

**NANYANG
TECHNOLOGICAL
UNIVERSITY**

SINGAPORE

**EXPERIMENTAL INVESTIGATION OF NON-ABELIAN
ARTIFICIAL GAUGE FIELDS: FROM $SU(2)$ TO $SU(3)$**

MADASU CHETAN SRIRAM

SCHOOL OF PHYSICAL AND MATHEMATICAL SCIENCES

2023

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SCHOOL OF PHYSICAL AND MATHEMATICAL SCIENCES

A thesis submitted to the
Nanyang Technological University in partial fulfilment
of the requirements for the degree of
Doctor of Philosophy

2023

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Madasu Chetan Sriram

Abstract

Gauge fields play a prominent role in modern physics from electromagnetic theory to standard model of particle physics. These are cutting edge mathematical tools to understand fundamental forces and interaction between sub-atomic particles. Though gauge fields are considered natural tools of study in high-energy physics, they can be extended to low-energy condensed matter physics to study the behaviour of quantum particles in special environments like electrons in a magnetic field. Apart from naturally occurring gauge fields, controllable artificial gauge fields can be used to tailor physical effects on quantum systems like neutral atoms, qubits, photons, anions etc.

This thesis reports the study of laser-induced artificial non-Abelian gauge fields interacting with an ultracold atomic wave packet in free space. In sharp contrast to the Abelian gauge fields, spatially uniform non-Abelian gauge fields can induce non-inertial motion of a particle. This motion is locked to the pseudo-spin state of the system and induces spin-orbit coupling. These non-Abelian gauge fields are experimentally realized by adiabatic evolution of the state of the atoms in the degenerate subspace of the coupling Hamiltonians. We produce SU(2) and SU(3) non-Abelian gauge fields using doubly degenerate dark state subspace of tripod and triply degenerate dark state subspace of double-tripod atom-light coupling schemes, respectively. Using the SU(2) gauge field in one dimension, we demonstrate the function of an atomtronic Datta-Das transistor (DDT) where spin polarized atoms enter the gate region and exit with their spin orientation controlled by the gate parameter analogous to the gate voltage in conventional field effect transistor. We demonstrate the geometric nature of the spin rotation in our DDT by showing insensitivity of the result to the input velocity and velocity dispersion of the atomic cloud. We have also implemented a Ramsey interferometric sequence to extract the phase of the output state of the DDT showing that the spin rotation by the DDT is coherent. In two dimensions, the dynamics of the wave packet in SU(2) gauge field show *Zitterbewegung* oscillations revealing spin Hall effect characteristics of the system. Whereas the dynamics of the wave packet in SU(3) gauge field show oscillations with multiple

frequencies emulating color-orbit coupling. These frequencies correspond to the energy separation between the eigenstates of the Hamiltonian. Using the $SU(3)$ Hamiltonian, we show two paths of reaching the same final state from the initial state demonstrating the existence of additional ladder operators in the $SU(3)$ system in addition to the well-known raising and lowering operators of angular momentum represented by $SU(2)$ symmetry. With this study, we realize a system that can be used to study the rich physics of $SU(3)$ Hamiltonians on table top experiments.

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Chapter 1

Introduction

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1.1 Introduction

In the presence of a strong magnetic field, the bulk of a 2D insulating material acts as an insulator but there exist certain special states of electrons on the boundaries of the material, known as edge states, that allow conduction leading to exactly quantized Hall conductance [1, 2] in integral units of e^2/h . Here, e is the charge of the electron and h is the Planck's constant. This effect is known as the integer quantum Hall effect [1, 3] discovered in 1980. Hall conductivity is observed to be independent of any local perturbations in the system and the number of edge states is given by a topological invariant implicating that the conductivity is topologically protected [4, 5]. Since the discovery of the integer quantum Hall effect, there has been a lot of interest in discovery and synthesis of new topological states of matter in condensed matter systems [6–8], photonics [9–11] and ultracold quantum simulators [12–15]. Quantum simulator is a unique platform where ultracold neutral atoms are placed in optical crystals formed by interfering lasers to experimentally test theories and synthesize new topological states. In this approach, well-controlled crystal structure of the material is defect-free and magnified in size compared to the naturally occurring materials that allow for a direct observation of reciprocal space characteristics of the material using time of flight imaging [12]. However, the problem with these systems is the absence of Lorentz force on the center of mass of the atoms as they are neutral. This problem is tackled by emulating the magnetic field by generating tunable artificial gauge fields. A widely used procedure to generate artificial gauge fields is by adiabatically dressing the atomic internal states with lasers [12, 16–20]. This method also offers a much higher range of magnetic field than that can be obtained by classical means [12].

Quantum Hall effect is explained by a single particle Hamiltonian without any interaction between electrons. Simulating single particle Hamiltonians with neutral atoms in free space offers insight into the topological features of the Hamiltonian. Therefore, synthesis of artificial gauge fields in free space to observe the wave packet dynamics provides a microscopic understanding of the system that can then be applied to periodic potentials [18]. With this aim in mind, spin-orbit coupling phenomena [21, 22], spin Hall effect [17] and topological features in free space [23–25] have been experimentally demonstrated using spatially non-uniform gauge fields that result in spin dependent coupling [16]. In these works, the spin-orbit coupling arises from the spatial non-uniformity of the gauge field that acts as an effective external magnetic field *i.e.*, a U(1) gauge field. In this thesis, we experimentally study the effect of spatially uniform SU(2) non-Abelian gauge fields on the dynamics of an atomic

wave packet. Though the gauge field is uniform, the non-Abelian nature of the gauge field gives rise to spin-orbit coupling Hamiltonian. The intrinsic spin-orbit coupling that causes anomalous Hall effect where Hall conductance is observed despite the absence of an external magnetic field [26] is explained by this kind of gauge field [18]. We observe *Zitterbewegung* like oscillations of the wave packet [27] evolving in the $\text{SU}(2)$ non-Abelian gauge field. Furthermore, we present preliminary results on the dynamics of the wave packet in the $\text{SU}(3)$ non-Abelian gauge field. $\text{SU}(3)$ non-Abelian gauge fields appearing in quantum chromodynamics [28] are much richer as the number of generators of the group are set of eight Gell-Mann matrices [29–32]. The study of dynamics can be used to extract topological features of the $\text{SU}(3)$ Hamiltonian and stands as a potential goal of our experiments.

1.2 ^{87}Sr : A Candidate for $\text{SU}(N)$ Quantum Simulations

Fermionic isotope of strontium ^{87}Sr (7% abundance) has no electronic spin in the ground state but has a nuclear spin of $I = 9/2$, the maximum found in the periodic table [33]. This offers 10 Zeeman levels in the ground state and any number of them can be coupled using lasers to achieve Hamiltonians of higher symmetry. Laser cooling and trapping techniques for ^{87}Sr are well established in last two decades to take advantage of the weakly allowed clock transition to build state of the art atomic clocks [34–38]. Taking advantage of the high measurement precision offered by an ultrastable laser, a spectroscopic observation of $\text{SU}(N \leq 10)$ symmetry in ^{87}Sr has been reported [39]. We use a degenerate fermi gas of ^{87}Sr atoms in our experiments to simulate $\text{SU}(2)$ and $\text{SU}(3)$ gauge fields by dressing three and five ground state Zeeman levels with lasers, respectively, under a large bias magnetic field. This study lays the groundwork for using alkaline-earth atoms as testbeds for simulating higher symmetry Hamiltonians.

1.3 Organization of Thesis

In chapter 2, the fundamentals of generation of artificial gauge fields using atom-light interaction are described. An overview of Aharonov-Bohm phase and Berry phase is described with emphasis on the similarity between magnetic vector potential and Berry connection. Later, the non-Abelian Berry connections, for tripod and double-tripod atom-light coupling schemes or equivalently $\text{SU}(2)$ and $\text{SU}(3)$ artificial gauge fields, respectively are derived. We also derive additional geometric scalar potentials that arise in these coupling schemes and

describe methods we use in calculating the theoretical expectations of velocities that are observed in the experiments.

In chapter 3, an overview of the current experimental setup and sample preparation of ultracold ^{87}Sr is described. We characterize the temperature of the sample and describe various spectroscopy methods that are employed in the determination of atomic resonances of the strontium intercombination line. We describe the implementation and characterization of a fluorescence imaging scheme that is used in extracting the momentum distribution of the ultracold gas interacting with artificial gauge fields. We describe the experimental implementation of the double-tripod scheme and characterization of the atom-light coupling by Rabi frequency measurements.

In chapter 4, implementation of an atomtronic Datta-Das transistor (DDT) using the $\text{SU}(2)$ non-Abelian gauge field in one dimension is described. We present the functionalities of a DDT and demonstrate them with our implementation. We describe a Ramsey interferometry sequence to extract the phase of the output superposition state of our DDT and reveal the effect of the geometric scalar potential.

In chapter 5, we show how that the geometric scalar potential can be canceled and obtain a pure spin-orbit coupling Hamiltonian. We present the results of *Zitterbewegung* dynamics of a wave packet under the influence of our $\text{SU}(2)$ non-Abelian gauge field. We summarize the preliminary results of dynamics of a wave packet in the $\text{SU}(3)$ non-Abelian gauge field simulating color-orbit coupling with two different initial state preparations. Finally, we show ladder-like operations between the basis states validating that our Hamiltonian actually belongs to $\text{SU}(3)$ group.

In chapter 6, we summarize the results of the thesis and provide an outlook of the research.



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Chapter 2

Theoretical Background

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2.1 Introduction

In quantum simulations using neutral atoms, the effect of electric field on (center of mass of) charged particles is simulated using AC Stark shift caused by far off-resonant lasers [1]. By engineering the intensity profile of the laser beams, harmonic potentials, uniform potential [2, 3] and optical lattices [4, 5] have been demonstrated. In the past decade, these techniques became primary tools in the field of quantum simulations. However, simulating the effect of magnetic fields on charged particles, recently developed, was missing due to the lack of net charge of the neutral atoms. In the past few years, simulating the effect of magnetic field *i.e.*, Lorentz force were developed by engineering effective Hamiltonians of the form

$$H_0(\mathbf{r}, \mathbf{p}) = \frac{(\mathbf{p} - \mathbf{A}(\mathbf{r}, t))^2}{2m} + \phi(\mathbf{r}, t) \quad (2.1)$$

where $\mathbf{A}(\mathbf{r}, t)$ and $\phi(\mathbf{r}, t)$ are the artificial vector potential and the scalar potential as functions of position $\mathbf{r}$ and time t , respectively. In this chapter, we aim to present basic underlying ideas in the generation of artificial gauge fields and derive system specific gauge potentials that are experimentally studied in this thesis.

This chapter is organized as follows: In section 2.2, we establish that the artificial gauge fields are the Berry connections with real space as the parameter space. To this end, we show the similarity in the mathematical structure of magnetic vector potential and Berry connection in quantum mechanical systems by describing the Aharonov-Bohm phase in section 2.2.1 and deriving Berry phase from adiabatic theorem in section 2.2.2 for non-degenerate Hamiltonians. In section 2.2.3, we discuss Berry connection for degenerate case as a candidate for non-Abelian gauge field. In section 2.2.4, we derive the projected Schrödinger equation for a system evolving in an n -dimensional degenerate subspace of the Hamiltonian. In section 2.3, we describe the atom-light interaction schemes that give rise to degenerate subspaces with two and three dark states *i.e.*, tripod scheme (in section 2.3.1) and double-tripod scheme (in section 2.3.3), respectively. In sections 2.3.2 and 2.3.4, we derive the SU(2) and the SU(3) artificial gauge fields associated with the tripod and the double-tripod schemes, respectively. In section 2.4, we provide a theoretical basis for simulating a momentum kick to the atoms at rest in order to explore the momentum space. In section 2.5, we describe our method of calculating theoretical expectations for the experimental results shown in chapter 5 and the chapter is concluded in section 2.6.

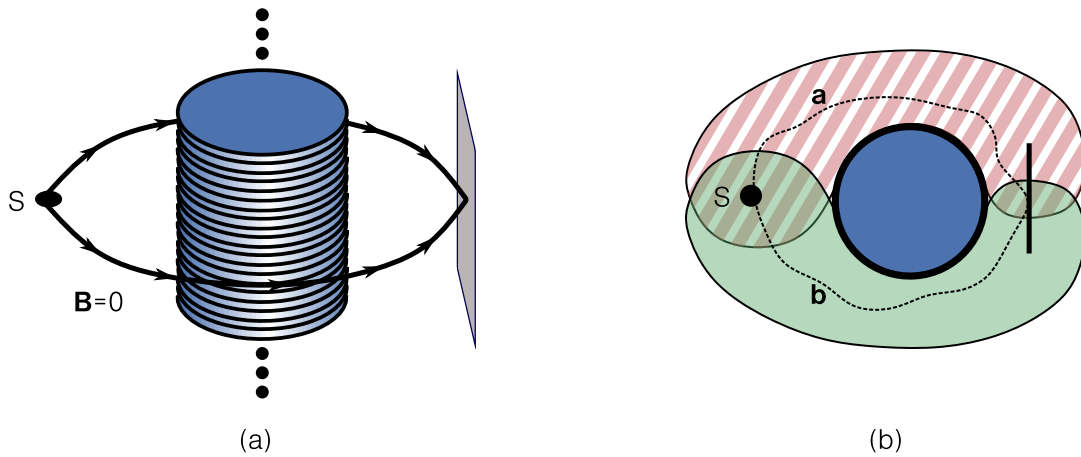


Figure 2.1: (a) Proposed experimental configuration where the Aharonov-Bohm effect can be observed. Charged particles from source S traverse through the zero magnetic field region around the infinite solenoid and form matter-wave interference pattern on the screen. Though magnetic field outside the solenoid is zero no matter the current in the coil, the interference pattern is shifted when current is non-zero. (b) Top view of the experimental setup showing two patches with different gauge functions containing the two paths.

2.2 Artificial Gauge Field: Berry Connection

In this section, we recap basic theoretical ideas used in quantum simulations of magnetism in free space. More rigorous and broad overviews on this topic can be found in references, [6–8].

2.2.1 Aharonov-Bohm Effect

In classical electrodynamics, electromagnetic potentials were introduced just as arithmetic tools. However, in quantum mechanics, the electromagnetic potentials are connected to the phase of the wave function of a charged particle that can be measured by interferometric methods. This idea was exemplified by the famous *Ehrenberg-Siday-Aharonov-Bohm effect* [9, 10]. Here, a charged particle traversing a region of zero electric and magnetic fields but a non-zero vector potential acquires different phases along different paths that depend on the local vector potential. This leads to a shift in the interference pattern at the intersection of the two paths. This phase is the hallmark of magnetism in quantum systems. In this section, we aim to describe the Aharonov-Bohm phase that is, in mathematical form, very similar to geometrical phases which form the basis of artificial magnetic fields.

Figure 2.1a shows the proposed set up for observing the Aharonov-Bohm effect. Charged particles with charge q emanating from a source S traverse around an infinite solenoid in

the zero magnetic field region to interfere and form a matter-wave interference pattern on a screen located away from the solenoid. When a current is allowed to flow in the solenoid, the magnetic field is non-zero inside the solenoid but it is still zero outside. However, the interference pattern on the screen is displaced by an amount that corresponds to a phase difference of

$$\Delta\phi_{AB} = 2\pi \times \frac{\Phi}{\Phi_0} \quad (2.2)$$

between the matter waves, where Φ is the flux through the surface bounded by the trajectories of the particles and $\Phi_0 = \frac{h}{q}$ is known as the flux quantum. This is remarkable as one could detect the presence of magnetic field by performing a measurement on charged particles that have never been in the region where the magnetic field exists. This phase does not depend on the velocity of the charged particles or their path as long as the two paths lie on either side of the solenoid. In this context, the Aharonov-Bohm phase can be considered as a topological phase as it is invariant under deformations of the trajectories of the particles. This phase is obviously gauge invariant as it is directly expressed in terms of the magnetic flux through the surface bounded by the trajectories of the particles. This can also be expressed in terms of local gauge field using Stokes' theorem [11] as follows

$$\Delta\phi_{AB} = \frac{2\pi}{\Phi_0} \iint_{\mathcal{S}} \mathbf{B} \cdot d\mathbf{S} = \frac{q}{\hbar} \oint_{\mathcal{C}} \mathbf{A} \cdot d\mathbf{l} \quad (2.3)$$

where $\mathcal{S}$ is the surface bounded by the trajectories and $\mathcal{C}$ is the contour containing the trajectories of the particles. One way of arriving at the Aharonov-Bohm phase in equation 2.2 is by working with the gauge field that is altered when the magnetic field is switched on. Consider that the gauge field, $\mathbf{A} = 0$ everywhere when the magnetic field, $\mathbf{B} = 0$ inside the solenoid. When the current in the solenoid is switched on, the gauge field outside the solenoid should be of the form

$$\mathbf{A}' = \vec{\nabla} \Lambda \quad (2.4)$$

such that $\mathbf{B} = \vec{\nabla} \times \mathbf{A}' = 0$, where Λ is a twice continuously differentiable scalar function known as the gauge function. Therefore, the action of switching on the current in the solenoid is equivalent to a gauge transformation, $\mathbf{A} = 0 \rightarrow \mathbf{A}' = \vec{\nabla} \Lambda$. Figure 2.1b shows top view of the Aharonov-Bohm setup where two patches of space **a** marked with red stripes and **b** shaded with green contain the paths of the particles from the source to the screen. Without loss of generality, one could define two gauge functions $\Lambda_{\mathbf{a}}$ and $\Lambda_{\mathbf{b}}$ respectively in these regions satisfying the equation 2.4. The wave functions of the particle traversing the regions **a** and **b**, say

$\psi_{\mathbf{a}}$ and $\psi_{\mathbf{b}}$ respectively acquire phase factors due to the gauge transformation [12] given by

$$\begin{aligned}\psi'_{\mathbf{a}} &= e^{\frac{iq\Lambda_{\mathbf{a}}}{\hbar}} \psi_{\mathbf{a}} \\ \psi'_{\mathbf{b}} &= e^{\frac{iq\Lambda_{\mathbf{b}}}{\hbar}} \psi_{\mathbf{b}}\end{aligned}\tag{2.5}$$

The matter wave interference pattern on the screen is related to the phase of

$$\begin{aligned}\psi'^*_{\mathbf{a}} \psi'_{\mathbf{b}} &= e^{\frac{iq(\Lambda_{\mathbf{b}} - \Lambda_{\mathbf{a}})}{\hbar}} \psi^*_{\mathbf{a}} \psi_{\mathbf{b}} = \exp \left[i \frac{q}{\hbar} \int_{\text{source}}^{\text{screen}} (\vec{\nabla} \Lambda_{\mathbf{b}} - \vec{\nabla} \Lambda_{\mathbf{a}}) \cdot d\mathbf{l} \right] \psi^*_{\mathbf{a}} \psi_{\mathbf{b}} \\ &= \exp \left[i \frac{q}{\hbar} \left(\int_{\text{source}}^{\text{screen}} \vec{\nabla} \Lambda_{\mathbf{b}} \cdot d\mathbf{l} + \int_{\text{screen}}^{\text{source}} \vec{\nabla} \Lambda_{\mathbf{a}} \cdot d\mathbf{l} \right) \right] \psi^*_{\mathbf{a}} \psi_{\mathbf{b}} \\ &= \exp \left[i \frac{q}{\hbar} \oint_{\mathcal{C}} \mathbf{A}' \cdot d\mathbf{l} \right] \psi^*_{\mathbf{a}} \psi_{\mathbf{b}} = \exp \left[i \frac{q}{\hbar} \iint_{\mathcal{S}} \mathbf{B} \cdot d\mathbf{S} \right] \psi^*_{\mathbf{a}} \psi_{\mathbf{b}} \\ &= e^{i\Delta_{AB}} \psi^*_{\mathbf{a}} \psi_{\mathbf{b}}\end{aligned}\tag{2.6}$$

Therefore, we recover the Aharonov-Bohm phase in equation 2.2. Note that this phase does not have a classical analog as the phase relies on the wave nature of the particles.

Instead of charged particles, one could imagine magnetic dipoles traversing a region around an electric field due to charges. The counterpart of Aharonov-Bohm phase in such a scenario is known as Aharonov-Casher phase [13] given by

$$\Delta\phi_{AC} = \frac{1}{\hbar c^2} \oint_{\mathcal{C}} \mathbf{E} \times \boldsymbol{\mu} \cdot d\mathbf{l}\tag{2.7}$$

where $\mathbf{E}$ is the electric field in the region and $\boldsymbol{\mu}$ is the magnetic moment of the particles. This phase was first observed in neutron interference experiments [14] and later in Josephson junction arrays [15].

2.2.2 Adiabatic Theorem and Berry Phase

The adiabatic theorem, originally stated by Max Born and Vladimir Fock [16], says that under the action of an adiabatic perturbation, a physical system initially in an eigenstate remains in its instantaneous eigenstate given that its eigenvalue is well separated from the rest of the eigen spectrum during the whole evolution. The instantaneous difference in the energy of the eigenstate and the next nearest eigen energy of the Hamiltonian is the deciding factor that determines if the perturbation is adiabatic or not. In this section, we describe the adiabatic theorem in its mathematical form and comment on the case where the eigen spectrum

of the Hamiltonian has a degeneracy.

Consider a time-independent Hamiltonian $H(\boldsymbol{\lambda})$ with non-degenerate eigenvalues which depends on a set of external parameters represented by a vector $\boldsymbol{\lambda}$ ¹. The time-dependent Schrödinger equation reads

$$H(\boldsymbol{\lambda})|n(\boldsymbol{\lambda})\rangle = E_n(\boldsymbol{\lambda})|n(\boldsymbol{\lambda})\rangle \quad (2.8)$$

where eigenvalues and eigenvectors of the Hamiltonian at every acceptable $\boldsymbol{\lambda}$ are labeled by n . Let us prepare the system in an eigenstate. The adiabatic theorem states that, as $\boldsymbol{\lambda}(t)$ is varied slowly such that its rate of change of $\dot{\boldsymbol{\lambda}}(t)$ is small with respect to the energy difference between the neighboring eigenstates (see 2.15), the system remains in the instantaneous eigenstate of the Hamiltonian though the eigenstate changes with the variation. To arrive at this, let us consider the time-dependent Schrödinger equation for the system

$$i\hbar \frac{\partial}{\partial t} |\psi(t)\rangle = H(\boldsymbol{\lambda}) |\psi(t)\rangle \quad (2.9)$$

where $|\psi(t)\rangle$ is the general state of the system which can be decomposed into the basis of instantaneous eigenstates $\{|m(\boldsymbol{\lambda})\rangle\}$ as

$$|\psi(t)\rangle = \sum_m c_m(t) e^{i\epsilon_m(t)} |m(\boldsymbol{\lambda})\rangle. \quad (2.10)$$

Here, $\epsilon(t)$ is the dynamical phase given by

$$\epsilon_m(t) = -\frac{1}{\hbar} \int_0^t E_m(\boldsymbol{\lambda}(t')) dt'. \quad (2.11)$$

We are interested in the evolution of the general state of the system $|\psi(t)\rangle$ which is encoded in the coefficients $c_m(t)$. By plugging in 2.10 in 2.9, we get

$$\sum_m \left(\dot{c}_m(t) e^{i\epsilon_m(t)} |m(\boldsymbol{\lambda})\rangle + c_m(t) e^{i\epsilon_m(t)} \nabla_{\boldsymbol{\lambda}} |m(\boldsymbol{\lambda})\rangle \cdot \dot{\boldsymbol{\lambda}} \right) = 0 \quad (2.12)$$

where $\nabla_{\boldsymbol{\lambda}}$ is the gradient with respect to the operator, $\boldsymbol{\lambda}$ in the parameter space. Taking inner

¹The external parameter space has to be a smooth manifold such that differentiation with respect to λ_i is well defined wherever we trace the system. This parameter space is also referred to as a configuration space.

product with $\langle n(\lambda) |$ gives us

$$\begin{aligned}\dot{c}_n(t) &= - \sum_m \left(c_m(t) e^{i(\epsilon_m(t) - \epsilon_n(t))} \langle n(\lambda) | \nabla_\lambda | m(\lambda) \rangle \cdot \dot{\lambda} \right) \\ &= i \frac{c_n(t)}{\hbar} \mathbf{A}(\lambda) \cdot \dot{\lambda} - \sum_{m \neq n} \left(c_m(t) e^{i(\epsilon_m(t) - \epsilon_n(t))} \langle n(\lambda) | \nabla_\lambda | m(\lambda) \rangle \cdot \dot{\lambda} \right)\end{aligned}\quad (2.13)$$

where $\mathbf{A}(\lambda) \equiv i\hbar \langle n(\lambda) | \nabla_\lambda | n(\lambda) \rangle$ is known as the *Berry connection* [17]². Under the adiabatic condition, the second term in equation 2.13 is proven to be close to zero. In order to see this, we can find the matrix elements of ∇_λ in the eigen basis using the time-independent Schrödinger equation in 2.8. We get

$$(E_m - E_n) \langle n(\lambda) | \nabla_\lambda | m(\lambda) \rangle = \langle n(\lambda) | \nabla_\lambda H(\lambda) | m(\lambda) \rangle \quad (2.14)$$

The second term in 2.13 is proportional to

$$\langle n(\lambda) | \nabla_\lambda | m(\lambda) \rangle \dot{\lambda} = \langle n(\lambda) | \nabla_\lambda H(\lambda) | m(\lambda) \rangle \frac{\dot{\lambda}}{(E_m - E_n)} \quad (2.15)$$

In the limit, $\dot{\lambda} \ll (E_m - E_n)$, we are only left with

$$\dot{c}_n(t) = \frac{i}{\hbar} c_n(t) \mathbf{A}(\lambda(t)) \cdot \dot{\lambda}(t) \quad (2.16)$$

During the evolution the coefficients $c_n(t)$ only depend on themselves and the path in the parameter space. If we initialize the system in the n^{th} i.e., $|\psi(0)\rangle = |n(\lambda(0))\rangle \implies c_n(0) = 1$, after time t

$$\begin{aligned}c_n(t) &= c_n(0) \exp \left(\frac{i}{\hbar} \int_0^t \mathbf{A}(\lambda(t')) \cdot \dot{\lambda}(t') dt' \right) \\ &= c_n(0) \exp \left(\frac{i}{\hbar} \int_{\lambda(0)}^{\lambda(t)} \mathbf{A}(\lambda) \cdot d\lambda \right)\end{aligned}\quad (2.17)$$

Therefore, if we start in an eigenstate of the Hamiltonian and the Hamiltonian is varied adiabatically, then we stay in the same eigenstate but acquire a geometrical phase factor. To remind again, this is valid only when the eigen energies are sufficiently far apart at all times during the evolution. This breaks down when the system has a level crossing during the evolution.

²In the original paper by M. Berry, Berry connection is defined as $\mathbf{A}(\lambda) \equiv i \langle n(\lambda) | \nabla_\lambda | n(\lambda) \rangle$ in relation to differential geometry. However, our definition resembling electromagnetic vector potential we have seen in section 2.1 is followed in the quantum simulations community and throughout this thesis.

The geometric phase acquired during adiabatic evolution of an eigenstate given by equation 2.17 is different from the well known dynamical phase. One apparent difference from the dynamical phase is that this phase does not depend on the time taken for the adiabatic evolution but depends on the path in the parameter space. Another difference is that the geometric phase is gauge dependent. If we were to choose another gauge where the instantaneous eigenstate differs by a phase factor *i.e.*, $|n'(\boldsymbol{\lambda})\rangle = e^{-\frac{i}{\hbar}\beta(\boldsymbol{\lambda})}|n(\boldsymbol{\lambda})\rangle$, then the geometric phase would differ by $\beta(\boldsymbol{\lambda}(t)) - \beta(\boldsymbol{\lambda}(0))$. Moreover, one could always choose a gauge where the geometric phase evaluates to zero except when the system is circulated through a closed contour in the parameter space. In this case, the phase factor acquired by the state of the system is gauge invariant and reads,

$$e^{i\gamma} = \exp\left(\frac{i}{\hbar} \oint_{\mathcal{C}} \mathbf{A}(\boldsymbol{\lambda}) \cdot d\boldsymbol{\lambda}\right) \quad (2.18)$$

where $\mathcal{C}$ is the contour in the parameter space. This gauge invariant phase γ is known as the *Berry phase*. The Berry phase is contour dependent and unique up to modulo 2π . The Berry phase resembles the Aharonov-Bohm phase of electrodynamics as shown in equation 2.2 for charge particles of charge $q = 1$. We also can define the equivalent of gauge invariant magnetic field strength for the Berry connection. This is known as the *Berry curvature* (mathematically, a tensor³) of the Berry connection defined as

$$D_{ij} = \frac{\partial A_j}{\partial \lambda_i} - \frac{\partial A_i}{\partial \lambda_j} \quad (2.19)$$

Berry phase solely depends on the geometry of the parameter space *i.e.*, its curvature. In terms of curvature, the Berry phase factor can be written as

$$e^{i\gamma} = \exp\left(\frac{i}{\hbar} \oint_{\mathcal{C}} \mathbf{A}(\boldsymbol{\lambda}) \cdot d\boldsymbol{\lambda}\right) = \exp\left(\frac{i}{\hbar} \int_{\mathcal{S}} \mathbf{D} \cdot d\mathbf{S}\right) \quad (2.20)$$

where $\mathcal{S}$ is a surface bounded by $\mathcal{C}$. Trivially, Berry phase is zero when the parameter space is one dimensional as there does not exist a surface in 1D.

³This can also be represented in a pseudo-vector form as $\mathbf{D} = \vec{\nabla}_{\boldsymbol{\lambda}} \times \mathbf{A}$ using $D_{ij} = \epsilon_{ijk} D_k$ so that it resembles the relationship between magnetic field and vector potential.

2.2.3 Berry Connection as Artificial Gauge Field

As we have seen in previous sections, the electromagnetic vector potential exactly resembles the Berry connection that appears in adiabatic evolution of quantum systems. In quantum simulation protocols with neutral atoms, this resemblance is taken advantage of in implementing the effect of Lorentz force on charged particles. As a simple toy model, adiabatic following of a dressed state of a two-level atom coupled to an electromagnetic radiation [7] in the parameter space spanned by the Rabi frequency and detuning of the coupling simulates a spin half system placed in a magnetic field. We could also generate the Lorentz force by rotating a cold gas of neutral atoms in rotationally symmetric potentials [18, 19]. In a rotating frame, the Coriolis force acts as the Lorentz force. In these works, the artificial gauge field or the Berry connection is a vector field that contributes a phase factor to the state of the system. So, the effect of these gauge fields on the state of the system is described by a unitary operation that belongs to $U(1)$ symmetry group just like the effect of electromagnetic vector potentials. However, when we apply the adiabatic theorem to a state that is one of the degenerate eigenstates, the Berry connection is no longer a vector field with scalar components but a vector field with matrix valued components. In such a scenario, the evolution of the state of the system is projected to local Hilbert space spanned by the degenerate eigenstates and the Berry connection is a vector field with matrix valued components. In the following section this projection into the degenerate subspace is described. In this case, the Berry curvature takes the form

$$D_{ij} = \frac{\partial A_j}{\partial \lambda_i} - \frac{\partial A_i}{\partial \lambda_j} - \frac{i}{\hbar} [A_i, A_j] \quad (2.21)$$

where $[\cdot, \cdot]$ is the usual commutator. At first sight this looks odd for two reasons: first, the Berry curvature contains a commutation between the components of the gauge field implying that it is not gauge invariant unlike the previous case and second, there exists a commutator between gauge field components that indicate spin like characteristics of the curvature. To get an intuitive sense of the equation, we can use the Landau levels that appear in the presence of a large magnetic field B_z as an example. The Hamiltonian that determines cyclotron orbits of a charged particle with charge q reads,

$$H_{QHE} = \frac{\Pi_x^2 + \Pi_y^2}{2m} \quad (2.22)$$

where $\Pi = \mathbf{p} - q\mathbf{A}$ is the mechanical momentum and $\mathbf{A}$ in this context is the electromagnetic vector potential that is related to the applied magnetic field B_z . This Hamiltonian can be solved just like the harmonic oscillator where $\mathbf{x}$ and $\mathbf{p}$ are replaced by Π_x and Π_y and the

commutation relationship $[\mathbf{x}, \mathbf{p}] = i\hbar$ replaced with $[\Pi_x, \Pi_y] = i\hbar q B_z$. If we make B_z as the subject in the later commutation relation, we have

$$\begin{aligned} B_z &= -\frac{i}{\hbar q} [p_x - qA_x, p_y - qA_y] \\ &= -\frac{i}{\hbar q} ([p_x, p_y] - [p_x, qA_y] - [p_y, -qA_x] + q^2[A_x, A_y]) \\ &= \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} - \frac{iq}{\hbar} [A_x, A_y] \end{aligned} \quad (2.23)$$

Here, we have used $[p_x, f(x, y)] = -i\hbar \frac{\partial f(x, y)}{\partial x}$. Therefore, we recover the equation 2.21 (with $q = 1$) for gauge independent equivalent of the magnetic field. In case of U(1) gauge potentials like electromagnetic vector potentials, the last term in the above equation is zero and we have usual $B_z = (\nabla \times \mathbf{A})_z$ relation from electromagnetism. In a general case however this is not true. If components of the Berry connection commute with each other, then the artificial gauge field is known as the Abelian gauge field where the Berry curvature is the artificial magnetic field. Spin-Hall effect in cold gasses has been simulated using spatially inhomogeneous Raman couplings in a Λ -scheme generating an artificial Abelian gauge field considering the two internal states as the pseudo-spin states [20]. On the other hand, if the components of resulting artificial gauge field do not commute with each other, then the gauge field is non-Abelian and we can observe effects of magnetic field-like phenomena even when the gauge field is uniform. In the later case, the geometric phase factor known as Wilczek-Zee phase or generally non-Abelian holonomy reads

$$e^{i\gamma} = \mathcal{P} \exp \left(\frac{i}{\hbar} \oint_{\mathcal{C}} \mathbf{A}(\boldsymbol{\lambda}) \cdot d\boldsymbol{\lambda} \right) = \mathcal{P} \exp \left(\frac{i}{\hbar} \int_{\mathcal{S}} \mathbf{D} \cdot d\mathbf{S} \right) \quad (2.24)$$

where $\mathcal{P}$ is the path-ordering operator. The Wilczek-Zee phase is path dependent, however, the trace of the Wilczek-Zee phase known as Wilson line or Wilson loop is path independent and gauge invariant⁴. This was originally proposed in high energy physics as a theoretical tool but was never really observed in high energy physics [21]. However, in condensed matter physics and synthetic quantum matter, these tools are of tremendous interest. Experimental measurement of the path-dependent Wilczek-Zee phase has been done in a two-band system in an optical lattice and characterized its matrix elements [21]. In free space, non-Abelian holonomic quantum gates have been experimentally realized recently [22] and Wilson lines were also characterized [23].

⁴In condensed matter literature, the terms Wilczek-Zee phase, Wilson line and Wilson loop are used synonymously.

2.2.4 Projected Schrödinger Equation in the Degenerate Subspace

The adiabatic theorem described in section 2.2.2 was extended to Hamiltonians with degenerate eigen spectra by F. Wilczek and A. Zee [24]. As stated previously, the state of the system initially in one of the degenerate eigenstates simply evolves in the subspace spanned by the degenerate eigenstates under adiabatic variation of the Hamiltonian in its parameter space. This evolution among the degenerate eigenstates can be theoretically described by the matrix valued Wilczek-Zee phase or equivalently by a matrix valued artificial gauge potential as stated previously. The degenerate dark states are functions of the parameters of the parameter space. In our case these parameters are space and time. This can arise from the dependence of phase and Rabi frequencies of the tripod lasers on space and time, respectively. The state of an atom evolving in the degenerate subspace spanned by n dark-states reads,

$$|\psi(\mathbf{r}, t)\rangle = \sum_{j=1}^n \chi_j(\mathbf{r}, t) |D_j(\mathbf{r}, t)\rangle. \quad (2.25)$$

In the absence of external potentials, this state evolves under the free particle's Hamiltonian, $H_0 = \frac{\mathbf{p}^2}{2m}$. This Hamiltonian evolution can also be described just by a column vector $\boldsymbol{\psi} = (\chi_1(\mathbf{r}, t), \chi_2(\mathbf{r}, t), \dots, \chi_n(\mathbf{r}, t))^T$ by absorbing the spatial and temporal dependence of the dark-states into the Hamiltonian of the form 2.1. This results in a $n \times n$ matrix Hamiltonian of the form 2.1 with matrix valued $\mathbf{A}$ that is at the heart of the experiments of this thesis. In this section, we show the projection of the Hamiltonian into a general dark state basis. Schrödinger equation for evolution of state $|\psi(\mathbf{r}, t)\rangle$ reads,

$$i\hbar \frac{\partial}{\partial t} \sum_{j=1}^n \chi_j(\mathbf{r}, t) |D_j(\mathbf{r}, t)\rangle = \frac{\mathbf{p}^2}{2m} \sum_{j=1}^n \chi_j(\mathbf{r}, t) |D_j(\mathbf{r}, t)\rangle. \quad (2.26)$$

Expanding this,

$$\begin{aligned} & i\hbar \sum_{j=1}^n \left(\frac{\partial}{\partial t} \chi_j(\mathbf{r}, t) |D_j(\mathbf{r}, t)\rangle + \chi_j(\mathbf{r}, t) \frac{\partial}{\partial t} |D_j(\mathbf{r}, t)\rangle \right) \\ &= \frac{-\hbar^2}{2m} \sum_{j=1}^n \left(\chi_j(\mathbf{r}, t) \nabla^2 |D_j(\mathbf{r}, t)\rangle + 2\nabla \chi_j(\mathbf{r}, t) \cdot \nabla |D_j(\mathbf{r}, t)\rangle + \nabla^2 \chi_j(\mathbf{r}, t) |D_j(\mathbf{r}, t)\rangle \right) \end{aligned} \quad (2.27)$$

Taking inner product with $\langle D_k(\mathbf{r}, t) |$ for $k = 1, 2, \dots, n$ on both sides results in n coupled differential equations of the form,

$$\begin{aligned} i\hbar \sum_{j=1}^n \left(\frac{\partial}{\partial t} \chi_j(\mathbf{r}, t) \delta_{kj} + \chi_j(\mathbf{r}, t) \langle D_k(\mathbf{r}, t) | \frac{\partial}{\partial t} | D_j(\mathbf{r}, t) \rangle \right) \\ = \frac{-\hbar^2}{2m} \sum_{j=1}^n \left(\chi_j(\mathbf{r}, t) \langle D_k(\mathbf{r}, t) | \nabla^2 | D_j(\mathbf{r}, t) \rangle + 2\nabla \chi_j(\mathbf{r}, t) \cdot \langle D_k(\mathbf{r}, t) | \nabla | D_j(\mathbf{r}, t) \rangle + \nabla^2 \chi_j(\mathbf{r}, t) \delta_{kj} \right) \end{aligned} \quad (2.28)$$

where δ_{kj} is the Kronecker delta function. Simplifying equation 2.28 results in Schrödinger equation for the k^{th} component of the vector $\boldsymbol{\psi}$ that reads,

$$\begin{aligned} i\hbar \frac{\partial}{\partial t} \chi_k(\mathbf{r}, t) = \frac{-\hbar^2}{2m} \nabla^2 \chi_k(\mathbf{r}, t) - \frac{1}{m} \sum_{j=1}^n -i\hbar \nabla \chi_j(\mathbf{r}, t) \cdot i\hbar \langle D_k(\mathbf{r}, t) | \nabla | D_j(\mathbf{r}, t) \rangle + \\ \frac{-\hbar^2}{2m} \sum_{j=1}^n \langle D_k(\mathbf{r}, t) | \nabla^2 | D_j(\mathbf{r}, t) \rangle \chi_j(\mathbf{r}, t) - i\hbar \sum_{j=1}^n \langle D_k(\mathbf{r}, t) | \frac{\partial}{\partial t} | D_j(\mathbf{r}, t) \rangle \chi_j(\mathbf{r}, t) \end{aligned} \quad (2.29)$$

Making the following definitions,

$$\mathbf{A}_{kj} \equiv i\hbar \langle D_k(\mathbf{r}, t) | \nabla | D_j(\mathbf{r}, t) \rangle \quad (2.30)$$

$$Q_{kj} \equiv \frac{-\hbar^2}{2m} \langle D_k(\mathbf{r}, t) | \nabla^2 | D_j(\mathbf{r}, t) \rangle \quad (2.31)$$

$$\omega_{kj} \equiv -i\hbar \langle D_k(\mathbf{r}, t) | \frac{\partial}{\partial t} | D_j(\mathbf{r}, t) \rangle \quad (2.32)$$

and substituting them in equation 2.29 gives,

$$i\hbar \frac{\partial}{\partial t} \chi_k(\mathbf{r}, t) = \frac{-\hbar^2}{2m} \nabla^2 \chi_k(\mathbf{r}, t) - \frac{1}{m} \sum_{j=1}^n -i\hbar \nabla \chi_j(\mathbf{r}, t) \cdot \mathbf{A}_{kj} + \sum_{j=1}^n Q_{kj} \chi_j(\mathbf{r}, t) + \sum_{j=1}^n \omega_{kj} \chi_j(\mathbf{r}, t) \quad (2.33)$$

Here, the matrix $\mathbf{A}$ is the Berry connection that will be interpreted from now on as the artificial gauge field and Q is the geometric scalar potential. These n coupled differential equations can be written as the following matrix equation

$$i\hbar \frac{\partial}{\partial t} \boldsymbol{\psi} = \left(\frac{\mathbf{p}^2}{2m} \otimes \mathbf{I}_n - \frac{\mathbf{A} \cdot \mathbf{p} \otimes \mathbf{I}_n}{m} + Q + \omega \right) \boldsymbol{\psi} \quad (2.34)$$

where $\mathbf{I}_n$ is the n -dimensional identity matrix in the dark state subspace. Since $\mathbf{A}$ and $\mathbf{p} \otimes \mathbf{I}_n \equiv \mathbf{p}$ commute with each other, the above equation can be rearranged to the following:

$$i\hbar \frac{\partial}{\partial t} \boldsymbol{\psi} = \left(\frac{(\mathbf{p} - \mathbf{A})^2}{2m} + W + \omega \right) \boldsymbol{\psi} \quad (2.35)$$

where $W = Q - \frac{A^2}{2m}$ is the net geometric scalar potential. This is the form of Hamiltonian (equation 2.1) we set out to simulate with neutral atoms.

2.3 Non-Abelian Gauge Fields Using Atom-Light Interaction

In this section, we describe the atom-light interaction schemes that give rise to SU(2) and SU(3) non-Abelian Berry connections in free space using a tripod and a double tripod scheme as proposed in [25, 26], respectively. As opposed to artificial gauge fields in lattices [27], the dynamical variables in these systems are relatively easy to calculate and offer a direct understanding of the single particle Hamiltonian. This is one of the motivations to experimentally observe the dynamics and apply it to periodic potentials. We start with atom-light coupling Hamiltonians in the bare state basis with Rabi frequencies and detunings as the parameters and arrive at artificial gauge potentials in the Hamiltonians in the dark state basis of the form of equation 2.35.

