Practical Active Noise Control: Restriction of Maximum Output Power

Woon-Seng Gan*, Dongyuan Shi* and Xiaoyi Shen*
* Digital Signal Processing Lab, School of Electrical and Electronic Engineering,
Nanyang Technological University, Singapore
E-mail: ewsgan@ntu.edu.sg

***Abstract*—This paper presents some recent algorithms developed by the authors for real-time adaptive active noise (AANC) control systems. These algorithms address some of the common challenges faced by AANC systems, such as speaker saturation, system divergence, and disturbance rejection. Speaker saturation can introduce nonlinearity into the adaptive system and degrade the noise reduction performance. System divergence can occur when the secondary speaker units are over-amplified or when there is a disturbance other than the noise to be controlled. Disturbance rejection is important to prevent the adaptive system from adapting to unwanted signals. The paper provides guidelines for implementing and operating real-time AANC systems based on these algorithms.**

Keywords: adaptive active noise control algorithm, output saturation, 2GD-FxLMS, leaky FxLMS

I. Introduction

Active noise control (ANC) system produces anti-noise to cancel out unwanted noise [1], [2], [3], [4], [5], [6], [7], [8], [9], [10]. However, adaptive ANC (AANC) in real-time [9], [11], [12], [13], [14] may fail if the audio amplifier is overdriven to saturation, causing the adaptive algorithm to diverge. Saturation distortion affects both the amplitude and phase of the secondary path in ANC [15], [16]. Some nonlinear algorithms based on neural networks or Volterra filters [17] can handle saturation distortion, but they are costly and complex to implement, and the vanishing gradient issue is a problem for real-world scenarios. Moreover, they do not solve the problem of insufficient power when the control signal exceeds the amplifier threshold. Thus, these nonlinear solutions are not practical for real applications and may only be used for partially overdriven actuators. Furthermore, the nonlinear adaptive algorithm without constraints is not a desired solution. A better approach is to limit the amplifier's output power and keep it within a certain power budget. Some of the common algorithms for this approach are (i) clipping or rescaling algorithms [18], which either truncate the output signal above the threshold or adjust the weight of the control filter; (ii) leaky-type filtered-reference Least Mean Square (FxLMS) algorithm [19], which adds a leak term or penalty factor to stabilize the algorithm, but requires trial and error to choose the optimal factor; (iii) A two-gradient FxLMS (2GD-FxLMS) [20] algorithm, which imposes a specific output constraint with no extra computation compared to the conventional FxLMS algorithm. Recent research has revealed new insights into constraint-FxLMS algorithms. This paper highlights the key differences and evaluates their merits and drawbacks. A typical block diagram of the ANC with saturation effect can be viewed in Fig. 1 and will be used throughout the discussion of different constraint FxLMS.

This research is supported by the Ministry of Education, Singapore, under its Academic Research Fund Tier 2 (Award No. MOE-T2EP50122-0018).

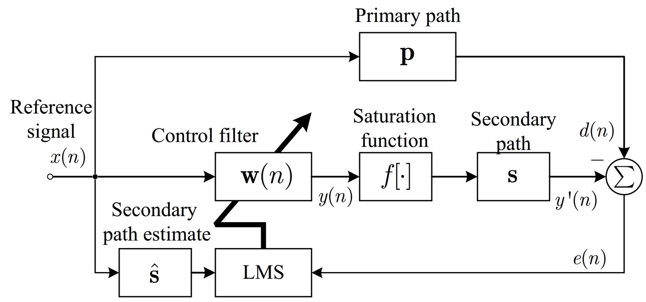

Fig. 1. Block diagram of the ANC with the saturation effect.

Before we move into the definition, explanation, and implementation of the various types of constraint FxLMS algorithms, we have defined a table of variables and their descriptions for ease of reading.

