

Integer programming formulation for convoy movement problem

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Abstract: This paper reports a study of the strategic network problem of routing military convoys between specific origin and destination pairs. Known as the Convoy Movement Problem (CMP), this problem is formulated as an integer linear program. The proposed mathematical model is evaluated on the basis of average number of iterations and average CPU times. LP-based lower bounds and heuristic based upper bounds were generated and used for evaluating the proposed model, particularly for large problem instances for which optimal solutions could not be obtained.

Keywords: military convoys; routing; scheduling; integer program; heuristics.

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1 Introduction

The problem of routing and scheduling military convoys between specific origin and destination pairs is known as the Convoy Movement Problem (CMP). Problems analogous to CMP include routing of Automated Guided Vehicles (AGV) in FMS environment (Batta et al., 1993), movement of luggage from different flights along a common automated transportation system to various pickup points (Bovet et al., 1991), strategic level routing of hazardous materials (Iakovou et al., 1999) and scheduling of trains on a single track (Higgins et al., 1996).

A defence establishment would typically need to move large number of personnel and equipment from their home bases to regions of conflict, threat or crisis, as rapidly as possible. In the process of movement, each military unit must be deployed as a convoy and certain precedence relationships must be followed among different units. Armoured Fighting Vehicles (AFVs) such as tanks and armoured personnel carriers are transported using specially designed trucks called *transporters*. At the time of deployment, convoys may use multiple modes of transportation. Road, Rail, Waterways and Airways are possible modes of transport. Convoys continue to move till they reach their respective destinations without halting *en-route*. They are allowed to cross/pass each other at junctions. Crossing or passing on roads is called a *conflict* and is not permitted to occur. This is because the roads along which convoys travel may not have the load bearing ability and width to accommodate two convoys at a time. The same is equally applicable in rail transportation also, as most of the towns are connected by single line railway networks.

The CMP can be stated as follows: Given a set of military convoys, and the origin and destination pair associated with each convoy, determine the routes and schedules for all the convoys on a multi-modal transportation network so as to minimise the sum of arrival times of the convoys at their respective destinations such that no convoy stops *en-route* and no two convoys cross or overtake each other along the roads.

The remainder of the paper is organised as follows. Section 2 provides a brief literature review. Section 3 gives a detailed description of the proposed mathematical formulation followed by a description of generation of bounds in Section 4. The characteristics of the generated test problem instances are discussed in Section 5. Section 6 contains a discussion of the computational results while Section 7 presents the conclusions.

2 Literature review

The models that address the problem of convoy routing and scheduling are categorised as military mobility models.

Bovet et al. (1991) proposed a convoy scheduling problem for scheduling the movements of a collection of convoys along the same road to reach different destinations. The movements were subjected to two key constraints: each convoy has a pre-specified time-

window for its departure and convoys may not pass or cross each other along the road. Two different formulations based on integer programming and on graphs were proposed. Montana et al. (1999) investigated the problem of scheduling the move of a large amount of military equipment from a fort or a depot to a port and used genetic algorithms to solve the problem.

Baker et al. (2002) described a large scale linear programming model NRMO (NPS/Rand Mobility Optimiser) for optimising military airlift. The model routes cargo and passengers through a specified transportation network with a given fleet of aircraft subject to many physical and policy constraints. The model captures important aspects such as aerial refuelling, tactical aircraft shuttles and constraints based on crew availability. McKinzie and Barnes (2003) reviewed a number of mobility models and observed that every model uses either cumbersome ineffective classical optimisation techniques or simplistic and ineffective greedy approaches to find solutions.

