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New observational method for prediction of one-dimensional consolidation settlement

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The Chapman–Richards model is adopted in this paper to best fit the Terzaghi's one-dimensional consolidation curve. The obtained formula fits the theoretical solution well, with a regression coefficient R^2 of 0·9995 and an error of less than 2%. By adopting this model, a new method to predict the ultimate settlement and the coefficient of consolidation using monitoring settlement data is proposed. The accuracy of the proposed method is verified against oedometer test and field monitoring data. For the cases verified, the ultimate settlement predictions from the proposed method are more reliable than those from the Asaoka's method for the different range of settlement data.

KEYWORDS: consolidation; ground improvement; monitoring

INTRODUCTION

Average degree of consolidation has been commonly used as a design specification for soil improvement using preloading (Hansbo, 1981; Holtz, 1987). Normally the average degree of consolidation is determined as the ratio of settlement at a given time over ultimate consolidation settlement. The settlement at a given time is measured and the ultimate consolidation settlement is predicted. As the soil properties are highly variable in the field, it is difficult to predict reliably the ultimate consolidation settlement using the e–log σ_v' as determined from one-dimensional oedometer tests (Holtz, 1987; Chung, 1999; Arulrajah *et al.*, 2003, 2004; Bo *et al.*, 2003; Chung *et al.*, 2014). Thus, in practice, the observational method is usually adopted to use the field monitoring data to predict the ultimate settlement. Two such methods are Asaoka (1978) and hyperbolic methods (Tan *et al.*, 1991; Tan, 1995; Tan & Chew, 1996; Chung *et al.*, 2009, 2014). Sometimes, the overall coefficient of consolidation is also determined from these methods.

The Asaoka's method was derived from Terzaghi's one-dimensional theory (Terzaghi *et al.*, 1996) in which the settlement plotted against time, δ–t, relationship can be written as

$$\delta = \delta_{\text{ult}}\left[1 - \exp\left(-\frac{12}{5}\frac{c_v}{H^2}t\right)\right] \quad (1)$$

where δ and δ_{ult} are the ground settlement at a given time t and the ultimate ground settlement, respectively; c_v is the coefficient of vertical consolidation; and H is the drainage path.

From equation (1), the degree of consolidation in the vertical direction U_v as calculated as $\delta/\delta_{\text{ult}}$ can be given as

$$U_v = 1 - \exp\left(-\frac{12}{5}T_v\right) \quad (2)$$

where $T_v = c_v t/H^2$ is the time factor.

When a series of settlements at different time intervals δ_1, $\delta_2,\ldots, \delta_n$ are selected from the monitoring data in such a way that δ_n is the settlement at time t_n and the sampling interval $\Delta t = t_n - t_{n-1}$ is a constant, the relationships between settlement δ_n and δ_{n-1} can be expressed as

$$\delta_{n+1} = \beta_0 + \beta_1\delta_n \quad (3)$$

where β_0 is the intercept and β_1 is the slope of the straight line in the δ_n against δ_{n-1} plot.

At the end of the primary consolidation, $\delta_n = \delta_{n-1} = \delta_{\text{ult}}$ and thus δ_{ult} can be calculated as

$$\delta_{\text{ult}} = \frac{\beta_0}{1 - \beta_1} \quad (4)$$

The coefficient of consolidation, c_v, can be estimated using (Balasubramaniam & Brenner, 1981)

$$c_v = -\frac{5}{12}\frac{H^2 \ln\beta_1}{\Delta t} \quad (5)$$

Equations (1)–(5) form the basis of Asaoka's method, which has often been used to predict ultimate settlement and coefficient of consolidation. However, this method underestimates the ultimate settlement and overestimates the coefficient of consolidation depending on the chosen sampling period (Asaoka, 1978; Edil *et al.*, 1991; Arulrajah *et al.*, 2003). This can be seen from the comparison in Fig. 1(b) where the difference between equation (1) and the Terzaghi's consolidation curve is relatively large and only becomes less than 6% when $U_v > 80\%$ or $T_v > 0{\cdot}55$. It should be stated that the curves shown in Fig. 1 are only for the case with uniform loading pressure and one-dimensional consolidation with either one-way or two-way drainage.