$\mathbf{w}(n)$: weight vector of the control filter	$e(n)$: error signal
$\mathbf{x}(n)$: reference signal vector	$\mathbf{x}'(n)$: filtered reference signal vector
μ: step size of the adaptive algorithm	γ: leaky factor
ρ^2: output power constraints	$\mathbf{s}$: impulse response of secondary path
L: length of the control filter	$d(n)$: disturbance
λ: Lagrangian factor	σ^2:variance of disturbance
y_o: optimal output	σ_x: variance of reference
$\mathbf{w}_\mathrm{o}$: optimal weight vector	η: degree of nonlinearity
$\mathbf{R}_{\mathrm{x}'}$: autocorrelation matrix of $\mathrm{x}'(n)$	$\mathbf{R}_\mathrm{y}$: autocorrelation matrix of $y(n)$
I: identity matrix	$\mathbf{P}_{\mathrm{dx}'}$: cross-correlation vector of $d(n)$ and $x'(n)$
λ_o: optimal Lagrangian factor	γ_o: optimal leaky factor
G_s: power gain of the secondary path	$S(e^{j\omega})$: transfer function of the secondary path

II. Clipping FxLMS

The clipping algorithm mainly truncates the parts of the output signal that exceeds a certain voltage level. However, this output amplitude constraint approach has a drawback of stability, distorts the attenuated noise, and may converge slowly. Therefore, in this paper, we will not consider this approach. A similar type of clipping algorithm is the nonlinear FxLMS algorithm [21], which consists of an exponential term that clips off the output signal when the control signal exceeds the output constraints.

Even though the clipping FxLMS algorithms can deal with an amplitude that exceeds the constraint, these simple approaches may not be ideal as these algorithms distort the attenuated noise, require an accurate model of the output-saturation model, and have significant residual distortion.

III. Output Constraint: Leaky-type FxLMS Algorithms

The leaky FxLMS (LFxLMS) algorithm [22], [23] introduces a leak term (or penalty coefficient) that constrains the control effort and stabilizes the algorithm. The general LFxLMS algorithm is given as

$$\mathbf{w}(n+1) = (1-\mu\gamma)\mathbf{w}(n) + \mu e(n)\mathbf{x}'(n), \tag{1}$$

$$e(n) = d(n) - f\left[\sum_{l=0}^{L-1} s_l \mathbf{w}^{\mathrm{T}}(n-l)\mathbf{x}(n-l)\right]. \tag{2}$$

In the LFxLMS algorithm, the selection of the leak factor plays a crucial role in balancing noise control and constraint satisfaction. However, due to the ad-hoc selection of the leak factor, the output signal may not always adhere to the imposed constraint. The time-domain version of the LFxLMS algorithm results in noise reduction errors and slower convergence. To address this issue, recent versions of the algorithm use frequency-domain constraints that only penalize frequency bins exceeding the constraint. Despite these improvements, the performance of the algorithm is still greatly influenced by the empirical choice of the leak factor.

Recently, an optimal leak factor selection method for output-constrained LFxLMS algorithm [23], [24] has been proposed. This latter technique ensures that the LFxLMS reaches optimal control within its specific output power constraint. A leak factor can be derived on-the-fly based on the application's measured primary and secondary acoustic paths. The cost function with a specific output constraint [25]

$$\begin{aligned} &\min_{\mathbf{w}} \mathrm{J}(\mathbf{w}) = \mathbb{E}\left[\left|d(n) - \sum_{l=0}^{L-1} s_l \mathbf{w}^{\mathrm{T}}(n-l)\mathbf{x}(n-l)\right|^2\right] \\ &\text{s.t. } g(\mathbf{w}) = \mathbb{E}\left[\left|\mathbf{w}^{\mathrm{T}}(n)\mathbf{x}(n)\right|^2\right] \le \rho^2, \end{aligned} \tag{3}$$

where $\mathbb{E}(\cdot)$ denotes the expectation of the argument. The Lagrangian function [26] based on (3) is defined as

$$\mathbb{L}(w,\lambda,\theta) = J(w) + \lambda\left[g(w) + \theta^2 - \rho^2\right]. \tag{4}$$

The Lagrangian factor λ is used in the above cost function to introduce constraints into the optimization problem, and the optimal Lagrangian factor, λ_{o} is derived as:

$$\lambda_{\mathrm{o}} = \frac{\mathbb{E}\left\{y'_{\mathrm{o}}(n)d(n)\right\} - \mathbb{E}\left\{y'_{\mathrm{o}}(n)^2\right\}}{\rho^2}. \tag{5}$$

