

Resilience-Oriented Distribution System Reconfiguration for Service Restoration Considering Distributed Generations

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Abstract—Significant evolution of distribution system, including distributed generation, microgrids and remote-controlled switches (RCSs), lead to novel and effective distribution system restoration (DSR) strategies. This paper proposes a resilience-oriented reconfiguration method to restore service to interrupted customers in distribution system as expeditious as possible after major blackouts. A structural model with virtual node and branches is provided to describe the radial structure constraint. The energy limits within microgrids are taken into considerations to account for scarcity of generation resources. The optimization problem is formulated as mixed-integer second-order cone programming (MISOCP), which employs a convex representation of a distribution network model on the basis of the conic quadratic format of the power flow equations. Case study is implemented on a modified 33-bus test system to verify the propose method.

Keywords—resilience, service restoration, distribution system reconfiguration, microgrids, distributed generator (DG).

I. INTRODUCTION

In recent years, extreme events such as natural disasters have catastrophic impacts on electricity infrastructure, resulting in significant economical and societal disruptions [1, 2]. The power system resilience is critically important as a key part of the infrastructure's defense against high-impact low-probability extreme events [3]. In the context of power systems, resilience is the ability to prepare and withstand extraordinary events, rapidly recover from such disruptive events, and adapt its operation and structure to prevent or mitigate the impact of similar events in the future [4]. From a pragmatic point of view, it is increasingly important for power industry to have coordinated and deliberate plans to restore the power system reliably, especially in response to ever present threat of natural disasters. This paper is mainly focused on the strategies to improve recovery and restoration, specifically, using existing distributed generations to restore critical loads in the distribution systems after a major disturbance to achieve resilient distribution network.

Distribution system restoration by reconfiguration has been extensively investigated in the literature [5]. Nevertheless,

power outages due to natural disasters have their unique features, multiple faults may occur and trigger widespread blackouts or even the substations collapse such that the distribution systems cannot be supplied by the main grids [6]. New techniques, such as distributed energy resources, microgrids, distribution automation, can be deployed to increase the resilience of distribution systems. The distribution system restoration is generally NP-hard [7] and computationally expensive. The conic relaxation technique has been recently proposed and widely studied, which relaxes the nonconvex power flow equations by using second order cones [8]. Furthermore, it is proved that the conic relaxation generally has no gap or small gap to the original exact power flow equations in most distributions systems [7]. Therefore, the restoration strategies through distribution network reconfiguration with distributed generation can be relaxed to a mixed integer convex programming that can be tractably solved, as opposed to the original nonconvex program.

In this paper, we propose a method for the resilience-oriented reconfiguration of a radial distribution system after a major disturbance. When multiple faults occur, the distribution systems can no longer be supplied by main grids for a period of time, the on-outage area will be optimally sectionalized into self-adequate MGs to autonomously supply electricity to as more customers as possible. Each microgrid is split by controlling the ON/OFF status of RCSs and energized by no more than one DG to keep radial network structure. By using conic relaxation, the optimization problem is formulated as mixed-integer second-order cone programming (MISOCP) that can be tractably dealt with. A modified 33-bus distribution test feeder is used to validate the proposed method.

The remainder of this paper is organized as follows. In Section II, the resilient strategies by network reconfiguration and sectionalization are analyzed. A structural model with virtual node and branches are provided. Section III formulates the MISOCP for the optimization of network topological structure and microgrids operation. In Section IV, numerical results based on a modified 33-bus test feeder including distributed generation are presented to test and verify our proposed method and the paper is summarized in Section V.

This work is supported by the Future Resilient System (FRS) at the Singapore-ETH centre (SEC), which is funded by the National Research Foundation of Singapore (NRF) under its Campus for Research Excellence and Technological Enterprise (CREATE) program.

II. NETWORK RECONFIGURATION WITH DISTRIBUTED ENERGY RESOURCES

A. Microgrids as Resilience Resources

Microgrids add active network components at the distribution level, which increase flexibility and reduce conventional power grid vulnerability [6]. Some recent studies have considered hierarchical control [9] or distributed control [10] for hybrid energy storage system in DC microgrid to ensure system reliability. Reconfiguration of the topology of the network with RCSs provides opportunities to restore the interrupted loads more quickly. A mixed-integer conic programming (MICP) formulation for the minimum loss distribution network reconfiguration is presented in [11]. A two-stage robust optimization model for the distribution network reconfiguration with load uncertainty is demonstrated in [12].

