Logistics Coordination in Vendor-Buyer Systems

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ABSTRACT

Nowadays, the global market and economy have made it difficult for companies to compete solely as individual entities. Many companies have realized the potential for achieving a competitive advantage through effectively coordinating different logistics participants in their supply chains. In the meanwhile, information technology and outsourcing have enabled companies to successfully operate a collaborative supply chain, in which each logistics participant focuses on only a few key strategic activities.

In this dissertation, we deal with the logistics coordination issues in two distinct vendor-buyer systems, where the vendor and buyer may represent any two upstream-downstream logistics participants that are independently managed, whether they belong to different companies or simply behave as such. For each vendor-buyer system studied, we investigate how the two parties can cooperate with each other in making decisions to increase both the system and the individual profitability. We have

(I). *Single-Channel Vendor-Buyer System with Complex Transportation Schemes*

In the single-channel vendor-buyer system studied, multiple products are shipped from a vendor to a buyer through a single channel. At the buyer, each product has a deterministic and constant demand; at the vendor, each product is supplied at the same rate as its demand. In the previous literature, it has been shown that system cost savings can be obtained through coordinating the vendor and buyer in such a single-channel system. Our research contributes to the literature by taking into account the industrial trend of outsourcing transportation to 3rd party logistics companies and by considering four complex transportation schemes. For each transportation scheme, we develop a vendor-buyer coordination model and investigate the optimal solution properties.

(II). *Dual-Channel Vendor-Buyer system with Minimum Purchase Commitment*

In the dual-channel vendor-buyer system studied, a single product is shipped from a vendor, through two distinct channels, to satisfy the stochastic demand at a buyer. The buyer replenishes its inventory with a *minimum purchase commitment* (MPC) as follows. In each time period, the buyer regularly places an order for a predetermined and fixed quantity, which is delivered from a distant vendor-owned central facility. The buyer also has option of placing an additional order for flexible quantity, which is delivered from a nearby vendor-owned regional facility. We develop two types of vendor-buyer coordination model and introduce a simulation-based method to quantitatively analyze each model.

Keywords: Logistics coordination, vendor-buyer system, integrated coordination, channel coordination, minimum purchase commitment, safety stock placement model.

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To the memory of my grandfather Wang Ming Fei (1928-2006)
and my grandmother Yang Zhan Hua (1919-2007)

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CHAPTER I

INTRODUCTION

How does one define *Supply Chain Management* or *Logistics Management*[1]? The answer has evolved in the last century since the term "*logistics*" was first associated with military during World War I (1905-1914), as a branch of war that pertains to the movement and supply for armies. By the end of World War II (1945), the logistics branch had been widely recognized as an effective way to ensure that the right military material is delivered to the right location at the right time. After the early 1950's, logistics management was accepted as an important activity to achieve successful operatios for a manufacturing company or service provider. In 1991, the international Council of Logistics Management (CLM) defined *Logistics Management* as "the process of planning, implementing, and controlling the efficient, effective flow and storage of goods, services, and related information from the point of origin to the point of consumption for the purpose of conforming to customer requirement".

The definition by CLM covers most activities and functions involved in today's logistics management except for an essential concept – *logistics coordination* – which has been raised by the increasingly global business world during the last two decades. Nowadays, the global market and economy have made it difficult for companies to compete solely as individual entities. Many companies have realized the potential for achieving competitive advantage through effectively coordinating different logistics participants in their supply chains. In the meanwhile, information technology and outsourcing have enabled companies to successfully operate with a collaborative supply chain, in which each logistics participant focuses on only a few specialized strategic activities.

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[1] We do not distinguish between *Supply Chain Management* and *Logistics Management* in this dissertation

In Simchi-Levi et al. (2003), *Supply Chain Management* was defined as "a set of approaches utilized to efficiently integrate suppliers, manufacturers, warehouses, and stores, so that merchandise is produced and distributed at the right quantities, to the right locations, and at the right time, in order to minimize system-wide costs while satisfying service level requirements". Compared with the earlier definitions of *Supply Chain Management* or *Logistics Management*, this definition by Simchi-Levi et al. emphasizes that suitable coordination mechanisms are essential to the development of an effective supply chain system, whose objective should be to ensure an optimal system-wide behavior, and preserve the autonomy and the local performance of the individual participant at the same time.

In 2006, the worldwide logistics cost was estimated at over 2 trillion dollars. In Singapore, the leading global logistics hub, logistics industry represents 8% of the country's GDP. The logistics industry's growth rate, however, is 11% compared to domestic GDP growth of 6% in 2006. More effective logistics coordination will definitely make a huge savings for the global and local production and transportation.

In this dissertation, we deal with logistics coordination issue in two distinct vendor-buyer systems: a *single-channel vendor-buyer system* and a *dual-channel vendor-buyer system*, where the vendor and buyer may represent any two upstream-downstream logistics participants that are independently managed, whether they belong to different companies or simply behave as such. For each vendor-buyer system studied, we focus on investigating how the two parties can cooperate with each other in making transportation and replenishment decisions to increase the supply chain profitability. Two types of vendor-buyer coordination mechanism are considered: *integrated coordination*, in which the vendor and buyer can fully cooperate with each other to maximize the system profit; and *channel coordination*, in which one party offers benefit to entice the other one to make decisions in a cooperative way.

Our research can serve as a building block and an assisting tool for several aspects in vendor-buyer coordination, supply chain network design and supply contracts negotiation. It also provides logistics managers the insights into the benefit of implementing certain logistics coordination scheme in their supply chain systems.

I.1 Scope of the Dissertation

I.1.1 Vendor-Buyer Coordination Models (Chapter II)

In Chapter II, we give a general discussion on the vendor-buyer coordination models studied in this dissertation, without specifying any vendor-buyer system parameter. We have

(I). *Integrated Coordination Model*

In an integrated coordination model, the vendor and buyer can fully cooperate with each other to make decisions that maximize the total system profit. The assumption of full cooperatio is usually valid in the practice of a *vendor-managed inventory* system; that is, the vendor is authorized to manage the buyer's inventories. Consequently, the vendor is responsible for making transportation and replenishment decisions to maximize the system profit. We show that the increased system profit needs to be allocated between the vendor and buyer so that the integrated coordination benefits both parties.

(II). *Channel Coordination Model*

In a channel coordination model, the party with less channel power initiates some coordination scheme to entice the other one to make decisions in a cooperative way that increases the individual profit for each party. We discuss using a purchase discount as the channel coordination scheme in a buyer-driven system, where the buyer has greater channel power than the vendor. The channel coordination automatically benefits both parties. Thus, no profit needs to be allocated between the vendor and buyer.

In addition to integrated coordination model and channel coordination model, we also discuss the *decentralized model* in which the vendor and buyer make decisions independently. As a consequence, the optimal system profit in a decentralized model is a lower bound for that of any corresponding coordination model. In our research, the decentralized model is used as a benchmark to evaluate the efficiency of certain vendor-buyer coordination mechanism.

For each vendor-buyer model discussed, we give the formulation with the profit-maximizing objective and investigate the properties of optimal individual and system profits. These discussion and observations are then used to support the studies on the specified logistics coordination issues in Chapter III and IV.

I.1.2 Single-Channel Vendor-Buyer Coordination Problems (Chapter III)

In Chapter III, we deal with a logistics coordination issue for a single-channel vendor-buyer system that consists of a vendor and a buyer. At the buyer, each product is demanded at a deterministic and constant demand rate; at the vendor, each product is supplied at the rate equal to its demand. These products are delivered from the vendor to the buyer through a single channel, which is characterized by a specified transportation scheme.

In the previous literature, it has been shown that coordinating the two parties' decisions in such a single-channel vendor-buyer system results in cost savings. In these works, however, the transportation cost is either ignored or assumed to be of a simple form. Our research contributes to the literature by incorporating more realistic transportation consideratios into the traditional single-channel vendor-buyer coordination problem. We take into account the industrial trend of outsourcing transportation to 3rd party logistics (3PL) companies and consider the situations where transportation is scheduled accounting for the transportation cost structure and transportation policy.

We consider two types of transportation cost structure as follows: *less-than-truckload incremental discount* (LID) *transportation cost structure*, which is meaningful when shipment quantity is less than the vehicle (or container) capacity; and *truckload discount* (TLD) *transportation cost structure*, which is meaningful when shipment quantity is close to or greater than the vehicle (or container) capacity. For the LID transportation cost structure, we assume that transportation is in the *less-than-truckload* (LTL) transportation mode and an incremental discount is provided to encourage larger shipment quantities. For the TLD transportation cost structure, we assume that transportation can be in both LTL and *truckload* (TL) transportation modes.

We consider two types of transportation policy as follows: *single-cycle continuous transportation policy* that requires all products to be shipped at a common frequency, which can take any positive value; and *multi-cycle discrete transportation policy* that allows each product to be partially shipped at different frequencies, which can only take discrete values from a given set.

By varying the transportation cost structure and transportation policy, we study four single-channel vendor-buyer coordination models. These models are shown as follows

	LID Trans. Cost	TLD Trans. Cost
Single-Cycle Continuous Trans. Policy	Model I (Section III.4.2)	Model III (Section III.5.2)
Multi-Cycle Discrete Trans. Policy	Model II (Section III.4.3)	Model IV (Section III.5.4)

For each single-channel vendor-buyer coordination model, the objective is to make optimal decisions for the vendor and buyer to minimize the system cost, which includes the inventory costs incurred at both locations and the transportation cost of shipping products between them. We also investigate the optimal solution properties for each model and give a numerical case study to present how this logistics coordination brings benefit to the single-channel vendor-buyer system with consideratio of the transportation costs and constraints.

I.1.3 Dual-Channel Vendor-Buyer System (Chapter IV)

In Chapter IV, we deal with a logistics coordination issue for Hewlett-Packard's ink cartridges supplies system, in which ink cartridges are shipped through central facilities and regional facilities to geographically scattered customers. We propose a dual-channel ink cartridges supplies system by utilizing additional channels to ship some amount of the ink cartridges from the central facilities directly to the customers, bypassing the intermediate regional facilities. In the proposed dual-channel ink cartridges supplies system, ink cartridges are supplied with a *Minimum Purchase Commitment* (MPC) agreement as follows.

At the beginning of each time period, a customer places *regular order* for a predetermined and fixed quantity, which is delivered from a central facility directly to the customer. We refer to such channel as *direct channel* because no intermediate regional facility is involved in shipping the regular orders. After receiving the regular order, the customer places no order if the resulting inventory position is greater than the order-up-to level, which is set to maintain a desired service level; otherwise the customer places a *supplementary order* from a regional facility to raise the inventory position to the order-up-to level. We refer to such channel as *indirect channel* because certain intermediate regional facility is involved in shipping the supplementary orders to the customer.

The Hewlett-Packard's ink cartridges supplies system is influenced by the proposed dual-channel supply strategy in the following aspects.

- *System supply cost*, which includes transportation cost and operating cost, is reduced because an efficient transportation mode (e.g. container loads) can be used for the direct deliveries and less ink cartridges are handled at the regional facilities.
- *Surplus inventories* are carried at the customers due to the commitment of placing a regular order for fixed quantity in each time period regardless of the realized demand.
- *Safety stocks* carried at the customers decrease because the surplus inventories can be used to satisfy the stochastic customer demand.
- *Safety stocks* carried at the central facilities and regional facilities decrease because some of the burden of handling demand variability is shifted to the customers.

For the purpose of simplicity, we consider a dual-channel vendor-buyer system that consists of a vendor-owned central facility, a vendor-owned regional facility, and a buyer. We investigate the logistics coordination issue concerning how the vendor and buyer can cooperate with each other in making an MPC agreement to improve the system profitability as well as their individual profitability. We develop and analyze both integrated coordination and channel coordination models, and introduce a simulation-based method to quantitatively analyze each model. A numerical case study of Hewlett-Packard's ink cartridges supplies business in Asia is also given to show how logistics coordination can bring benefit.

I.2 Organization of the Dissertation

The remainder of the dissertation is organized as follows. In Chapter II, we give an introductory discussion on two types of vendor-buyer coordination models. In Chapter III, we study the logistics coordination issue for a single-channel vendor-buyer system where multiple products are delivered accounting for specified transportation cost structure and transportation policy. In Chapter IV, we study the logistics coordination issue for a dual-channel vendor-buyer system where a single product is delivered through two supply channels with a minimum purchase commitment.

CHAPTER II

VENDOR-BUYER COORDINATION MODELS

In this chapter, we give an introductory discussion on the two types of vendor-buyer coordination model studied in this dissertation: an *integrated coordination model* in which the vendor and buyer can fully cooperate with each other in making decisions to maximize the total system profit, and a *channel coordination model* in which one party provides a certain coordination scheme to entice the other one to make decisions in a cooperative way that increases the individual profit for both parties. In addition, we also discuss a benchmark model, referred to as the *decentralized model*, in which there is no logistics coordination between the vendor and buyer.

We discuss these vendor-buyer models in a general context without specifying the system parameters (e.g. customer demand uncertainty, transportation cost structure etc). For each model, we give the profit-maximizing formulation and the optimal properties of individual and system profits. We also introduce the practice in which the corresponding coordination mechanism could be meaningful. These discussion and observations are then used to support the studies on the specified logistics coordination issues in Chapter III and IV.

This chapter is organized as follows. The decentralized model and channel power assumption are discussed in the next section. In Section II.2, we discuss the integrated coordination model and one of its major practices: *vendor-managed inventory* (VMI) system. In Section II.3, we discuss the channel coordination model and one of its major practices: purchase discount. This chapter concludes with a summary in section II.4.

II.1 Decentralized Model

In a *decentralized model*, there is no coordination between the vendor and buyer, which may represent any two upstream-downstream logistics participants that are independently managed. To analyze a decentralized model, we need a channel power assumption about which party has the power to drive the channel and to initiate the transaction to maximize its individual profit. Varying the channel power assumption, we have two types of decentralized model: vendor-driven decentralized model and buyer-driven decentralized model.

Vendor-Driven Decentralized Model

In a vendor-driven decentralized model, the vendor has the greater channel power and makes decisions (e.g. supply, replenishment, manufacturing etc) independently to maximize its individual profit. Consequently, the buyer has to make decisions (e.g. replenishment, selling etc) subject to the vendor's optimal decisions. The vendor-driven assumption usually makes sense in the situations where the vendor and buyer are two independent departments (or facilities) in the same company. For instance, the vendor may represent a warehouse owned by the logistics department and the buyer may represent a retail outlet owned by the marketing department. A vendor-driven decentralized model consists of two sub-problems as follows.

Vendor-Driven Decentralized Model

Vendor's problem		Buyer's problem	
max	E[Vendor's Profit]	*max*	E[Buyer's Profit]
s.t.	Vendor's constraints	*s.t.*	Vendor's optimal decisions
			Buyer's constraints

As illustrated above, the two sub-problems are solved in sequence. First, the vendor's problem aims to make the optimal decisions (e.g. manufacturing, replenishment, supply etc) for the vendor to maximize its individual profit. The buyer's problem, in turn, aims to make the optimal decisions (e.g. replenishment and selling price etc) for the buyer to maximize its individual profit, subject to the buyer's optimal decisions obtained in the vendor's problem.

Buyer-Driven Decentralized Model

In a buyer-driven decentralized model, the buyer has the greater channel power to initiate the replenishment decision to maximize its individual profit. The vendor needs to make decisions subject to the buyer's optimal decisions. The buyer-driven assumption usually makes sense in situations where the vendor and buyer belong to different companies. A buyer-driven decentralized model consists of two sub-problems as follows.

Buyer-Driven Decentralized Model

Buyer's problem		Vendor's problem	
max	E[Buyer's Profit]	*max*	E[Vendor's Profit]
s.t.	Buyer's constraints	*s.t.*	Buyer's optimal decisions
			Vendor's constraints

As illustrated above, the buyer makes decisions to maximize its individual profit in the buyer's problem. The buyer's optimal decisions, in turn, become one constraint in the vendor's problem, whose objective is to maximize the vendor's profit through making decisions on manufacturing, replenishment, supply etc.

Without any coordination between the vendor and buyer, a decentralized vendor-buyer system is profit inefficient because of *double marginalization*, which refers to the fact that the total system profit is divided between the two independent parties. Thus each party makes decisions considering only a portion of the total system profit. As a consequence, the decentralized model can be used as a benchmark to evaluate the efficiencies of the coordination models discussed in the following Section II.2 and Section II.3.

We use π_D, π_D^v and π_D^b to denote the total system profit, the vendor's profit and the buyer's profit, respectively in the optimal solution of a decentralized model. Due to the double marginalization, the optimal system profit π_D of a decentralized model is a lower bound for the optimal system profit of any corresponding coordination model.

II.2 Integrated Coordination Model

In an *integrated coordination model* (also known as *centralized coordination model*), the vendor and buyer fully cooperate with each other to make decisions that maximize the system profit. The assumption of full cooperatio usually makes sense in the practice of *vendor-managed inventory* (VMI) system, where the vendor, usually a manufacturer but sometimes a distributor, is authorized to manage the inventory levels at the buyer.

In a VMI system, the operatios of the vendor and buyer can be integrated through information sharing by using the information technologies such as *electronic data interchange* (EDI) or internet-based protocols. The vendor can use this information to plan production, schedule deliveries, and manage inventory levels at the buyer. As a consequence, system cost will likely be reduced while capacity utilization will be increased. These benefits of VMI have been widely recognized in different industries, especially in the retail industry. Many successful retailers such as Wal-Mart, Kmart and JCPenny were the pioneer firms to adopt VMI system. The popularity of VMI has led to the claim that it is the wave of the future and this concept will revolutionize the tactical decision in the distribution channel (see Andel, 1996, Burke, 1996 and Cottrill, 1997). For further discussion on the VMI system, we refer the reader to Xu et al. (2001), Waller et al. (1999) and Yao et al. (2005).

An integrated coordination model can be expressed as follows

Integrated Coordination Model

max E[Total System Profit]

s.t. Buyer's constraints

Vendor's constraints

We use π_I, π_I^v and π_I^b to denote the system profit, the vendor's profit and the buyer's profit in the optimal solution of an integrated coordination model. We have the following observations for any integrated coordination model.

(I). $\pi_I \geq \pi_D$. The optimal system profit π_I of an integrated coordination model is an upper bound on the optimal system profit π_D of the corresponding decentralized model.

This inequality comes from the double marginalization of the decentralized model as aforementioned (see §II.1). This is also consistent with the intuition that logistics coordination always benefits vendor-buyer system.

(II). $\pi_I^b \le \pi_D^b$ (Buyer-driven system). For a buyer-driven vendor-buyer system, the optimal buyer's profit π_I^b of the integrated coordination model is a lower bound for the counterpart π_D^b of the decentralized model.

This inequality can be interpreted by the fact that the dominant party can initiates the transactions to maximize its individual profit in a decentralized system. We have the inequality of $\pi_D^v \ge \pi_I^v$ for the case of a vendor-driven vendor-buyer system.

(III). $\pi_I^v \ge \pi_D^v$ (Buyer-driven system). For a buyer-driven vendor-buyer system, the optimal vendor's profit π_I^v of a buyer-driven integrated coordination model is an upper bound on the counterpart π_D^v of the corresponding buyer-driven decentralized model.

This inequality comes from the observations I and II. According to the similar argument, we have the inequality of $\pi_D^b \le \pi_I^b$ for the case of a vendor-driven vendor-buyer system.

(IV). *Participation Constraints*. This is referred to the fact that the integrated coordination should benefit both individual parties so that they intend to participate. However, from the observation II and III, we can see that the integrated coordination always damages the dominant party (e.g. the vendor in a vendor-driven system) and benefits the other one. Thus, the amount of increased system profit should be allocated between the two parties by some means so that the participation constraints are satisfied.

We use Π to denote the amount of profit allocated to the dominant party. For a buyer-driven integrated system, the participation constraints are given as follows.

$$\begin{aligned} &\pi_I^b - \pi_D^b - \Pi > 0 \\ &\pi_I^v - \pi_D^v + \Pi > 0 \end{aligned} \tag{2.1}$$

A simple way is to implement an *equal allocation scheme*, under which the increased system profit $\pi_I - \pi_D$ is equally allocated between the vendor and buyer. We have

$$\Pi = \frac{1}{2}\left[\left(\pi_I^v - \pi_D^v\right) - \left(\pi_I^b - \pi_D^b\right)\right]. \tag{2.2}$$

In practice, the vendor and buyer usually negotiate a contract to decide the allocated profit amount Π that satisfies the participation constraints (2.1).

In Chapter III and IV, we study the integrated coordination models for both the single-channel vendor-buyer system and the dual-channel vendor-buyer system, respectively.

II.3 Channel Coordination Model

For the integrated vendor-buyer coordination discussed in Section II.2, certain strategic alliance like a VMI system is required so that the vendor and buyer intend to share their information with each other to make decisions that maximize the total system profit. Such strategic alliance and information sharing, however, may not be desirable or feasible in some situations. In addition, profit allocation could be seen as price discrimination, and this is legally prohibited in some regions and countries. In those cases, we can resort to another coordination mechanism called *channel coordination* to increase the system profitability.

The term "*channel coordination*" was first coined in marketing literature to refer to the activities of improving the system profitability through certain channel coordination schemes (e.g. purchase agreement) between the vendor and buyer (see Tsay et al. 2000). In a *channel coordination model*, the party with less channel power initiates some channel coordination scheme to entice the other one to make decisions in a cooperative way that increases the individual profit for each party. A buyer-driven channel coordination model consists of two interacting sub-problems as follows.

Buyer-Driven Channel Coordination Model

Buyer's problem		Vendor's problem	
max	E[Buyer's Profit]	max	E[Vendor's Profit]
s.t.	Vendor's optimal decisions	s.t.	Buyer's optimal decisions
	Channel coordination scheme		Vendor's constraints
	Buyer's constraints		

As illustrated above, the two interacting sub-problems are solved as follows. The buyer's problem aims to make optimal decisions to maximize its individual profit subject to the channel coordination scheme provided by the vendor. The vendor's problem aims to make optimal decisions to channel coordination scheme that maximizes its individual profit subject to the buyer's optimal decisions. We assume that the buyer is a ratioal player so that its optimal decisions are predictable in the vendor's problem.

We use π_C, π_C^v and π_C^b to denote the system profit, the vendor's profit and the buyer's profit, respectively, in the optimal solution of a channel coordination model. We have observations as follows.

(I). $\pi_D \le \pi_C \le \pi_I$. The optimal system profit π_C in a channel coordination model is a lower bound for the counterpart π_I of the corresponding integrated coordination model, and an upper bound on the counterpart π_D of the corresponding decentralized model.

The first inequality comes from the double marginalization of the decentralized model as aforementioned (see §II.1). The second inequality is consistent with the intuition that "full coordination" outperforms "partial coordination" in benefiting the vendor-buyer system.

(II). $\pi_C^b \ge \pi_D^b$, $\pi_C^v \ge \pi_D^v$. The optimal buyer's profit π_C^b and the optimal vendor's profit π_C^v of a channel coordination model are upper bounds on the counterparts of the corresponding decentralized model.

These two inequalities constitute the *participation constraints* of channel coordination model; that is, a channel coordination scheme should benefit both the vendor and the buyer so that each individual party is willing to participate.

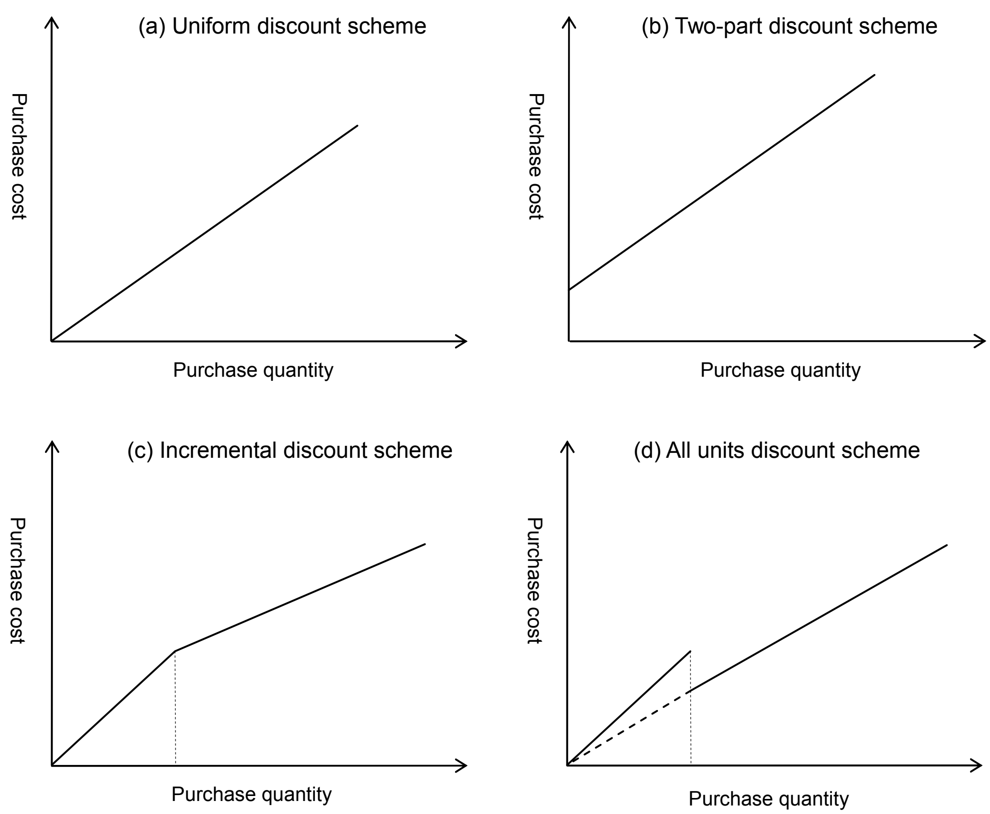

Figure 1. Four purchase discount schemes in channel-vendor coordination

In this dissertation, we consider using *purchase discount* as the channel coordination scheme: the vendor offers a discount based on the purchase quantity to entice the buyer to make decisions that increase both parties' individual profits. There are four popular purchase discount schemes as shown in Figure 1.

(I): *Uniform discount scheme* (see Figure 1-a): purchase discount is offered at a constant per-unit rate, independent of the purchase quantity. Uniform discount scheme is usually used to encourage the buyer to accept specified purchase contract (e.g. to accept a longer lead time, to accept a cheaper transportation mode) so that the vendor's profit is increased.

(II). *Two-part discount scheme* (see Figure 1-b), an additional fixed purchase cost is charged as long as the purchase quantity is positive.

(III). *Incremental discount scheme* (see Figure 1-c), the purchase discount is applied to the purchase quantities beyond given breakpoint.

(IV). *All units discount scheme* (see Figure 1-d), the purchase discount is applied to the entire quantity once it exceeds a given breakpoint.

Monahan (1984) studies a channel coordination model from the vendor's perspective for establishing an optimal all units discount scheme to maximize its individual profit, with the assumption that the vendor's fixed order cost is larger than the buyer's. Monahan argues that the vendor can benefit from less frequent orders from the buyer, which comes from the purchase discount. Banerjee (1986) extends and generalizes Monahan's work to take into account inventory cost. Lal and Staelin (1984) analyze a continuous decaying purchase discount scheme, which can be seen as an extreme case of the incremental discount scheme with infinite breakpoints.

II.4 Summary

In this Chapter, we gave an introductory discussion on the two types of vendor-buyer coordination model studied in this dissertation: an *integrated coordination model* in which the vendor and buyer can fully cooperate with each other in making decisions to maximize the total system profit, and a *channel coordination model* in which one party provides a certain coordination scheme to entice the other one to make decisions in a cooperative way that increases the individual profit for both parties. In addition, we also discussed a benchmark model, referred to as the *decentralized model*, in which there is no logistics coordination between the vendor and buyer.

For each model discussed, we gave the profit-maximizing formulation and the optimal properties of individual and system profits and introduced the practice in which the corresponding coordination mechanism could be meaningful. These discussion and observations are used to support the studies on the specified logistics coordination issues in Chapter III and IV. Four integrated coordination model are developed for the single-channel vendor-buyer system studied in Chapter III. An integrated coordination model and a channel coordination model are developed for dual-channel vendor-buyer system in Chapter IV.

Both the integrated coordination model and channel coordination model discussed are relatively simple in terms of the coordination mechanism. In practical situations, companies always coordinate by contracting on a set of transfer payments so that each party's objective becomes aligned with the system's objective. We refer to Cachon (2003) for more discussion on the vendor-buyer coordination using contracts. Our research, however, can still provide useful insights to logistics managers on designing contracts to coordination their vendor-buyer systems.

CHAPTER III

SINGLE-CHANNEL VENDOR-BUYER SYSTEM WITH COMPLEX TRANSPORTATION SCHEMES

In this chapter, we deal with a logistics coordination issue for a single-channel vendor-buyer system that consists of a vendor and a buyer. At the buyer, each product is demanded at a deterministic and constant demand rate; at the vendor, each product is supplied at the rate equal to its demand. These products are delivered from the vendor to the buyer through a single channel, which is characterized by a specified transportation scheme.

In the previous literature, it has been shown that coordinating the two parties' decisions in such a single-channel vendor-buyer system results in cost savings. In these works, however, the transportation cost is either ignored or assumed to be of a simple form. Our research contributes to the literature by incorporating more realistic transportation consideratios into the traditional single-channel vendor-buyer coordination problem. We take into account the industrial trend of outsourcing transportation to 3rd party logistics (3PL) companies and consider the situations where transportation is scheduled accounting for the transportation cost structure and transportation policy.

We consider two types of transportation cost structure as follows: *less-than-truckload incremental discount* (LID) *transportation cost structure*, which is meaningful when shipment quantity is less than the vehicle (or container) capacity; and *truckload discount* (TLD) *transportation cost structure*, which is meaningful when shipment quantity is close to or greater than the vehicle (or container) capacity. For the LID transportation cost structure, we assume that transportation is in the *less-than-truckload* (LTL) transportation mode and an incremental discount is provided to encourage larger shipment quantities. For the TLD transportation cost structure, we assume that transportation can be in both LTL and *truckload* (TL) transportation modes.