The optimal Lagrangian factor, which includes the variance of the output signal from the control filter and the output filtered that passes through the secondary path (includes DAC, amplifier, secondary speaker, and the acoustic transfer path from speaker to error microphone), is shown below:

$$\lambda_{\mathrm{o}} = \frac{\sigma_{y_o}^2 \sum_{l=0}^{L-1} s_l^2}{\rho^2}\left(\sqrt{\frac{\sigma_d^2}{\sum_{l=0}^{L-1} s_l^2 \sigma_{y_o}^2}} - 1\right). \tag{6}$$

To further simplify the above expression, we can introduce a new term, known as the degree of nonlinearity of the system, η^2:

$$\eta^2 = \max\left(\frac{\sigma_d^2}{\sum_{l=0}^{L-1} s_l^2 \rho^2}, 1\right), \tag{7}$$

which leads to

$$\lambda_{\mathrm{o}} = \sum_{l=0}^{L-1} s_l^2(\eta - 1). \tag{8}$$

The optimal control filter with output constraint is given as:

$$\mathbf{w}_{\mathrm{o}} = \left(\lambda_{\mathrm{o}}\sigma_{\mathrm{x}}^2\mathbf{I} + \mathbf{R}_{\mathrm{x}'}\right)^{-1}\mathbf{P}_{\mathrm{dx}'}. \tag{9}$$

The optimal control filter using LFxLMS is given as:

$$\mathbf{w}_{\mathrm{o}} = \left(\gamma\mathbf{I} + \mathbf{R}_{\mathrm{x}'}\right)^{-1}\mathbf{P}_{\mathrm{dx}'}. \tag{10}$$

Therefore, comparing these two optimal control filters, we have a sufficient condition (under white noise):

$$\gamma = \lambda_{\mathrm{o}}\sigma_{\mathrm{x}}^2. \tag{11}$$

In practical cases, we have to replace the sufficient condition as:

$$\gamma = \lambda_{\mathrm{o}}\mathbf{R}_{\mathrm{x}}. \tag{12}$$

Substituting the optimal Lagrangian factor into the above yields the optimal leak factor as shown:

$$\gamma_{\mathrm{o}} = G_{\mathrm{s}}(\eta - 1)\mathbf{R}_{\mathrm{x}}, \tag{13}$$

where $G_s = \sigma_{\mathrm{y}'_{\mathrm{o}}}^2/\rho^2$ is the power gain of the secondary path and can be written as the energy of the secondary path if the control signal is narrowband over $[\omega_1, \omega_2]$ and the degree of nonlinearity of the system are:

$$G_{\mathrm{s}} = \frac{1}{2\pi}\int_{\omega_1}^{\omega_2}\left|S\left(e^{j\omega}\right)\right|^2 d\omega, \tag{14}$$

$$\eta^2 = \max\left(\frac{\sigma_{\mathrm{d}}^2}{G_{\mathrm{s}}\rho^2}, 1\right). \tag{15}$$

All the above statistic terms can be measured from the ANC system before its normal control operation. It is important to note that the LFxLMS algorithm with optimal leak factor is useful in ensuring that the solution does not violate the constraint, and at the same time, obtains a satisfying noise reduction performance. Since the optimal LFxLMS does not allow the control filter to operate beyond the constraint, there is no need to include any system saturation model in the derivation. Furthermore, unlike other algorithms, no primary path is required in the implementation of the optimal LFxLMS algorithm.

As mentioned, there is still a drawback of the above optimal LFxLMS algorithm as several parameters can only be obtained from the control signal's statistical observation before its normal control operation. As the autocorrelation matrix of reference input, secondary path, disturbance power, and the secondary path may change during real-time control, the pre-derived leaky term may not be suitable during its real-time control process.

A more practical solution, which is based on inverse adaptive modeling [27], [28], can estimate the optimal LFxLMS for different types of noise distribution in practice. This latter method also does not require the assumption of the type of noise signal. The power gain of the secondary path, G_s can be approximated as the ratio of the variance of disturbance and its corresponding control signal.

$$G_s = \frac{\sigma_{\mathrm{d}}^2}{\sigma_{\hat{\mathrm{y}}_{\mathrm{d}}}^2}. \tag{16}$$