In this paper, during the fault period T , the DGs can dynamically sectionalize the distribution network into separate microgrids and continue supplying electricity to critical customers until the restoration of the main grid is accomplished. This paper adopts and further develops the MISOCP model in [11] to the service restoration, specifically, the distribution network is reconfigured by sectionalizing the interrupted area into multiple self-adequate MGs to optimally restore the system without violating any operation constraints.

B. A Structural Model with Virtual Node

The distribution system can be modeled as an undirected graph $G = (\mathcal{N}, \mathcal{L})$ [13], where \mathcal{N} and \mathcal{L} are the sets of nodes and edges, respectively. Buses with DGs installed are called source nodes and buses only carrying load are deemed as load nodes. Some faulted islands need to be removed from the set of nodes \mathcal{N} and edges \mathcal{L} . For the preprocessed radial distribution network, the set of nodes and edges are represented by $\tilde{\mathcal{N}}, \tilde{\mathcal{L}}$.

This paper proposes a virtual node n_0 and virtual branches set \mathcal{L}_0 added into $\tilde{\mathcal{N}}$ and $\tilde{\mathcal{L}}$, to ensure that a distribution network corresponds to a spanning tree. The virtual node has the following properties [14]: (1) The virtual node is connected to every source node (main substation or where DGs installed) through virtual branches, as shown in Fig. 1 and Fig. 2. (2) The virtual node does not generate, consume or transmit electric power. Similarly, virtual branches do not transfer electric power either.

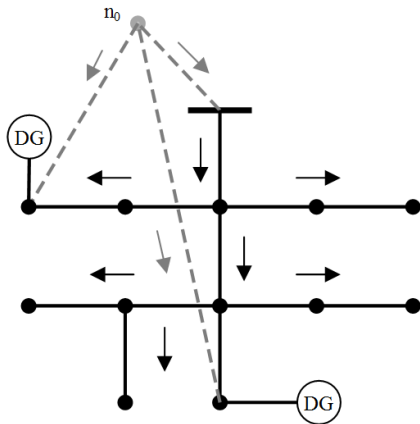


Fig. 1. Distribution system model with the virtual node

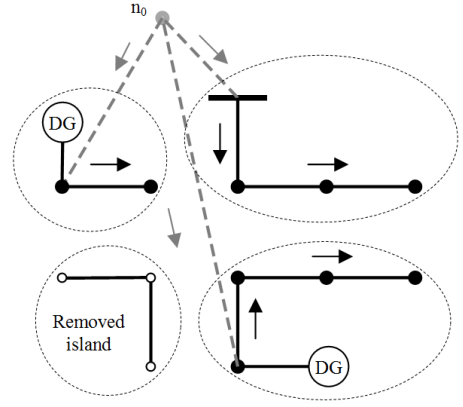


Fig. 2. Example of network reconfiguration.

C. Energy Constraints

After a major disturbance, generation resources within microgrids, such as fuel reserves (FR) or battery state of charge, are usually limited and hard to supplement during and right after a natural disaster, which significantly affects the generation availability [15]. In [16], the lifelines of DGs are used to evaluate microgrid availability during disasters. Publication [15] introduces the continuous operating time to assess the duration for which the microgrid is able to serve critical loads with limited energy reserves. Therefore, a practical service restoration strategy should also take considerations of the scarcity of generation resources.

III. PROBLEM FORMULATION

This section illustrates the optimization formulation for the service restoration. The problem is formulated as MISOCP that ensures the convexity of the problem for distribution system service restoration.