We consider two types of transportation policy as follows: *single-cycle continuous transportation policy* that requires all products to be shipped at a common frequency, which can take any positive value; and *multi-cycle discrete transportation policy* that allows each product to be partially shipped at different frequencies, which can only take discrete values from a given set.

By varying the transportation cost structure and transportation policy, we study four single-channel vendor-buyer coordination models. These models are shown as follows

	LID Trans. Cost	TLD Trans. Cost
Single-Cycle Continuous Trans. Policy	Model I (Section III.4.2)	Model III (Section III.5.2)
Multi-Cycle Discrete Trans. Policy	Model II (Section III.4.3)	Model IV (Section III.5.4)

For each single-channel vendor-buyer coordination model, the objective is to make optimal decisions for the vendor and buyer to minimize the system cost, which includes the inventory costs incurred at both locations and the transportation cost of shipping products between them. We also investigate the optimal solution properties for each model and give a numerical case study to present how this logistics coordination brings benefit to the single-channel vendor-buyer system with consideratio of the transportation costs and constraints.

The rest of this chapter is organized as follows. We address the research motivation in the next section. We review the relevant literature in Section III.2. The problem notation, assumptions, and formulations are given in Section III.3. We study four distinct single-channel vendor-buyer integrated coordination models in Section III.4 and III.5. A numerical case study is given in Section III.6 to show how the logistics coordination can benefit a single-channel vendor-buyer sytem. Finally, conclusions, managerial implications, and future research are given in Section III.7.

III.1 Research Motivation

Hong Kong, 21 March 2007 -- Hong Kong-based global consumer goods exporter Li & Fung Limited announced strong full-year results for 2006. Li & Fung reported robust turnover and profit growth of 22% and 23% respectively for 2006. Group turnover was HK$68,010 million (US$8.7 billion), 22% higher than 2005. Profit increased by 23% to HK $2,202 million. Strong growth momentum driven by organic business expansion and acquisitions continues into 2007. The Group has made good progress in line with the Plan's target of achieving a turnover of US$10 billion by the end of 2007.

Mr. William K. Fung, Group Managing Director of Li & Fung Limited, commented, "2006 was a year of solid growth for Li & Fung. The Group has continued to grow organically and gain market share in the markets in which it operates.

"The strong results also demonstrated that the Group has a very scalable business in which growth rates have not slowed down despite achieving a very high turnover. We see an increasing number of retailers worldwide recognizing the strategic value of partnering with Li & Fung to enable them to focus on their core competencies."

Founded in Guangzhou, the PRC in 1906, the Li & Fung Limited is a multinational export sourcing company. The company owns an extensive global sourcing network of over 70 offices covering 40 plus countries and territories around the world. With a growing network of nearly 10,000 international suppliers, Li & Fung explores the world to find quality-conscious, cost-effective manufacturers in order to provide the highest-quality goods, exceptional value and reliable, on-time delivery to its customers.

In addition to the wide range and variety of consumer products available through its sourcing network, Li & Fung also leverages its strengths in managing the entire supply chain - from product design and development, through raw material and factory sourcing, production planning and management, quality assurance and export documentation, to shipping consolidation.

In this research, the logistics network of Li & Fung is simplified as a single-channel vendor-buyer system, in which multiple products are delivered from a vendor (Li & Fung's outsourcing customers) to a buyer (Li & Fung's sourcing customers) through a single channel, which is characterized by a specified transportation scheme. The vendor and buyer may represent any two upstream-downstream logistics participants that are independently managed, whether they belong to different companies or simply behave as such. A typical single-channel vendor-buyer system is presented in Figure 2, where inventories are carried at both the vendor and buyer.

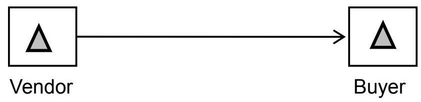

Figure 2. Single-channel vendor-buyer system

For such a single-channel vendor-buyer system, we deal with the logistics coordination problem that aims to make optimal shipment (or replenishment) decisions for each product to minimize the total system cost, which includes the inventory costs incurred at both locations and transportation cost of shipping products between them.

The single-channel vendor-buyer coordination problem models could also be meaningful and of interest in several practical cases as follows.

(I). <u>*Transportation Consolidation*</u>

In the case of transportation consolidation, products are supplied at different origins (e.g. factories) and demanded at different destinations (e.g. retailers). From the origins, products are shipped to a consolidation site and consolidated into the shipments to a breakbulk site, and then shipped to different destinations.

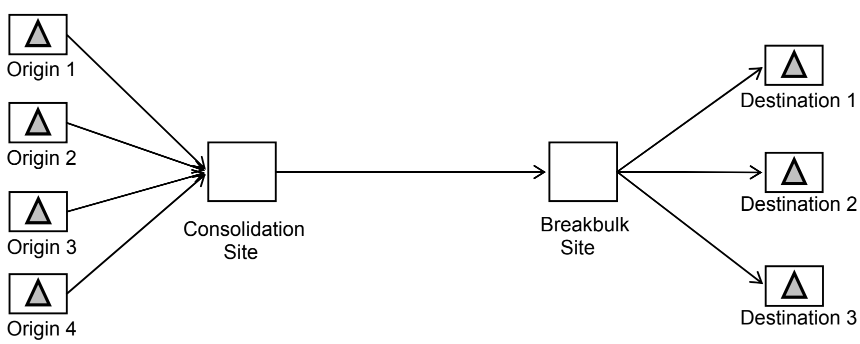

Figure 3. An example of transportation consolidation

An example of transportation consolidation is presented in Figure 3, where different products are shipped from four origins to three destinations. The origins and destinations are located far from each other so that directly delivering products between them will result in a high system transportation cost. By using a consolidation site and a breakbulk site, the logistics manager aims to achieve the economies of scale in the long-distance transportation between the two sites. As shown in Figure 3, we assume no inventories are carried at the consolidation and break bulk sites. This assumption is valid in practice since products are usually carried for less than 24 hours at the consolidation and break bulk sites. In addition, we neglect the short-distance transportation costs for shipping products between the origins and consolidation site, and between the break bulk site and destinations

For such a supply chain system with transportation consolidation, the logistics manager needs to decide and coordinate the shipment (or replenishment) schedule that minimizes the total system cost, which includes the inventory costs incurred at the origins and destinations and the transportation cost of shipping products between the consolidation and breakbulk sites. Thus, we can treat the transportation consolidation problem as a single-channel vendor-buyer coordination problem.

Speranza and Ukovich (1992) study a transportation consolidation problem about the IBM product distribution system in Italy, which is based on outsourcing production to external suppliers scattered on a large geographic area. A large number of products are shipped from different suppliers to the consolidation center at Padua, and then shipped to the central warehouse in Vimercate, about 250 kilometers from Padua. Speranza and Ukovich develop a single-channel vendor-buyer coordination model (also known as single-link model) to decide the optimal transportation schedule for shipping products between the consolidation center and the central warehouse. They also discuss using their model to evaluate the strategy of building new consolidation centers in other areas.

(II). *Network Decomposition*

In the case of network decomposition, we deal with a large scale supply chain network with dozens of origins and destinations. Then we might need to use multiple consolidation and breakbulk sites so that each origin and destination site can be served by at least one nearby consolidation and breakbulk site, respectively. The logistics manager needs to make the optimal shipment routing and schedule decisions that minimize the total system cost. Making simultaneous decisions on the shipment routing and schedule, however, could be prohibitive due to the system complexity. A two-step algorithm can be used to obtain an approximate solution as follows.

In the first step, we decompose the whole supply chain network to single shipping links based on a feasible shipment routing schedule. For each shipping link, we can obtain the local optimal shipment schedule by solving a single-channel vendor-buyer coordination problem. Then, an approximate global optimal solution can be identified by enumerating all the feasible shipment routing schedules. An example of network decomposition is shown as follows.

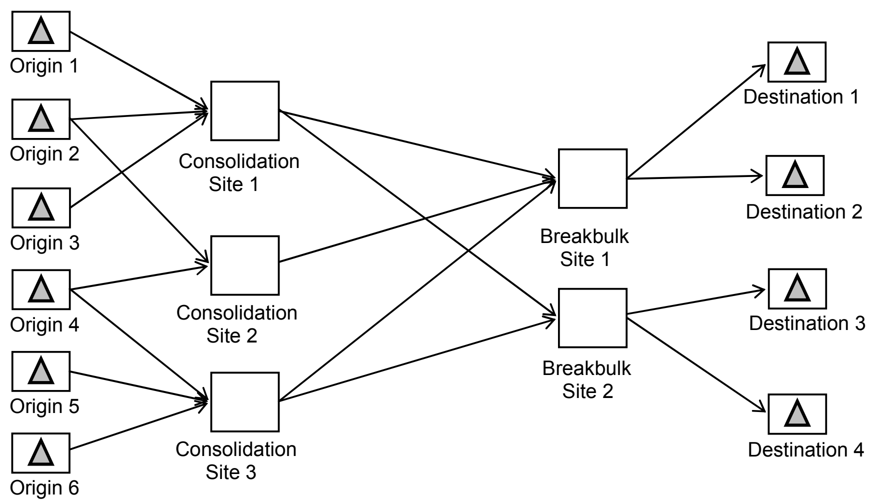

Figure 4. An example of network decomposition

Blumenfeld et al. (1987) introduce a network decomposition problem for the huge distribution system of General Motors in the United States and Canada, which includes 20,000 suppliers, 160 assembly plants, and 11,000 dealers. They decompose the whole distribution system into hundreds of single shipping links and get the local optimal shipment schedule for each shipping link. In addition, they simplify the problem by considering all the products shipped on each link as a composite product, which is characterized with the average weighted unit volume and the average weighted holding cost rate for all products assigned to the shipping link. They show that a total system cost savings of 26% has been obtained by implementing the network decomposition approach in the distribution system of General Motors.

(III). *Inbound Transportation Management*

The single-channel vendor-buyer coordination model can also be used to help the logistics manager to evaluate the procurement strategies of *free-on-board* (FOB) *destination* and *free-on- board* (FOB) *origin* and to manage the inbound transportation.

In the survey of Gentry (1991), the 678 respondents use *FOB destination* for 42.6% of their purchase transactions; that is, their vendors are responsible for shipping products and charge the transportation cost as part of the product price. However, there is the opportunity for companies to reduce procurement costs by purchasing *FOB origin*; that is, the buyers are responsible for collecting and shipping products from their vendors. Buxbaum (1995) describes a successful case of implementing the inbound transportation management in a $27 billion company GTE. In 1993, GTE paid an $84 million transportation bill out of $5 billion purchase. To reduce the procurement cost, the GTE logistics department audited suppliers' freight invoices and found markups of up to 60% in shipping charges. GTE decided to purchase *FOB origin* from all its suppliers, and control the inbound shipments, freight dispatch and so on. In 1994, GTE succeeded in reducing procurement costs by $14 million.

The single-channel vendor-buyer coordination problem can provide the managerial insights how the *FOB origin* procurement can benefit the buyer by making replenishment decision with consideratio of both transportation cost and inventory cost.

In the rest of this research, we focus on the single-channel vendor-buyer system consisting of a vendor and a buyer as presented in Figure 2. All the results and observations can be applied to the practical cases discussed in this section.

III.2 Literature Review

In this section, we review the literature in the single-channel vendor-buyer integrated coordination models. We categorize the literature into two types based on the different system cost structure assumption.

(I). *Vendor-Buyer Coordination Modes: Replenishment Cost + Inventory Cost*

In the first type of literature, the system cost includes the inventory costs and replenishment costs incurred at both the vendor and buyer. Goyal (1976) studies such a single-channel vendor-buyer coordination problem as follows,

- At the buyer, a single product is demanded at deterministic and constant rate d.
- The buyer periodically replenishes its inventory in quantity Q_b from the vendor, which in turn periodically replenishes its inventory in quantity Q_v from an external supplier. The replenishment quantity Q_v is an integer multiple of the replenishment quantity Q_b; that is, $Q_v = mQ_b$ (integer m). In Goyal (1976), the inventory levels at the vendor and buyer have the patterns as illustrated in Figure 5.

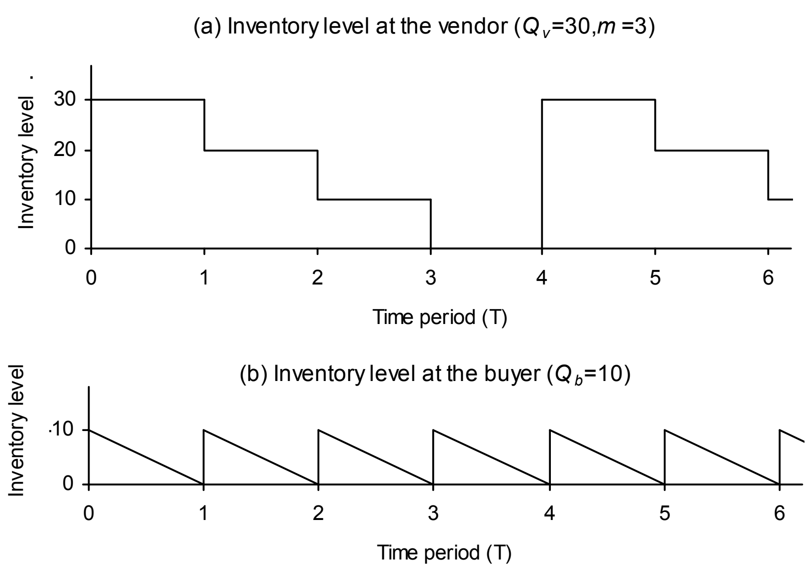

Figure 5. An example of the inventory levels in Goyal (1976)

- At the vendor, the fixed replenishment cost S_v is assumed to be greater than the counterpart S_b incurred at the buyer: $S_v > S_b$.
- At the vendor, inventory holding cost rate h_v is assumed to be smaller than the counterpart h_b at the buyer: $h_v < h_b$.

The Goyal's problem aims to make the optimal decisions on the buyer's replenishment quantity Q_b and on the integer m to minimize the total system cost $C_{sys}(Q_b,m)$, which can be expressed as follows,

$$C_{sys}(Q_b,m)=\frac{S_b d}{Q_b}+\frac{Q_b h_b}{2}+\frac{S_v d}{mQ_b}+\frac{h_v(m-1)Q_b}{2}. \quad (3.1)$$

In (3.1), the first two terms represent the replenishment cost and inventory cost incurred at the buyer, and the last two terms represent the replenishment cost and inventory cost incurred at the vendor. Goyal provides a two-step exact algorithm to first derive the optimal system cost as a function of the integer m; then, to enumerate all the possible values of integer m to get the optimal replenishment quantity Q_b and integer m.

Goyal also develops a buyer-driven decentralized model, in which the buyer replenishes with the *EOQ* quantity of $\sqrt{2dS_b/h_b}$. Then, the vendor needs to make decisions on its replenishment quantity Q_v, which satisfies the relationship of $Q_v = m\sqrt{2dS_b/h_b}$. Goyal finds that the buyer replenishes more frequently in the decentralized model and, in consequence, reduces his own costs whilst increasing the vendor's costs by a greater amount.

Toptal et al. (2003) extend the Goyal's problem by considering both inbound and outbound transportation costs incurred at the vendor. They assume that both the inbound transportation and the outbound transportation are by truckload (TL) mode, in which a constant transportation cost of C is charged for every journey performed by one truck with given capacity P. Thus, the total system cost $C_{sys}(Q_b,m)$ can be expressed as follows,

$$C_{sys}(Q_b,m)=\frac{S_b d}{Q_b}+\frac{Q_b h_b}{2}+\left\lceil\frac{Q_b}{P}\right\rceil\frac{Cd}{Q_b}+\frac{S_v d}{mQ_b}+\frac{h_v(m-1)Q_b}{2}+\left\lceil\frac{mQ_b}{P}\right\rceil\frac{Cd}{mQ_b}. \quad (3.2)$$

In (3.2), the first two terms represent the buyer's replenishment cost and inventory cost, and the fourth and fifth terms represent the vendor's replenishment cost and inventory cost. The third and last terms represent the outbound and inbound TL transportation costs at the vendor. They develop heuristic algorithms to obtain the approximate solutions with promising error bounds of 25% and 6%. Using the approximate solutions as upper bounds, they give a procedure to get the exact optimal solution in finite time.

(II). *Vendor-Buyer Coordination Models: Transportation Cost + Inventory Cost*

In the second type of literature, the system cost includes the inventory costs at both the vendor and buyer and the transportation cost of shipping the product(s) between them. Speranza and Ukovich (1994) study such a single-channel vendor-buyer coordination problem (also known as *single-link problem*) as follows.

- At the buyer, multiple products are demanded at deterministic and constant rates, and periodically replenished from the vendor.
- At the vendor, the products are continuously produced at the rates equal to their demands. In Speranza and Ukovich (1992), the inventory levels at the vendor and buyer have the patterns as illustrated in Figure 6.

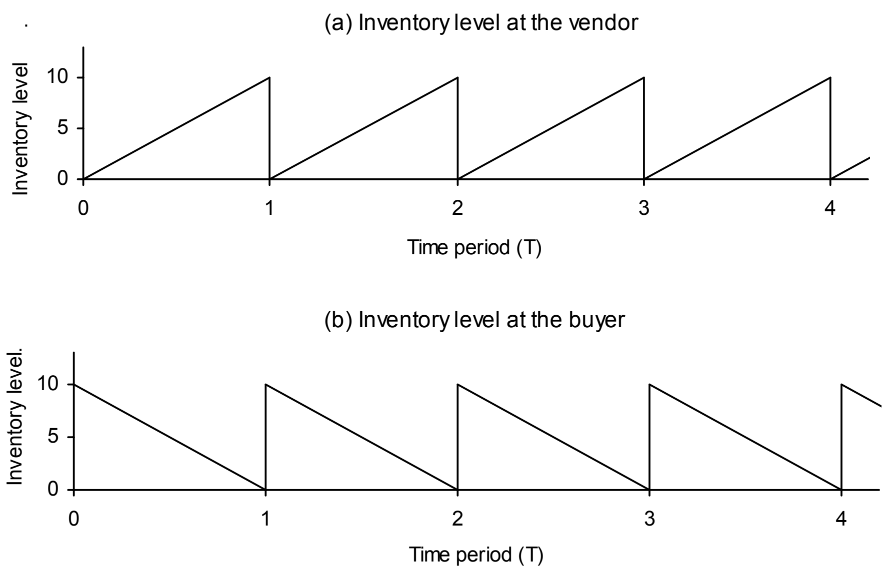

Figure 6. An example of the inventory levels in Speranza and Ukovich (1992)

- The products are shipped in trucks with a given capacity P and one journey performed by a single truck costs the constant transportation cost C.
- The shipments are scheduled at discrete time periods, e.g., a certain time of the day or a certain day of the week.
- No fixed production and replenishment costs are considered.

The single-link problem aims to make shipment decision for each product to minimize the total system cost, which includes TL transportation cost and inventory costs. The single-link problem is similar to the one studied in Section III.4.4 except that we consider an additional less-than-truckload (LTL) transportation mode.

Speranza and Ukovich (1996) prove that the classical single-link problem is NP-hard and develop a branch-and-bound algorithm to efficiently solve the small and medium size problem instances. Bertazzi et al. (2000) derive dominance rules for the single-link problem solution to allow a tightening of the bounds on the problem variables and improve the efficiency of the branch-and-bound algorithm introduced in Speranza and Ukovich (1996). They also give some heuristics and compare them with two different modifications of the EOQ-based solution for the continuous shipment case.

Bertazzi et al. (1997) extend the single-link problem by considering a supply chain network consisting of one vendor and multiple buyers. They present different heuristics by solving a single-link problem for each of the given buyers first, and then on improving the solution through local search techniques. Bertazzi and Speranza (2002) give a unifying framework for the identification of optimal continuous and discrete frequency cases for the single-link problem. They show that the optimal solutions of the different frequency restriction policies can be ranked and the distance between the optimal costs can be unlimited in the worse case. For a more detailed literature review on the single-link problem, we refer the interested reader to Bertazzi and Speranza (2002) and the survey paper of Bertazzi and Speranza (1999).

In all the literature reviewed above, the transportation cost is either ignored or assumed to be associated with a simple truckload (TL) mode in these single-channel vendor-buyer coordination problems. Our research contributes to the literature by considering the situation where products can be shipped in two alternative transportation modes of *less-than-truckload* (LTL) and *truckload* (TL). In particular, we allow an incremental discount to encourage larger shipments in LTL transportation mode. To the best of our knowledge, such single-channel vendor-buyer coordination problem has not been studied before.

III.3 Notation, Assumptions, and Formulations

In this section, we introduce the notation, assumptions and formulations of the single -channel vendor-buyer coordination problem. The notation is as follows.

Table 1. Notation for the single-channel vendor-buyer system with complex transportation scheme

Product Parameters	
k, K	Index and set of products, $k \in K$.
d_k	Demand rate of product k (quantity per time period).
v_k	Volume per unit of product k.
h_k^v, h_k^b	Holding cost rates of product k at the vendor and buyer ($ per unit per time period).
Transportation Scheme: Transportation Policy Parameters	
f	Shipment frequency.
j, J	Index and set of the discrete shipment frequencies, $j \in J$.
f_j	The jth shipment frequency in the given discrete set, $j \in J$.
n, N	Index and set of time periods, $n \in N$ and $\lvert N \rvert = 1/\gcd\{f_j : \forall j \in J\}$.
δ_j^n	Binary coefficient indicating whether $f_j \cdot n =$ integer.
Transportation Scheme: Transportation Cost Structure Parameters	
r, R	Index and set of LID shipment quantity breakpoints, $r \in R$.
P_r	The rth LID shipment quantity breakpoint (volume).
S_r, α_r	The rth fixed and variable LID transportation costs, given $P_{r-1} \le Q < P_r$.
P	Capacity of given vehicle (or container).
S, α	Fixed and variable TLD transportation costs ($ per unit volume).
C	Full truckload transportation cost.
η	Total number of vehicles (or containers) used.
$G(Q)$	Transportation cost function.
Decision Variables	
Q	Shipment quantity (for Model I & III).
x_k^j	Fraction of product k shipped at the jth discrete frequency f_j (for Model II & IV).

III.3.1 Single-Channel Vendor-Buyer System

In the single-channel vendor-buyer system (see Figure 2), multiple products are demanded at the buyer, supplied at the vendor, and shipped through a single channel between the two locations. We use k and K to denote the index and set of the products. Each product is characterized by four parameters: demand rate d_k, volume v_k, and holding cost rates h_k^v and h_k^b.

(I). *Demand Rate d*

We use d_k to denote the demand rate of product k at the buyer. For each product, we assume the demand rate d_k is deterministic, constant, and price-insensitive. This demand assumption is a reasonable approximation for medium and long term planning contexts. We also note that the single-channel vendor-buyer coordination problems with dynamic and stochastic demands have received attention in recent research, e.g., see Chan et al. (2000) and Ben-Daya (2004).

(II). *Volume v*

We use v_k to denote the physical volume of unit product k and assume all the products have the same density. The transportation cost is charged based on the volume, rather than quantity or weight. We ignore the shipment category issue, in which transportation cost structure differs according to the average density value of products shipped. The shipment category issue is studied by Hall (1991), Fleischmann (1993), Barnhart et al. (1993) and Lapierre et al. (2004). The objective of these works is to investigate how to make decisions on the mix of heavy and light products to improve the utilization of transportation capacity.

(III). *Holding Cost Rates h*

At the vendor and buyer, the inventory cost is proportional to the average inventory level and holding cost rate. We denote by h_k^v and h_k^b the holding cost rates of product k at the vendor and buyer, respectively. In addition, we assume that carrying the same amount of product at the buyer costs more than at the vendor; that is, $h_k^v < h_k^b$. We use h_k to denote the sum of h_k^v and h_k^b, that is, $h_k = h_k^v + h_k^b$.

III.3.2 Transportation Scheme: Dual-Mode Transportation Cost Structure

As aforementioned, our research contributes to the literature by taking into account the industrial trend of outsourcing transportation to 3rd party logistics (3PL) companies, and by considering the situations where transportation is scheduled accounting for the transportation cost structure and transportation policy.

We consider a dual-mode transportation cost structure, in which products can be shipped in two transportation modes: *less-than-truckload* (LTL) mode and *truckload* (TL) mode. In addition, we assume there is an incremental discount that encourages larger shipments in LTL transportation mode. Such a dual-mode transportation cost structure with LTL incremental discount is presented in the Figure 7.

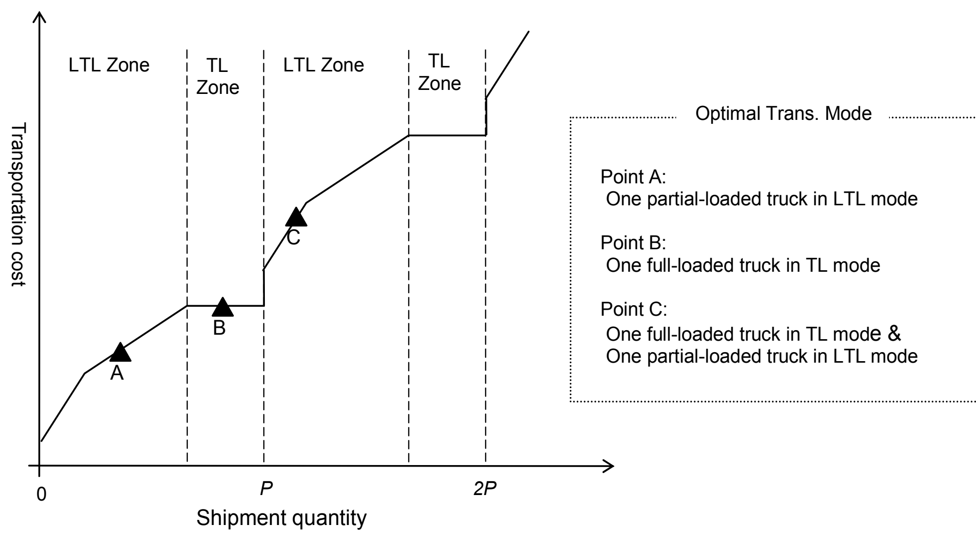

Figure 7. An example of dual-mode transportation cost structure with LTL incremental discount

The dual-mode transportation cost structure can be interpreted as follows. When the shipment quantity is less than the truckload *P* (e.g. point A in Figure 7), it is optimal for the shipper to choose the LTL transportation mode and pay the LTL incremental discount transportation cost. Once the shipper has paid for the full truckload transportation cost, it is optimal to choose TL transportation mode and, in consequence, there is no charge for the remaining shipment quantities in that truckload (e.g. point B in Figure 7). Once the first truck is full loaded, the shipper chooses a combination of the two transportation modes by shipping the excess quantity in LTL transportation mode (e.g. point C in Figure 7), and so forth.

By using the dual-mode transportation cost structure, the logistics manager can make simultaneous decisions on the optimal shipment quantity and the corresponding transportation mode selected. In addition, the dual-mode transportation cost structure is also meaningful in the situation where the shipper owns a private freight fleet and intends to outsource a certain amount of shipment in LTL mode to an external freight company when its private freight fleet is fully utilized.

However, the complexity of the dual-mode transportation cost structure makes it difficult to formulate and solve the single-channel vendor-buyer coordination problem with a dual-mode transportation cost structure. For the purpose of simplicity, we decompose the dual-mode transportation cost structure into two simple transportation cost structures as follows.

(I). *Less-Than-Truckload Incremental Discount (LID) Transportation Cost Structure*

In the case of less-than-truckload incremental (LID) transportation cost structure, the shipment quantities are less than truckload *P*, and the shipper mainly deals with the trade-off between the inventory cost and the transportation discount, e.g., to ship a larger quantity to obtain more transportation discount while increasing the inventory cost, or to ship a smaller quantity to decrease inventory cost but with less transportation discount. We discuss and formulate the LID transportation cost structure in Section III.4.1, and study two single-channel vendor-buyer coordination models (Model I&II) with LID transportation cost structure in Section III.4.2 and Section III.4.3.

(II). *Truckload Discount (TLD) Transportation Cost Structure*

In the case of truckload discount (TLD) transportation cost structure, the shipment quantities are close to or larger than the truckload *P*, and the shipper mainly deals with the transportation mode selection, e.g., to ship all the products in full loads to decrease the transportation cost, or to ship some amount of the products in partial loads to decrease the inventory cost. In addition, we ignore the LTL transportation discount issue and focus on the transportation mode selection issue. We discuss and formulate the TLD transportation cost structure in Section III.5.1, and study two single-channel vendor-buyer coordination models (Model III&IV) with TLD transportation cost structure in Section III.5.2 and Section III.5.4.

III.3.3 Transportation Scheme: Transportation Policies

In addition to the transportation cost structures discussed in Section III.3.2, we also consider two transportation policies, *single-cycle continuous transportation policy* and *multi-cycle discrete transportation policy*, which dictate when and how shipments can be scheduled.

(I). *Single-Cycle Continuous Transportation Policy*

Under the single-cycle continuous transportation policy, all the products need to be periodically shipped at a common frequency, which can take any positive value. Thus, we can treat all the products as a composite product characterized by demand rate d, unit volume v and holding cost rate h. We have

$$d = \sum_{k \in K} d_k \ . \tag{3.3}$$

$$v = \sum_{k \in K} v_k d_k \Big/ \sum_{k \in K} d_k \ , \tag{3.4}$$

$$h = \sum_{k \in K} \left[\left(h_k^v + h_k^b \right) d_k \right] \Big/ \sum_{k \in K} d_k \ . \tag{3.5}$$

The composite product demand rate d is equal to the sum of product demand rate d_k. The composite product unit volume v and holding cost rate h are equal to the average weighted unit volume and holding cost rate of all the products. We use f and Q to denote the shipment frequency and shipment quantity. We have

$$Q = \frac{1}{f} vd \ . \tag{3.6}$$

Under the single-cycle continuous transportation policy, a single-channel vendor-buyer coordination problem's objective is to find the optimal shipment quantity Q^* (or shipment frequency f^*) that minimizes the total system cost.

