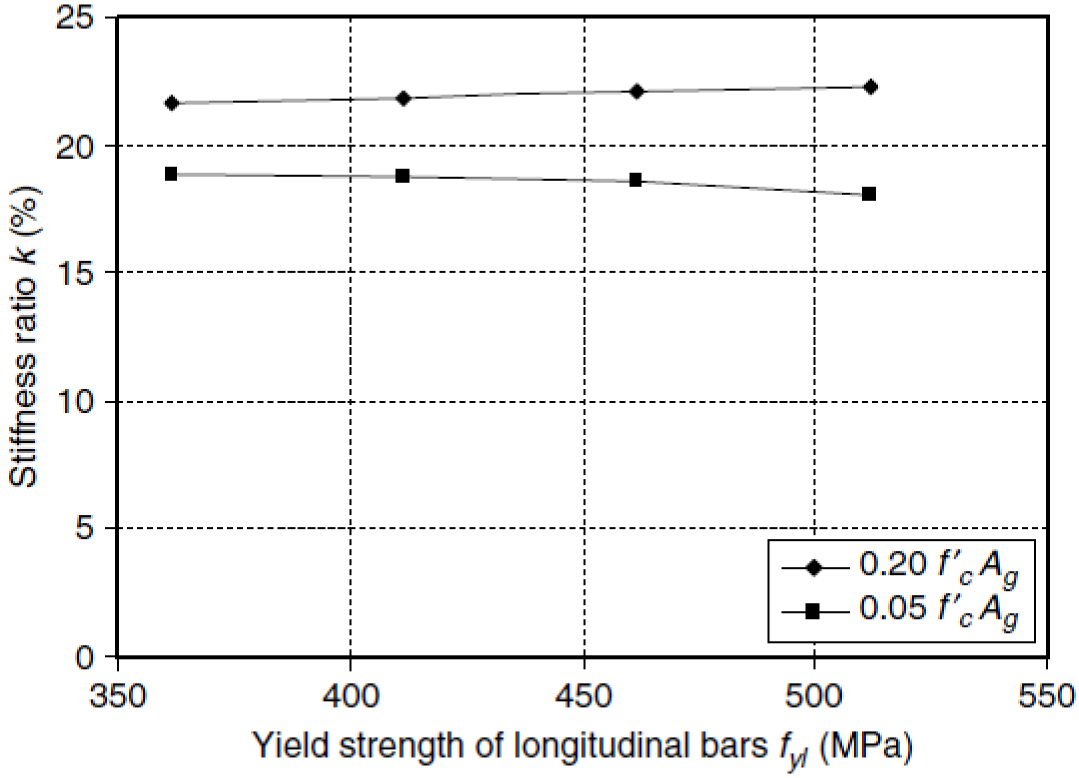

25
20
15
10
5
0
0.1
0.2
0.3
0.4
0.5
Stiffness ratio k (%)
Transverse reinforcement ratio ρv (%)
0.20 f'c Ag
0.05 f'c Ag