A. Constraints for Reconfiguration in MISOCP

1) Spanning Tree Constraints

The spanning tree has a characteristic that every node except the root node (virtual node n_0) has exactly one parent node. Introduce two binary variables β_{ij} and β_{ji} corresponding to each line ℓ , whose connection status is depicted by the variable α_ℓ . This can be denoted as following [11]:

$$\sum_{\ell \in \{\mathcal{L}_0, \tilde{\mathcal{L}}\}} \alpha_\ell = N_n - 1 \quad (1)$$

$$\beta_{ij} + \beta_{ji} = \alpha_\ell, \forall \ell \in \{\mathcal{L}_0, \tilde{\mathcal{L}}\} \quad (2)$$

$$\sum_{j \in \emptyset(i)} \beta_{ij} = 1, \forall i \in \tilde{\mathcal{N}} \quad (3)$$

$$\beta_{ij} \in \{0, 1\}, \forall i \in \tilde{\mathcal{N}}, j \in \emptyset(i) \quad (4)$$

$$0 \leq \alpha_\ell \leq 1, \forall \ell \in \tilde{\mathcal{L}} \quad (5)$$

$$\beta_{0j} = 0, \forall j \in \emptyset(n_0) \quad (6)$$

$$\beta_{j0} = 1, \forall j \in \emptyset(n_0) \quad (7)$$

Where $\emptyset(i)$ denotes the set of nodes connected to i by a line. N_n is the number of nodes in $\{n_0, \tilde{\mathcal{N}}\}$. The premise for the radial network is denoted by (1). Equation (2) suggests that a line ℓ is

in the spanning tree ($\alpha_\ell = 1$) if either node j is the parent of node i ($\beta_{ij} = 1$), or node i is the parent of node j ($\beta_{ji} = 1$). Equation (3) requires that every node other than the virtual node n_0 has certainly one parent, whereas the virtual node n_0 as the root node has no parent.

2) Real and Reactive Power Constraints

A convex relaxation can be attained by reformulating the problem in terms of continuous variables as a convex second-order cone program. Define the following new variables for each time point:

$$u_i^t = (V_i^t)^2 / \sqrt{2}, \forall i \in \tilde{\mathcal{N}}, t \in \mathbb{T} \quad (8)$$

$$R_{ij}^t = V_i^t V_j^t \cos(\theta_i^t - \theta_j^t), \forall (i, j) \in \tilde{\mathcal{L}}, t \in \mathbb{T} \quad (9)$$

$$T_{ij}^t = V_i^t V_j^t \sin(\theta_i^t - \theta_j^t), \forall (i, j) \in \tilde{\mathcal{L}}, t \in \mathbb{T} \quad (10)$$

Where V_i^t and θ_i^t are the magnitude and phase angle of the complex voltage at bus i for each time point t . In terms of these three new variables, the real and reactive power injection are expressed as following:

$$P_{ii}^t = \sum_{j \in \mathcal{O}(i)} P_{ij}^t = P_{Gi}^t - \gamma_i^t P_{Di}^t, \forall i \in \tilde{\mathcal{N}}, t \in \mathbb{T} \quad (11)$$

$$Q_{ii}^t = \sum_{j \in \mathcal{O}(i)} Q_{ij}^t = Q_{Gi}^t - \gamma_i^t Q_{Di}^t, \forall i \in \tilde{\mathcal{N}}, t \in \mathbb{T} \quad (12)$$

where

$$P_{ij}^t = \sqrt{2} g_{ij} u_i^t - g_{ij} R_{ij}^t - b_{ij} T_{ij}^t \quad (13)$$

$$Q_{ij}^t = -\sqrt{2} (b_{ij} + b_{shij}/2) u_i^t + b_{ij} R_{ij}^t - g_{ij} T_{ij}^t \quad (14)$$

$$2u_i^t u_j^t \geq (R_{ij}^t)^2 + (T_{ij}^t)^2, R_{ij}^t \geq 0 \quad (15)$$

and γ_i^t is a binary variable indicating whether the load is connected to bus or curtailed, since in distribution system one cannot feed partially the load behind a MV/LV transformer. Equation (15) is the rotated conic quadratic constraints.

The conic quadratic network model given by (11)-(15) is only associated with variables R_{ij}^t , T_{ij}^t and u_i^t . The voltage magnitudes V_i^t and phase angle θ_i^t in (8)-(10) can be retrieved based on the solution of (11)-(15).