The single-cycle continuous transportation policy, however, may produce an impractical shipment schedule to implement such as shipping products every 1.4142 days (see discussion in Hall (1985a) and Muckstadt and Roundy(1993)). In addition, the shipment departure timing may be restricted by the freight company, for instance, a fixed departure schedule of every Monday morning for the sea freight from New York to Singapore. Furthermore, to ship all the products at a common frequency may not be a good strategy since the expensive products should be replenished more frequently than the cheaper ones. In these situations, a multi-cycle discrete transportation policy could be meaningful and of interest.

(II). *Multi-Cycle Discrete Transportation Policy*

Under the multi-cycle discrete transportation policy, each product is allowed to be partially shipped at different frequencies, which should take discrete values from a given set. We use j and J to denote the index and set of the given discrete shipment frequencies, and use f_j to denote the value of the jth frequency. Let the decision variable x_k^j denote the fraction of product k shipped (or replenished) at the jth frequency f_j. We assume each product can be continuously divided so that x_k^j can take any value between 0 and 1. Thus we have

$$0 \le x_k^j \le 1 \quad \text{and} \quad \sum_{j \in J} x_k^j = 1. \tag{3.7}$$

Then, a single-channel vendor-buyer coordination problem's objective is to find the optimal replenishment schedule X_k^j that minimizes the total system cost. Across consecutive time periods, the shipment quantities have a cycle pattern since products are periodically replenished in discrete frequencies. Thus, to calculate the average transportation cost, we only need to consider a finite time horizon $|N|$, which is equal to the reciprocal *greatest common divisor* (gcd) of all the shipment frequencies:

$$|N| = 1/lcm\{1/f_j : \forall j \in J\}. \tag{3.8}$$

We use n to denote the index of the time period, which is assumed to start in 0 and end in $|N|$-1.

To illustrate the notation introduced above, we give an example of shipping four products A, B, C, and D with parameters as follows.

	A	B	C	D
Demand rate d_k (per day)	4	5	2	2
Unit volume v_k	2	1.5	2.5	1
Shipment frequency f (1/day)	1	1/2	1/4	1/8
Replenishment order size	8	15	20	16

Based on the parameters given above, the shipment quantity pattern is shown in Figure 8. We observe that the cyclic shipment quantity pattern repeats every 8 time periods, which is equal to the value of $|N| = 1/\text{gcd}(1,\ 1/2,\ 1/4,\ 1/8) = 8$. Thus, to calculate the long term average system transportation cost, we only need to consider a finite time horizon of any consecutive 8 time periods (e.g. time period 0 - 7).

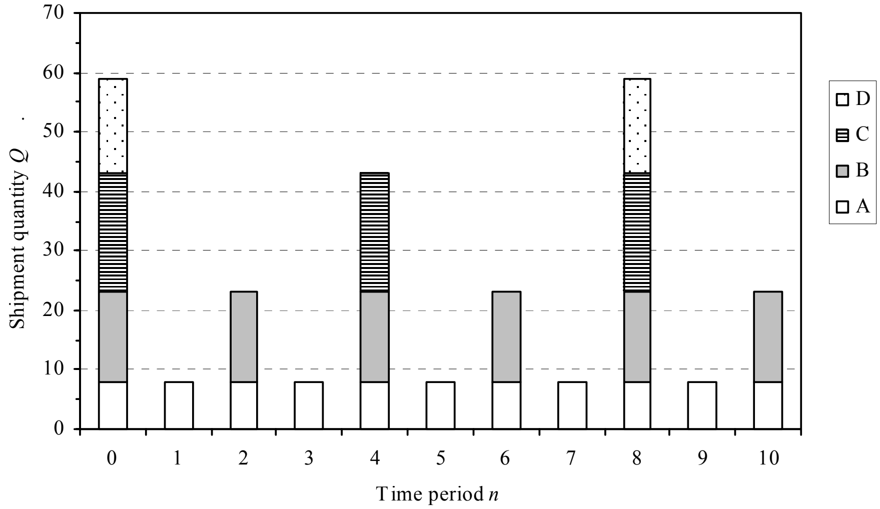

Figure 8. An example of shipment quantity pattern with multi-cycle discrete transportation policy

To calculate the average transportation cost, we also need the assumption about how the products are consolidated into shipments in each time period; for instance, whether the deliveries of all the products can be outsourced to a single external freight company or only the deliveries in the same frequency can be outsourced to a single external freight company. We consider two types of shipment consolidation rule: *frequency consolidation rule* and *time period consolidation rule*.

(II.1). *Frequency Consolidation Rule in Multi-Cycle Discrete Transportation Policy*

In the frequency consolidation rule, only the products with the same shipment frequency can be consolidated into one shipment in each time period. We use Q_j to denote the shipment quantity with the jth shipment frequency f_j, and $G(\cdot)$ to denote the transportation cost function. We denote by C_{trans} the average system transportation cost per time period. We have

$$Q_j = \sum_{k \in K} \frac{1}{f_j} d_k v_k x_k^j, \qquad \forall j \in J \tag{3.9}$$

$$C_{trans} = \sum_{j \in J} G(Q_j) f_j \tag{3.10}$$

For the numerical example shown in Figure 8, we have

$$Q_j : [Q_1 = 8,\ Q_2 = 15,\ Q_3 = 20,\ Q_4 = 16,\ ], \tag{3.11}$$

$$C_{trans} = \left[G(8) + \frac{1}{2} G(15) + \frac{1}{4} G(20) + \frac{1}{8} G(16) \right]. \tag{3.12}$$

(II.2). *Time Period Consolidation Rule in Multi-Cycle Discrete Transportation Policy*

In the time period consolidation rule, all the products can be consolidated into one shipment in each time period. We use Q_n to denote the total shipment quantity in time period n. We use a binary coefficient δ_f^n to denote if the shipment at the rth frequency f_j happens in time period n. In addition, we assume that all the products are shipped in time period 0. Thus we have

$$Q_n = \sum_{j\in J}\sum_{k\in K}\frac{1}{f_j}d_k v_k x_k^f \delta_f^n, \qquad \forall n\in N \tag{3.13}$$

$$C_{trans} = \frac{1}{|N|}\sum_{n\in N}G(Q_n) \tag{3.14}$$

For the numerical example shown in Figure 8, we have

$$Q_n : [Q_0 = 59,\ Q_1 = 8,\ Q_2 = 23,\ Q_3 = 8,\ Q_4 = 43,\ Q_5 = 8,\ Q_6 = 23,\ Q_7 = 8], \tag{3.15}$$

$$C_{trans} = \frac{1}{8}[4G(8) + 2G(23) + G(43) + G(59)]. \tag{3.16}$$

By assuming all the products are shipped in certain time period (e.g. time period 0), we ignore the *shipment phase issue* in which the shipments with different frequencies can be scheduled in different time periods. The shipment phase issue could be meaningful when the transportation capacity is limited. For instance, if the product B is delivered in time period 1 with the rest of the products delivered in time period 0, we have the shipment quantities Q_n: [Q_0=44, Q_1=23, Q_2=8, Q_3=23, Q_4=28, Q_5=23, Q_6=8, Q_7=23] and the average transportation cost C_{trans}=1/8[2G(8)+4G(23)+G(28)+G(44)]. We can see that the maximum shipment quantity decreases from 59 to 44 when the product B is delivered in time period 1.

In our research, we assume the transportation capacity is unlimited. We conjecture that it is optimal to ship all the products in certain time period if the transportation cost function $G(\cdot)$ is concave. We do not give the proof since it is beyond the scope of our research

Besides the transportation policies discussed above, another possible transportation policy may allow each product to be partially shipped at different frequencies, which can take any continuous value. Obviously, such multi-cycle continuously policy increases the problem complexity. We observe that the multi-cycle discrete policy may provide a reasonably close solution when the minimum discrete value is small and the given set is large.

III.3.4 Four Single-Channel Vendor-Buyer Coordination Models

By varying the transportation cost structure and transportation policy, we have four transportation schemes. We study the single-channel vendor-buyer integrated coordination model for each transportation scheme, as follows.

	LID Trans. Cost	TLD Trans. Cost
Single-Cycle Continuous Trans. Policy	Model I (Section III.4.2)	Model III (Section III.5.2)
Multi-Cycle Discrete Trans. Policy	Model II (Section III.4.3)	Model IV (Section III.5.4)

These four single-channel vendor-buyer models could be meaningful and of interest in the practical situations as follows.

Model I: LTL trans. cost structure + single-cycle continuous trans. policy

As we discussed, this model could be meaningful when the transportation is outsourced to an external transportation provider and the transportation quantity is relatively small. The unit volume costs of products should not vary a lot so that it is not necessary to ship the products in different frequencies. For instance, Model I could be used to optimize the delivery quantity between a central warehouse and a local wholesaler for several types of electronics products.

Model II: LTL trans. cost structure + multi-cycle discrete trans. policy

In Model II, each product is allowed to be partially shipped at different frequencies, which should take discrete values. The possible discrete frequency values may depend on the shipment schedule of the external transportation provider. For instance, Model II could be used to optimize the delivery schedule of long distant sea freight for several types of maintenance parts, which differ a lot in their values.

Model III: TLD trans. cost structure + single-cycle continuous trans. policy

In Model III, the transportation quantities are relatively large and transportation discount is offered to encourage the full-truck loaded shipment. The unit volume costs of products should not vary a lot so that it is not necessary to ship the products in different frequencies. For instance, Model III could be used to optimize the long distant delivery quantity for a single type of electronic product.

Model IV: TLD trans. cost structure + multi-cycle discrete trans. policy

In Model IV, the transportation quantities are relatively large and transportation discount is offered to encourage the full-truck loaded shipment. The unit volume costs of products should not vary a lot so that it is not necessary to ship the products in different frequencies. Each product is allowed to be partially shipped at different frequencies, which should take discrete values. The possible discrete frequency values may depend on the shipment schedule of the external transportation provider. For instance, the Model IV could be used to optimize the delivery schedule for a trade company to supply dozens of different products by sea freight.

In Section III.4., we assume the shipment quantities are less than the vehicle (or container) capacity and transportation cost is charged according to the less-than-truckload incremental discount (LID) structure. In Section III.5, we assume the shipment quantities are close to or larger than the vehicle (or container) capacity and transportation cost is charged according to the truckload discount (TLD) structure.

III.4 Single-Channel Vendor-Buyer Coordination Models with LID Transportation

In this section, we study two single-channel vendor-buyer coordination models with the assumption that the shipment quantities are less than the vehicle (or container) capacity. Thus, all the products are shipped in a single LTL transportation mode, and shipments are charged according to a *less-than-truck incremental discount* (LID) *transportation cost structure*.

This section is organized as follows. We introduce and formulate the LID transportation cost structure in Subsection III.4.1. Two single-channel vendor-buyer coordination models are introduced and studied in Subsection III.4.2 and III.4.3, with consideratio of single-cycle continuous transportation policy and multi-cycle discrete transportation policy, respectively.

III.4.1 Less-Than-Truckload Incremental Discount (LID) Transportation Cost Structure

For the case of LID transportation cost structure, the shipment quantities are less than the given vehicle capacity and an incremental quantity discount is applied to the additional shipment quantities beyond the predetermined breakpoints (see Figure 9).

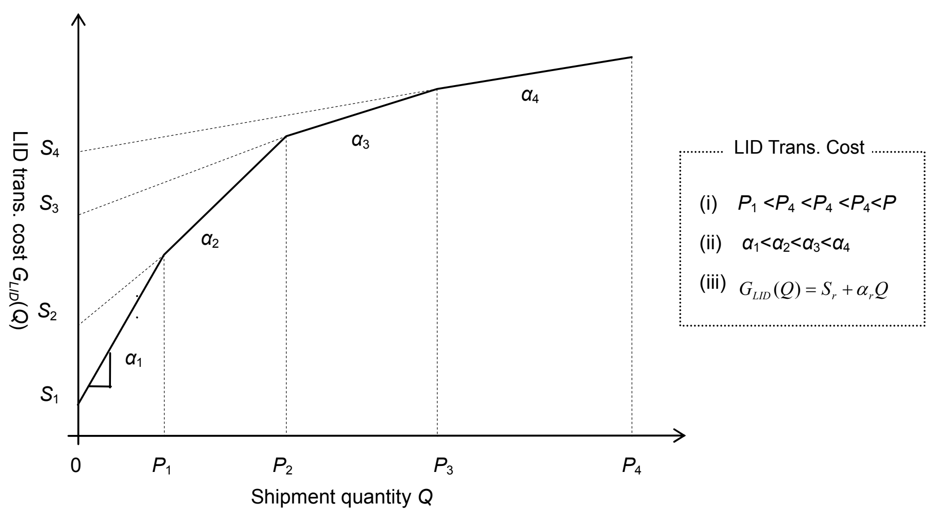

Figure 9. Less-than-truckload incremental discount (LID) transportation cost structure

As presented in Balakrishnan and Graves (1989), the LID transportation cost structure can be modeled as a piece-wise linear and concave function of the shipment quantity Q. We use $G_{LID}(Q)$ to denote the LID transportation cost function with the following assumptions.

- Let r and R be the index and set of LID quantity breakpoints, (r =0, 1,…, R).
- Let P_r denote the value of rth quantity breakpoint. P_0= 0 and P_R should be not greater than the truckload P.
- Let α_r denote the variable transportation cost per unit for those quantities between breakpoints $P_{r\text{-}1}$ and P_r. The α_r can be interpreted as the slope of the corresponding rth line segment in Figure 9.
- Let S_r be the y-intercept of the linear extention of the rth line segment in Figure 9. Thus, the LID transportation cost is equal to the sum of a fixed cost S_r and a variable cost $\alpha_r Q$, given the shipment quantity $Q \in (P_{r-1}, P_r]$. The LID transportation cost function $G_{LID}(Q)$ can be formulated as a piece-wise linear concave cost function as follows,

$$G_{LID}(Q) = S_r + \alpha_r Q \text{, given } Q \in (P_{r-1}, P_r]. \tag{3.17}$$

We use $\Delta G_{LID}(Q)$ to denote the incremental LID transportation cost function, which is defined as the transportation cost for an additional unit volume to the shipment quantity Q.

$$\Delta G_{LID}(Q) = \frac{\partial [G_{LID}(Q)]}{\partial Q} = \alpha_r, \quad \text{given } Q \in (P_{r-1}, P_r]. \tag{3.18}$$

The incremental LID transportation cost function $\Delta G_{LID}(Q)$ is shown in Figure 10.

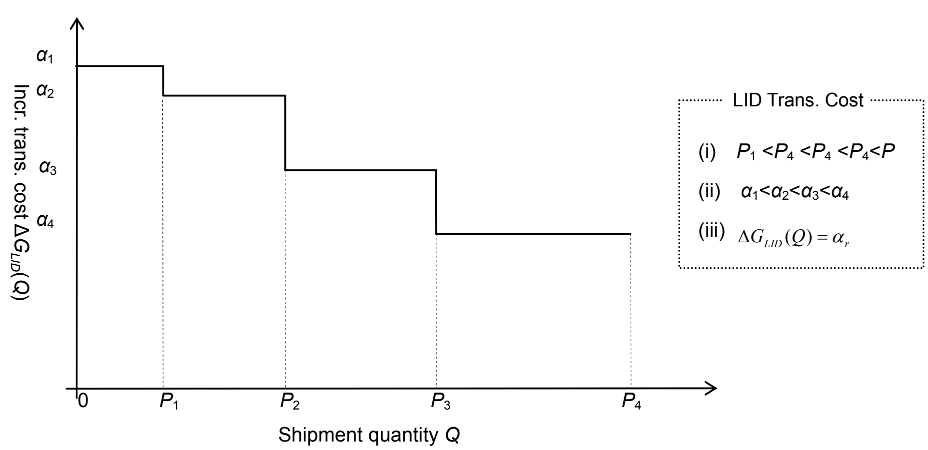

Figure 10. Incremental LID transportation cost structure

We use $\overline{G}_{LID}(Q)$ to denote the average LID transportation cost function, which is defined as the average transportation cost per unit volume. We have:

$$\overline{G}_{LID}(Q) = \frac{G_{LID}(Q)}{Q} = \frac{S_r}{Q} + \alpha_r \text{, given } Q \in (P_{r-1}, P_r]. \tag{3.19}$$

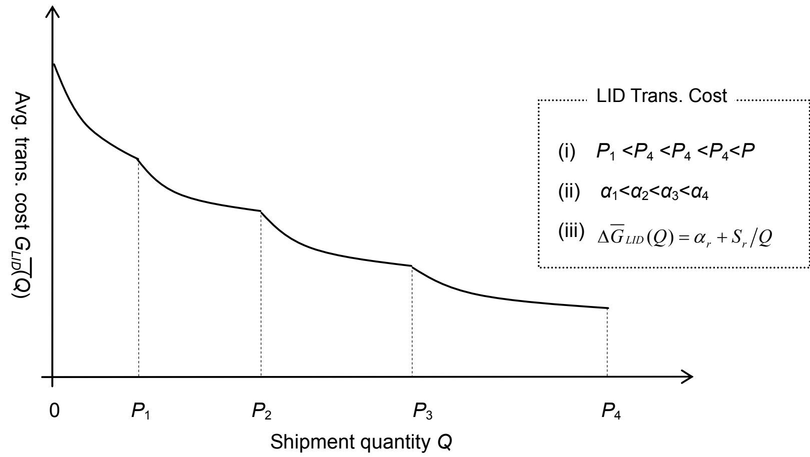

Figure 11. Average LID transportation cost structure

As illustrated in Figure 11, the average LID transportation cost function $\overline{G}_{LID}(Q)$ is a decreasing function of the shipment quantity Q.

III.4.2 Model I: A Model with LID Transportation Cost Structure and Single-Cycle Continuous Transportation Policy

The first single-channel vendor-buyer coordination model is characterized by

- *Single less-than-truckload (LTL) transportation mode.*
- *Less-than-truckload incremental (LID) transportation cost structure.*
- *Single-cycle continuous transportation policy.*

In Model I, all the products should be shipped at a common frequency, which can take any positive value. Thus, all the products can be treated as a composite product, which is characterized by demand d, unit volume v, and holding cost h. The objective is to find the optimal shipment quantity Q that minimizes the system cost per time period $TC_I(Q)$. We have

$$TC_I(Q) = vd\overline{G}_{LID}(Q) + \frac{hQ}{2v}. \tag{3.20}$$

The first term represents the transportation cost per time period, and the second term represents the inventory cost per time period. The average system cost $TC_I(Q)$ can be rewritten as (3.21), and shown in Figure 12.

$$TC_I(Q) = \left(\frac{S_r}{Q} + \alpha_r\right)vd + \frac{hQ}{2v}, \text{ given } Q \in \left(P_{r-1}, P_r\right] \tag{3.21}$$

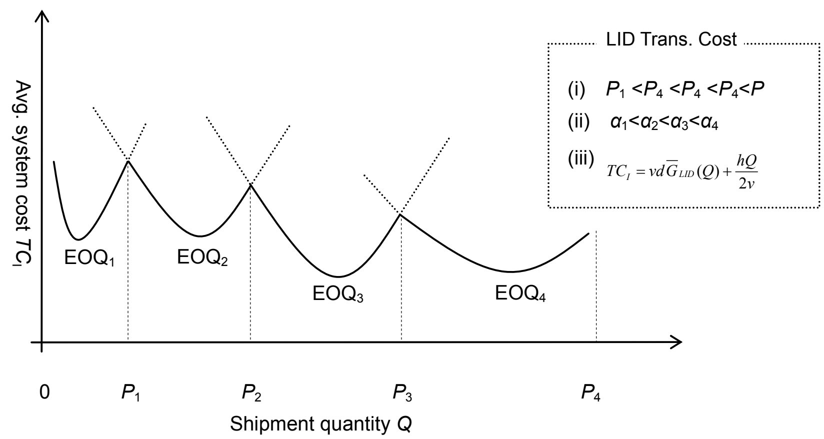

Figure 12. Average system cost function $TC_I(Q)$ function for model I

From the piece-wise convexity of average system cost $TC_I(Q)$ shown in figure 12, we conjecture that we only need to enumerate the breakpoints P_r and EOQ values to find the optimal shipment quantity Q_I^*. The EOQ values are

$$EOQ_r = v \cdot \sqrt{(2S_r d)/h}, \qquad \forall r \in R \tag{3.22}$$

We do not need to enumerate the truckload P since we assume that the shipment quantities are less than P. Furthermore, we have the following proposition.

PROPOSITION III.1. For the Model I, the optimal shipment quantity Q_I^* never takes the value of breakpoints P_r.

PROOF. This can be simply proved by contradiction. Suppose the optimal shipment quantity $Q_I^* = P_r \ (r \in R)$, then the average system cost $TC_I(Q)$ should be decreasing in the half open range $(P_r\text{-}\varepsilon, P_r]$ and increasing in the half open range $(P_r, P_r\text{+}\varepsilon]$, where ε is a very small number. Thus, we have the inequalities of $EOQ_r \geq P_r$ and $EOQ_{r+1} \leq P_r$, which are equivalent to the following inequality.

$$v \cdot \sqrt{(2S_r d)/h} \geq v \cdot \sqrt{(2S_{r+1} d)/h}\ . \tag{3.23}$$

From (3.23), we have the inequality $S_r \geq S_{r+1}$, which contradicts the LID transportation cost assumption that $S_r < S_{r+1}, \quad \forall r \in R$. Therefore, Proposition III.1 is true. □

Therefore, the optimal shipment quantity Q_I^* only takes the value of a certain EOQ value. We can solve the Model I by enumerating all the EOQ values:

$$Q_I^* = \underset{Q=EOQ_r\ (r \in R)}{\arg\min}\ TC_I(Q) \tag{3.24}$$

III.4.3 Model II: A Model with LID Transportation Cost Structure and Multi-Cycle Discrete Transportation Policy

The second single-channel vendor-buyer coordination model is characterized by

- *Single less-than-truckload (LTL) transportation mode.*
- *Less-than-truckload incremental (LID) transportation cost structure.*
- *Multi-cycle discrete transportation policy.*
- *Time period consolidation rule.*

In Model II, each product can be partially replenished at different frequencies, which take discrete values from a given set. We define decision variable x_k^j as the fraction of the demand for product k that is shipped (or replenished) at frequency f_j. Thus, the objective is to find the optimal solution X_k^j that minimizes the average system cost per time period $TC_{\text{II}}(X_k^j)$.We introduce a mixed integer programming (MIP) formulation for Model II.

For each pair of shipment frequency $j \in J$ and time period $n \in N$, we define the binary coefficient δ_j^n to denote if the shipment with frequency f_j is scheduled in the time period n. We define the binary variable y_n^r to denote if the shipment quantity Q_n falls in the half open range of (P_{r-1}, P_r]. Variable Q_n^r is equal to shipment quantity Q_n if the latter falls in the half open range of (P_{r-1}, P_r]. These variables are:

x_k^j = fraction of product k shipped at the jth frequency f_j, $\sum_{j \in J} x_k^j = 1, \forall k \in K$.

δ_j^n = 1, if shipment with frequency f_j is scheduled in time period n,
= 0, otherwise.

Q_n = the shipment quantity delivered in time period n.

y_n^r = 1, if shipment quantity $Q_n \in (P_{r-1}, P_r]$ in time period n,
= 0, otherwise.

Q_n^r = Q_n, if shipment quantity $Q_n \in (P_{r-1}, P_r]$ in time period n,
= 0, otherwise.

Min $\sum_{k\in K}\sum_{j\in J}\left(\frac{1}{2f_j}h_k d_k x_k^j\right)+\frac{1}{|N|}\sum_{n\in N}\sum_{r\in R}\left(S_r y_n^r+\alpha_r Q_n^r\right)$ (3.25)

s.t. $\sum_{j\in J} x_k^j = 1$, $\forall k\in K$ (3.26)

$\sum_{k\in K}\sum_{j\in J}\left(\frac{1}{f_j}d_k v_k x_k^j \delta_j^n\right)=\sum_{r\in R} Q_n^r$, $\forall n\in N$ (3.27)

$Q_n^r \le P_r y_n^r$, $\forall n\in N,\ r\in R$ (3.28)

$Q_n^r \ge P_{r-1} y_n^r$, $\forall n\in N,\ r\in R$ (3.29)

$\sum_{r\in R} y_n^r \le 1$, $\forall n\in N$ (3.30)

$0 \le x_k^j \le 1$, $\forall j\in J,\ k\in K$ (3.31)

$y_n^r \in \{0,1\}$, $\forall n\in N,\ r\in R$ (3.32)

$Q_n^r \ge 0$, $\forall n\in N,\ r\in R$ (3.33)

The first term in objective (3.25) represents the system inventory cost per time period, and the second term represents the system transportation cost per time period. Constraints (3.26) ensure that each product is completely assigned shipment frequencies, and constraints (3.27) set the total shipment quantity each time period. Constraints (3.28)-(3.30) are used to model the piece-wise linear and concave LID transportation cost structure. Constraints (3.28) and (3.29) make sure that if LID transportation cost index *r* is used in time period *n*, then the shipment quantity must fall in its associated interval (P_{r-1}, P_r]. Finally constraints (3.32) indicate that at most one cost range can be selected for each time period.

The MIP problem described above is more complex than the single-link problem studied in Speranza and Ukovich (1996), which is proved to be NP-hard. Thus, the Model II is at least NP-hard and, we can improve a solution procedure by implementing a branch-and-bound algorithm with the following optimal solution properties III.2 and III.3.

We use $f_{\min}^k$ and $f_{\max}^k$ to denote the minimum and maximum of the shipment frequencies assigned to product *k* in the optimal solution X_k^j. We have

$f_{\min}^k = \min\left(f_j : x_k^j > 0\right)$ $\forall k\in K$ (3.34)

$f_{\max}^k = \max\left(f_j : x_k^j > 0\right)$ $\forall k\in K$ (3.35)

PROPOSITION III.2. For the Model II, let the products be indexed in an increasing order of the ratio h_k/v_k; that is, $(h_1/v_1)<(h_2/v_2)<\ldots<(h_k/v_k)$. Then, the optimal solution has the following relationship on the shipment frequencies of products,

$$f^1_{\min} \le f^1_{\max} \le f^2_{\min} \le f^2_{\max} \le \ldots \le f^K_{\min} \le f^K_{\max} \tag{3.36}$$

PROOF. To prove (3.36), it is sufficient to prove the following statement: product a should not be delivered more frequently than product b given that $(h_a/v_a)<(h_b/v_b)$. That is

$$f^a_{\max} \le f^b_{\min}\text{, given } (h_a/v_a)<(h_b/v_b) \quad \forall a,b \in K. \tag{3.37}$$

The statement (3.37) can be proved by contradiction as follows. Suppose there exists an optimal solution X^j_k where we have $f^a_{\max} > f^b_{\min}$ for products a and b. We can construct a new feasible solution X'^j_k with a very small value ε as follows. For the proportion $\varepsilon/(v_a d_a)$ of demand d_a that shipped at frequency $f^a_{\max}$ (the ith frequency, $i \in J$) and the proportion $\varepsilon/(v_b d_b)$ of demand d_b that shipped at frequency $f^b_{\min}$ (the lth frequency, $l \in J$), we change their shipment frequencies to each other. That is, we reduce x^i_α by $\varepsilon/(v_a d_a)$ and increase x^l_a by $\varepsilon/(v_a d_a)$ for product a, and for product b, we reduce x^l_b by $\varepsilon/(v_b d_b)$ and increase x^i_b by $\varepsilon/(v_b d_b)$. Thus, the shipment quantity Q_n in the new feasible solution X'^j_k remains the same as the one in the original solution X^j_k, so does the system transportation cost. As a consequence, the system cost difference Δ is only attributable to the inventory cost for the amount $\varepsilon/(v_a d_a)$ of product a and the amount $\varepsilon/(v_b d_b)$ of product b. We have

$$\begin{aligned}\Delta &= \left[\frac{1}{2f^b_{\min}}\frac{\varepsilon}{v_a d_a} d_a h_a + \frac{1}{2f^a_{\max}}\frac{\varepsilon}{v_b d_b} d_b h_b\right] - \left[\frac{1}{2f^a_{\max}}\frac{\varepsilon}{v_a d_a} d_a h_a + \frac{1}{2f^b_{\min}}\frac{\varepsilon}{v_b d_b} d_b h_b\right] \\ &= -\frac{1}{2}\varepsilon\left(\frac{h_b}{v_b}-\frac{h_a}{v_a}\right)\left(\frac{1}{f^b_{\min}}-\frac{1}{f^a_{\max}}\right) < 0\end{aligned} \tag{3.38}$$

The first term represents the inventory cost for the amount $\varepsilon/(v_a d_a)$ of product a and the amount $\varepsilon/(v_b d_b)$ of product b in the new solution X'^j_k, and the second term represents the counterpart in the original solution X^j_k. The inequality holds due to the assumptions of $f^a_{\max} > f^b_{\min}$ and $(h_a/v_a)<(h_b/v_b)$. Thus the new solution X'^j_k outperforms the original solution X^j_k, which contradicts the optimality of original solution X^j_k. Statement (3.37) is true. Therefore, Proposition III.2 is true. □

Since the proof of Proposition III.2 does not need any specified assumption of transportation cost structure, it is true for a type II single-channel vendor-buyer coordination model, regardless of the transportation cost structure.

PROPOSITION III.3. For the Model II, it is optimal to assign the single shipment frequency to each product. Furthermore, it is optimal to assign the single shipment frequency to all the products of the same ratio of h_k/v_k. That is,

$$x_k^j \in \{0,1\}, \qquad \forall j \in J, k \in K; \tag{3.39}$$

$$x_a^j = x_b^j, \qquad \text{given } (h_a/v_a) = (h_b/v_b) \quad \forall a,b \in K. \tag{3.40}$$

PROOF: Since each product can be divided and shipped at different shipment frequencies, we can treat different shipment frequencies for a single product as equivalent to having different products with the same ratio of h_k/v_k being shipped at different frequencies. Therefore, to prove Proposition III.3, it is sufficient to prove the statement that "*for all the products with the same ratio of* h_k/v_k, *it is optimal to assign them the same shipment frequency*". And this is proved by contradiction as follows.