As there is no closed form equation, the U_v plotted against T_v relationship in Terzaghi's one-dimensional consolidation theory has been written into two simplified equations depending on whether U_v is greater than 0·53 (Taylor, 1942; Leonards, 1962). As this is not convenient in engineering practice and cannot provide unique values of the factors relevant to consolidation, other single approximate equations have also been proposed. One of them is proposed by Chung *et al.* (2014) as follows

$$T_v = -0{\cdot}39\ln\left(\frac{1 - U_v^{1{\cdot}0102}}{0{\cdot}8283}\right) \quad (6)$$

or

$$U_v = [1 - 0{\cdot}8283\exp(-2{\cdot}564T_v)]^{0{\cdot}99} \quad (7)$$

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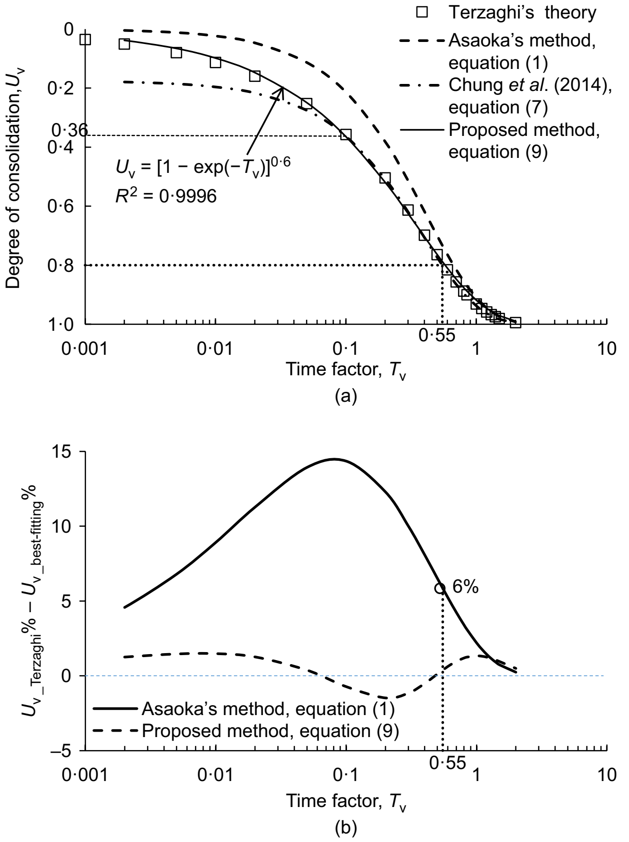

Fig 1. Using the Chapman–Richards model to best fit the Terzaghi's one-dimensional consolidation curve: (a) simulated curves; (b) differences of best-fitted U_v to Terzaghi's theory

Equation (7) is plotted in Fig. 1(a). It can be seen that it only fits Terzaghi's consolidation curve when $U_v > 36\%$.

In this paper, a formula is presented to simulate Terzaghi's one-dimensional consolidation curve. Based on the formula, an observational method is proposed to predict the ultimate settlement and the coefficient of consolidation from analysis of settlement data collected during one-dimensional consolidation. The effectiveness of the proposed method was verified using laboratory oedometer tests and field monitoring data.

FORMULA OF U_V–T_V CURVE

The Chapman–Richards model is adopted in this study to best fit the U_v–T_v curve derived from Terzaghi's consolidation theory. The mathematical expression of the Chapman–Richards equation was derived by Richards (1959) and Chapman (1961) and can be written as (Ratkowsky, 1990)

$$y = \eta[1 - \kappa \exp(-\mu t)]^{\lambda} + \varepsilon \tag{8}$$

where η is the amplitude of the curve; ε is the offset from zero; κ, μ and λ are rate constants; and exp() is the base of the natural logarithm.

When equation (8) is used for curve fitting Terzaghi's consolidation curve, the constants η, κ and ε are predetermined as 1·0, 1·0 and 0·0, respectively, because the maximum limit of U_v is 1·0 and the consolidation curve starts from $T_v = 0$ and $U_v = 0$. The values of μ and λ are obtained using the 'Solver' function in the Microsoft Excel program by setting a target to maximise R^2. The obtained curves and equations are shown in Fig. 1(a). The errors involved in the curve fitting in U_v are also shown in Fig. 1(b). The R^2 for the curve fitting is 0·9996 and the errors are less than 2%. The equation for the U_v–T_v relationship can be rewritten as

$$U_v = [1 - \exp(-2T_v)]^{0.6} \tag{9}$$

Combining equation (9) and $U_v = \delta/\delta_{ult}$ gives the δ–t relationship in one-dimensional consolidation as

$$\delta = \delta_{ult}\left[1 - \exp\left(-\frac{2c_v}{H^2}t\right)\right]^{0.6} \tag{10}$$