But to predict the control signal from the disturbance, we need to estimate the inverse model $c(n)$ of the secondary path, as shown in Fig. 2, and the predicted control signal (or anti-noise) is $\hat{y}_d = \mathbf{c}_{\mathrm{o}}^{\mathrm{T}}\mathbf{d}(n)$. From these equations, we can estimate the power gain across a frame of K samples

$$G_{\mathrm{s}} = \frac{\sum_{k=0}^{K} d^2(n-k)}{\sum_{k=0}^{K} \hat{y}_{\mathrm{d}}^2(n-k)}, \tag{17}$$

and the optimal leak term is given as

$$\gamma_{\mathrm{o}} = \frac{\sum_{k=0}^{K} d^2(n-k)}{\sum_{k=0}^{K} \hat{y}_{\mathrm{d}}^2(n-k)}(\eta-1)\mathbf{R}_{\mathrm{x}}. \tag{18}$$

Our proposed algorithm OLFxLMS [27] can be applied to different ANC applications and handling different types of noise, with only a slight increase in computational complexity compared to other algorithms.

A recent paper [29] introduces the modified FxLMS algorithm to continuously estimate the disturbance power and the power of the secondary path, and thus, a time variable penalty factor can be derived iteratively. This latter approach expands the ability of the constraint-based leaky FxLMS algorithm to handle dynamic noise sources in real-time, and still be able to constraint within its maximum power output.

IV. Output Constraint: gradient project based FxLMS Algorithms

In this class of output constraint FxLMS algorithm, modifying the direction of the error gradient, or gradient projection, does not alter the cost function. The search is usually taken along or below the constraint contour. For example, the rescaling FxLMS algorithm [30], which rescales the weight and output signal of the control filter, is a form of gradient projection algorithm. However, this algorithm increases computational load and can lead to a reduction in sampling frequency for practical implementation. To combine the modification of search direction and imposing a constraint, we can derive an optimal solution, which is referred to as the output constraint two-gradient descent (2GD) FxLMS algorithm [20], [31]. Several variants of the 2GD FxLMS take into account faster convergence, minimum mean square error, and extension to multiple channels. A pseudocode of the baseline 2GD-FxLMS algorithm is listed in Table I.

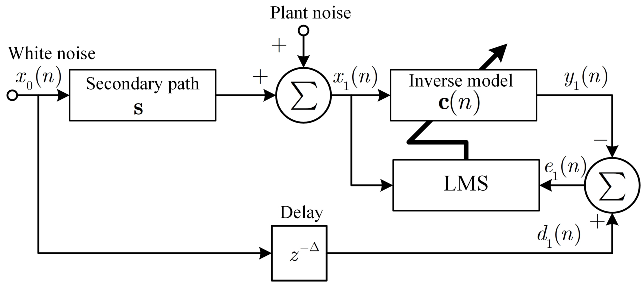

Fig. 2. Block diagram of adaptive inverse modeling for the secondary path.

TABLE I
PSEUDOCODE OF 2GD-FXLMS ALGORITHM

Two gradient FxLMS algorithm(2GD-FxLMS)
Input: Reference signal $x(n)$ and error signal $e(n)$ **Output**: Clipped output signal $y_{\text{out}}(n)$
Step 1: $y(n) = \mathbf{w}^{\mathrm{T}}(n)\mathbf{x}(n)$; $\mathbf{x}'(n) = \hat{s}^{\mathrm{T}}(n)\mathbf{x}(n)$. **Step 2**: If $\lvert y(n)\rvert \leq \rho$ $\mathbf{w}(n+1) = \mathbf{w}(n) + \mu_1 e(n)\mathbf{x}'(n)$; else $\mathbf{w}(n+1) = \mathbf{w}(n) - \mu_2 y(n)\mathbf{x}(n)$. **Step 3**: If $y(n) > \rho$, $y_{\text{out}} = \rho$; else if $y(n) < -\rho$, $y_{\text{out}} = -\rho$; else $y_{\text{out}} = y(n)$.