3) Branch Connection Status Constraints

Two variables for each line ℓ , $u_i^{\ell,t}$ and $u_j^{\ell,t}$ are introduced. These variables are zero when the line is disconnected ($\alpha_\ell = 0$) and are set to the values at the nodes i and j (u_i^t and u_j^t) when the line is connected ($\alpha_\ell = 1$):

$$0 \leq u_i^{\ell,t} \leq \frac{V_{i\max}^2}{\sqrt{2}} \alpha_\ell \quad (16)$$

$$0 \leq u_j^{\ell,t} \leq \frac{V_{j\max}^2}{\sqrt{2}} \alpha_\ell \quad (17)$$

$$0 \leq u_i^t - u_i^{\ell,t} \leq \frac{V_{i\max}^2}{\sqrt{2}} (1 - \alpha_\ell) \quad (18)$$

$$0 \leq u_j^t - u_j^{\ell,t} \leq \frac{V_{j\max}^2}{\sqrt{2}} (1 - \alpha_\ell) \quad (19)$$

Therefore, the (13) – (15) can be rewritten as:

$$P_{ij}^t = \sqrt{2} g_{ij} u_i^{\ell,t} - g_{ij} R_{ij}^t - b_{ij} T_{ij}^t \quad (20)$$

$$Q_{ij}^t = -\sqrt{2} (b_{ij} + b_{shij}/2) u_i^{\ell,t} + b_{ij} R_{ij}^t - g_{ij} T_{ij}^t \quad (21)$$

$$2u_i^{\ell,t} u_j^{\ell,t} \geq (R_{ij}^t)^2 + (T_{ij}^t)^2, R_{ij}^t \geq 0 \quad (22)$$

4) Voltage Magnitude Constraints

$$\frac{V_{i\min}^2}{\sqrt{2}} \leq u_j^{\ell,t} \leq \frac{V_{i\max}^2}{\sqrt{2}} \quad (23)$$

5) Line Current Constraints

$$(I_\ell^t)^2 = \sqrt{2} A_{ij} u_i^{\ell,t} + \sqrt{2} B_{ij} u_j^{\ell,t} - 2C_{ij} R_{ij}^t + 2D_{ij} T_{ij}^t \leq I_{\ell\max}^2 \quad (24)$$

Where

$$A_{ij} = g_{ij}^2 + (b_{ij} + b_{shij}/2)^2, B_{ij} = g_{ij}^2 + b_{ij}^2 \quad (25)$$

$$C_{ij} = g_{ij}^2 + b_{ij}(b_{ij} + b_{shij}/2), D_{ij} = g_{ij} b_{shij}/2 \quad (26)$$

6) Generation Capacity and Energy Constraints

$$P_{Gk\min} \leq P_{Gk}^t \leq P_{Gk\max}, \forall k \in \mathcal{K}, t \in \mathbb{T} \quad (27)$$

$$Q_{Gk\min} \leq Q_{Gk}^t \leq Q_{Gk\max}, \forall k \in \mathcal{K}, t \in \mathbb{T} \quad (28)$$

$$\sum_{t \in \mathbb{T}} P_{Gk}^t \Delta t \leq E_{Gk}^{\text{total}}, \forall k \in \mathcal{K} \quad (29)$$

Where \mathcal{K} describes the set of microgrids presented by each DG's node number.

B. Objective

The goal of the optimization is to maximize the service restoration to loads on distribution feeders weighted by their priorities. Let w_i signify the priority weight in relation to the load at node i , which are randomly weighted between 0 and 10. It is noted that topological structure reconfiguration is a one-shot decision making problem at the initial time of restoration whereas DGs can be dispatched at every time step. The optimization problem is formulated as an MISOCP:

$$\text{maximize } \sum_{t \in \mathbb{T}} \left(\sum_{i \in \tilde{\mathcal{N}}} w_i \gamma_i^t \Delta t - \sum_{k \in \mathcal{K}} P_{Gk}^t \right) \quad (30)$$

subject to:

- 1) Spanning tree constraints: (1)-(7);
- 2) Real and reactive power constraints: (11), (12), (20), (21);
- 3) Branch connection status constraints: (16)-(19);
- 4) Rotated conic quadratic constraints: (22);
- 5) Voltage magnitude constraints: (23);
- 6) Line current constraints: (24)-(26);
- 7) Generation capacity and energy constraints: (27)-(29);

The first term in (30) depicts the maximum load to be pick up, the second item illustrates the minimum network loss [11], which ensures the balance of supplying electricity and loss reduction. Meanwhile, this penalty term would enforce (22) to be binding at the optimum as it is proved that the solution of optimal power flow with conic relaxation is exact if all

inequality constraints (22) are converged to the equality ones [7].