PROPOSED OBSERVATIONAL METHOD

As the settlement, δ, plotted against time, t, behaviour of a clay layer follows a uniform relationship with respect to δ_{ult}, c_v and H in one-dimensional consolidation, equation (10) is further processed to be used as an observational model to predict δ_{ult}, c_v using monitored settlement data. Using the same procedure as in Asaoka's method, such as selecting settlement δ_1, δ_2,…,δ_n at the constant sampling period $\Delta t = t_n - t_{n-1}$ from the settlement curve, gives the expressions of settlement δ_n, δ_{n-1} and their responding time t_n, t_{n-1} as shown in equations (11) and (12), respectively.

$$\delta_n = \delta_{ult}\left[1 - \exp\left(-\frac{2c_v}{H^2}t_n\right)\right]^{0.6} \tag{11}$$

$$\delta_{n-1} = \delta_{ult}\left[1 - \exp\left(-\frac{2c_v}{H^2}t_{n-1}\right)\right]^{0.6} \tag{12}$$

Combining equations (11) and (12) gives the relationship between the settlements δ_n and δ_{n-1} as

$$\delta_n^{1.667} = \alpha + \beta\delta_{n-1}^{1.667} \tag{13}$$

$$\alpha = (1 - \beta)\delta_{ult}^{1.667} \tag{14}$$

$$\beta = \exp\left(-\frac{2c_v}{H^2}\Delta t\right) \tag{15}$$

Equation (13) shows that the $\delta_n^{1.667}$ against $\delta_{n-1}^{1.667}$ plot is a straight line with slope α and intercept β. The procedure is similar to Asaoka's method except in using $\delta_n^{1.667}$ for the graph. From equation (14) the ultimate settlement δ_{ult} can be derived as

$$\delta_{ult} = \left(\frac{\alpha}{1 - \beta}\right)^{0.6} \tag{16}$$

Using equation (15), the coefficient of consolidation c_v can be derived as

$$c_v = -\frac{H^2 \ln\beta}{2\Delta t} \tag{17}$$

VERIFICATION USING OEDOMETER TESTING DATA

Effect of data range used for prediction

To investigate the influence of data range on the accuracy of the proposed method, Terzaghi's U_v–T_v solution is adopted with predefined parameters $c_v = 1$, $H = 1$, $\delta_{ult} = 1$ and the range of settlement data from U_0 to U_{30}, U_0 to U_{60} and U_0 to U_{90}.

The proposed method as in equations (16) and (17) is used to predict δ_{ult} and c_v, and the results are shown in Table 1. The value of δ_{ult} and c_v predicted using the settlement data in

Table 1. Effect of sampling period and range of settlement data on the predictions of the proposed and Asaoka's method (for $c_v = 1$, $h = 1$ and $\delta_{ult} = 1$)

Proposed method		β	α	R^2	N_{90}	c_v	Error: %	δ_{ult}	Error: %
$\Delta T = 0{\cdot}005$	U_0–U_{30}	0·9592	0·0123	0·9995	41	4·166	316·6	0·487	−51·3
	U_0–U_{60}	0·9889	0·0100	1·0000	152	1·116	11·6	0·939	−6·1
	U_0–U_{90}	0·9910	0·0097	1·0000	187	0·907	−9·3	1·041	4·1
$\Delta T = 0{\cdot}01$	U_0–U_{30}	0·9216	0·0240	0·9986	21	4·082	308·2	0·492	−50·8
	U_0–U_{60}	0·9777	0·0199	0·9998	75	1·128	12·8	0·934	−6·6
	U_0–U_{90}	0·9820	0·0192	1·0000	94	0·908	−9·2	1·039	3·9
$\Delta T = 0{\cdot}025$	U_0–U_{30}	0·8382	0·0555	0·9980	10	3·530	253·0	0·526	−47·4
	U_0–U_{60}	0·9450	0·0490	0·9993	30	1·131	13·1	0·933	−6·7
	U_0–U_{90}	0·9557	0·0473	0·9999	38	0·906	−9·4	1·040	4·0
Asaoka's method		β_1	β_0	R^2	j_{90}	c_v	Error: %	δ_{ult}	Error: %
$\Delta T = 0{\cdot}005$	U_0–U_{30}	0·8122	0·0565	0·9821	11	17·334	1633·4	0·301	−69·9
	U_0–U_{60}	0·9472	0·0314	0·9972	42	4·520	352·0	0·595	−40·5
	U_0–U_{90}	0·9755	0·0213	0·9994	93	2·067	106·7	0·868	−13·2
$\Delta T = 0{\cdot}01$	U_0–U_{30}	0·6848	0·0977	0·9693	6	15·776	1477·6	0·310	−69·0
	U_0–U_{60}	0·8964	0·0612	0·9922	21	4·557	355·7	0·591	−40·9
	U_0–U_{90}	0·9516	0·0421	0·9983	46	2·067	106·7	0·870	−13·0
$\Delta T = 0{\cdot}025$	U_0–U_{30}	0·4985	0·1750	0·9750	3	11·603	1060·3	0·349	−65·1
	U_0–U_{60}	0·7699	0·1373	0·9778	9	4·358	335·8	0·597	−40·3
	U_0–U_{90}	0·8844	0·1006	0·9936	19	2·047	104·7	0·870	−13·0