When the average output power of the control filter is within the constraint ($\mathbb{E}\left[y^2(n)\right] \leq \rho^2$), the weight update is given by

$$\mathbf{w}(n+1) = \mathbf{w}(n) - \frac{1}{2}\mu\frac{\nabla J(\mathbf{w})}{\lVert\nabla J(\mathbf{w})\rVert}. \tag{19}$$

In contrast, when the average output power exceeds the constraint ($g(\mathbf{w}) > \rho^2$), the weight update changes to

$$\mathbf{w}(n+1) = \mathbf{w}(n) - \frac{1}{2}\mu\frac{\nabla g(\mathbf{w})}{\lVert\nabla g(\mathbf{w})\rVert}. \tag{20}$$

By replacing the normalized gradient and average power constraint with the instantaneous gradient and amplitude constraint, the update equation can now be expressed as

$$\begin{cases} \mathbf{w}(n+1) = \mathbf{w}(n) + \mu_1 e(n)\mathbf{x}'(n), & \mathbb{E}\left\{y(n)^2\right\} \leq \rho^2 \\ \mathbf{w}(n+1) = \mathbf{w}(n) - \mu_2 y(n)\mathbf{x}(n), & \mathbb{E}\left\{y(n)^2\right\} > \rho^2 \end{cases} \tag{21}$$

The main advantage of the 2GD-FxLMS algorithm is its ability to continuously update even when the output of the control filter exceeds the amplifier threshold; therefore, it possesses

TABLE II
COMPARATIVE EVALUATION OF THE DIFFERENT TYPES OF CONSTRAINT-BASED FXLMS ALGORITHMS

Algorithms:	**2GD-FxLMS**	**Optimal Leaky FxLMS**
Advantages	· Light computational load · A simple operator to switch between modes · Smaller residual error · Perform better in broadband noise · Enhance system stability and quality of anti-noise. · Able to implement a light-weight multi-channel 2GD-FxLMS.	· An optimal leaky FxLMS can avoid output saturation distortion and provides good noise reduction. · Online estimation of optimal leak factor that can handle different types of noise.
Disadvantages	· Noise reduction performance is usually inferior to other constraint-based algorithms. · May result in slower convergence, but can be solved using momentum and variable step size.	· Higher computational cost may not be desired for MCANC. Can use MOV-FxLMS [29] to reduce complexity. · More steps in determining the optimal leak term, but can be done online.
Usages	In practical real-time implementation of MCANC for open aperture and spatial ANC for controlling high-amplitude acoustic noise sources.	

the switching mechanism, and its computational complexity is the same as the iterative FxLMS algorithm. To put it simply, the 2GD-FxLMS forces the amplifier and speaker to operate in their linear region. Several real-time implementations of the 2GD FxLMS algorithms have been tested in air ducts.

There are several recent improvements in the 2GD FxLMS algorithm, namely:

(i) **Momentum 2GD-FxLMS** algorithm [32] with variable step size. We introduce a momentum factor to improve the convergence speed of the baseline 2GD-FxLMS algorithm and added a computation-effective variable step size approach to further reduce the steady-state error due to the switching nature of the 2GD-FxLMS.

(ii) **Multi-channel 2GD-FxLMS (or 2GD-MCFxLMS)** [33] is an extension of the single-channel 2GD-FxLMS algorithm. However, with the increase in the number of multiple output amplifiers, the output saturation nonlinearity also increases in the multiple-channel compared to the single-channel 2GD-FxLMS algorithm. Additionally, having multiple channel ANC (MCANC) applications means more effort is required to maintain the system components. Constantly subjecting the system to high-amplitude anti-noise reproduction can shorten the lifespan of the amplifiers/speakers and increase maintenance costs.

V. CONCLUSIONS

Table II provides a comprehensive comparison of various constraint-based FxLMS algorithms. These algorithms effectively address the issues of actuator overdriving and nonlinearity caused by saturation. Both the 2GD-FxLMS and optimal Leaky FxLMS algorithms employ a similar constraint-based approach to derive the adaptive FxLMS algorithm.

Typically, imposing a strong constraint on the FxLMS algorithm directs more effort towards achieving the desired output power. However, this approach compromises the level of noise reduction. In other words, constraint-based FxLMS algorithms are tailored to specific requirements and struggle to perform optimally in meeting both objectives simultaneously. Therefore, this paper proposes deriving an optimal leak factor that strikes a balance between noise reduction and satisfying power constraints. We provide a summary of two existing approaches, namely the 2GD-FxLMS and optimal leaky FxLMS algorithms, highlighting their unique properties, advantages, disadvantages, and practical applications.