IV. NUMERICAL RESULTS

In this paper, the proposed reconfiguration method for service restoration is validated via a 33-bus test system [17]. The programming is implemented using MATLAB 2015b. Optimization model has been established using YALMIP [18] and solved by MOSEK [19].

A. Test system

In this modified 33-bus test feeder, five DGs are connected to distribution network. The substation and six lines are at fault due to disastrous events, as shown in Fig. 3. The information of DGs is summarized in TABLE I. The loads are assumed to be products of basic load components and multipliers. Fig. 4 and Fig. 5 describes the load profile and weight factors. It should be noted that all the configurations are for demonstration and can be adjusted for a variety of scenarios. The optimization horizon T is assumed to be 5 hours. A 1-hour time step Δt is adopted in simulations. The topological structure will be reconfigured at the time of restoration. DGs can be dispatched and load status can be altered every hour.

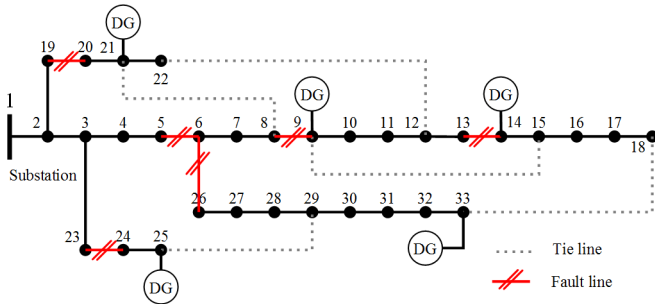


Fig. 3. A modified 33-bus test system with five DGs and multiple faults.

TABLE I. LOCATIONS AND SIZES OF DGs

No. of DG	Bus #	P_{max} (kW)	Q_{max} (kVar)	E_{total} (kWh)
5	9, 14, 21, 25, 33	450	218	1575

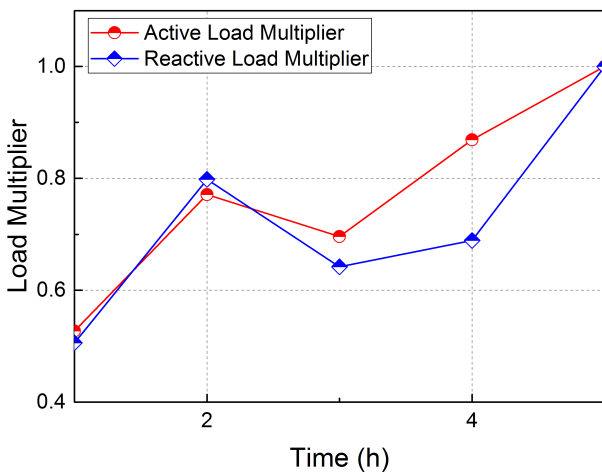


Fig. 4. Load profile multipliers.

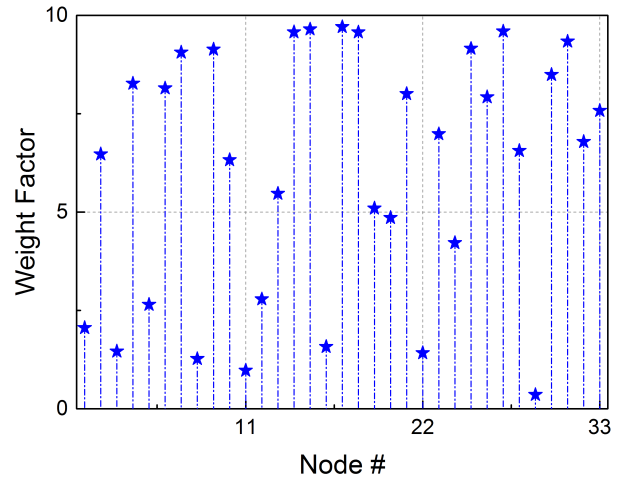


Fig. 5. Load point weight factors.

B. Simulation Results

In this scenario, an isolated zone has been partitioned after multiple faults happen in the distribution system, which cannot be supplied with electricity anyway. The remaining disrupted area will receive support from micro turbines to implement restoration by sectionalization into 5 microgrids, which can be accomplished by changing the status of sectionalizing and tie switches, as shown in Fig. 6. The simulation takes 167.35 s, in view of the large combinatorial space of the problem (i.e. binary variables indicating switches status and load curtailment status for all 5 time steps), the running time are acceptable.