Chardaire et al. (2005) gave a formal specification for the convoy movement problem and established that the corresponding feasibility problem is *NP*-Complete. They developed an integer programming model based on a time-space network for a simplified version of CMP and applied Lagrangean Relaxation to solve the model. They acknowledged the existence of constraints against crossing and overtaking, but simplified their model by dropping them. The authors described the use of Lagrangean Relaxation to solve the model and their results were encouraging. Tuson and Harrison (2005) demonstrated that a straight forward reformulation of the model proposed by Chardaire et al. (2005) rendered the realistic instances amenable to solution by simple heuristics. The authors stated that the *NP*-hardness is a worst case measure of the problem's time complexity and real world problems may not necessarily be hard. Akgun and Tansel (2007) introduced the Deployment Planning Problem (DPP) that may be roughly defined as the problem of planning the physical movement of military units, stationed at geographically dispersed locations, from their home bases to their destinations, subject to the constraints on scheduling and routing and on various types of transportation assets. The authors proposed a mixed integer program model to solve the DPP. Gopalan and Narayanaswamy (2008) considered an online version of the CMP where convoy demands arise dynamically over time. Assuming convoys to be of zero length, i.e., particle convoys, they proposed approximation algorithms for solving the problem.

Literature is yet to provide a mathematical model that incorporates all the characteristics associated with the CMP. Some aspects of the problem have been addressed in the literature on freight transportation (Crainic and Laporte 1997, Crainic 2003), on vehicle routing and scheduling (Toth and Vigo, 2002) and on rail transportation (Cordeau et al., 1998; Higgins et al., 1996). In this paper, we develop an integer programming mathematical model for the convoy movement problem. It is an adaptation of the model proposed by Higgins et al. (1996) for scheduling trains on single track. We have also drawn ideas from Fisher (1976, 1981), Fisher and Kedia (1990), Erlenkotter (1978), Guignard (1988), Held and Karp (1970, 1971), Shapiro (1971), Beasley (1985, 1990), Schank et al. (1991) and Kress (2001) while developing the formulation.

3 Integer programming formulation

3.1 Assumptions

The following is the list of assumptions made:

- the underlying network is a bi-modal network of highways and railways consisting of nodes that represent cities and bi-directional arcs connecting them
- all convoys have equal priority
- all the nodes have sufficient infrastructure for accommodating more than one convoy at a time
- transporters are always available
- all convoys are ready at their respective home bases at time 0.

3.2 Sets and indices

Q : Set of all the nodes

A : Set of arcs

M : Set of transportation modes

S : Set of source nodes

D : Set of destination nodes

C : Set of military convoys need to be moved

K : Very large integer.

3.3 Input parameters

q_s^c : Node of convoy c , $c \in C$, $q_s^c \in S$

q_d^c : Destination node of convoy c , $c \in C$, $q_d^c \in D$

$T_{qq'}$: Number of time units required to traverse between nodes q and q' , $(q, q') \in Q$

h : Minimum headway distance to be maintained (in time units).

3.4 Variables

AT_q^c : Arrival time of convoy c at node q

DT_q^c : Departure time of convoy c at node q

$$A_{qq'}^{mc} = \begin{cases} 1 & \text{If convoy } c \text{ traverses between nodes } q \text{ and } q' \text{ by mode } m \\ 0 & \text{otherwise.} \end{cases}$$

$$X_{qq'}^{mcc'} = \begin{cases} 1 & \text{If convoy } c \text{ traverses between nodes } q \text{ and } q' \text{ before convoy } c' \text{ by mode } m \\ 0 & \begin{cases} \text{If convoy } c' \text{ traverses between nodes } q \text{ and } q' \\ \text{before convoy } c \text{ by mode } m \\ \text{(or)} \\ \text{If arc } qq' \text{ is not on the path of either convoy } c \text{ or } c' \text{ or both} \end{cases} \end{cases}$$

$$Y_{qq'}^{mcc'} = \begin{cases} 1 & \begin{cases} \text{If convoy } c \text{ traverses between nodes } q \text{ and } q' \\ \text{before-convoy } c' \text{ traverses between nodes } q' \text{ and } q \text{ by mode } m \end{cases} \\ 0 & \begin{cases} \text{If convoy } c' \text{ traverses between nodes } q' \text{ and } q \\ \text{before-convoy } c \text{ traverses between } q \text{ and } q' \text{ by mode } m \\ \text{(or)} \\ \text{If arc } qq' \text{ is not on the path of convoy } c \text{ or } c' \text{ or both.} \end{cases} \end{cases}$$

Among the six decision variables, AT_q^c and DT_q^c are integer variables and the rest are binary variables. The mathematical formulation follows.