2.3.1 Tripod Scheme

The tripod scheme, an extension of the well-known three-level Λ -system, consists of three ground states coupled to one excited state as shown in figure 2.2a. The Rabi frequencies of these couplings are Ω_1, Ω_2 and Ω_3 for σ^+ , π and σ^- (see figure 2.2c) transitions, respectively. Using appropriate unitary transformations and applying rotating-wave approximation (RWA), the time-dependence of the internal state Hamiltonian due to laser fields can be removed [28, 29]. This internal state Hamiltonian in $\{|e\rangle, |1\rangle, |2\rangle, |3\rangle\}$ basis reads,

$$H_I = \frac{\hbar}{2} \begin{pmatrix} 0 & \Omega_1(\mathbf{r}, t)e^{i\phi_1} & \Omega_2(\mathbf{r}, t)e^{i\phi_2} & \Omega_3(\mathbf{r}, t)e^{i\phi_3} \\ \Omega_1(\mathbf{r}, t)e^{-i\phi_1} & -2\delta_1 & 0 & 0 \\ \Omega_2(\mathbf{r}, t)e^{-i\phi_2} & 0 & -2\delta_2 & 0 \\ \Omega_3(\mathbf{r}, t)e^{-i\phi_3} & 0 & 0 & -2\delta_3 \end{pmatrix} \quad (2.36)$$

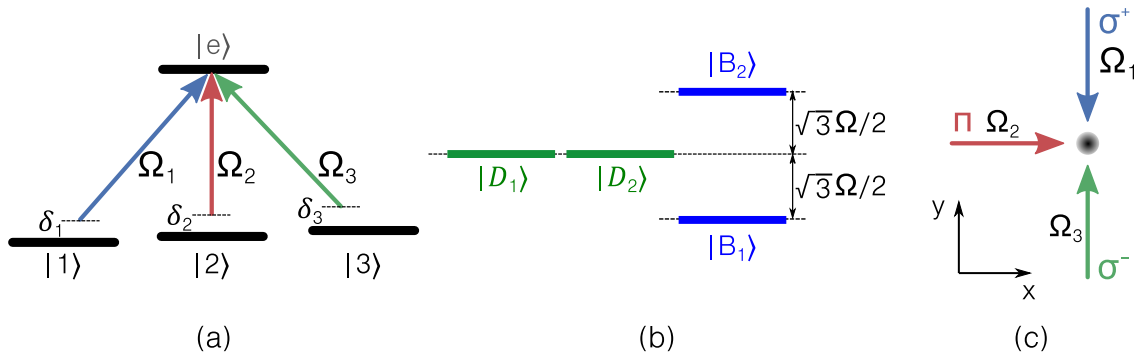


Figure 2.2: (a) Atom-light coupling schematic of a tripod scheme showing polarizations and Rabi frequencies of coupling lasers. (b) Dressed state picture of the atom-light coupling showing two degenerate dark states and bright states when the Rabi frequencies of the tripod lasers are equal, they are at resonance and $\Omega = \sqrt{|\Omega_1|^2 + |\Omega_2|^2 + |\Omega_3|^2}$. (c) The configuration of tripod laser beams showing propagation directions that gives rise to a non-Abelian gauge field implemented in the experiments described in chapters 4 and 5.

where δ_1, δ_2 and δ_3 are the detunings and ϕ_1, ϕ_2 and ϕ_3 are the phases of the σ^+ , π and σ^- lasers, respectively. In two-photon resonance conditions *i.e.*, $\delta_1 = \delta_2 = \delta_3 = 0$, the Hamiltonian in 2.36 has two null eigen energies and two non-zero eigen energies. Consequently, in the dressed state picture, this leads to two degenerate dark-states, the eigenstates of zero eigen energy and two bright-states of non-zero eigen energy [28, 30] as schematically shown in figure 2.2b. These dark-states are superpositions of only ground states and do not contain the excited state $|e\rangle$. Hence, these states are protected from spontaneous emission from the excited state and have a long lifetime. These dark-states can be chosen in terms of the laser parameters as

$$\begin{aligned} |D_1\rangle &= \sin\varphi e^{i(\phi_3-\phi_1)} |1\rangle - \cos\varphi e^{i(\phi_3-\phi_2)} |2\rangle \\ |D_2\rangle &= \cos\vartheta (\cos\varphi e^{i(\phi_3-\phi_1)} |1\rangle + \sin\varphi e^{i(\phi_3-\phi_2)} |2\rangle) - \sin\vartheta |3\rangle \end{aligned} \quad (2.37)$$

where the mixing angles $\varphi = \arctan\left(\frac{|\Omega_2|}{|\Omega_1|}\right)$ and $\vartheta = \arccos\left(\frac{|\Omega_3|}{\sqrt{|\Omega_1|^2 + |\Omega_2|^2 + |\Omega_3|^2}}\right)$ ⁵. With large Rabi frequencies, the bright and dark states can be adiabatically decoupled. Hence, by preparing an initial state as a coherent superposition of dark states, the state of the system evolves in the subspace spanned by the degenerate dark-states $|D_1\rangle$ and $|D_2\rangle$ [31]. These two dark states form the pseudo-spin- $\frac{1}{2}$ basis for the experiments described in chapters 4 & 5 where we show action of SU(2) artificial gauge field on the evolution of the pseudo-spin- $\frac{1}{2}$ system.

⁵The dependence of Rabi frequencies on $\mathbf{r}$ and t are dropped for notational simplicity

2.3.2 SU(2) Gauge Field

In this section, the SU(2) artificial gauge field that is generated in the experiments of this thesis is derived. Theoretical works on generating spin-orbit coupling with the tripod-scheme have also described this in references [7, 25, 31]. Nonetheless, we derive it for the specific cases that are necessary to understand the results of chapters 4 and 5. Here, we also establish the coordinate system and the notation that is used for the rest of this thesis.

The tripod scheme discussed in section 2.3.1 is implemented using three Gaussian laser beams of which two beams are counter-propagating and the other is propagating perpendicular to the counter-propagating beams as shown in figure 2.2c. As the size of the Gaussian beams is much larger than the size of the atomic cloud (see table 3.2), these can be modeled by plane waves in their interaction zone with the atoms. Considering the Rabi frequencies coupling the tripod levels as plane waves, the dark-states of the tripod scheme in equation 2.37 reads,

$$\begin{aligned} |D_1\rangle &= \sin\varphi e^{i(2ky - (\delta_1 - \delta_3)t)} |1\rangle - \cos\varphi e^{i(k(y-x) - (\delta_2 - \delta_3)t)} |2\rangle \\ |D_2\rangle &= \cos\vartheta (\cos\varphi e^{i(2ky - (\delta_1 - \delta_3)t)} |1\rangle + \sin\varphi e^{i(k(y-x) - (\delta_2 - \delta_3)t)} |2\rangle) - \sin\vartheta |3\rangle \end{aligned} \quad (2.38)$$

where k is the wave number of the tripod lasers. With these definitions of dark states, the geometric vector potential $\mathbf{A}$ in equation 2.30 reads,

$$\begin{aligned} \mathbf{A} &= \hbar k \begin{pmatrix} \cos^2\varphi & -\cos\vartheta \sin 2\varphi/2 \\ -\cos\vartheta \sin 2\varphi/2 & \cos^2\vartheta \sin^2\varphi \end{pmatrix} \hat{\mathbf{e}}_x - \hbar k \begin{pmatrix} 1 + \sin^2\varphi & \cos\vartheta \sin 2\varphi/2 \\ \cos\vartheta \sin 2\varphi/2 & \cos^2\vartheta (1 + \cos^2\varphi) \end{pmatrix} \hat{\mathbf{e}}_y + \\ &\quad \hbar \cos\vartheta \begin{pmatrix} 0 & -i\vec{\nabla}\varphi \\ i\vec{\nabla}\varphi & 0 \end{pmatrix} \end{aligned} \quad (2.39)$$

Here, $\vec{\nabla}\varphi = 0$ as the Rabi frequencies of the tripod lasers are uniform over the size of the atomic cloud. The geometric scalar term Q in 2.31 reads,

$$Q = -\frac{\hbar^2 k^2}{m} \begin{pmatrix} 1 + \sin^2\varphi & \cos\vartheta \sin 2\varphi/2 \\ \cos\vartheta \sin 2\varphi/2 & \cos^2\vartheta (1 + \cos^2\varphi) \end{pmatrix} \quad (2.40)$$

and the matrix ω (defined in 2.32) reads,

$$\omega = -\hbar \begin{pmatrix} \delta_1 \sin^2 \varphi + \delta_2 \cos^2 \varphi - \delta_3 & \cos \vartheta \sin 2\varphi (\delta_1 - \delta_2)/2 \\ \cos \vartheta \sin 2\varphi (\delta_1 - \delta_2)/2 & \cos^2 \vartheta (\delta_1 \cos^2 \varphi + \delta_2 \sin^2 \varphi - \delta_3) \end{pmatrix} + \hbar \cos \vartheta \begin{pmatrix} 0 & -i \frac{\partial \varphi}{\partial t} \\ i \frac{\partial \varphi}{\partial t} & 0 \end{pmatrix} \quad (2.41)$$

When the tripod lasers are set to resonance, the matrix Q breaks the degeneracy of the dark states. For constant Rabi frequencies $\frac{\partial \varphi}{\partial t} = 0$, we can set the detunings such that $\omega + Q = 0$ and find that the Hamiltonian of the system

$$H = \frac{\mathbf{p}^2}{2m} - \frac{\mathbf{p} \cdot \mathbf{A}}{m} \quad (2.42)$$

resembles the spin-orbit coupling Hamiltonian. For equal Rabi frequencies of the tripod beams $\varphi = \pi/4$ and $\vartheta = \cos^{-1}(\sqrt{1/3})$ and eigenvalues of this Hamiltonian are

$$E_{\pm} = \frac{p_x^2 + p_y^2}{2m} - \frac{\hbar k(p_x - 3p_y)}{3m} \pm \frac{\sqrt{p_x^2 + 3p_y^2}}{3m} \quad (2.43)$$

Figure 2.3a shows the eigenvalues of this Hamiltonian (removing the identity part as it can be gauged away by a gauge transformation) as a function of p_x and p_y . This reveals a Dirac point at $\mathbf{p} = 0$. The eigenvalues of our real symmetric Hamiltonian are equivalent to the eigenvalues of a spin- $\frac{1}{2}$ in a magnetic field $\mathbf{B} = (B_x, 0, B_z)$ where the roles of B_x and B_z is played by linear combinations of p_x and p_y [32]. However, the dispersion is asymmetric. So it can be seen as a spin- $\frac{1}{2}$ whose magnetic moment depends on the direction of the magnetic field. Therefore, if we were to make a second adiabatic approximation where we prepare the system in an eigenstate and adiabatically vary its $\mathbf{p}$ to make a closed loop around the Dirac point, then the state will acquire a geometric phase of $\pm\pi$ [32] (depending on the state) equivalent to a winding number of ± 1 similar to graphene in the first Brillouin zone. Note that the Berry connection in the second adiabatic evolution is Abelian and the Berry curvature at $\mathbf{p} = 0$ diverges as in the case of a Dirac monopole [32].

2.3.3 Double-Tripod Scheme

The second type of atom-light coupling we implement is the double-tripod configuration with five ground states and two excited states. Figure 2.4a shows the energy level diagram of the double-tripod configuration. The Hamiltonian (after applying rotating wave approximation) in the bare state basis $\{|1\rangle, |2\rangle, |3\rangle, |4\rangle, |5\rangle, |e_1\rangle, |e_2\rangle\}$ reads,

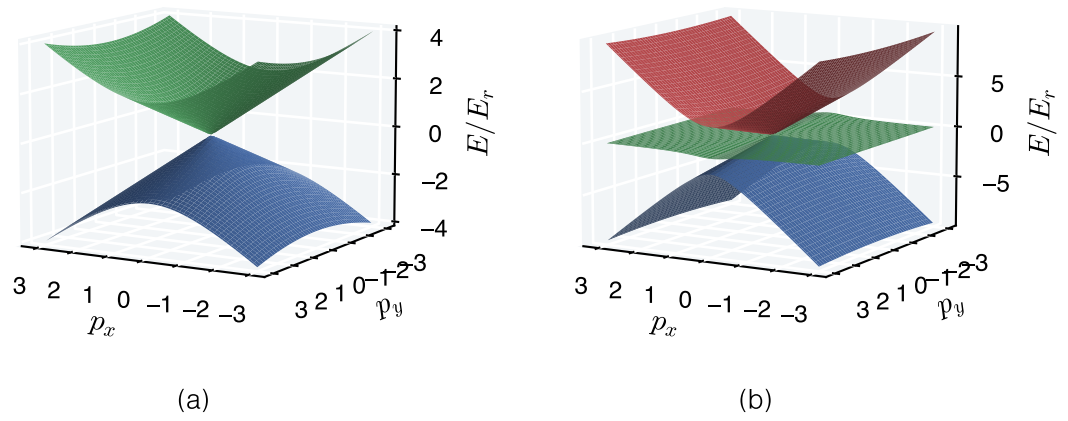


Figure 2.3: Dispersion relation of SU(2) (a) and SU(3) (b) Hamiltonians neglecting the terms proportional to the identity. p_x and p_y are in the units of recoil momentum $p_r = \hbar k$ and $E_r = \hbar^2 k^2 / 2m$.

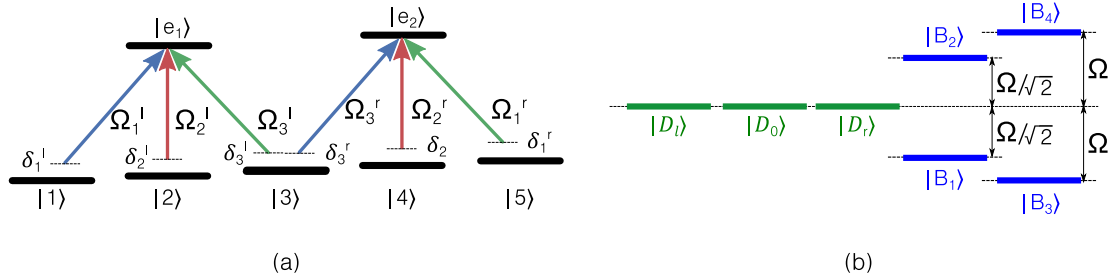


Figure 2.4: (a) Atom-light coupling schematic of a double-tripod scheme showing ground and excited state Zeeman sub-levels and targeting lasers. (b) Dressed state picture of the atom-light coupling showing dark and bright states for the case of equal Rabi frequencies (Ω) for all the targeting lasers.

$$H_I = \frac{1}{2} \begin{pmatrix} -2\delta_1^l & 0 & 0 & 0 & 0 & \Omega_1^l e^{i\phi_1^l} & 0 \\ 0 & -2\delta_2^l & 0 & 0 & 0 & \Omega_2^l e^{i\phi_2^l} & 0 \\ 0 & 0 & -(\delta_3^l + \delta_3^r) & 0 & 0 & \Omega_3^l e^{i\left(\frac{\delta_3^l - \delta_3^r}{2} + \phi_3^l\right)} & \Omega_3^r e^{i\left(\frac{\delta_3^l - \delta_3^r}{2} + \phi_3^r\right)} \\ 0 & 0 & 0 & -2\delta_2^r & 0 & 0 & \Omega_2^r e^{i\phi_2^r} \\ 0 & 0 & 0 & 0 & -2\delta_1^r & 0 & \Omega_1^r e^{i\phi_1^r} \\ \Omega_1^l e^{-i\phi_1^l} & \Omega_2^l e^{-i\phi_2^l} & \Omega_3^l e^{-i\left(\frac{\delta_3^r - \delta_3^l}{2} + \phi_3^l\right)} & 0 & 0 & 0 & 0 \\ 0 & 0 & \Omega_3^r e^{-i\left(\frac{\delta_3^l - \delta_3^r}{2} + \phi_3^l\right)} & \Omega_2^r e^{-i\phi_2^r} & \Omega_1^r e^{-i\phi_1^r} & 0 & 0 \end{pmatrix} \quad (2.44)$$

In the dressed state picture, this Hamiltonian with lasers at resonance gives rise to three degenerate dark states and four bright states as shown in figure 2.4b for equal Rabi frequencies of the tripod lasers [26].

2.3.4 SU(3) Gauge Field

The double-tripod configuration is implemented by producing sidebands using EOMs in the path of the tripod beams (see figure 3.12). The three degenerate dark-states with the orienta-

tion of the tripod beams shown in figure 2.2c can be chosen as [26]

$$\begin{aligned}
|D_l\rangle &= \sin\varphi_l e^{4iky - i(\delta_1^l - \delta_3^l)t} |1\rangle - \cos\varphi_l e^{ik(-x+3y) - i(\delta_2^l - \delta_3^l)t} |2\rangle \\
|D_0\rangle &= \frac{1}{\alpha_0} [\cot\vartheta_l (\cos\varphi_l e^{4iky - i(\delta_1^l - \delta_3^l)t} |1\rangle + \sin\varphi_l e^{ik(-x+3y) - i(\delta_2^l - \delta_3^l)t} |2\rangle) - e^{2iky} |3\rangle + \\
&\quad \cot\vartheta_r (\sin\varphi_r e^{ik(-x+y) - i(\delta_2^r - \delta_3^r)t} |4\rangle + \cos\varphi_r e^{-i(\delta_1^r - \delta_3^r)t} |5\rangle)] \\
|D_r\rangle &= \sin\varphi_r e^{-i(\delta_1^r - \delta_3^r)t} |5\rangle - \cos\varphi_r e^{ik(-x+y) - i(\delta_2^r - \delta_3^r)t} |4\rangle
\end{aligned} \tag{2.45}$$

where $\varphi_a = \arctan(|\Omega_2^a|/|\Omega_1^a|)$, $\vartheta = \arccos\left(\frac{|\Omega_3^a|}{\sqrt{|\Omega_1^a|^2 + |\Omega_2^a|^2 + |\Omega_3^a|^2}}\right)$ for $a = l, r$ corresponding respectively to left and right tripod lasers and $\alpha_0 = \sqrt{1 + \cot^2\vartheta^l + \cot^2\vartheta^r}$. Here Ω_i^a are the Rabi-frequencies and δ_i^a are the detunings of the double tripod lasers. Using these dark-states, the matrices 2.30, 2.31 and 2.32 read,

$$\begin{aligned}
\mathbf{A} &= \hbar k \begin{pmatrix} \cos^2\varphi_l & -\cot\vartheta_l \sin 2\varphi_l / 2\alpha_0 & 0 \\ -\cot\vartheta_l \sin 2\varphi_l / 2\alpha_0 & (\cot^2\vartheta_l \sin^2\varphi_l + \cot^2\vartheta_r \sin^2\varphi_r) / \alpha_0^2 & -\cot\vartheta_r \sin 2\varphi_r / 2\alpha_0 \\ 0 & -\cot\vartheta_r \sin 2\varphi_r / 2\alpha_0 & \cos^2\varphi_r \end{pmatrix} \hat{\mathbf{e}}_x - \\
&\quad \hbar k \begin{pmatrix} (3 + \sin^2\varphi_l) & \cot\vartheta_l \sin 2\varphi_l / 2\alpha_0 & 0 \\ \cot\vartheta_l \sin 2\varphi_l / 2\alpha_0 & (\cot^2\vartheta_l (3 + \cos^2\varphi_l) + \cot^2\vartheta_r \sin^2\varphi_r + 2) / \alpha_0^2 & -\cot\vartheta_r \sin 2\varphi_r / 2\alpha_0 \\ 0 & -\cot\vartheta_r \sin 2\varphi_r / 2\alpha_0 & \cos^2\varphi_r \end{pmatrix} \hat{\mathbf{e}}_y + \\
&\quad - \frac{i\hbar}{\alpha_0} \begin{pmatrix} 0 & -\cot\vartheta_l \vec{\nabla}\varphi_l & 0 \\ \cot\vartheta_l \vec{\nabla}\varphi_l & 0 & \cot\vartheta_r \vec{\nabla}\varphi_r \\ 0 & -\cot\vartheta_r \vec{\nabla}\varphi_r & 0 \end{pmatrix}
\end{aligned} \tag{2.46}$$

$$\mathbf{Q} = -\frac{\hbar^2 k^2}{m} \begin{pmatrix} (5 + 3\sin^2\varphi_l) & 3\cot\vartheta_l \sin 2\varphi_l / 2\alpha_0 & 0 \\ 3\cot\vartheta_l \sin 2\varphi_l / 2\alpha_0 & (\cot^2\vartheta_l (5 + 3\cos^2\varphi_l) + \cot^2\vartheta_r \sin^2\varphi_r + 2) / \alpha_0^2 & -\cot\vartheta_r \sin 2\varphi_r / 2\alpha_0 \\ 0 & -\cot\vartheta_r \sin 2\varphi_r / 2\alpha_0 & \cos^2\varphi_r \end{pmatrix} \tag{2.47}$$

$$\begin{aligned}
\omega &= -\hbar \begin{pmatrix} \delta_1^l \sin^2\varphi_l + \delta_2^l \cos^2\varphi_l - \delta_3^l & \frac{1}{2\alpha_0} (\cot\vartheta_l \sin 2\varphi_l (\delta_1^l - \delta_2^l)) & 0 \\ \frac{1}{2\alpha_0} (\cot\vartheta_l \sin 2\varphi_l (\delta_1^l - \delta_2^l)) & \frac{1}{\alpha_0^2} \sum_{a=\{l,r\}} \cot^2\vartheta_a (\delta_2^a \sin^2\varphi_a + \delta_1^a \cos^2\varphi_a - \delta_3^a) & \frac{1}{2\alpha_0} (\cot\vartheta_r \sin 2\varphi_r (\delta_1^r - \delta_2^r)) \\ 0 & \frac{1}{2\alpha_0} (\cot\vartheta_r \sin 2\varphi_r (\delta_1^r - \delta_2^r)) & \delta_1^r \sin^2\varphi_r + \delta_2^r \cos^2\varphi_r - \delta_3^r \end{pmatrix} \\
&\quad - \frac{i\hbar}{\alpha_0} \begin{pmatrix} 0 & -\cot\vartheta_l \frac{\partial\varphi_l}{\partial t} & 0 \\ \cot\vartheta_l \frac{\partial\varphi_l}{\partial t} & 0 & \cot\vartheta_r \frac{\partial\varphi_r}{\partial t} \\ 0 & -\cot\vartheta_r \frac{\partial\varphi_r}{\partial t} & 0 \end{pmatrix}
\end{aligned} \tag{2.48}$$

Similar to the SU(2) case, for constant Rabi frequencies $\frac{\partial \varphi}{\partial t} = 0$, we can set the detunings such that $\omega + Q = 0$ and find that the Hamiltonian of the system resembles the spin-orbit coupling Hamiltonian for a spin-1 particle or potentially a particle with characteristic of a higher symmetry like color or flavor. Since the Hamiltonian is of size 3×3 , it belongs to SU(3) group generated by Gell-Mann matrices in the most general case. For equal Rabi frequencies of the double-tripod scheme, we have $\varphi^l = \varphi^r = \pi/4$, $\vartheta^l = \vartheta^r = \cos^{-1} \sqrt{1/3}$ and $\alpha_0 = \sqrt{2}$ and the Hamiltonian reads

$$H = \frac{p_x^2 + p_y^2}{2m} \otimes \mathbf{I}_3 - \frac{p_x \hbar k}{m} \begin{pmatrix} 1/2 & -1/4 & 0 \\ -1/4 & 1/4 & -1/4 \\ 0 & -1/4 & 1/2 \end{pmatrix} - \frac{p_y \hbar k}{m} \begin{pmatrix} -7/2 & -1/4 & 0 \\ -1/4 & -2 & 1/4 \\ 0 & 1/4 & -1/2 \end{pmatrix} \quad (2.49)$$

Removing the trace, the Hamiltonian reads

$$H' = -\frac{p_x \hbar k}{m} \begin{pmatrix} 1/12 & -1/4 & 0 \\ -1/4 & -2/12 & -1/4 \\ 0 & -1/4 & 1/12 \end{pmatrix} - \frac{p_y \hbar k}{m} \begin{pmatrix} -3/2 & -1/4 & 0 \\ -1/4 & 0 & 1/4 \\ 0 & 1/4 & 3/2 \end{pmatrix} \quad (2.50)$$

The dispersion relations of this Hamiltonian as a function of p_x and p_y are illustrated in figure 2.3b.

From inspection, it is clear that the Hamiltonian in equation 2.50 cannot be written as a linear combination of components of angular momentum J_x , J_y and J_z operators for spin-1⁶. However, one could argue that there may exist some set of unitary transformations that transform these operators that could potentially lead to full decomposition of the Hamiltonian 2.49 in SU(2) transformed operators (spin-1 J'_x , J'_y and J'_z operators). Therefore, it is important to refute this argument to claim that we simulate a Hamiltonian with SU(3) symmetry. A true signature of SU(3) symmetry is the existence of non-vanishing higher order Casimir invariant. Casimir invariants, $C_n = \text{Tr}(H^{n+1})$ for $n = 1$ and 2, where H is a traceless Hamiltonian, are the constants of motion associated with the symmetry (SU($n+1$)) of the system. In case of SU(2) symmetry, the square of the angular momentum operator, $C_1 = J^2$ is the only non-vanishing Casimir invariant. For a system to possess SU(3) symmetry we need to have two

⁶For reference: $J_x = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$, $J_y = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}$ and $J_z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$

non-vanishing Casimir invariants [33–35] C_1 and C_2 that read

$$C_1 = \text{Tr}(H'^2) = \frac{19}{4} \left(\frac{p_y}{m} \right)^2 + \frac{7}{24} \left(\frac{p_x}{m} \right)^2 \quad (2.51)$$

$$C_2 = \text{Tr}(H'^3) = \frac{p_x}{m^3} \left(\frac{p_y^2}{32} + \frac{5p_x^2}{144} \right) \quad (2.52)$$

for the Hamiltonian in equation 2.50. By inspection, C_2 vanishes on the line $p_x = 0$ in 2D momentum space. Thus, the system is generally governed by the Hamiltonian of SU(3) symmetry in the $p_x - p_y$ plane except on the line $p_x = 0$. Though, at first sight, A_y cannot be decomposed into J_x, J_y and J_z , it can be decomposed in a transformed angular momentum basis. With this information, we solidify the following claim: Observation of three frequencies in the eigen spectrum of the Hamiltonian, consequently in the dynamics, is a mark of the SU(3) system. This can be argued as follows: Consider a 3×3 Hamiltonian and diagonalize it. By imposing tracelessness condition it can be of the form,

$$H_1 = \hbar \begin{pmatrix} \omega & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\omega \end{pmatrix} \quad (2.53)$$

or

$$H_2 = \hbar \begin{pmatrix} \omega_1 + \omega_2 & 0 & 0 \\ 0 & -\omega_1 + \omega_2 & 0 \\ 0 & 0 & -2\omega_2 \end{pmatrix} \quad (2.54)$$

The Hamiltonians of the form H_1 (Zeeman splitting) belong to SU(2) group as $C_2 = 0$. Dynamics of the systems governed by these Hamiltonians will have at most two frequencies, ω and 2ω . But the Hamiltonians of the form H_2 belongs SU(3) group as $C_2 = 6(\omega_1^2 - \omega_2^2)\omega_2 \neq 0$ in general (unless $\omega_1 = \omega_2$ or $\omega_2 = 0$, then class of Hamiltonians H_2 reduces to the class of Hamiltonians H_1). Therefore, if we show three frequencies $2\omega_1$, $(\omega_1 - \omega_2)$, and $(\omega_1 + \omega_2)$ in the dynamics then we prove that the symmetry group of our system is SU(3). This is indeed the case as shown in section 2.5.

2.4 Galilean Boost

In previous sections, we have derived the effective Hamiltonians of our system for different atom-light coupling schemes. However, our atomic sample is localized in the momentum space at $\mathbf{p} = 0$ with a spread, $\sigma_{\mathbf{p}} \ll \hbar k$, where $\hbar k$ is the recoil momentum of tripod transitions.

To explore the momentum space, we introduce a change of origin of momentum space using the matrix ω in the form of a Galilean boost. In this section, we revisit non-relativistic Galilean boosts in quantum mechanics. Lagrangian of a classical charged particle with velocity $\mathbf{v}$ in lab frame and charge $q = 1$ in an electromagnetic field is given by [36]

$$\mathcal{L}(\mathbf{r}, \mathbf{v}) = T - U = \frac{1}{2} m \mathbf{v}^2 - \phi(\mathbf{r}) + \mathbf{v} \cdot \mathbf{A}(\mathbf{r}) \quad (2.55)$$

where $T = \frac{1}{2} m \mathbf{v}^2$ is the kinetic energy and $U = \phi(\mathbf{r}) - \mathbf{v} \cdot \mathbf{A}(\mathbf{r})$ is the net electromagnetic potential energy. Here, $\phi(\mathbf{r})$ is the electric scalar potential and $\mathbf{A}$ is the magnetic vector potential. The conjugate momentum

$$\mathbf{p} = \frac{\partial \mathcal{L}}{\partial \mathbf{v}} = m \mathbf{v} + \mathbf{A}(\mathbf{r}) \quad (2.56)$$

The first term is the mechanical momentum and the second term is known as field momentum since it is due to the magnetic vector potential $\mathbf{A}$. In a moving frame of reference with velocity $\mathbf{v}_0$, the velocity of the charged particle is $\mathbf{v}' = \mathbf{v} - \mathbf{v}_0$ and Lagrangian $\mathcal{L}' = \frac{1}{2} m \mathbf{v}'^2 - \phi(\mathbf{r} - \mathbf{v}_0 t) + (\mathbf{v} - \mathbf{v}_0) \cdot \mathbf{A}$, since $\mathbf{A}' = \mathbf{A}$. The conjugate momentum in the moving frame reads,

$$\mathbf{p}' = \frac{\partial \mathcal{L}'}{\partial \mathbf{v}'} = m \mathbf{v}' + \mathbf{A} = m(\mathbf{v} - \mathbf{v}_0) + \mathbf{A} = \mathbf{p} - m \mathbf{v}_0 \quad (2.57)$$

To simulate a momentum kick of $-m \mathbf{v}_0$, we can artificially add $-\mathbf{v}_0 \cdot \mathbf{A}$ to the scalar potential using the ω matrix changing the momentum origin to $m \mathbf{v}_0$. In quantum mechanics though, this adds an additional global phase to the wave function. In summary, a change of reference frame to a moving frame with velocity $\mathbf{v}_0$ leads to the following changes [37]:

$$\begin{aligned} \mathbf{r}' &= \mathbf{r} - \mathbf{v}_0 t, t' = t \\ \mathbf{A}' &= \mathbf{A} \\ \nabla' &= \nabla, \frac{\partial}{\partial t'} = \frac{\partial}{\partial t} + \mathbf{v}_0 \cdot \nabla \\ \phi'(\mathbf{r}', t') &= \phi(\mathbf{r}, t) - \mathbf{v}_0 \cdot \mathbf{A} \\ \psi(\mathbf{r}', t') &= \exp\left(\frac{i}{\hbar}(-m \mathbf{v}_0 \cdot \mathbf{r} + \frac{1}{2} m \mathbf{v}_0^2 t)\right) \psi(\mathbf{r}, t) \end{aligned} \quad (2.58)$$

This structure is already visible in our Hamiltonian. Consider that the atoms are moving with a velocity $\mathbf{v}_0 = v_{0x} \mathbf{e}_x + v_{0y} \mathbf{e}_y$. The resonant transitions of the tripod and double-tripod schemes will now be detuned by $-\mathbf{k} \cdot \mathbf{v}$ due to the Doppler shift. With these detunings, the first term in the matrix ω in equations 2.32 (for SU(2) system) and 2.48 (for SU(3) system) is equivalent to $v_{0x} A_x + v_{0y} A_y$. This method of changing the coordinate systems significantly reduces the

experimental difficulty as it allows for scanning the momentum space. Moreover, the momentum kick depends on two-photon detunings that can be accurately controlled, allowing for a well-defined momentum.

2.5 Dynamics in SU(3) Gauge Field

The dynamics in the presence of the SU(2) gauge field is well described in a previous thesis [38]. In this section, we concentrate on the wave-packet dynamics in the presence of the SU(3) gauge field derived in section 2.3.4. We ignore the trace of the Hamiltonian as it is irrelevant to the dynamics. The Hamiltonian in equation 2.50 can be decomposed into Gell-Mann matrices as follows:

$$\begin{aligned} H &= -\frac{p_x \hbar k}{m} \mathbf{a} \cdot \boldsymbol{\lambda} - \frac{p_y \hbar k}{m} \mathbf{b} \cdot \boldsymbol{\lambda} \\ &= -\frac{p_x \hbar k}{m} \sum_{m=1}^8 a_m \lambda_m - \frac{p_y \hbar k}{m} \sum_{m=1}^8 b_m \lambda_i \end{aligned} \quad (2.59)$$

where $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \dots, \lambda_8)$ are the Gell-Mann matrices⁷, $\mathbf{a} = (-\frac{1}{4}, 0, \frac{1}{8}, 0, 0, -\frac{1}{4}, 0, -\frac{1}{8\sqrt{3}})$ and $\mathbf{b} = (-\frac{1}{4}, 0, -\frac{3}{4}, 0, 0, \frac{1}{4}, 0, -\frac{3\sqrt{3}}{4})$ ⁸. The Gell-Mann matrices are

$$\begin{aligned} \lambda_1 &= \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \\ \lambda_4 &= \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \\ \lambda_6 &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} \end{aligned} \quad (2.60)$$

The above Hamiltonian is traceless, real and symmetric. Its eigenvalues as a function of p_x and p_y are real and look complicated in form. We resort to numerical evaluation of eigenvalues. Nevertheless, we can solve for the dynamics using Heisenberg picture after diago-

⁷Trace normalization condition of the Gell-Mann matrices is $Tr(\lambda_i \lambda_j) = 2\delta_{ij}$, where δ_{ij} is the Kronecker delta function.

⁸The decomposition can be obtained using the normalization condition as $\mathbf{a} = Tr(A_x \boldsymbol{\lambda})/2$ and $\mathbf{b} = Tr(A_y \boldsymbol{\lambda})/2$

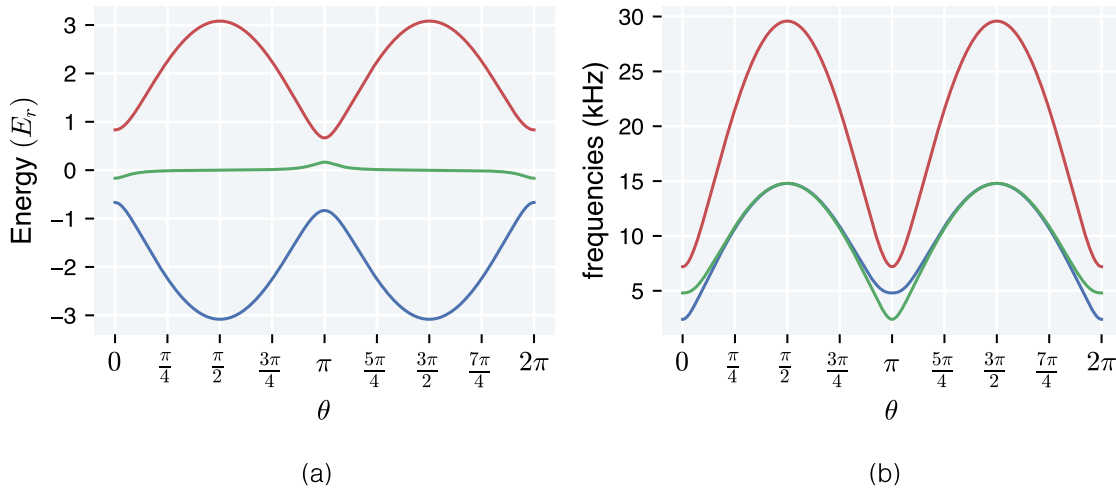


Figure 2.5: (a) Energies of the Hamiltonian on the circular contour $p_x^2 + p_y^2 = (\hbar k)^2$ as a function of $\theta = \tan^{-1} \frac{p_y}{p_x}$. (b) Frequencies of dynamics as a function of θ .

nalization of the Hamiltonian. A general traceless diagonal Hamiltonian generated by SU(3) group with eigen energies $\hbar\omega_1, \hbar\omega_2$ and $\hbar\omega_3$ can be expressed in terms of diagonal Gell-Mann matrices as

$$H^d = \hbar \begin{pmatrix} \omega_1 & 0 & 0 \\ 0 & \omega_2 & 0 \\ 0 & 0 & \omega_3 \end{pmatrix} = \hbar(e_3\lambda_3 + \sqrt{3}e_8\lambda_8) \quad (2.61)$$

where $e_3 = (\omega_1 - \omega_2)/2$ and $e_8 = -\omega_3/2 = (\omega_1 + \omega_2)/2$. Heisenberg's equation of motion for Gell-Mann matrices is

$$\begin{aligned} i \frac{d\lambda_m}{dt} &= [\lambda_m, e_3\lambda_3 + \sqrt{3}e_8\lambda_8] \\ i \frac{d\lambda_m}{dt} &= e_3[\lambda_m, \lambda_3] + \sqrt{3}e_8[\lambda_m, \lambda_8] \end{aligned} \quad (2.62)$$

Using the commutation relations between Gell-Matrices $[\lambda_l, \lambda_m] = 2i \sum_n f^{lmn} \lambda_n$ where f^{lmn} are the structure constants⁹, we get a system of eight equations represented in a matrix equa-

⁹Structure constants $f^{lmn} = -\frac{i}{4} \text{Tr}(\lambda_l[\lambda_m, \lambda_n])$ are the generalization of Levi-Civita ϵ_{lmn} in SU(2).

tion for vector $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \dots, \lambda_8)^T$ as follows

$$\dot{\boldsymbol{\lambda}} = \begin{pmatrix} 0 & -2e_3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2e_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -(e_3 + 3e_8) & 0 & 0 & 0 \\ 0 & 0 & 0 & e_3 + 3e_8 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -(3e_8 - e_3) & 0 \\ 0 & 0 & 0 & 0 & 0 & (3e_8 - e_3) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \boldsymbol{\lambda} \quad (2.63)$$

The solution to the above initial value problem is

$$\boldsymbol{\lambda}(t) = \begin{pmatrix} \lambda_1(0) \cos(2e_3 t) - \lambda_2(0) \sin(2e_3 t) \\ \lambda_1(0) \sin(2e_3 t) + \lambda_2(0) \cos(2e_3 t) \\ \lambda_3(0) \\ \lambda_4(0) \cos((e_3 + 3e_8)t) - \lambda_5(0) \sin((e_3 + 3e_8)t) \\ \lambda_4(0) \sin((e_3 + 3e_8)t) + \lambda_5(0) \cos((e_3 + 3e_8)t) \\ \lambda_6(0) \cos((3e_8 - e_3)t) - \lambda_7(0) \sin((3e_8 - e_3)t) \\ \lambda_6(0) \sin((3e_8 - e_3)t) + \lambda_7(0) \cos((3e_8 - e_3)t) \\ \lambda_8(0) \end{pmatrix} \quad (2.64)$$

This solution is in the eigen basis of the Hamiltonian H and should be changed to the dark state basis by appropriate unitary transformation. Nevertheless, we obtain the frequencies involved in the dynamics from equation 2.64 as $|\omega_1 - \omega_2|$, $|2\omega_1 + \omega_2|$ and $|\omega_1 + 2\omega_2|$ or equivalently $|\omega_1 - \omega_2|$, $|\omega_1 - \omega_3|$ and $|\omega_2 - \omega_3|$. Figure 2.5a shows the eigen energies of our traceless Hamiltonian 2.50 and figure 2.5b shows the frequencies involved in the dynamics of the atoms on a circular contour $p_x^2 + p_y^2 = (\hbar k)^2$ as a function of angle $\theta = \tan^{-1} \frac{p_y}{p_x}$. We notice a symmetry in the eigen energies and thus in frequencies when the angle θ is rotated by π . This can be explained by the π rotational symmetry in the laser geometry. Note that we have three frequencies at $\theta = 0$ and $\theta = \pi$. As we approach $\theta = \frac{\pi}{2}$ or $\theta = \frac{3\pi}{2}$ equivalent to $p_x = 0$, the three frequencies reduce to two where the Casimir invariant $C_2 = 0$.

Since $H^d = U^\dagger H U$ where U is the unitary matrix with columns as the eigenvectors of H , the initial expectation values of $\boldsymbol{\lambda}(0) = \text{Tr}(|\psi'\rangle \langle \psi'| \boldsymbol{\lambda})/2 = \text{Tr}(U^\dagger |\psi\rangle \langle \psi| U \boldsymbol{\lambda})/2$. Here, $|\psi'\rangle$ and $|\psi\rangle$ are the state vectors in eigen basis and dark state basis respectively. Though part of this

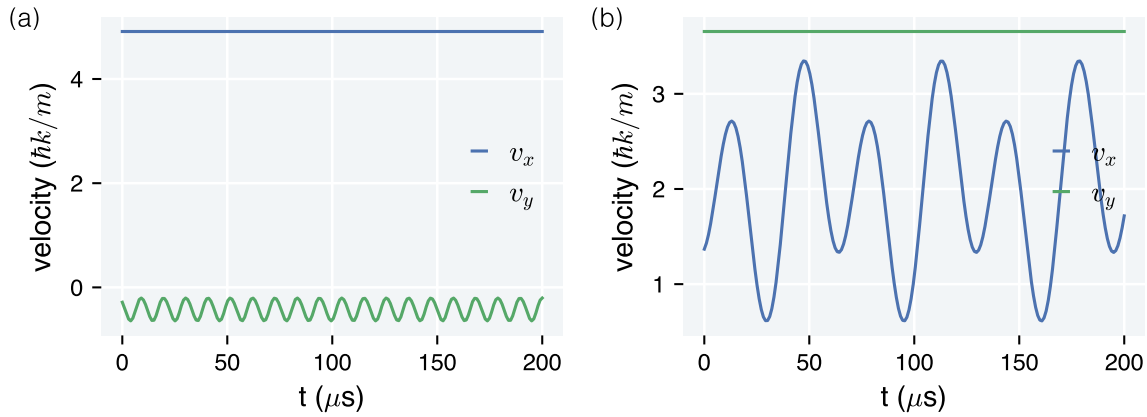


Figure 2.6: Evolution of components of velocity of the cloud initialized in $\psi = \frac{|D_0\rangle + 2|D_r\rangle}{\sqrt{5}}$ zero temperature for a momentum kick of (a) $\mathbf{p} = 4\hbar k \hat{\mathbf{e}}_x$ and (b) $\mathbf{p} = 4\hbar k \hat{\mathbf{e}}_y$.

method is numerical, all the relevant measurement quantities can be obtained with relative ease with this algorithmic approach. One of the measurements is the velocity of the atomic cloud as a function of evolution time. The expectation of velocity operator reads

$$\begin{aligned} \langle \mathbf{v}(t) \rangle &= \langle \psi | \mathbf{v}(t) | \psi \rangle = \frac{\mathbf{p} - \langle \psi | \mathbf{A}(t) | \psi \rangle}{m} \\ &= \frac{\mathbf{p} - \mathbf{p}_0 - (\mathbf{a}' \cdot \boldsymbol{\lambda}(t) \hat{\mathbf{e}}_x + \mathbf{b}' \cdot \boldsymbol{\lambda}(t) \hat{\mathbf{e}}_y)}{m} \end{aligned} \quad (2.65)$$

where $\mathbf{p}_0 = \left(\frac{\text{Tr}(A_x)}{3} \hat{\mathbf{e}}_x + \frac{\text{Tr}(A_y)}{3} \hat{\mathbf{e}}_y \right)$, $\mathbf{a}' = \text{Tr}(U^\dagger A_x U \boldsymbol{\lambda})/2$ and $\mathbf{b}' = \text{Tr}(U^\dagger A_y U \boldsymbol{\lambda})/2$. In a similar way, expectation value of any operator as a function of time can be obtained by using the evolution of $\boldsymbol{\lambda}(t)$ given that the state of the system in dark state basis is known. Though the trace of the gauge field does not affect the frequencies of the dynamics, note that it defines the initial condition for the velocity. Figure 2.6 shows the components of the velocity of the atoms at zero temperature evolving in the gauge field as a function of time for momentum kicks of $\mathbf{p} = 4\hbar k \hat{\mathbf{e}}_x$ and $\mathbf{p} = 4\hbar k \hat{\mathbf{e}}_y$. The component of velocity along the momentum kick does not oscillate. This can be explained by looking at the acceleration of the atoms in the Heisenberg picture:

$$\begin{aligned} \frac{dv_x}{dt} &= -\frac{i}{\hbar m^2} p_x [A_x, A_y] \\ \frac{dv_y}{dt} &= -\frac{i}{\hbar m^2} p_y [A_y, A_x] \\ \Rightarrow \frac{d\mathbf{v}}{dt} &= -\frac{i}{\hbar m^2} \mathbf{p} \times \mathbf{A} \times \mathbf{A} \end{aligned} \quad (2.66)$$

This indicates that the acceleration is always perpendicular to the momentum kick $\mathbf{p}$. This shows the spin Hall effect nature of the dynamics where momentum kick direction $\hat{\mathbf{p}}$ plays the role of direction of the magnetic field [39]. Temperature dependence of the evolution of an observable is numerically obtained by averaging expectation value of observable over a Boltzmann distribution or more precisely Fermi-Dirac distribution of $\mathbf{p}$ [40].