Furthermore, because of the scarcity of generation resources, some non-critical load would be curtailed by opening load switches, and picking up critical loads with greater weight factor will be the top priority. TABLE II provides the generation dispatch results and load shedding in each MG for each time step. The results indicate that MG1 and MG2 are self-adequate without any load curtailment, whereas MG3, MG4 and MG5 need to curtail non-critical loads to keep power balance. In particular, MG 4 encounters severe scarcity of generation resource due to the fact that in the neighboring area of node 25, the load capacity is much greater than generation capacity with only 500 kW.

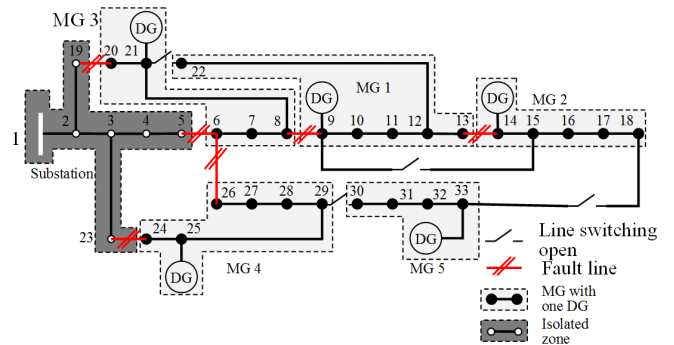


Fig. 6. Microgrids formation results for the 33-bus test system.

TABLE II. GENERATION DISPATCHES AND LOAD SHEDDING RESULTS

Time	MG #	DG #	Schedule (kW)	Load Shed (Bus #)	Load Shed (kW)
1	MG 1	DG 9	198.2	N/A	N/A
	MG 2	DG 14	206.4	N/A	N/A
	MG 3	DG 21	185.3	8, 20	152.9
	MG 4	DG 25	31.6	24, 25, 26, 28, 29	569.3
	MG 5	DG 33	222.0	30	105.4
2	MG 1	DG 9	290.4	N/A	N/A
	MG 2	DG 14	302.4	N/A	N/A
	MG 3	DG 21	341.2	8	154.2
	MG 4	DG 25	370.2	24, 26, 28, 29	508.9
	MG 5	DG 33	325.2	30	154.2
3	MG 1	DG 9	262.1	N/A	N/A
	MG 2	DG 14	273.0	N/A	N/A
	MG 3	DG 21	307.9	8	139.3
	MG 4	DG 25	334.3	24, 26, 28, 29	459.6
	MG 5	DG 33	293.5	30	139.3
4	MG 1	DG 9	327.3	N/A	N/A
	MG 2	DG 14	341.0	N/A	N/A
	MG 3	DG 21	384.6	8	173.8
	MG 4	DG 25	417.2	24, 26, 28, 29	573.5
	MG 5	DG 33	366.5	30	173.8
5	MG 1	DG 9	377.0	N/A	N/A
	MG 2	DG 14	392.9	N/A	N/A
	MG 3	DG 21	353.1	8, 20	290.0
	MG 4	DG 25	420.0	24, 26, 27, 28, 29	720.0
	MG 5	DG 33	362.2	30, 33	260.0

V. CONCLUSION

This paper presents a method to enhance the distribution system resilience against extreme events by sectionalization into microgrids after a major disaster. The DGs schedule their available energy to feed the critical loads within each microgrid while satisfying self-adequacy, topological structure and operating constraints. A virtual node and virtual branches are introduced to represent the radial structure of a distribution system.

The optimization problem is formulated as mixed-integer second-order cone programming, which can be efficiently modeled by YALMIP and solved by MOSEK. The formulation employs a convex representation of the distribution network model on the basis of the conic quadratic format of the power flow equations. Case study on the modified 33-bus test system indicates that the proposed measure can optimally utilize the

available resources to restore critical loads and therefore minimize the catastrophic impacts of natural hazards on electricity service.

ACKNOWLEDGMENT

The MOSEK Optimization has been used in this research thanks to the MOSEK free academic license.

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