3.5 Objective function

The objective is to minimise the sum of arrival times of convoys at their respective destinations.

$$\text{Minimise } \sum_c AT_{q_d^c}^c. \quad (1)$$

3.6 Constraints

1 Network flow constraints

This constraint ensures each convoy starts moving from its home base and reaches its destination without terminating its journey at any of the intermediary nodes.

$$\sum_{q'} \sum_m A_{qq'}^{mc} - \sum_{q''} \sum_m A_{q''q}^{mc} = \begin{cases} 1 & \text{if } q = q_s^c \\ 0 & \text{if } q \neq q_s^c \text{ or } q_d^c \\ -1 & \text{if } q = q_d^c \end{cases} \quad \forall c \in C, m \in M, (q, q', q'') \in Q, q \neq q' \text{ and } q \neq q''. \quad (2)$$

2 Travel time constraints

This set of constraints ensures that each convoy takes the stipulated time to travel between any pair of nodes. In addition, it ensures that the arrival time of a convoy at its destination is never less than the sum of individual arc travel times.

$$\left. \begin{aligned} AT_{q'}^c + K(1 - A_{qq'}^{mc}) &\geq T_{qq'}^m A_{qq'}^{mc} + DT_q^c \\ AT_{q'}^c - K(1 - A_{qq'}^{mc}) &\leq T_{qq'}^m A_{qq'}^{mc} + DT_q^c \\ AT_{q_a}^c &\geq \sum_q \sum_{q'} T_{qq'}^m A_{qq'}^{mc} \end{aligned} \right\} \forall c \in C, m \in M, (q, q') \in Q \text{ and } q \neq q'. \quad (3)$$

3 No-overtaking constraints

This set of constraints ensures that when two convoys travel along the same arc in the same direction, a minimum headway time is always maintained between them.

$$\left. \begin{aligned} KX_{qq'}^{mcc'} + K(1 - A_{qq'}^{mc}) + K(1 - A_{qq'}^{mc'}) + AT_{q'}^c &\geq AT_{q'}^{c'} + h(1 - X_{qq'}^{mcc'}) \\ K(1 - X_{qq'}^{mcc'}) + K(1 - A_{qq'}^{mc}) + K(1 - A_{qq'}^{mc'}) + AT_{q'}^{c'} &\geq AT_{q'}^c + hX_{qq'}^{mcc'} \end{aligned} \right\} \quad (4)$$

$(c, c') \in C, m \in M,$
 $\forall (q, q') \in Q, c \neq c',$
 $q \neq q' \text{ and } c' > c.$

4 No-crossing constraints

This set of constraints ensures no two convoys cross each other on an arc while travelling along the same arc in opposite direction.

$$\left. \begin{aligned} KY_{qq'}^{mcc'} + K(1 - A_{qq'}^{mc}) + K(1 - A_{q'q}^{mc'}) + DT_q^c &\geq AT_{q'}^{c'} \\ K(1 - Y_{qq'}^{mcc'}) + K(1 - A_{qq'}^{mc}) + K(1 - A_{q'q}^{mc'}) + DT_{q'}^{c'} &\geq AT_q^c \end{aligned} \right\} \quad (5)$$

$\forall (c, c') \in C, m \in M, (q, q') \in Q$
 $c \neq c', q \neq q' \text{ and } c' > c.$

5 No-stoppage constraint

This constraint ensures that once a convoy starts from its home base, it does not stop *en-route* till it reaches its destination.