Suppose there exists a unique optimal solution X_k^j in which products a and b are assigned the different single shipment frequencies j_a and j_b ($j_a \neq j_b$). The expected average system cost associated with solution X_k^t is:

$$TC(X_k^j) = \sum_{k \in K} \frac{1}{2 f_{j_k}} h_k d_k + \frac{1}{|N|} \sum_{n \in N} [S_n + \alpha_n Q_n] \tag{3.41}$$

Let's construct two feasible solutions X'^j_k and X''^j_k identical to the optimal solution X_k^j except for the shipment frequencies assigned to the product a and b:

$$X'^j_k: \; j_b' = j_a, \qquad \text{and } j_k' = j_k \;\; \forall k \neq b \tag{3.42}$$

$$X''^j_k: \; j_a'' = j_b, \qquad \text{and } j_k'' = j_k \;\; \forall k \neq a \tag{3.43}$$

We use N_k to denote the set of time periods at which product k is shipped ($N_k \in N$). Thus, product a and b should have the different shipment time period sets: N_a and N_b in the origin solutions X_k^j. The expected average system cost associated with the new solution X'^j_k can be written as

$$
\begin{aligned}
TC(X'^{j}_{k}) &= \sum_{k\in K}\frac{1}{2f_{j'_k}}h_k d_k + \frac{1}{|N|}\sum_{n\in N}G_{LID}(Q'_n)\\
&= \frac{1}{2f_{j_a}}h_b d_b + \sum_{k\in K\setminus\{b\}}\frac{1}{2f_{j_k}}h_k d_k + \frac{1}{|N|}\{\sum_{n\in N\setminus(N_a\cup N_b)}G_{LID}(Q_n) + \sum_{n\in N_a\setminus N_b}G_{LID}(Q_n+\frac{1}{f_{j_a}}v_b d_b)\\
&+ \sum_{n\in N_b\setminus N_a}G_{LID}(Q_n-\frac{1}{f_{j_b}}v_b d_b) + \sum_{n\in N_a\cap N_b}G_{LID}(Q_n+v_b d_b(\frac{1}{f_{j_a}}-\frac{1}{f_{j_b}}))\}\\
&\le \frac{1}{2f_{j_a}}h_b d_b + \sum_{k\in K\setminus\{b\}}\frac{1}{2f_{j_k}}h_k d_k + \frac{1}{|N|}\{\sum_{n\in N\setminus(N_a\cup N_b)}G_{LID}(Q_n) + \sum_{n\in N_a\setminus N_b}[G_{LID}(Q_n)+\frac{1}{f_{j_a}}\alpha_n v_b d_b] \qquad (3.44)\\
&+ \sum_{n\in N_b\setminus N_a}[G_{LID}(Q_n)+\alpha_n(-\frac{1}{f_{j_b}}v_b d_b)] + \sum_{n\in N_a\cap N_b}[G_{LID}(Q_n)+\alpha_n v_b d_b(\frac{1}{f_{j_a}}-\frac{1}{f_{j_b}})]\}\\
&\le \frac{1}{2}h_b d_b(\frac{1}{f_{j_a}}-\frac{1}{f_{j_b}}) + \sum_{k\in K}\frac{1}{2f_{j_k}}h_k d_k + \frac{1}{|N|}\{\sum_{n\in N}G_{LID}(Q_n) + \sum_{n\in N_a}\frac{1}{f_{j_a}}\alpha_n v_b d_b - \sum_{n\in N_b}\frac{1}{f_{j_b}}\alpha_n v_b d_b\}
\end{aligned}
$$

In (3.44), the first inequality holds because of the concavity and monotonicity of the LID transportation cost structure $G_{LID}(Q)$; that is:

$$
G_{LID}(Q_n+\delta) \le G_{LID}(Q_n) + \alpha_r\delta, \qquad \text{given } Q_n \in (P_{r-1}, P_r]. \qquad (3.45)
$$

Then the difference in expected average system costs between solution X'^{j}_{k} and X^{j}_{k} is:

$$
\begin{aligned}
\Delta' &= TC(X'^{j}_{k}) - TC(X^{j}_{k})\\
&\le \frac{1}{2}\left(\frac{1}{f_{j_a}}-\frac{1}{f_{j_b}}\right)h_b d_b + \frac{1}{|N|}\left(\sum_{n\in N_a}\frac{1}{f_{j_a}}\alpha_n v_b d_b - \sum_{n\in N_b}\frac{1}{f_{j_b}}\alpha_n v_b d_b\right) \qquad (3.46)\\
&= d_b v_b\left[\frac{h_b}{2v_b}\left(\frac{1}{f_{j_a}}-\frac{1}{f_{j_b}}\right) + \frac{1}{|N|}\left(\sum_{n\in N_a}\alpha_n\frac{1}{f_{j_a}} - \sum_{n\in N_b}\alpha_n\frac{1}{f_{j_b}})\right)\right]
\end{aligned}
$$

For the similar argument, we have the difference in expected average system costs between solution X''^{j}_{k} and X^{j}_{k} as follows.

$$
\begin{aligned}
\Delta'' &= TC(X''^{j}_{k}) - TC(X^{j}_{k})\\
&\le \frac{1}{2}\left(\frac{1}{f_{j_b}}-\frac{1}{f_{j_a}}\right)h_a d_a + \frac{1}{|N|}\left(\sum_{n\in N_b}\frac{1}{f_{j_b}}\alpha_n v_a d_a - \sum_{n\in N_a}\frac{1}{f_{j_a}}\alpha_n v_a d_a\right) \qquad (3.47)\\
&\le -d_a v_a\left[\frac{h_a}{2v_a}\left(\frac{1}{f_{j_a}}-\frac{1}{f_{j_b}}\right) + \frac{1}{|N|}\left(\sum_{n\in N_a}\alpha_n\frac{1}{f_{j_a}} - \sum_{n\in N_b}\alpha_n\frac{1}{f_{j_b}})\right)\right] = -\frac{d_a v_a}{d_b v_b}\Delta'
\end{aligned}
$$

In (3.47), the last equality holds because of the assumption $(h_a/v_a) = (h_b/v_b)$. Since we have $\min(\Delta', \Delta'') = \min(\Delta', -\frac{d_a v_a}{d_b v_b}\Delta') \le 0$, the original solution X^{j}_{k} does not outperform both of the new feasible solutions X'^{j}_{k} and X''^{j}_{k}. And this contradicts the initial assumption that X^{j}_{k} is the unique optimal solution. Therefore, Proposition III.3 is true.□

Given the Proposition III.2 and III.3, a type II single-channel vendor-buyer coordination model should have the optimal solution that satisfies the following properties.

(I). Each product should be shipped (or replenished) at a single discrete frequency, even though we permit a product to be shipped at multiple frequencies. This optimal solution property holds for any single-channel vendor-buyer coordination model with concave transportation cost structure and multi-cycle discrete transportation policy.

(II). All products with the same ratio h_k/v_k should be shipped at the same frequency.

(III). The product with lower value of h_k/v_k should never be shipped more frequently than the product with higher value of h_k/v_k. The optimal properties II and III reflect the trade-off between the inventory cost charged on product unit and the transportation cost charged on product volume.

In practice, we can group the products with similar values of h_k/v_k to reduce the problem complexity. The optimal solution property (III) can be used to improve the branch-and-bound algorithm for solving the MIP problem ((3.25)-(3.33)).

III.5 Single-Channel Vendor-Buyer Coordination Models with TLD Transportation

In this section, we study two single-channel vendor-buyer coordination models with the assumption that the shipment quantities are close or greater than the vehicle (or container) capacity. Thus, the products could be shipped in either single one or a combination of *less-than-truckload* (LTL) and *truckload* (TL) transportation modes. The shipments are charged according to a *truckload discount* (TLD) *transportation cost structure*.

This section is organized as follows. We introduce the TLD transportation cost structure in Subsection III.5.1. Two single-channel vendor-buyer coordination models are studied in Subsection III.5.2 and III.5.4. An extended topic on the dependence between transportation mode and shipment quantity of the Model III is discussed in Subsection III.5.3.

III.5.1 Truckload Discount (TLD) Transportation Cost Structure

In the TLD transportation cost structure, the shipment quantities are supposed to be close or greater than the capacity of given vehicle (or container) and the shipments can be in *less-then-truckload* (LTL) and *truckload* (TL) transportation modes as follows.

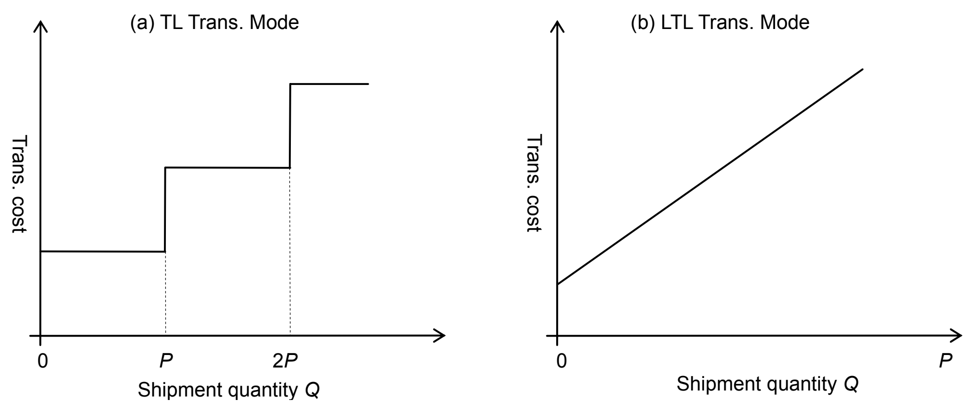

Figure 13. Two transportation modes in TLD transportation cost structure

In the TL transportation mode (see Figure 13-a), the transportation cost is proportional to the number of journeys performed by a vehicle. Let P denote the vehicle capacity in volume and C denote the corresponding transportation cost for one journey performed by a truck. We use $G_{TL}(Q)$ to denote the stepwise TL transportation cost function:

$$G_{TL}(Q) = \lceil Q/P \rceil C. \tag{3.48}$$

In the LTL transportation mode (see Figure 13-b), transportation cost consists of fixed and variable components. We use S to denote the fixed transportation cost and use α to denote the variable transportation cost. We have a linear transportation cost function $G_{LTL}(Q)$:

$$G_{LTL}(Q) = S + \alpha Q. \tag{3.49}$$

We also assume that the inequality $\alpha > (C/P)$ holds to ensure that the TL transportation mode could be selected in certain situations. Given these TL and LTL transportation modes, a logistics manager needs to make simultaneous decisions on the shipment quantity Q, the LTL shipment quantity Q_{LTL}, and the TL shipment quantity Q_{TL}. We have

$$\text{Min} \quad G_{TL}(Q_{TL}) + G_{LTL}(Q_{LTL}) \tag{3.50}$$

$$\text{s.t.} \quad Q_{TL} + Q_{LTL} = Q \tag{3.51}$$

$$Q_{TL}, Q_{LTL} \geq 0 \tag{3.52}$$

The above shipment quantity decision and transportation mode selection can be integrated using a *truckload discount* (TLD) transportation cost structure (see Figure 14).

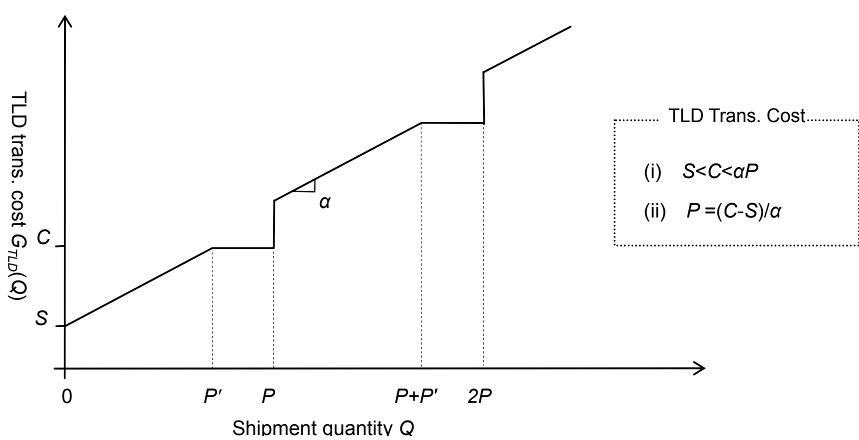

Figure 14. Truckload discount (TLD) transportation cost structure

The TLD transportation cost structure can be interpreted as follows. A truckload consists of P volume units. The freight company charges a fixed transportation cost of S and a variable transportation cost of α per volume unit up until the shipper has paid for the full truckload transportation cost of C ($S < C < S + \alpha P$), at which point, there is no charge for the remaining shipment quantities in that truckload. Once the first truckload is full, the shipper again pays the fixed and variable transportation costs, and so forth. We use P' to denote the *free shipping point* in the TLD transportation cost structure, which is defined as $P' = (C - S)/\alpha$.

We use η to define the total amount of trucks used for the shipment quantity Q, which is defined as $\eta = \lceil Q/P \rceil$. The TLD transportation cost function $G_{TLD}(Q)$ can be written as

$$G_{TLD}(Q) = \begin{cases} (\eta-1)C + S + [Q-(\eta-1)P]\alpha, & \text{if } (\eta-1)P < Q \le (\eta-1)P + P' \\ \eta C & \text{, if } (\eta-1)P + P' < Q \le \eta P \end{cases} \quad (3.53)$$

The average transportation cost function $\overline{G}_{TLD}(Q)$ is shown in Figure 15. We observe that $\overline{G}_{TLD}(Q)$ decreases within the half open ranges $((\eta-1)P, \eta P]$ and reaches its minimum (C/P) when shipments are full truckloaded.

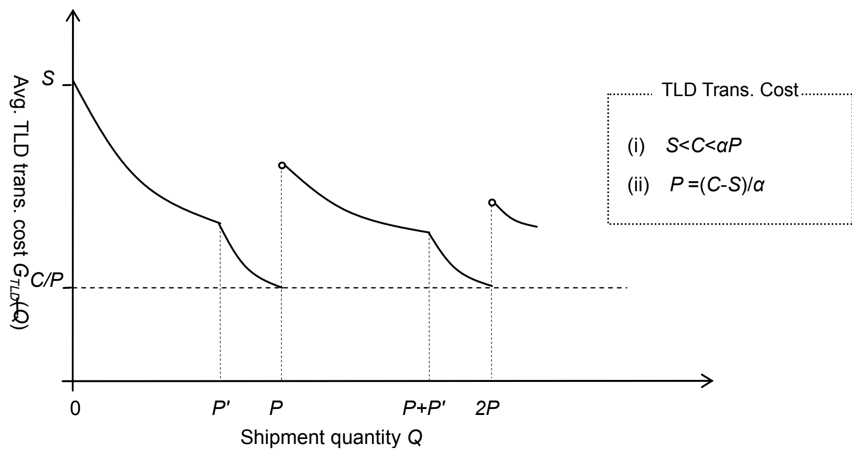

Figure 15. Average TLD transportation cost function $\overline{G}_{TLD}(Q)$

The TLD transportation cost structure incorporates the transportation mode selection into the shipment quantity decision. It can also be useful for the freight company to evaluate the strategy of providing multiple transportation modes. To neglect the LTL fixed cost S in the TLD cost structure, we have a common price discount structure practiced by retailers when they make seasonal promotions by advertising like "*buy 3 and get 1 free*" or "*buy a dozen and get 20% off*". We may also use such a price discount structure to achieve channel coordination when a vendor is responsible for delivering the products in TL transportation mode. In this situation, the vendor intends to encourage the buyers to order in full truckloads to obtain the transportation cost savings. For more discussions on the TLD cost structure, we refer to the works of Dolan (1987) and Elhedhli and Benli (2004).

III.5.2 Model III: A Model with TLD Transportation Cost Structure and Single-Cycle Continuous Transportation Policy

The third integrated single-channel vendor-buyer coordination model is characterized by

- *Two transportation modes of less-than-truckload (LTL) and truckload (TL).*
- *Truckload discount (TLD) transportation cost structure.*
- *Single-cycle continuous transportation policy.*

In Model III, all the products should be shipped at a common frequency, which can take any positive value. Thus, all the products can be treated as a composite product, which is characterized by demand d, unit volume v, and holding cost h. The objective is find the optimal shipment quantity Q_{III}^* that minimizes the system cost per time period $TC_{III}(Q)$. We have

$$TC_{III}(Q) = vd\overline{G}_{TLD}(Q) + \frac{hQ}{2v}. \qquad (3.54)$$

The first term represents the average transportation cost, and the second term represents the average inventory cost. An example of the average total system cost is shown in Figure 16.

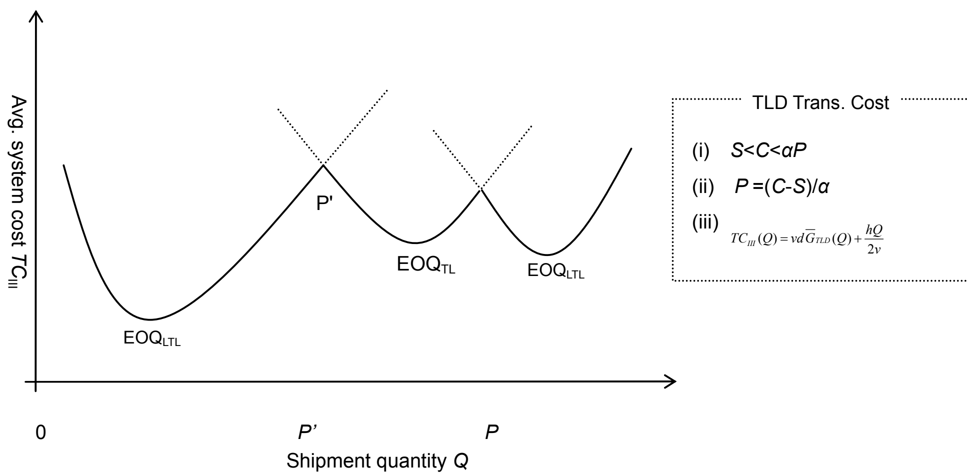

Figure 16. Average total system cost function $TC_{III}(Q)$ for model III

From the Figure 16, we conjecture that we only need to enumerate the breakpoints P' and P and EOQ values to find the optimal shipment quantity Q_{III}^*. Now, we give an optimal solution property that allows an efficient algorithm to find the optimal shipment quantity Q_{III}^*.

<u>PROPOSITION III.4.</u> For the Model III, the optimal shipment quantity Q_{III}^* should not be greater than a single truckload P.

PROOF. Consider the shipment strategy of delivering products in single full truckload P and another shipment strategy of delivering products in the shipment quantity Q' larger than P. For the two shipment strategies, we have the system cost functions as follows.

$$TC_{III}(P) = \frac{vdC}{P} + \frac{hP}{2v} \tag{3.55}$$

$$TC_{III}(Q') = vd\overline{G}_{TLD}(Q') + \left(h/2v\right)Q'. \tag{3.56}$$

From (3.55) and (3.56), we have the cost difference between the two shipment strategies,

$$\Delta = TC_{III}(Q') - TC_{III}(P) = vd\left[\overline{G}_{TLD}(Q') - \left(C/P\right)\right] + \left(h/2v\right)\left(Q' - P\right) \tag{3.57}$$

As aforementioned, C/P is the minimum average transportation cost; that is

$$C/P \le \overline{G}_{TLD}(Q'), \quad \forall Q' > P \tag{3.58}$$

Thus, the cost difference Δ is always positive as long as Q' is larger than P. The optimal shipment quantity Q^*_{III} is never greater than P. Therefore, the Proposition III.4 is true. □

According to Proposition III.4, we only need to enumerate a finite set of possible values of the optimal shipment quantity Q^*_{III}. We have

(I). When the optimal shipment quantity $Q^*_{III} \in (0, P')$, we have

$$Q^*_{III} = EOQ_{LTL} = v\sqrt{2Sd/h} \tag{3.59}$$

$$TC_{III}(EOQ_{LTL}) = \sqrt{2hdS} + vd\alpha \tag{3.60}$$

(II). When the optimal shipment quantity $Q^*_{III} \in (P', P)$, we have

$$Q^*_{III} = EOQ_{TL} = v\sqrt{2Cd/h} \tag{3.61}$$

$$TC_{III}(EOQ_{TL}) = \sqrt{2hdC}\,. \tag{3.62}$$

(III). When the optimal shipment quantity $Q^*_{III} = P$, we have

$$TC_{III}(P) = \left(vdC/P\right) + \left(h/2v\right)P. \tag{3.63}$$

(IV). The optimal shipment quantity $Q^*_{III} \ne P'$. This can be proved as follows. When $Q^*_{III} = P'$, we have two inequalities of $EOQ_{LTL} \ge P'$ and $EOQ_{TL} \le P'$. Thus, we have $EOQ_{LTL} \ge EOQ_{TL}$ and $S \ge C$, which contradicts the TLD transportation cost assumption.

In the next section, we discuss further the dependence between the transportation mode and the shipment quantity Q^*_{III} in the optimal solution of Model III.

III.5.3 Dependence between Transportation Mode and Shipment Quantity in the Optimal Solution of Model III

In this section, we investigate the dependence between the optimal transportation mode and optimal shipment quantity for Model III. We use $\Gamma(M)$ and Q^*(M) to denote the optimal system cost and the optimal transportation quantity as functions of the product parameter ratio *M*=*v*/*h*, respectively. By fixing the TLD transportation parameters [*P*, *C*, *S*, *α*] and the volume demand *v*·*d*, we having the following relationships regarding how the shipment quantity Q^*(M) and the system cost $\Gamma(M)$ depend on the transportation mode.

LTL transportation: $$\Gamma(M)=\sqrt{2Svd/M}+vd\alpha,\quad Q^*(M)=\sqrt{2SvdM} \qquad (3.64)$$

Partial TL transportation: $$\Gamma(M)=\sqrt{2Cvd/M},\quad Q^*(M)=\sqrt{2CvdM} \qquad (3.65)$$

Full TL transportation: $$\Gamma(M)=vd\,C/P+P/(2M),\quad Q^*(M)=P \qquad (3.66)$$

We define three threshold values of M_1, M_2 and M_3 as follows.

(I). When $M=M_1$, the LTL transportation and the partial TL transportation lead to the same optimal system cost $\Gamma(M)$. We have

$$\sqrt{2\mathrm{Svd}/M_1}+vd\alpha=\sqrt{2\mathrm{Cvd}/M_1} \quad\Rightarrow\quad M_1=\frac{2\left(\sqrt{C}-\sqrt{S}\right)^2}{vd\alpha^2}. \qquad (3.67)$$

(II). When $M=M_2$, the partial TL transportation and the full TL transportation lead to the same optimal system cost $\Gamma(M)$. We have

$$\sqrt{2Cvd/M_2}=vd\,C/P+P/2M_2 \quad\Rightarrow\quad M_2=P^2/2Cvd. \qquad (3.68)$$

(III). When $M=M_3$, the LTL transportation and the full TL transportation lead to the same optimal system cost $\Gamma(M)$. We have

$$\sqrt{2Svd/M_3}+vd\alpha=\frac{vdC}{P}+\frac{P}{2M_3} \quad\Rightarrow\quad M_3=\frac{P^2}{2vd\left(\sqrt{S+P\alpha-C}+\sqrt{S}\right)^2}. \qquad (3.69)$$

Next, we investigate the dependence between the optimal transportation mode and the optimal shipment quantity in two different cases differing on the relationship of M_1 and M_2.

III.5.3.1 Case of $M_1 \le M_2$

In the case of $M_1 \le M_2$, the TLD transportation cost parameters should satisfies the inequality $2\left(C-\sqrt{CS}\right) \le \alpha P$. An example of the function Γ(*M*) is illustrated as Figure 17.

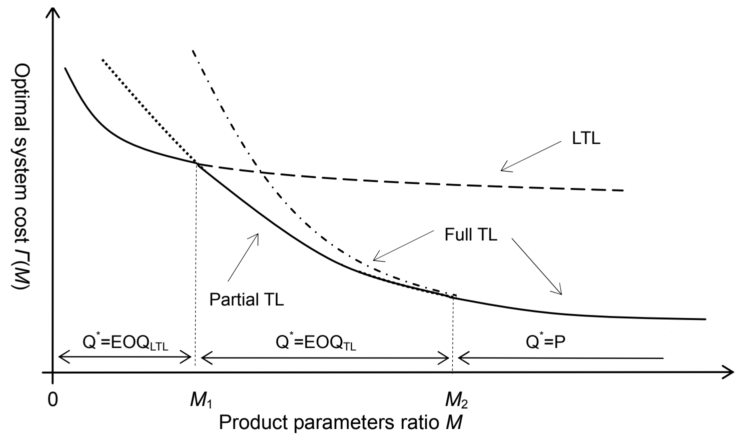

Figure 17. Optimal system cost function Γ(*M*) when $M_1 \le M_2$

(I). When the product parameter ratio *M* falls in the half open range $(0,\ M_1]$, we have the LTL transportation, the shipment quantity $Q^*(M) = \sqrt{2SvdM}$ and the system cost $\Gamma(M) = \sqrt{2Svd/M} + vd\alpha$ in the optimal solution.

(II). When the product parameter ratio *M* falls in the open range (M_1, M_2), we have the partial TL transportation, the shipment quantity cost $\Gamma(M) = \sqrt{2Cvd/M}$ and the system quantity $Q^*(M) = \sqrt{2CvdM}$ in the optimal solution.

(III). When the product parameter ratio *M* falls in the half open range $[M_2, +\infty)$, we have the full TL transportation, the shipment quantity cost $\Gamma(M) = vd\,C/P + P/(2M)$ and the system quantity $Q^*(M) = P$ in the optimal solution.

In the case of $M_1 \le M_2$, the optimal shipment quantity function $Q^*(M)$ is show in Figure 18. We can observe that the optimal shipment quantity function $Q^*(M)$ is discontinuous in the range of $\left(0, \frac{2\left(\sqrt{C}-\sqrt{S}\right)\sqrt{S}}{\alpha}\right] \cup \left(\frac{2\left(\sqrt{C}-\sqrt{S}\right)\sqrt{C}}{\alpha}, P\right]$. This observation is consistent with our previous analysis that the optimal shipment quantity Q^*_{III} never takes the value of the free shipping point *P*'.

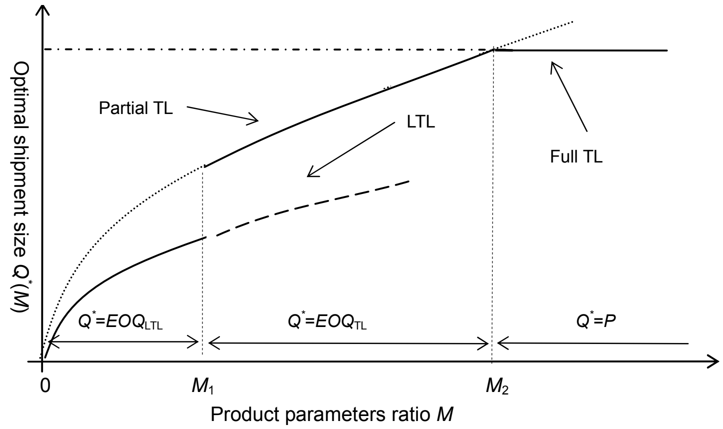

Figure 18. Optimal shipment quantity function $Q^*(M)$ when $M_1 \leq M_2$

Hall (1985b) studies the dependence between the optimal transportation mode and the optimal shipment quantity for several numerical examples, which can be generalized in the case of $M_1 \leq M_2$. However, Hall does not give a systematic analysis as we do. In addition, Hall does not study the other case of $M_1 > M_2$, which is discussed in the next section.

III.5.3.2 Case of $M_1 > M_2$

In the case of $M_1 > M_2$, the TLD transportation cost parameters should satisfies the inequality $2\left(C-\sqrt{CS}\right) > \alpha P$. An example of the function $\Gamma(M)$ is illustrated in Figure 19.

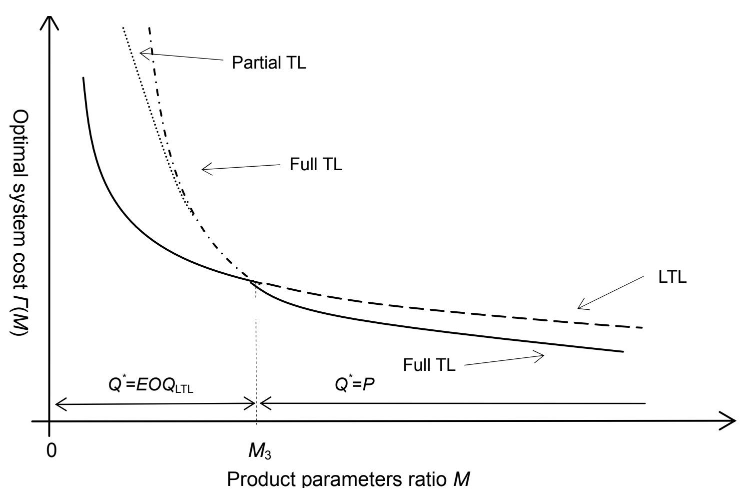

Figure 19. Optimal system cost function $\Gamma(M)$ when $M_1 < M_2$

(I). When the product parameter ratio M falls in the open range $(0, M_3)$, we have the LTL transportation, the shipment quantity $Q^*(M) = \sqrt{2SvdM}$ and the system cost $\Gamma(M) = \sqrt{2Svd/M} + vd\alpha$ in the optimal solution.

(II). When the product parameter ratio M falls in the half open range $[M_3, +\infty)$, we have the full TL transportation, the shipment quantity P and the system cost $\Gamma(M) = vd\,C/P + P/(2M)$ in the optimal solution.

In the case of $M_1 > M_2$, we notice that only LTL and full TL transportation are used in the optimal solution. Thus, the shippers never inflate their shipment quantities so that partial TL transportation is used. The optimal system quantity $Q^*(M)$ is shown in Figure 20.