the range from U_0 to U_{30} and sampling period $\Delta T = 0{\cdot}025$ is underestimated by 47·4% and overestimated by 253%, respectively. When the data in the range from U_0 to U_{60} are used, δ_{ult} is underestimated by 6·7% and c_v is overestimated by 13·1%, which is acceptable for common engineering application. When the settlement data in the range from U_0 to U_{90} are used, the errors are further reduced to 4% overestimation for δ_{ult} and 9·4% underestimation for c_v.

For comparison, Asaoka's method as in equations (4) and (5) is also adopted to predict δ_{ult} and c_v. The percentages of error between this method and the theoretical values are also computed. The results are given in Table 1. Using the early settlement data from U_0 to U_{30} and sampling period $\Delta T = 0{\cdot}025$, Asaoka's method underestimates δ_{ult} by 65·1% and overestimates c_v by 1060·3%. Using the settlement data from U_0 to U_{60}, δ_{ult} is underestimated by 40·3% and c_v is overestimated by 335·8%, which are still not acceptable. Even when settlement data from U_0 to U_{90} are used, δ_{ult} is still underestimated by 13% and c_v is overestimated by 104·7%. Compared with Asaoka's method, the proposed method is much improved in terms of accuracy in prediction as well as the data range required (up to 60% in the proposed method) to make a reasonable prediction.

Effect of sampling period

Some previous studies indicated that the accuracy of Asaoka's method is affected by the choice of sampling period Δt (Edil *et al.*, 1991; Long *et al.*, 2013). To evaluate the influence of sampling period on the accuracy of prediction, Edil *et al.* (1991) proposed a parameter j_{95} to define the number of samples to reach a 95% degree of consolidation. The suggested sampling period should give a j_{95} value between 10 and 30. Similarly, Tan & Chew (1996) used j_{90} to define the number of time increments to give a 90% degree of consolidation, which is shown as

$$j_{90} = \frac{\ln(1 - U_{90})}{\ln \beta_1} \tag{18}$$

In this paper, a similar parameter N_{90} is adopted, which is defined as the number of samples to achieve a 90% degree of consolidation

$$\frac{c_v N_{90} \Delta t}{H^2} = T_{90} \tag{19}$$

Combining equations (15) and (19) and $T_{90} = 0{\cdot}848$ gives the expression of N_{90} as

$$N_{90} = -\frac{1{\cdot}696}{\ln \beta} \tag{20}$$

Terzaghi's theoretical U_v–T_v curve is adopted to investigate the effect of sampling period ΔT on the predictions of the proposed method as well as Asaoka's method. The ΔT values used in the calculations are 0·005, 0·01 and 0·025 as reported in Table 1. It appears that the selection of ΔT does not have much of an effect on the c_v and δ_{ult} values obtained from the proposed method. When using the settlement data from U_0 to U_{90}, the proposed method underestimates c_v by about 9% and overestimates δ_{ult} by about 4% for all of the three ΔT values of 0·005, 0·01 and 0·025.