2.6 Conclusion

In conclusion, we described basic theoretical ideas employed in quantum simulations of magnetic phenomena with neutral atoms starting from Aharonov-Bohm phase and Berry phase in connection to the artificial magnetism. We have derived the reduced Schrödinger equation in the degenerate subspace of the Hamiltonian when it is adiabatically evolved. We have also discussed the two atom-light coupling schemes *viz.*, tripod scheme and double-tripod scheme, that lead to degenerate subspaces of two and three dark states. We derived the corresponding artificial SU(2) and SU(3) gauge fields in these coupling schemes and additional scalar potentials that arise due to the configuration of lasers. We have shown the theoretical basis for exploring the momentum space using Galilean boosts to extract characteristic dynamics of the system. We discussed a semi analytical method that is developed in order to calculate the theoretical expectation values of the observables that are discussed in the later chapters.



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Chapter 3

Experimental Apparatus and Methods

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3.1 Introduction

In this chapter, an overview of the experimental apparatus, sample preparation and relevant measurement techniques used in the experiments is presented. The main experimental apparatus used to perform the experiments of this thesis is about ten years old and is well-described in reference [1] and in subsequent PhD theses [2–4]. Nevertheless, the apparatus has been upgraded enabling the experiments described in this thesis. Of these upgrades, the vertical imaging system used to acquire fluorescence images and optical system to implement the double-tripod coupling scheme are the significant changes that need description. In this chapter, these are described along with a brief overview of the existing system to have a comprehensive description however limiting mainly to the details that are absent in [1–4].

In section 3.2, the description of the experimental vacuum chamber with emphasis on the pressure at various points is provided. In section 3.3, a summary of ultracold sample preparation is described with details on characterization of optical pumping and final sample after evaporative cooling. In section 3.4, diagnostic tools like spectroscopy methods used in navigating the Zeeman states of the intercombination line and fluorescence imaging used to obtain relative populations of the ground state Zeeman sublevels are presented. In section 3.5, the optical laser system designed to target the double-tripod scheme is characterized. In section 3.6, the experimental system to implement STIRAP techniques used to redistribute the ground state populations in a controlled manner is described. In section 3.7, measurement of Rabi frequencies of double-tripod configuration by directly observing Rabi flopping is described. In section 3.8, a spectroscopy probing the dressed states of the double-tripod configuration is presented. Finally the chapter is concluded with a summary in section 3.9.

3.2 Vacuum Chamber

Wall-less confinement of atoms enabled by optical and magneto-optical traps is a primary requirement of modern experiments using ultracold atoms. In conventional terms, these trapped atoms are the sample of study. The common way of isolating this sample from environment is by producing the sample in an ultra high vacuum (UHV) environment. The experiments of this thesis are performed using the sample produced in the vacuum chamber shown in figure 3.1 with its important constituents labeled. The source of strontium atoms is the oven, a tube surrounded with a heating coil, which is heated to and maintained at 580 °C to create a vapor with sufficient density of strontium atoms from a solid strontium reser-

voir. This is clamped to the vacuum chamber at a 6-way cross. The four co-planar arms of the 6-way cross orthogonal to the oven are fitted with viewports that allow for optical collimation of the atomic beam emanating from the oven using 2D molasses. This region of the chamber, continuously pumped by an ion pump (not shown in the figure), is at a pressure of 7×10^{-9} mbar, the highest in the whole chamber. The one remaining arm of the 6-way cross is connected to the Zeeman slower by a narrow tube of inner diameter 1 cm and length 10 cm. This narrow tube serves two purposes: First, it rejects divergent atoms (with high transverse velocity) from entering the Zeeman slower. Second, it acts as a differential pumping tube. The 32 cm long Zeeman slower tube is connected to the hexagonal 2D magneto optical trap (MOT) chamber through one of its four 1.5" viewports. The viewport diametrically opposite to the Zeeman slower is fitted with a sapphire viewport that serves as input for the Zeeman slower laser beam. This sapphire viewport is maintained at 375 °C to prevent deposition of strontium that renders the viewport opaque to the Zeeman slower laser beam. The 2D MOT chamber, continuously pumped by an ion pump, is at a pressure of 5×10^{-10} mbar. This chamber is connected to the science chamber by a differential pumping tube that makes an angle of 30° with the Zeeman slower tube. The science chamber, where final ultracold sample is prepared and probed, is an octagonal glass cell with two 4" windows and seven accessible 1.5" windows with broadband anti-reflection coating for 460-1100 nm light on both internal and external surfaces. The science chamber is also continuously pumped by an ion pump and is maintained below the measurement threshold of the ion pump i.e., 10^{-10} mbar. The science chamber is occasionally treated with a titanium sublimation pump to help maintain low pressure. The low pressure in the science chamber reduces background gas collisions with the strontium atoms and provides a lifetime of about 50 s for atoms in our dipole trap [4].

3.3 Ultracold Sample Preparation

As shown in section 2.3, the magnitude of the artificial gauge field generated using tripod scheme is of the order of recoil momentum $\hbar k$, where k is the wave number of the tripod lasers. Therefore, if the temperature of the gas is larger than the corresponding recoil temperature ($\frac{\hbar^2 k^2}{mk_B} = 460$ nK), the single particle dynamics in the presence of the gauge field are obscured by the aggregate dynamics of atoms of various momentum classes. In order to limit the temperature effect, we prepare an ultracold sample of fermionic strontium atoms at sub-recoil temperatures (≈ 55 nK). At these temperatures, the gas is Fermi degenerate. However, quantum degeneracy of the sample is not a necessary condition for our study as we study

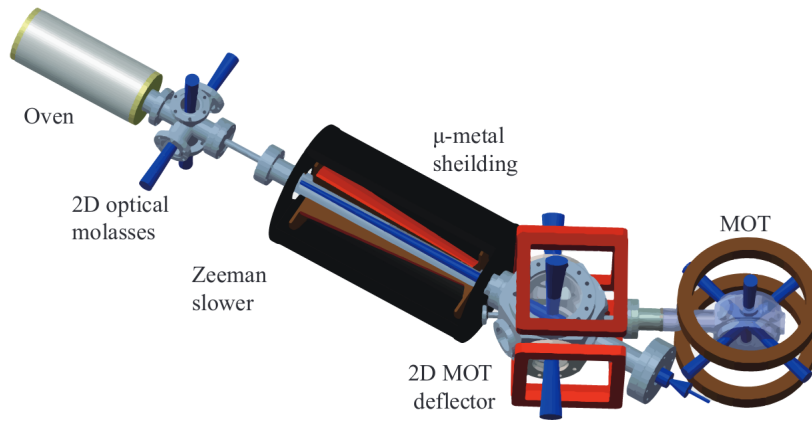


Figure 3.1: Schematic of ultra-high vacuum chamber along with key components of laser cooling and trapping apparatus used in preparing ultracold sample of strontium atoms. Figure reproduced from reference [1] with kind permission of The European Physical Journal D(EPJD)

single particle dynamics. In our system, quantum degeneracy of the sample is a consequence of two key experimental details: First, a gas at a sub-recoil temperature for aforementioned reason. Second, a better signal-to-noise ratio of our spin sensitive imaging system with a low temperature and large number of atoms. Keeping these conditions in mind, we prepared a degenerate sample with $T/T_F = 0.26$ ($T_F \approx 212\text{ nK}$). In this section, a brief account of this preparation is given. Detailed accounts on achieving quantum degenerate samples of strontium atoms and their characterization can be found in [4–8].

3.3.1 Two Stage MOT

An effusive beam of strontium atoms is produced by solid strontium in a oven maintained at 580°C . These atoms are transversely cooled and collimated using a 2D molasses before they enter the Zeeman slower tube. Here, they are longitudinally cooled in a magnetic field gradient designed to compensate for the Doppler shift caused by slowing of the atoms. After the Zeeman slower, atoms enter 2D MOT stage which cools and deflects the colder atoms into the UHV science chamber for 3D MOT. The laser beams required for aforementioned functions excluding Zeeman slower stage are derived from a commercial 461 nm Toptica laser with an output power of 550 mW frequency locked to the atomic resonance using a secondary oven. Zeeman slower beam with a power of 150 mW is obtained from a second Toptica SHG Pro fed with 360 mW of power at 922 nm from the output of injection locked diode laser amplified by a tapered amplifier. See appendix A for details on the Zeeman slower beam.

Atoms diverted to the UHV science chamber are first captured by a 3D MOT implemented

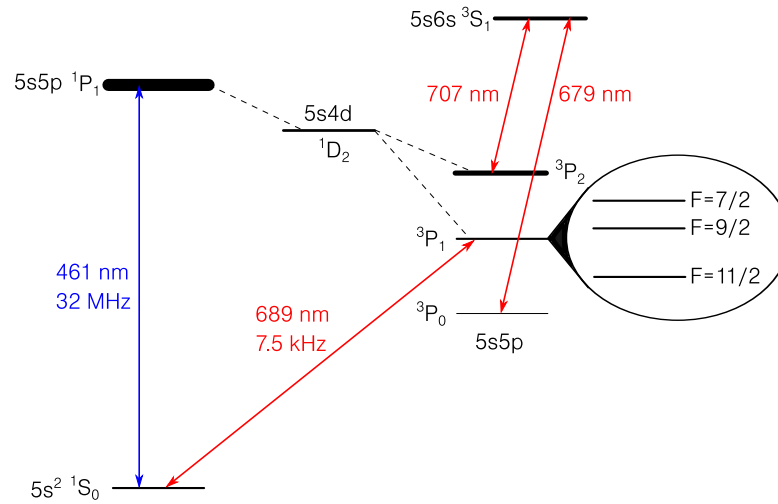


Figure 3.2: Energy level diagram of Sr showing transitions used in laser cooling and trapping. Lines in the ellipse show the zoomed in view of the hyperfine structure splitting of $^3P_1$ in ^{87}Sr .

using the singlet $^1S_0 \rightarrow ^1P_1$ transition (refer figure 3.2) at 461 nm and quadrupole magnetic field with an axial gradient of 43 G/cm. The high cycling rate of this transition allows for high rate of scattering of light with the atoms imparting large radiation pressure force. Laser beams of approximately 1 cm in waist size with a combined power of 80 mW are used for this MOT to capture a significant proportion of the atoms entering the science chamber. This transition is not closed and the atoms leak to $^1D_2$ and subsequently to triplet states $^3P_1$ and $^3P_2$. The atoms from $^3P_1$ radiatively decay to $^1S_1$ however atoms in $^3P_2$ had to be repumped using additional transitions in $^3S_1$ using repumper lasers at 707 nm and 679 nm. These repumper lasers excite atoms to $^3S_1$ forcing them to return to $^3P_1$ and thereby to $^1S_1$ closing the MOT cycle. Refer to references [3, 4] for more details on the the implementation of repumping. Once enough number of atoms are accumulated (about a 10^8 atoms), then the power of the laser beams is ramped down to further cool atoms at the center of the MOT at a temperature of about 2 mK. With this first stage of MOT known as *Blue-MOT* is completed.

The second stage of MOT is using the $^1S_0 \rightarrow ^3P_1$ narrow intercombination line at 689 nm called *Red-MOT*. The narrow transition allows for recoil limited cooling in a MOT [9, 10]. However, the narrow linewidth of the transition significantly increases the complexity of the laser system required to implement a MOT. Figure 3.3 shows the schematic of the red laser system. The master laser at 689 nm is a homemade external cavity diode laser system locked to a home-built ULE cavity using Pound-Drever-Hall technique to reduce the line width to 1 kHz [11, 12]. This is then locked to the resonance of $m_J = 0 \rightarrow m_J = 0$ transition of $^1S_0 \rightarrow ^3P_1$ of

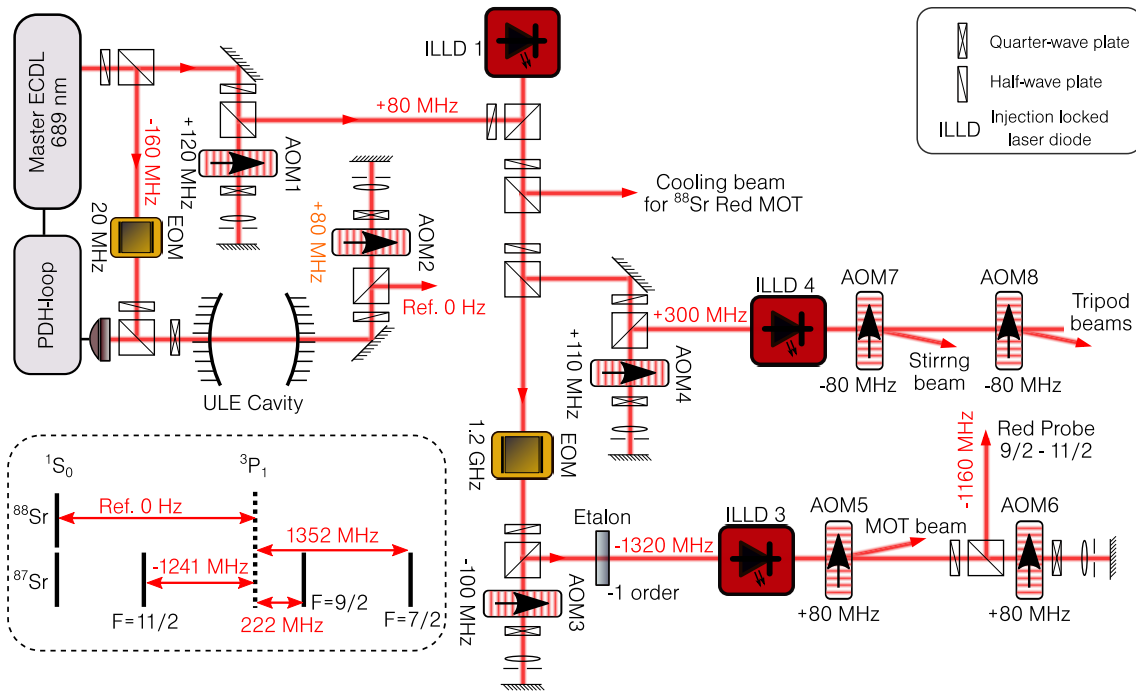


Figure 3.3: Schematic of laser system targeting various hyperfine levels of the intercombination line in ^{87}Sr .

^{88}Sr by saturated absorption spectroscopy on the atomic beam of the secondary source. This is done by using an acousto-optic modulator (AOM 2 in the figure 3.3) in double pass configuration that scans and locks its frequency at the resonance to compensate for slow drifts in the cavity resonance. In figure 3.3, assuming the frequency of the AOM 2 as 80 MHz (shown in amber in the figure), the frequency detuning with respect to the atomic resonance of the intercombination line of the laser beams at various points in the path are labeled. As the frequency of AOM 2 is drifting during a typical day, this frequency drift is used in calculating all other detunings in the control software (Refer to [3] for more details).

The excited state $3P_1$ is further split into three hyperfine levels in ^{87}Sr as shown in figure 3.2 and inset of figure 3.3 shows the frequency shifts of the hyperfine levels from the reference. Two of the three hyperfine manifolds are used in the Red-MOT: $F = 11/2$ manifold for cooling and trapping using MOT beams and $F = 9/2$ manifold for stirring the population in the ground state to counter the optical pumping due to the cooling beam [9, 10]. At the end of Blue-MOT, the Red-MOT laser beams are turned on with their linewidths artificially broadened by modulating the frequency of the AOMs to capture a wide range of velocity classes to form a broadband Red-MOT [13, 14]. The linewidths of the Red MOT beams is subsequently

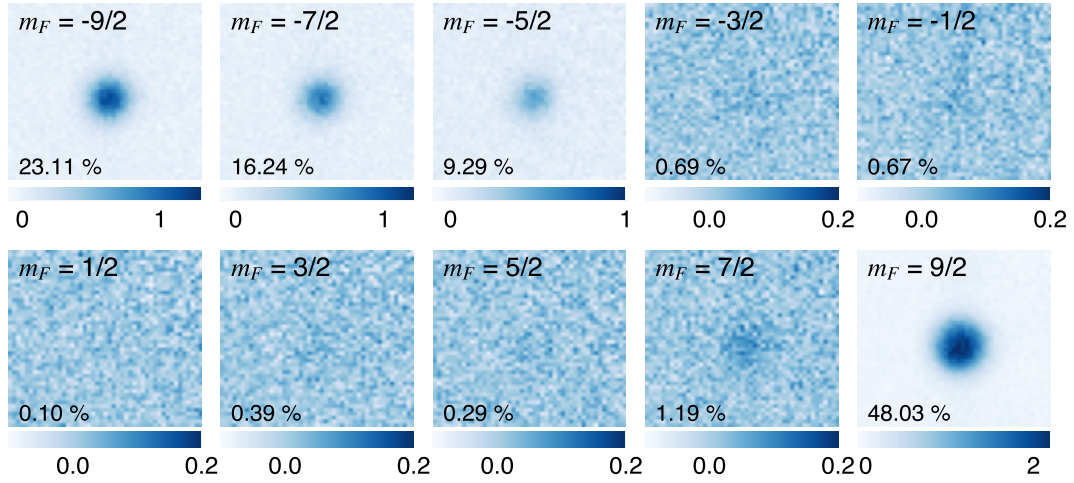


Figure 3.4: Optical pumping in the ground state Zeeman levels before evaporative cooling. The relative population distribution is measured with spin sensitive absorption imaging using the $|^1S_0, F = 9/2\rangle \rightarrow |^3P_1, F = 11/2\rangle$ intercombination line in the presence of a bias magnetic field of 10 G. The error in the estimation of percentages of populations in the ground states is 2%.

reduced simultaneously as their optical power is ramped down and a single band Red-MOT is formed with 20×10^6 atoms at a temperature of about $3 \mu K$.

3.3.2 Optical Pumping

At the end of single band Red-MOT, we load the atoms into a crossed beam optical dipole trap by turning on high power dipole trap beams of wavelength 1064 nm. We trap 2.5×10^6 atoms at a temperature of about $4.5 \mu K$ with powers of 4.8 W and 4.3 W for the dipole trap beams that produce a trap depth of $65 \mu K$. At this level into the sample preparation, the atoms in ground state are equally distributed in 10 magnetic sublevels of the ground state. Now, we perform optical pumping using $|^1S_0, F = 9/2\rangle \rightarrow |^3P_1, F = 9/2\rangle$ to facilitate for adiabatic mapping into the dark state subspace and also to minimize unwanted coupling of atoms in certain ground state Zeeman sublevels with the sidebands of tripod lasers (see section 3.5). The atoms in a few negative m_F sublevels as shown in figure 3.4 are retained to aid elastic collisions in evaporative cooling. We use a bias field of $B = 23.5$ G during the optical pumping sequence to lift the degeneracy in the excited state separating its Zeeman sublevels by 2 MHz *i.e.*, $\approx 270\Gamma$. First, we use a σ^+ laser beam of our tripod laser system, described in section 3.5, with a power of $200 \mu W$ to target $|^1S_0, F = 9/2, m_F\rangle \rightarrow |^3P_1, F = 9/2, m_F + 1\rangle$ where $m_F = 1/2, 3/2, 5/2$ and $7/2$ one after another so that atoms in the positive m_F sublevels is optically pumped to the stretched state $|^1S_0, F = 9/2, m_F = 9/2\rangle$. Next, we use a σ^- laser beam

to target $|^1S_0, F=9/2, m_F\rangle \rightarrow |^3P_1, F=9/2, m_F-1\rangle$ to pump out the atoms in $m_F = -1/2$ and $-3/2$. A typical experimental signal showing distribution of atoms in the ground state Zeeman levels after optical pumping procedure is shown in figure 3.4. These images are acquired with spin sensitive absorption imaging using $|^1S_0, F=9/2\rangle \rightarrow |^3P_1, F=11/2\rangle$ transitions averaged with three independent repetitions of the experiment. The optical depth of the absorption image of $|^1S_0, F=9/2, m_F=7/2\rangle$ is indicative of residual population is an artifact in our imaging system projecting the population in $|^1S_0, F=9/2, m_F=7/2\rangle$ due to the angle (about 20°) of the probe beam with the quantization axis. The final distribution in the Zeeman manifold corresponds to 50(3)% atoms in $|^1S_0, F=9/2, m_F=9/2\rangle$ state and the rest in the extreme three negative m_F sublevels.

3.3.3 Evaporative Cooling

After optical pumping, we begin evaporative cooling of the sample by continuously reducing the optical power of the dipole trap beams in order to lower the trap depth. The characterization of evaporative cooling is detailed in [4, 15]. To produce a sample with reproducible number of atoms and temperature, we implemented an intensity servo for the power of the dipole trap beams. The implementation details of this intensity servo are given in appendix B. After 7.5 s of evaporative cooling, we prepare a degenerate sample of 8×10^4 atoms at a temperature of about 55 nK. Figure 3.5a shows the linear relationship between squared size of the sample and squared time of flight. The temperature extracted from by fitting the time of flight images to the Gaussian distribution yields 51 nK and 57 nK along two orthogonal axes in the image plane. However, in quantum degenerate regime the temperature is slightly overestimated by the Gaussian fitting. The degeneracy parameter T/T_F is obtained by fitting the time of flight images to the Fermi-Dirac distribution [16]. Figure 3.5b shows the curve fitting of azimuthal averaged optical density of the cloud with the Fermi-Dirac distribution yielding $T/T_F = 0.26(2)$. This value of T/T_F obtained from absorption imaging with a 461 nm probe beam sets the upper bound for the degeneracy parameter for the sample in $|^1S_0, F=9/2, m_F=9/2\rangle$ as the atoms in negative m_F ground states contribute more to the Gaussian distribution [5]. Figure 3.5c shows the residue of the fitting to Gaussian and Fermi-Dirac distribution. In the limit of large velocities (beyond 10 mm/s), both the distributions are almost identical as shown by the residue converging to zero. However, at lower velocities, the Fermi-Dirac distribution fits the experimental data much better than the Gaussian distribution as indicated by the residue. Due to azimuthal averaging of the optical depth, the noise

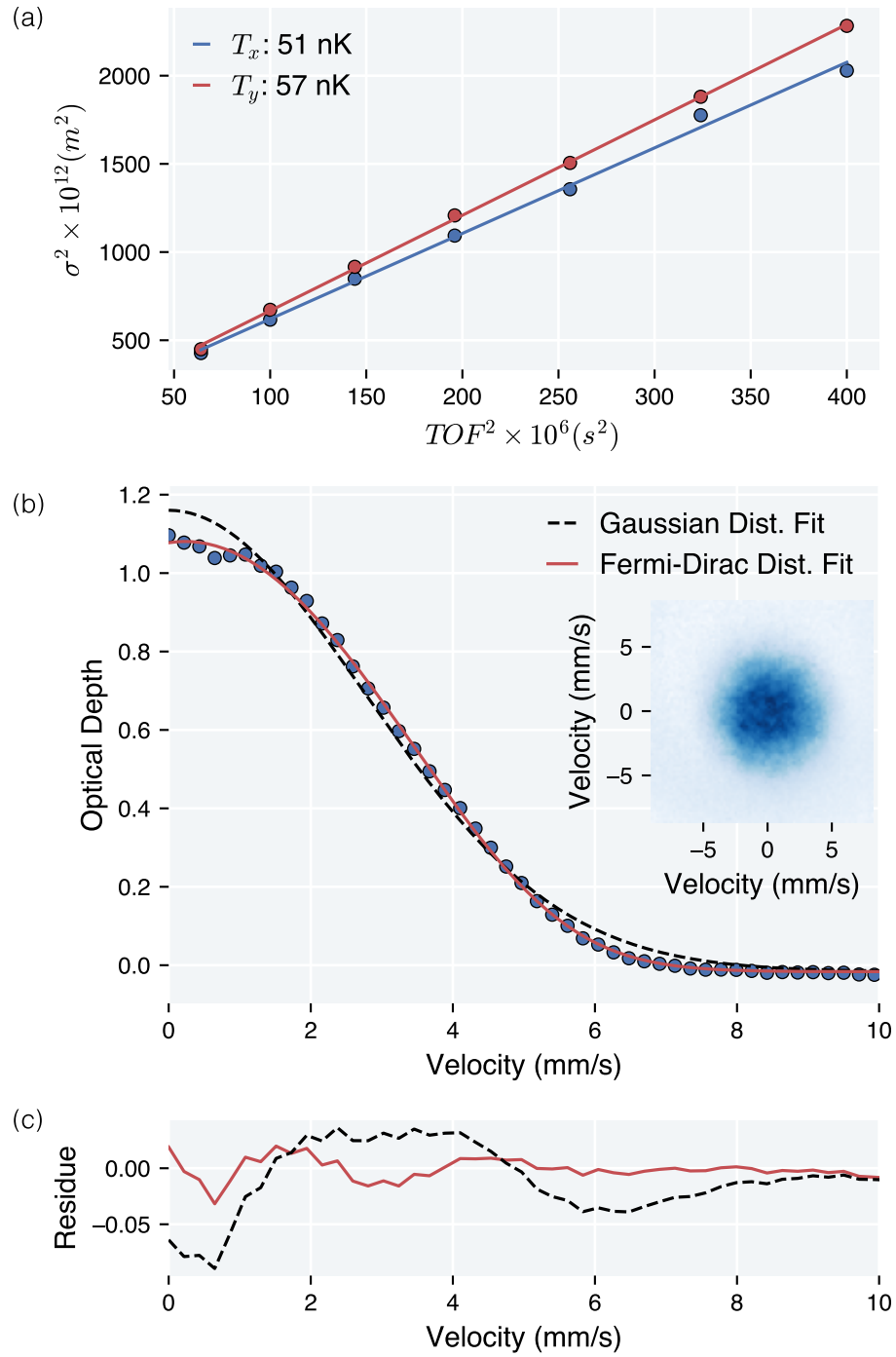


Figure 3.5: (a) Expansion of the cloud over time of flight. (b) Azimuthal averaged cloud density at 14 ms time of flight fit with Gaussian fit (dashed black line) and Fermi-Dirac distribution (plain red line). Inset shows the absorption image of the circular cloud at this time of flight. (c) Residue = experimental data - value of fit function for the fitting in subplot (b). The plain red line and dashed black line represent residues of the data from Fermi-Dirac distribution and Boltzman distribution respectively.

in the tails is smaller than it is at the center as the signal from a large number of pixels is averaged compared to few pixels at the center. That is why, the residue at low velocities deviates from zero but by a smaller amount in comparison to Gaussian distribution's residue.

3.4 Diagnostics

In this section, we provide an overview of the diagnostic and measurement tools we use in characterizing the atomic sample and atom-light interaction parameters. These tools are well known (see [17, 18]) and here I describe some implementation details pertaining to the current experimental system. In characterizing the ultracold sample that we have discussed in previous sections, we use absorption imaging using 461 nm probe to estimate the temperature of the gas and number of atoms in the sample. We also use absorption imaging using narrow 689 nm probe to extract distribution of populations of the atoms among the Zeeman sublevels of the ground state to characterize the optical pumping. The former is relatively simple and the frequency drifts due to environmental changes are negligible as the transition $|^1S_0\rangle \rightarrow |^1P_1\rangle$ is broad with a linewidth of $2\pi \times 32 \text{ MHz}$ ¹ much higher than the thermal broadening caused by the temperature of the cloud. The later however is more challenging because of its narrow linewidth. Even in the presence of frequency servo on the atomic resonance, we notice that resonant frequencies drift by about 10 kHz over the day. So the resonances are continuously tracked by performing spectroscopy everyday at least once. In the following subsection, this spectroscopy method is described.

3.4.1 Spectroscopy

The transitions of the intercombination line that are of interest are $|^1S_0, F = 9/2\rangle \rightarrow |^3P_1, F = 11/2\rangle$ and $|^1S_0, F = 9/2\rangle \rightarrow |^3P_1, F = 9/2\rangle$. The spectroscopy of the first transition are performed by direct measurement of absorption by the cloud and is used for magnetic field calibration and finding resonances to measure ground state Zeeman sublevels for characterizing the optical pumping. Whereas the spectroscopy of the second transition is performed by an indirect measurement, known as atom-loss spectroscopy that is used in implementing tripod and double-tripod configurations. In this subsection the details of these spectra are described.

¹since the hyperfine splitting in the excited state cannot be resolved the scattering cross section is multiplied by a factor approximately equal to 2.

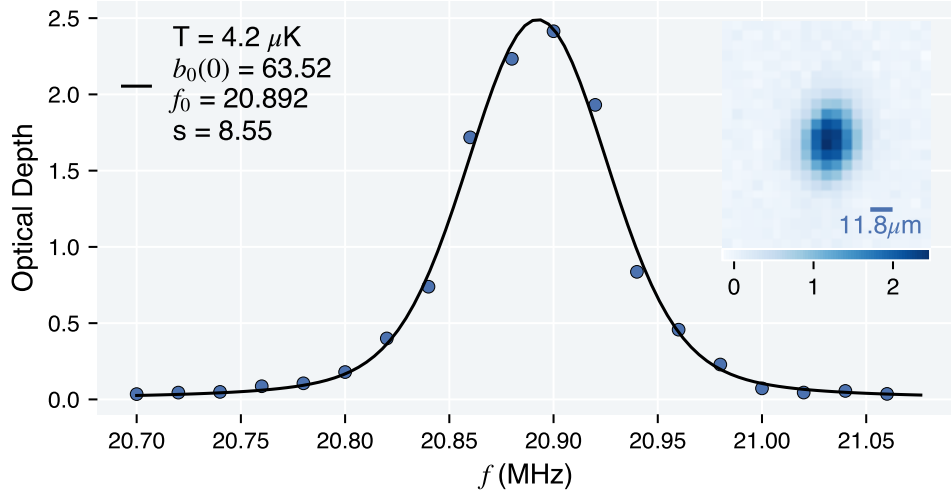


Figure 3.6: Spectrum of $|^1S_0, F=9/2, m_F=9/2\rangle \rightarrow |^3P_1, F=11/2, m_F=11/2\rangle$ transition using σ^+ probe beam before optical pumping sequence. The data is fit to a Voigt profile with temperature, resonant optical depth at zero temperature, resonant frequency of the transition and the saturation parameter of the probe as free parameters. Inset shows the absorption image of the cloud at resonance to indicate the SNR of the imaging system.

The absorption spectroscopy of $|^1S_0, F=9/2, m_F=9/2\rangle \rightarrow |^3P_1, F=11/2, m_F=11/2\rangle$ shown in figure 3.6 is performed after loading into dipole trap and before the evaporation ramp down in the presence of a bias magnetic field of 10 G by shining a σ^+ polarized probe beam whose detuning is controlled by AOM3 in figure 3.3 and AOM6 in single path configuration. The excited state $|^3P_1, F=11/2\rangle$ has a g-factor of 3/11 that lead to large Zeeman shift at large magnetic field (for example 67 G during the tripod sequence, see appendix C). In this case, we use AOM6 in double path configuration to add additional frequency to target the resonances. Absorption images at frequencies uniformly spread over 360 kHz encompassing the expected resonant frequency are acquired. These are fit with a 2D Gaussian function and peak optical depth is determined. The experimental data points in figure 3.6 are peak optical depth (OD) as a function of detuning of the probe (from resonance at 0 bias field). The data is then fit with Voigt profile

$$b_{\bar{v}}(\delta) = b_0(0) \sqrt{\frac{\pi}{8}} \frac{1}{1+s} \frac{\Gamma \sqrt{1+s}}{k \bar{v}} \Re \left[w \left(\frac{\delta + i \Gamma \sqrt{1+s}/2}{\sqrt{2} k \bar{v}} \right) \right] \quad (3.1)$$

where $b_0(0)$ is the resonant optical depth at zero temperature, $\bar{v} = \sqrt{k_B T/M}$ is the width of the velocity distribution, $b_{\bar{v}}(\delta)$ is the thermal optical depth as a function of detuning $\delta = 2\pi(f - f_0)$, s is the saturation parameter of the probe, Γ is the natural linewidth of the transition

and $\Re(w(x))$ is the real part of Faddeva function [2, 19]. The free parameters are the resonant optical depth at zero temperature $b_0(0)$, temperature T of the cloud, resonance frequency of the transition f_0 and the saturation parameter s (within 10% strict bounds of the estimation by measuring the probe power) of the probe. In essence, this fit quantifies the thermal and power broadening of the spectrum and thus verifies the experimental conditions. The temperature extracted from the fit is independently verified by time of flight imaging using blue probe. From the extracted saturation parameter, it is to be noted that the spectroscopy is done using a saturated probe. The reason for this choice is to have a high SNR of the imaging system without power broadening as far as $\sqrt{1+s} < k\bar{v}/\Gamma \sim 4$.

The indirect spectroscopy technique for measuring the resonance of $|^1S_0, F=9/2\rangle \rightarrow |^3P_1, F=9/2\rangle$ transitions is known as atom-loss spectroscopy [4, 20, 21]. This method is used to obtain the single photon resonance of the tripod lasers in the presence of 67 G bias magnetic field. We force atoms away by applying a radiation pressure force from the σ^+ tripod beam targeting $|^1S_0, F=9/2, m_F=5/2\rangle \rightarrow |^3P_1, F=9/2, m_F=7/2\rangle$ and measure the population of $|^1S_0, F=9/2, m_F=5/2\rangle$ at 10 G bias field probing resonance by observing maximum loss. Here, we switch off the dipole trap abruptly to eliminate the differential light shift due to the dipole trap in the measurement before switching on the tripod beam for 60 μs and recapture the atoms again for population measurement. Figure 3.7a shows the pixel sum of the cloud ($\propto OD$) as a function of frequency. The line shape of this spectrum is slightly more complicated than the previous spectroscopy as there are more than one light sources interacting with the atoms. We fit the experimental data points to an inverted Gaussian function to extract the resonance frequencies. Similar procedure is followed to extract spectrum of $|^1S_0, F=9/2, m_F=1/2\rangle \rightarrow |^3P_1, F=9/2, m_F=3/2\rangle$ shown in figure 3.7b that is essential for implementing the double tripod scheme. Due to release and recapture of the atoms from the dipole trap, we loose atoms from the trap and therefore the SNR for this spectrum is worse compared to the previous case. This leads to an error of approximately 10 kHz in the width of the Gaussian.

3.4.2 Fluorescence Imaging

In the previous sections, the characterization of the atomic sample and measurements with the atoms performed by absorption imaging using both 461 nm and 689 nm probes in the horizontal plane (where as the image plane is vertical) is described. In this section, we describe a fluorescence imaging scheme implemented to extract cloud expansion in the horizontal im-

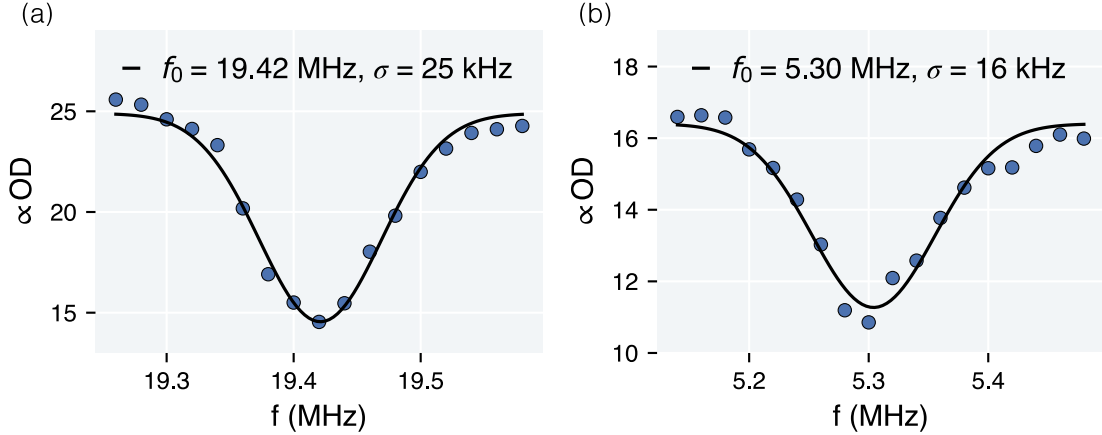


Figure 3.7: (a) Spectrum of $|^1S_0, F=9/2, m_F=5/2\rangle \rightarrow |^3P_1, F=9/2, m_F=7/2\rangle$. (b) Spectrum of $|^1S_0, F=9/2, m_F=1/2\rangle \rightarrow |^3P_1, F=9/2, m_F=3/2\rangle$. The data is fitted to a Gaussian profile to extract resonance frequency. The error in the width of the Gaussian is approximately 10 kHz.

age plane using $^1S_0 \rightarrow ^1P_1$ transition. The primary signal in our experiments described in chapters 4&5 is the measurement of populations of the Zeeman ground states. This is usually performed using spin-sensitive absorption imaging on the $^1S_0 \rightarrow ^3P_1$ intercombination line in the presence of a magnetic field [3]. However, the SNR of the absorption imaging with red probe is not very good. As the experimental sequence involves evaporative cooling, duration of each experimental cycle is as long as 12 seconds. The duration of the experiment would be very long if we were to average multiple runs of the experiment and causes inefficient data collection due to stability issues, for example, unlocking of the injection locked diode lasers and shot-to-shot fluctuations in number of atoms. For the tripod and double-tripod experiments involving adiabatic elimination of excited states, another spin-sensitive measurement can also be performed using the blue transition, $^1S_0 \rightarrow ^3P_1$. Due to the absence of spontaneous emission during adiabatic coupling of tripod lasers with the atoms, the interaction of lasers with the atoms is completely coherent through Raman transitions. These Raman transitions separate the atoms in different ground states of the tripod scheme in the momentum space [22–25]. Initially, we start with atoms at rest in $m_F = 9/2$ i.e., $|^1S_0, F=9/2, m_F=9/2, \vec{p}=0\rangle$. For an atom to be in $m_F = 5/2$ after the gauge field lasers are turned on, it has to absorb a photon from σ^- beam and emit a photon into σ^+ beam. For the geometry of tripod lasers shown in figure 2.2c, its final state is $|^1S_0, F=9/2, m_F=5/2, \vec{p}=2p_r\hat{e}_y\rangle$, where $p_r = \hbar k$ is the recoil momentum. Similarly, for an atom to be in $m_F = 7/2$, it has to absorb a photon from σ^- beam and emit a photon into π beam and hence its state is $|^1S_0, F=9/2, m_F=7/2, \vec{p}=p_r(\hat{e}_y - \hat{e}_x)\rangle$. Therefore, after sufficient time of flight, the initial atomic cloud splits into three as shown in figure 3.8a. From these images, populations of all the bare states can be easily calculated by

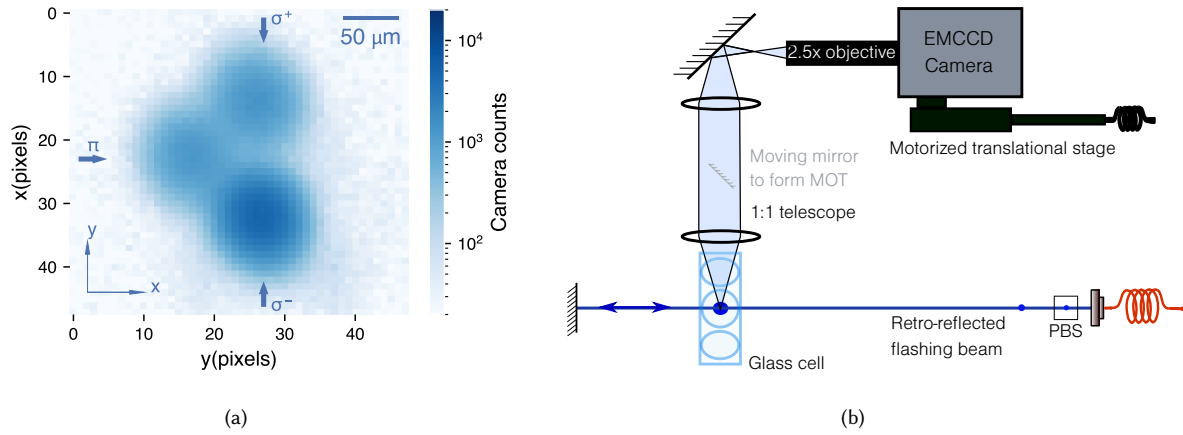


Figure 3.8: Vertical fluorescence imaging system. (a) Fluorescence image with a time of flight of 9 ms showing clear separation of m_F states in momentum space. (b) Schematic of the fluorescence imaging system used to measure the bare state populations in the experiment. Tripod laser beams and the flashing beam lie in the same plane.

appropriate fitting in a single run of the experiment. Since the relative populations of the bare states is sufficient to extract the state of the system in dark-state basis, this method is also robust against shot-to-shot fluctuations in the total number of atoms, drifts in the power of the flashing beam and calibration of fluorescence to number of atoms.

The schematic of the fluorescence imaging system is shown in figure 3.8b. A linearly polarized (normal to the plane of the paper in figure 3.8b) retro-reflected laser beam resonant with the $^1S_0 \rightarrow ^1P_1$ transition is pulsed for a short time ($10 \mu\text{s}$). Horizontal polarization of the flashing beam is favorable as the dipole radiation pattern is perpendicular to the flashing beam polarization and hence maximizes the fluorescence collected in the vertical direction. The cloud of fluorescing atoms is imaged using a 1:1 telescope constructed with lenses of focal length 100 mm and a numerical aperture of 0.2 at the working distance of a microscope objective. The objective used is 2.5x Infinity Achromat with a magnification of 2.5, resolving power of $5.2 \mu\text{m}$, numerical aperture of 0.064 and depth of focus of $130 \mu\text{m}$. The collected fluorescence is imaged on an EMCCD camera (Andor iXon 897 Life, pixel size $16 \mu\text{m}$) mounted on a motorized translation stage (driven by Thorlabs Z825B actuator) to adjust the focus for different times of flight. There are three main components in this imaging system that determine the SNR of the final image apart from the shot-to-shot fluctuations of the number of atoms. They are the properties of the flashing beam, alignment of the camera with respect to the imaging system and the readout noise of the EMCCD. In this section, the parameters of

these components is summarized.