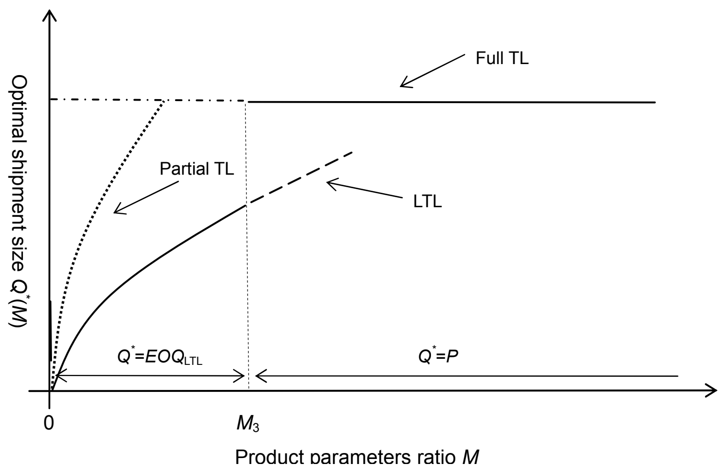

Figure 20. Optimal shipment quantity function $Q^*(M)$ when $M_1 > M_2$

From the Figure 20, we can observe that the optimal shipment quantity function $Q^*(M)$ is discontinuous in $\left(0, \dfrac{P\sqrt{S}}{\sqrt{S+\alpha P - C} + \sqrt{S}}\right] \cup (P)$, which is consistent with our previous analysis that the optimal shipment quantity $Q^*(M)$ never takes the value of free shipping point P'.

The dependence between the transportation mode and the shipment quantity in the optimal solution of Model III is shown in Table 2.

Table 2. Dependence between transportation mode and shipment quantity in the optimal solution of Model III

	Product parameter M $M=v/h$	Optimal trans. mode	Optimal shipment quantity Q	Optimal system cost
$2\left(C-\sqrt{CS}\right) \le \alpha P$	$M \in (0, M_1]$	*LTL*	$v\sqrt{(2Sd)/h}$	$\sqrt{2hdS} + vd\alpha$
	$M \in (M_1, M_2)$	*partial TL*	$v\sqrt{(2Cd)/h}$	$\sqrt{2hdC}$
	$M \in [M_2, +\infty)$	*full TL*	P	$vd\,C/P + P/2M$
$2\left(C-\sqrt{CS}\right) > \alpha P$	$M \in (0, M_3)$	*LTL*	$v\sqrt{(2Sd)/h}$	$\sqrt{2hdS} + vd\alpha$
	$M \in [M_3, +\infty)$	*full TL*	P	$vd\,C/P + P/2M$

III.5.4 Model IV: A Model with TLD Transportation Cost Structure and Multi-Cycle Discrete Transportation Policy

The fourth single-channel vendor-buyer coordination model is characterized by

- *Two transportation modes of less-than-truckload (LTL) and truckload (TL).*
- *Truckload discount (TLD) transportation cost structure.*
- *Multi-cycle discrete transportation policy.*
- *Frequency consolidation rule.*

In Model IV, we allow each product to be partially replenished at different frequencies, which take discrete values from a given set. We define decision variable x_k^j as the fraction of the demand for product *k* that is shipped at the *j*th frequency f_j. We define the integer variable η_j as the total number of trucks used for the shipment at the *j*th frequency f_j. We use the binary variable y_j to denote if the LTL transportation is used for the shipment with the *j*th frequency f_j. We use the variable z_j^{LTL} and z_j^{TL} to denote the shipment quantities associated with the *j*th frequency f_j and the LTL and TL transportation modes, respectively. We have

x_k^j = the fraction of the demand for product *k* that is shipped at the *j*th frequency f_j, $\sum_{j\in J} x_k^j = 1, \forall k$.

Q_j = the total shipment quantity associated with the frequency f_j.

η_j = the total number of trucks used to ship the quantity Q_j, $\eta_j = \lceil Q_j/P \rceil$.

y_j = 1, if $Q_j \in \left(\left(\eta_j - 1\right)P, \left(\eta_j - 1\right)P + P'\right]$, which implies that LTL transportation is used for the shipment at the frequency f_j.

= 0, if $Q_j \in \left(\left(\eta_j - 1\right)P + P', \eta_j P\right]$, which implies that only TL transportation is used for the shipment at the frequency f_j.

z_j^{LTL} = $Q_j - (\eta_j - 1)P$, if shipment quantity $Q_j \in \left(\left(\eta_j - 1\right)P, \left(\eta_j - 1\right)P + P'\right]$

= 0, otherwise

z_j^{TL} = $Q_j - (\eta_j - 1)P$, if shipment quantity $Q_j \in \left(\left(Q_j - 1\right)P + P', Q_j P\right]$

= 0, otherwise

Given the above parameters, the TLD transportation cost $G_{TLD}(Q)$ can be expressed as:

$$G_{TLD}(Q_j) = \left(\eta_j - y_j\right)C + y_j S + z_j^{LTL}\alpha \tag{3.70}$$

Model IV can be formulated as an MIP problem as follows.

$$\text{Min} \quad \sum_{k\in K}\sum_{j\in J}\left(\frac{1}{f_j}h_k d_k x_k^j\right)+\sum_{j\in J} f_j\left[\left(\eta_j-y_j\right)C+y_j S+z_j^{LTL}\alpha\right] \tag{3.71}$$

$$\text{s.t.} \quad \sum_{j\in J} x_k^j=1 \qquad \forall k\in K \tag{3.72}$$

$$\sum_{k\in K}\frac{1}{f_j}v_k d_k x_k^j=(\eta_j-1)P+z_j^{LTL}+z_j^{TL} \qquad \forall j\in J \tag{3.73}$$

$$0\le z_j^{LTL}\le P' y_j \qquad \forall j\in J \tag{3.74}$$

$$P'(1-y_j)\le z_j^{TL}\le P\left(1-y_j\right) \qquad \forall j\in J \tag{3.75}$$

$$0\le x_k^j\le 1 \qquad \forall j\in J, k\in K \tag{3.76}$$

$$y_j\in\{0,1\} \qquad \forall j\in J \tag{3.77}$$

$$z_j^{LTL}, z_j^{TL}\ge 0 \qquad \forall j\in J \tag{3.78}$$

$$\eta_j \ \text{integer} \qquad \forall j\in J \tag{3.79}$$

In the objective function (3.71), the first term represents the system inventory cost per time period and the second term represents the system transportation cost per time period. Constraints (3.72) ensure that, the whole demand of each product is assigned to different shipment frequencies. The relationships among the TLD transportation variables x_k^j, η_j, z_j^{LTL} and z_j^{TL} are defined in constraints (3.73). Constraints (3.74) and (3.75) specify that if LTL transportation is used for the shipment at the frequency f_j, the binary variable y_j must be 1; otherwise y_j must be 0. For the optimal solution to the Model IV, we have the following propositions.

<u>PROPOSITION III.5.</u> In the optimal solution of Model IV, partial TL transportation should always be associated with the greatest frequency.

PROOF. This can be proved by contradiction. Suppose there is an optimal solution X_k^j where partial TL transportation is associated with the shipment frequency f_l, which is greatest than another shipment frequency f_i. Let ε be a small volume quantity that is shipped at frequency f_i in the optimal solution X_k^j. We can construct a new feasible solution X'^j_k by shipping the quantity ε at frequency f_l in the new solution X'^j_k so that the number of trucks with frequency f_l remains the same as the optimal solution X_k^j.

Thus, the transportation cost in the new feasible solution X'^{j}_{k} is no greater than the one in the optimal solution X^{j}_{k} since shipping the quantity ε at the frequency f_l. costs zero. The inventory cost in the new feasible solution X'^{j}_{k} is less than the one in the optimal solution X^{j}_{k} since the quantity ε is replenished more frequently. Therefore, the new feasible solution X'^{j}_{k} outperforms the original solution X^{j}_{k}, which contradicts to the assumption of the optimality of the solution X^{j}_{k}. Therefore, Proposition III.5 is true □

PROPOSITION III.6. In the optimal solution of Model IV, there is at most one shipment frequency at which LTL transportation is used.

PROOF. This can be proved by contradiction. Suppose there are two shipment frequencies f_l and f_i ($f_i < f_l$) at which LTL transportation is used. Let ε be a very small volume quantity that is shipped at frequency f_i in the optimal solution X^{j}_{k}. We can construct a new feasible solution X'^{j}_{k} by shipping the quantity ε at frequency f_l in the new solution X'^{j}_{k} so that the number of trucks with frequency f_l remains the same as the optimal solution X^{j}_{k}. Thus, the system inventory cost decreases because the quantity of ε is replenished more frequently in the new feasible solution X'^{j}_{k}. The system transportation cost does not increase because shipping the quantity ε costs more than α per volume unit in the original solution X^{j}_{k}, while costs not more than α per volume unit in the new solution X'^{j}_{k}. Therefore, the new solution X'^{j}_{k} outperforms the original solution X^{j}_{k} . Proposition III.6 is true□

The optimal solution for Model IV has an upper bound UB and a lower bound LB with the limited gap. Consider a feasible solution of shipping all the products every time period. The system cost can be seen as an upper bound on the optimal solution. Then, the shipment quantity is vd and we have an upper bound UB as follows,

$$UB = G_{TLD}(vd) + \tfrac{1}{2}hd \qquad (3.80)$$

From the previous analysis, we know that the minimum possible transportation cost per unit volume is C/P in the TLD transportation cost structure. In addition, the minimum possible inventory cost per unit incurs when the products are shipped every time period. Then, we have a lower bound LB as follows.

$$LB = Cvd / P + \frac{1}{2} hd \tag{3.81}$$

Thus, the gap between upper bound *UB* and lower bound *LB* is $G_{LTD}(vd) - Cvd/P$. Given the TLD transportation cost structure assumption of $S < C < \alpha P$. This gap has a limit of

$$UB - LB \leq C \frac{(P - P')}{P} \tag{3.82}$$

Where *P'* denotes the free shipping point in the TLD transportation cost structure and equals to $(C - S) / \alpha$.

Given the Proposition III.5, III.6, and the bounds, a type IV single-channel vendor-buyer coordination model should have the optimal solution that satisfies the following properties.

(I). If LTL transportation is used, it should be assigned to a single shipment frequency.

(II). If partial TL transportation is used, it should be assigned to the largest shipment frequency in the optimal solution.

(III). The product with lower value of h_k / v_k should never be shipped more frequently than the product with higher value of h_k / v_k.

The Model IV can be seen as an extension of the *single-link model* studied in Speranza and Ukovich (1994), in which only the TL transportation is considered.

III.6 Numerical Case Study

In this section, we give a numerical case to show how the logistics coordination benefits a single-channel vendor-buyer system based on a hypothetical company, *Yuanf*a *Export & Sourcing Limited* (YES), which provides the service of a one-stop shop from product design and development, raw material and factory sourcing, production planning and management, and quality assurance to shipment consolidations. Headquartered in Hong Kong, YES's major business covers the garments, furnishing, gifts, home products, toys, electronics, sporting goods and travel goods etc.

In this numerical case study, a Holland import company places an order of six products, which include garments, toys, and home electronics. YES outsources the production to three different China manufacturers: a garments factory in Guang Zhou, a toy factory in Dong Guan and a consumer electronics factory in Shen Zhen. We have the following parameters,

- For each product, indexed from 1 to 6, the demand rate is deterministic, constant, and price-insensitive; the supply rate is equal to its demand rate.
- These products have the volume dimensions as follows [Toys: 20x20x20, 40x20x20, Electronics: 40x40x10, 40x40x25, Garments: 40x20x4, 40x25x 4] (in *cm*).
- The holding cost is incurred based on the annual interest rate of 25% (50 weeks in a year).
- YES owns all the raw materials and products at factories, and is responsible for shipping the products from factories to Hong Kong in truck freight, then shipping products to Amsterdam (Holland) in sea freight.
- The sea freight is in standard 40´ dry freight container, which has the usable cubic capacity of $68m^3$ and the maximum net weight of 26.4 tons.
- YES and the Holland import company can cooperate to make replenishment and transportation decisions to minimize the system cost, which includes the inventory costs incurred at both sides and the sea freight transportation cost of shipping products from Hong Kong to Amsterdam. We ignore the truck freight transportation cost by assuming the products are always delivered in full truckloads.

In the next two subsections, we study the four distinct single-channel vendor-buyer coordination models for the numerical case.

III.6.1 Model I & II

In this section, we study the type I and II single-channel vendor-buyer coordination models with the assumption that the shipment quantities are less than the given container capacity. We have the following transportation and product parameters.

- The less-than-truckload incremental discount (LID) transportation cost structure is defined by the breakpoints P_r of [0, 10, 20, 30, 50, 68](in m^3), the variable costs α_r of [45, 38, 32, 28, 0] and the fixed cost S_1 of 80 (see Figure 21).

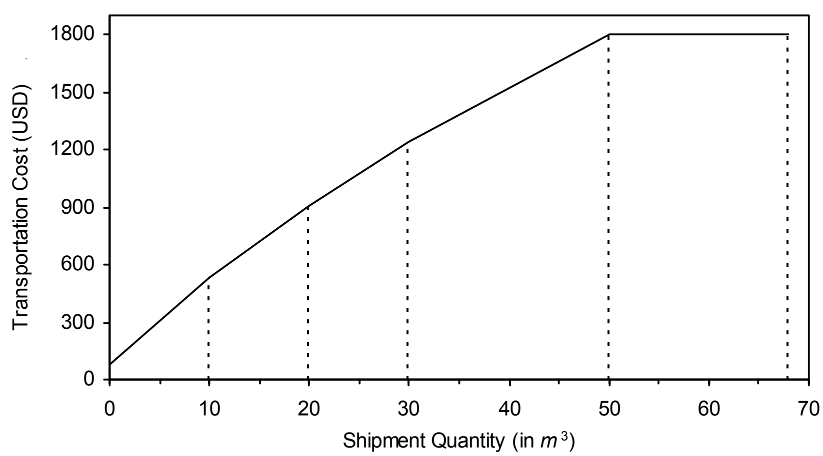

Figure 21. LID transportation cost structure in the numerical case of single-channel vendor-buyer coordination problem

- The products are demanded at the rates of [250, 125, 125, 50, 625, 500] (in unit/week).
- We randomly generate 10 sets of product holding cost rates with the assumption that these holding cost rates are uniformly distributed in a specified range as follows [0.32-0.8, 0.64-1.6, 1.6-3.2, 4-8, 1.92-3.84, 2.4-4.8 ($/yr)].

For Model I, we consider the single-cycle continuous transportation policy that requires all products to be shipped at a common frequency. For Model II, we consider the multi-cycle discrete transportation policy that allows each product to be partially shipped at different discrete frequencies. In addition, we also study another model in which all products are required to be shipped at the common discrete frequency. We refer to this model as Model 0. The numerical results for these three models are shown in Table 3.

Table 3. Numerical results for the single-channel vendor-buyer coordination model I & II

Product	Shipment Period $1/f$ (in week)						Annual Cost (USD)		
	1	2	3	4	5	6	Inv. Cost	Tran. Cost	Total Cost
Scenario 1:	Holding cost rates = [0.56, 1.20, 2.96, 6.00, 2.13, 2.90].								
Model 0	2	2	2	2	2	2	7,480	26,050	33,530
Model I	1.41	1.41	1.41	1.41	1.41	1.41	5,296	28,096	33,392
Model II	2	2	2	2	2	2	7,480	26,050	33,530
Scenario 2:	Holding cost rates = [0.52, 0.88, 2.80, 4.80, 2.21, 3.70].								
Model 0	1	1	1	1	1	1	4,060	30,300	34,360
Model I	1.35	1.35	1.35	1.35	1.35	1.35	5,518	28,318	33,836
Model II	4	4	4	4	1	1	6,550	27,350	33,900
Scenario 3:	Holding cost rates = [0.68, 1.20, 1.60, 6.40, 3.12, 2.72].								
Model 0	1	1	1	1	1	1	4,150	30,300	34,450
Model I	1.34	1.34	1.34	1.34	1.34	1.34	5,578	28,378	33,957
Model II	4	4	4	4	1	1	6,670	27,350	34,020
Scenario 4:	Holding cost rates = [0.44, 1.20, 1.76, 5.80, 2.37, 3.34].								
Model 0	1	1	1	1	1	1	3,920	30,300	34,220
Model I	1.38	1.38	1.38	1.38	1.38	1.38	5,422	28,222	33,644
Model II	4	4	4	4	1	1	6,230	27,350	33,580
Scenario 5:	Holding cost rates = [0.36, 0.80, 2.72, 4.40, 2.77, 4.68].								
Model 0	1	1	1	1	1	1	4,820	30,300	35,120
Model I	1.24	1.24	1.24	1.24	1.24	1.24	6,012	28,812	34,824
Model II	4	4	4	4	1	1	7,070	27,350	34,420
Scenario 6:	Holding cost rates = [0.64, 1.20, 2.72, 6.80, 2.53, 4.78].								
Model 0	1	1	1	1	1	1	4,220	30,300	34,520
Model I	1.33	1.33	1.33	1.33	1.33	1.33	5,625	28,425	34,051
Model II	2	2	2	2	1	1	5,210	29,250	34,460
Scenario 7:	Holding cost rates = [0.36, 1.12, 2.64, 4.80, 3.54, 3.80].								
Model 0	1	1	1	1	1	1	4,910	30,300	35,210
Model I	1.23	1.23	1.23	1.23	1.23	1.23	6,068	28,868	34,936
Model II	4	4	4	4	1	1	7,310	27,350	34,660
Scenario 8:	Holding cost rates = [0.68, 0.64, 2.88, 5.80, 3.46, 2.56].								
Model 0	1	1	1	1	1	1	4,340	30,300	34,640
Model I	1.31	1.31	1.31	1.31	1.31	1.31	5,705	28,505	34,210
Model II	4	4	4	4	1	1	7,040	27,350	34,390
Scenario 9:	Holding cost rates = [0.68, 1.60, 2.88, 6.00, 3.28, 3.46].								
Model 0	1	1	1	1	1	1	4,810	30,300	35,110
Model I	1.24	1.24	1.24	1.24	1.24	1.24	6,006	28,806	34,812
Model II	2	2	1	2	1	1	5,480	29,600	35,080
Scenario 10:	Holding cost rates = [0.40, 0.88, 1.92, 5.40, 3.62, 3.78].								
Model 0	1	1	1	1	1	1	4,870	30,300	35,170
Model I	1.24	1.24	1.24	1.24	1.24	1.24	6,043	28,843	34,887
Model II	4	4	4	4	1	1	7,030	27,350	34,380

We have 10 sets of scenarios with the same product demand rates and different holding cost rates. We give the optimal shipment period, system inventory cost, system transportation cost, and total system cost for each set. From the numerical results presented in Table 3, we have the following observations.

(I). In 6 out of the 10 scenarios, the Model I results in the least system cost. The optimal solution of Model I, however, usually leads to a shipment (or replenishment) schedule hard to implement, e.g. to ship all the products every 1.35 week. Such a shipment schedule could be infeasible in the situations where the freight company has restrictions on departure timing, e.g. a sea freight company could have a fixed departure schedule of every Monday morning for the shipment from Hong Kong to Amsterdam.

(II). In all the scenarios, the Model 0 results in the largest system cost. The optimal solution of Model 0 can be obtained by simply rounding the type I solution to the neighboring discrete value that leads to the smallest system cost.

(III). In all the scenarios, the Model II always outperforms the Model 0 because we allow the products to be shipped at different discrete frequencies. The optimal solutions are consistent with our previous analysis that toys and electronics are not shipped more frequently than the garments because the garments have the higher price per unit volume (h/v).

III.6.2 Model III & IV

In this section, we consider the scenarios in which shipment quantities are close to or larger than the given container capacity. The product holding cost rates remain the same and the demand rates vary in these scenarios. We study the type III and IV single-channel vendor-buyer coordination models with the following parameters.

- Truckload discount (TLD) cost structure is defined by fixed cost S of 100, variable cost α of 34 and full container charge C of 1,800 (USD). The full container capacity P is 68 and free shipping point P' is 50 (in m^3). The TLD transportation cost structure is illustrated in Figure 22.

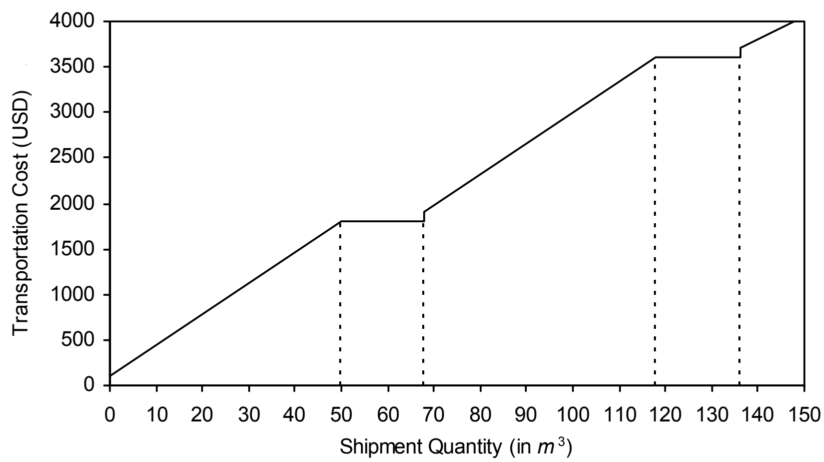

Figure 22. TLD transportation cost structure in the numerical case of single-channel vendor-buyer coordination problem

- The products have the holding cost rates of [0.5, 1, 2, 5, 3, 4] ($/yr).
- We generate 10 scenarios in which all the products have the same demand rate, which takes the value from 750 to 1200 with the increment of 50 (unit/week).

We study the Model III with the single-cycle continuous transportation policy that requires all products to be shipped at a common continuous frequency, and the Model IV with the multi-cycle discrete transportation policy that allows each product to be partially shipped at different discrete frequencies. The numerical results for the two models are shown in Table 4.

Table 4. Numerical case results for single-channel vendor-buyer coordination model III & IV

	Shipment Period 1/*f* (in week)						Annual Cost (USD)		
Product	1	2	3	4	5	6	Inv. Cost	Tran. Cost	Total Cost
Scenario 1: Product demand rates = [750, 750, 750, 750, 750, 750].									
Model III	1.03	1.03	1.03	1.03	1.03	1.03	12,118	86,558	98,676
Model IV	1	1	1	1	1	1	11,625	90,000	101,625
Scenario 2: Product demand rates = [800, 800, 800, 800, 800, 800].									
Model III	0.97	0.97	0.97	0.97	0.97	0.97	12,118	92,329	104,447
Model IV	1 (.725) 6 (.275)	1	1	1	1	1	12,950	93,825	106,775
Scenario 3: Product demand rates = [850, 850, 850, 850, 850, 850].									
Model III	0.91	0.91	0.91	0.91	0.91	0.91	12,118	98,099	110,218
Model IV	1 (.100) 6 (.133) 12 (.767)	1	1	1	1	1	17,077	98,687	115,764
Scenario 4: Product demand rates = [900, 900, 900, 900, 900, 900].									
Model III	0.86	0.86	0.86	0.86	0.86	0.86	12,118	103,870	115,989
Model IV	6	1 (.772) 6 (.228)	1	1	1	1	17,303	105,000	122,303
Scenario 5: Product demand rates = [950, 950, 950, 950, 950, 950].									
Model III	0.82	0.82	0.82	0.82	0.82	0.82	12,118	109,641	121,760
Model IV	4	1 (.523) 6 (.467)	1	1	1	1	17,601	112,500	130,101
Scenario 6: Product demand rates = [1000, 1000, 1000, 1000, 1000, 1000].									
Model III	0.77	0.77	0.77	0.77	0.77	0.77	12,118	115,411	127,530
Model IV	4	1 (.300) 2 (.275) 4 (.425)	1	1	1	1	18,654	117,490	136,144
Scenario 7: Product demand rates = [1050, 1050, 1050, 1050, 1050, 1050].									
Model III	0.74	0.74	0.74	0.74	0.74	0.74	12,118	121,182	133,301
Model IV	4	1 (.098) 2 (.781) 4 (.121)	1	1	1	1	19,144	124,902	144,046
Scenario 8: Product demand rates = [1100, 1100, 1100, 1100, 1100, 1100].									
Model III	0.7	0.7	0.7	0.7	0.7	0.7	12,118	126,952	139,071
Model IV	6	4 (.879) 6 (.121)	1 (.914) 4 (.086)	1	1	1	24,125	127,500	151,625
Scenario 9: Product demand rates = [1150, 1150, 1150, 1150, 1150, 1150].									
Model III	0.67	0.67	0.67	0.67	0.67	0.67	12,118	132,723	144,841
Model IV	2	2	1 (.746) 2 (.254)	1	1	1	20,229	135,000	155,229
Scenario 10: Product demand rates = [1200, 1200, 1200, 1200, 1200, 1200].									
Model III	0.64	0.64	0.64	0.64	0.64	0.64	12,118	138,494	150,612
Model IV	2 (.725) 6 (.275)	2	1 (.592) 2 (.408)	1	1	1	22,138	140,321	162,459

We have 10 sets of scenarios with the same holding cost rates and the different product demand rates. We give the optimal shipment period, system inventory cost, system transportation cost, and total system cost for each set. From the numerical results presented in Table 4, we have the following observations.

(I). In all the scenarios, the Model III always outperforms the Model IV. In the optimal solutions of the Model III, all the products are shipped at a common frequency that leads to the shipments in single full truckload. The optimal solution of Model III, however, usually leads to a shipment (or replenishment) schedule hard to implement, e.g. to ship all the products every 0.64 week.

(II). For Model IV, the optimal solutions are consistent with our previous analysis that toys and electronics are not shipped more frequently than the garments because the garments have the higher price per unit volume (h/v).

III.7 Conclusions, Managerial Implications, and Future Research

III.7.1 Conclusions

In this chapter, we dealt with the logistics coordination issue in a single-channel vendor-buyer system, where multiple products are delivered from a vendor to a buyer through a single channel. Our research contributes to the literature by incorporating the practical transportation consideratios into the traditional single-channel vendor-buyer coordination problem. We take into account the industrial trend of outsourcing transportation to 3rd party logistics (3PL) companies, and consider the situations where transportations are scheduled subject to certain transportation cost structure and transportation policy.

We considered two types of transportation cost structure: *less-than-truckload incremental discount* (LID) *transportation cost structure*, which is meaningful when shipment quantities are less than the vehicle (or container) capacity; and *truckload discount* (TLD) *transportation cost structure*, which is meaningful when shipment quantities are close to or greater than the vehicle (or container) capacity. In addition, we considered two types of transportation policy: *single-cycle continuous transportation policy* that requires all products to be shipped at a common frequency, which can take any continuous value; and *multi-cycle discrete transportation policy* that allows each product to be partially shipped at different frequencies, which take discrete values from a given set.

For the different transportation cost structures and transportation policies, we studied four distinct single-channel vendor-buyer integrated coordination models, with the objective of making optimal transportation and replenishment decisions to minimize the total system cost. For each model, we developed a mathematical formulation and investigated the optimal solution properties. A numerical case study was given to present how the logistics coordination brings cost savings to the single-channel vendor-buyer system with consideratio of complex transportation schemes.

III.7.2 Managerial Implications

This research provides logistics managers with a tool for considering the practical transportation issues in their decisions. The results and observations give insights as follows,

(I). *Multi-Cycle Shipment Scheduling*

When the products have the different ratio of h/v, shipping the products at the same frequency may result in an ineffective shipment schedule. It is reasonable to ship the products as different frequencies: the expensive products with higher ratio of h/v should be shipped more frequently than the cheaper products with lower ratio of h/v.

(II). *Dual-Mode Transportation*

Our research shows that shipping product in multiple transportation mode may result in cost savings. E.g., a company owning a private fleet could ship the products in full truckloads and outsource LTL transportation to an external freight company so that transportation cost is minimized and private vehicles are fully utilized.

(III). *Complex Production Scheme or Price Scheme*

We can interpret the LID and LTD transportation cost structures as a production cost structure or price structure. Thus, all the results and observations also apply to the situations where the vendor and buyer cooperate with each other to minimize the system cost, which includes inventory cost, production cost or procurement cost. .

III.7.3 Future Research

This chapter presented several analytical models for discussing the benefits of vendor-buyer coordination in single-channel vendor-buyer system. A numerical case with industry data could provide further evidence to support our analysis. In addition, future research could also focus on the following aspects.

(I). *Channel coordination models*

We studied four integrated coordination models in this research. A reasonable future research direction could be to deal with the channel coordination issue in the single-channel vendor-buyer system. When the vendor and buyer can not fully cooperate with each other, the vendor may desire to offer a purchase discount to entice the buyer to make cooperative decisions so that both parties obtain benefits.

(II). *Dynamic, stochastic, or price-sensitive demand*

Although the assumption of deterministic, constant, and price-insensitive demand is a reasonable approximation for medium and long term planning contexts, our research could be limited in the seasonal products or short planning context. Especially, given the demand is price sensitive, the vendor and buyer can cooperate with each other to make decisions on the selling price to increase the demand so that the total system profit is increased. Thus, to consider dynamic, stochastic, or price-sensitive demand in single-channel vendor-buyer coordination problem could be a good future research topic.

(III). *Inbound/outbound transportation management*

In this research, the system cost includes inventory cost incurred at both locations and the transportation cost of shipping products between them. However, in some situations, we might want to consider the inbound transportation to the vendor. Thus, single-channel vendor-buyer coordination problem with consideratio of both inbound and outbound transportation costs could be a potential direction for the future research.

CHAPTER IV

DUAL-CHANNEL VENDOR-BUYER SYSTEM WITH MINIMUM PURCHASE COMMITMENT

In this chapter, we deal with a logistics coordination issue for the Hewlett-Packard's ink cartridges supplies system, in which ink cartridges are shipped through central facilities and regional facilities to geographically scattered customers. We propose a dual-channel ink cartridges supplies system to utilize additional channels, through which some amount of the ink cartridges are shipped from the central facilities directly to the customers, bypassing the intermediate regional facilities. In addition, we consider a *Minimum Purchase Commitment* (MPC) agreement in the proposed dual-channel ink cartridges supplies system as follows.