A laboratory oedometer test at loading step of 2–4 kPa, as reported by Tan & Chew (1996) was used to evaluate the proposed method and the effect of Δt on the prediction. The tested soil was kaolin and the size of the specimen was 75 mm in diameter and 25 mm high. Drainage was permitted only at the top. The coefficient of consolidation c_v was calculated as 0·613 mm²/s based on Taylor's t_{90}, and as 0·513 mm²/s based on Casagrande's t_{50}. The settlement data were taken from Tan & Chew (1996) with $\delta_{ult} = 1$ mm. Analyses using six different sampling periods of $\Delta t = 10$, 30, 50, 100, 200 and 300 s were carried out. The plots obtained using the proposed method are shown in Fig. 2(a). The obtained α, β, R^2 and computed c_v, δ_{ult}, N_{90} are reported in Table 2. It can be seen that the computed c_v, δ_{ult} are stable, although the magnitudes of α, β and R^2 are different for different Δt. The calculated coefficients of consolidation, c_v, are 12·3% to 17·8% lower than those from Taylor's t_{90}, or −1·7% to 4·8% different than those from c_v using Casagrande's t_{50}. For Δt ranging from 10 to 300 s, the estimated δ_{ult} are approximately the same as those measured with overestimates of only 0·5–1·0%. The N_{90} does not have any obvious effect on the accuracy of the computed c_v

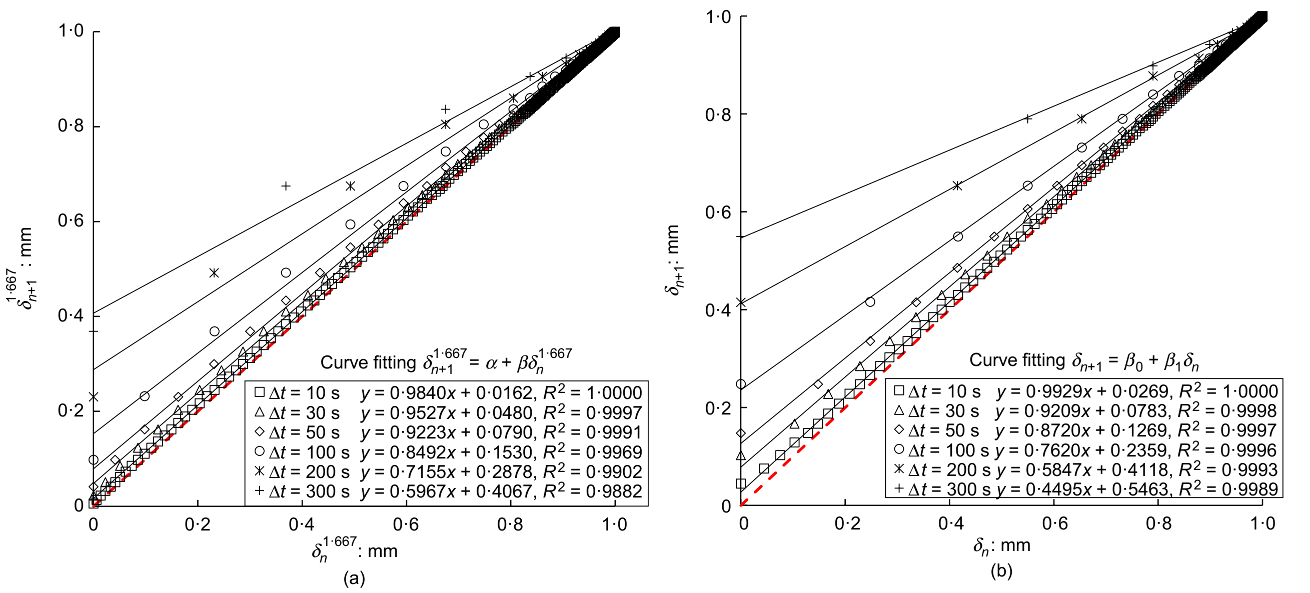

Fig. 2. Influence of sampling period on the plots using (a) proposed and (b) Asaoka's method (data from Tan & Chew (1996))

Table 2. Effect of sampling period on the predictions of the proposed and Asaoka's method (for loading step from 2 kPa to 4 kPa, $c_v = 0{\cdot}513$ mm²/s from Casagrande t_{50}, and $c_v = 0{\cdot}613$ mm²/s from Taylor's t_{90}, $H = 25$ mm and observed $\delta_{ult} = 1$ mm, data from Tan & Chew (1996))