The flashing beam lies in the plane of tripod lasers and it is polarized nearly parallel ($\approx 5^\circ$) to x-axis shown in figure 3.8a. This beam passes through a PBS before going through the science chamber to maintain the polarization. This beam is also slightly focusing to account for the losses at the surfaces of the science chamber² and the retro-reflecting mirror such that incoming beam and retro-reflected beam have similar intensities. The panels in the first row of the figure 3.9 show the fluorescence images as a function of flashing time of the probe when retro reflection is blocked. The bottom panels show the fluorescence images when the retro-reflection of the probe is active showing reduced mechanical action as radiation pressure forces are balanced. Even though the flashing beam is retro-reflected, there is still a mechanical action on the atoms as the shining time is increased. Figure 3.10a shows the total fluorescence count recorded on the camera as a function of the flashing time of the probe. Ideally, the fluorescence should keep increasing linearly with the duration of the flashing beam. The fluorescence increases linearly up to 40 μs of flashing time and saturates. The fluorescence image for 40 μs and 80 μs with retro-reflected flashing beam are very similar to each other as also indicated by total fluorescence in figure 3.10a. This indicates that by 40 μs of shining, all the atoms are almost flushed away due to heating. Note that the dipole trap is switched off during the imaging. The duration of the flash pulse is chosen to be 10 μs such that the mechanical action on the atoms is marginal with no distortion of the shape of the cloud.

The intensity of the flashing beam (from power of single beam) is maintained well above the saturation intensity ($\approx 15I_s$) of the blue transition (42 mW cm^{-2}) to maximize the fluorescence. Figure 3.10b shows the variation of fluorescence as a function of estimated saturation parameter of the flashing beam for a flashing duration of 10 μs . The fit of fluorescence with the function $\frac{a \times s}{1+b \times s}$ yields $b = 0.36$. This indicates that effective saturation parameter is 0.36 times the estimated saturation parameter. Reason for this low effective saturation is attributed to *interference* between flashing beam and its retro-reflection.

The optical imaging system consisting the telescope and the microscope objective of the camera is aligned using a vertical laser beam hitting the atoms and passing through the centers of the telescope lenses. As indicated in the figure 3.8b, vertical MOT beams and the telescope for fluorescence imaging share the same space (the top windows of the science cham-

²although the science chamber windows are broadband AR coated, the coating is not that efficient at 461 nm

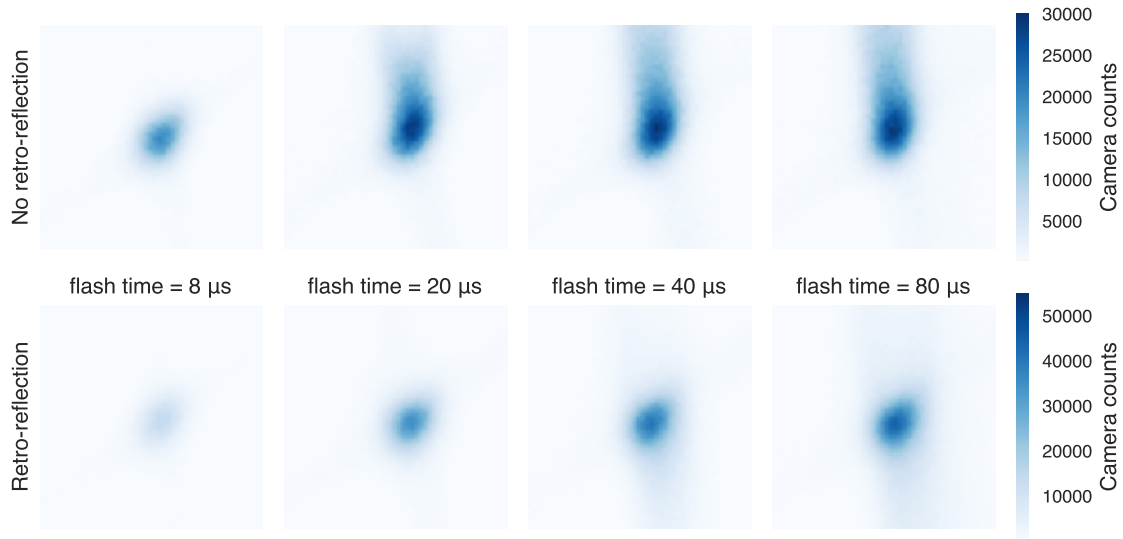


Figure 3.9: The figure showing the effect of retro-reflection of the flashing beam. Without the retro-reflection of the flashing beam, the shining time cannot be larger than $20\ \mu\text{s}$ as the atoms are rapidly flushed away. Whereas with the retro-reflection, the mechanical action on the atoms is reduced and allows for a larger shining time and hence a larger signal. The colorbars indicate the total counts on the EMCCD.

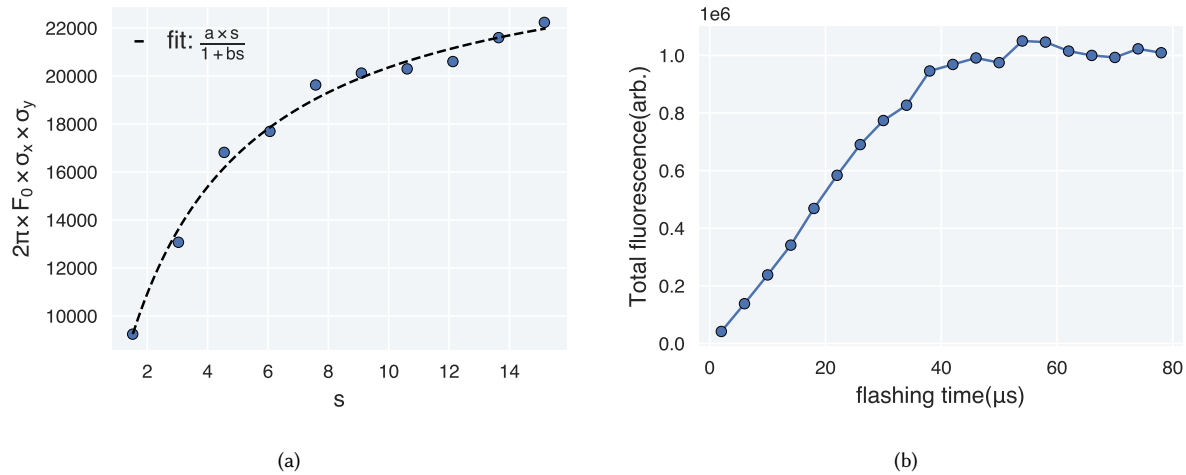


Figure 3.10: Optimization of flashing beam for fluorescence imaging with retro-reflected flashing beam. a) Fluorescence as a function of saturation parameter of the flashing beam. F_0, σ_x, σ_y are the amplitude, sizes in x and y directions, respectively of a 2D Gaussian fit of the image. b) Fluorescence as a function of flashing time of the probe.

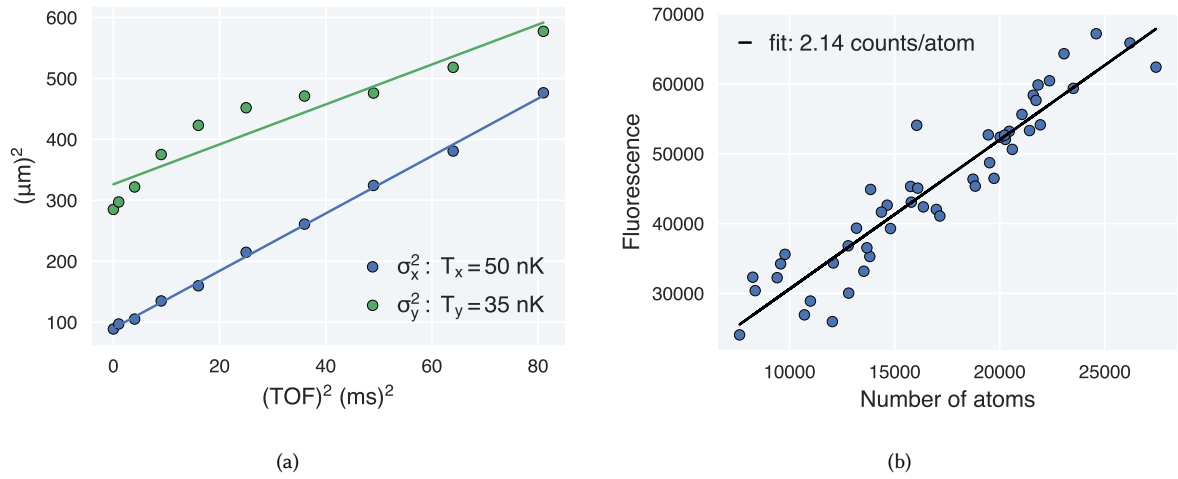


Figure 3.11: Characterization of the fluorescence imaging system. a) Temperature measurement of atoms after evaporation using the fluorescence imaging system. b) Number of atoms calibration of fluorescence indicating linear dependence of fluorescence on number of atoms.

ber) in the apparatus. So, we placed a mirror that can be moved in during the MOT and away from the fluorescence path during the evaporation before the imaging part. Vertical MOT beams are also expanded by a telescope very close to the science chamber and because of this, the laser beam used for alignment cannot be as small as desired. This caused difficulty in aligning the microscope objective and the EMCCD camera. After reasonable alignment of the optical system, the focus for different times of flight are determined by varying the position of the camera. The fluorescence images indicate astigmatism in the optical system. However, the foci in the orthogonal directions lie within the depth of focus of the objective ($130 \mu\text{m}$) rendering the defect not so critical for our imaging purpose. Using the determined focus positions for different times of flight, temperature of the atoms after evaporation is measured (shown in figure 3.11a) and verified from absorption imaging using a different imaging system. The square of sizes in y-direction (the direction of flashing beam) are a bit deviant from straight line due to non-uniform heating from the flashing beam across the beam.

Finally, the camera is configured to the slowest shift speed of the CCD available (1 MHz) to minimize the readout noise. An EM gain of 50 is also used to amplify the signal in all the experiments. Figure 3.11b shows the variation of fluorescence with the number of atoms obtained by usual absorption imaging. This is a check that fluorescence observed changes linearly with number of atoms. This also indicates the SNR ratio we have in our system, 7.26 taking into account the shot-to-shot fluctuation in the number of atoms and probe power fluctuations. However, this SNR is not of interest as we are measuring the relative populations of the ground

$\langle f \rangle$	$\langle \sigma_c \rangle$	$\langle NEA \rangle$	$\langle SNR \rangle_{\text{image at 9ms TOF}}$	$\langle SNR \rangle_{\text{system}}$
0.2	14.3	72.0	64.3	7.3

Table 3.1: Performance measures of the fluorescence images acquired.

states. Therefore, the SNR of the images acquired is the important parameter.

To characterize the fluorescence images acquired with this imaging system, consider the fluorescence from a Gaussian cloud of atoms given by

$$F(x, y) = F_0 e^{-\left(\frac{(x-x_0)^2}{2\sigma_x^2} + \frac{(y-y_0)^2}{2\sigma_y^2}\right)} + C \quad (3.2)$$

where F_0, σ_x, σ_y and C are the amplitude, size in x-direction, size in y-direction and a constant offset, respectively of the Gaussian centered at (x_0, y_0) . The fluorescence images are background corrected by using another image where there are no atoms. From the fit with the function in equation 3.2 we extract total fluorescence signal, the volume under the Gaussian (in counts $\times$ pixel²), $\mathfrak{F} = 2\pi F_0 \sigma_x \sigma_y$. To quantify the fluorescence signal acquired by the camera, we repeatedly measure number of atoms and acquire the corresponding fluorescence image for 60 times. Number of atoms, N_{atoms} is obtained from 60 absorption images. From this, average fluorescence per atom per pixel, $\langle f \rangle = \langle \frac{\mathfrak{F}}{N_{atoms} 2\pi \sigma_x \sigma_y} \rangle$ can be calculated. Average noise equivalent number of atoms (NEA) per pixel can be estimated using,

$$\langle NEA \rangle = \frac{\langle \sigma_c \rangle}{\langle f \rangle} \quad (3.3)$$

where $\langle \sigma_c \rangle$ is the standard deviation of counts in pixels far away from the Gaussian cloud averaged over different shots. Mean SNR of the image, $\langle SNR \rangle_{\text{image}} = \frac{\langle F_0 \rangle}{\sigma_c}$ while mean SNR of the total system is $\langle SNR \rangle_{\text{system}} = \frac{\langle f \rangle}{\sigma_f}$, where σ_f is the standard deviation of f calculated over different shots of the experiment. These parameters for our imaging system are summarized in the table 3.1.

In summary, a fast and reliable fluorescence imaging system with a noise floor of 72 atoms/pixel, that is used to extract the Zeeman states' population in the experiments is described. Various performance metrics of the fluorescence images acquired with the imaging system is also characterized in this section.

3.5 Double-Tripod Laser System

Atom-light coupling of a double tripod scheme described in section 2.3.3 is implemented by using the Zeeman sublevels $|^1S_0, F = 9/2, m_F > 0\rangle$ as ground states and $|^3P_1, F = 9/2, m_F = 3/2\rangle$ and $|^3P_1, F = 9/2, m_F = 5/2\rangle$ as excited states in the presence of a large bias field $B_{bias} = 67$ G (see figure 3.12a). The ground states are nuclear spin states and hence are weakly coupled to the external magnetic field. At 67 G of magnetic field, the separation between consecutive Zeeman states is $g_F|\Delta m_F|\mu_B B_{bias} = 12.4$ kHz where $|\Delta m_F| = 1$ is the difference in azimuthal quantum number of the Zeeman states, $g_F = 1.3 \times 10^{-4}$ is the Landé g-factor and μ_B is Bohr magneton. Whereas excited states have electronic spin that is strongly coupled to the bias field giving rise to a large shift among various Zeeman sublevels. At this bias field, the linear Zeeman shift between consecutive Zeeman states $g_F\Delta m_F\mu_B B_{bias} = 5.68$ MHz where $|\Delta m_F| = 1$ and $g_F = 2/33$. The excited state has a significant higher order contribution leading to unequal separation between the excited state Zeeman sublevels. The frequency separations between the excited state Zeeman sublevels are found by doing atom-loss spectroscopy (see section 3.4.1 and [4] of various $|^1S_0, F = 9/2, m_F\rangle \rightarrow |^3P_1, F = 11/2, m_F + 1\rangle$ transitions verified by the numerical determination of electric quadrupole effect summarized in appendix C. The frequency separation between the transitions $|^1S_0, F = 9/2, m_F = 1/2\rangle \rightarrow |^3P_1, F = 9/2, m_F = 3/2\rangle$ (detuned by 5.30 MHz w.r.t resonance at zero bias field) and $|^1S_0, F = 9/2, m_F = 5/2\rangle \rightarrow |^3P_1, F = 9/2, m_F = 7/2\rangle$ (detuned by 19.42 MHz w.r.t resonance at zero bias field) is found to be 14.12 MHz. To target the transitions of the double tripod scheme,

- First, we target the σ^+ and σ^- transitions of the right tripod scheme (connecting the $m_F = 9/2$ ground state that is initially populated) with two laser beams controlled by σ^+ and σ^- AOMs as shown in figure 3.12b with -80 and -80.0248 MHz, respectively. The frequency difference of 24.8 kHz is to account for the ground state shifts in the bias magnetic field. The single photon detuning of this Λ -scheme is controlled by AOM3 in double path configuration in figure 3.3 that is frequency shifted by 19.42 MHz at resonance.
- Next, we use σ^+ and σ^- EOMs driven with a modulation frequency of 14.12 MHz whose -1 order sidebands are used to target the σ^+ and σ^- transitions of the left tripod respectively.
- Finally, we use a laser beam controlled by π -AOM at 94.132 MHz to target the π transition of the left tripod. The EOM in the path of the π beam is also modulated at 14.12 MHz such that +1 order sideband targets the π transition of the right tripod. This way

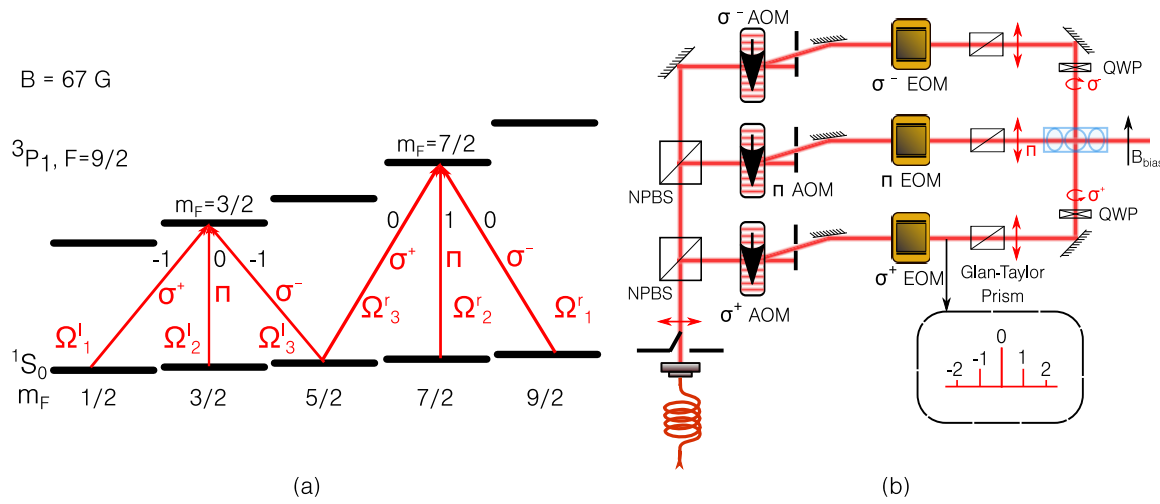


Figure 3.12: (a) Energy level scheme of double-tripod configuration showing the sideband index and polarization of the laser beams. (b) Schematic showing the implementation of the double-tripod scheme using AOMs and EOMs. Inset shows the schematic spectrum of light after EOM.

of targeting the π transition is implemented in order to compensate for the disparity in Clebsch-Gordan coefficients (see figure D.1b) between the π transitions and achieve flexible range of modulation depths to control the Rabi-frequencies of the tripod lasers. Practically, we address the transition with the smallest CG coefficient with the carrier to minimize the modulation index necessary to reach equal Rabi frequency.

The tripod AOMs are driven by amplified signal from Analog Digital 9852 direct digital synthesis (DDS) to accurately control the detuning of tripod lasers. The tripod EOMs are driven by Rigol DG4102 function generators and then amplified by RF amplifiers that can be triggered during the tripod laser sequence. The total power of each beam is controlled by the RF power supplied to the AOMs and the distribution among the sidebands is controlled by the RF power provided to the EOMs by modifying the modulation depth of the EOMs. This implementation of the double tripod configuration offers accurate controllability of the experimental parameters *viz.*, Rabi frequencies and detunings of the lasers. In the following subsection characterization of the modulation depth provided by the EOMs is described.

3.5.1 Calibration of modulation depth of the EOMs

The EOMs used to produce the sidebands are driven by high power RF amplifiers fed with the output of arbitrary waveform generators. We calibrate the modulation depth β as a function of the input voltages to the RF amplifiers using Michelson's interferometer shown in figure

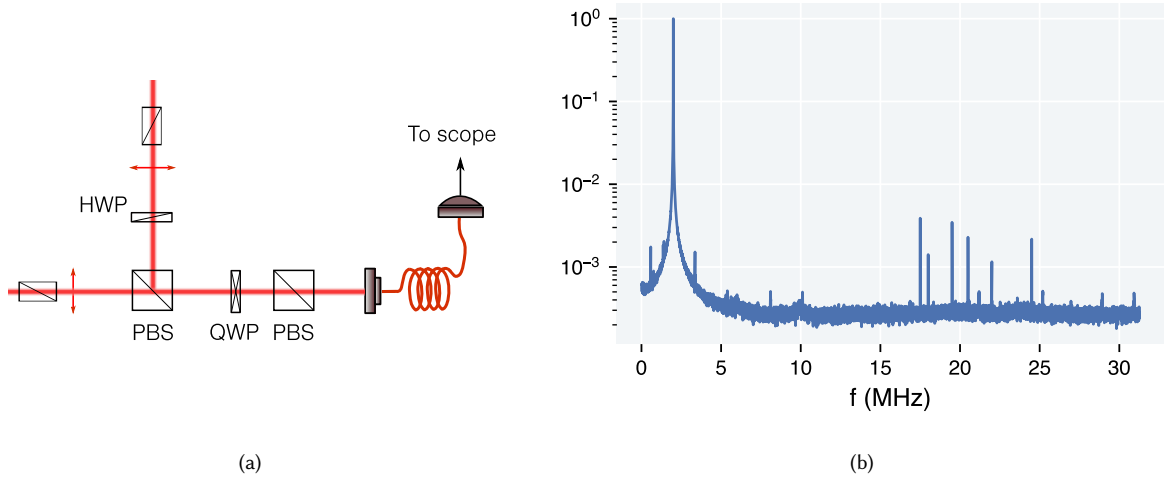


Figure 3.13: (a) Optical system for performing the interferometry among the tripod beams. The tripod beams are collected after the Glan-Taylor prism using flipping mirrors. (b) Normalized spectrum of the beat signal with 1 MHz frequency difference between the tripod AOMs and EOMs switched off.

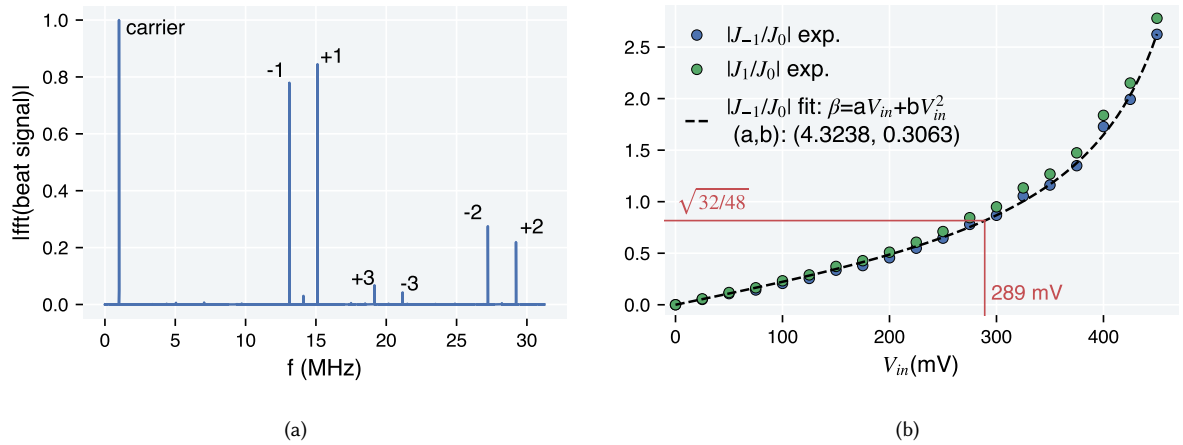


Figure 3.14: Calibration of σ^+ EOM. (a) Spectrum of the beat signal normalized to carrier strength when $V_{in} = 275$ mV. Third order sidebands are aliased due to maximum frequency limitation of 32 MHz. (b) Ratios of first order sidebands generated by σ^+ EOM as a function of input voltage to the RF amplifiers. This data is fit with a quadratic polynomial of V_{in} to extract the calibration.

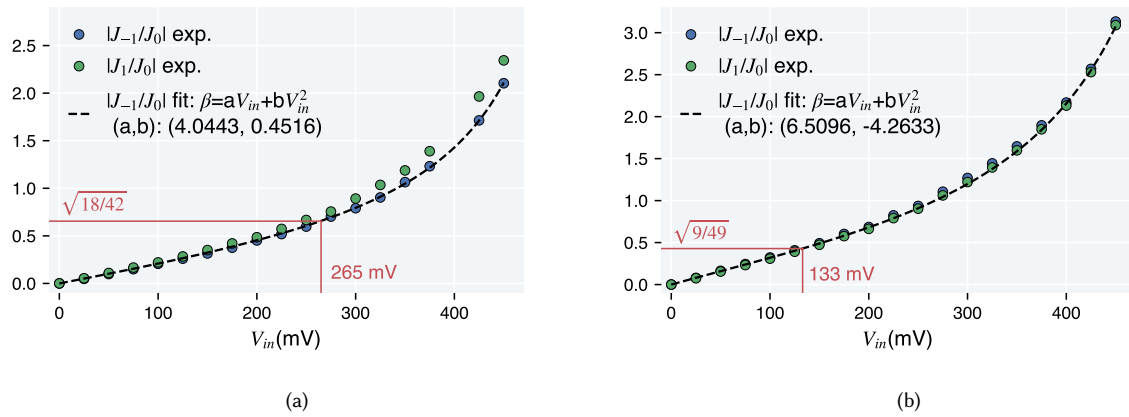


Figure 3.15: (a) Ratios of first order sidebands generated by σ^- EOM as a function of input voltage to the RF amplifiers. (b) Ratios of first order sidebands generated by π EOM as a function of input voltage to the RF amplifiers.

3.13a. We set the frequency difference between the tripod AOMs of the beating beams and extract the beat signal. Figure 3.13b shows the beat signal when EOMs are switched off. The EOMs are phase modulators (Thorlabs EO-PM-NR-C1) and their frequency response of the electric field of the laser when modulated with a sinusoidal voltage with frequency Ω_0 is given by Jacobi-Anger expansion that reads

$$E(\omega) = E_0 e^{i(\omega t + \beta \sin \Omega_0 t + \phi)} = E_0 \sum_{n=0}^{n=\infty} J_n(\beta) e^{i(\omega + n\Omega_0)t + i\phi} \quad (3.4)$$

where ω and E_0 are the carrier frequency and carrier amplitude, respectively and J_n is n^{th} -order Bessel function. To characterize a particular EOM, we interfere the modulated laser, with balanced optical powers, a reference laser beam with slightly different carrier frequency then the intensity of the output beat signal is

$$I \propto 2|E_0|^2 \sum_{n=0}^{n=\infty} J_n(\beta) \cos((\omega - \omega_{ref} + n\Omega_0)t + \phi) \quad (3.5)$$

where ω_{ref} is the frequency of the reference signal. We set $\omega - \omega_{ref}$ to be 1 MHz to differentiate the intensity of positive and negative sidebands where ω is set to resonance frequency of the appropriate double-tripod transition. Figure 3.14a shows the spectrum of the beat signal normalized to the strength of carrier where the positive order and negative order sidebands are separated by 2 MHz. Peak at 14.12 MHz (modulation frequency) is due to polarization misalignment with the principle axis of EOMs that appears as amplitude modulation after Glan polarizer. The third order sidebands are aliased to a lower frequency due to a limited

Nyquist frequency of 32 MHz during measurement. Figure 3.14b shows the ratios of strengths of first order side bands and carrier signal as a function of input voltage to the RF amplifier. These ratio corresponds to $J_{\pm 1}(\beta(V_{in}))/J_0(\beta(V_{in}))$ and negative sideband ratio is fit to β as a quadratic polynomial in V_{in} to extract the calibration coefficients a and b. The +1 order sideband is far blue detuned and is left out with out any effect. Though the RF amplifiers driving identical EOMs are identical commercial products, their gain is not the same. Figures 3.15a and 3.15b shows this calibration for σ^- and π EOMs respectively for appropriate sidebands. The Rabi frequency of the coupling to atoms is related to intensity I of the lasers as

$$\Omega = \Gamma \sqrt{\frac{(CG)^2 I}{I_s}} \quad (3.6)$$

where Γ is the linewidth, I_s is the saturation intensity and CG is the Clebsch-Gordan coefficient of the transition. For equal Rabi frequencies of both σ^+ transitions the ratio of sideband to carrier should be

$$\frac{J_{-1}(\beta)}{J_0(\beta)} = \frac{(CG)_0}{(GC)_{-1}} = \sqrt{\frac{32}{48}} \quad (3.7)$$

The input voltage required to achieve this modulation is 289 mV. The voltages required for equal Rabi frequencies σ^+ and π transitions 265 mV and 133 mV, respectively.

3.5.2 Second Harmonic Cancellation

Evidently, there exists higher order sidebands that couple unwanted transitions and modify the state of the system evolving in double-tripod configuration. They are -2 order sidebands of σ^+ and σ^- transitions and -1 order of the π transition. The -1 order sideband of π laser targets $|^1S_0, F=9/2, m_F=-1/2\rangle \rightarrow |^3P_1, F=9/2, m_F=-1/2\rangle$ off-resonantly due to second order Zeeman shift in the excited state and cannot be fully eliminated. Moreover, the ground state $|^1S_0, F=9/2, m_F=-1/2\rangle$ does not contain any population during the start of the tripod sequence. However, the -2 order sideband of σ^- can depopulate atoms in $|^1S_0, F=9/2, m_F=1/2\rangle$ state to $|^1S_0, F=9/2, m_F=-1/2\rangle$, part of the double-tripod configuration, by two-photon resonant Raman scattering. This can be avoided by canceling the -2 order sideband of the σ^- laser by adding a modulation at the second harmonic of the modulation frequency with an optimal phase difference with respect to the first harmonic modulation. Figure 3.16 shows the spectrum of the beat signal after canceling the -2 order. This second harmonic cancellation slightly modifies the strength of all the sidebands and this change has

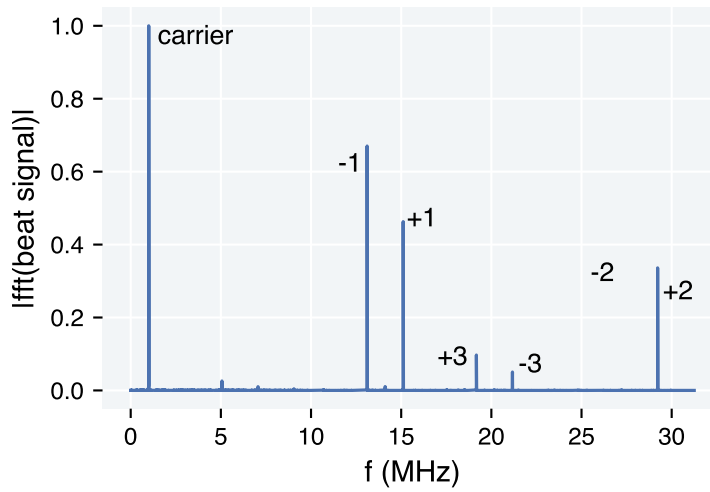


Figure 3.16: Second harmonic cancellation of σ^- laser. Spectrum is shown for 240 mV of first harmonic modulation to maintain the equal Rabi frequency condition for carrier and -1 order side band and 173 mV of second harmonic modulation with a phase difference of -87° with respect to the first harmonic.

to be taken into account for determining the ratio of the Rabi-frequencies. Spectrum is shown for 240 mV of first harmonic modulation to maintain the equal Rabi frequency condition for carrier and -1 order side band and 173 mV of second harmonic modulation with a phase difference of -87° with respect to the first harmonic.

3.6 STIRAP Techniques

Before moving on to measure the Rabi frequencies of the tripod beams, in this section we show the transfer of population from $|^1S_0, F = 9/2, m_F = 9/2\rangle$ to positive m_F ground states by using stimulated Raman adiabatic passage (STIRAP) technique to individually measure the Rabi frequencies of the double-tripod transitions. As the dipole trap is switched off during the measurements, it is favorable to limit the duration of STIRAP sequence before measuring the Rabi frequency. However, we are limited by the lab control system with a time resolution of $2 \mu\text{s}$ and it is difficult to produce smooth adiabatic pulses required for efficient STIRAP transfer in the least amount of time of flight with this system. In order to solve this problem, we modify the RF system that controls the AOM as shown in figure 3.17. As stated earlier, the RF frequency is generated by a DDS with set frequency controlled by the lab control software. The output of the DDS passes through a voltage controlled attenuator that is used to set the peak power of the RF and then through two fast RF switches (Mini-Circuits ZFSWHA-1-20+) in

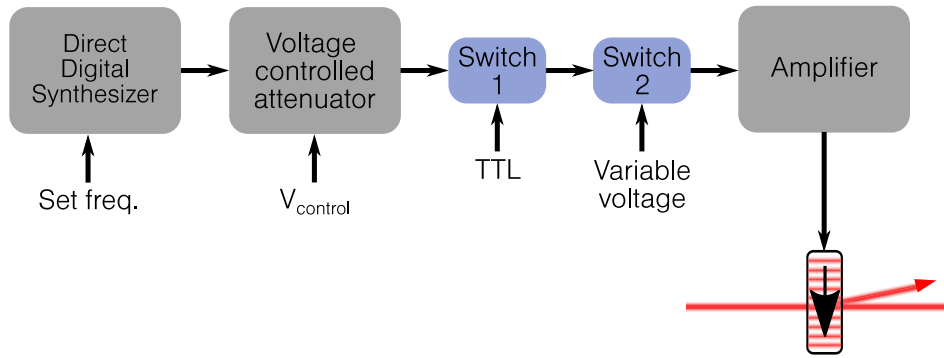


Figure 3.17: Block diagram showing the RF electronics system for generating Gaussian pulses for STIRAP and atomtronics Datta-Das transistor experiment.

series before the signal is fed to the RF amplifier. The first switch is used to abruptly switch on and off the RF power that is necessary for switching the tripod beams and the second switch offers fast continuous control by variable switching voltage. The variable voltage to smoothly vary the RF power output is supplied with an arbitrary generator that can be triggered on demand. The power of the diffracted beam relative to the input optical power of the AOM is calibrated with the voltage supplied to the switch. In this way, we can control the power of the first order beam in arbitrary pulse shape. Figure 3.18 shows the Gaussian pulses generated using this technique with a set temporal standard deviation, σ_t of $8 \mu\text{s}$. The fit to a Gaussian curve yields close approximation of the set value for σ_t . The center of the Gaussian pulses are also well defined. This technique is used for all the adiabatic variations of optical power in the experiments.

With these Gaussian pulses, we perform STIRAP from $|^1S_0, F = 9/2, m_F = 9/2\rangle$ to $|^1S_0, F = 9/2, m_F = 5/2\rangle$ or $|^1S_0, F = 9/2, m_F = 7/2\rangle$ within a duration of $44 \mu\text{s}$. Figures 3.19 c&d show the momentum distribution of the atoms after a time of flight of 9 ms. The atoms at $p = 0$ correspond to the 50% of the total number of atoms in negative m_F states. These two STIRAP sequences contain only two Gaussian pulses. However, to transfer the atoms from $|^1S_0, F = 9/2, m_F = 9/2\rangle$ to $|^1S_0, F = 9/2, m_F = 3/2\rangle$ we need to perform two STIRAP procedures. First, from $|^1S_0, F = 9/2, m_F = 9/2\rangle$ to $|^1S_0, F = 9/2, m_F = 5/2\rangle$ using σ^+ and σ^- beams with both EOMs switched off followed by a STIRAP from $|^1S_0, F = 9/2, m_F = 5/2\rangle$ to $|^1S_0, F = 9/2, m_F = 3/2\rangle$ using π beam and -1 order σ^- beam. During the second STIRAP the π EOM is switched off while σ^- EOM is switched on. This takes a total time of $92 \mu\text{s}$ and the figure 3.19b shows the final momentum distribution. From the images, it is evident that the transfer is coherent and non-adiabatic effects are minimum as

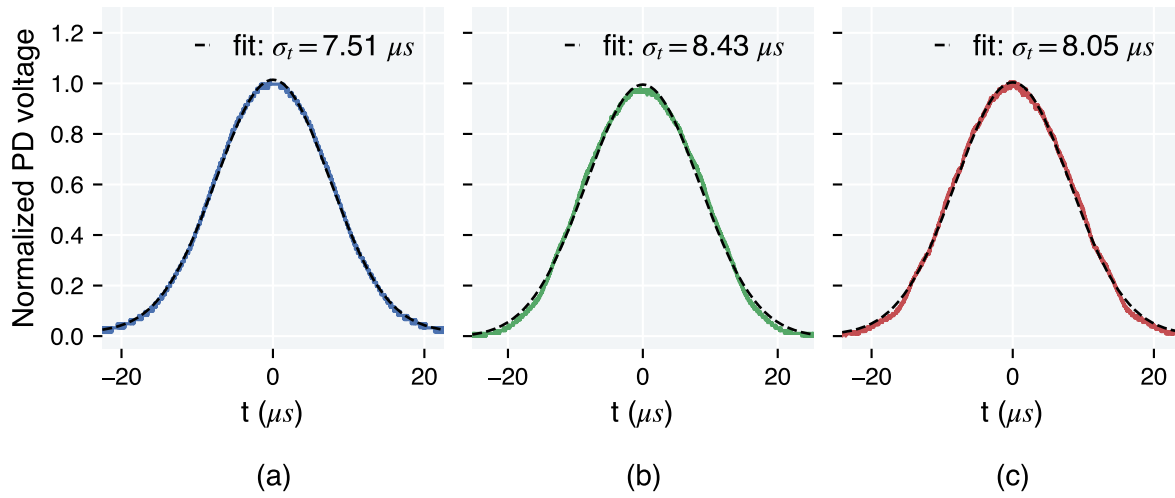


Figure 3.18: Generation of Gaussian pulses. Normalized power of the Gaussian pulses σ^+ (a), σ^- (b) and π (c) beams as a function of time for $\sigma_t = 8 \mu s$.

there are no randomly scattered atoms away from the expected momenta.

The transfer from $|^1S_0, F = 9/2, m_F = 9/2\rangle$ to $|^1S_0, F = 9/2, m_F = 1/2\rangle$ is performed by adiabatically the following dark state of the double- Λ scheme (formed by switching off π lasers of the double tripod scheme shown in figure 3.12) that reads

$$|D\rangle = -\frac{\Omega_3^l \Omega_1^r |1\rangle - \Omega_1^l \Omega_1^r |3\rangle + \Omega_1^l \Omega_3^r |5\rangle}{\sqrt{|\Omega_3^l \Omega_1^r|^2 + |\Omega_1^l \Omega_1^r|^2 + |\Omega_1^l \Omega_3^r|^2}} \quad (3.8)$$

This can be achieved by σ^+ and σ^- STIRAP Gaussian pulses with EOMs turned on. Initially the dark state $|D\rangle$ is $|5\rangle \equiv |^1S_0, F = 9/2, m_F = 9/2\rangle$ and by the of STIRAP pulses, the dark state $|D\rangle$ is $|1\rangle \equiv |^1S_0, F = 9/2, m_F = 1/2\rangle$. Figure 3.19a shows the final momentum distribution after the 44 μs STIRAP sequence. In conclusion, we demonstrate good control on the initial population distribution of the double tripod system.

3.7 Rabi frequency measurement

In section 3.5.1, we showed how equal relative Rabi frequency for transitions targeted by carrier and the sideband can be achieved by setting appropriate modulation depth of the EOMs. However, the absolute Rabi frequency is still unknown. In this section, we show how we exper-

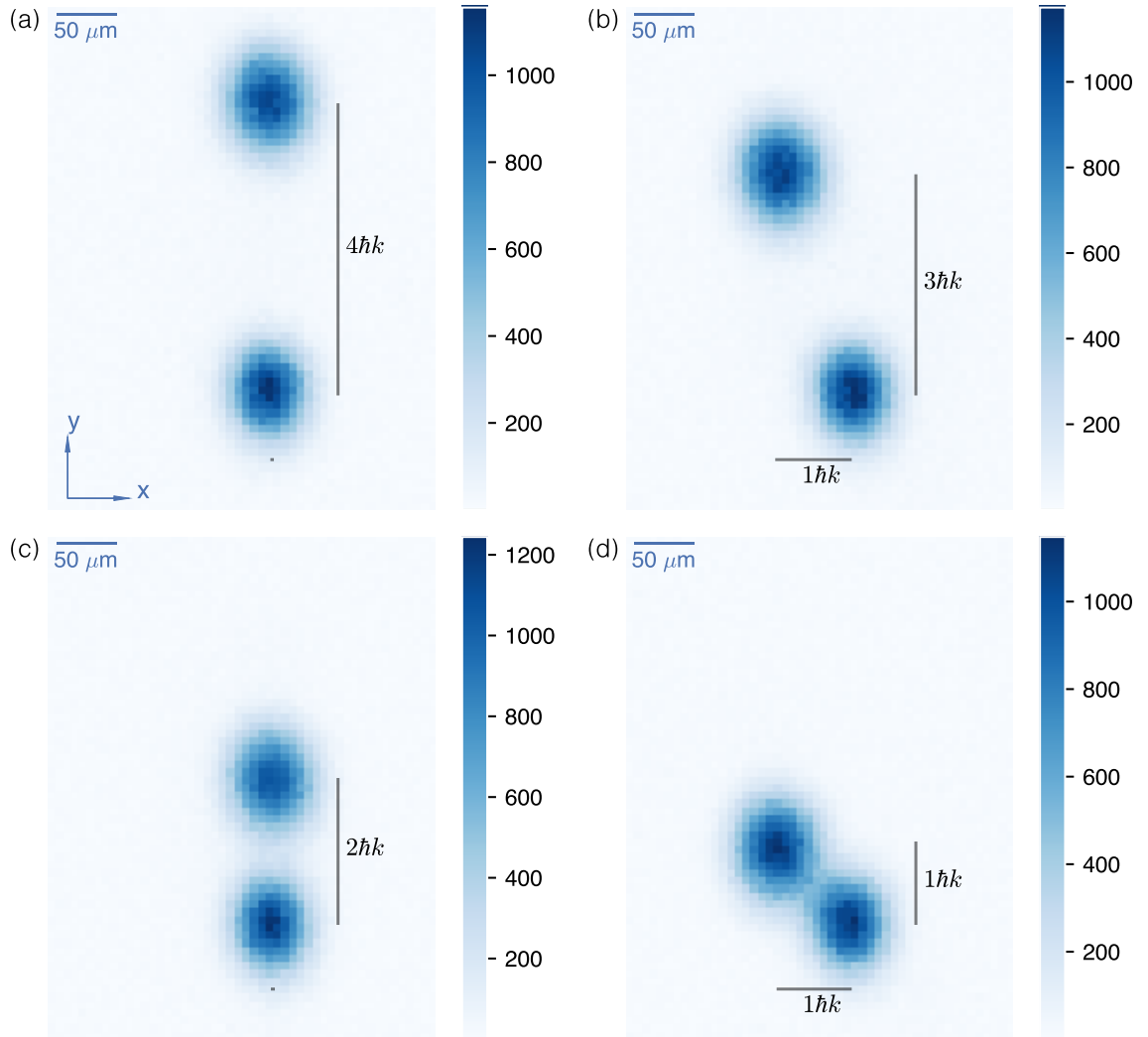


Figure 3.19: STIRAP to various ground state m_F levels. (a) Population of $m_F = 9/2$ is transferred to $m_F = 1/2$ using σ^+ and σ^- beams with both EOMs switched on. (b) Population of $m_F = 9/2$ is transferred to $m_F = 3/2$ using two STIRAP sequences: first to $m_F = 5/2$ using σ^+ and σ^- beams with both EOMs switched off and then from $m_F = 5/2$ to $m_F = 3/2$ using the carrier of π beam and -1 sideband of σ^- . (c) Population of $m_F = 9/2$ is transferred to $m_F = 5/2$ using σ^+ and σ^- beams with both EOMs switched off. (d) Population of $m_F = 9/2$ is transferred to $m_F = 7/2$ using +1 sideband of π beam and carrier of σ^- .