The focus on real-time implementation in this study is of great value to practitioners, as it enables them to incorporate the optimal leak factor and 2GD into their adaptive ANC (FxLMS) systems. This applied research approach effectively bridges the gap between academic studies and practical implementations, facilitating more efficient noise control in real-world applications.

REFERENCES

[1] Sen M Kuo and Dennis R Morgan, *Active noise control systems*, vol. 4, Wiley, New York, 1996.

[2] Stephen J Elliott and Philip Arthur Nelson, “Active noise control,” *IEEE signal processing magazine*, vol. 10, no. 4, pp. 12–35, 1993.

[3] Yoshinobu Kajikawa, Woon-Seng Gan, and Sen M Kuo, “Recent advances on active noise control: open issues and innovative applications,” *APSIPA Transactions on Signal and Information Processing*, vol. 1, 2012.

[4] Colin N Hansen, *Understanding active noise cancellation*, CRC Press, 1999.

[5] Feiran Yang, Yin Cao, Ming Wu, Felix Albu, and Jun Yang, “Frequency-domain filtered-x lms algorithms for active noise control: a review and new insights,” *Applied Sciences*, vol. 8, no. 11, pp. 2313, 2018.

[6] Jihui Zhang, Huiyuan Sun, Prasanga N Samarasinghe, and Thushara D Abhayapala, “Active noise control over multiple regions: Performance analysis,” in *ICASSP 2020-2020 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*. IEEE, 2020, pp. 8409–8413.

[7] Cheng-Yuan Chang, Xiu-Wei Liu, Sen M Kuo, et al., “Active noise control for centrifugal and axial fans,” *Noise Control Engineering Journal*, vol. 68, no. 6, pp. 490–500, 2020.

[8] Xiaoyi Shen, Woon-Seng Gan, and Dongyuan Shi, “Multi-channel wireless hybrid active noise control with fixed-adaptive control selection,” *Journal of Sound and Vibration*, vol. 541, pp. 117300, 2022.

[9] Dongyuan Shi, Woon-Seng Gan, Bhan Lam, Rina Hasegawa, and Yoshinobu Kajikawa, “Feedforward multichannel virtual-sensing active control of noise through an aperture: Analysis on causality and sensor-actuator constraints,” *The Journal of the Acoustical Society of America*, vol. 147, no. 1, pp. 32–48, 2020.

[10] Bhan Lam, Dongyuan Shi, Woon-Seng Gan, Stephen J Elliott, and Masaharu Nishimura, “Active control of broadband sound through the open aperture of a full-sized domestic window,” *Scientific reports*, vol. 10, no. 1, pp. 1–7, 2020.

[11] Jordan Cheer and Stephen J. Elliott, "Multichannel control systems for the attenuation of interior road noise in vehicles," *Mechanical Systems and Signal Processing*, vol. 60-61, pp. 753–769, 2015.

[12] Yoshinobu Kajikawa, Woon-Seng Gan, and Sen Maw Kuo, "Recent applications and challenges on active noise control," in *2013 8th International Symposium on Image and Signal Processing and Analysis (ISPA)*. IEEE, 2013, pp. 661–666.

[13] Xiaoyi Shen, Dongyuan Shi, Woon-Seng Gan, and Santi Peksi, "Adaptive-gain algorithm on the fixed filters applied for active noise control headphone," *Mechanical Systems and Signal Processing*, vol. 169, pp. 108641, 2022.

[14] Junwei Ji, Dongyuan Shi, Zhengding Luo, Xiaoyi Shen, and Woon-Seng Gan, "A practical distributed active noise control algorithm overcoming communication restrictions," in *ICASSP 2023-2023 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*. IEEE, 2023, pp. 1–5.

[15] Sen M Kuo, Hsien-Tsai Wu, Fu-Kun Chen, and Madhu R Gunnala, "Saturation effects in active noise control systems," *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 51, no. 6, pp. 1163–1171, 2004.

[16] Dongyuan Shi, Chuang Shi, and Woon-Seng Gan, "Effect of the audio amplifier's distortion on feedforward active noise control," in *2017 Asia-Pacific Signal and Information Processing Association Annual Summit and Conference (APSIPA ASC)*. IEEE, 2017, pp. 469–473.