Proposed method	Δt: s	β	α	R^2	N_{90}	c_v	Error: % Casagrande	Error: % Taylor	δ_{ult}	Error: % Proposed method
	10	0·9840	0·0162	1·0000	105	0·504	−1·7	−17·8	1·007	0·7
	30	0·9527	0·0480	0·9997	35	0·505	−1·6	−17·7	1·009	0·9
	50	0·9223	0·0790	0·9991	21	0·506	−1·5	−17·5	1·010	1·0
	100	0·8492	0·1530	0·9969	10	0·511	−0·4	−16·7	1·009	0·9
	200	0·7155	0·2878	0·9902	5	0·523	2·0	−14·7	1·007	0·7
	300	0·5967	0·4067	0·9882	3	0·538	4·8	−12·3	1·005	0·5
Asaoka's method	**Δt: s**	**β_1**	**β_0**	**R^2**	**j_{90}**	**c_v**	**Error: % Casagrande**	**Error: % Taylor**	**δ_{ult}**	**Error: % Asaoka's method**
	10	0·9729	0·0269	1·0000	84	0·715	39·5	16·7	0·993	−0·7
	30	0·9209	0·0783	0·9998	28	0·715	39·4	16·7	0·990	−1·0
	50	0·8720	0·1268	0·9997	17	0·713	39·1	16·4	0·991	−0·9
	100	0·7620	0·2359	0·9996	8	0·708	38·0	15·5	0·991	−0·9
	200	0·5847	0·4118	0·9993	4	0·699	36·2	14·0	0·992	−0·8
	300	0·4495	0·5463	0·9989	3	0·694	35·3	13·2	0·992	−0·8

and δ_{ult}. However, the larger the value of Δt, the lower the regression coefficient R^2, suggesting the adoption of $N_{90} > 20$ to achieve a high value of R^2. For comparison, Asaoka's plots using different sampling periods and settlement data are shown in Fig. 2(b) and Table 2. The calculated c_v are about 15% and 40% larger than that calculated from Taylor's t_{90} and Casagrande's t_{50}, respectively. The results also show that δ_{ult} is always slightly overestimated, by 1·0% in the worst case.

COMPARISON USING DATA FROM FIELD TESTS

The proposed method is applied to two well-documented case records and the predictions are compared with those from the Asaoka's method. The first case is the Changi East reclamation project in Singapore, as reported by Choa *et al.* (2001), Chu *et al.* (2009) and Bo *et al.* (2005). A pilot test was conducted on four, 50 m square subzones to investigate the effect of prefabricated vertical drain spacing. One subzone, termed lot X, was used as a control zone with no drain installed. A 10 m high sand fill surcharge was applied on the surface of the site. The loading history and monitored settlement data are shown in Fig. 3. The clay stratum was about 42·5 m deep with a 2 m thick interlayer of sand and many horizontal sand seams (Chu *et al.*, 2009). The laboratory tests indicate the c_v of the clay ranges from 0·5 to 2·3 m²/year. More details of the site information, location of the field instrumentation and construction procedure can be found elsewhere (Bo *et al.*, 2005; Chu *et al.*, 2009).

Figure 4(a) shows the plots used to predict δ_{ult} and c_v using the proposed method. A sampling period of 20 days was adopted, with the initial settlement data taken as the settlement at $t_0 = 260$ days (see point A in Fig. 3), which was the beginning of the full load. In the determination of the c_v, the total clay thickness was taken as 22·0 m (Chu *et al.*, 2009). The coefficient of consolidation c_v obtained from the proposed method is 6·87 m²/year, which is higher than that from the laboratory oedometer test, as expected. The obtained δ_{ult} from the proposed method is 1·977 m. Based on $U_v = \delta/\delta_{ult}$, the U_v at $t = 760$ days is calculated as 39%

using the proposed method, which matches with the U_v calculated using pore water pressure data (Chu *et al.*, 2009). For comparison, the Asaoka's plot using the same sampling period and settlement data is shown in Fig. 4(b). The coefficients of consolidation c_v and δ_{ult} obtained from Asaoka's method are 21·03 m^2/year and 1·226 m, respectively. The U_v at $t = 760$ days from Asaoka's method is 62·8%. For the presented case history with low degree of