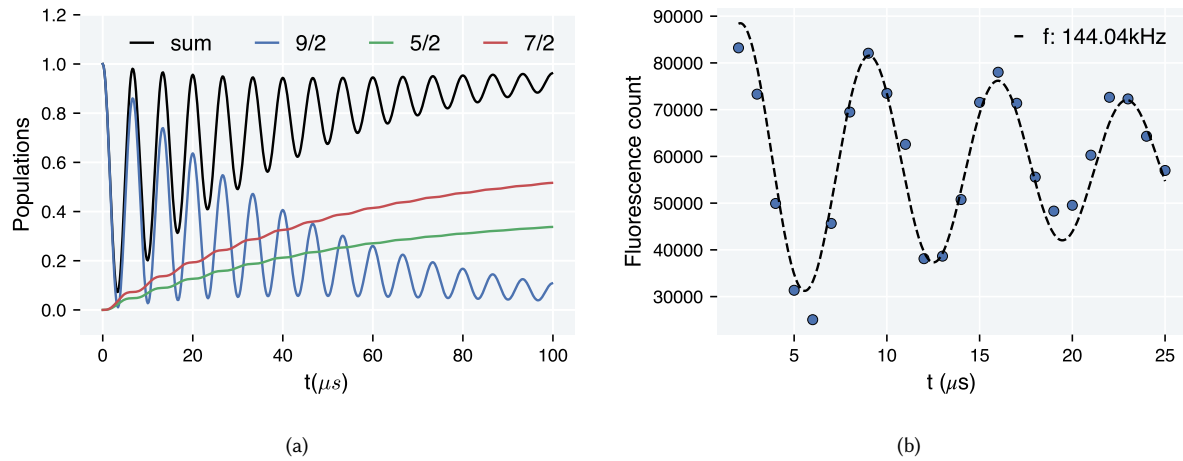


Figure 3.20: (a) Populations of tripod ground states when excited with a Rabi pulse. Sum of populations of the ground states (black plain line) is the experimental signal. (b) Sample experimental signal of Rabi flopping of $|^1S_0, F = 9/2, m_F = 9/2\rangle \rightarrow |^1S_0, F = 9/2, m_F = 7/2\rangle$ transition fit with an exponentially decaying sinusoidal function with two decay constants to extract the Rabi frequency.

imentally measure the Rabi frequencies by directly observing the Rabi flopping of the atoms. We prepare the atoms in appropriate ground state using the STIRAP techniques described in previous section and abruptly excite the atoms with a tripod laser. When the atoms undergo Rabi flopping, we abruptly switch off the exciting laser and immediately measure the ground state population as a function of excitation duration. Since we need to immediately measure the population, we cannot have a time of flight and so we only measure the sum of fluorescence from atoms in all the Zeeman ground states. Moreover, the tripod transitions are not closed *i.e.*, an atom in excited state can decay into three Zeeman ground states connecting the excited state, and hence the atoms are optically pumped out of the initial ground state to the other two Zeeman ground states connected to the excited state. We numerically simulate the evolution of populations of tripod ground states using optical Bloch equations when population is initialized in stretched state $|^1S_0, F = 9/2, m_F = 9/2\rangle$ and excited with a σ^- pulse. Figure 3.20a shows the evolution of populations in relevant ground states and the sum of ground state populations simulated at zero temperature. From this, we can extract the Rabi frequency of the exciting laser.

Figure 3.20b shows the experimental signal of Rabi flopping of atoms due to excitation of $|^1S_0, F = 9/2, m_F = 9/2\rangle \rightarrow |^1S_0, F = 9/2, m_F = 9/2\rangle$ transition. The experimental data is fit with an exponentially decaying sinusoidal and Rabi frequency is extracted. The effective SNR in this case is reduced by shot-to-shot fluctuations in the atom count unlike the time of flight

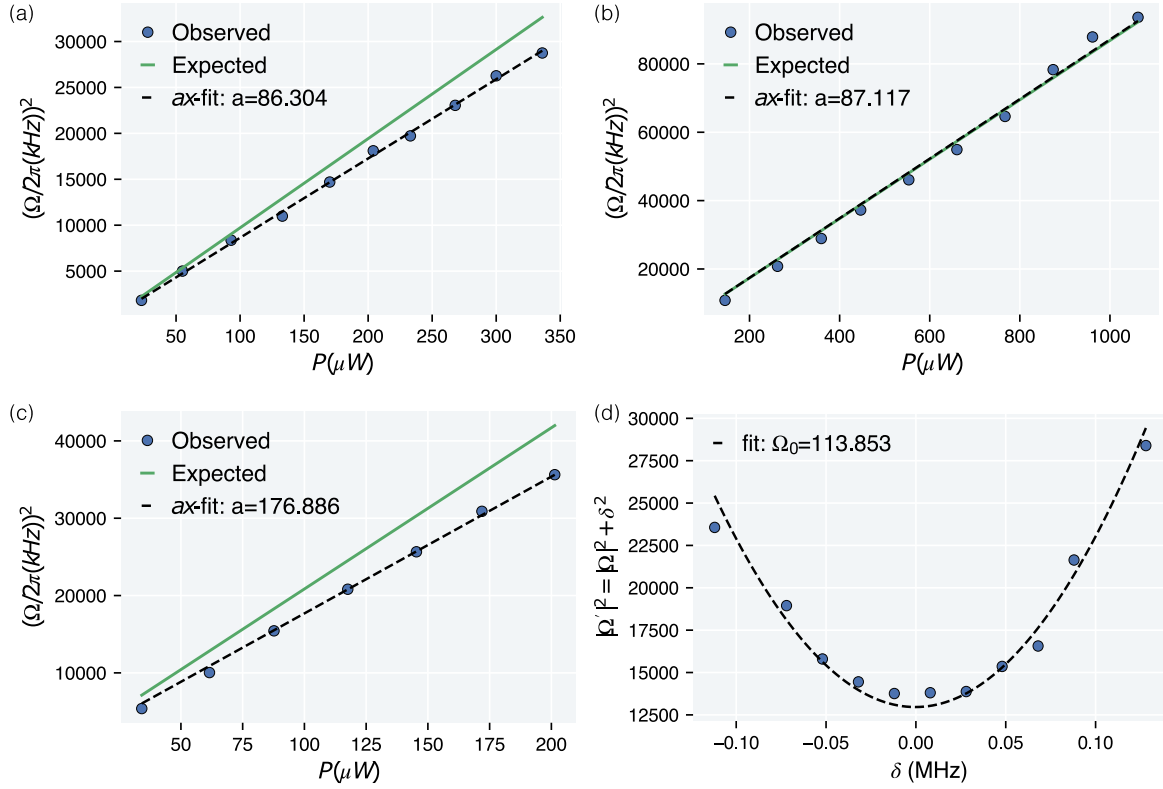


Figure 3.21: (a) Squared Rabi frequency $|\Omega_3^r|^2$ as a function of optical power of σ^+ beam. (b) Squared Rabi frequency $|\Omega_1^r|^2$ as a function of optical power of σ^- beam. (c) Squared Rabi frequency $|\Omega_2^r|^2$ as a function of optical power of π beam (power of +1 order). (d) Square of the generalized Rabi frequency $|\Omega'|^2$ of the $|^1S_0, F=9/2, m_F=5/2\rangle \rightarrow |^1S_0, F=9/2, m_F=3/2\rangle$ transition as a function of detuning of the exciting laser by varying the modulation frequency of the σ^- laser.

	σ_x (mm)	σ_y (mm)
σ^+	1.405	1.362
σ^-	1.103	1.135
π	1.266	1.120

Table 3.2: Gaussian beam sizes of the tripod beams.

images. Moreover, the imaging is performed immediately after the excitation in the 67G bias field. This reduces the fluorescence signal due to Zeeman splitting of $^1S_1$ state. Nevertheless, this technique is proven to be useful in experimentally measuring the Rabi frequencies. We measure the Rabi frequencies of tripod transitions and verify that the tripod beams are hitting the atoms at their center by comparing the expected peak Rabi frequency estimated using the relation 3.6 and experimentally observed Rabi frequencies. The beam sizes of the tripod beams used in the calculation is tabulated in table 3.2. Figures 3.21a,b&c show the linear relationship between squared Rabi frequency and the optical power of the σ^+ , σ^- and π beams, respectively targeting right tripod transitions. The green plain lines in these plots are the estimated peak Rabi frequencies. The mismatch is due to slight misalignment of the tripod beams with the atoms *i.e.*, the atoms are not located at the center of the beams. In figure 3.21d, we show the square of the generalized Rabi frequency, $|\Omega'|^2 = |\Omega|^2 + \delta^2$ as a function of the detuning δ of the exciting tripod laser and fit it with a quadratic function of δ . In conclusion, we have developed a method to directly measure the Rabi frequencies of the double-tripod transitions by observing Rabi flopping and check that the optical system that is implemented is in order for the gauge field experiments.

3.8 Dressed State Spectroscopy

As shown in section 3.5.1, there exist unwanted sidebands generated by phase modulation of tripod EOMs. These side bands are far detuned (detuning of the order of MHz while $\Gamma = 2\pi \times 7.5$ kHz). It is expected that the sidebands give rise to light shifts (AC Stark shifts) of various levels in the double-tripod scheme. The light shifts of the excited states are not significant as they only cause single photon detuning that does not influence the dark-state manifold. However, the light shifts of the ground states cause two-photon detuning that is important for the experiments described in chapter 2.3.4. For a two-level atom interacting with off-resonant electromagnetic radiation, light shift of the ground state in the weak coupling approximation reads [26],

$$\delta_g = \frac{\delta}{4\delta^2 + \Gamma^2} \Omega^2 \quad (3.9)$$

$m_F = 1/2$	$m_F = 3/2$	$m_F = 5/2$	$m_F = 7/2$	$m_F = 9/2$
-8.63	-3.74	-2.28	4.73	5.23

Table 3.3: Estimation of ground state light shifts (in kHz) for a 230 kHz of Rabi frequency for all the double-tripod transitions.

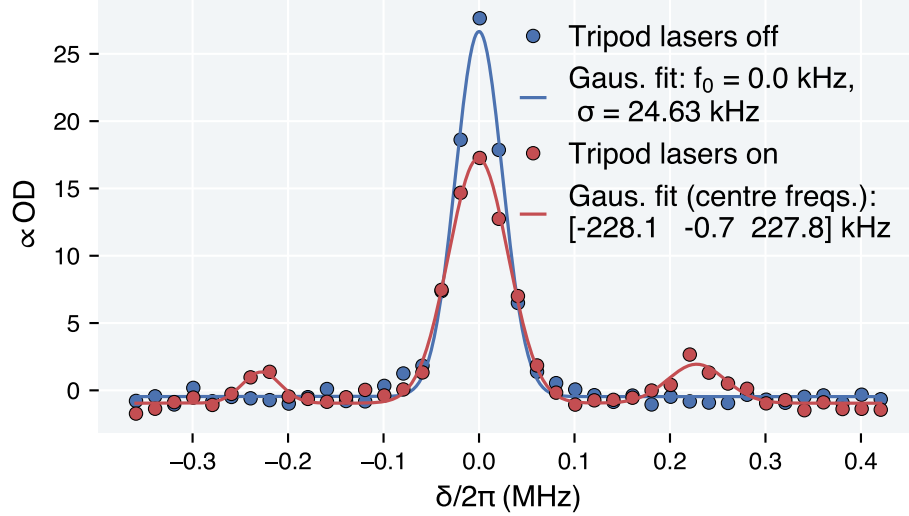


Figure 3.22: Experimental spectra of the $|^1S_0, F = 9/2, m_F = 5/2\rangle \rightarrow |^3P_1, F = 11/2, m_F = 7/2\rangle$ transition with the tripod lasers off (blue circles) and on (red circles). The legend indicates the fit values for the experimental data.

where, δ and Ω are the detuning and Rabi frequency of the radiation field, respectively and Γ is the linewidth of the excited state of the atom. Before we experimentally measure the light shifts, an estimation of the ground state light shifts is made by summing up light shifts due to individual off-resonant sidebands when the double-tripod transitions are coupled with a Rabi frequency of 230 kHz (experimental parameter) using equation 3.9. These estimates are summarized in table 3.3. It should be noted that the light shifts are of the order of excited state linewidth and hence should have little influence on the experimental results. Moreover, it is the two-photon detunings that influence the dynamics among the dark states.

To experimentally measure these light shifts, we use a σ^+ polarized weak probe ($\Omega \gg \Omega_{probe}$) targeting $|^1S_0, F = 9/2, m_F\rangle \rightarrow |^3P_1, F = 11/2, m_F + 1\rangle$. In the experiment, after dropping atoms from the dipole trap, we shine the double-tripod lasers by switching on the AOMs and the EOMs abruptly along with the probe beam for 80 μ s and vary the frequency of the probe beam across the resonance of the $|^1S_0, F = 9/2, m_F\rangle \rightarrow |^3P_1, F = 11/2, m_F + 1\rangle$ transition, where m_F is the target ground state for which light shift is measured. The shift in the

State	m_F	$ \langle m_F B_1 \rangle ^2$	$ \langle m_F B_2 \rangle ^2$	$ \langle m_F B_3 \rangle ^2$	$ \langle m_F B_4 \rangle ^2$
$^1S_0, F = 9/2$	1/2	0.125	0.125	0.0625	0.0625
	3/2	0.125	0.125	0.0625	0.0625
	5/2	0	0	0.25	0.25
	7/2	0.125	0.125	0.0625	0.0625
	9/2	0.125	0.125	0.0625	0.0625
$^3P_1, F = 9/2$	3/2	0.25	0.25	0.25	0.25
	7/2	0.25	0.25	0.25	0.25

Table 3.4: Populations of various bare states in bright states when the Rabi frequency of all the double tripod transitions is equal. The state $|^1S_0, F = 9/2, m_F = 5/2\rangle$ is present only in the bright states $|B_3\rangle$ and $|B_4\rangle$ apart from the dark state $|D_0\rangle$.

resonance of this transition gives us the light shift of the ground state. This is a well known technique used to measure the Autler-Towns splitting in pump-probe experiments using Λ -system [27, 28]. To start with, we extract dressed state spectrum of double-tripod configuration by probing the $|^1S_0, F = 9/2, 5/2\rangle \rightarrow |^3P_1, F = 11/2, 7/2\rangle$ transition. Figure 3.22 shows a typical spectroscopy data. The red and blue circles are the experimental data of the $|^1S_0, F = 9/2, m_F = 5/2\rangle \rightarrow |^3P_1, F = 11/2, m_F = 7/2\rangle$ resonance when the double-tripod lasers (including the unwanted side-bands due to EOMs) are turned on and off, respectively. During this experiment, the Rabi frequency of the double-tripod laser is set to 230 kHz. The spectrum when the double-tripod lasers are switched-on shows two lateral peaks apart from the central resonance. This data is fit to sum of three Gaussians and the frequencies of the peaks are extracted. These lateral peaks are interpreted to be the bright states $|B_3\rangle$ and $|B_4\rangle$ which are separated by $-\Omega$ and Ω from the dark states. This interpretation is in correspondence with the frequency separation between the centers of the peaks (-227.4 kHz and 228.5 kHz). With this interpretation, one expects that the other two bright states $|B_1\rangle$ and $|B_2\rangle$ should also be present in the spectrum. However, this is not the case as the component of the ground state $|^1S_0, F = 9/2, m_F = 5/2\rangle$ in $|B_1\rangle$ and $|B_2\rangle$ is zero. Table 3.4 shows the populations of various bare states of the double-tripod scheme in the bright states that result from equal Rabi frequency coupling. The statistical uncertainty in determining the centers of the peaks from this experimental data is 4.5 kHz. Unfortunately, this precision is not sufficient to exactly measure the estimated light shift of $m_F = 5/2$ ground state *i.e.*, -2.28 kHz. Nevertheless, this spectrum confirms that the experimental parameters are well under control. We perform spectroscopy of other ground states and do not find statistically significant and reproducible light shifts. With this, we conclude that the light shifts of the ground states are insignificant (lower than the estimated values) for the double-tripod experiments.

3.9 Conclusion

In conclusion, this chapter provides all the details pertaining to the sample preparation and characterization and measurement methods along with the description of experimental apparatus employed in the measurements. We started with description of vacuum chamber and production of ultracold sample with a overview of various steps involved. We present the measurement of number of atoms in the sample and its temperature. We presented spectroscopic techniques used in locating the resonances of various Zeeman levels of the inter-combination line. We characterized the newly developed vertical fluorescence imaging used in acquiring the data for the experiments described in the following chapters. We described the implementation and control system of double-tripod configuration and calibrations of the relevant experimental quantities. We demonstrated the control on the initial ground state populations using STIRAP techniques. We fully characterized the experimental parameters involved in Rabi frequency measurements and verified with the theoretical expectations. We performed dressed state spectroscopy and find that the light matter coupling in double-tripod scheme is in order with the experimental systems. With the experimental system well characterized we are set to describe the results of the experiments.



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Chapter 4

Atomtronic Datta-Das transistor

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4.1 Introduction

Atomtronics is a sub-field of atomic, molecular and optical (AMO) physics that aims to design and create devices with ultracold neutral atoms as the carriers instead of electrons and holes in standard electronics. Unlike electronic devices where only the charge of the carriers holds information in the form of electric current and voltage, atomtronic devices offer capabilities to store and manipulate information in internal as well as external states of the atom. However, there exist spintronics devices where the spin degree of freedom of the electrons is put to use. Besides, neutral atoms possess complicated internal structure that can give rise to carriers of higher spin. These additional degrees of freedom offer exciting avenues to create new devices and sensors based on quantum coherence or entanglement that have no classical electronics' analogues. Though there existed devices that require a large laboratory-like environment such as atom-interferometers [1] and atomic fountain clocks [2], it was impractical to conceive such devices to be of large scale practical utility. Borrowing laser cooling and trapping techniques developed in AMO physics and utilizing cutting edge modern fabrication technologies developed in the last couple of decades, it is now possible to produce compact (not as compact as conventional electronics) atomtronic devices (see [3] for a recent review). Examples of such atomtronic devices include: batteries with chemical potential as a source for neutral currents [4], capacitors being storage of atoms with exponential charge and discharge characteristics [5], switches and quantized supercurrents [6–8], transistors using triple well potential [9] and Josephson junctions [10, 11]. Guiding and manipulating ultracold gases in wall-less guides to form circuits for atomic currents are also important capabilities demonstrated in lensing [12], and curved guiding beyond adiabatic regime [13].

In the footsteps of the above mentioned experiments, we implemented an equivalent of a Datta-Das transistor (DDT) [14] for atomtronic circuits that has not been experimentally realized in conventional electronics. DDT is the spintronics version of a field effect transistor (FET) with a semiconductor gate region sandwiched between ferromagnetic source and drain. This was thought to revolutionize the spintronic devices allowing for development of compact low power memory devices that are efficient compared to the existing integrated circuits [15]. The working principle of a DDT, as shown in figure 4.1, is that spin polarised electrons from the source enter the gate region where their spins precess due to the spin-orbit coupling controlled by the gate voltage [16]. The contact between the gate and the drain acts as a spin filter that allows only certain orientation of spins to pass through into the drain region, allowing for a controlled source to drain spin current. Effectively, in optics terms, a

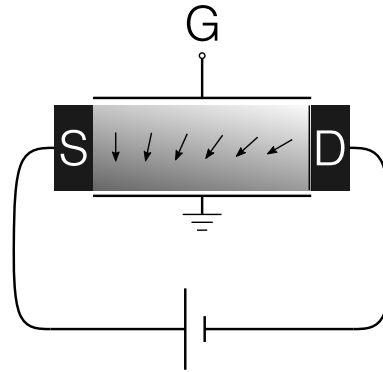


Figure 4.1: Schematic of Datta-Das transistor, an electronic analog of electro-optic modulator. The source and drain are ferromagnetic contacts to a semiconductor gate region with spin-orbit coupling connected to gate terminal using a Schottky contact.

DDT is the combination of a Pockels cell and a polarizer for the spin of electrons. Implementing a DDT in solid state systems proved to be challenging due to multiple difficulties, *viz.* inefficient electronic spin polarization in the source, difficulties in spin injection into the gate region, depolarization of spins due to scattering phenomena and insufficient strength of spin-orbit coupling in the gate region [17]. However, in atomtronic systems, these limitations are lifted significantly by efficient optical pumping techniques and spin-orbit coupling generated using atom-light interaction, and an almost defect-free environment of ultra high vacuum and optical lattices. In this chapter, we use the $SU(2)$ gauge field derived in chapter 2 to generate spin-orbit coupling in one dimension necessary to implement atomtronic DDT. However, due to the use of only one component of non-Abelian gauge field in equation 2.39, the effect produced in this case can be considered due to an Abelian gauge field akin to experiment in Ref. [18]. The experiments described in this chapter provide a simpler picture of the $SU(2)$ gauge field we have, and makes the effect of geometric scalar potential explicit that is compensated for the experiments described in the next chapter.

This chapter is organized as follows: operation of our atomtronic DDT is described in section 4.2. In section 4.3, short reminder of the experimental sequence is given along with calibration of gate parameter that is used as a control parameter in the experiment. In section 4.4, main results showing controllability of pseudo-spin rotation and sensitivity characterization of our DDT are described. In section 4.5, we demonstrate Ramsey-like interferometry using two DDTs in series. We discuss the effect of the geometric scalar field on the phase of the Ramsey-like fringes in section 4.5.1. Finally, the chapter is concluded in section 4.6.

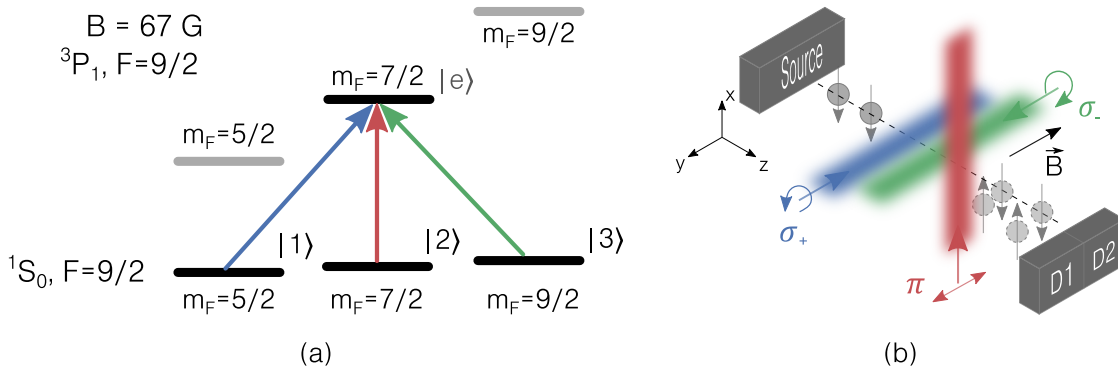


Figure 4.2: (a) Implementation of tripod scheme in ^{87}Sr . (b) Schematic of atomtronic DDT showing source, drain and gate region made of tripod beams. After traversing through the gate region, atoms in state $|D_1\rangle \equiv |1\rangle$ are deflected from their initial path and are collected at drain D1 while atoms in state $|D_2\rangle \equiv |3\rangle$ maintain their course and are collected at the drain D2.

4.2 Operation of the Atomtronic Datta-Das Transistor

In our experiment, atomtronic DDT is implemented using a degenerate Fermi gas coupled to lasers using the tripod scheme described in section 2.3.1. Spin-polarized atoms enter the interaction region (equivalent to gate region) consisting of three Gaussian beams and exit with their spin orientation controlled by the peak Rabi-frequency (thereby optical power) of the σ^- beam (equivalent to gate voltage) [19], as shown in the figure 4.2. An atom, prepared in the state $|3\rangle$, departs towards the interaction region at a constant velocity $\mathbf{v} = v_z \hat{\mathbf{e}}_z$. The atom crosses the tripod beams with Gaussian shape that propagate along different direction in the xy -plane with polarizations satisfying the electric dipole transition selection rules as shown in figure 4.2. The beam propagation axes are equally spaced with along the z -axis and their coupling Rabi frequencies can be described as

$$\Omega_i(z) = |\Omega_{0i}| e^{-(z-Z_i)^2/4\sigma_z^2} \quad (4.1)$$

where $i = 1, 2$ and 3 correspond to σ^+ , π and σ^- laser beams, respectively. Ω_{0i} is the value of the peak Rabi frequency, and σ_z is the spatial standard deviation of the intensity of the Gaussian beams along z -axis. The centers of the Gaussian beams along z -axis, Z_1, Z_2 and Z_3 , are defined as $3.5\sigma_z$, $(3.5+2\eta)\sigma_z$ and $(3.5+\eta)\sigma_z$, respectively with origin at the source of the DDT. Here, η is a dimensionless parameter called separation parameter defining the separation between the axis of propagation of the tripod beams in units of σ_z . We simulate the motion of the atoms into the gate region by keeping the atoms at rest and modulating the Rabi frequencies of the tripod beams in the form of Gaussian pulses as shown in figure 4.3. The technical

advantages of doing this change of the frame of reference are: first, the cold atomic sample preparation sequence produces atoms at rest in lab frame and we do not have to set them in motion with uniform velocity by any additional techniques. Second, the characterization of our DDT to the physical parameters of the gate region requires good control of beam waists and the separation between them. This difficult task, given there are three Gaussian beams involved, can be very easily tackled by changing the temporal behavior of the Rabi frequencies in the laboratory frame. By varying different parameters of the gate region we show the geometric nature of the spin rotation operation. In the proper inertial frame of the atoms *i.e.*, the laboratory frame, the Rabi-frequency amplitude of the tripod beams have the form of Gaussian pulses parameterized as

$$\Omega_i(t) = |\Omega_{0i}| e^{-(t-t_i)^2/4\sigma_t^2} \quad (4.2)$$

with $t = z/v_z$ and z the atomic center-of-mass position. Ω_{0i} is the value of peak Rabi frequency, and σ_t is its temporal standard deviation of the intensity of Gaussian pulses. The times $t_1 = 3.5\sigma_t$, $t_2 = t_1 + 2\eta\sigma_t$ and $t_3 = t_1 + \eta\sigma_t$ correspond to the three mean times of the Gaussian pulses, see figure 4.3a. After the pulse sequence, the atoms are in a controlled superposition of the two pseudo-spin states. Due to difference in transverse velocity of the two pseudo-spin states, a spin filter is naturally built into the system at the output of the gate region allowing the design of the drain to be positioned such that only a certain velocity class of the atoms are collected as shown in figure 4.2b. For instance, in figure 4.2b, the atoms in pseudo-spin down state $|D_1\rangle$ are deflected from their initial velocity and are collected at drain D1 while atoms in pseudo-spin up state $|D_2\rangle$ maintain their course to be collected at the drain D2.

4.3 Experimental Sequence

In this section, we recap the experimental sequence before jumping into the results. Atoms are laser-cooled using two-stage MOT sequence: First, the blue MOT using 461 nm fast transition. Second, the red MOT using narrow intercombination line at 689 nm. Then about 2.5×10^6 atoms are transferred to crossed beam optical dipole trap. Optical pumping of atoms in ground state Zeeman levels and a distribution of atoms as shown in figure 3.4 is achieved. Evaporative cooling is performed by lowering trap-depth to reach a sample of 8×10^4 atoms at a temperature of about 50 nK. The optical dipole trap is switched-off and the bias magnetic field strength is increased to 67.4 G [20]. Now, the pulse sequences of the tripod lasers shown

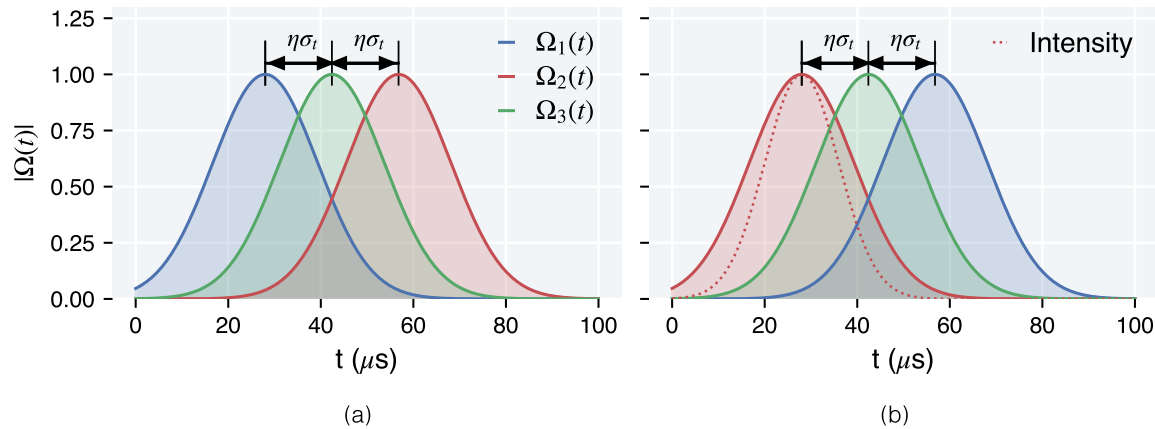


Figure 4.3: Rabi frequencies of tripod lasers in the atomic frame. (a) Order of pulses that simulates a stream of atoms with velocity along $\hat{e}_z$ as shown in figure 4.2b (b) Order of pulses that simulates a stream of atoms with velocity along $-\hat{e}_z$

in figure 4.3 are applied. After that the atoms are allowed to freely fall under gravity for 9 ms and an intense blue probe is shone at the atoms to acquire fluorescence images (see section 3.4.2). This is the signal of our experiment and all this results shown in this chapter are obtained by analyzing these images. Note that half of the final number of atoms is expected to be in negative m_F Zeeman states. The fluorescence contributed by these atoms is systematically measured once and removed during the analysis.

4.3.1 Calibration of Ratio of Rabi-frequencies

An important experimental quantity used in the results of the atomtronic DDT is the ratio of Rabi frequencies between the tripod lasers (used in the definition of the gate parameter ξ). At the time the data for this experiment is acquired, the direct measurement of Rabi frequencies discussed in section 3.7 was not yet developed. However, we calibrated the ratio of Rabi frequencies using the three-level Λ -system formed by switching off one of the tripod lasers. The dark-state of a three-level Λ -system coupled by two lasers with Rabi-frequencies Ω_a and Ω_b reads

$$|D\rangle = \frac{\Omega_b |g_a\rangle - \Omega_a |g_b\rangle}{\sqrt{|\Omega_a|^2 + |\Omega_b|^2}} \quad (4.3)$$

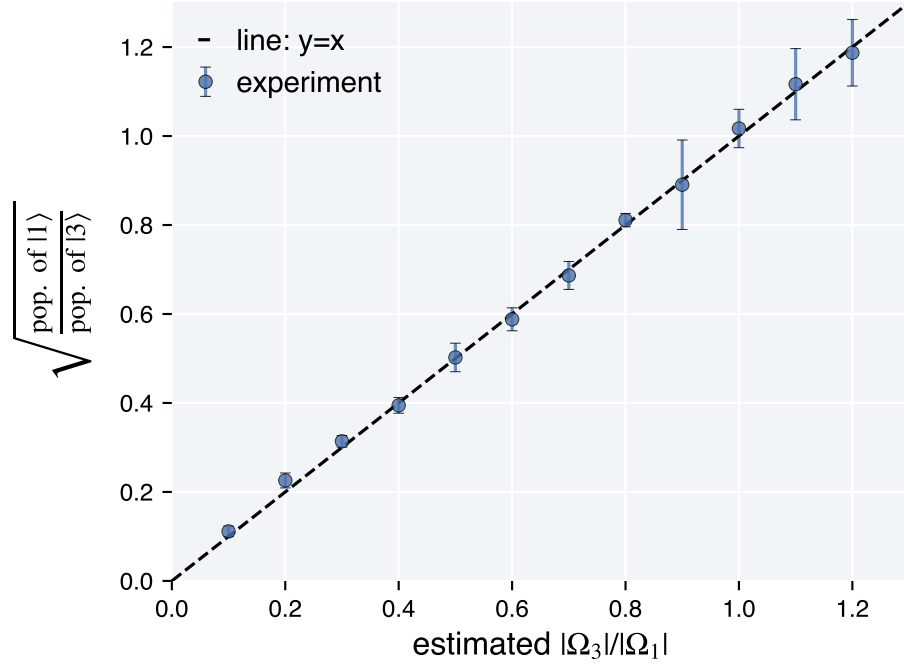


Figure 4.4: Calibration of ratio of Rabi frequencies $\frac{|\Omega_3|}{|\Omega_1|}$

where $|g_a\rangle$ and $|g_b\rangle$ are the two ground states. By populating the dark-state and measuring the population of ground states, the ratio of Rabi frequencies can be obtained using

$$\frac{|\Omega_a|}{|\Omega_b|} = \sqrt{\frac{\text{population of } |g_b\rangle}{\text{population of } |g_a\rangle}} \quad (4.4)$$

Consider the Λ -system formed by the ground states $|1\rangle$ and $|3\rangle$ and excited state $|e\rangle$. Atoms are initially prepared in $|3\rangle$. We populate the dark-state of this Λ -system by abruptly switching on the σ^+ beam and adiabatically turning on the σ^- beam. By measuring the populations of the ground states $|1\rangle$ and $|3\rangle$ after the full ramp up of the σ^- beam, we measure the ratio of Rabi-frequencies $\frac{|\Omega_1|}{|\Omega_3|}$. Using this technique we calibrate the relative Rabi-frequencies of the tripod beams. First, we set the power of σ^+ laser such that the estimated Rabi-frequency is 260 kHz for the measured beam sizes. We then make the ratio $\frac{|\Omega_1|}{|\Omega_3|} = 1$ by adjusting the power of the σ^- laser. From this power, we estimate the powers required for various values of the ratio and verify them by measuring the populations of the ground state as shown in figure 4.4. Similarly, using the Λ -system formed by the ground states $|2\rangle$ and $|3\rangle$ and excited state $|e\rangle$, we make the ratio $\frac{|\Omega_2|}{|\Omega_3|} = 1$ by adjusting the power of the π laser at the power of σ^- beam where $\frac{|\Omega_1|}{|\Omega_3|} = 1$. This way we ensure $|\Omega_1| = |\Omega_2|$ and these are left unchanged for the rest of the

experiment.

4.4 Results

The aim of this experiment was to demonstrate a working atomtronic DDT and characterize its performance. The key results of this experiment are:

- Demonstration of controllability of pseudo-spin rotation using a geometric operation.
- Characterization of the sensitivity of our DDT with respect to geometric parameters of the gate region.
- Demonstration of robustness of the function of our DDT to velocity dispersion of the atomic current.

These results are discussed in the following subsections.

4.4.1 Controlled Pseudo-spin Rotation

In a DDT, the source-to-drain spin current is controlled by the gate voltage as stated earlier. In our atomtronic DDT, we define a gate parameter $\xi = |\Omega_{03}/\Omega_{01}|$, analogous to the gate voltage, that controls the source-to-drain spin current. In this section we demonstrate the controllability of the pseudo-spin rotation as a function of the gate parameter for two pulse sequences shown in figure 4.3a&b. The first scenario corresponds to atoms prepared in state $|3\rangle$ crossing the tripod beams in the order σ^+, σ^- and π and the later corresponds to the opposite order. During the interaction with tripod lasers, three phenomena occur to a certain extent simultaneously due to the pulse sequence. First, the atoms initially in state $|3\rangle$ are adiabatically transferred into the dark-state manifold, in particular to the dark state $|D_2\rangle$. Second, the state of the atoms evolve in the dark-state manifold due to the gauge field and forms a coherent superposition of the two dark states. Finally, the atoms evolving in the dark-state manifold are adiabatically returned back to the bare states of the atom. These occur coherently as the bright states are prevented from being populated and the superposition is not disturbed. The populations of the tripod ground states after the pulse sequence shown in figure 4.3a with $\sigma_t = 8 \mu\text{s}$ and $\eta = 1.8$ as a function of ξ (keeping $|\Omega_{02}| = |\Omega_{01}|$) are shown in figure 4.5a. Clearly, after the pulse sequence the atoms do not populate the state $|2\rangle$. This can be understood by inspecting the limits,

$$\begin{aligned}
\lim_{z \rightarrow +\infty} |D_1(z)\rangle &= \lim_{\phi \rightarrow \frac{\pi}{2}, \vartheta \rightarrow \frac{\pi}{2}} |D_1(\phi, \vartheta)\rangle = |1\rangle \\
\lim_{z \rightarrow +\infty} |D_2(z)\rangle &= \lim_{\phi \rightarrow \frac{\pi}{2}, \vartheta \rightarrow \frac{\pi}{2}} |D_2(\phi, \vartheta)\rangle = |3\rangle
\end{aligned}
\tag{4.5}$$

where $\vartheta = \cos^{-1} \left(\frac{|\Omega_3|}{\sqrt{|\Omega_1|^2 + |\Omega_2|^2 + |\Omega_3|^2}} \right)$ and $\phi = \tan^{-1} \left(\frac{|\Omega_1|}{|\Omega_2|} \right)$ are the mixing angles. These are functions of time in the proper frame of the atoms. In figure 4.7a, these mixing angles are plotted as functions of time for the pulse sequence shown in figure 4.3a i.e., $\sigma_t = 8 \mu\text{s}$ and $\eta = 1.8$. Therefore, measuring the final populations of the states $|1\rangle$ and $|3\rangle$ yields the superposition between the dark-states. On the other hand, any population transfer among the bare states also alters the velocity of the center-of-mass of the atoms as the interactions with the tripod beams are mediated by coherent Raman transitions involving a net change in the velocity of the atoms as the beams have different propagation directions [21–24]. Therefore, an atom initially in the state $|3\rangle$ is coherently transferred to the state $|1\rangle$ by absorbing one photon from the σ^- beam and undergoing stimulated emission of one photon into the σ^+ beam. During this process, the atoms acquires a net momentum of $2p_r \hat{e}_y$, where $p_r = m v_r$ is the recoil momentum. Due to this velocity redistribution between photons and atoms and the sub-recoil temperature of the atoms, after a time of flight atoms in different bare states (and thus different dark states) have clearly resolved mean momenta. Utilizing this, the populations shown in figure 4.5 are extracted from the fluorescence images taken after 9 ms of time of flight [25]. These time of flight images are shown in figure 4.5b for various values of ξ to illustrate this. Here, the distribution with a mean momentum of $2p_r \hat{e}_y$ is identified to be in the state $|1\rangle$, whereas the distribution with unchanged mean momentum is associated to the state $|3\rangle$. This momentum-pseudo-spin coupling offers a natural solution for the spin filter for the final spin current as the atomic clouds with different spin components are spatially separated after a time-of-flight.

By reversing the temporal order of the pulses *i.e.*, reversing σ^+ and π beams as shown in figure 4.3b, one can reassign the role of state $|1\rangle$ to state $|2\rangle$. The population of the tripod ground states in this situation is depicted in figure 4.6a that shows the identical role played by the state $|2\rangle$ as that of $|1\rangle$ in the previous case. In this scenario, the atoms are transferred to the state $|2\rangle$ with a net momentum of $p_r(\hat{e}_y - \hat{e}_x)$ and found population at this momentum (see figure 4.6b) corresponding to the state $|2\rangle$, whereas here the state $|1\rangle$ remains unpopulated

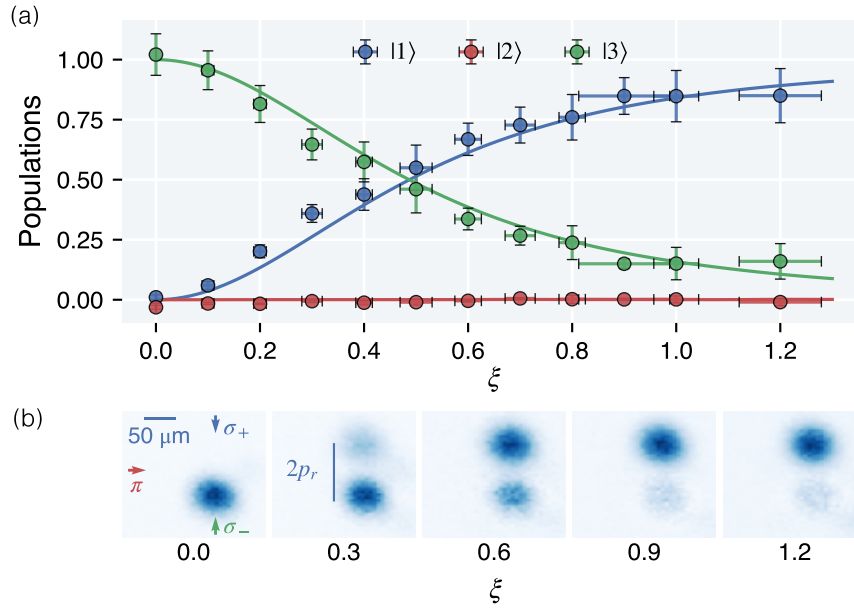


Figure 4.5: (a) Populations of the tripod ground states after the atoms exit the gate region for the pulse order shown in figure 4.3a. Here dark-state $|D_1\rangle$ corresponds to the bare-state $|1\rangle$. (b) Fluorescence images showing the atomic clouds after a time of flight of 9 ms indicating coherent interaction with the tripod lasers.

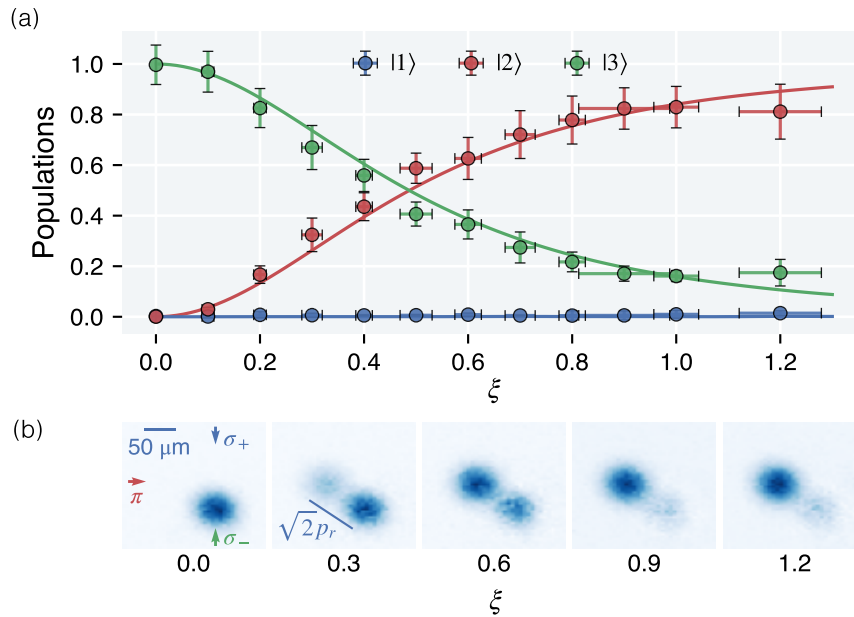


Figure 4.6: (a) Populations of the tripod ground states after the atoms exit the gate region for the pulse order shown in figure 4.3b. Here dark-state $|D_1\rangle$ corresponds to the bare-state $|2\rangle$. (b) Fluorescence images showing the atomic clouds after a time of flight of 9 ms indicating coherent interaction with the tripod lasers.

at the DDT output. This can be understood simply by inspecting the asymptotic limit of the dark-state $|D_1\rangle$ shown in equation 4.6.

$$\begin{aligned}\lim_{z \rightarrow +\infty} |D_1(z)\rangle &= \lim_{\phi \rightarrow 0, \vartheta \rightarrow \frac{\pi}{2}} |D_1(\phi, \vartheta)\rangle = |1\rangle \\ \lim_{z \rightarrow -\infty} |D_1(z)\rangle &= \lim_{\phi \rightarrow \frac{\pi}{2}, \vartheta \rightarrow \frac{\pi}{2}} |D_1(\phi, \vartheta)\rangle = |2\rangle\end{aligned}\tag{4.6}$$

The limiting conditions in equation 4.6 occur at $t=0$ and $t=100 \mu s$ respectively are illustrated in figure 4.7b. Therefore, two output momentum ports can be accessed simply swapping σ^+ and π beams, as it was previously pointed out with a similar beam configuration in [26]. One notes that this swapping between states $|1\rangle$ and $|2\rangle$ also occurs by changing the propagation direction of the atoms from $\hat{e}_z$ to $-\hat{e}_z$. This feature could also be used to enforce unidirectional flow of the atomic current in an atomtronic circuit. From here on, we study only the case depicted in figure 4.2b simulated by the pulse sequence in figure 4.3a and expect that the corresponding study for the pulse sequence in figure 4.3b would yield the same results.