$$DT_q^c - AT_q^c = 0, \quad \forall c \in C, q \notin S \text{ and } q \notin D. \quad (6)$$

6 Binding constraints

(a) For convoys traversing along an arc in the same direction

This set of constraints ensures that a minimum headway time is maintained between two convoys if and only if an arc under consideration is common to both of them.

$$\begin{aligned} X_{qq'}^{mcc'} &\leq A_{qq'}^{mc} \quad \forall (c, c') \in C, m \in M, (q, q') \in Q, c \neq c', q \neq q' \text{ and } c' > c \\ X_{qq'}^{mcc'} &\leq A_{qq'}^{m'c'} \quad \forall (c, c') \in C, m \in M, (q, q') \in Q, c \neq c', q \neq q' \text{ and } c' > c. \end{aligned} \quad (7)$$

(b) For convoys traversing an arc in opposite directions

In order to decide which convoy traverses a particular arc first, the following set of constraints ensures that the arc is part of both the convoys' routes.

$$\begin{aligned} Y_{qq'}^{mcc'} &\leq A_{qq'}^{mc} \quad \forall (c, c') \in C, m \in M, (q, q') \in Q, c \neq c', q \neq q' \text{ and } c' > c \\ Y_{qq'}^{mcc'} &\leq A_{q'q}^{m'c'} \quad \forall (c, c') \in C, m \in M, (q, q') \in Q, c \neq c', q \neq q' \text{ and } c' > c. \end{aligned} \quad (8)$$

(7) The following set of constraints ensures that the convoys are ready at their respective home bases at time 0 and convoys do not move after reaching their destination.

$$\begin{aligned} AT_{q_s^c}^c &= 0, \quad \forall c \in C \text{ and } q_s^c \in S \\ DT_{q_d^c}^c &= K, \quad \forall c \in C \text{ and } q_d^c \in D. \end{aligned} \quad 7$$

4 Lower and upper bounds

The Convoy Movement Problem is a proven *NP*-Hard problem. Hence, optimal solutions can be obtained for problem instances upto a certain size beyond which it may not be computationally viable. For such instances, the performance of the heuristic is evaluated with reference to the lower bound.

4.1 Lower bound

A well-known method of generating a lower bound is Linear Programming (LP) relaxation in which the integrality requirement is dropped and the resulting LP is solved exactly using a standard algorithm.

4.2 Upper bound

For the objective of minimisation, upper bounds are generated using heuristic techniques. In the present problem, conflicts arise between convoys only at two specific instances: The conflicts are resolved on a first-come-first-serve basis.

Situation 1: Two convoys attempt to traverse a bi-modal arc in opposite directions at the same time along same mode of transportation.

Resolving Mechanism: In Figure 1, let EE' be a bi-modal (modes $m1$ and $m2$) conflicting edge. Suppose convoy $C1$ reaches node E before $C2$ reaches node E' : Convoy 'C1' is allowed to traverse arc EE' using the faster mode of transport and direct convoy 'C2' to traverse $E'E$ by the slower mode of transport.

Situation 2: Two convoys competing to traverse along a uni-modal arc in opposite directions at the same time using same mode of transportation.

Resolving Mechanism: In Figure 2, let EE' be a uni-modal conflicting edge. Suppose convoy $C2$ reaches node E' before convoy $C1$ reaches node E : Convoy $C2$ is allowed to traverse arc $E'E$ and convoy $C1$ is forbidden from traversing along arc EE' ; accordingly, its traverse time on that arc is set to ∞ . An alternative route is found between the same nodes using Dijkstra's algorithm.

4.2.1 Step-by-step procedure

- 1 Solve the problem using Dijkstra's shortest path algorithm.
- 2 Examine the obtained solution for conflicts. If there is no conflict, terminate the heuristic by reporting this solution as final. Else, proceed to step 3.
- 3 Resolve all the conflicts that arise using the proposed resolving mechanisms and then record the final solution.