[17] Li-Zhe Tan and Jean Jiang, "Filtered-x second-order volterra adaptive algorithms," *Electronics letters*, vol. 33, no. 8, pp. 671–672, 1997.

[18] Xiaojun Qiu and Colin H Hansen, "A study of time-domain fxlms algorithms with control output constraint," *The Journal of the Acoustical Society of America*, vol. 109, no. 6, pp. 2815–2823, 2001.

[19] Orlando José Tobias and Rui Seara, "Leaky-fxlms algorithm: Stochastic analysis for gaussian data and secondary path modeling error," *IEEE Transactions on speech and audio processing*, vol. 13, no. 6, pp. 1217–1230, 2005.

[20] DongYuan Shi, Woon-Seng Gan, Bhan Lam, and Chuang Shi, "Two-gradient direction fxlms: An adaptive active noise control algorithm with output constraint," *Mechanical Systems and Signal Processing*, vol. 116, pp. 651–667, 2019.

[21] Sen M Kuo and Hsien-Tsai Wu, "Nonlinear adaptive bilinear filters for active noise control systems," *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 52, no. 3, pp. 617–624, 2005.

[22] Lifu Wu, Xiaojun Qiu, and Yecai Guo, "A generalized leaky fxlms algorithm for tuning the waterbed effect of feedback active noise control systems," *Mechanical Systems and Signal Processing*, vol. 106, pp. 13–23, 2018.

[23] Dongyuan Shi, Bhan Lam, Woon-Seng Gan, and Shulin Wen, "Optimal leak factor selection for the output-constrained leaky filtered-input least mean square algorithm," *IEEE Signal Processing Letters*, vol. 26, no. 5, pp. 670–674, 2019.

[24] Dongyuan Shi, Woon-Seng Gan, Bhan Lam, and Xiaoyi Shen, "Comb-partitioned frequency-domain constraint adaptive algorithm for active noise control," *Signal Processing*, vol. 188, pp. 108222, 2021.

[25] Farzin Taringoo, Javad Poshtan, and Mohammad Hossein Kahaei, "Analysis of effort constraint algorithm in active noise control systems," *EURASIP Journal on Advances in Signal Processing*, vol. 2006, pp. 1–9, 2006.

[26] Brian Beavis and Ian Dobbs, *Optimisation and stability theory for economic analysis*, Cambridge university press, 1990.

[27] Dongyuan Shi, Woon-Seng Gan, Bhan Lam, and Xiaoyi Shen, "Optimal penalty factor for the mov-fxlms algorithm in active noise control system," *IEEE Signal Processing Letters*, vol. 29, pp. 85–89, 2021.

[28] Dongyuan Shi, Woon-Seng Gan, Bhan Lam, and Xiaoyi Shen, "A frequency-domain output-constrained active noise control algorithm based on an intuitive circulant convolutional penalty factor," *IEEE/ACM Transactions on Audio, Speech, and Language Processing*, vol. 31, pp. 1318–1332, 2023.

[29] Chung Kwan Lai, Dongyuan Shi, Bhan Lam, and Woon-Seng Gan, "Mov-modified-fxlms algorithm with variable penalty factor in a practical power output constrained active control system," *IEEE Signal Processing Letters*, 2023.

[30] Hui Lan, Ming Zhang, and Wee Ser, "A weight-constrained fxlms algorithm for feedforward active noise control systems," *IEEE Signal Processing Letters*, vol. 9, no. 1, pp. 1–4, 2002.

[31] Dongyuan Shi, Woon-Seng Gan, Bhan Lam, and Shulin Wen, "Practical consideration and implementation for avoiding saturation of large amplitude active noise control," *Proc. 23rd Int. Congr. Acoust*, pp. 6905–6912, 2019.

[32] Xiaoyi Shen, Dongyuan Shi, Zhengding Luo, Junwei Ji, and Woon-Seng Gan, "A momentum two-gradient direction algorithm with variable step size applied to solve practical output constraint issue for active noise control," in *ICASSP 2023-2023 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*. IEEE, 2023, pp. 1–5.

[33] Dongyuan Shi, Bhan Lam, Xiaoyi Shen, and Woon-Seng Gan, "Multi-channel two-gradient direction filtered reference least mean square algorithm for output-constrained multichannel active noise control," *Signal Processing*, p. 108938, 2023.