At the beginning of each time period, a customer places *regular order* for the predetermined and fixed quantity, which is delivered from a central facility directly to the customer. We refer to such channel as *direct channel* because no intermediate regional facility is involved in shipping the regular orders. After receiving the regular order, the customer places no order if the resulting inventory position is greater than the order-up-to level, otherwise the customer places a *supplementary order* from a regional facility to raise the inventory position to the order-up-to level. We refer to such channel as *indirect channel* because certain intermediate regional facility is used to ship the supplementary orders.

The Hewlett-Packard's ink cartridges supplies system is influenced by the proposed dual-channel supply strategy in the following aspects.

- *System supply cost*, which includes transportation cost and operating cost, is reduced because an efficient transportation mode (e.g. container loads) can be used for the direct deliveries and less ink cartridges are handled at the regional facilities.
- *Surplus inventories* are carried at the customers due to their commitment of placing a regular order for the predetermined and fixed quantity in each time period regardless of the realized demand.
- *Safety stocks* at the customers decrease because the surplus inventories can be used to satisfy the stochastic customer demand.
- *Safety stocks* at the central facilities and the regional facilities decrease because some of the burden of handling demand variability is shifted to the customers.

In this research, we investigate the above impacts of implementing the dual-channel supply strategy, and study the logistics coordination issue concerning how Hewlett-Packard and its customers can cooperate with each other in making an MPC agreement to improve the system profitability as well as their individual profitability.

The rest of this chapter is organized as follows. We address the research motivation in the next section. We review the relevant literature in Section IV.2. The problem notation, assumptions, and formulations are given in Section IV.3. In Section IV.4 and IV.5, we introduce a simulation-based method to quantitatively analyze the impacts of implementing a dual-channel MPC supply strategy. In Section IV.6, we develop two vendor-buyer coordination models, and investigate the optimal solution properties for each one. In Section IV.7, we give a numerical case study based on Hewlett-Packard's ink cartridges supplies system in Asia. Finally, conclusions, implications, and future research are presented in Section IV.8.

IV.1 Research Motivation

Palo Alto, Calif., May 16th 2006 – Hewlett-Packard (HP) announced financial results for the second fiscal quarter with net revenue of $22.6 billion, representing growth of 5% year over year. "HP delivered another solid quarter," said Mark Hurd, HP CEO and president. "We grew revenue, expanded margins, and generated record cash flow. At the same time, we continued to remain focused on executing our strategy and investing in the company's long-term success."

Founded by William Hewlett and David Packard in 1939, and merged with Compaq in 2002, HP today ranks as the 2nd largest IT company and the 11th largest company in the world, with an estimated net revenue of $90 billion in 2006. Organized by production and function, HP has three major business segments as follows.

Technology Solutions Group (TSG), which is an aggregation of three smaller business segments: *Enterprise Storage and Servers*, *HP Services*, and *Software*. TSG's mission is to provide IT solutions to allow customers to manage and transform their business.

Personal Systems Group (PSG), which is the 2nd largest vendor of personal computers in the world based on unit volume and annual revenue. PSG provides personal computers, workstations, handheld computing devices, digital entertainment systems and other related accessories, software and services for the commercial and consumer markets.

Imaging and Printers Group (IPG), which is a leading global provider of imaging and printing solutions across customer segments from individual consumers, small and medium businesses to large enterprises. IPG's offering includes inkjet printers, LaserJet printers, digital photography and entertainment, graphics and imaging and printer supplies.

The net revenues and operating profits of HP in the first two fiscal quarters of 2006 are presented in the following table (*Source*: HP financial report 2006).

	(in millions)	1st Quarter 2006	2nd Quarter 2006
Net Revenue	TSG	8,487	8,301
	PSG	6,977	7,449
	IPG	6,724	6,545
	Others	366	364
	Total HP	22,554	22,659
Operating Profit	TSG	670	628
	PSG	248	293
	IPG	1,041	973
	Others	(10)	5
	Total HP	1,949	1,899

In the above table, we can notice that IPG is the most profitable business segment in HP, which contributes about 50% of the total operating profit and 30% of the total net revenue. In the second fiscal quarter of 2006, the profit margin of IPG is increased to 15.5% from the 12.7% a year earlier, representing a growth of 22%. The high profitability and remarkable growth partially come from the sales strategy of "selling low-profit printers and making money from supplying high-profit ink cartridges". About one year ago, HP cut the prices on some of its most popular printers – business inkjets and all-in-one models – to increase the sales. Now the owners of those printers need to purchase a lot of replacement ink cartridges, which contribute higher profit margin than the printers do.

In this research, we focus on studying the logistics coordination issue for HP's ink cartridges (also known internally as pens) supplies business, which covers approximately 15 product families and over 250 manufacturing stock keeping units (See Billington et al., 2004). These ink cartridges are supplied to customers through a supply chain network as follows.

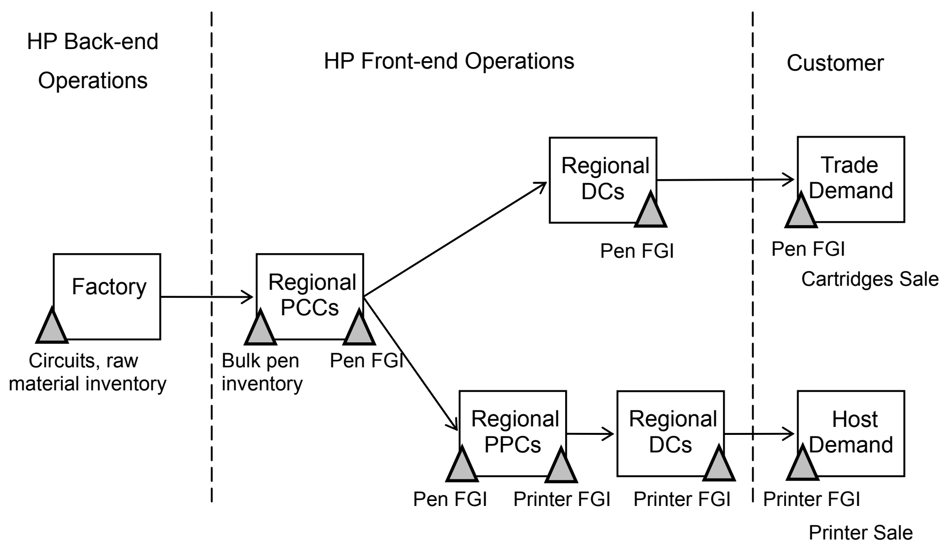

Figure 23. Ink cartridges supplies network in Hewlett-Packard[2]

As presented above, HP owns all the manufacturing facilities and distribution facilities in the ink cartridges supplies network. From upstream suppliers, manufacturing components (e.g. circuits and raw materials) are shipped to and stored at the *factory*, where these components are assembled into bulk ink cartridges. From the factory, bulk ink cartridges are shipped to different regional *pen completion centers* (PCCs), where bulk ink cartridges are localized and

[2] This figure is cited from Billington et al. 2004

packed into ink cartridges finished good inventories (FGIs) according to specified regional requirements. At the regional PCCs, inventories of both bulk ink cartridges and finished ink cartridges are carried to decouple the customer-facing operatios from the long manufacturing lead times at the factory, and to buffer the system from the order variability associated with the dozens of packaging configuratios. From the regional PCCs, finished ink cartridges are shipped to support two types of customer demand stream.

(I). *Trade demand stream* is referred to the demand that comes from the ink cartridges replacement. For a particular ink cartridge model, its trade demand mainly depends on the total amount of all the compatible printers sold in the market and the average ink cartridge depletion rates. To support trade demand stream, finished ink cartridges are shipped from the regional PCCs to the regional DCs, and then shipped to the customers (see Figure 23).

(II). *Host demand stream* is referred to the demand that comes from the new printer sales. For a particular ink cartridge model, its host demand is equivalent to the total demand of all the compatible printers. To support host demand stream, finished ink cartridges are shipped from the regional PCCs to the printer postponement centers (PPCs), where printers are localized and bundled with starter kits that include ink cartridges. From the PPCs, finished printers are shipped to and stored at the regional DCs, and then shipped to the customers (see Figure 23).

Our discussion with IPG logistics managers indicates that they are interested in implementing some new supply strategy to overhaul the ink cartridges supplies system. Based on their suggestions and our analysis, we propose two new supply strategies as follows.

(I). <u>Dual-Channel Supply Strategy with Minimum Purchase Commitment</u>

In the existing ink cartridges supplies (trade demand) system of HP, each end customer receives deliveries from a regional DC that is supplied by a regional PCC (see Figure 23). Such supply strategy aims to provide satisfactory delivery lead times because the regional DCs are usually located near to the geographically scattered customers.

The IPG logistics managers suggest that delivering some amount of ink cartridges directly from the regional PCCs to some big customers by an efficient transportation mode (e.g. container loads) may result in cost savings. With such direct deliveries, however, the customers may need to carry more safety stocks to maintain a desired service level due to the long lead times. To eliminate the impact of the long lead times on the safety stocks at the customers, we propose a *Minimum Purchase Commitment* (MPC) that requires the customers to regularly purchase predetermined and fixed quantities for the direct deliveries from the regional PCCs. Thus, ink cartridges are supplied through the proposed dual-channel system as follows.

In each time period, a customer places a regular order for predetermined and fixed quantity from a regional PCC. If the resulting inventory position is greater than the order-up-to level, no more order is placed; otherwise the customer can place another order from a regional DC to raise the inventory position up to the order-up-to level.

The dual-channel supply strategy with MPC aims to reduce the system supply cost by carefully treating demand uncertainty as follows. For the portion of the demand that is likely to be certain, ink cartridges can be directly shipped from the regional PCCs to the customers to achieve a lower transportation cost. For the portion of the demand that is likely to be uncertain, the ink cartridges are delivered from the regional DCs to achieve demand pooling.

(II). <u>A Tailored Postponement Supply Strategy</u>

In the existing ink cartridges supplies system of HP, a *postponement* strategy is used to manufacture bulk ink cartridges at the factory and postpone the customization (e.g. localizing and packaging) to the regional PCCs. The postponement strategy aims to consolidate the demand variability associated with the dozens of localizing and packaging configuratios at the regional PCCs and, in consequence, all activities prior to the customization require the aggregate forecasts of bulk ink cartridges, which are more accurate than the individual forecasts of finished ink cartridges.

There is, however, a cost associated with the postponement strategy because customizing ink cartridges at the regional PCCs typically costs more than that at the factory. Thus, postponement strategy may not be very effective for those ink cartridges with stable demands since the benefit from forecast aggregation is small whereas customization cost is high.

We propose a *tailored postponement* strategy as follows. At the factory, some amount of bulk ink cartridges are customized into finished ink cartridges and shipped in predetermined and fixed quantities directly to different regional DCs. The rest of the ink cartridges are customized at the regional PCCs and then shipped to different regional DCs.

The tailored postponement supply strategy aims to reduce the system manufacturing cost by carefully treating demand uncertainty as follows. For the portion of the demand that is likely to be certain, postponed customization at the regional PCCs provides little value in terms of increasing forecast accuracy. Thus, the customization can be done at the factory to achieve a lower manufacturing cost. For the portion of the demand that is likely to be uncertain, the customization is postponed to the regional PCCs to achieve the benefit from the forecast aggregation.

In the rest of this research, we focus on studying the proposed dual-channel supply strategy for the ink cartridges supplies system of HP. All the analysis and results can also be applied to the tailored postponement supply strategy discussed above.

IV.2 Literature Review

In this section, we review the literature on inventory models for the dual-channel system that consists of a *standing channel* and a *flexible channel*. The term "*channel*" refers to a set of parameters with which a buyer places orders and a vendor fulfills these orders. The vendor and buyer may represent any two upstream-downstream logistics participants in a supply chain system. Typically, a standing channel and a flexible channel are distinguished with three parameters of *price terms*, *quantity terms,* and *responsiveness terms*. We have

(I). A standing channel is usually associated with low supply cost, long lead time, and restricted order quantity and timing. Thus, through a standing channel, a buyer may be required to place orders for restricted quantity and timing, and a vendor may tend to provide discount due to the decreased supply cost.

(II).A flexible channel is usually associated with high supply cost, short lead time, and flexible order quantity and timing. Thus, through a flexible channel, a buyer may be allowed to place orders for flexible quantity and timing, and a vendor may tend to charge more due to the increased supply cost.

By designing a dual-channel system that consists of a standing channel and a flexible channel, logistics managers intend to achieve a reasonable balance between the system cost and the system responsiveness.

In one of the earliest works on dual-channel inventory model, Rosenshine and Obee (1976) consider a situation where a buyer replenishes its inventory through a single channel with long lead time and, in consequence, a high level of safety stock needs to be carried to maintain a desired service level. Rosenshine and Obee propose a dual-channel procurement strategy as follows. In each time period, the buyer places a *standing order* with the predetermined and fixed quantity Q. If the resulting inventory position is below the order-up-to level S, the buyer places an *emergency order* with zero lead time to raise the inventory position up to S. They assume the buyer has limited capacity so that extra inventory has to be sold off at a given salvage price. By assuming a finite discrete set for the stochastic demand, they formulate the buyer's inventory levels as the discrete states of a Markov chain. With the dual-channel procurement strategy, no safety stock needs to be carried and, in consequence, the buyer's cost is reduced although extra emergency order cost and salvage cost are charged.

Anupindi and Akella (1993a) study a dual-channel system in which the buyer commits to purchase a fixed quantity of Q every time period, and has the option of placing a supplementary order with adjustable lead time and extra per-unit purchase cost to satisfy the stochastic demand. Anupindi and Akella show that the optimal policy is an S-type policy in a periodic review and finite planning horizon setting. Moinzadeh and Nahmias (2000) consider a similar dual-channel system as Anupindi and Akella (1993a) does, but study it in a continuous review and infinite planning horizon setting. The buyer commits to purchase a fixed quantity of Q every time period, and has the option of adjusting the delivery quantity upwards just prior to a delivery, but must pay a fixed premium to do so. Moinzadeh and Nahmias show that the equations one must solve to find the optimal order-up-to level S are intractable. They develop an effective diffusion approximation that is coupled with the solution to a deterministic version of the problem. They also give a computational study to find the cost discount that equalizes the expected costs for the dual-channel system and the traditional single-channel system for a large parameter set.

In the paper of Janssen and de Kok (1999), they use the moment-iteration method introduced in De Kok (1989) to analyze the inventory level at the buyer in a dual-channel system and develop an algorithm to calculate the optimal order-up-to level and committed purchase quantity to minimize the buyer's cost a service constraint.

Thomas and Hackman (2003) develop a simulation-based method to approximate the expected inventory level at the buyer in a dual-channel system as quadratic functions of the committed minimum purchase quantity and the resell price. They show that the approximation method enables closed-form solutions with reasonable errors.

Bassok and Anupindi (1997) analyze a dual-channel system with total minimum purchase commitment, in which a buyer commits to purchase a minimum cumulative quantity over a finite planning horizon. They show that the optimal replenishment policy is characterized by the order-up-to levels and a single period standard newsboy problem with no commitment but with discounted price. For more details in literature on multi-supply problem, we refer to Minner (2003) and Cachon (2003).

In this research, we consider and analyze a dual-channel system that differs from the aforementioned literature in the following two aspects.

(I). *Vendor-Buyer Coordination Issue*

All the previous works study the dual-channel system from the individual perspective of the buyer. These works focus on investigating the impact of the dual-channel procurement strategy on the buyer's inventory level and estimating the optimal inventory policy for the buyer. In this research, we study the dual-channel system from the perspectives of both the buyer and the vendor. We incorporate the logistics coordination issue into the traditional dual-channel inventory model by investigating how the vendor and buyer can cooperate in making supply/purchase decisions to improve the system profitability as well as the individual profitability. In particular, we consider two types of vendor-buyer coordination mechanisms: *integrated coordination*, in which the buyer and vendor can fully cooperate with each other to make decisions to increase the system profit; and *channel coordination*, in which the vendor offers purchase discount to entice the buyer to make decisions in a cooperative way.

(II). *Stimulation-Based Approximation Method*

The analytical methods discussed in the previous works, e.g. diffusion approximation in Moinzadeh and Nahmias (2000) and moment-iteratio method in Janssen and de Kok (1999), can provide reasonable accuracy in analyzing the dual-channel inventory model. However, these methods need complex modeling techniques and heavy computation. The simulation-based approximation method introduced in Thomas and Hackman (2003) can only be used to estimate the inventory level in some specified cases. In this research, we introduce a simulation-based approximation method to quantitatively analyze inventory level at the buyer and the safety stock levels at the buyer and vendor in the general cases where the customer demand is *iid* normally distributed in each time period.

IV.3 Notation, Assumptions, and Formulations

In this section, we introduce the notation, assumptions and formulations for the dual-channel vendor-buyer system. The notation is as follows.

Table 5. Notation for the dual-channel vendor-buyer system with minimum purchase commitment

n, N	Index and set of time period (e.g. a day or a week), $n \in N$.
D	Random variable for the customer demand in each time period.
μ, σ	Mean and standard deviation of the random demand D.
D_n	Customer demand in the time period n.
Q	Minimum purchase commitment (MPC) quantity (or regular order quantity).
z	Standardized MPC quantity, $z = (\mu - Q)/\sigma$.
q_n	Supplementary order quantity placed in the time period n.
O_n	Total order quantity placed in the time period n, $O_n = Q + q_n$.
L	Replenishment lead time (in time periods).
S	Order-up-to level.
IL_n	Inventory level at the beginning of the time period n.
IP_n	Inventory position at the beginning of the time period n.
SS	Safety stock level.
h	Holding cost rate ($ per unit per time period).
c_1, c_2	Channel supply cost rates ($ per unit) for the direct and indirect channels.
p	Unit purchase price ($ per unit) for supplementary orders.
λ	Purchase price discount (in percentage) for regular orders.
α	Service level (the probability of no stockout happens in each time period).
β	Fill rate (the proportion of demand that is satisfied from inventory on-hand).
$k(z)$	Surplus inventory coefficient function.
$\psi_{\alpha,L}(z)$	Safety stock coefficient function for buyer.
$\varphi_{\alpha,L}(z)$	Safety stock coefficient function for vendor.

In this research, we study a logistics coordination issue in a dual-channel vendor-buyer system, which represents a simplified HP ink cartridges (trade demand) supplies system as follows.

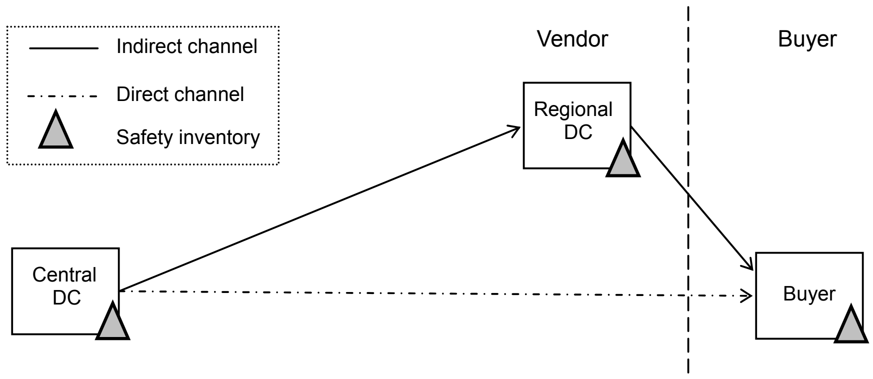

Figure 24. Dual-channel vendor-buyer system

As shown above, the dual-channel vendor-buyer system consists of three facilities: a vendor-owned central DC, a vendor-owned regional DC, and a buyer. A single product is delivered through two channels to satisfy the stochastic demand at the buyer: a *direct channel* which is from the central DC to the buyer; and an *indirect channel* which is from the central DC to the regional DC, and then to the buyer. The dual-channel vendor-buyer system assumptions and formulations are given as follows.

(I). *Common replenishment time period*

All the three facilities periodically replenish their inventories at a common replenishment time period (e.g. one day or one week), which is a given parameter rather than a decision variable. We use n and N to denote the index and set of the time period: $n = 0, 1, \ldots, N\text{-}2, N\text{-}1$.

(II). *Stochastic customer demand D*

We use D to denote the random variable of the customer demand observed at the buyer, which is *iid* normally distributed with mean μ and standard deviation (STD) σ in each time period. We assume the customer demand D is price-insensitive and use D_n to denote the customer demand realized in time period n.

Although not necessary, we assume the customer demand D is relatively steady with a small coefficient of variance (σ/μ) that is less than 1. This assumption can be justified by our previous discussion that the ink cartridge trade demand comes from replacements, which mainly depend on the total amount of the compatible printers sold and the depletion rate.

(III). *Dual-channel MPC supply strategy*

In the dual-channel vendor-buyer system presented in Figure 24, the single product is supplied through the direct channel and the indirect channel as follows.

At the beginning of time period n, the buyer places a *regular order* for the predetermined and fixed quantity Q through the direct channel. The regular order is purchased with a percentage purchase discount λ, which is offered by the vendor to encourage regular orders. Note that the regular order quantity Q should be smaller than the demand mean μ; otherwise the buyer's inventory level will be raised without bound in an infinite horizon.

After receiving the regular order, the buyer places no order if the resulting inventory position is not less than a predetermined order-up-to level S; otherwise the buyer can place a *supplementary order* for quantity q_n through the indirect channel to raise the inventory position up to the order-up-to level S. The supplementary order is purchased at the unit price P, without any purchase discount offered by the vendor.

The direct channel and indirect channel are shown in the following table.

Table 5. Dual-channel vendor-buyer system

	Direct channel	Indirect channel
Supply quantity	Predetermined and fixed	Flexible
Supply timing	Each time period	At most once each time period
Supply price	Discounted price	Normal price
Supply lead time	Long	Short
Push or pull	Push	Pull

(IV). *Replenishment lead time L*

We use L_b, L_{cdc} and L_{rdc} to denote the replenishment lead times at the buyer (for the supplementary orders placed to the regional DC), the central DC (for the orders placed to the external supplier) and the regional DC (for the orders placed to the central DC), respectively. We ignore the replenishment lead time of the regular orders since it has no impact on the safety stock carried at the buyer. All these replenishment lead times are assumed to be deterministic.

(V). *Service level α and safety stock level SS*

At each facility, safety stock is carried to maintain a desired service level α at the buyer, which is defined as the probability that no stockout happens in each time period. For the amount of demand beyond this service level, other ways such as spot market or expediting can be used. Thus, the dual-channel vendor-buyer system is designed to satisfy the customer demand D with an upper bound, which is determined by the service level α. We use SS_b, SS_{rdc}, and SS_{cdc} to denote the safety stock levels at the buyer, the central DC and the regional DC, respectively.

(VI). *Holding cost rate h*

At each facility, inventory cost is incurred proportionally to the average inventory level and the cumulated product cost. We use h_{cdc}, h_{rdc} and h_b to denote the holding cost rates at the buyer, the central DC and the regional DC, respectively. For the sake of simplicity, we assume the same holding cost rate h_b for the inventories of both regular orders and supplementary orders, despite the cumulated product costs are different. We also assume the cumulated product cost is increasing along the supply chain: $h_{cdc} < h_{rdc} < h_b$.

(VII). *Channel supply cost rate c*

The system supply cost includes the transportation cost of shipping product, and the operating cost incurred at the intermediate regional DC (e.g. loading and unloading costs, labor cost etc). We assume that the supply cost is incurred proportionally to the quantity delivered through each channel. We use c_1 and c_2 to denote the channel supply cost rate of the direct channel and the indirect channel. We use c_3 to denote the supply cost rate for those demand that is satisfied by spot market or expediting. We use β to denote the fill rate that is defined as the proportion of demand that is satisfied from inventory on hand. Thus, the expected average system supply cost per time period C_{supply} can be expressed as follows.

$$C_{supply} = c_1 Q + c_2\left(\mu\beta - Q\right) + c_3(1-\beta)\mu \tag{4.1}$$

In (4.1), the first term c_1Q represents the cost of supplying the MPC (or regular order) quantity Q through the direct channel; the second term $c_2(\mu\beta\text{-}Q)$ represents the cost of supplying the average supplementary order quantity $\mu\beta\text{-}Q$ through the indirect channel; the third term $c_3(1\text{-}\beta)\mu$ represents the cost of supplying the quantity $(1\text{-}\beta)\mu$ that is satisfied in certain "backup" way like spot market or expediting. By making an approximation, we assume the supply cost rate c_3 is equal to the indirect channel supply cost rate c_2. Thus, the expected average system supply cost C_{supply} can be rewritten as a linear function of the MPC quantity Q.

$$C_{supply} = c_2\mu - \left(c_2 - c_1\right)Q \tag{4.2}$$

In (4.2), the first term $c_2\mu$ can be interpreted as the expected average system supply cost in traditional single-channel system, and the second term $-(c_2-c_1)Q$ can be interpreted as the system supply cost savings by delivering the MPC quantity Q through the direct channel. We assume $c_1<c_2$, which implies that utilizing the direct channel always improves the system cost structure. This assumption can be justified by the fact that utilizing the direct channel may reduce the supply cost due to an efficient transportation mode used, the shorter transportation distance, and the reduced operating cost incurred at the intermediate regional DC.

(VIII). *System cost structure*

For the dual-channel vendor-buyer system studied, we consider four system cost components: system supply cost, surplus inventory cost at the buyer, safety stock cost at the buyer and safety stock cost at the vendor. The dual-channel MPC supply strategy influences these cost components as follows.

- *System supply cost*, which includes transportation cost and operating cost, is reduced because an efficient transportation mode (e.g. container loads) can be used for the direct deliveries and less inventory are handled at the regional facilities.
- *Surplus inventory cost* is incurred at the buyer due to the commitment of placing a regular order for fixed quantity in each time period regardless of the realized demand.
- *Safety stock cost* is reduced at the buyer because the surplus inventories can be used to satisfy the stochastic customer demand.
- *Safety stock costs* are reduced at the central DC and regional DC because some of the burden of handling demand variability is shifted to the buyer.

We have already discussed the impact of the dual-channel MPC supply strategy on the system supply cost. In the next two sections, we will focus on investigating the impacts of the dual-channel MPC supply strategy on the inventory levels at each facility.

IV.4 Surplus Inventory in the Dual-Channel Vendor-Buyer System with MPC

In the dual-channel vendor-buyer system studied, the vendor and buyer make a *Minimum Purchase Commitment* (MPC) agreement of delivering a predetermined and fixed quantity Q through the direct channel in each time period. Such MPC agreement can result in surplus inventory carried at the buyer when the realized demand is less than the committed purchase quantity Q. With the assumption that demand D is *iid* normally distributed in each time period, no closed-form analytical solution is known for calculating the surplus inventory, and even exact calculation is prohibitive. In this research, we resort to quantitatively estimating surplus inventory level with a simulation-based method, rather than giving an analytical solution.

This section is organized as follows. In Subsection IV.4.1, we formulate and analyze the surplus inventory at the buyer by comparing the dual-channel system with a traditional single-channel system. In Subsection IV.4.2, we investigate how the long-term expected average surplus inventory level depends on the system parameters. In Subsection IV.4.3, a simulation-based method is introduced to quantitatively estimate surplus inventory level.

In addition, we discuss two further topics on surplus inventory in Appendices. In Appendix A.1, we analyze the surplus inventory with the demand assumptions of truncated normal distribution, uniform distribution, and general discrete distribution. In Appendix A.3, we introduce another simulation-base method to analytically estimate surplus inventory level using piece-wise quadratic estimation functions.

IV.4.1 Single-Channel System VS Dual-Channel System

Now, let's compare the dual-channel system with a traditional single-channel system to demonstrate how the MPC results in the surplus inventory carried at the buyer.

Single-Channel Vendor-Buyer System

In a traditional single-channel system, the buyer replenishes its inventory through a single channel according to an order-up-to S policy. At the beginning of time period n, the buyer orders O_n to raise the inventory position IP_n up to the level S. Thus, the order quantity O_n always equals the realized demand D_{n-1} at the previous time period n-1. We have

$$O_n = D_{n-1} \tag{4.3}$$

$$IP_n = S = SS_b + \left(L_b + 1\right)\mu \tag{4.4}$$

In (4.4), the inventory position IP_n is equivalent to the order-up-to level S, which includes two components: the safety stock SS_b that the buyer needs to carry to maintain a desired service level α, and $(L_b+1)\mu$ that the buyer needs to order to fulfill the expected average demand during the next (L_b+1) time periods. The buyer's average inventory level $\overline{IL}_b$ can be expressed as

$$\overline{IL}_b = SS_b + \frac{1}{2}\mu \tag{4.5}$$

Dual-Channel Vendor-Buyer System with MPC

In the dual-channel system, the buyer replenishes its inventory through both the direct and the indirect channel according to a modified order-up-to policy with *minimum purchase commitment* (MPC). At the beginning of time period n, the buyer places a regular order in the predetermined and fixed quantity Q through the direct channel. Thus, the inventory position IP_n may exceed the order-up-to level S by an overshoot when the MPC quantity Q is larger than D_{n-1}, which is the realized demand during the previous time period n-1. We refer to such inventory overshoot as *surplus inventory* and use SI_n to denote the surplus inventory level at the buyer in the time period n. We have

$$O_n = \max\left(D_{n-1}, Q\right) \tag{4.6}$$

$$IP_n = Max\left\{S, IP_{n-1} + Q - D_{n-1}\right\} = SS_b + (L_b + 1)\mu + SI_n \tag{4.7}$$

As shown in (4.6), the buyer's total order quantity O_n takes the maximum value of the realized demand D_{n-1} and regular order quantity Q. The inventory position IP_n includes three components: the surplus inventory SI_n, the safety stock SS_b, and the expected average demand $(L_b+1)\mu$. The buyer's average inventory level $\overline{IL_b}$ can be expressed as follows.

$$\overline{IL_b} = SS_b + \frac{1}{2}\mu + SI \,. \tag{4.8}$$

Now, we give a numerical example to show how the inventory level and inventory position at the buyer differ in a single-channel system and a dual-channel system. The system parameters are given as follows.

- Stochastic demand D is *iid* normally distributed with mean μ=8 and STD σ=4.
- Buyer's replenishment lead time L_b=2 time periods.
- In both systems, replenishments are governed by an order-up-to policy with S = 30.
- In the dual-channel system, the buyer purchases committed Q=7 in each time period.