Fig. 5



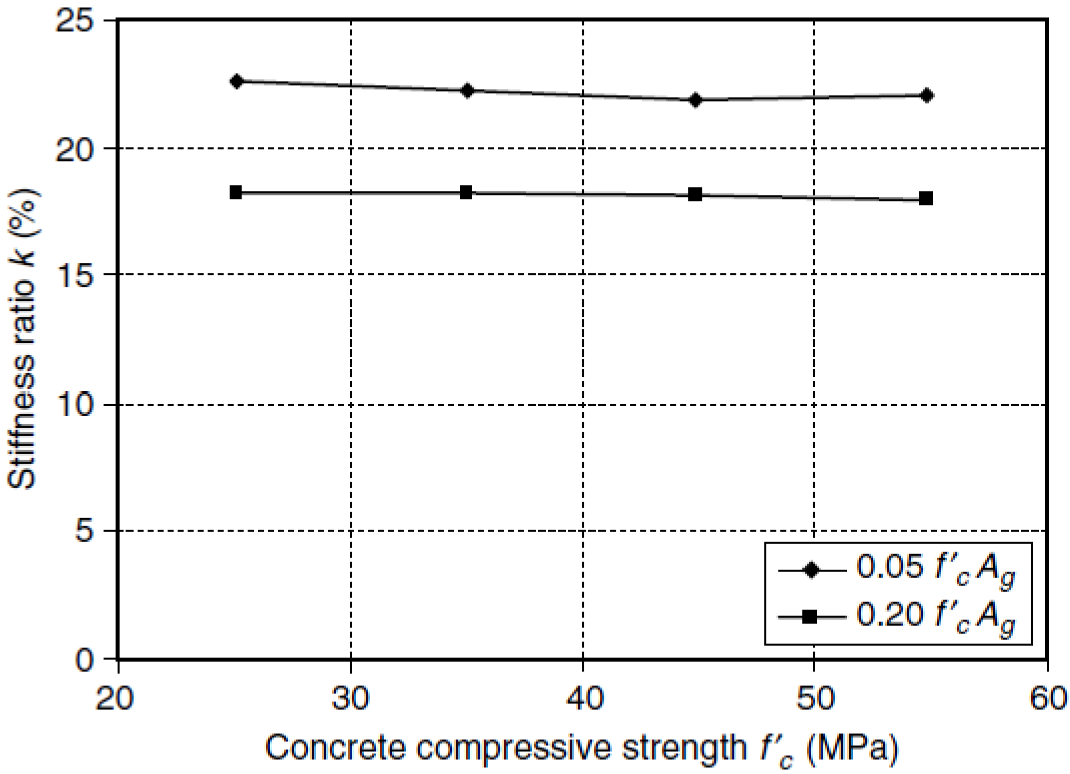
25
20
15
10
5
0
1.5
2
2.5
3
3.5
Stiffness ratio k (%)
Longitudinal reinforcement ratio ρ_l (%)
$0.20\ f'_c A_g$
$0.05\ f'_c A_g$

Fig. 6



Stiffness ratio k (%)
25
20
15
10
5
0
350
400
450
500
550
Yield strength of longitudinal bars f_{yl} (MPa)
0.20 $f'_c A_g$
0.05 $f'_c A_g$

Fig. 7



Stiffness ratio k (%)
25
20
15
10
5
0
20
30
40
50
60
Concrete compressive strength f'_c (MPa)
$0.05\ f'_c A_g$
$0.20\ f'_c A_g$