5 Test problem instances

Since information pertaining to defence planning and logistics is classified, hypothetical problem instances were generated to evaluate the efficacy of the proposed formulation. The problems could be classified as instances with and without known optimal solutions. A total of 12 networks with problem sizes ranging from ten cities and three convoys to 100 cities and 30 convoys were generated. An *Arc Density Factor* of 0.15 and Node-Convoy ratio of [3.5 – 5] was assumed for all the networks. These are computed as shown below:

$$\text{Arc Density Factor } (\rho) = \frac{\text{Degree of a node}}{\text{Total number of nodes}}$$

$$\text{Node Convoy Ratio} = \frac{\text{Number of Nodes}}{\text{Number of Convoys}}.$$

To evaluate the robustness of the formulation, three different scenarios were introduced for each network, using a factor called the '*Identical Destination Factor*', defined as follows:

$$\text{Identical Destination Factor } (\theta) = \frac{\text{Number of identical destination nodes}}{\text{Total number of convoys}}.$$

An identical destination factor '0' indicates that no more than one convoy heads towards any destination; $\theta = 1$ indicates that all the convoys head towards a common destination.

6 Results and discussions

The models were coded in C++ language and solved using ILOG CPLEX 9.0 Concert Technology. The computational times reported are in CPU seconds on a Zenith PC working on Intel Pentium 4, 3 GHz processor with 1 GB of RAM. All the results obtained are summarised in Tables 1–3. Tables 1 and 2 pertain to problem instances for which CPLEX could give optimal solutions. Table 3 pertains to problem instances without optimal solutions.

Table 1 reports the Best case performance, Worst case performance and Average case performance of the formulation in terms of average CPU time and average number of iterations upto a problem size of 50 cities and ten convoys. There does not appear to be a significant difference between best case and worst case performances for smaller problems. As the problem size increases, the difference in performance grows exponentially. Figure 3 depicts a graph plotted between Network size and Average CPU time for $\theta = 0.5$.

In Table 2, the quality of the LP bounds and upper bounds are reported with respect to the optimal solutions obtained. In Table 3, for problem instances without optimal solutions, we have reported the performance of the proposed heuristic evaluated with respect to the lower bound for four large networks with 15 instances each.

7 Conclusions

The strategic problem of routing military convoys between specific origin and destination pairs within a network has been considered in this study. Formulated as an integer linear program, it is found amenable to optimisation for problem-sizes up to 50 cities and ten convoys, in terms of the amount of computational time required. Since it is not possible to find optimal solutions to larger problems, the proposed heuristic is evaluated against the lower bound for such instances. Viewed against the encouraging results obtained in this paper, the proposed model marks a significant improvement over the existing models in the literature.