In Table 7 and Figure 25, we present the inventory levels and positions at the buyer during 7 consecutive time periods in both systems.

Table 7. An example of the inventory level and position at the buyer in a single-channel system and a dual-channel system

	Time period (T)	0	1	2	3	4	5	6
	Demand D_n	4	11	9	12.5	5.5	8	
Single Chann	Order quantity O_n		4	11	9	12.5	5.5	8
	Inventory level IL_n	10	16	15	10	8.5	12	16.5
	Inventory position IP_n	30	30	30	30	30	30	30
Dual Channel	Order quantity O_n		7	8	9	12.5	7	7
	Inventory level IL_n	10	16	15	13	8.5	12	16.5
	Inventory position IP_n	30	33	30	30	30	31.5	30.5
	Surplus inventory SI_n	0	3	0	0	0	1.5	0.5

In the dual-channel system, positive surplus inventory SI_1 and SI_5 are carried due to that the MPC quantity Q is larger than the realized demands D_0 and D_4. We have

$$\begin{aligned} SI_1 &= Max\{0, SI_0 + Q - D_0\} = Max\{0, 0+7-4\} = 3 \\ SI_5 &= Max\{0, SI_4 + Q - D_4\} = Max\{0, 0+7-5.5\} = 1.5 \end{aligned} \,. \tag{4.9}$$

Positive surplus inventory SI_6 is carried due to that surplus inventory SI_5 is carried from the previous time period 5. We have

$$SI_6 = Max\{0, SI_5 + Q - D_5\} = Max\{0, 1.5 + 7 - 8\} = 0.5\,. \tag{4.10}$$

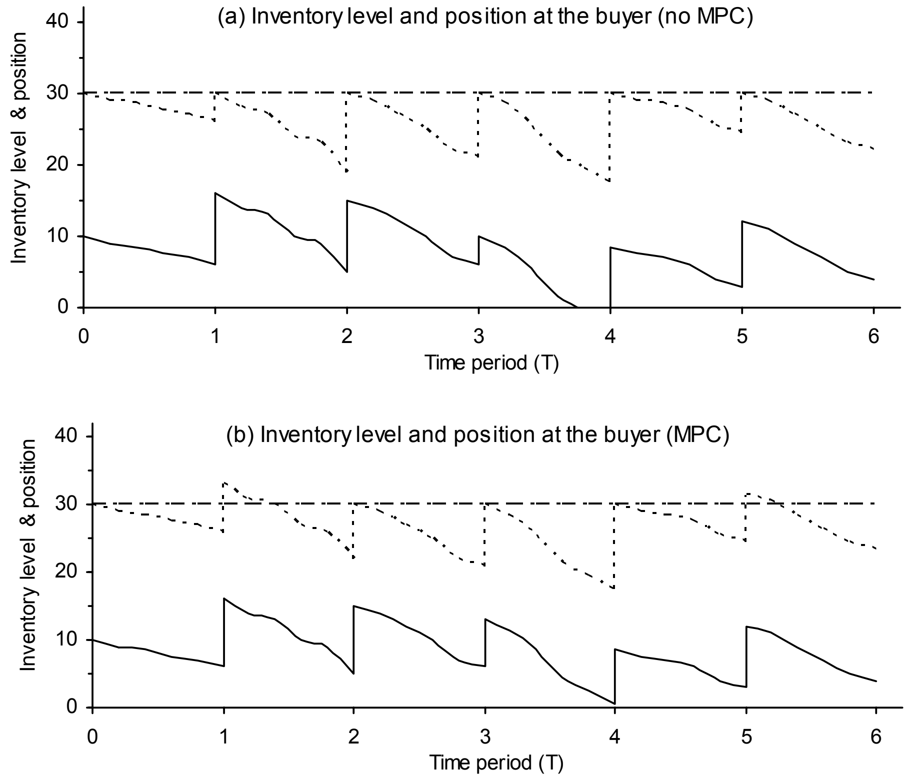

Figure 25. An example of the inventory level and position at the buyer in a single-channel system and a dual-channel system

From Table 7, we can calculate that the buyer's average inventory level is 12.64 in the dual-channel system, which is higher than the counterpart of 11.93 in the single-channel system. The surplus inventory can then be used to satisfy the stochastic customer demand and, in consequence, the service level is improved in the dual-channel system. For instance, stock-out happens in the time period 3 in the single-channel system (see Figure 25), while no stock-out happens in the time period 3 in the dual-channel system due to the surplus inventory carried.

IV.4.2 Simulation-Based Method

In Section IV.4.1, we gave a numerical example to show how the MPC agreement results in the surplus inventory carried at the buyer. In this section, we introduce a simulation-based method to estimate the surplus inventory level *SI*.

Analytical Computation of Surplus Inventory

First, let's look at the analytical computation of the surplus inventory level. By rewriting (4.7), we get a Lindley type iterative equation for the surplus inventory level SI_n as follows.

$$SI_n = Max\{0, SI_{n-1} + Q - D_{n-1}\}. \tag{4.11}$$

From the equation (4.11), we notice that the surplus inventory *SI* is dependent on the MPC quantity *Q* and the stochastic demand *D*, and independent of the order-up-to level *S*. The equation (4.11) also shows that the surplus inventory level *SI* is equivalent to the customer waiting time in a single-stage GI/D/1 queue, where the customers arrive with general independent inter-arrival times *D* and are served by a single *first-come-first-served* (FCFS) server with deterministic service time *Q*.

Unfortunately, it is well known that there is no closed-form analytical solution for the average customer waiting time in a single-stage GI/D/1 queue.

Exact Computation of Surplus Inventory

Now we give an example for the exact computation of expected average surplus inventory given that stochastic demand *D* has a probability density function $f(\cdot)$, the MPC quantity is *Q* and the starting surplus inventory level at the beginning of time period 0 is SI_0. For the cases of time horizon N=2 and N=3, we have

$$SI(N=2) = \frac{1}{2}SI_0 + \frac{1}{2}\int_{-\infty}^{SI_0+Q}(SI_0 + Q - D_0)f(D_0)dD_0 \tag{4.12}$$

$$\begin{aligned} SI(N=3) = {} & \frac{1}{3}SI_0 + \frac{1}{3}\int_{-\infty}^{SI_0+Q}(SI_0 + Q - D_0)f(D_0)dD_0 \\ & + \frac{1}{3}\int_{SI_0+Q}^{+\infty}\int_{-\infty}^{Q}(Q - D_1)f(D_1)dD_1 f(D_0)dD_0 \\ & + \frac{1}{3}\int_{-\infty}^{SI_0+Q}\int_{-\infty}^{SI_0+2Q-D_1}(SI_0 + 2Q - D_1 - D_0)f(D_1)dD_1 f(D_0)dD_0 \end{aligned} \tag{4.13}$$

The four terms in (4.13) represent, respectively, the starting surplus inventory level at the beginning of time period 0, the expected surplus inventory level at the beginning of time period 1, the expected surplus inventory level at the beginning of time period 2 when SI_1=0, and the expected surplus inventory level at the beginning of time period 2 when $SI_1>0$.

As shown in (4.12) and (4.13), even a moderate value of time horizon N can lead to a complex convolution so that the computation is prohibitive. Not mention we might need to enumerate the possible values of the MPC quantity Q to find the optimal solution.

In our research, rather than giving an analytical solution or an exact numerical solution, we resort to quantitatively estimating the expected average surplus inventory level SI in a simulation-based approximation method, which is based on the proposition given as follows.

Simulation-Based Estimation of Surplus Inventory

PROPOSITION IV.1. When the starting surplus inventory level SI_0 is zero and the stochastic demand D is *iid* normally distributed in each time period, the expected average surplus inventory level SI can be expressed as a product of the demand STD σ and a surplus inventory coefficient function k, which is a function of the time horizon N and the standardized MPC quantity $z=(\mu-Q)/\sigma$. That is

$$SI=\sigma\cdot k\left(N,z\right) \tag{4.14}$$

PROOF. To prove (4.14), it is sufficient to prove that the variable $\left(SI_n/\sigma\right)$ has the same probability density function $g_n(\cdot)$ in any dual-channel system with the same standardized MPC quantity z. For two dual-channel systems with the demand parameters $D'\left(\mu',\sigma'\right)$ and $D"\left(\mu",\sigma"\right)$, the statement (4.14) can be rewritten as follows.

$$g_n^{'}\left[\left(\frac{SI_n^{'}}{\sigma'}\right)=x\right]=g_n^{"}\left[\left(\frac{SI_n^{"}}{\sigma"}\right)=x\right],\quad given\quad \frac{\mu'-Q'}{\sigma'}=\frac{\mu"-Q"}{\sigma"}\qquad \forall x\geq 0, n\in N \tag{4.15}$$

The statement (4.15) can be proved by induction:

When n = 0, (4.15) holds according to the assumption of $SI_0^{'}=SI_0^{"}=0$.

When n = 1, we have

$$g_1^{'}\left[\left(\frac{SI_1^{'}}{\sigma'}\right)=x\right]=\Pr\left(D_0^{'}=Q-\sigma' x\right)=\Phi\left(-x+\frac{Q-\mu'}{\sigma'}\right) \tag{4.16}$$

$$g_1^{"}\left[\left(\frac{SI_1^{"}}{\sigma"}\right)=x\right]=\Pr\left(D_0^{"}=Q-\sigma" x\right)=\Phi\left(-x+\frac{Q-\mu"}{\sigma"}\right) \tag{4.17}$$

Where $\Phi(\cdot)$ is the PDF of the standard normal distribution *Norm*(0,1). Thus, the statement (4.15) holds when n = 1. Now, suppose (4.15) also holds when $n = m$ ($1 < m < N$-1); that is

$$g_m^{'}\left[\left(\frac{SI_m^{'}}{\sigma'}\right)=x\right]=g_m^{"}\left[\left(\frac{SI_m^{"}}{\sigma"}\right)=x\right], \qquad m\in[2,N-2] \tag{4.18}$$

When $n = m+1$, we e have

$$\begin{aligned} g_{m+1}^{'}\left[\left(\frac{SI_{m+1}^{'}}{\sigma'}\right)=x\right] &= \int_0^{+\infty}\Pr\left(D_m^{'}=SI_m^{'}+Q-\sigma' x\right)g_m^{'}(\frac{SI_m^{'}}{\sigma'})d(\frac{SI_m^{'}}{\sigma'}) \\ &= \int_0^{+\infty}\Phi\left(\frac{SI_m^{'}}{\sigma'}-x+\frac{Q-\mu'}{\sigma'}\right)g_m^{'}(\frac{SI_m^{'}}{\sigma'})d(\frac{SI_m^{'}}{\sigma'}) \\ &= \int_0^{+\infty}\Phi\left(y-x+\frac{Q-\mu'}{\sigma'}\right)g_m^{'}(y)d(y) \end{aligned} \tag{4.19}$$

$$\begin{aligned} g_{m+1}^{"}\left[\left(\frac{SI_{m+1}^{"}}{\sigma"}\right)=x\right] &= \int_0^{+\infty}\Pr\left(D_m^{"}=SI_m^{"}+Q-\sigma" x\right)g_m^{"}(\frac{SI_m^{"}}{\sigma"})d(\frac{SI_m^{"}}{\sigma"}) \\ &= \int_0^{+\infty}\Phi\left(\frac{SI_m^{"}}{\sigma"}-x+\frac{Q-\mu"}{\sigma"}\right)g_m^{"}(\frac{SI_m^{"}}{\sigma"})d(\frac{SI_m^{"}}{\sigma"}) \\ &= \int_0^{+\infty}\Phi\left(y-x+\frac{Q-\mu"}{\sigma"}\right)g_m^{"}(y)d(y) \end{aligned} \tag{4.20}$$

Thus, the statement (4.15) holds when $n = m+1$.

By induction, the statement (4.15) holds for any value n. Therefore, Proposition IV.1 is true.□

<u>PROPOSITION IV.2.</u> The expected average surplus inventory level is increasing when the time horizon *N* increases. And the long term expected average converges to a constant level when time horizon *N* is increasing to infinite (see Figure 26).

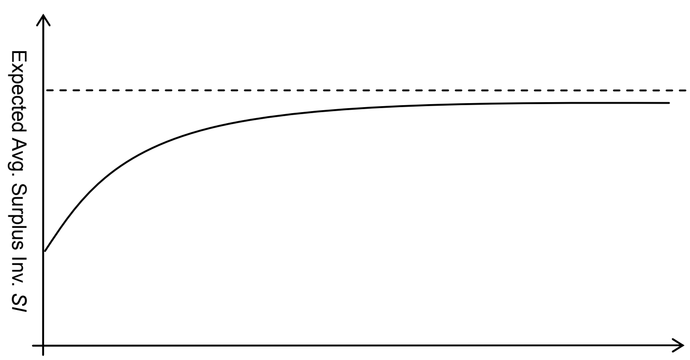

Figure 26. Effect of time horizon *N* on the expected average surplus inventory *SI*

PROOF. The proof of the convergence of surplus inventory level is the same as the proof of the convergence of average customer waiting time in a single-stage G/G/1 queue. We will not give the proof since this is out of the scope of this research.

According to the Proposition IV.1 and IV.2, the long-term expected average surplus inventory SI can be expressed as (4.21).

$$SI = \sigma \cdot k(z) \tag{4.21}$$

In (4.21), the long-term expected average surplus inventory SI depends on the demand standard deviation σ, the standardized MPC quantity z and the surplus inventory coefficient function $k(z)$. Thus, to decide SI, it is sufficient to know the value of the surplus inventory coefficient function $k(z)$, whether analytically or quantitatively.

IV.4.3 Surplus Inventory Coefficient Function *k*(*z*)

According to the discussion in Section IV.4.2, the surplus inventory coefficient function *k*(*z*) is unique for any case where the demand *D* is *iid* normally distributed in each time period. Now, we introduce a simulation-based method to evaluate the value of the surplus inventory coefficient function *k*(*z*), which can be used to quantitatively estimate the long-term average surplus inventory level *SI*.

By modeling the dual-channel inventory system as a single-stage GI/D/1 queue, we conduct simulations in Simul8™ with the following parameters.

- The time horizon *N* is 20,000 (time periods).
- In each time period, the demand *D* is *iid* normally distributed with the mean μ of 400 and the standard deviation σ of 100.
- The starting surplus inventory SI_0 is zero.
- The MPC quantity *Q* varies between 300 and 400 with an incremental of 1; that is, the standard MPC quantity *z* varies between 0 and 1 with an increment of 0.01.
- Each simulation trial includes 1,000 random runs.

In Table 8, we show the 100 simulation trial results of the surplus inventory coefficient function *k*(*z*). Each trial result has a 99% confidence interval that is at most 1% of its mean

Table 8. The simulation results of surplus inventory coefficient function *k*(*z*)

k(*z*)	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.00	79.45	40.19	23.33	15.83	11.98	9.478	7.794	6.591	5.687	5.005
.10	4.443	3.968	3.608	3.305	3.028	2.785	2.571	2.398	2.246	2.099
.20	1.964	1.848	1.743	1.651	1.561	1.475	1.399	1.333	1.269	1.211
.30	1.152	1.103	1.055	1.010	0.969	0.928	0.891	0.856	0.822	0.791
.40	0.761	0.733	0.706	0.681	0.656	0.632	0.610	0.590	0.571	0.550
.50	0.531	0.514	0.498	0.483	0.466	0.451	0.437	0.424	0.411	0.398
.60	0.386	0.375	0.363	0.353	0.342	0.332	0.322	0.312	0.303	0.295
.70	0.287	0.278	0.270	0.263	0.256	0.249	0.242	0.235	0.228	0.223
.80	0.216	0.211	0.205	0.199	0.194	0.189	0.184	0.179	0.174	0.169
.90	0.165	0.161	0.156	0.152	0.148	0.144	0.141	0.137	0.133	0.130

Given the simulation results presented in Table 8, we introduce a simulation-based method to estimate the long-term expected average surplus inventory level *SI* as follows:

(I).For any dual-channel system with the demand process *D~Norm*(σ, μ) and the MPC quantity *Q*, calculate the standardized MPC quantity $z = (\mu - Q)/\sigma$.

(II).If the value *z* falls in Table 8, estimate the surplus inventory coefficient function *k*(*z*) using the simulation result.

(III).If the value *z* does not fall in Table 8, estimate the surplus inventory coefficient function *k*(*z*) by assuming a linear function of *k*(*z*) between the value *z*'s two neighboring values in Table 8. We have

$$k(z) \approx k(z_i) + \frac{z - z_i}{z_j - z_i}\left[k(z_j) - k(z_i)\right], \quad given \; z_i < z < z_j \quad \text{and} \quad (z_j - z_i) = 0.01 \qquad (4.22)$$

(IV).According to the equation (4.21), evaluate the long-term expected average surplus inventory level $SI = \sigma \cdot k(z)$.

Given the simulation results and the linear interpolation estimation method, the surplus inventory coefficient function *k*(*z*) is shown in Figure 27 as follows

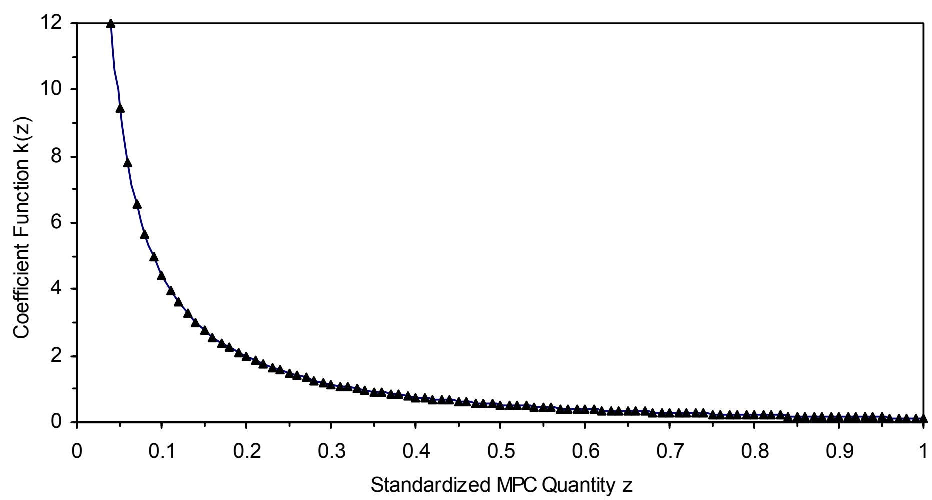

Figure 27. Surplus inventory coefficient function *k*(*z*)

From the Figure 27, we have two observations on the long-term surplus inventory coefficient function $k(z)$ as follows.

(I). *Exponential trend*

The surplus inventory coefficient function $k(z)$ is exponentially increasing as the standardized MPC quantity z is decreasing to zero. This exponential trend can be interpreted as follows. Decreasing MPC quantity z increases the probability of MPC quantity Q being larger than demand D and, in consequence, and increases the probability of the surplus inventories being built up over consecutive time periods. Such surplus inventory accumulation results in the exponential trend of the surplus inventory coefficient function $k(z)$.

(II). *Reasonable Range of the optimal standardized MPC quantity z*

From the Figure 27, we notice that the surplus inventory coefficient function $k(z)$ is equal to 79.45 and 0.130 when the standardized MPC quantity z is 0 and 1. The long term average surplus inventory will go to infinite when the standardized MPC quantity z drops below 0. Thus, it is enough to search the optimal standardized MPC quantity z^* within the range of [0, 1]; that is, a reasonable MPC quantity Q should fall in the range of $[\mu\text{-}\sigma, \mu]$.

In this section, we introduced a simulation-based method to quantitatively estimate the long-term expected average surplus inventory SI using the surplus inventory coefficient function $k(z)$. In Appendix A.3, we introduce another simulation-based method to estimate the surplus inventory SI in piece-wise quadratic format as follows.

$$SI \approx \sigma A_k^r z^2 + \sigma B_k^r z + \sigma C_k^r, \qquad \text{when} \quad z \in (Z_r, Z_{r+1}] \tag{4.23}$$

where Z_r is the breakpoint value of [0.1, 0.2, 0.3, 1].

IV.5 Safety Stocks in the Dual-Channel Vendor-Buyer System with MPC

In the dual-channel vendor-buyer system, each location periodically replenishes its inventory based on an order-up-to policy and carries safety stock to maintain a service level α, which is defined as the probability of no stockout happens in each time period. For the amount of realized demand beyond this service level, other ways such as spot market or expediting is used. The proposed dual-channel MPC supply strategy influences the system safety stock cost in the ways as follows.

At the buyer, the surplus inventory can be used to satisfy the stochastic customer demand so that maintaining the certain service level requires a lower order-up-to level than that in the traditional single-channel system. Thus, less safety stock needs to be carried at the buyer.

At the regional DC and the central DC, some of the burden of handling demand variability is shifted to the customers due to the MPC agreement. Therefore, less safety stocks need to be carried at the regional DC and the central DC.

In this section, we study the above impacts on the safety stocks in the dual-channel vendor-buyer system, with the assumption that demand *D* is *iid* normally distributed in each time period. For the same argument in Section IV.4, we resort to quantitatively estimating the safety stock level with a simulation-based approximation method, rather than giving an analytical solution or exact solution.

This section is organized as follows. We study the safety stocks at the buyer, the central DC and the regional DC in subsection IV.5.1 and IV.5.2. In Subsection IV.5.3, we introduce a simulation-based method to quantitatively estimate the safety stocks required to maintain a certain service level at each location. . In Appendix A.3, we introduce another simulation-based method to estimate the safety stocks using piece-wise quadratic functions.

IV.5.1 Safety Stock at the Buyer

In this section, we analyze the safety stock at the buyer in the traditional single-channel vendor-buyer system and the dual-channel vendor-buyer system, where inventory is periodically replenished according to an order-up-to policy. For the purpose of generality, we assume the supplementary orders have a positive lead time L_b, although it is assumed to be zero in the HP ink cartridges supplies system.

Single-Channel Vendor-Buyer System

In the traditional single-channel vendor-buyer system, the buyer's inventory position is always raised up to the order-up-to level S at the beginning of each time period. The order-up-to level S is determined to satisfy the total customer demand during the next L_b+1 time periods, subject to a desired service level α. Given the assumption that demand D is *iid* normally distributed in each time period, the total demand during the next L_b+1 time periods is normally distributed with the mean of $(L_b+1)\mu$ and the standard deviation of $\sigma \cdot \sqrt{L_b+1}$. Thus, to maintain a desired service level α, the safety stock level SS_b should be determined by

$$\Pr(\sum_{m=n}^{n+L_b} D_m \le IP) = \phi\left(\frac{IP-(L_b+1)\mu}{\sigma\sqrt{L_b+1}}\right) = \phi\left(\frac{SS_b}{\sigma\sqrt{L_b+1}}\right) = \alpha \qquad (4.24)$$

where $\phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}} dy$, which is the cumulative function of the standard normal distribution with mean of 0 and STD of 1. Thus, the safety stock level SS_b can be easily determined by a coefficient function $\chi(\alpha)$. We have

$$SS = \sigma \cdot \sqrt{L_b+1} \cdot \chi(\alpha) \qquad (4.25)$$

Dual-Channel Vendor-Buyer System

In the dual-channel vendor-buyer system with minimum purchase commitment, the buyer's inventory position is raised up to the level $S+SI_n$ at the beginning of time period n. Here, surplus inventory level SI_n is a random variable. The amount of inventory $S+SI_n$ should be used to satisfy the total demand during the next L_b+1 time periods, subject to a desired service level α. We have

$$\Pr(\sum_{m=n}^{n+L_b} D_m \le IP) = \phi\left(\frac{IP-(L_b+1)\mu}{\sigma\sqrt{L_b+1}}\right) = \phi\left(\frac{SI_n+SS_b}{\sigma\sqrt{L_b+1}}\right) = \alpha \qquad (4.26)$$

From the equations (4.24) and (4.26), we observe that less safety stock SS (or lower order-up-to level S) is required to maintain the same service level α at the buyer in the dual-channel vendor-buyer system. In addition, we have the following proposition.

PROPOSITION IV.3. In a dual-channel vendor-buyer system where the stochastic demand D is *iid* normally distributed in each time period, the safety stock level at the buyer SS_b can be determined by the following equation.

$$SS_b = \sigma \cdot \sqrt{L_b + 1} \cdot \psi(\alpha, z, L_b, N) \tag{4.27}$$

Where the coefficient function ψ depends on the service level α, the standardized MPC quantity z, the lead time L_b and the time horizon N.

PROOF. Consider any two dual-channel vendor-buyer systems with: the *iid* normally distributed customer demands $D' \sim Norm(\mu', \sigma')$ and $D'' \sim Norm(\mu'', \sigma'')$, the replenishment leadtime $L_b^{'}$ and $L_b^{''}$, and the same other system parameters of α, SI_0, z and N. At the beginning of time period n, the buyers should maintain safety stock levels $SS_b^{'}$ and $SS_b^{''}$ as follows

$$\Phi\left(\frac{SI_n^{'} + SS_b^{'}}{\sigma^{'}\sqrt{L_b^{'} + 1}}\right) = \Phi\left(\frac{SI_n^{''} + SS_b^{''}}{\sigma^{''}\sqrt{L_b^{''} + 1}}\right) = \alpha \tag{4.28}$$

Based on Proposition IV.1, the surplus inventory levels $SI_n^{'}$ and $SI_n^{''}$ have the *PDF* as

$$g'\left[\left(SI_n^{'}/\sigma'\right) = x\right] = g''\left[\left(SI_n^{''}/\sigma''\right) = x\right] \tag{4.29}$$

From (4.28) and (4.29), the safety stock levels $SS_b^{'}$ and $SS_b^{''}$ should satisfy

$$\frac{SS_b^{'}}{SS_b^{''}} = \frac{\sigma^{'}\sqrt{L_b^{'} + 1}}{\sigma^{''}\sqrt{L_b^{''} + 1}} \tag{4.30}$$

Therefore, Proposition IV.3 is true.□

Given a service level α, a demand STD σ and a replenishment lead time L_b, we can compute the required safety stock level SS_b with a coefficient function $\psi_{\alpha,L}(z)$ as follows

$$SS_b = \sigma \cdot \sqrt{L_b + 1} \cdot \psi_{\alpha,L}(z) \tag{4.31}$$

IV.5.2 Safety Stock at the Vendor

In this section, we analyze the safety stock at the vendor (central DC and regional DC) in the traditional single-channel vendor-buyer system and the dual-channel vendor-buyer system, where inventory is periodically replenished according to an order-up-to policy.

Single-Channel Vendor-Buyer System

In the traditional single-channel vendor-buyer system, the buyer always orders the amount of the realized demand during the previous time period. Thus, both the regional DC and the central DC observe an order process that is exactly the same as the demand process D. As a consequence, the safety stock level at the vendor SS_v can be determined by the same coefficient function $\chi(\alpha)$ as the buyer's safety stock level SS_b does. We have

$$SS_v = \sigma \cdot \sqrt{L_v} \cdot \chi(\alpha) \tag{4.32}$$

Dual-Channel Vendor-Buyer System

In the dual-channel vendor-buyer system with minimum purchase commitment, the regional DC and central DC's inventory positions are always raised up to the order-up-to level S at the beginning of each time period. The order process observed at the vendor, however, is different from the demand process D. Under the minimum purchase commitment, the buyer places an order for the smaller quantity of Q or (D_n-SI_n) at the beginning of time period n; that is

$$O_n = \min(Q, D_n - SI_n) \tag{4.33}$$

Thus, at the beginning of time period n, the inventory position IP_v should be used to satisfy the total order quantity during the next L_v time period. We have

$$SI_{n+1} = SI_n + O_n - D_n \tag{4.34}$$

$$\begin{aligned} O_n &= SI_{n+1} - SI_n + D_n \\ O_{n+1} &= SI_{n+2} - SI_{n+1} + D_{n+1} \\ \dots \dots &= \dots \dots \\ O_{n+L_v-1} &= SI_{n+L_v} - SI_{n+L_v-1} + D_{n+L_v-1} \end{aligned} \quad \Rightarrow \sum_{m=n}^{m=n+L_v-1} O_m = \sum_{m=n}^{m=n+L_v-1} D_m + SI_{n+L_v} - SI_n \tag{4.35}$$

$$\begin{aligned} \Pr(\sum_{m=n}^{n+L_v-1} O_m \le IP_v) &= \Pr(\sum_{m=n}^{n+L_v-1} D_m \le (SS_v + SI_n - SI_{n+L_v} + L_v\mu)) \\ &= \Phi\left(\frac{SS_v + SI_n - SI_{n+L_v}}{\sigma\sqrt{L_v}}\right) = \alpha \end{aligned} \tag{4.36}$$

PROPOSITION IV.4. In a dual-channel vendor-buyer system where the stochastic demand *D* is *iid* normally distributed in each time period, , the safety stock level at the vendor SS_v can be determined by the following equation.

$$SS_v = \sigma \cdot \sqrt{L_v} \cdot \varphi(\alpha, z, L_b, N) \qquad (4.37)$$

Where the coefficient function φ depends on the service level α, the standardized MPC quantity z, the lead time L_b and the time horizon N.

PROOF. Similar to the proof of Proposition IV.3.

Given a service level α, demand STD σ and replenishment lead time L_b, we can compute the required safety stock level SS_v with a coefficient function $\varphi_{\alpha,L}(z)$ as follows

$$SS_v = \sigma \cdot \sqrt{L_v} \cdot \varphi_{a,L}(z) \qquad (4.38)$$

Compare the equations (4.26) and (4.36), we notice that the vendor requires more safety stock to maintain a service level than the buyer does. That is

$$\varphi(\alpha, z, L_b, N) \geq \psi(\alpha, z, L_v, N), \qquad \forall L_b + 1 = L_v \qquad (4.39)$$

From the above analysis, however, it is still not clear whether the proposed dual-channel MPC supply strategy can reduce the required safety stock level at the vendor, since the demand process D_m, surplus inventory levels SI_n and SI_{n+L} are dependent on each other. Rather than giving a rigorous proof, we discuss the following situations.

(I). In the case of a "*large*" total demand realized during the time periods n to $n+L_v$-1, the surplus inventory SI_{n+L} is more likely to be zero. Thus, the total order quantity $\sum D_m - SI_n$ is likely to be smaller than the total demand quantity $\sum D_m$.