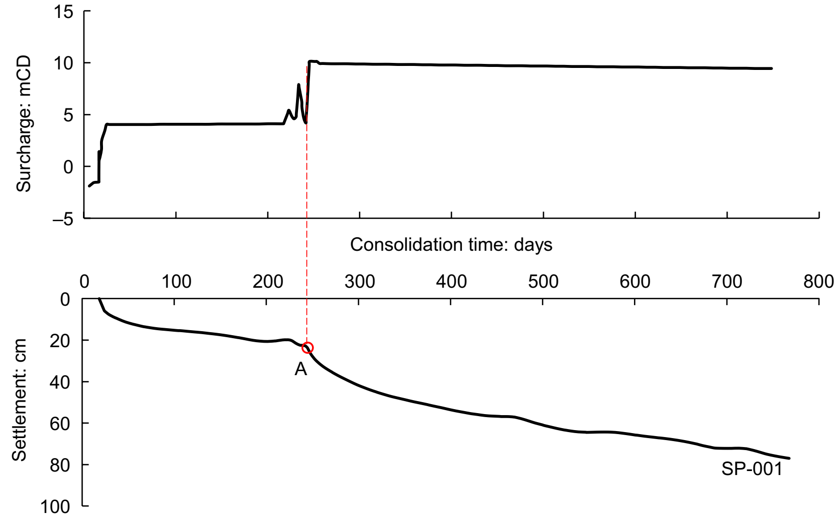

Fig. 3. Loading history and monitored settlement data at lot *X* for the Changi East land reclamation project (modified after Chu *et al.* (2009); mCD = m above chart datum)

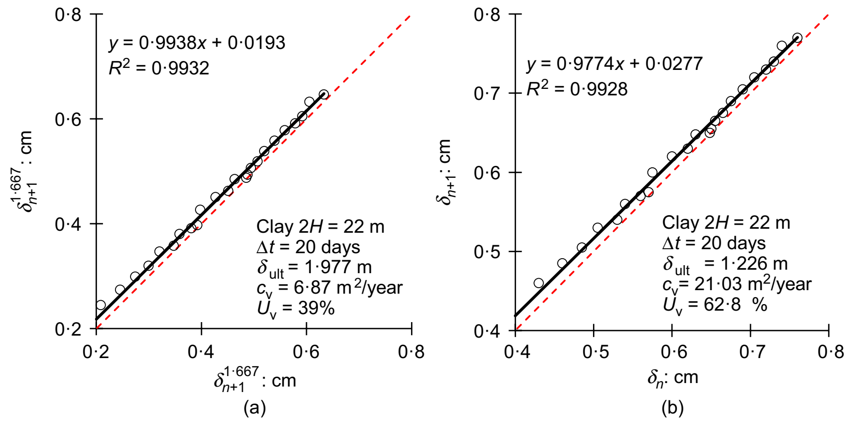

Fig. 4. Plots to predict c_v and δ_{ult} using (a) proposed and (b) Asaoka's method

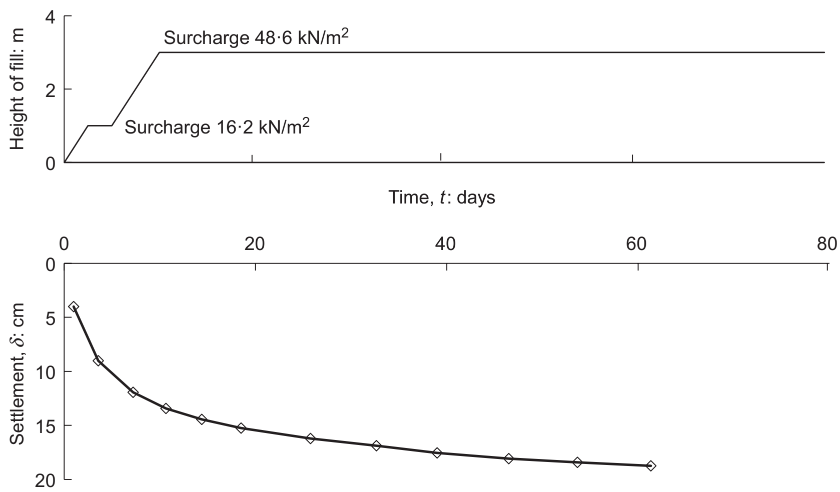

Fig. 5. Loading history and monitored settlement data for the embankment constructed on a typical sabkha formation (after Dhowian *et al.*, 1987)