References

- Akgun, I. and Tansel, B.C. (2007) 'Optimization of transportation requirements in the deployment of military units', *Computers and Operations Research*, Vol. 34, No. 4, pp.1158–1176.
- Baker, S.F., Morton, D.P., Rosenthal, R.E. and Williams, L.M. (2002) 'Optimizing military airlift', *Operations Research*, Vol. 50, No. 4, pp.582–602.
- Batta, R., Krishnamurthy, N. and Karwan, M.H. (1993) 'Developing conflict-free routes for automated guided vehicles', *Operations Research*, Vol. 41, No. 6, pp.1077–1090.
- Beasley, J.E. (1985) 'A note on solving large p-median problems', *European Journal of Operational Research*, Vol. 21, pp.270–273.
- Beasley, J.E. (1990) 'A Lagrangean heuristic for set covering problems', *Naval Research Logistics*, Vol. 37, pp.151–164.
- Bovet, J., Constantin, C. and de Werra, D. (1991) 'A convoy scheduling problem', *Discrete Applied Mathematics*, Vol. 30, No. 1, pp.1–14.
- Chardaire, P., McKeown, G.P., Harrison, S.A. and Richardson, S.B. (2005) 'Solving a Time-space network formulation for the Convoy Movement Problem', *Operations Research*, Vol. 53, No. 2, pp.219–230.
- Cordeau, J.F., Toth, T. and Vigo, D. (1998) 'A survey of optimization models for train routing and scheduling', *Transportation Science*, Vol. 32, No. 4, pp.380–404.
- Crainic, T. (2003) 'A survey of optimization models for long-haul freight transportation', in Hall, R.W. (Ed.): *Handbook of Transportation Science*, 2nd ed., pp.415–516.
- Crainic, T. and Laporte, G. (1997) *Planning Models for Freight Transportation in Design and Operation of Civil and Environmental Engineering Systems*, Interscience, Wiley, New York, pp.343–394.
- Erlenkotter, D. (1978) 'A dual-based procedure for uncapacitated facility location', *Operations Research*, Vol. 26, pp.992–1009.
- Fisher, M.L. (1976) 'A dual algorithm for the one-machine scheduling problem', *Math Programming*, Vol. 11, pp.229–251.
- Fisher, M.L. (1981) 'The Lagrangean relaxation method for solving integer programming problems', *Management Science*, Vol. 27, No. 1, pp.1–18.
- Fisher, M.L. and Kedia, P. (1990) 'Optimal solution of set covering/partitioning problems using dual heuristics', *Management Science*, Vol. 36, No. 6, pp.674–688.

Gopalan, R. and Narayanaswamy, N.S. (2008) 'Analysis of algorithms for an online version of the convoy movement problem', Accepted for publication in the *Journal of the Operational Research Society*.

Guignard, M.A. (1988) 'Lagrangian dual ascent algorithm for simple plant location problems', *European Journal of Operational Research*, Vol. 35, pp.193–200.

Held, M. and Karp, R.M. (1970) 'The travelling salesman problem and minimum spanning trees', *Operations Research*, Vol. 18, pp.1138–1162.

Held, M. and Karp, R.M. (1971) 'The travelling salesman problem and minimum spanning trees: Part II', *Math Programming*, Vol. 1, pp.6–25.

Higgins, A., Kozan, E. and Ferriera, L. (1996) 'Optimal scheduling of trains on a single line track', *Transportation Research*, Vol. 30, No. 2, Part-B, pp.147–161.

Iakovou, E., Douligieris, C., Li, H. and Yudhbir, L. (1999) 'A maritime global route model for hazardous materials', *Transportation Science*, Vol. 33, No. 1, pp.34–48.

Kress, M. (2001) 'Efficient strategies for transporting mobile forces', *Journal of the Operational Research Society*, Vol. 52, No. 3, pp.310–317.

McKinzie, K. and Barnes, J.W. (2003) 'A review of strategic mobility models supporting the defense transportation system', *Mathematical and Computer Modeling*, Vol. 39, pp.839–868.

Montana, D., Bidwell, G., Vidaver, G. and Herrero, J. (1999) 'Scheduling and route selection for military land moves using genetic algorithms', *IEEE proceedings of the 1999 congress on Evolutionary Computation*, Vol. 2, pp.1123–1128.

Schank, J., Mattock, M., Sumner, G., Greenberg, I., Rothenberg, J. and Stucker, J.P. (1991) *A Review of Strategic Mobility Models and Analysis*, Report R-3926-JS, RAND Corporation, Santa Monica, CA.

Shapiro, J.E. (1971) 'Generalized Lagrange multipliers in integer programming', *Operations Research*, Vol. 19, pp.68–76.

Toth, T. and Vigo, D. (2002) *The Vehicle Routing Problem*, Philadelphia, SIAM.

Tuson, A.L. and Harrison, S.A. (2005) 'Problem difficulty of real instances of convoy planning', *Journal of the Operational Research Society*, Vol. 56, pp.763–775.