Fig. 8

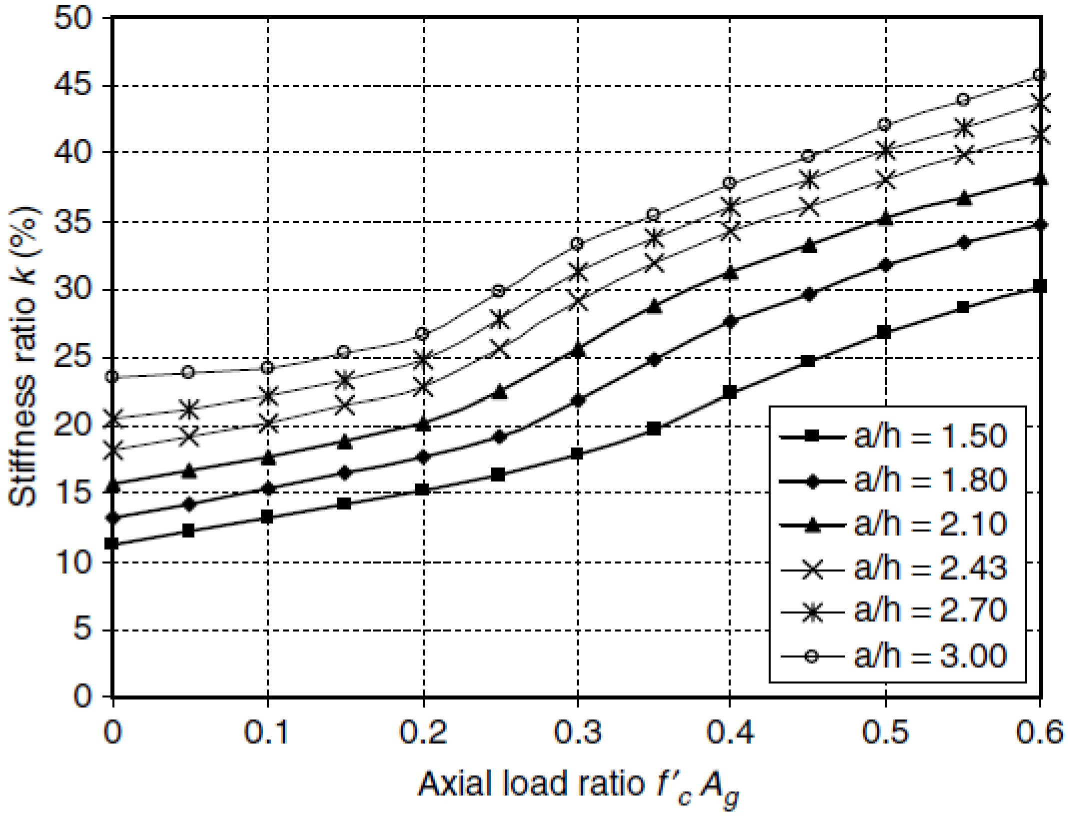

Stiffness ratio k (%)
Axial load ratio f′c Ag
0
5
10
15
20
25
30
35
40
45
50
0
0.1
0.2
0.3
0.4
0.5
0.6
a/h = 1.50
a/h = 1.80
a/h = 2.10
a/h = 2.43
a/h = 2.70
a/h = 3.00