For qualitative description of our atomtronic DDT device with well known stimulated Raman adiabatic passage (STIRAP) phenomena in a Λ -system, we identify three important arguments. First, the temporal variation of the pulses needs to be slow enough such that the dynamics remain adiabatic, consequently the excited state $|e\rangle$ is not populated at any time. Hence, no spontaneous emission occurs and the evolution remains coherent and independent of the total time of the pulse sequence. Second, if the final pulse is Ω_2 , it means that the output state does not contain the state $|2\rangle$ akin to STIRAP experiments [27]. Therefore, the state $|2\rangle$ can be viewed as an ancilla, and the states $|1\rangle$ and $|3\rangle$ form the pseudo-spin basis of the DDT as previously explained. Finally, the second pulse Ω_3 is coupled to the input state $|3\rangle$ and its amplitude acts as a spin-orientation control knob. If the pulse amplitude is zero, the input state will not be coupled and the pseudo-spin retains its initial orientation. On the other limiting case where Ω_{03} is large, the state should be fully transferred to $|1\rangle$, corresponding to a π -rotation of the pseudo-spin. In the intermediate range, the experiment depicted in figure 4.5a indicates non-zero populations in both states $|1\rangle$ and $|3\rangle$. We test that evolution is coherent by applying, immediately after the first pulse sequence, an second pulse sequence in the reversed order, namely π , σ^- and σ , and observe that more than 85% of the population returns back to the initial input state $|3\rangle$. This point is discussed in detail in section 4.5.

To quantify the results, we perform a Galilean transformation to a frame moving at veloc-

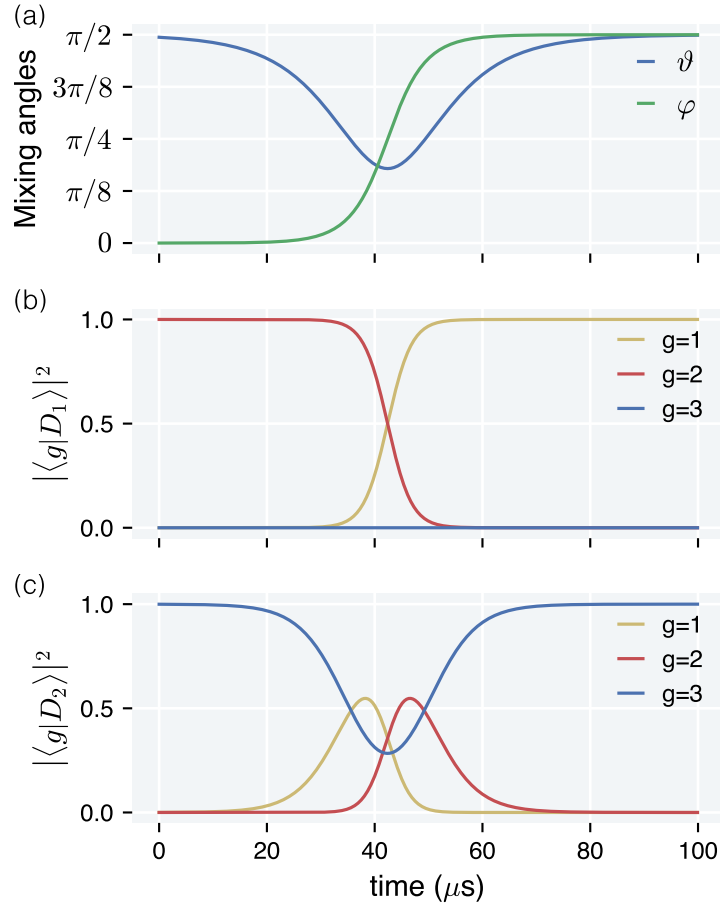


Figure 4.7: (a) Mixing angles for pulse sequence in figure 4.3a as a function of time. (b) Squared projections of dark-state $|D_1\rangle$ on tripod ground states. (c) Squared projections of dark-state $|D_2\rangle$ on tripod ground states.

ity $\mathbf{v} = -v_z \hat{\mathbf{e}}_z$ [19, 25]. In the limiting case where $|\mathbf{v}| \gg v_r$, $\eta\sigma_t \ll (k\sigma_v)^{-1}$ and $(kv_r)^{-1}$, the dynamics of the system are predominantly governed by the Galilean boost term $-\mathbf{v} \cdot \mathbf{A}$ in the Hamiltonian (see section 2.4). The first two inequalities imply that the atomic wave-packet velocity is well defined and does not significantly change during the interaction with the tripod beams as the strength of the gauge field components in the transverse direction is of the order of recoil momentum. The second inequality implies that the atoms, within the interaction time, do not significantly explore the phases of the tripod beams due to transverse motion. As we will show in section 4.4.3, this condition usually, but not always, holds true in our experiment. If both inequalities are fulfilled, and the temporal coherence of the atomic wave-packet is much shorter than the temporal extension of a tripod beam pulses, we have $\langle -\mathbf{v} \cdot \hat{\mathbf{A}} \rangle \gg \langle (-i\hbar\nabla - \hat{\mathbf{A}})^2 \rangle / 2m, \langle W \rangle$, where $\langle \dots \rangle$ stands for the expectation value. The effective Hamiltonian reduces to

$$H = v_z A_z \quad (4.7)$$

where gauge field component reads

$$\hat{A}_z(t) = \hbar \cos \vartheta(t) \frac{\partial \varphi(t)}{\partial z} \hat{\sigma}_y = \hbar \cos \vartheta(t) \frac{\partial \varphi(t)}{v_z \partial t} \hat{\sigma}_y \quad (4.8)$$

The evolution operator for the pseudo-spin reads

$$\hat{U}(t_f) = \exp \left[-i \int_0^{t_f} \cos \vartheta(t) \frac{\partial \varphi(t)}{\partial t} \hat{\sigma}_y dt \right] \quad (4.9)$$

where $\hat{\sigma}_y$ is the Pauli matrix along y and t_f is the interaction duration. Using equation 4.9 and definition of dark-states, we extract the bare state populations as represented by the plain curves in figures 4.5a and 4.6a. From equation 4.9, it is clear that the rotation of the pseudo-spin occurs about the y -axis. This unitary operation is geometric in the sense that the pseudo-spin rotation depends neither on the velocity of the atoms v_z nor explicitly on the Rabi-frequencies (but depends only on their ratios).

4.4.2 Sensitivity

In this section, we characterize the sensitivity of the pseudo-spin rotation of our atomtronic DDT to the geometrical parameters of the gate region, in particular the separation parameter. The pseudo-spin rotation can be characterized by the polar angle θ on the Bloch sphere where

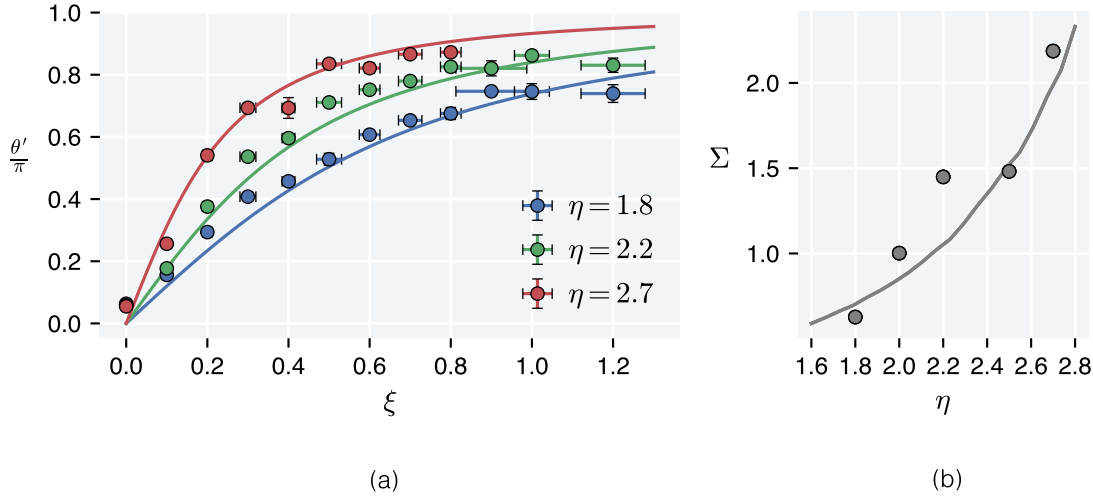


Figure 4.8: (a) Supplementary Bloch angle $\theta' = \pi - \theta$ as a function of the gate parameter for different values of η . (b) Sensitivity of the spin rotation as a function of the separation parameter η for $\sigma_t = 8\mu s$.

north pole corresponds to $|D_1\rangle$ and south pole corresponds to $|D_2\rangle$. By definition,

$$\frac{\theta}{2} \equiv \arctan \left(\frac{|\langle D_2 | \psi \rangle|}{|\langle D_1 | \psi \rangle|} \right) = \arctan \left(\sqrt{\frac{|\langle 3 | \psi \rangle|^2}{|\langle 1 | \psi \rangle|^2}} \right) \Big|_{t=100\mu s} \quad (4.10)$$

for a measurement after the tripod interaction (see figures 4.7b&c). Here $|\psi\rangle$ is the state of the system in pseudo-spin basis. In figure 4.8a, supplementary to this polar angle ($\theta' = \pi - \theta$) is plotted as function of ξ for three values of the separation parameter η . We observe the pseudo-spin rotation to be more sensitive to changes in the gate parameter ξ for larger η . For a more precise analysis of this phenomena, we have defined the sensitivity of the spin rotation with respect to η as the slope

$$\Sigma(\eta) = \left. \frac{\Delta \theta'}{\Delta \xi} \right|_{\theta=\pi/2} \quad (4.11)$$

Figure 4.8b shows that the sensitivity of the spin rotation increases monotonically with η . This can be understood by looking at the unitary transformation in equation 4.9 that depends on the derivative of the mixing angle φ . For large η the lasers well separated and the derivative of φ is large leading to a higher sensitivity to the changes in the gate parameter ξ . For practical atomtronics DDT operations, the ξ value shall be the main control parameter, and one could choose a configuration with a large η value for spin modulation purposes and a low η value for a precise control of the spin orientation.

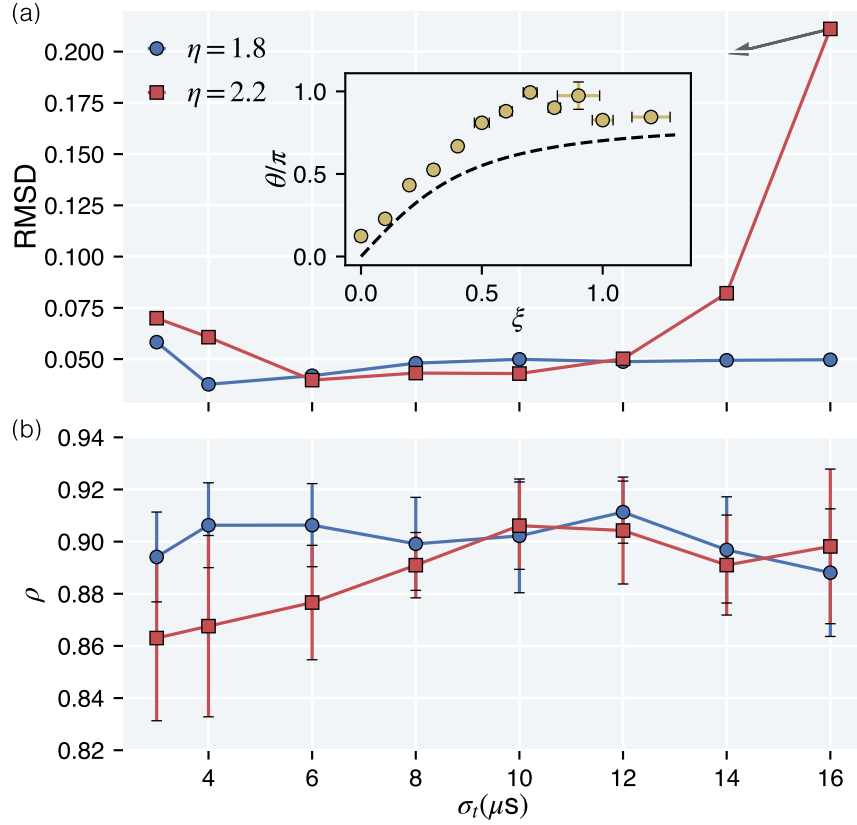


Figure 4.9: (a) RMSD of experimental data with the theoretical model. Inset shows a sample experimental data for $\sigma_t = 16 \mu\text{S}$ and $\eta = 2.2$. (b) Tracking of spontaneous emission in the experiment due to non-adiabaticity.

4.4.3 Robustness to velocity dispersion

Clearly from the unitary transformation 4.9, the pseudo-spin rotation does not depend on the input velocity of the atoms. Due to the unavoidable velocity dispersion of the atomic gases, this feature of our DDT is of fundamental interest. We illustrate this point in figure 4.9a, where we plot the root-mean-squared deviation (RMSD) of the experimental data to the theoretical prediction using equation 4.9 as a function of σ_t for two values of η . The root-mean-squared deviation (RMSD) of Bloch angle θ is defined as

$$\text{RMSD} = \frac{1}{\pi} \sqrt{\langle (\theta'_{\text{exp}} - \theta'_{\text{theory}})^2 \rangle} \quad (4.12)$$

where θ'_{exp} and θ'_{theory} are the supplementary Bloch angles calculated from the experiment and theory given by equation 4.9. Here, the average is taken over different values of the gate parameter ξ . The control parameter σ_t , which compresses or expands the interaction zone, is understood to be inversely proportional to the velocity of the atoms. To directly compare the

experimental data and the theory prediction, we recall that all the experimental data shown so far (in figures 4.5, 4.6 and 4.8) correspond to $\sigma_t = 8\mu\text{s}$ and $\eta = 1.8$. The RMSD is minimum and close to zero for a range covering a factor 2 to 3 in value of σ_t showing that atoms with a broad range of velocities can be targeted by our DDT. For the $\eta = 2.2$ case, RMSD increases for both larger and smaller values of σ_t . This is due to the breakdown of the model approximations in these two limiting cases: failure of adiabatic approximation at high velocities and comparable transverse velocity during the interaction at low velocities. Non-adiabatic processes populate the bright states and hence the excited state $|e\rangle$ which leads to incoherent spontaneous emission. Spontaneous emission will change the net momentum of atoms in a random manner. This random spontaneous decay mechanism can be traced in the experiment by monitoring the total population of the two output distributions with well-defined net momentum shown in figures 4.5b and 4.6b with respect to the input $|3\rangle$ population. If the ratio

$$\rho = \frac{\text{output population in well-defined gaussian peaks}}{\text{input population}} \quad (4.13)$$

decays significantly, it means that the incoherent decay processes are at play. For short σ_t value, we note that the value of ρ decreases (see figure 4.9b), which indicates that some spontaneous emission events have taken place, and the adiabatic approximation might no longer be valid here. For large σ_t values, the adiabatic condition should hold but we observe that the experimental data clearly deviates from the model prediction (see inset of figure 4.9a). In this case, the inequality $\eta\sigma_t \ll (k\sigma_v)^{-1}$ does not hold, and the velocity spread of the atoms in xy -plane leads to exploration of extra vector potential components that are sensitive to relative phase variations of the tripod lasers during the interaction. Effect of these additional components of the gauge field should also be considered [28,29] in the theory. Thus from figure 4.9a, we note that a smaller value of η helps to broaden the model validity range in the large σ_t limit.

We support these arguments by numerically solving the well known Lindblad master equation for open quantum systems considering the spontaneous emission from the excited state [30]. For the tripod system in figure 4.2a, this equation for the density matrix ρ reads,

$$\dot{\rho} = -i[H, \rho] + \begin{pmatrix} -\Gamma\rho_{ee} & -\Gamma\rho_{e1}/2 & -\Gamma\rho_{e2}/2 & -\Gamma\rho_{e3}/2 \\ -\Gamma\rho_{1e}/2 & \Gamma C_1^2\rho_{ee} & 0 & 0 \\ -\Gamma\rho_{2e}/2 & 0 & \Gamma C_2^2\rho_{ee} & 0 \\ -\Gamma\rho_{3e}/2 & 0 & 0 & \Gamma C_3^2\rho_{ee} \end{pmatrix} \quad (4.14)$$

where H is the Hamiltonian from equation 2.36 and $C_1^2 = \frac{32}{99}$, $C_2^2 = \frac{49}{99}$ and $C_3^2 = \frac{18}{99}$ are the

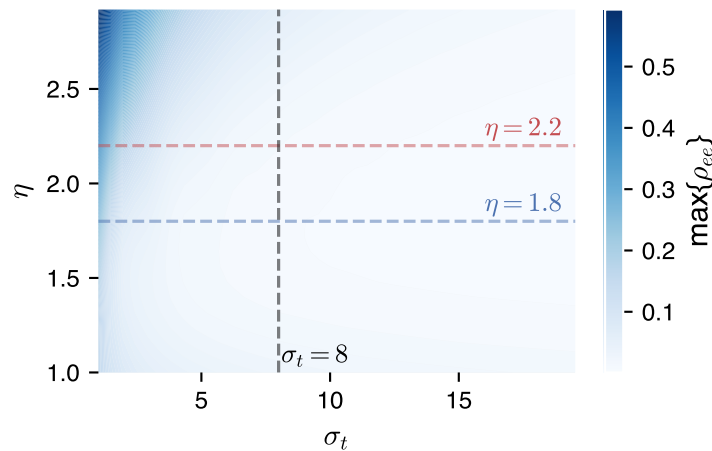


Figure 4.10: Numerically obtained maximum excited state population for the tripod pulse sequence shown in figure 4.3a for a wide parameter space of σ_t and η .

squared Clebsch-Gordan coefficients of σ^+ , π and σ^- transitions, respectively. Here, Γ is the linewidth of the tripod transitions. In figure 4.10, we plot the maximum of the excited state population during the pulse sequences for various values of σ_t and η . This indicates the region of validity of adiabatic approximation in the parameter space. The parameters for the experimental results shown so far are indicated to illustrate the validity (also partly invalidity) of adiabatic approximation in our experiment. The evolution tends to be not adiabatic for smaller σ_t and larger η as we concluded from the experimental data.

4.5 Geometric Ramsey Interferometry

In previous sections, our main effort was to show the coherent spin rotation that is achieved in our DDT. In the Bloch sphere representation, we focused on the control of the polar angle θ with our gate parameter ξ . In this section we probe the phase difference between the dark states i.e., the azimuthal angle ϕ on the Bloch sphere. We do this by performing Ramsey interferometry using two DDT pulse sequences that are equivalent to two $\frac{\pi}{2}$ -pulses [31]. However, the results of this experiment indicate that this is not exactly correct in our system due to a specific geometric scalar term that was not experimentally shown earlier. We show the role of various terms in our Hamiltonian in modifying the phase of the superposition of the dark states.

A typical Ramsey interferometry [32] sequence used to measure the atomic resonance ω_0

consists of two $\frac{\pi}{2}$ -pulses which are generated from the same source with frequency ω that are separated in time by a free evolution time T . The first pulse puts the atoms initially in the ground state into a coherent superposition of ground and excited states with equal probabilities. During the free evolution time T , the phase of the superposition state rotates with atomic resonance frequency ω_0 while the phase of the probe field rotates with frequency ω . Therefore, after the free evolution, the phase difference between the atomic state and probe field is accumulated to $(\omega_0 - \omega)T$. If this phase difference is an integer multiple of 2π , then the second $\frac{\pi}{2}$ -pulse is a mere extension of the first $\frac{\pi}{2}$ -pulse to form a π -pulse where full transfer from the ground state to excited state occurs. On the other hand, if this phase difference is an odd integer multiple of π , then the second pulse acts as a $-\frac{\pi}{2}$ -pulse where the superposition state is reverted back to the ground state. For a fixed free evolution time T , the population of the excited state can be seen as an interference pattern in frequency domain. The principle maximum occurs at $\omega = \omega_0$ and first minima occur at $\omega = \omega_0 \pm \pi/T$. Another interference pattern can also be observed for a fixed ω , by the scanning the free evolution time T where the first maximum occurs at $T = 0$ and first minimum occurs at $T = \frac{\pi}{|\omega_0 - \omega|}$. In Ramsey interferometry experiments, the atoms are at rest all along the pulse sequence. However, if the velocity of the atoms along the propagation direction of the probe pulses changes due to the pulses, then the phase accumulation due to the Doppler shifts has to be accounted for. This phase is probed to measure the inertial acceleration that alters the velocity and hence the Doppler shifts. These experiments fall under the broad category of inertial interferometers.

Our experiment falls under the category of the later inertial interferometry as the mean momentum of the atomic state is non-zero after the first DDT pulse sequence. In our experiment, interferometry is performed using the pulse sequence shown in figure 4.11a. Each set of three pulses correspond to a $\frac{\pi}{2}$ -pulse. The centers of the Gaussian pulses that compose the first $\frac{\pi}{2}$ -pulse indicated in the superscripts are parameterized as $t_1^{(1)} = t_3^{(1)} - \eta\sigma_t$, $t_2^{(1)} = t_3^{(1)} + \eta\sigma_t$ and $t_3^{(1)} = 4\sigma_t$. Similarly, the centers of the Gaussian pulses corresponding to the second $\frac{\pi}{2}$ -pulse indicated in the superscripts are parameterized as $t_3^{(2)} = t_3^{(1)} + 8\sigma_t + t_{delay}$, $t_2^{(2)} = t_3^{(2)} - \eta\sigma_t$ and $t_1^{(2)} = t_3^{(2)} + \eta\sigma_t$. The 2 in the superscript is not to be confused as a square. Here, σ_t and η bear the same meaning as before and t_{delay} is the delay between the pulses that plays the role of the free evolution time T in the Ramsey interferometry. For the data described in this section, we chose $\sigma_t = 2.5\mu s$ and $\eta = 1.8$ to minimize the total duration of the experiment and thereby minimize the effects of decoherence mechanisms. Though the end of first $\frac{\pi}{2}$ -pulse is a little ambiguous, we consider that the end of first Ω_3 Gaussian pulse marks the end of the first $\frac{\pi}{2}$ -pulse. This is safe to assume as this time is same for both the pulses. Note that the

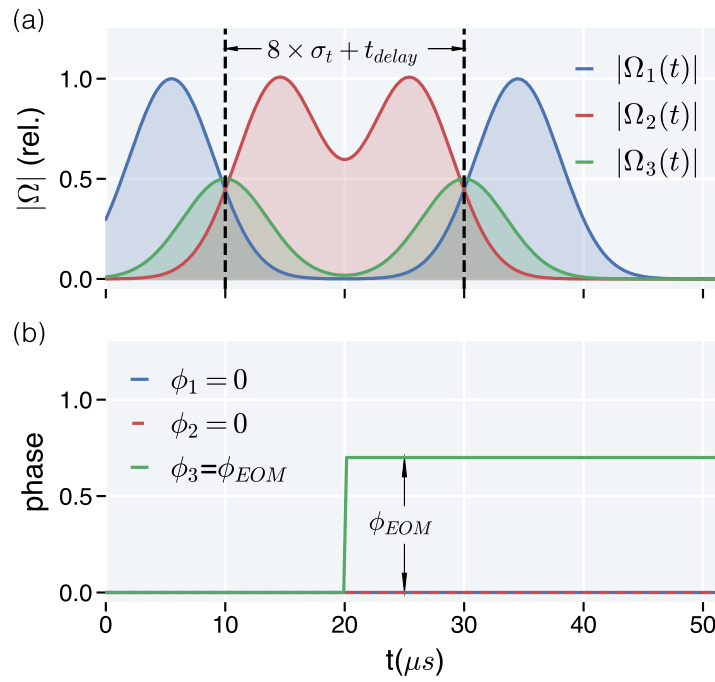


Figure 4.11: Ramsey inteferometry time sequence. (a) Rabi frequencies of tripod lasers as a function of time with $\sigma_t = 2.5 \mu s$ and $\eta = 1.8$. First three gaussian pulses (upto 20 (μs)) from a $\frac{\pi}{2}$ -pulse and later three pulses form a $\frac{\pi}{2}$ -pulse. (b) Phase changes introduced to the tripod lasers during the sequence. Only phase of σ^- beam is jumped after the first $\frac{\pi}{2}$ -pulse using an EOM in the path of the σ^- laser beam.

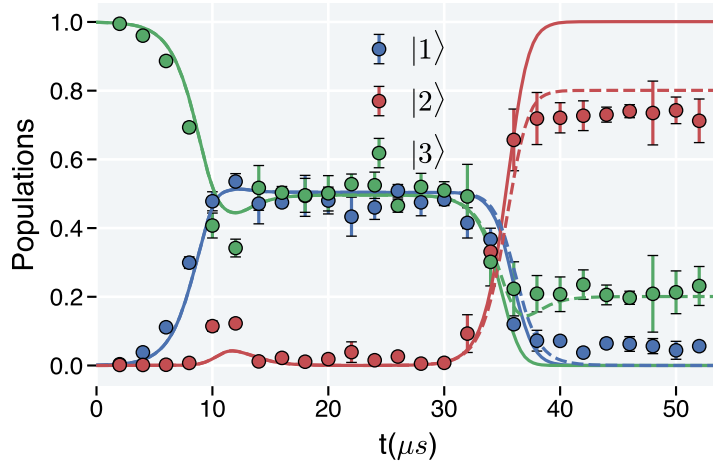


Figure 4.12: Temporal evolution of the bare state populations for time sequence shown in figure 4.11 with $t_{delay} = 6 \mu s$. Thick lines are calculated at zero temperature and dashed lines are calculations $T = 50$ nK. Populations deviate from the zero temperature curves only after the second pulse as damping due to transverse velocity distribution of the atomic wave packet affects the phase difference between the dark-states.

order of second set of Gaussian pulses is reverse to that of the first set of Gaussian pulses. This change in the order is implemented to facilitate the adiabatic switching of the second $\frac{\pi}{2}$ -pulse. After the first $\frac{\pi}{2}$ -pulse, the atoms are in a superposition of bare states $|1\rangle$ and $|3\rangle$ while the state $|2\rangle$ is left unpopulated. Turning on the Ω_1 pulse after first $\frac{\pi}{2}$ -pulse would couple the atoms in state $|1\rangle$ to the excited state $|e\rangle$ and the picture discussed so far in terms of degenerate dark states is no longer valid. So, we first turn on the Gaussian pulse Ω_2 and then couple the other states into the dark state subspace.

Figure 4.12 shows the evolution of bare state population as a function of time for the pulse sequence shown in figure 4.11a for $t_{delay} = 6 \mu s$. Effectively, this is the result of a π -pulse. After the first $\frac{\pi}{2}$ -pulse, equal superposition between $|D_1\rangle$ and $|D_2\rangle$ is observed as the populations of bare states $|1\rangle$ and $|3\rangle$ are equal. After the second $\frac{\pi}{2}$ -pulse, a full transfer to the dark state $|D_1\rangle$ is expected to populate only the state $|2\rangle$. However, this interferometric signal being sensitive to the phase of the initial superposition that has a spread due to temperature of the atoms results in averaged phase for the final state. By thermal averaging, we see a good match with the numerical curves.

4.5.1 Effect of Geometric Scalar Potential

The projection of the Hamiltonian to the dark state subspace resulted in a geometric vector and scalar potentials as shown in section 2.2.4. The action of DDT in previous section was

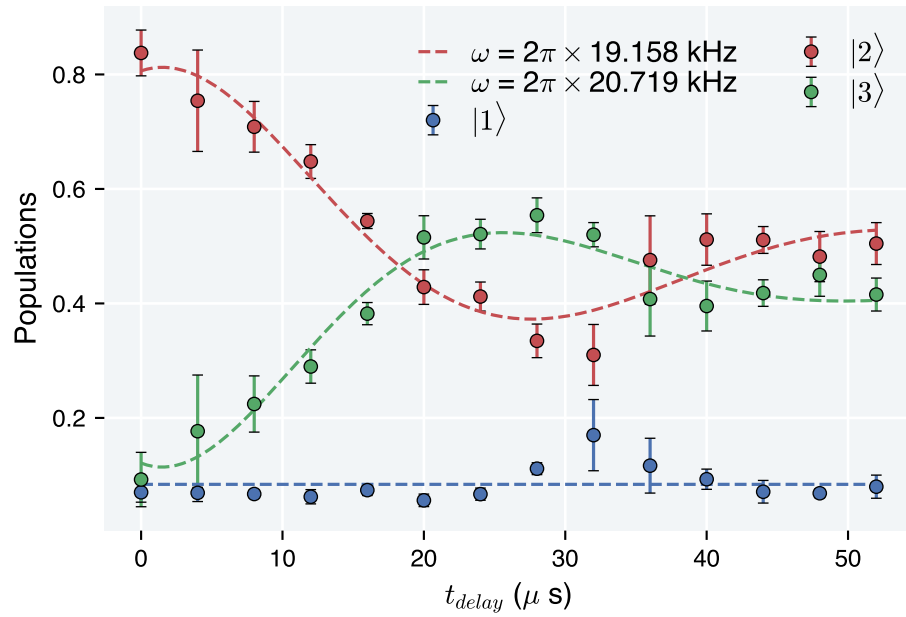


Figure 4.13: Populations of the tripod ground states as a function of delay between the pulses, t_{delay} . Dashed lines are the fits to exponentially decaying sinusoid function. The frequencies extracted from fits suggests that the oscillation between the dark states occur with $4\omega_r$ equivalent to a momentum of $2\hbar k$.

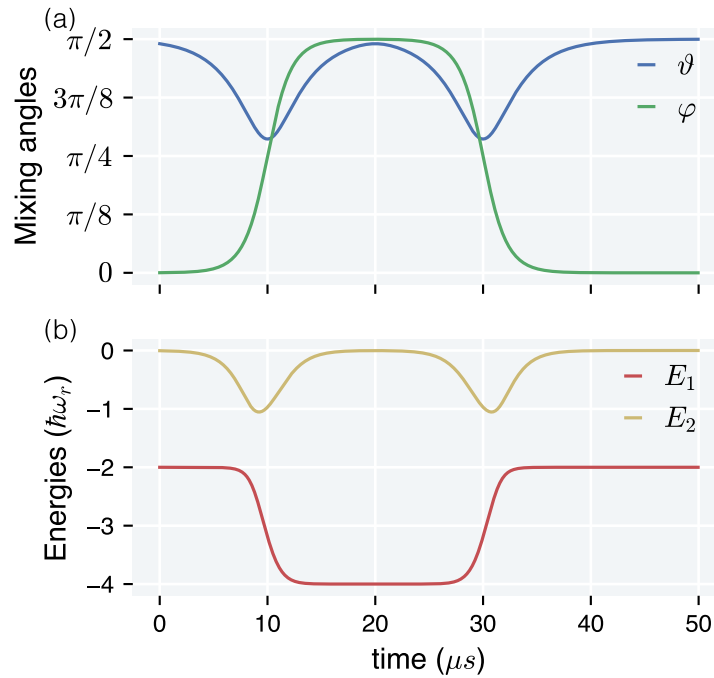


Figure 4.14: (a) Mixing angles of the Rabi frequencies as a function of time for the Ramsey interferometry sequence. (b) Eigen energies of the geometric scalar potential. During the free evolution time the energy difference between the dark states is $4\hbar\omega_r$, where ω_r is the recoil frequency.

explained by the geometric vector potential alone. One of the assumptions in the model was to neglect the effect of scalar term on the resulting superposition state as the energy due to vector potential is significantly higher than the energy scale of the scalar term. However, this is not a valid assumption when probing the phase of the superposition. Effect of this geometric scalar potential was first experimentally observed in [33] and is not considered as a particularly useful tool since there exist other ways of creating a scalar potential for example using AC Stark shifts. Nevertheless, the effect of this scalar potential should be considered to explain the data in our experiment. Figure 4.13 shows the bare state populations as a function of free evolution time t_{delay} in our interferometry experiment. As we scan the delay between the pulses, we observe a damped oscillation of the population of the dark states. These can be considered as the Ramsey fringes in the time domain. Fitting these oscillating populations to an exponentially damped sinusoid function yields approximately equal frequencies for both the dark states. For a pinned atom, this oscillation is not expected as the two dark states are degenerate. However, in this case the Doppler shifts due to exchange of momentum with the photons of tripod beams has to be accounted. It turns out that the action of the scalar potential on the dark states accounts for this momentum exchange. The Hamiltonian including the scalar term reads

$$H = \nu_z A_z + Q \quad (4.15)$$

where $-\nu$ is the velocity of the frame of reference and Q is the scalar field. For the configuration of our laser beams, this scalar field reads

$$Q = -\hbar\omega_r \begin{pmatrix} 2(1 + \sin^2 \varphi) & \cos \vartheta \sin 2\varphi \\ \cos \vartheta \sin 2\varphi & 2\cos^2 \vartheta (1 + \cos^2 \varphi) \end{pmatrix} \quad (4.16)$$

where $\omega_r = \frac{\hbar k^2}{2m}$ is the recoil frequency. Figure 4.14a shows the temporal evolution of mixing angles ϑ and φ for the Ramsey pulse sequence. Eigen energies of the scalar term during the pulse sequence is shown in figure 4.13b. When the mixing angle φ is 0 or $\frac{\pi}{2}$, the scalar term matrix is diagonal. Therefore, the eigen vectors of the scalar field in these conditions are just the dark states. At time $t = 0 \mu s$, the eigen values of the scalar field matrix are $-2\hbar\omega_r$ and 0 corresponding to the dark states $|D_1\rangle$ and $|D_2\rangle$, respectively. Here, the dark states $|D_1\rangle$ and $|D_2\rangle$ are just the bare states $|2\rangle$ and $|3\rangle$. As discussed before, the velocity of $|2\rangle$ is $\nu_r(\hat{e}_y - \hat{e}_x)$ (with magnitude $\sqrt{2}\nu_r$) while the velocity of $|3\rangle$ is 0. This means that the eigen energies of the scalar field matrix at $t = 0$ are just the kinetic energies (with a negative sign) of the instantaneous dark states. Similarly at time $t = 20 \mu s$ i.e., after the first $\frac{\pi}{2}$ -pulse, the dark states $|D_1\rangle$ and $|D_2\rangle$ are just the bare states $|1\rangle$ and $|3\rangle$ with velocities $2\nu_r\hat{e}_y$ and 0. The kinetic energies due to

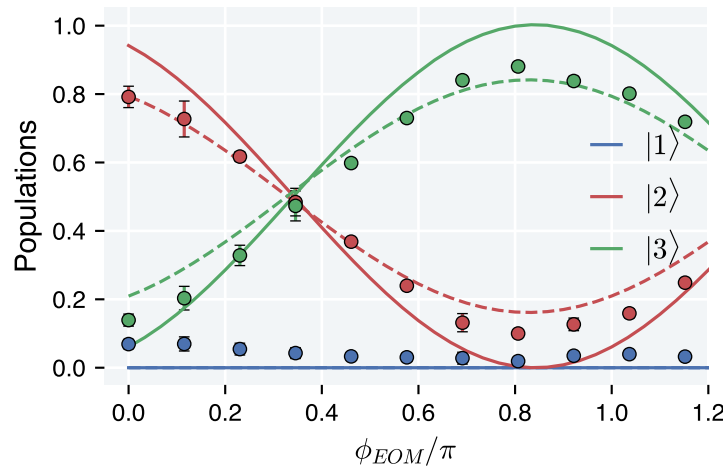


Figure 4.15: Populations of the ground states as a function of the phase jump after the first $\frac{\pi}{2}$ -pulse. Phase difference at $\phi_{EOM} = 0$ is due to the presence of scalar term and transverse velocity distribution of the atomic wave packet. Thick lines are calculated at zero temperature and dashed lines are calculations $T = 50$ nK. These are also analogous to the Ramsey fringes.

these velocities are indeed the eigen energies of the scalar term. During the free evolution time, these energies are constant. So, when we scan the free evolution time t_{delay} , we should observe oscillations with a frequency that corresponds to the difference of the eigen energies of the scalar term. Substituting the recoil frequency of 4.8 kHz for the tripod transitions, the frequency of this oscillation is expected to be 19.2 kHz. The values extracted from fitting the experimental data are 19.16 kHz and 20.72 kHz shown in figure 4.12 are very close to this frequency. Therefore, the momentum exchange with photons during the pulses is accounted by adding the scalar term to the Hamiltonian.

As these oscillations are damped out very fast due to temperature of the gas, we find an alternative method to show our Hamiltonian indeed accounts for all the physical effects. In this method, we fix the free evolution time $t_{delay} = 0$ and deliberately impose an artificial phase on the laser field Ω_3 using an electro optic modulator (EOM) after the first $\frac{\pi}{2}$ -pulse (see figure 4.11b). The action of this phase on the state can be represented by the unitary matrix

$$M = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\phi_{EOM}} \end{pmatrix} \quad (4.17)$$

In this way, we limit the total time of the experiment and hence the associated dephasing yet probe the phase of the coherent state after the first $\frac{\pi}{2}$ -pulse. Figure 4.15 shows the populations of bare states as a function of the artificially imposed phase. The initial phase difference at

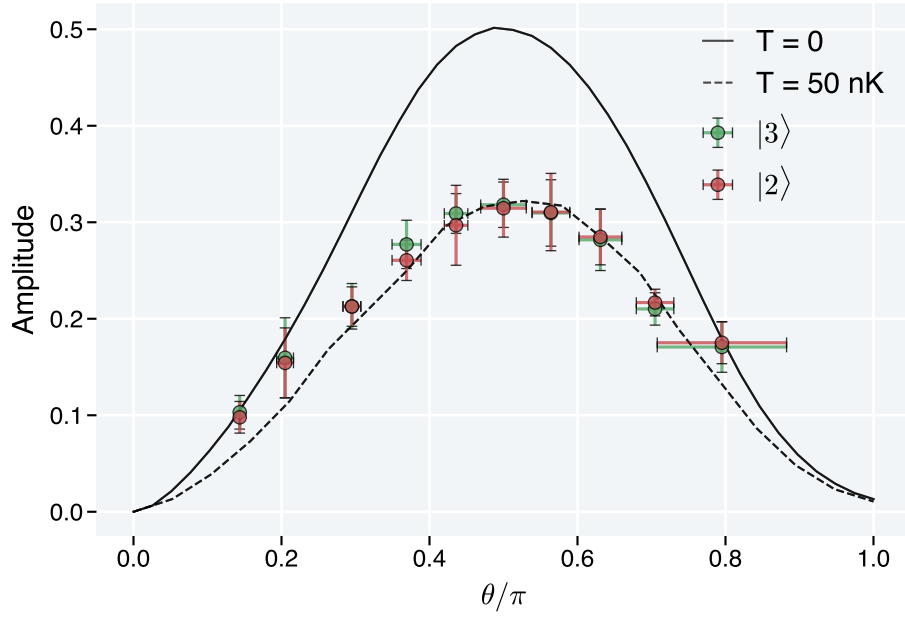


Figure 4.16: Amplitudes of the populations in the Ramsey-like fringes as a function of Bloch angle θ after the first θ -pulse

$\phi_{EOM} = 0$ is caused by the time dependent scalar field during the pulses. As additional phase ϕ_{EOM} is added, the total phase difference is just algebraic sum of initial phase difference and ϕ_{EOM} . The thick theory line are extracted from the following unitary operation considering the scalar field and the phase jump

$$\hat{U}(t_f) = M \exp \left[\int_0^{t_f} -i \left(\cos \vartheta(t) \frac{\partial \varphi(t)}{\partial t} \hat{\sigma}_y - \omega_r (\cos \vartheta \sin 2\varphi \hat{\sigma}_x + ((1 + \cos^2 \vartheta) \sin^2 \varphi - \cos 2\vartheta) \hat{\sigma}_z) \right) dt \right] \quad (4.18)$$

where final time $t_f = 50 \mu s$ and $\hat{\sigma}_x, \hat{\sigma}_y$ and $\hat{\sigma}_z$ are the usual Pauli matrices. The dashed lines are obtained by thermal averaging for a gas at a temperature of 50 nK. We see a good match with the theoretical expectation showing our interpretation of the scalar term is indeed correct. We also perform two θ -pulses in consecutive order for various values of θ by controlling superposition probabilities using ξ . The amplitudes of the fringes observed for $|D_1\rangle \equiv |2\rangle$ and $|D_2\rangle \equiv |3\rangle$ are shown in figure 4.16. The amplitudes of the fringes are maximum for equal superposition at $\theta = \frac{\pi}{2}$ as expected.

4.6 Conclusion

In conclusion, we have simulated an atomtronics DDT using a spin-polarized ultracold gas coupled to a sequence of three optical pulse beams targeting the tripod scheme. We have

shown that the function of our DDT is independent of velocity of the atoms covering the typical velocity dispersion of an ultracold gas. We also characterized the behaviour of our DDT using two key control parameters of the system *viz.*, the gate parameter ξ and the separation parameter η . The first controls the spin current while the later controls the rate of change of spin current with respect to the first showing that our configuration is suitable for both precise spin amplitude control and spin modulation purposes. We remind that this coherent rotation of the pseudo-spin occurs due to the action of a time dependent or a spatially inhomogeneous Abelian gauge field, simply because only one component of the gauge field is in action. We also demonstrate the momentum exchange with photons of the tripod beams cause momentum-spin dependence and conduct to a spatial separation of the pseudo-spin components like in Stern-Gerlach experiment, but of different nature compared to a spin-Hall effect, which depends on the atom's velocity in the gauge field [34]. This experiment demonstrates the use of geometric gauge potential to impart motional changes to the atomic wave packet in a coherent manner. It demonstrates the robustness of the result of the experiment to fluctuations in the parameters. This approach can be extended by designing complex gauge fields interacting with matter waves to produce superposition states with a desired velocity distribution in atomtronics devices. We then implemented a Ramsey interferometry sequence using two DDTs in series and observe interference fringes in time domain revealing the effect of scalar term. This scalar potential breaks the degeneracy of the two dark states and hence a dynamical phase is accumulated as a function of free evolution time as observed in the experiment. To minimize the effect of this scalar potential one could co-propagate σ^+ and σ^- beams as done in [31, 35]. In chapter 5, we show that one could completely cancel the scalar term by adding appropriate detunings to the tripod lasers. Finally, we note that our experiment was performed with a degenerate Fermi gas of strontium atoms, but can be generalized to other types of fermions or bosons as long as one could isolate a resonant tripod scheme, for example like in Rb atoms [36].