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Figure 3	Network size vs. average CPU time for $\theta = 0.5$ (see online version for colours)

<i>Network</i>	<i>No. of nodes</i>	<i>No. of convoys</i>	<i>Identical destination factor (θ)</i>	<i>No. of problems solved</i>	<i>Best case performance</i>		<i>Worst case performance</i>		<i>Average performance</i>	
					<i>CPU time</i>	<i>No. of iterations</i>	<i>CPU time</i>	<i>No. of iterations</i>	<i>CPU time</i>	<i>No. of iterations</i>
N1	10	3	0.0	10	0.09	8	0.57	67	0.39	41
			0.5	10	0.17	9	0.70	162	0.36	44
			1.0	10	0.20	15	0.60	94	0.41	42
N2	15	4	0.0	10	0.38	35	1.66	159	0.93	73
			0.5	10	0.37	34	2.25	277	1.21	124
			1.0	10	0.58	45	2.54	346	1.30	148
N3	20	5	0.0	10	0.65	85	4.68	411	2.07	191
			0.5	10	0.82	73	8.05	480	3.54	207
			1.0	10	0.88	86	7.99	498	3.43	229
N4	25	6	0.0	10	1.74	114	33.92	944	10.04	389
			0.5	10	2.36	130	26.25	703	11.17	425
			1.0	10	1.64	106	42.39	1769	22.10	506
N5	30	7	0.0	10	4.85	193	70.80	1045	23.91	428
			0.5	10	13.32	314	60.94	716	27.98	461
			1.0	10	17.88	339	121.07	2251	58.06	989
N6	35	8	0.0	10	6.60	229	284.5	2368	33.03	459
			0.5	10	6.47	226	308.5	2579	63.34	716
			1.0	10	8.66	206	274.55	3308	86.23	818
N7	40	9	0.0	10	29.60	447	255.52	1399	121.80	817
			0.5	10	37.99	465	536.06	6986	143.27	1427
			1.0	10	42.46	413	824.18	7143	228.75	1840
N8	50	10	0.0	10	29.33	575	297.93	2835	172.02	1734
			0.5	10	42.21	624	723.15	5771	354.43	2339
			1.0	10	44.34	592	1204.61	8982	540.11	3780

Table 1

<i>Problem size</i>	<i>No. of problem instances solved</i>	<i>Average quality of LP bound*</i>	<i>Average quality of Heuristic solution[#]</i>
10 × 3	30	0.51	1.39
15 × 4	30	0.92	2.04
20 × 5	30	1.70	2.83
25 × 6	30	2.23	3.94
30 × 7	30	3.41	5.88
35 × 8	30	3.72	5.29
40 × 9	30	5.29	6.50
50 × 10	30	5.84	8.05

*Quality of LP bound = ((Optimal solution – LP bound)/Optimal solution) × 100%.

[#]Quality of Heu. Solution = (Heu.soln – Optimal solution)/Optimal solution × 100%.

Table 2

<i>Problem size</i>	<i>No. of problem instances solved</i>	<i>Performance of the proposed heuristic</i>
60 × 15	15	13.38
75 × 20	15	12.66
85 × 25	15	13.51
100 × 30	15	15.68

*% gap = ((Heuristic solution – LP bound)/LP bound) × 100%.