Fig. 9

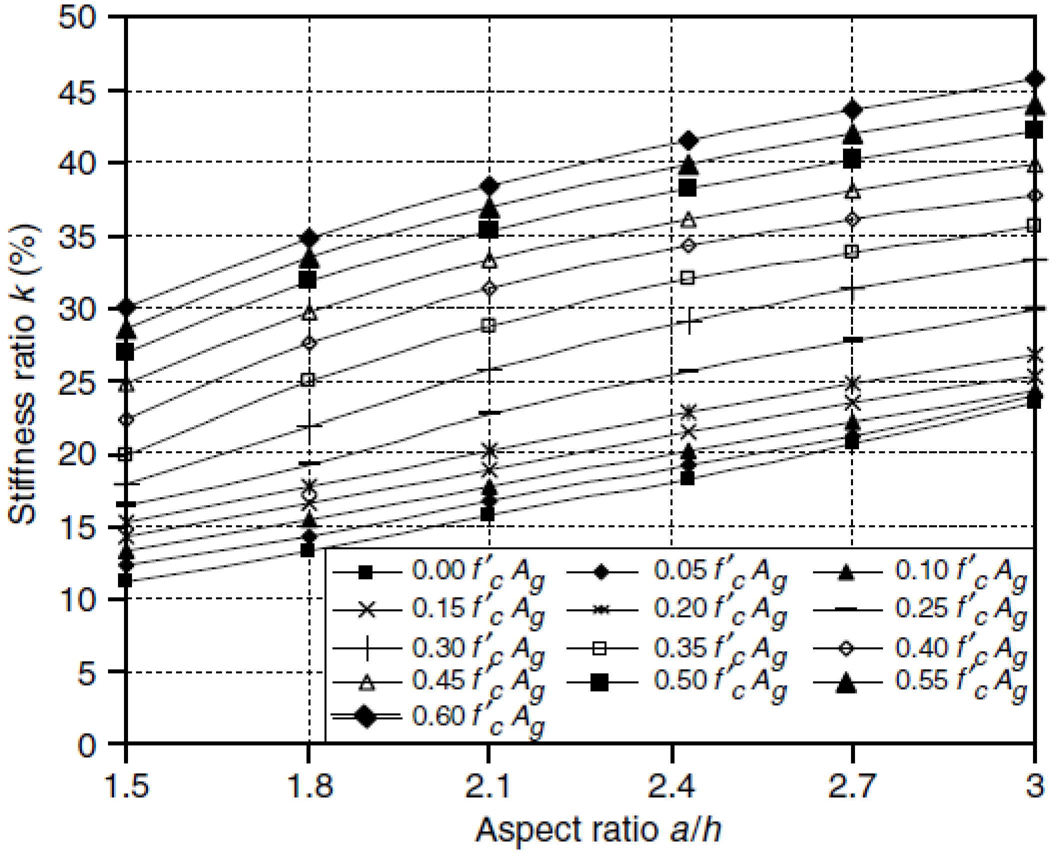
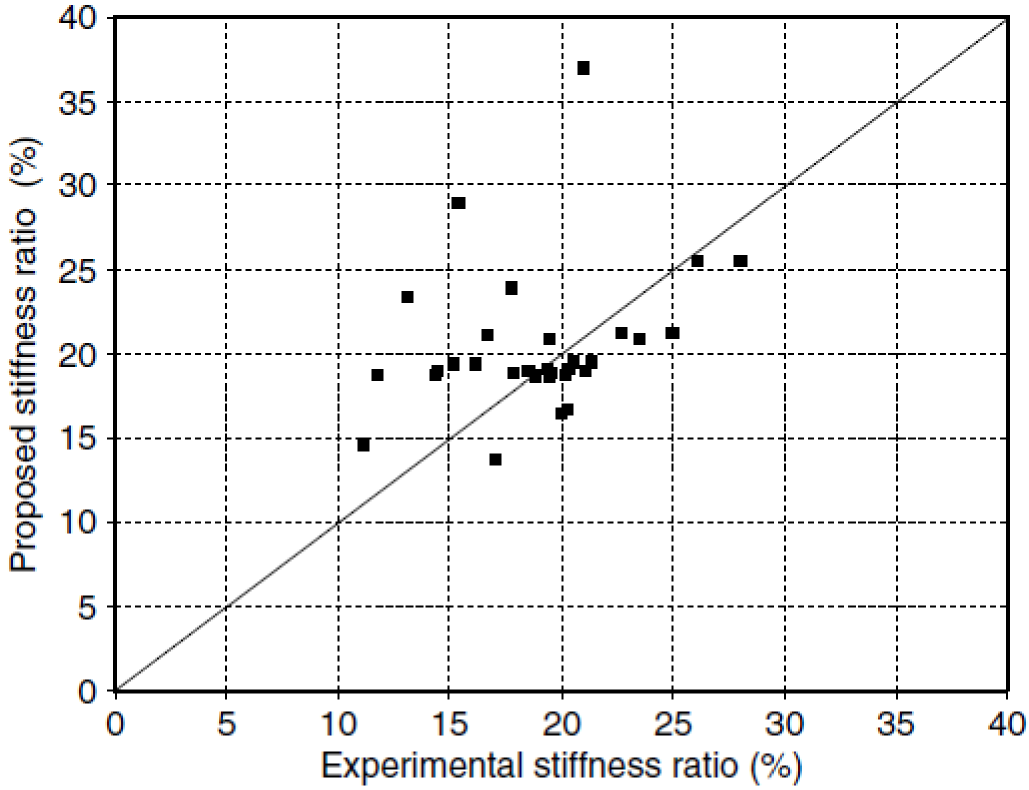

Stiffness ratio k (%)
Aspect ratio a/h
50
45
40
35
30
25
20
15
10
5
0
1.5
1.8
2.1
2.4
2.7
3
0.00 f′c Ag
0.05 f′c Ag
0.10 f′c Ag
0.15 f′c Ag
0.20 f′c Ag
0.25 f′c Ag
0.30 f′c Ag
0.35 f′c Ag
0.40 f′c Ag
0.45 f′c Ag
0.50 f′c Ag
0.55 f′c Ag
0.60 f′c Ag

Fig. 10



Proposed stiffness ratio (%)
Experimental stiffness ratio (%)
0
5
10
15
20
25
30
35
40

Fig. 11