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Chapter 5

Dynamics in Non-Abelian gauge field

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5.1 Introduction

The generation of artificial gauge fields using atom-light interaction in free space has been presented in chapter 2 where different terms of the gauge field can be controlled by coupling parameters. This allows for controlled study and simulation of magnetism in the single quantum particle limit. The gauge fields that belong to SU(2) symmetry have been realized experimentally in free space [1, 2] as well as in optical lattices [3, 4] that mimic the condensed matter systems exhibiting integer [5] and fractional quantum Hall effects [6]. However, the dynamics of neutral atoms in non-Abelian gauge fields have not been experimentally explored. The dynamics in SU(2) non-Abelian gauge fields is governed by a Hamiltonian resembling spin-orbit coupling (SOC) Hamiltonian where the dark states of the atom-light coupling form the pseudo-spin basis [7–10]. Whereas dynamics of neutral atoms in SU(3) non-Abelian gauge field resemble color-orbit coupling (COC) that appears in quantum chromodynamics (QCD) [11]. Wave-packet dynamics of a SOC system is predicted to exhibit an oscillation similar to the *Zitterbewegung* of the Dirac equation, *i.e.*, a trembling motion of a particle associated with a quantum Rabi flopping of the pseudo-spin between the eigen states of the Hamiltonian [9, 10, 12, 13]. We have shown theoretically in section 2.5 that dynamics in SU(3) system exhibits oscillatory behavior with three frequencies. Experimental study of dynamics of neutral atoms exhibiting COC like phenomena has not been reported. However, in recent years theoretical proposals for simulating SU(N) systems for $N > 2$ both in free space and optical lattices have been proposed [11, 14–17] with emphasis on exploring topological properties.

In this chapter, we describe the experimental observation of dynamics of wave packet in non-Abelian gauge fields. In section 5.2, we provide an overview of dynamics in SU(2) non-Abelian gauge field where *Zitterbewegung*-like oscillations are observed. We start by explaining how a pure spin-orbit coupling Hamiltonian is achieved in the experiment by canceling the geometric scalar term in section 5.2.1. We present the dynamics result in section 5.2.2 and end SU(2) study with description of spin Hall characteristics of the dynamics 5.2.3. In section 5.3, we summarize our preliminary results on the dynamics of the wave-packet in SU(3) gauge field. We first explain the cancellation the scalar term in section 5.3.1. In section 5.3.2, we describe and characterize the initial states that are used in the dynamics experiment. In section 5.3.3, we show the dynamics results with various values of the momentum kick and show that the oscillations in dynamics consist of multiple frequencies at most three. In section 5.4, we demonstrate existence of additional ladder operators in SU(3) system than the well known raising and lowering operators of angular momentum. In section 5.5, we conclude the chap-

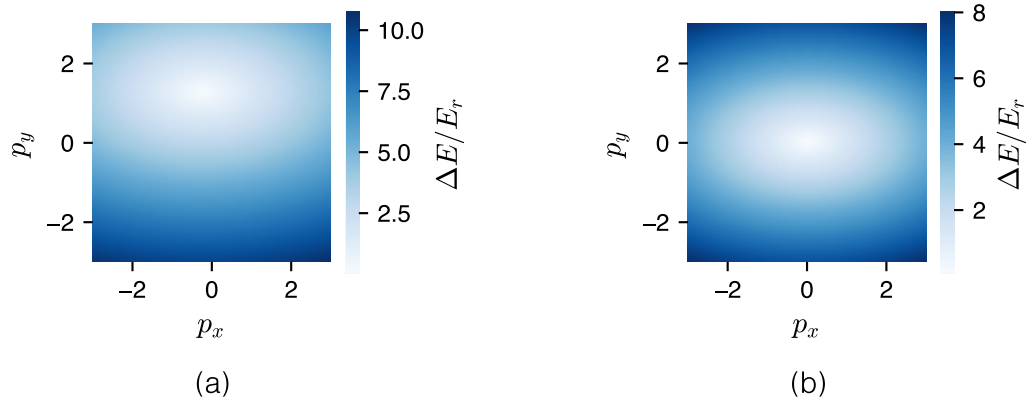


Figure 5.1: (a) Difference between the dispersion branches in momentum space when the scalar field is present. (b) Difference between the dispersion branches in momentum space when the scalar field is canceled. The minimum occurs at the Dirac point located at $\mathbf{p} = 0$.

ter.

5.2 Dynamics in SU(2) Gauge Field

The SU(2) Hamiltonian we generate with non-Abelian gauge field using tripod scheme is a mixture of both Rashba [18] and Dresselhaus [19] spin-orbit coupling interactions. The study of wave packet dynamics in this Hamiltonian has been described in the thesis [20] and subsequently in Ref. [21]. Here, an overview of the results is provided focusing mainly on the ideas relevant to the dynamics in SU(3) Hamiltonian.

5.2.1 Scalar Potential Cancellation

The geometric scalar potential that arises due to the configuration of tripod lasers is of the same order of magnitude as $\mathbf{p} \cdot \mathbf{A}/m$ when $\mathbf{p} \approx \hbar \mathbf{k}$ (see equation 2.40). As previously discussed

in section 4.5.1, this scalar field breaks the degeneracy between the dark states when the tripod lasers satisfy two-photon resonance condition in the rest frame. Since the transfer of population among the bare states is coherent and adiabatic, when the atoms occupy bare states $|1\rangle$ and $|2\rangle$, the tripod transitions are Doppler shifted. These motional shifts are the origin of the scalar term [22]. These shifts can be nullified by appropriately detuning the tripod lasers such that $Q + \omega = 0$ and the result is a pure SOC Hamiltonian:

$$H = \frac{\mathbf{p}^2}{2m} - \frac{\mathbf{p} \cdot \mathbf{A}}{m} \quad (5.1)$$

Figures 5.1a&b show the difference in energy between the eigen states of the Hamiltonians $H + Q$ and H as a function of $\mathbf{p}$, respectively. By canceling the scalar term in SU(2) Hamiltonian, we positioned the Dirac point at $\mathbf{p} = 0$. We perform the experiment for the case of equal Rabi frequencies of tripod lasers *i.e.*, $\varphi = \frac{\pi}{4}$ and $\vartheta = \cos^{-1} \frac{1}{\sqrt{3}}$. The detunings required for the tripod lasers are $\delta_1 = -4\omega_r$, $\delta_2 = -2\omega_r$ and $\delta_3 = 0$ where $\omega_r = \frac{p_r^2}{2m\hbar} = 2\pi \times 4.8$ kHz is the recoil frequency. When an atom is transferred from $|3\rangle$ to $|1\rangle$ it acquires a momentum of $2p_r \hat{\mathbf{e}}_y$ and therefore for atoms in $|1\rangle$ the σ^+ tripod laser connecting $|1\rangle$ to the excited state $|e\rangle$ is blue detuned by $4\omega_r$. The scalar cancellation detuning $\delta_1 = -4\omega_r$ cancels this blue shift and brings the σ^+ tripod laser into resonance. Similarly, when an atom is transferred from $|3\rangle$ to $|2\rangle$ it acquires a momentum of $p_r(\hat{\mathbf{e}}_y - \hat{\mathbf{e}}_x)$ and therefore for atoms in $|2\rangle$ the π tripod transition is blue detuned by $2\omega_r$. The scalar cancellation detuning $\delta_2 = -2\omega_r$ cancels this blue shift and brings the σ^+ tripod laser into resonance. That is why, the scalar detunings do not depend on the Rabi frequency ratios or mixing angles ϑ and φ . In summary, the scalar field cancellation nullifies the effect of Doppler shifts due to motional effects among the three ground states of the tripod scheme.

5.2.2 Zitterbewegung Dynamics

We map initial population in the bare state $|3\rangle$ into the dark state subspace by adiabatically turning on the tripod lasers in the following order:

- turning on σ^+ and π beams with equal Rabi frequencies abruptly at $t=0$. Therefore, the mixing angle $\varphi = \frac{\pi}{4}$ for the whole gauge field sequence. Therefore, the term proportional to $\frac{\partial \varphi}{\partial t}$ in the detuning matrix ω in equation 2.41 is zero.
- adiabatically ramping the power of σ^- beam in $10 \mu\text{s}$. This ramps up the mixing angle ϑ from 0 to $\cos^{-1} \frac{1}{\sqrt{3}}$.

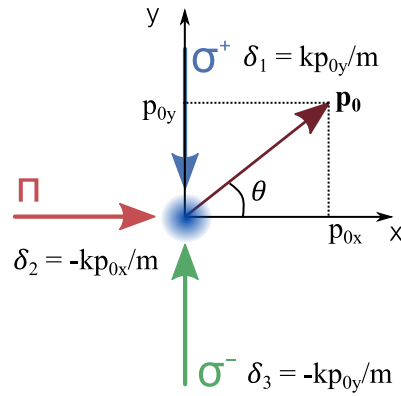


Figure 5.2: Schematic showing the detunings of the tripod lasers when atoms are imparted with a momentum kick of $\mathbf{p}_0$. This momentum kick is simulated by detuning the tripod lasers by the shown amounts. This is equivalent to going to a moving frame of reference with velocity $-\mathbf{p}_0/m$ by adding $-\frac{\mathbf{p}_0 \cdot \mathbf{A}}{m}$ to the scalar term as shown in section 2.4 using the detuning matrix.

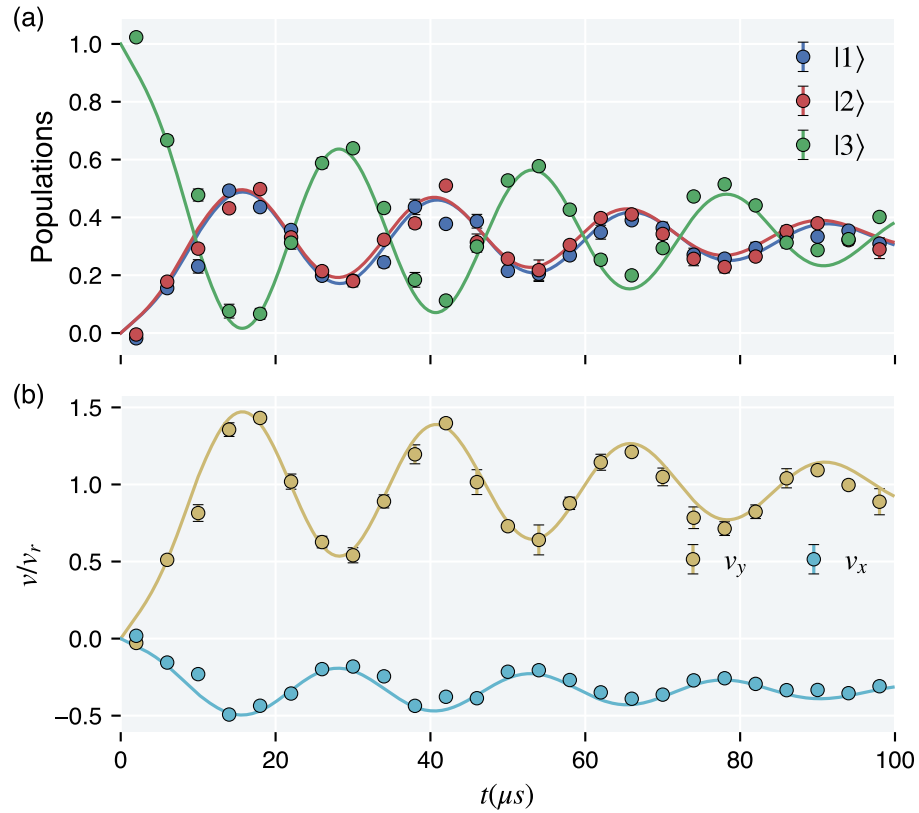


Figure 5.3: Dynamics of the wave packet with a momentum kick of $4\sqrt{2}\hbar k$ at an angle $\theta = 0.1\pi$ (a) Evolution of bare state population in uniform gauge field (b) Evolution of velocity of the the wave packet by subtracting the ballistic velocity due to the kick.

With this ramping sequence, we map the system into the dark state that is connected to $|3\rangle$ at $t=0$ i.e., $|D_2\rangle = \frac{1}{\sqrt{6}}(e^{-2iky}|1\rangle + e^{ik(x-y)}|2\rangle - 2|3\rangle)$ [20, 23]. During this mapping sequence, the scalar field remains canceled for the previously stated detunings. After the ramping sequence we maintain the Rabi frequencies as constant. To explore the momentum space, we add the Galilean boost term $-\frac{\mathbf{p}_0 \cdot \mathbf{A}}{m}$ where $\mathbf{p}_0$ is the momentum kick or boost using the detuning matrix ω equivalently simulating the Doppler shifted tripod laser beams. Figure 5.2 shows the momentum kick and the Doppler shifts for the tripod lasers. The detuning matrix with these detunings read

$$\begin{aligned} \omega &= \hbar \begin{pmatrix} (\delta_1 - \delta_3) \sin^2 \varphi + (\delta_2 - \delta_3) \cos^2 \varphi & \cos \vartheta \sin 2\varphi (\delta_1 - \delta_2)/2 \\ \cos \vartheta \sin 2\varphi (\delta_1 - \delta_2)/2 & \cos^2 \vartheta ((\delta_1 - \delta_3) \cos^2 \varphi + (\delta_2 - \delta_3) \sin^2 \varphi) \end{pmatrix} \\ &= \frac{\hbar k}{m} \begin{pmatrix} 2p_{0y} \sin^2 \varphi + (p_{0y} - p_{0x}) \cos^2 \varphi & \cos \vartheta \sin 2\varphi (p_{0y} + p_{0x})/2 \\ \cos \vartheta \sin 2\varphi (p_{0y} + p_{0x})/2 & \cos^2 \vartheta (2p_{0y} \cos^2 \varphi + (p_{0y} - p_{0x}) \sin^2 \varphi) \end{pmatrix} \\ &= -\frac{p_{0x}A_x + p_{0y}A_y}{m} = -\frac{\mathbf{p}_0 \cdot \mathbf{A}}{m} \end{aligned} \quad (5.2)$$

We add these detunings to the tripod lasers along with the scalar cancellation detunings and initiate the ramping sequence. We scan the duration in the gauge field and abruptly switch off the tripod lasers and observe the momentum distribution after a time of flight of 9 ms using fluorescence imaging. Figure 5.3a shows the evolution of bare state populations as a function of time in the uniform gauge field produced by equal Rabi frequencies of tripod lasers when the imparted momentum kick is $p_0 = 4\sqrt{2}$ along $\theta = \pi/10$. These populations are extracted from the fluorescence images in the same way as explained in chapter 4. Figure 5.3b shows the evolution of velocity of center of mass of the atoms oscillating showing the *Zitterbewegung* oscillations. The center of mass velocity is extracting by using the relations:

$$\begin{aligned} v_x &= -v_r P_2 \\ v_y &= v_r (2 \times P_1 + P_2) \end{aligned} \quad (5.3)$$

where $v_r = \frac{\hbar k}{m}$ is the recoil velocity and P_1 and P_2 are the population of $|1\rangle$ and $|2\rangle$, respectively. The ballistic velocity due to momentum kick is cleverly subtracted in equation 5.3 showing only the velocity oscillations due to the presence of gauge field. The effective Hamiltonian that describes the dynamics of atoms can be considered to be ω in equation 5.2. We characterize the frequencies and amplitude of these oscillation as a function of p_0 and θ presented in a previous thesis [20] and Ref. [21]. Since the difference between the eigen energies of ω are not cylindrically symmetric about $\mathbf{p}_0 = 0$, we observe an asymmetry in the oscillation frequencies.

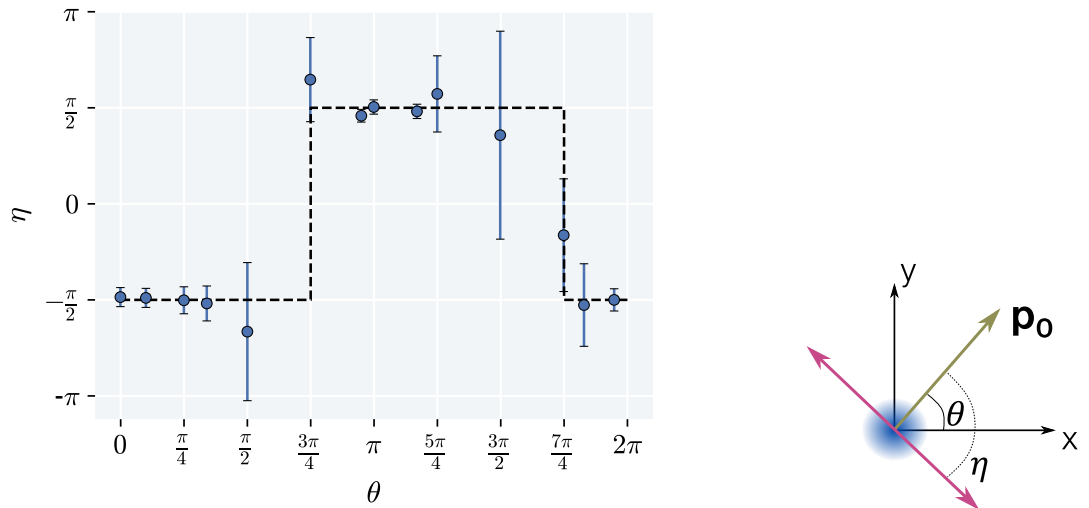


Figure 5.4: Angle made by the acceleration of the atoms with the direction of momentum kick indicating spin Hall characteristics of the *Zitterbewegung* oscillations.

This Hamiltonian resembles the two level atom coupled to electromagnetic radiation. The two levels are the dark states and the off-diagonal terms are analogous to the Rabi frequency of the coupling. Maximum amplitude of oscillations occurs when the initial state $|D_2\rangle$ is equal superposition of the eigen states of the Hamiltonian. This occurs at $\theta = \pi/10$ (the case shown in figure 5.3).

5.2.3 Spin Hall Characteristics

Spin Hall effect is a phenomena where spin accumulates on the lateral boundaries of a current carrying conductor [24,25]. The origin of spin Hall characteristics in our system is the intrinsic spin-orbit interaction resulting from presence of non-Abelian gauge field. A characteristic of wave packet dynamics in the non-Abelian gauge field is that the oscillations occur in a plane normal to the momentum kick. As already introduced in section 2.5, the acceleration due to the presence of non-Abelian uniform gauge field reads

$$\frac{d\mathbf{v}}{dt} = -\frac{i}{\hbar m^2} \mathbf{p}_0 \times \mathbf{A} \times \mathbf{A} \quad (5.4)$$

The momentum kick $\mathbf{p}_0$ can be considered as the drift momentum of the carriers due to electric current in spin Hall effect. Figure 5.4 shows the angle η extracted from the oscillations of the velocity as a function of angle of the momentum kick θ . We observe that the oscillation of velocity is always perpendicular to the momentum kick. Large errors in η for certain values

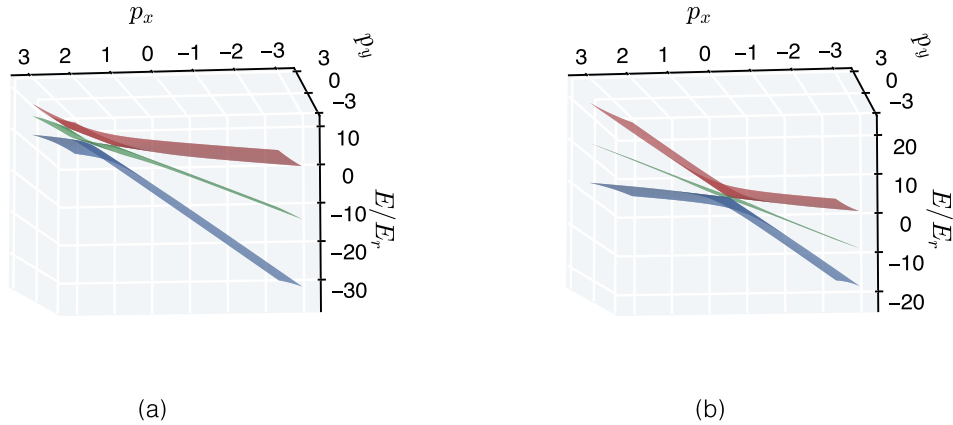


Figure 5.5: (a) The dispersion branches of the Hamiltonian when the scalar field is present. (b) The dispersion branches of the Hamiltonian when the scalar field is canceled. The Dirac-like point located at $\mathbf{p} = 0$ where all three branches of the dispersion intersect.

of θ are due to the small amplitude of oscillation at these angles [21]. The sign change in the angle η is due to the phase difference between the components of the velocity.

5.3 Dynamics in SU(3) Gauge Field

As opposed to SU(2) system where we observed *Zitterbewegung* oscillations with single frequency, we observe oscillations with multiple frequencies in the SU(3) system. These oscillations consist of three frequencies as shown in section 2.5 that correspond to the energy difference between the eigen states the Hamiltonian that resembles COC [11]. In this section we describe the experimental observation of these oscillations.

5.3.1 Scalar Potential Cancellation

In section 5.2.1, we have shown how we cancel the scalar term in SU(2) system using the de-tuning matrix. We cancel the scalar term of SU(3) system in the same way. Effect of geometric

δ_1^l	δ_2^l	δ_3^l	δ_3^r	δ_2^r	δ_1^r
$-8\omega_r$	$-2\omega_r$	$4\omega_r$	$-4\omega_r$	$-2\omega_r$	0

Table 5.1: Detunings of the double tripod lasers that result in the cancellation of SU(3) scalar potential.

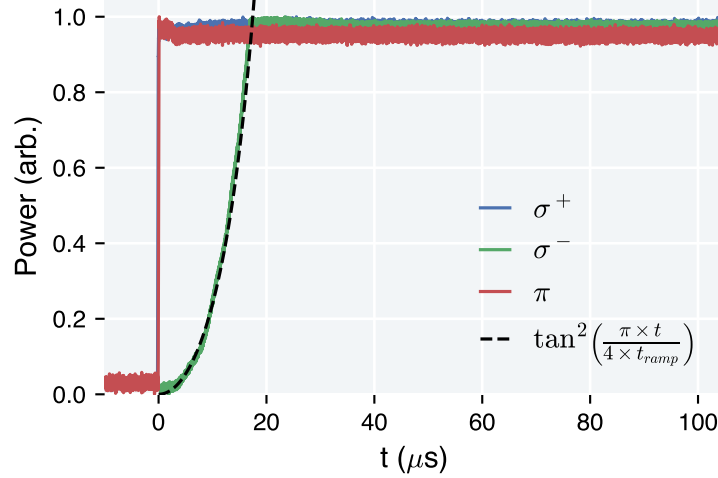


Figure 5.6: Normalized powers of the tripod beams during the experiment. Initial ramping sequence maps the atoms in bare state $|5\rangle$ into dark state subspace.

scalar potential in SU(3) system is slightly more complicated than that in the SU(2) system. In SU(2) system, the scalar potential breaks the degeneracy between the dark states at $\mathbf{p} = 0$ effectively shifting the Dirac point. However, in SU(3) system due to the presence of three branches in the dispersion relation, the scalar term not merely shifts the dispersion but also lead to three Dirac points separated in momentum space. Figure 5.5a shows the dispersion branches of the Hamiltonian with out kinetic energy in the presence of scalar potential. After canceling the scalar term, this condition is eliminated by making all the branches intersect at a single point in momentum space at $\mathbf{p} = 0$. The detunings required to cancel the scalar term in SU(3) system are tabulated in table 5.1. The right tripod detunings are same detunings as in the case of SU(2) scalar field cancellation. Left tripod detunings are different especially $\delta_3^l = 4\omega_r$ to counter the Doppler shift of $-4\omega_r$ for σ^- laser's -1 order sideband. These detunings are set by adjusting the modulation frequency of EOMs and RF frequency of the AOMs.

5.3.2 Initial State Characterization

In the double-tripod system, the mapping of bare states of the atom into the dark state basis can be done in multiple ways. We initialize the system into the uniform gauge field in two

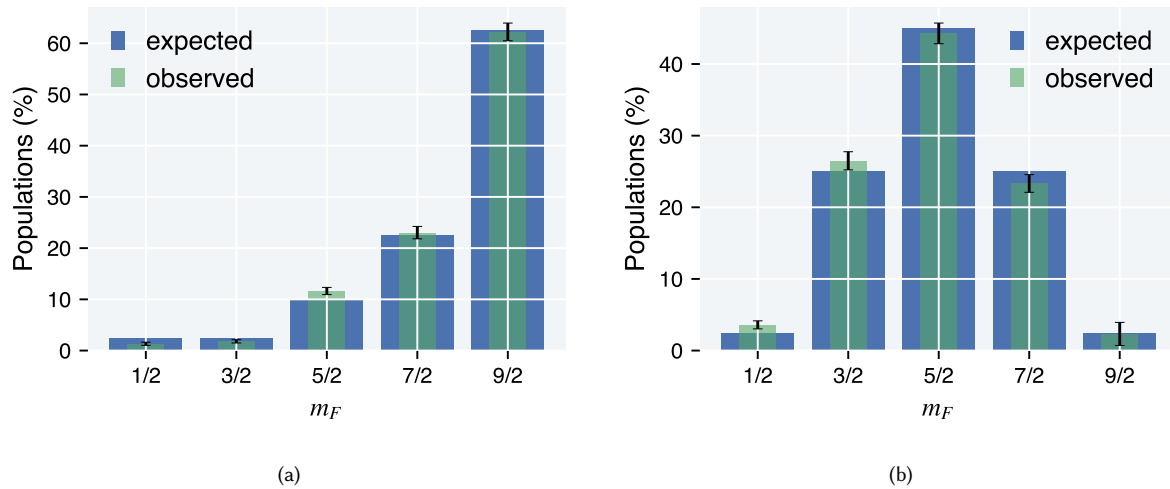


Figure 5.7: Initial state preparations. (a) Bare state populations of the initial state $|D_+\rangle$ prepared by abruptly switching on σ^+ and π beams when atoms are in $|5\rangle$ and ramping σ^- beams. (b) Bare state populations of the initial state $|D_3\rangle$ prepared by abruptly switching on π beams when atoms are in $|3\rangle$ and ramping σ^+ and σ^- beams.

initial states. This is done to verify that the frequencies in the dynamics of the wave packet are independent of the initial state (see section 5.3.3). The first state that we prepare is $|D_+\rangle = \frac{1}{\sqrt{5}}(|D_0\rangle + 2e^{i\pi/3}|D_r\rangle)$. This state is prepared using a similar ramping sequence as in SU(2) system explained in section 5.2.2. First, we switch on the σ^+ and π tripod beams abruptly and ramp up the σ^- beams, both carrier and the sideband, slowly in 18 μ s. Figure 5.6 shows the total powers of the tripod beams during the tripod sequence. We ramp the σ^- lasers' power such that their Rabi frequencies follow the equations:

$$\begin{aligned} |\Omega_1^r| &= |\Omega_2^r| \tan\left(\frac{\pi \times t}{4 \times t_{ramp}}\right) \\ |\Omega_3^l| &= |\Omega_2^l| \tan\left(\frac{\pi \times t}{4 \times t_{ramp}}\right) \end{aligned} \quad (5.5)$$

This function of the ramp is chosen such that $\frac{\partial \varphi_r}{\partial t} = -\frac{\pi}{4t_{ramp}}$ the ramp part of the matrix ω in equation 2.48. Figure 5.7a shows the populations of the bare states after the above mentioned ramping sequence. The second initial state we prepare is $|D_3\rangle = -\frac{2}{11}(\frac{6}{5}|D_l\rangle - |D_0\rangle + \frac{6}{5}|D_r\rangle)$. This is prepared in two steps: first, the atoms in $|5\rangle$ are transferred to $|3\rangle$ using STIRAP sequence. Second, π beams are switched on abruptly and both σ^+ and σ^- transitions are ramped up according to the tangent function in equation 5.5. Figure 5.7b shows the populations of bare states after this ramping sequence.

Both the initial states shown in 5.7a&b are prepared with no momentum kick. When an

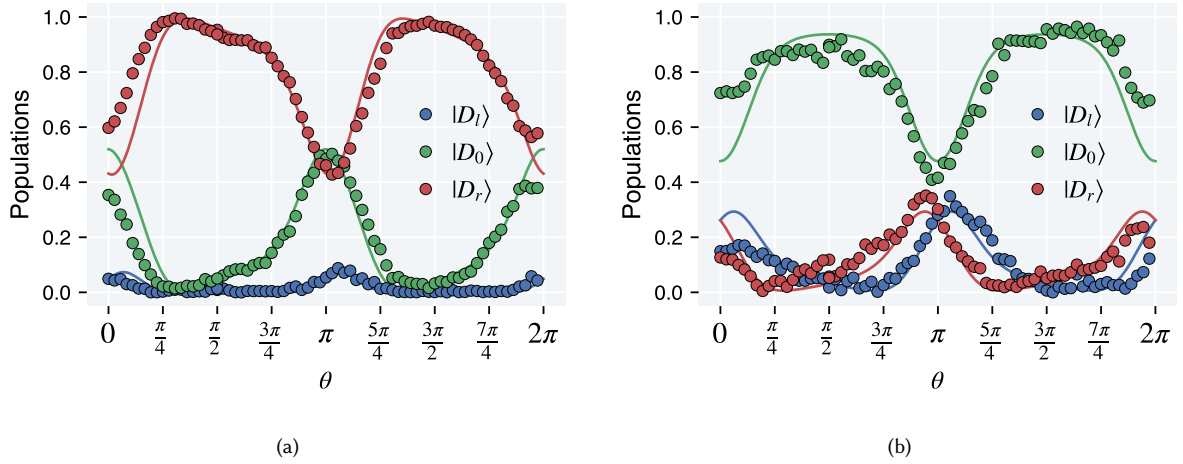


Figure 5.8: (a) Dark state populations after the $|D_+\rangle$ preparation ramping sequence with a momentum kick of $4\hbar k$ as a function of kick angle θ . (b) Dark state populations after the $|D_3\rangle$ preparation ramping sequence with a momentum kick of $4\hbar k$ as a function of kick angle θ .

additional momentum kick is imparted using the detuning matrix, the initial state is further rotated by the $-\frac{\mathbf{p}_0 \cdot \mathbf{A}}{m}$ term. Figure 5.8a&b shows the population of dark states after the ramping sequence as a function of the kick angle θ when a momentum kick of $4\hbar k$ is applied. The plain lines are obtained by solving the Schrödinger equation

$$i\hbar \frac{\partial}{\partial t} |\psi\rangle = \left[\frac{\mathbf{p}^2}{2m} - \frac{(\mathbf{p} + \mathbf{p}_0) \cdot \mathbf{A}}{m} + \omega \right] |\psi\rangle \quad (5.6)$$

during the ramping as the Hamiltonian is time dependent. We observe that the state prepared after the ramping sequence is slightly asymmetric at angles separated by 180° . This can be attributed to imperfect scalar cancellation due to residual light shifts of the order of $\omega_r = 2\pi \times 4.8$ kHz. This imperfection shifts the degeneracy point at $\mathbf{p} = 0$. This can be compensated by slightly adjusting the scalar cancellation detunings.

5.3.3 Dynamics

After the scalar term cancellation, we observe the evolution of bare state populations with a momentum kick in the gauge field produced with equal Rabi frequencies of 225 kHz for the double-tripod transitions similar to the SU(2) case. We add a momentum kick of $\mathbf{p}_0$ using the detuning matrix and the effective Hamiltonian of the system reads,

$$H = \frac{\mathbf{p}^2}{2m} - \frac{(\mathbf{p} + \mathbf{p}_0) \cdot \mathbf{A}}{m} \quad (5.7)$$

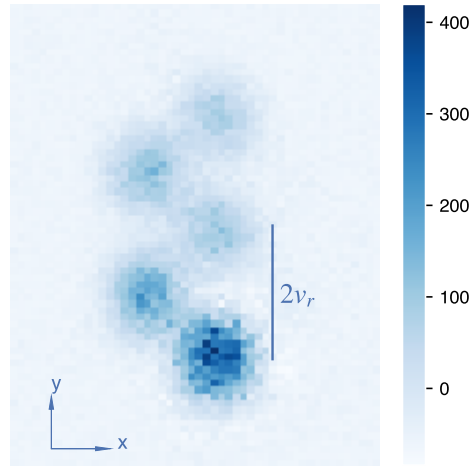


Figure 5.9: Fluorescence image showing the expansion of the atomic cloud in the presence of the gauge field after a time of flight of 9 ms. Various clouds of atoms show the population of the bare states distributed according to their velocity.

From the bare state populations, we extract the components of velocity of the cloud $\mathbf{v} = v_x \hat{\mathbf{e}}_x + v_y \hat{\mathbf{e}}_y$ in the moving frame using

$$\begin{aligned} v_x &= -v_r(P_2 + P_4) \\ v_y &= v_r(4 \times P_1 + 3 \times P_2 + 2 \times P_3 + P_4) \end{aligned} \quad (5.8)$$

where v_r is the recoil velocity and P_i are the populations of bare states $|i\rangle$ for $i=1, 2, 3, 4$ and 5. Figure 5.9 shows the typical fluorescence image acquired after a time of flight of 9 ms. These images are fit with sum of five 2D Gaussians and all five bare state populations are extracted simultaneously. Figures 5.10a&b show the evolution of population of bare states with $|D_+\rangle$ as the initial state and a momentum kick of $4\hbar k$ is imparted to the atoms along $\theta = 0$ *i.e.*, along $\hat{\mathbf{e}}_x$. Figure 5.10c shows the components of velocity at this momentum kick. The plain lines are the numerically obtained results. We can see qualitative agreement with the theory lines but there exist some mismatch in the frequencies of oscillations. This mismatch is attributed to the inaccuracies in the detunings due to residual light shifts that effectively results in a slightly larger momentum kick than the detunings correspond to. Apart from that, we observe the spin Hall effect (more aptly color Hall effect) behavior like SU(2) case as the component of velocity along the momentum kick direction does not vary implying acceleration along the kick is zero. Figure 5.11 shows the evolution of populations and velocity for the same initial state when the momentum kick is $4\hbar k$ along $\theta = 180^\circ$ *i.e.*, along $-\hat{\mathbf{e}}_x$. The COC term is anti-symmetric about θ . However, the dynamics are not affected by the overall neg-

ative sign of the Hamiltonian. Therefore, the dynamics should be same at angles separated by 180° as the results in figures 5.10&5.11 qualitatively match. The angles around $\theta = 0^\circ$ or $\theta = 180^\circ$ is chosen for the experiment as the three frequencies in the dynamics are small and well separated compared to other angles (see figure 2.5b).

To extract multiple frequencies from the dynamics, we take the discrete Fourier transform (using fft) of the velocity components. As the oscillation signal damps down due to temperature, the frequencies of the dynamics for smaller values of momentum kick (for example $\mathbf{p}_0 = 4\hbar k\hat{\mathbf{e}}_x$) cannot be resolved in the fft. So, we repeat the experiment with a larger momentum kick of $\mathbf{p}_0 = 8\hbar k\hat{\mathbf{e}}_x$. Figures 5.12&5.13 shows the fft of velocity components when the system is initialized in states $|D_+\rangle$ and $|D_3\rangle$, respectively, with $\mathbf{p}_0 = 8\hbar k\hat{\mathbf{e}}_x$. The dashed lines are cubic interpolated values of the fft of the velocities obtained by numerical simulation. The vertical dashed black lines represent the frequencies corresponding to the difference between different eigen energies of the Hamiltonian. The amplitudes of different frequencies depend on the components of the initial state on the eigen states of the Hamiltonian. When the system is initialized in the state $|D_+\rangle$ only two peaks are predominantly visible in the fft. However, when the initial state is $|D_3\rangle$, all three frequencies are visible in the fft of the velocity. However, there exist a large deviation between experimental data and numerical expectation in the amplitudes of the peaks. This is due to the slight mismatch in the frequencies and noise in the observation. If we slightly change the angle of the momentum kick, for example $\theta = 5^\circ$, we find a better match with the numerics. This is shown in figure 5.14.

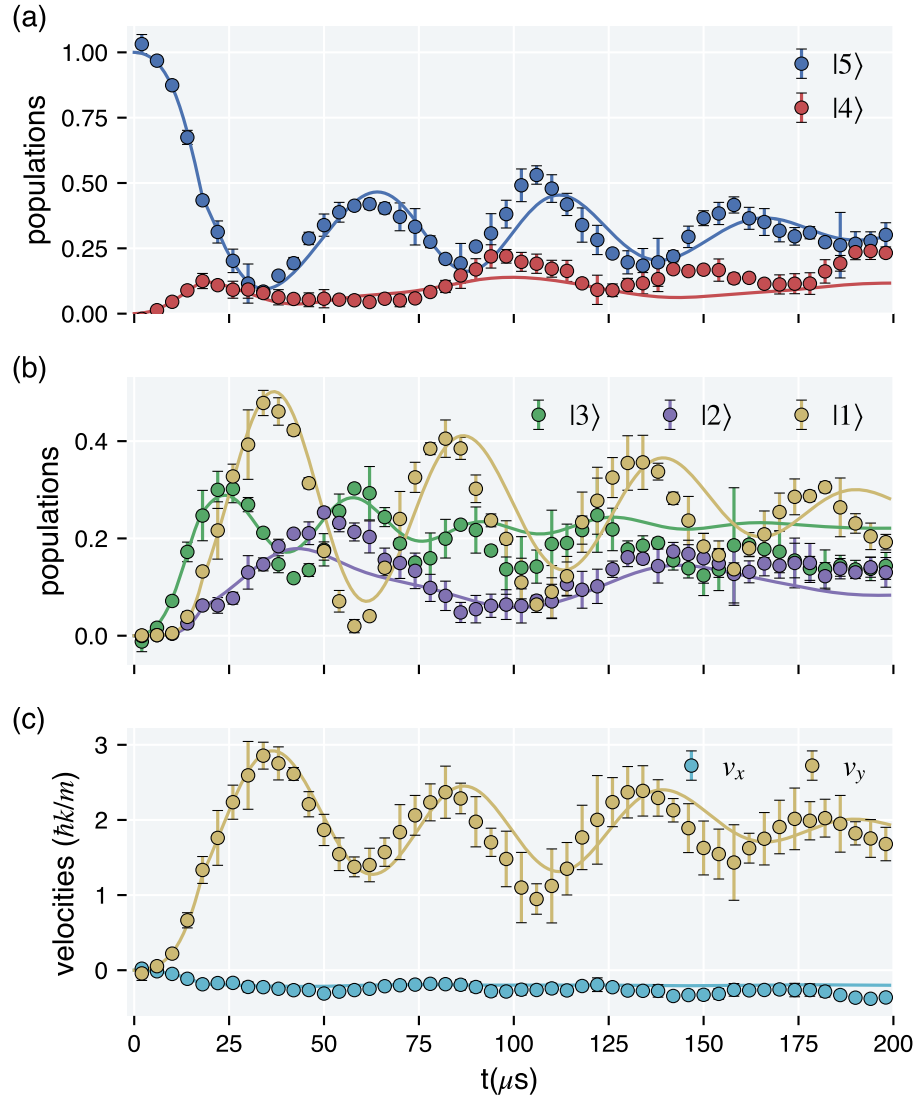


Figure 5.10: Evolution of bare state populations in SU(3) gauge field (a)&(b) and components of velocity for a momentum kick of $4\hbar k$ at an angle $\theta = 0^\circ$.

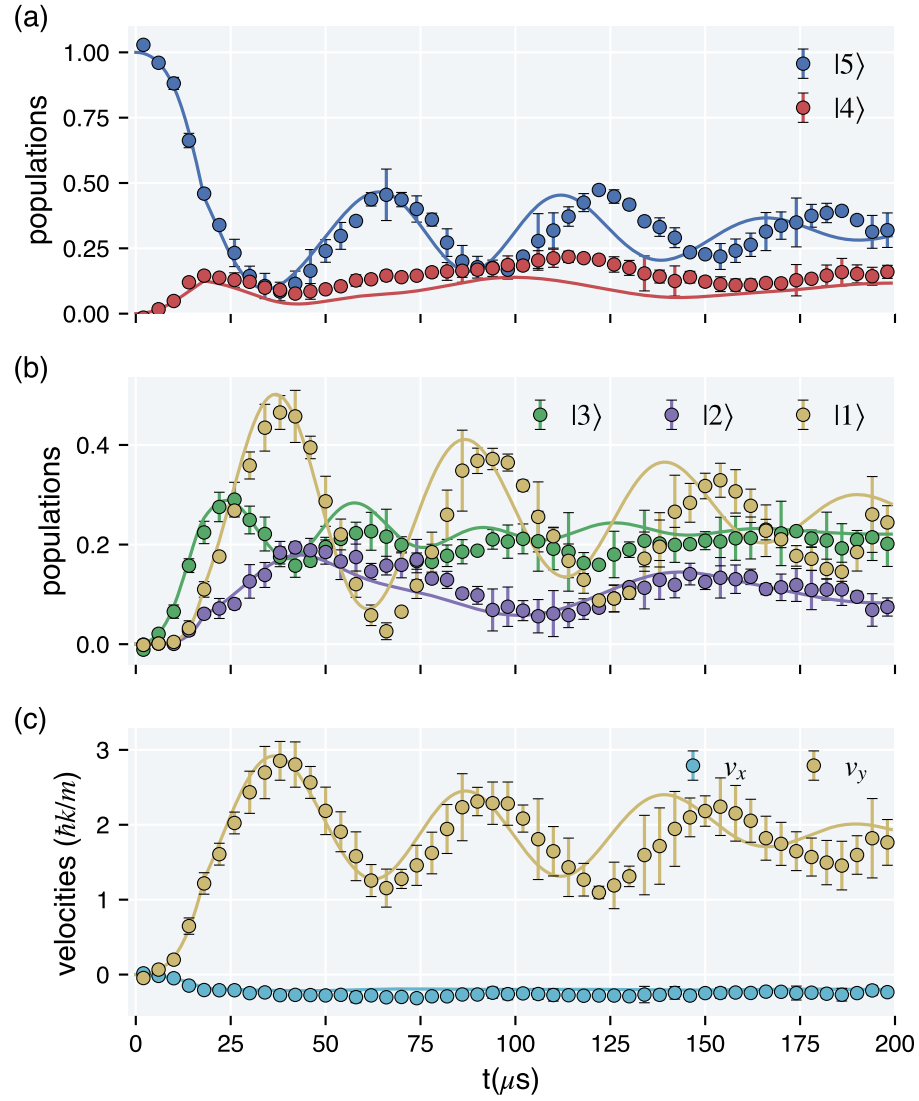


Figure 5.11: Evolution of bare state populations in SU(3) gauge field (a)&(b) and components of velocity for a momentum kick of $4\hbar k$ at an angle $\theta = 180^\circ$.