Table 3

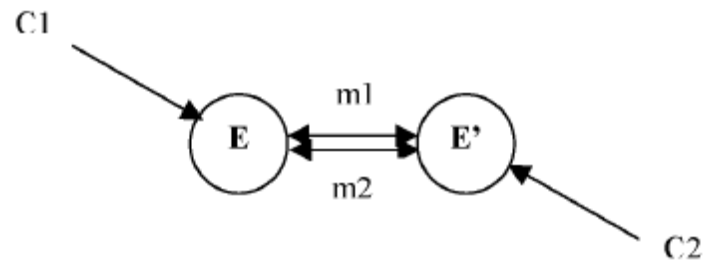


Figure 1

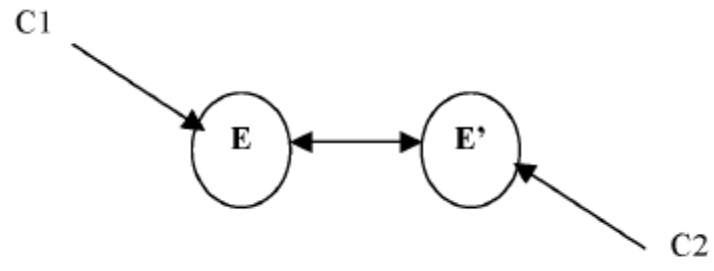


Figure 2

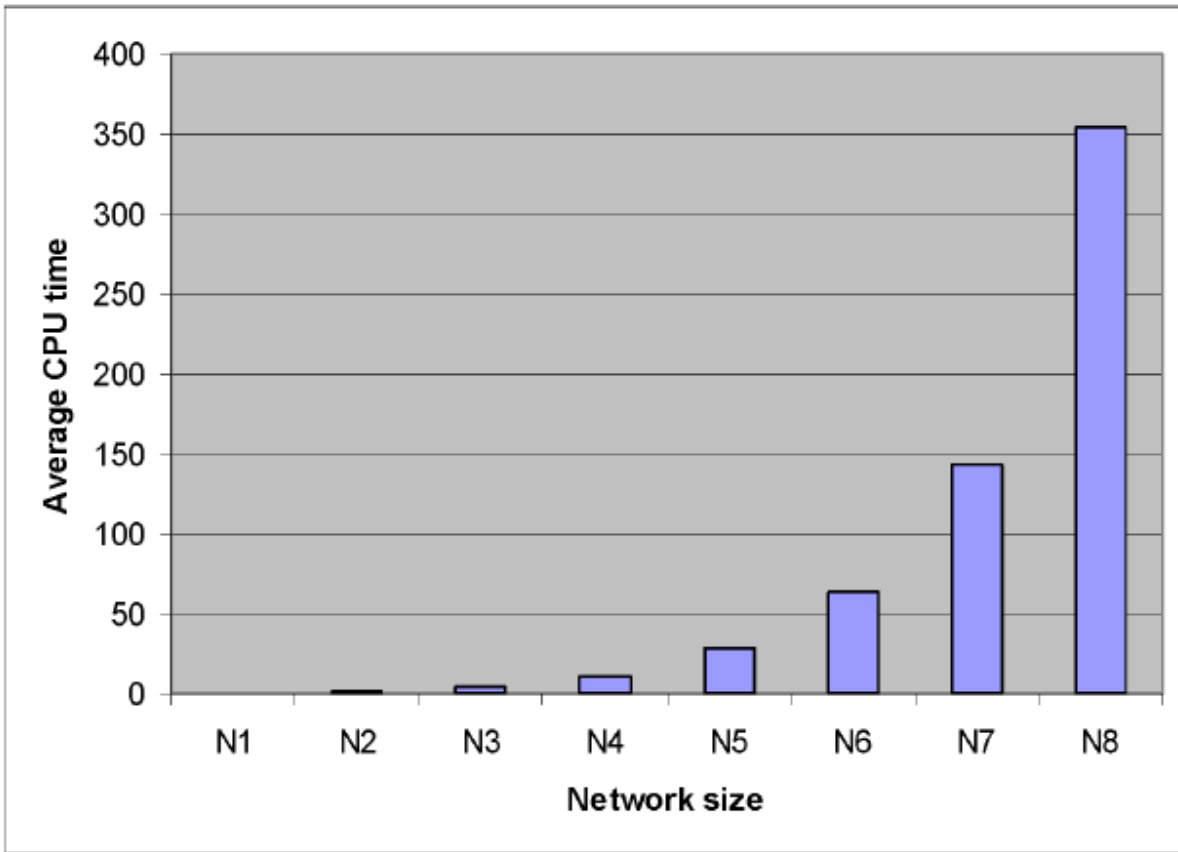


Figure 3