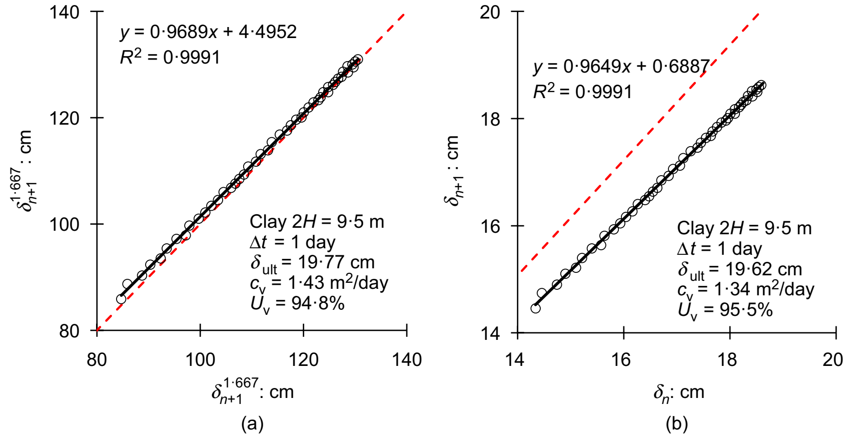

Fig. 6. Plots to predict c_v and δ_{ult} using (a) proposed and (b) Asaoka's method

consolidation, the proposed method gives better results than Asaoka's method.

The second field site is from the town of Jazan situated on the southeast coast of the Kingdom of Saudi Arabia. The soil profile at the site comprised 0·7–2·2 m thick sabkha crust (mixture of fine sand and silt), 6·0–16·5 m thick compressible sabkha complex (varies from non-plastic fine sand to highly plastic organic clays) and sabkha base (Dhowian *et al.*, 1987). The loading history and monitored settlement are shown in Fig. 5. Based on the settlement data, c_v of 1·86 m^2/day was back-calculated by assuming the sabkha was 9·5 m thick with double drainage (Dhowian *et al.*, 1987). The monitored settlement data shown in Fig. 5 are used to plot Figs 6(a) and 6(b) employing the proposed method and Asaoka's method, respectively. The predictions from the two methods agree well with each other. The hyperbolic method was also used by Al-Shamrani (2005) to analyse the settlement data and the δ_{ult} obtained was 17·2 cm, which was lower than that from either Asaoka's method or the proposed method.

CONCLUSIONS

The Chapman–Richards model is adopted in this paper to best fit Terzaghi's one-dimensional consolidation curve. The obtained formula fits the theoretical solutions well, with a regression coefficient of 0·9995 and errors less than 2%. The derived formula could be used as an observational method to predict the ultimate settlement of compressible soil undergoing one-dimensional consolidation, as well as to back-calculate the coefficient of consolidation of the soil.

The accuracy of the proposed observational method is verified against oedometer testing data and field settlement monitoring data. The settlement prediction made using the proposed method is more accurate compared with that using Asaoka's method. The settlement prediction can become reasonably accurate when data in the range corresponding to the degree of consolidation up to 60% are available. The analysis shown in this paper also indicates the proposed method is not much affected by the choice of sampling interval Δt. However, the number of sampling points N_{90} should be greater than 20 in order to achieve a higher regression coefficient for the least-squares linear regression for 90% degree of consolidation.

NOTATION

c_v	coefficient of vertical consolidation
H	drainage path
j_{95}	number of samples to reach 95% degree of consolidation
N_{90}	number of samples to achieve 90% degree of consolidation
R^2	regression coefficient
T_v	non-dimensional time factor
t	consolidation time
t_{50}	time for completion of 50% degree of consolidation
t_{90}	time for completion of 90% degree of consolidation
U_v	degree of consolidation
α	intercept of straight line in $\delta_n^{1\cdot667}$ against $\delta_{n-1}^{1\cdot667}$ plot
β	slope of straight line in $\delta_n^{1\cdot667}$ against $\delta_{n-1}^{1\cdot667}$ plot
β_0	intercept of straight line in δ_n against δ_{n-1} plot
β_1	slope of straight line in δ_n against δ_{n-1} plot
Δt	sampling period
δ	ground settlement at consolidation time t
δ_{ult}	ultimate ground settlement
$\eta, \varepsilon, \kappa, \mu, \lambda$	constants in Chapman–Richards equation

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