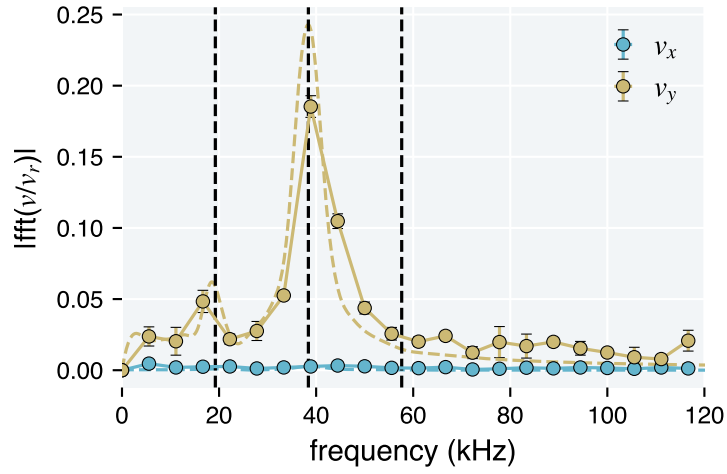


Figure 5.12: FFT of the velocity evolution in SU(3) gauge field when initialized in $|D_+\rangle$ with a kick of $\mathbf{p}_0 = 8\hbar k \hat{\mathbf{e}}_x$ i.e., $\theta = 0^\circ$. The vertical dashed black lines represent the frequencies corresponding to the difference between different eigen energies of the Hamiltonian.

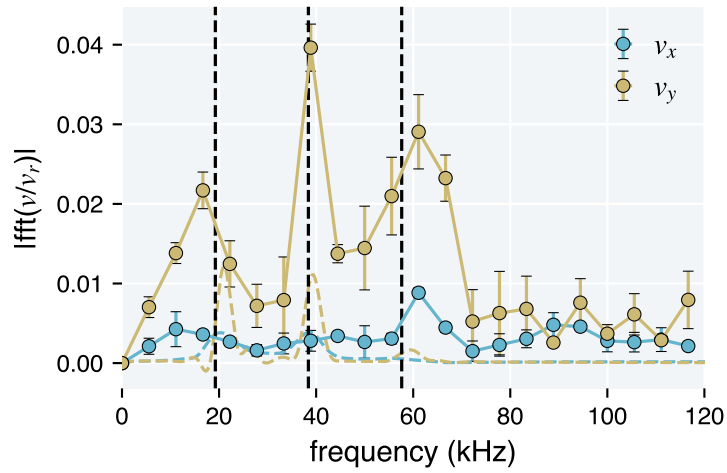


Figure 5.13: FFT of the velocity evolution in SU(3) gauge field when initialized in $|D_3\rangle$ with a kick of $\mathbf{p}_0 = 8\hbar k \hat{\mathbf{e}}_x$ i.e., $\theta = 0^\circ$. The vertical dashed black lines represent the frequencies corresponding to the difference between different eigen energies of the Hamiltonian.

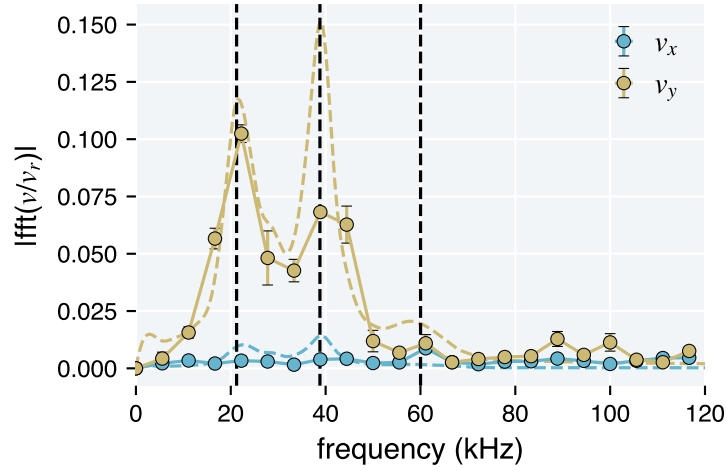


Figure 5.14: FFT of the velocity evolution in SU(3) gauge field when initialized in $|D_3\rangle$ with a kick of $8\hbar k$ along $\theta = 5^\circ$. The vertical dashed black lines represent the frequencies corresponding to the difference between different eigen energies of the Hamiltonian.

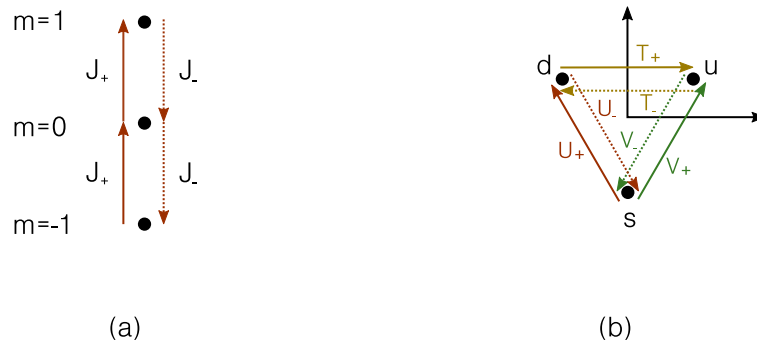


Figure 5.15: Ladder operations. (a) SU(2) ladder operations of spin angular momentum. (b) SU(3) ladder operators of color charge of quarks from QCD.

5.4 Ladder Operations

Spin angular momentum represented by SU(2) matrices has two ladder operators, raising and lowering ladder operators. The operators represented by SU(3) matrices like color charge have two quantum numbers with six ladder operators, three raising and three lowering operators illustrated in figure 5.15 [26]. Therefore, starting from an initial state, we can reach a final state in two possible paths in contrast to angular momentum. One direct ladder operator(raising/lowering) and another, combination of two other ladder(two raising/ two lowering) operators. In this section, we demonstrate existence of more than two ladder operators in our system. The detuning matrix in $\{|D_l\rangle, |D_0\rangle, |D_r\rangle\}$ basis for equal Rabi frequencies of double-tripod transitions reads

$$\omega = -\hbar \begin{pmatrix} \frac{\delta_1^l + \delta_2^l}{2} & \frac{\delta_1^l - \delta_2^l}{4} & 0 \\ \frac{\delta_1^l - \delta_2^l}{4} & \frac{\delta_1^l + \delta_2^l + \delta_1^r + \delta_2^r}{8} & \frac{\delta_1^r - \delta_2^r}{4} \\ 0 & \frac{\delta_1^r - \delta_2^r}{4} & \frac{\delta_1^r + \delta_2^r}{2} \end{pmatrix} \quad (5.9)$$

At $\mathbf{p} = 0$, this term is the only color-dependent term in the Hamiltonian. This is equivalent to the Hamiltonian of three level system with two couplings. However, the coupling strength between the dark states is determined by the detunings of the double-tripod transitions. By inspecting the Hamiltonian in 5.11, we note that the direct coupling between $|D_l\rangle$ and $|D_r\rangle$ is zero. To have all three coupling between the dark states, we perform a change of basis from $\{|D_l\rangle, |D_0\rangle, |D_r\rangle\} \rightarrow \{|D_l\rangle, |D'_+\rangle, |D'_-\rangle\}$ where

$$\begin{aligned} |D'_+\rangle &= \frac{1}{\sqrt{2}}(|D_0\rangle + e^{i\pi/4}|D_r\rangle) \\ |D'_-\rangle &= \frac{1}{\sqrt{2}}(|D_0\rangle - e^{i\pi/4}|D_r\rangle) \end{aligned} \quad (5.10)$$

In the new basis the detuning matrix ω reads,

$$\omega' = -\hbar \begin{pmatrix} \frac{\delta_1^l + \delta_2^l}{2} & \frac{\delta_1^l - \delta_2^l}{4\sqrt{2}} & \frac{(\delta_1^l - \delta_2^l)e^{i\pi/4}}{4\sqrt{2}} \\ \frac{\delta_1^l - \delta_2^l}{4\sqrt{2}} & \frac{\delta_1^l + \delta_2^l + 9\delta_1^r + \delta_2^r}{16} & \frac{e^{i\pi/4}}{4} \left(\frac{\delta_1^l + \delta_2^l}{4} - \frac{3(\delta_1^r + \delta_2^r)}{4} \right) \\ \frac{(\delta_1^l - \delta_2^l)e^{-i\pi/4}}{4\sqrt{2}} & \frac{e^{-i\pi/4}}{4} \left(\frac{\delta_1^l + \delta_2^l}{4} - \frac{3(\delta_1^r + \delta_2^r)}{4} \right) & \frac{\delta_1^l + \delta_2^l + \delta_1^r + 9\delta_2^r}{16} \end{pmatrix} \quad (5.11)$$

This Hamiltonian is equivalent to a three level system coupled by three lasers forming a closed loop. Unlike the detuning matrix of SU(2) where any combination of detunings lead to an effective momentum kick (two two-photon detunings in two dimensions), the detuning matrix

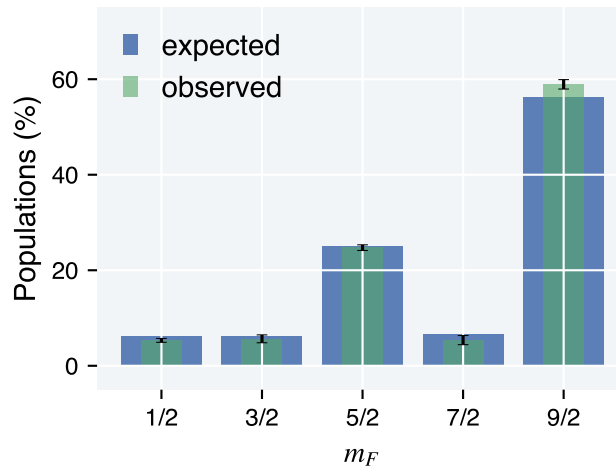


Figure 5.16: Observed populations for the bare states that correspond to the initial state $|D'_+\rangle = \frac{1}{\sqrt{2}}(|D_0\rangle + e^{i\pi/4}|D_r\rangle)$

5.11 offers a additional degrees of freedom as it is composed of more than two detunings. Using this feature, we aim to perform population transfer among the dark states to demonstrate existence of more than two ladder operators, raising and lowering operators, that are available in SU(2) spin systems. We do this by performing full transfer of population from the initial state to the final state in two ways:

- a direct transfer of population from initial state $|D'_+\rangle$ to the final state $|D'_-\rangle$ without populating the remaining state. This constitutes a direct π -pulse between these states.
- an indirect transfer of population from initial state $|D'_+\rangle$ to the final state $|D'_-\rangle$ by partially populating $|D_l\rangle$.

Before we describe these results, we show how we extract the state of the system in dark state basis from the bare state populations. For equal Rabi frequencies of tripod lasers, the state of the system in dark state basis can be completely determined by measuring the population of bare states. The dark states for equal Rabi frequencies read

$$\begin{aligned}
 |D_l\rangle &= \frac{1}{\sqrt{2}}(|1\rangle - |2\rangle) \\
 |D_0\rangle &= \frac{1}{2\sqrt{2}}(|1\rangle + |2\rangle - 2|3\rangle + |4\rangle + |5\rangle) \\
 |D_r\rangle &= \frac{1}{\sqrt{2}}(-|4\rangle + |5\rangle)
 \end{aligned}
 \tag{5.12}$$

$$\tag{5.13}$$

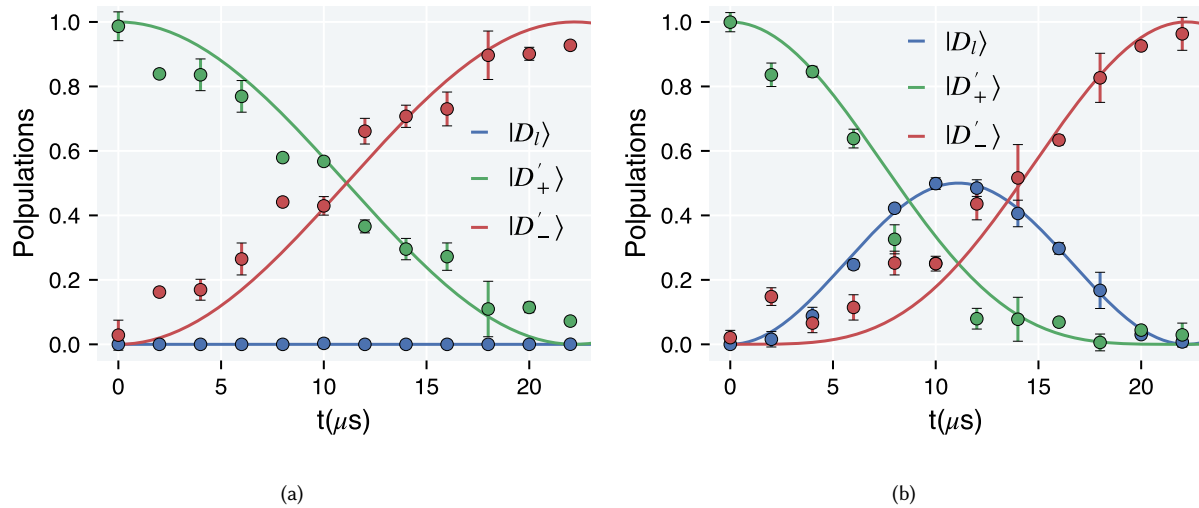


Figure 5.17: Population transfer from initial state to final state in two paths: (a) a direct π -pulse (b) an indirect π -pulse.

Using the fact that population of $|3\rangle$ is proportional to only $|D_0\rangle$, we can extract the state of the system in dark state basis. The state in dark state basis can be written as $|\psi\rangle_D = d_l e^{i\alpha} |D_l\rangle + d_0 |D_0\rangle + d_r e^{i\beta} |D_r\rangle$ satisfying $\sqrt{d_l^2 + d_0^2 + d_r^2} = 1$. The parameters of this state in terms of bare state populations reads

$$\begin{aligned}
 d_l &= \sqrt{p_1 + p_2 - p_3/2} \\
 d_0 &= \sqrt{2p_3} \\
 d_r &= \sqrt{p_4 + p_5 - p_3/2} \\
 \alpha &= \cos^{-1} \left(\frac{p_1 - p_2}{\sqrt{2p_3} \sqrt{p_1 + p_2 - p_3/2}} \right) \\
 \beta &= \cos^{-1} \left(\frac{p_5 - p_4}{\sqrt{2p_3} \sqrt{p_4 + p_5 - p_3/2}} \right)
 \end{aligned} \tag{5.14}$$

Once we know the state in this basis, we can perform change of basis to $\{|D_l\rangle, |D'_+\rangle, |D'_-\rangle\}$ using the same change of basis matrix used to transform the detuning matrix from ω to ω' .

We prepare the atoms in the initial state $|D'_+\rangle$ by abruptly turning on σ^+ beams and simultaneously ramping up π and σ^- beams. Figure 5.16 shows the bare state populations of this state. Figure 5.17a shows the evolution of population of dark states of $\{|D_l\rangle, |D'_+\rangle, |D'_-\rangle\}$ basis when transfer from initial state $|D'_+\rangle$ to $|D'_-\rangle$ performed. This result is obtained by setting $\delta_1^l = \delta_2^l = 0$ and $\delta_1^r = \delta_2^r = 2\pi \times 30$ kHz. In this case, only the lower 2×2 matrix coupling

$|D'_+\rangle$ and $|D'_-\rangle$ are non-zeros. This corresponds to the direct π -pulse. The indirect π -pulse is performed by setting $\delta_1^l = -\delta_2^l = 2\pi \times 45$ kHz and $\delta_1^r = \delta_2^r = 0$. Figure 5.17b shows the population of dark states during this indirect pulse. Hence, we demonstrate two possible paths of reaching the same final state from the initial state.

5.5 Conclusion

In conclusion, we have presented the results of dynamics of a wave packet in non-Abelian gauge fields. We first described scalar field cancellation using detuning matrix and showed results of *Zitterbewegung* dynamics in SU(2) gauge field when a momentum kick is imparted to the atoms using detuning matrix. We demonstrated the spin Hall characteristics of this dynamics similar to spin Hall effect in graphene. We described the dynamics in SU(3) gauge field with two initial state and show multiple frequencies in the dynamics as anticipated. Finally, we demonstrated the two pathways in which atoms in an initial state were transferred to a final state revealing SU(3) nature of our Hamiltonian.



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Chapter 6

Summary and Outlook

In summary, we have presented the experimental study of artificial non-Abelian gauge fields with ultracold atoms coupled to lasers in tripod and double-tripod configurations proposed in [1, 2]. We started with Aharonov-Bohm effect and equivalence of Aharonov-Bohm phase and Berry phase in adiabatic evolution. This equivalence in non-degenerate systems is extended to systems with degenerate states by F. Wilczek and A. Zee revealing the non-Abelian structure of the artificial gauge field [3]. We presented a theoretical overview on synthesis of SU(2) and SU(3) artificial non-Abelian gauge fields using atom-light interactions in tripod and double-tripod configurations. We presented the essential expressions and theoretical tools that are used to calculate the expectation values of quantities *i.e.*, velocity and populations, that are observed in the experiment.

We described the experimental system that is used to produce an ultracold sample of strontium atoms at sub-recoil temperature of ≈ 50 nK and methods used to characterize the atom-light coupling used in the synthesis of artificial gauge fields. We presented overview of vacuum chamber, laser systems used for trapping and cooling the atoms in a two stage magneto-optical trap using high rate $461 \text{ nm } ^1S_0 \rightarrow ^1P_1$ transition and $689 \text{ nm } ^1S_0 \rightarrow ^3P_1$ intercombination line. We described optical pumping that is required for state preparation in the dark states and evaporative cooling that is implemented to produce a Fermi-degenerate sample of $T/T_F = 0.26(2)$. We also provide an overview of various spectroscopy tools that are employed in determining the transition resonances. We described the fluorescence imaging system that is implemented to extract the populations of the ground state Zeeman sub-levels during the gauge fields sequence and characterized its performance. We described, in detail, the implementation of double-tripod configuration using sidebands produced by the EOMs and its characterization. We presented a direct measurement of Rabi frequencies of the tripod

transitions and cross examined with the expected values determined by optical powers and beam sizes. We also demonstrated the use of STIRAP technique to arbitrarily transfer populations among the ground state Zeeman sublevels that is essential for control of initial state preparations in the gauge field experiments. Finally, we perform a dressed state spectroscopy to verify that the atom-light coupling is well controlled by the characterized laser parameters.

We presented the results of atomtronic Datta-Das transistor, originally proposed in [4], that is implemented using one dimensional SU(2) gauge field. We demonstrated the control of spin rotation with respect to the gate parameter and characterized the performance of the DDT with respect to the geometrical properties of the tripod lasers in the gate region. One such metric is the sensitivity of the spin rotation with respect to the gate parameter as a function of the separation parameter. We observe a monotonic increase in the sensitivity. The key result is the robustness of the spin rotation to the input velocity of the atoms into the DDT. We demonstrated a built-in spin filter that separated the atoms in different spins into different momenta. Later, we performed a geometric Ramsey interferometry mimicking two DDTs in series to extract the phase of the superposition after the first DDT spin rotation. We perform a $\frac{\pi}{2}$ -pulse using first DDT and another $\frac{\pi}{2}$ -pulse with the second DDT with a phase knob between them controlled by an EOM in the path of σ^- laser. We add an additional phase between DDTs using the phase knob and show the Ramsey interference fringes. With this, we also reveal, experimentally, the effect of SU(2) geometric scalar potential. We showed that the contrast of the fringes as a function of initial superposition ratio controlled by the gate parameter.

We demonstrated the cancellation of geometric scalar potential that appears in the SU(2) system using detuning matrix and obtained a pure spin-orbit coupling Hamiltonian. When we added a momentum kick to the atoms in this Hamiltonian, we observed *Zitterbewegung* dynamics as proposed in theory works [5, 6]. This result is obtained by observing the oscillations in the velocity of the atomic wave packet from time of flight images. From these oscillations obtained for various angles of the momentum kick, direction of the acceleration due to the presence of non-Abelian gauge field is obtained. We showed that the acceleration is always perpendicular to the imparted momentum kick in line with the spin Hall effect [7]. Later, we showed the oscillations in the velocity of the wave packet in SU(3) gauge field. For this part of the experiment, we prepared two different initial states into which the system is initialized and evolution is recorded. These oscillations are composed of three frequencies that correspond to the eigen energies of the Hamiltonian. We demonstrated two possible

paths for transfer of population between two chosen states. First is a direct transfer without involving the third available state and second, an indirect transfer using the third state as an intermediate state. This demonstrated that our system consists of additional ladder operators than those exist for $SU(2)$ angular momentum implying that our system truly simulates $SU(3)$ symmetric group.

From this study, we realize our system can be used to study the rich physics of $SU(3)$ Hamiltonians on table top experiments. The $SU(3)$ detuning matrix does not restrict the resulting dispersion to a mere shift of the singularity in the momentum space like the $SU(2)$ detuning matrix. This offers an avenue of experimentation for realizing dispersion relations that have multiple singularity points in the momentum space. These kind of studies offer observation of potential topological features in the engineered Hamiltonians [8, 9]. Another approach to modify the dispersion relation is by periodic modulation of parameters of the atom-light coupling. This can be used to introduce additional Gell-Mann matrices in the Hamiltonian and control the system. In the dynamics experiments, we fix the same momentum kick and observe the evolution. To extract topological and geometric properties of the system, it is very useful to be able to perform a loop in the momentum space. This can be done by dynamically changing the detunings of the double-tripod scheme. This needs further upgrade to the electronics control system for the DDSs controlling the frequency shift of the tripod AOMs. Another apparent challenge that can be noted in the experiments is the effect of residual light shifts. A way of dealing with this is by accurately modeling the atom-light coupling and accounting for these light shifts.



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Appendix A

Laser System for the Zeeman Slower Beam

The laser beam for Zeeman slower is obtained by amplifying the 922 nm laser beam derived from the main Toptica DL-SHG Pro's auxiliary output. The power of auxiliary output is 2.6 mW and is fiber-coupled to the Zeeman slower beam's optical circuit. Figure [A.1](#) shows the optical circuit used to set the detuning of the Zeeman slower beam and amplify its power. The seeding beam with of power 1.7 mW passes through the two AOMs that shift the frequency by -135 MHz with respect to the main diode's frequency. This seeding beam injects a diode laser with an output power of 50 mW before the second Faraday isolator. A power of 40 mW injects the Eagleyard Tapered Amplifier running at 2.5 A and generates an output of approximately 1 W. After some beam shaping and resizing and a pair of Faraday isolators, a power of 700 mW enters the fiber to Toptica SHG Pro. Approximately 360 mW of 922 nm light enters the SHG cavity to generate 150 mW of 461 nm laser beam that is used as the Zeeman slower beam.



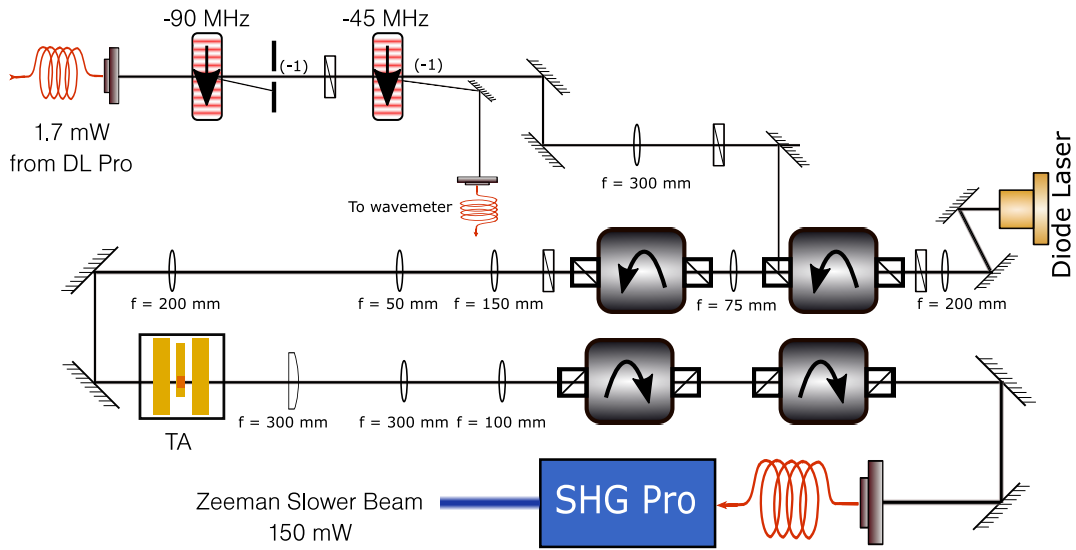


Figure A.1: Injection locked laser system amplified by tapered amplifier to produce a high power Zeeman slower beam.

Appendix B

Intensity Servo for Dipole Trap Beams

The crossed beam optical dipole trap is formed with two 1064 nm laser beams derived from high power (≈ 20 W) Coherent Mephisto MOPA laser. Two AOMs are used to switch on and off the dipole trap beams. The RF power applied to the AOM is also used to control the optical power in the first order diffraction. The first order is coupled into a fiber and sent to the science chamber. Due to thermal effects in AOM crystals and the optics, the optical power of the ODT beams at the output end of the fibre varies slowly. Especially, when the optical power is changed during the evaporation. The consistency of evaporation ramp and final power of the dipole trap beams at the end of evaporation is critical for a reproducible sample of atoms. To achieve this, we lock the power of lasers by using a PID servo. Figure B.1 shows the schematic of power lock of one of the ODT beams. After the beam passes through the science cell, it is split by a non-polarizing beam-splitter (NPBS) and 90% of the power is dumped into a beam-dump. The remaining 10% is further divided into two by another NPBS for two photo-detectors (PDA-36A). The signal from one of the photo-detectors, called in-loop photo-detector, is used by the PID controller (Koheron PI200) to generate the error signal. The set point is set using an analog voltage supply that can be controlled by the lab control program. The output of the PID controller is amplified 4 times and fed to the RF controller of the AOM. This is to avoid the sign flip in the PID controller that is needed if RF power is increased beyond the RF power at which maximum first order power is observed. The performance of the power lock is monitored by the monitor photo-detector. Figure B.2 shows relative intensity noise (RIN) indicating the performance of the power lock at low power (400 mW) and high power (4.3 W). The red and blue curves indicate RIN when the servo is on and off, respectively. The grey curves are the noise spectrum obtained from a thermal lamp maintaining the same voltage on the detectors that was observed with the laser. The lamp is driven by AC line and that explains the huge peak at 100 Hz. The performance is clearly limited by the noise of the

detectors in the low frequency region. The power lock doesn't reduce the noise in the lower power regime, however it reduces high frequency noise in the high power regime. This result indicate the low intensity noise of the dipole trap lasers. In any case, the servo-loop is used for controlling the ramp during evaporative cooling.

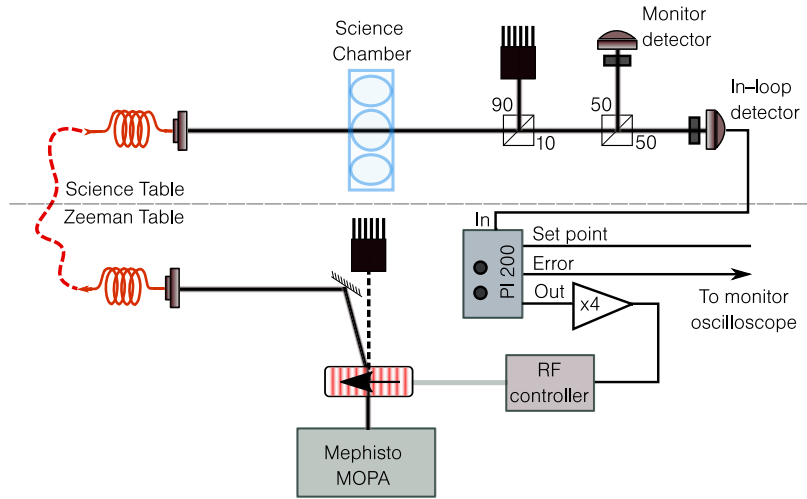


Figure B.1: Schematic showing implementation of intensity servo for dipole trap beams.



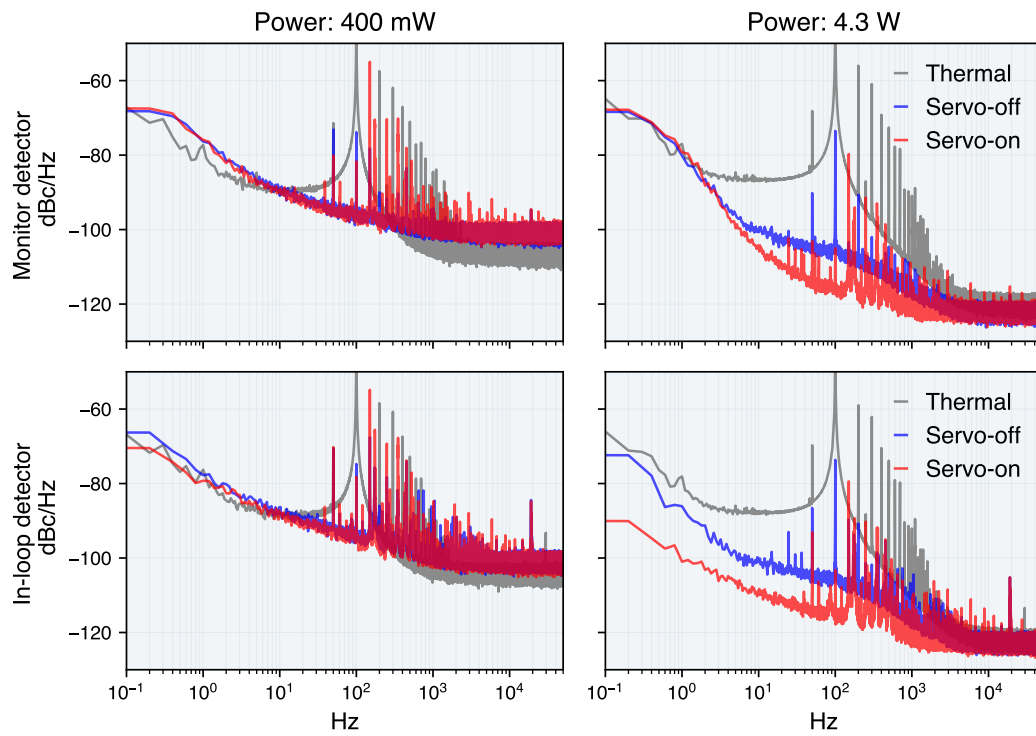


Figure B.2: Performance of power lock of the ODT beams. The performance is limited by the signal to noise ratio of the detector.

Appendix C

Zeeman Splitting of $^3P_1$ of ^{87}Sr

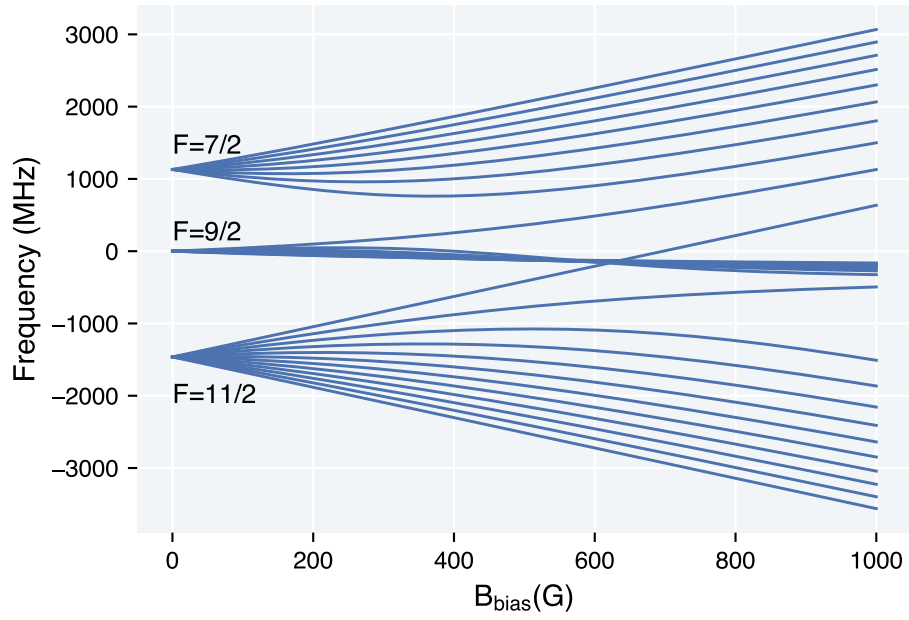
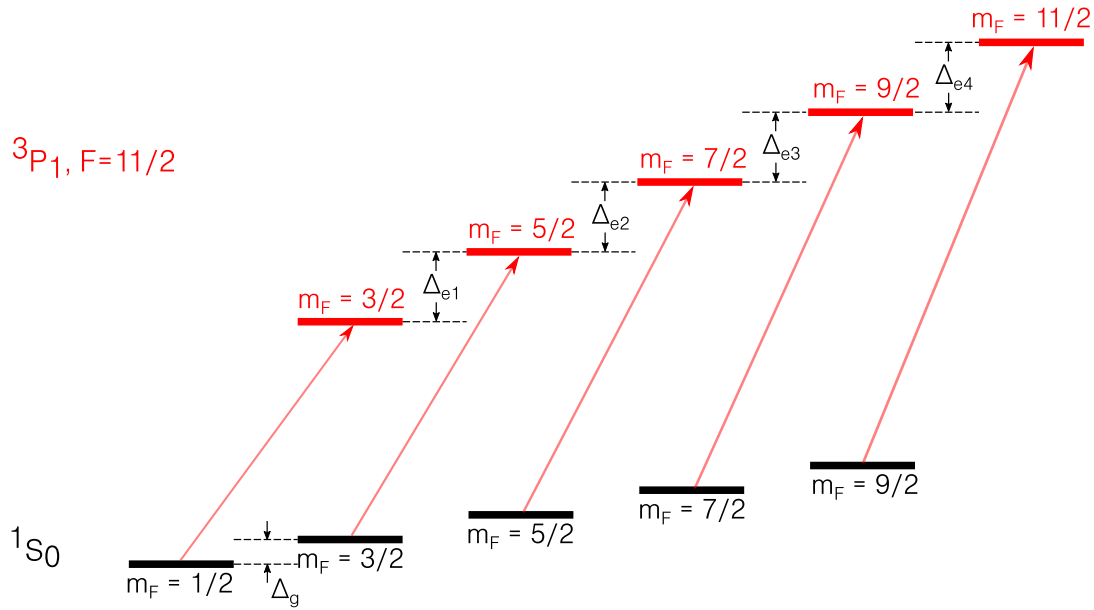
The hyperfine interaction splitting of the $^3P_1$ in ^{87}Sr is of the order of 1 GHz. At large magnetic fields, the linear Zeeman splitting is comparable to the hyperfine splitting and hence the linear approximation breaks down. To exactly calculate the splitting of different levels at large magnetic field intensities, the hyperfine interaction and the Zeeman splitting should be treated simultaneously. For this, the Hamiltonian including the electric quadrupole effect reads [1, 2],

$$H = A_{hfs} \mathbf{I} \cdot \mathbf{J} + B_{fs} \mathbf{L} \cdot \mathbf{S} + Q \frac{3\mathbf{I} \cdot \mathbf{J}(2\mathbf{I} \cdot \mathbf{J} + 1) - 2IJ(I+1)(J+1)}{4IJ(2I-1)(2J-1)} + (g_I I_z + g_L L_z + g_S S_z) \mu_B B_{bias} \quad (\text{C.1})$$

where $A_{hfs} = 260.084$ MHz, is the hyperfine splitting constant, $B_{fs} = 5.58 \times 10^6$ MHz is the fine structure splitting constant, $Q = -35.685$ MHz is the electric quadrupole splitting constant, $g_I = -1.32 \times 10^{-4}$, is the nuclear spin g-factor, $g_L = 1$ and $g_S = 2$. The electric quadrupole term is necessary for accurate calculation of the hyperfine splitting at $B_{bias} = 0$. This Hamiltonian is diagonalized and figure C.1 shows the frequencies of m_F levels of the $^3P_1$ with respect to $F = 9/2$ manifold at $B_{bias} = 0$ as a function of magnetic field. The separation between these levels is used to calibrate the bias magnetic field.

For this purpose, the Zeeman splitting of magnetic sublevels of $^3P_1 F = 11/2$ has been measured at $B_{bias} \approx 67$ G using absorption spectroscopy. Figure C.2 shows the transitions that have been measured. The frequencies of these resonances are tabulated in table C.1. From these frequencies, the frequency separation between the excited states (see figure C.2)

$$\Delta_{ei}^{exp} = f_{i+1} - f_i + \Delta_g \quad (\text{C.2})$$

Figure C.1: Breit-Rabi diagram of $^3P_1$ hyperfine levels.Figure C.2: Schematic showing the transitions of the intercombination line of ^{87}Sr used in calibrating the magnetic field.

for $i = 1$ to 4 are calculated. Here, $\Delta_g = 12.4$ kHz is the calculated ground state Zeeman splitting

Label	$^1S_0, F = 9/2, m_F$	$^3P_1, F = 11/2, m_F$	frequency (MHz)
f_1	1/2	3/2	32.650
f_2	3/2	5/2	59.025
f_3	5/2	7/2	85.851
f_4	7/2	9/2	113.14
f_5	9/2	11/2	140.912

Table C.1: Frequencies of resonances of transitions shown in figure C.2.

at $B_{bias} = 67$ G. Now, these separations are used to minimize the following cost function

$$C(B_{bias}) = \sum_{i=1}^4 (\Delta_{ei}^{exp} - \Delta_{ei}^{num}(B_{bias}))^2 \quad (C.3)$$

with B_{bias} as the free parameter. Here, Δ_{ei}^{num} is the numerically evaluated separation by diagonalizing the Hamiltonian in equation C.1. This optimization yields an optimum value of B_{bias} as 67.07 G at a minimum value of 1.95×10^{-5} for C. This corresponds to an RMS deviation ($\sqrt{C/4}$) of Δ_{ei}^{exp} of 2.2 kHz from the numerical values. At this magnetic field of 67.07 G, the Zeeman splittings of the excited states are numerically evaluated and the frequencies of the transitions shown in figure C.2 are tabulated in table C.2. The mean difference between

Label	f^{num} (MHz)	Δ_{ei}^{num} (MHz)	f^{exp} (MHz)	Δ_{ei}^{exp} (MHz)	$f^{num} - f^{exp}$ (MHz)
f_1	32.660	–	32.650	–	0.010
f_2	59.038	26.378	59.025	26.375	0.014
f_3	85.862	26.824	85.851	26.826	0.011
f_4	113.150	27.288	113.140	27.289	0.010
f_5	140.922	27.772	140.912	27.772	0.010

Table C.2: Frequencies of resonances of transitions shown in figure C.2.

numerically evaluated value and the experimental value (last column in C.2), 11 kHz is the offset. This offset exists due to imperfection and drift in the reference spectroscopy. In fact, an offset of 70 kHz was already compensated while determining f^{exp} .

With this bias field, the frequencies of relevant transitions to $F=9/2$ manifold are tabulated in table C.3. From these values we find the frequency of the EOMs for double tripod configuration as 14.121 MHz.



$^1S_0, F=9/2, m_F$	$^3P_1, F=9/2, m_F$	f^{num} (MHz)
1/2	3/2	5.305
3/2	5/2	11.986
5/2	7/2	19.426

Table C.3: Frequencies of resonances of transitions shown in figure C.2.

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Appendix D

Clebsch-Gordan Coefficients of $^1S_0 \rightarrow ^3P_1$ of ^{87}Sr

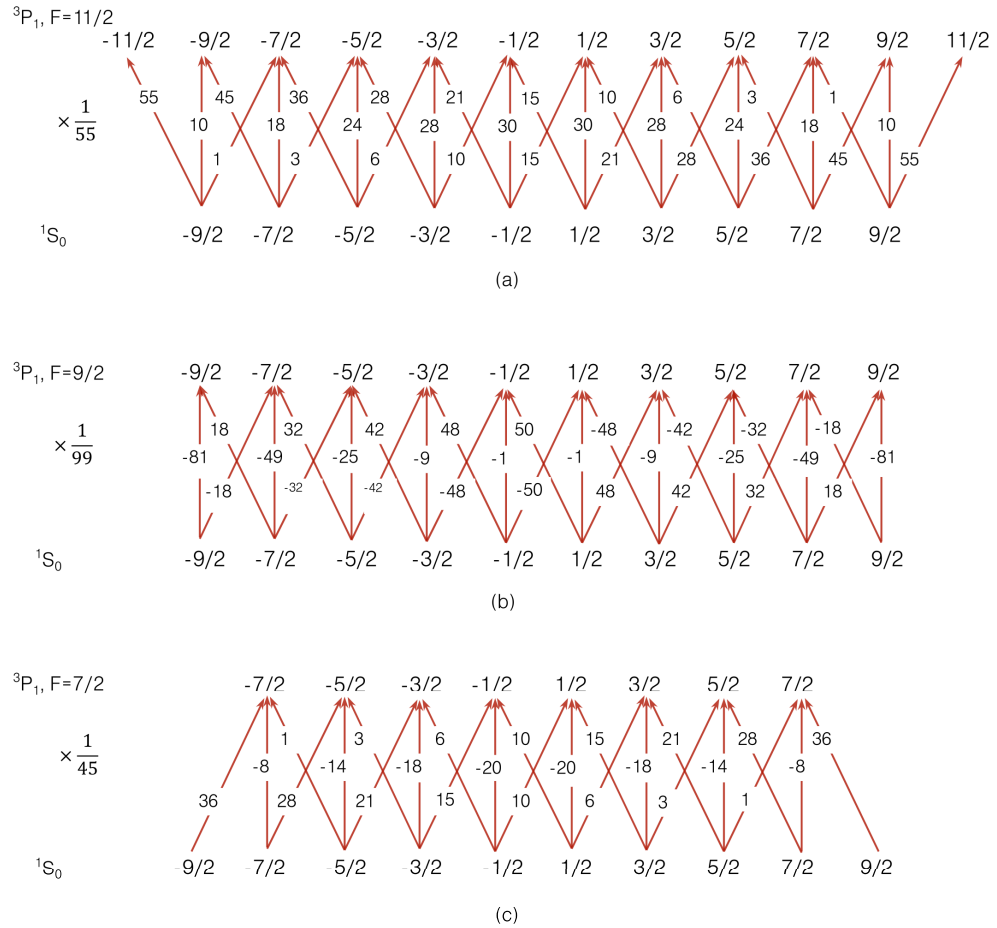


Figure D.1: Squared Clebsch-Gordan coefficients (retaining the sign) of various transitions of the intercombination line.



Appendix E

List of Publications

1. [Chetan Sriram Madasu](#), Chang Chi Kwong, David Wilkowski, Kanhaiya Pandey, "Homodyne detection of a two-photon resonance assisted by cooperative emission", Phys. Rev. A 105, 013713, (2022)
2. M. Hasan, [Ch.S. Madasu](#), K.D. Rathod, C.C. Kwong and D. Wilkowski, "Evolution of an ultracold gas in a non-Abelian gauge field: finite temperature effect", Quantum Electron. 52, 532, (2022)
3. [Chetan Sriram Madasu](#), Mehedi Hasan, Ketan Damji Rathod, Chang Chi Kwong, David Wilkowski, "Datta-Das transistor for atomtronic circuits using artificial gauge fields", Phys. Rev. Research 4, 033180, (2022)
4. Mehedi Hasan, [Chetan Sriram Madasu](#), Ketan D Rathod, Chang Chi Kwong, Christian Miniatura, Frederic Chevy, David Wilkowski, "Wave-packet Dynamics in Synthetic Non-Abelian Gauge Fields", Phys. Rev. Lett. 129, 130402, (2022)