(II). In the case of a "*small*" total demand realized during the time periods n to $n+L_v$-1, the surplus inventory SI_{n+L} is more likely to be positive. Thus, the total order quantity $\sum D_m - SI_n + SI_{n+L}$ is likely to be larger than the total demand quantity $\sum D_m$

Thus, in the dual-channel vendor-buyer system, the order process observed at the vendor is smoother than that in the single-channel vendor-buyer system and, in consequence, less safety stock is required at the vendor to maintain the same service level.

IV.5.3 Safety Stock Coefficient Functions $\psi(z)$ and $\varphi(z)$

We conducted simulations with VBA programming in Microsoft Excel™ to get the long-term safety stock coefficient functions $\psi_{\alpha,L}(z)$ and $\varphi_{\alpha,L}(z)$ with the following parameters:

- The demand process D is *iid* normally distributed with mean μ of 400 and standard deviation σ of 100 in each time period.
- The MPC quantity Q varies between 300 and 390 with an incremental of 1; that is, the standard MPC quantity z varies between 0.1 and 1 with an increment of 0.01.
- The starting surplus inventory SI_0 is zero, and the time horizon N is 20,000.
- For the safety stock coefficient functions $\psi_{\alpha,L}(z)$ and $\varphi_{\alpha,L}(z)$, we consider the service levels of [98%, 95%, 90%] and the lead times of [1, 3, 5, 7, 15, 25].

Each simulation trial includes 2,000 random runs. Each trial result has a 99.5% confidence interval that is at most 1% of its mean.

Table 9. The simulation results of safety stock coefficient function $\psi_{98\%,1}(z)$

$\psi_{98\%}(z)$	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.10	1.273	1.317	1.358	1.394	1.426	1.455	1.482	1.507	1.531	1.552
.20	1.572	1.591	1.608	1.624	1.639	1.654	1.668	1.681	1.693	1.705
.30	1.716	1.726	1.737	1.746	1.756	1.765	1.773	1.782	1.789	1.797
.40	1.804	1.811	1.818	1.824	1.831	1.837	1.842	1.848	1.854	1.859
.50	1.864	1.869	1.874	1.878	1.883	1.887	1.891	1.895	1.899	1.903
.60	1.907	1.910	1.914	1.918	1.921	1.924	1.927	1.930	1.933	1.936
.70	1.939	1.942	1.944	1.947	1.950	1.952	1.954	1.957	1.959	1.961
.80	1.963	1.965	1.967	1.969	1.971	1.973	1.975	1.977	1.979	1.981
.90	1.982	1.984	1.986	1.987	1.989	1.990	1.992	1.993	1.994	1.996

Table 10. The simulation results of safety stock coefficient function $\varphi_{98\%,1}(z)$

$\psi_{98\%}(z)$	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.10	1.210	1.257	1.301	1.340	1.374	1.406	1.435	1.462	1.486	1.509
.20	1.530	1.550	1.569	1.586	1.603	1.619	1.633	1.647	1.659	1.672
.30	1.684	1.695	1.706	1.716	1.725	1.735	1.744	1.753	1.761	1.769
.40	1.777	1.784	1.792	1.799	1.805	1.812	1.818	1.825	1.831	1.837
.50	1.843	1.848	1.853	1.858	1.863	1.868	1.872	1.877	1.881	1.885
.60	1.889	1.893	1.897	1.901	1.904	1.908	1.911	1.914	1.917	1.921
.70	1.924	1.927	1.930	1.932	1.935	1.938	1.940	1.943	1.945	1.948
.80	1.950	1.953	1.955	1.957	1.959	1.962	1.964	1.966	1.968	1.970
.90	1.972	1.974	1.976	1.977	1.979	1.981	1.982	1.984	1.985	1.987

In Table 9 and 10, we show the simulation results of $\psi_{98\%,1}(z)$ and $\varphi_{98\%,1}(z)$. More simulation results are given in the Appendix A.4.

For any value of the standardized MPC quantity z, safety stock coefficient functions $\psi_{\alpha,L}(z)$ and $\varphi_{\alpha,L}(z)$ can be estimated by assuming a linear function between any two neighboring values in Table 9 and 10. Thus, we have

$$\psi_{\alpha,L}(z)=\psi_{\alpha,L}(z_i)+\frac{z-z_i}{z_j-z_i}\left[\psi_{\alpha,L}(z_j)-\psi_{\alpha,L}(z_i)\right], \quad given \;\; z_i<z<z_j \tag{4.40}$$

$$\varphi_{\alpha,L}(z)=\varphi_{\alpha,L}(z_i)+\frac{z-z_i}{z_j-z_i}\left[\varphi_{\alpha,L}(z_j)-\varphi_{\alpha,L}(z_i)\right], \quad given \;\; z_i<z<z_j \tag{4.41}$$

The simulation results in Table 9 and 10 are also shown in Figure 28.

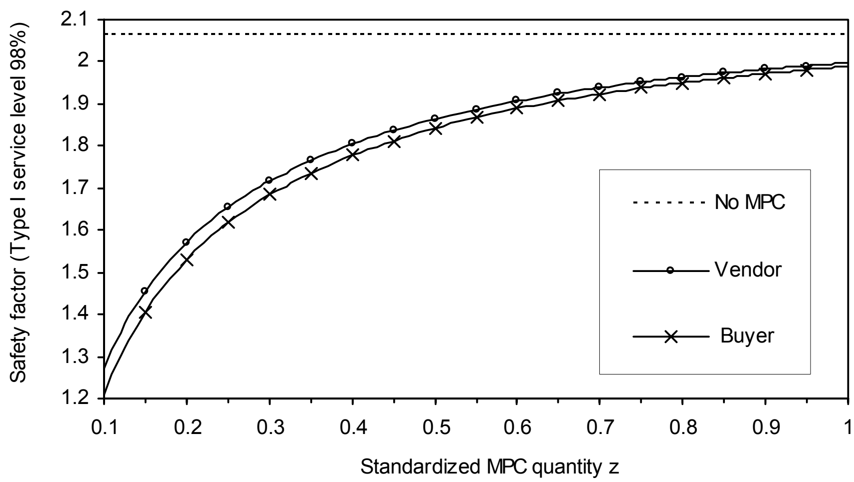

Figure 28. Simulation results of safety stock coefficient functions $\psi_{98\%,1}(z)$ and $\varphi_{98\%,1}(z)$

From the Figure 28, we observe that the required surplus inventory levels are increasing when the standardized MPC quantity z decreases. In addition the required surplus inventory levels are converging to 2.055, which is the safety stock factor to maintain a service level of 98% in the traditional single-channel vendor-buyer system.

In Appendix A.2, we give more simulation results on the coefficient functions $\psi_{\alpha,L}(z)$ and $\varphi_{\alpha,L}(z)$. In Appendix A.3, we introduce another method to estimate the coefficient functions $\psi_{\alpha,L}(z)$ and $\varphi_{\alpha,L}(z)$ in piece-wise quadratic format as follows.

$$\psi_{\alpha,L}(z)\approx A^r_{\psi,\alpha,L}z^2+B^r_{\psi,\alpha,L}z+C^r_{\psi,\alpha,L}, \qquad \text{when} \;\; z\in(Z_r,Z_{r+1}] \tag{4.42}$$

$$\varphi_{\alpha,L}(z)\approx A^r_{\varphi,\alpha,L}z^2+B^r_{\varphi,\alpha,L}z+C^r_{\varphi,\alpha,L}, \qquad \text{when} \;\; z\in(Z_r,Z_{r+1}] \tag{4.43}$$

IV.6 Dual-Channel Vendor-Buyer Coordination Models

In this section, we develop the integrated coordination model and the channel coordination model for the dual-channel vendor-buyer system studied, using the simulation-based approximation methods introduced in Section IV.4 and IV.5.

IV.6.1 Integrated Coordination Model

In the *integrated coordination model*, the buyer and vendor fully cooperate with each other to make decision on the MPC quantity Q that minimizes the total system cost. We use C_{sys} to denote the expected average system cost per time period. We have

$$C_{sys} = \left[c_2\mu - (c_2 - c_1)Q\right] + (\frac{1}{2}\mu + SI + SS_b)h_b + SS_{RDC}h_{RDC} + SS_{CDC}h_{CDC} \qquad (4.44)$$

In (4.44), the first term represents the system supply cost of shipping product through both indirect and direct channels. The second term represents the buyer's inventory cost, which includes the cycle inventory cost $0.5\mu h_b$, the surplus inventory cost $SI\cdot h_b$, and the safety stock cost $SS_b\cdot h_b$. The last two terms represent the safety stock costs at the regional DC and central DC. With the simulation-based methods introduced Section IV.4 and IV.5, we can rewrite the expected average system cost C_{sys} as a function of the standardized MPC quantity z as follows.

$$\begin{aligned} C_{sys}(z) = \mu(c_1 + \frac{1}{2}h_b) + \sigma\Big\{(c_2 - c_1)z + \left[k(z) + \sqrt{L_b + 1}\psi_{\alpha, L_b+1}(z)\right]h_b \\ + \sqrt{L_{RDC}}\varphi_{a, L_{RDC}}(z)h_{RDC} + \sqrt{L_{CDC}}\varphi_{a, L_{CDC}}(z)h_{CDC}\Big\} \end{aligned} \qquad (4.45)$$

Therefore, the objective of an integrated coordination model is to find the optimal standardized MPC z^* that minimizes the expected average system cost $C_{sys}(z)$.

$$z^* = \arg\min_{0<z\le 1} C_{sys}(z) \qquad (4.46)$$

As aforementioned, the feasible optimal standardized MPC quantity z^* should take a value within the half open range (0, 1]; that is, the optimal MPC quantity Q^* should be nonnegative and smaller than the demand mean μ. Now we discuss the effects of the given problem parameters on the optimal solution of the integrated coordination model.

(I). *Effect of demand parameters μ and σ on the optimal solution* z^*

From the expression of the expected average system cost C_{sys} in (4.45), we observe that the first term is proportional to the demand mean μ but remains as a constant when the standardized MPC quantity z varies, and the second term is a product of the demand STD σ and a value that does not depend on the demand parameters μ *and* σ. Thus, the optimal solution z^* is independent of the demand parameters μ *and* σ.

(II). *Effect of supply cost rates on the optimal solution* z^*

The supply cost rates c_1 and c_2 contribute to the expected average system cost C_{sys} in that the supply cost savings per unit (c_2-c_1) can be obtained by shipping product through the direct channel. When the value of (c_2-c_1) increases, the optimal MPC quantity Q^* should also increase and, in consequence, the optimal standardized MPC quantity z^* decreases.

(III). *Effect of holding cost rate h on the optimal solution* z^*

From the previous analysis, we know that the MPC agreement increases the inventory level at the buyer and decreases the inventory levels at the regional DC and central DC. Thus, the optimal solution z^* increases when the holding cost rate h_b increases, and decreases when the holding cost rate h_{rdc} or h_{cdc} increases.

According to the discussion in Section II.2, the integrated coordination always benefits the vendor and damages the buyer. Thus, the increased system profit (or decreased system cost) needs to be allocated between the vendor and buyer so that the buyer is willing to participate in implementing the integrated coordination. We use Π to denote the amount of system profit that needs to be shifted from the vendor to the buyer under the *equal allocation scheme*. We have

$$\begin{aligned}\Pi = &\frac{1}{2}\sigma k(z)h_b - \frac{1}{2}\sqrt{L_b+1}\sigma\left[\psi_{\alpha,L_b+1}(\mu/\sigma) - \psi_{\alpha,L_b+1}(z)\right]h_b + \frac{1}{2}(c_2-c_1)(\mu-\sigma z)\\ &+\frac{1}{2}\sqrt{L_{RDC}}\sigma\left[\varphi_{a,L_{RDC}}(\mu/\sigma) - \varphi_{a,L_{RDC}}(z)\right]h_{RDC}\\ &+\frac{1}{2}\sqrt{L_{CDC}}\sigma\left[\varphi_{a,L_{CDC}}(\mu/\sigma) - \varphi_{a,L_{CDC}}(z)\right]h_{CDC}\end{aligned} \tag{4.47}$$

In (4.47), the first two terms represent the half of the increased buyer's cost, and the last three terms present the half of the decreased vendor's cost. When the profit allocation is in the form of purchase discount, the vendor should offer a purchase discount for the MPC quantity Q at percentage λ, which is defined as

$$\lambda = \Pi / Q^* \tag{4.48}$$

IV.6.2 Channel Coordination Model

In the *channel coordination model*, the vendor offers a purchase discount λ to entice the buyer to make replenishment decisions in a cooperative way. The channel coordination model consists of two interacting problems as follows.

Buyer's Problem

In the buyer's problem, the buyer makes replenishment decisions to minimize its individual cost, which includes the inventory cost and procurement cost. We use C_b to denote the expected average buyer's cost per time period. We have

$$C_b = p\mu - \lambda pQ + \left(\tfrac{1}{2}\mu + SI + SS_b \right) h_b \tag{4.49}$$

The first two terms represent the cost of purchasing product through both the direct channel and the indirect channel. The third term represents the inventory cost at the buyer. With the simulation-based method introduced in Section IV.4 and IV.5, we can rewrite the buyer's cost C_b as a function of the standardized MPC quantity z as follows.

$$C_b(z) = p\mu - \lambda p(\mu - \sigma z) + \left[\tfrac{1}{2}\mu + \sigma k(z) + \sigma\sqrt{L_b + 1}\ \psi_{\alpha, L_b+1}(z) \right] h_b \tag{4.50}$$

In (4.50), the purchase discount λ is a given parameter rather than a decision variable. Thus, the objective of buyer's problem is to find the optimal standardized MPC quantity z_λ^* minimizes the buyer's cost C_b.

$$z_\lambda^* = \arg\min_{0<z\leq 1} C_b(z) \tag{4.51}$$

Vendor's Problem

In the vendor's problem, the vendor makes decision on the price discount λ to minimize its individual cost, which includes the safety stock costs at the central DC and regional DC, the system supply cost, and the price discount offered to the buyer. The expected average buyer's cost C_v can be expressed as follows

$$C_v = c_2\mu - (c_2 - c_1)Q_\lambda^* + SS_{RDC}h_{RDC} + SS_{CDC}h_{CDC} + \lambda pQ_\lambda^* \tag{4.52}$$

The first two terms represent the system supply cost. The third and fourth terms represent the safety stock costs at the central DC and the regional DC. The last term represents the price discount offered to the buyer's committed purchase quantity Q_λ^*.

Using the simulation-based method introduced in Section IV.4 and IV.5, we can rewrite the expected vendor's average cost C_v as a function of the purchase discount λ.

$$C_v(\lambda) = c_2\mu - (c_2 - c_1)(\mu - \sigma z_\lambda^*) + \sqrt{L_{RDC}}\sigma\varphi_{\alpha,L_{RDC}}(z_\lambda^*)h_{RDC} + \sqrt{L_{CDC}}\sigma\varphi_{\alpha,L_{CDC}}(z_\lambda^*)h_{CDC} + \lambda p(\mu - \sigma z_\lambda^*) \tag{4.53}$$

Note that, in (4.53), we assume that the buyer is a rational decision maker so that the buyer's committed purchase quantity Q_λ^* is predictable. Thus, the objective of the vendor's problem is to find the optimal purchase discount λ^* to minimize its individual cost. We have

$$\lambda^* = \arg\min_{0 \le \lambda < 1} C_v(\lambda) \tag{4.54}$$

Now we discuss the effects of the given problem parameters on the optimal solution of the channel coordination model.

(I). *Effect of demand parameters μ and σ on the buyer's optimal solution* z_λ^*

For the buyer's problem in the channel coordination model, the expected average buyer's cost C_b can be rewritten as

$$C_b(z) = (p\mu - \lambda p\mu + \tfrac{1}{2}\mu h_b) + \sigma\left[\lambda pz + k(z)h_b + \sqrt{L_b + 1}\ \psi_{\alpha,L_b+1}(z)h_b\right] \tag{4.55}$$

Where the first term is proportional to the demand mean μ but remains as a constant when the standardized MPC quantity z varies, and the second term is a product of the demand STD σ and a part that does not depend on the demand parameters μ *and* σ. Thus, the optimal solution z_λ^* is independent of the demand parameters μ *and* σ.

(II). *Effect of purchase discount λ on the buyer's optimal solution* z_λ^*

The purchase discount λ contributes to the buyer's C_b in that cost savings λ can be obtained by purchasing per unit through the direct channel. When the purchase discount λ increases, the optimal MPC quantity Q^* should also increases and, in consequence, the optimal solution z^* decreases.

(III). *Impact of holding cost rate h_b on the buyer's optimal solution* z_λ^*

From the previous analysis, we know that the MPC agreement will increase the inventory level at the buyer. Thus, the optimal solution z_λ^* increases when the holding cost rate h_b increases.

(IV). *Impact the dual-channel MPC supply strategy on the system inventory*

As aforementioned, in both coordination models, the proposed dual-channel MPC supply strategy always reduces the inventories (safety stocks) at the central DC and regional DC. The safety stock reduction at these vendor-owned locations is due to the less order variability.

At the buyer, the proposed dual-channel MPC supply strategy always increases the inventory. This is due to that the increased surplus inventory is always larger than the decreased safety stock at the buyer. This is also consistent with the intuition that the buyer cannot decrease its inventory by committing to purchase a minimum quantity every time period, given the demand process is not affected by the MPC agreement.

The impact of the dual-channel MPC supply strategy on the total system inventory, which includes the safety stocks at all the locations and the surplus inventory at the buyer, depends on the system structure and the MPC quantity. In all of our numerical cases, the total system inventory increases because the increased surplus inventory at the buyer is larger than the decreased safety stocks in the system. To consider a general multi-stage vendor-buyer system, however, we can foresee that the system inventory could decrease when the number of upstream vendor-owned facilities is increasing so that the decreased system safety stocks increases.

IV.7 Numerical Case Study

In this section, we study the numerical case for a dual-channel vendor-buyer system, which is based on the distribution of the HP black ink cartridge (product code: 15).

As shown in the right figure, the HP 15 black ink cartridge is distributed to a retailer in Beijing through two channels. Through a direct channel, the orders are delivered directly from a HP-owned regional PCC in Singapore. Through an indirect channel, the orders are delivered from a HP-owned regional DC in Shanghai, where the ink cartridge is stored and supplied from the upstream regional PCC. Sea freight is used for the outbound shipments (dashed lines in the figure) at the regional PCC. Truck freight is used for the shipments (solid line in the figure) between the regional DC and the retailer. The case parameters are given as follows.

- At the retailer, the ink cartridge has an *iid* normally distributed demand $D \sim Norm$ (1000, 400) per week. The demand D is independent of the retailing price, which is \$30.
- Through the indirect channel, the retailer purchases the ink cartridge at the price P of \$27. For the orders delivered through the direct channel, a purchase discount λ is offered.
- The ink cartridge has the cumulated product costs of \$27, \$23, and \$22 at the retailer, the regional DC, and the regional PCC, respectively.
- Holding cost is incurred based on the annual interest rate = 25% and 50 weeks in a year.
- Supply cost rates are c_1=\$0.8 for the direct channel, and c_2=\$1.2 for the indirect channel.
- Each location replenishes its inventory every week.
- At the retailer, the target service level α is 98%.
- The inbound replenishment leadtime at the PCC (L_{CDC}) is 5 weeks, and the inbound replenishment leadtime at the regional DC (L_{RDC}) is 3 weeks.

For the dual-channel vendor-buyer system described above, we study both the integrated coordination model and the channel coordination model in Section IV.6.1 and Section IV.6.2.

IV.7.1 Integrated Coordination Model

In the *integrated coordination model*, the buyer and vendor can fully cooperate with each other to make decision on MPC quantity Q so that the total system cost is minimized.

Effect of demand parameters μ and σ on the optimal solution z^*

In Figure 29, we show the total system cost C_{sys} with the demand parameters [(μ=1000, σ=400), (μ=1000, σ=450), (μ=800, σ=500), (μ=800, σ=550)]. We have the following observations.

(I). The total system cost C_{sys} is a convex function of the standardized MPC quantity z and is exponentially increasing when the value z is getting close to zero. This trend is mainly due to the exponentially increasing of the surplus inventory cost.

(II). For all the four sets of demand parameters, the optimal standardized MPC quantity z^* is 0.248. This confirms our finding that the optimal standardized MPC quantity z^* is independent of the demand mean and the demand STD.

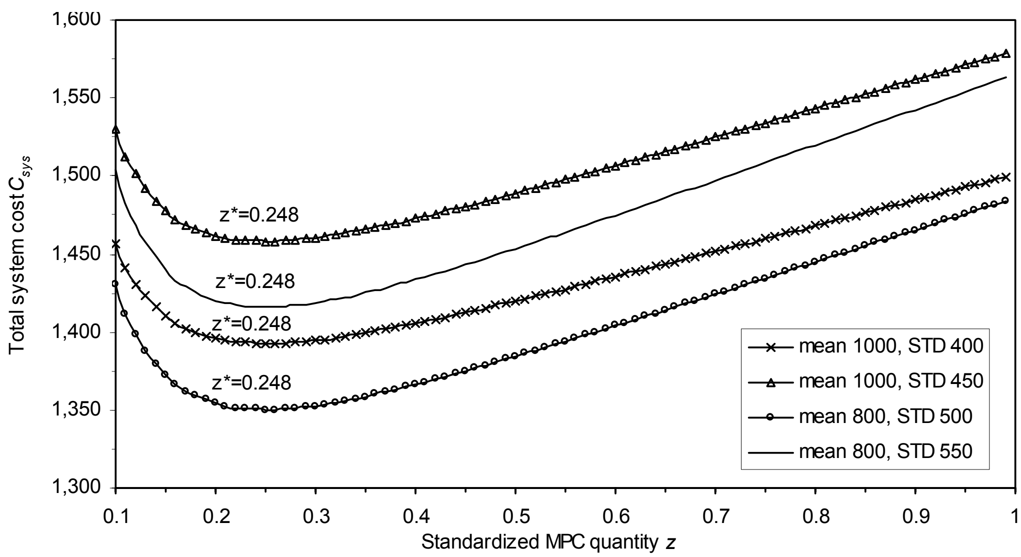

Figure 29. Effect of demand mean μ and STD σ on the optimal solution z^*

For the set of demand parameters (μ = 1000, σ = 400), we have

- The optimal value z^* is 0.248 and the optimal MPC quantity Q^* is 900.
- The minimum total system cost C^*_{sys} is \$1392 per week.
- The surplus inventory holding cost at the buyer is \$39.8 per week.
- The system supply cost is \$840 per week.
- The safety stock holding costs at CDC, RDC and buyer are \$176,\$139,\$89 per week.

Effect of supply cost rates on the optimal solution z^*

In Figure 30, we demonstrate the effect of the supply cost rates c_1 and c_2 on the optimal solution z^* in integrated coordination model. The four sets of the parameters are [(c_1=0.8, c_2=1), (c_1=0.8, c_2=1.2), (c_1=0.8, c_2=1.4), (c_1=0.8, c_2=1.6)].

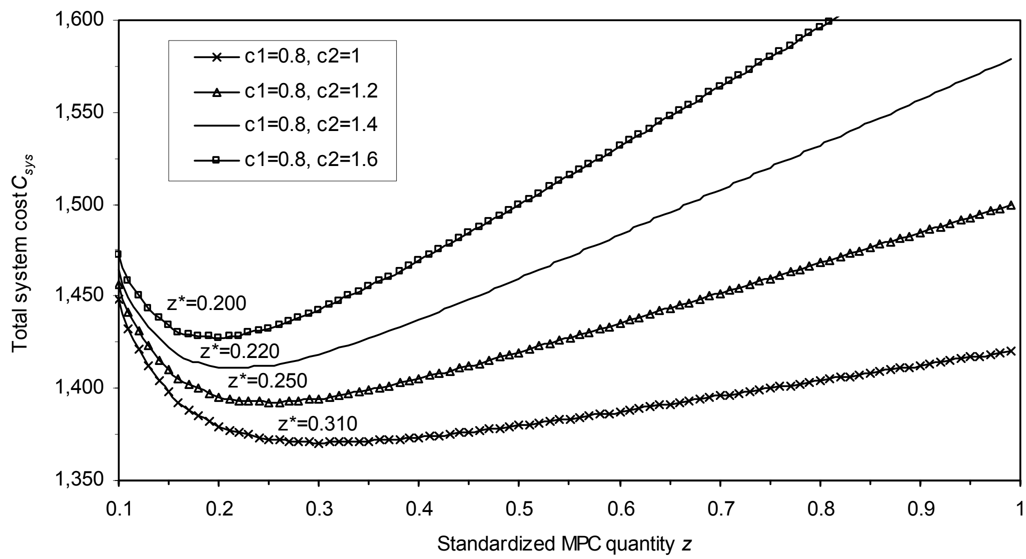

Figure 30. Effect of supply cost rates c_1 and c_2 on the optimal solution z^*

From the Figure 30, we have the following observations.

(I).When the value (c_2-c_1) increases, the optimal MPC quantity Q^* increases and, in consequence, the optimal standardized MPC quantity z^* decreases. This observation confirms our analysis in Section IV.6.1.

(II). When the standardized MPC quantity z increases, the system cost difference is increasing mainly due to the increasing cost difference in supplying product in cost c_2.

Effect of buyer's hodling cost rate on the optimal solution z^*

In Figure 31, we demonstrate the effect of the holding cost rate h_b on the optimal solution z^* in integrated coordination model. The four sets of parameters are [(h_b=5), (h_b=6), (h_b=7), (h_b=8)](\$/unit year).

From the simulation results, we observe that the optimal solution z^* increases when holding cost rate h_b increases. This observation confirms our analysis in Section IV.6.1.

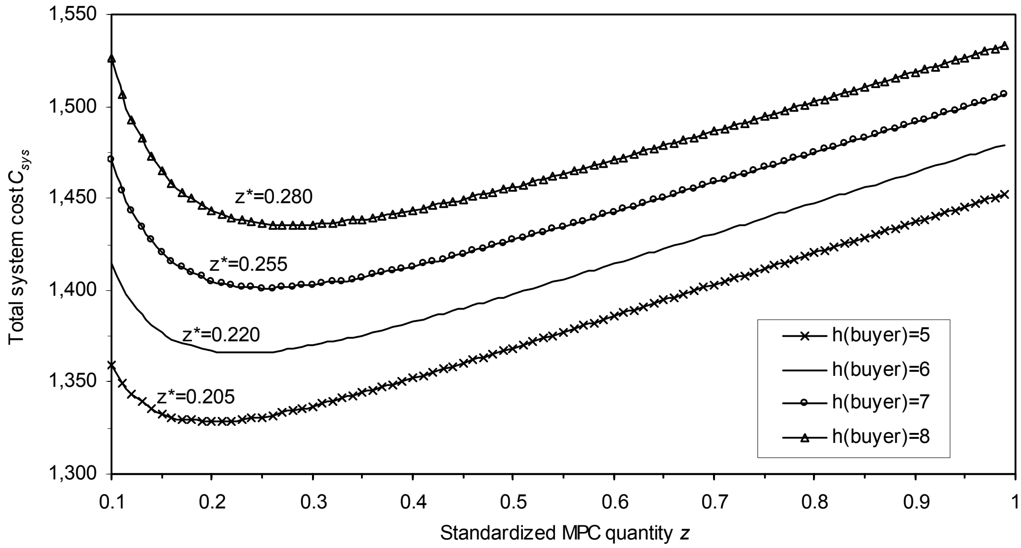

Figure 31. Effect of holding cost rate h_b on the optimal solution z^*

Effect of vendor's hodling cost rate on the optimal solution z^*

In Figure 32, we demonstrate the effect of holding cost rate h_{cdc} on the optimal solution z^* in integrated coordination model. The four sets of parameters are [(h_{cdc} =5), (h_{cdc} =6), (h_{cdc} =7), (h_{cdc} =8)] (\$/unit year). We observe that the optimal standardized MPC quantity z^* increases when the holding cost rate h_{CDC} decreases. This observation confirms our analysis in Section IV.6.1.

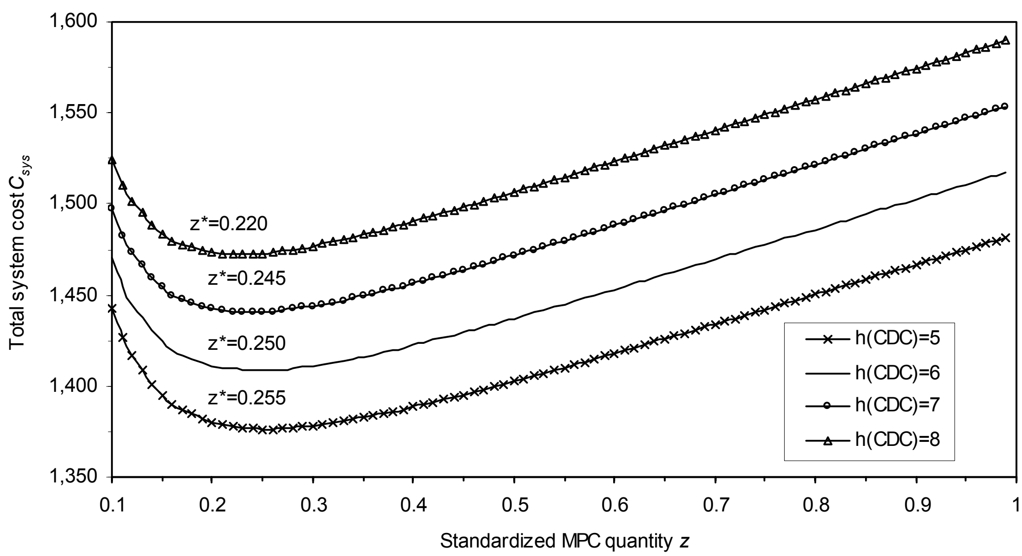

Figure 32. Effect of holding cost rate h_{cdc} on the optimal solution z^*

IV.7.2 Channel Coordination Model

In the *channel coordination model*, the vendor offers a purchase discount λ to entice the buyer to make replenishment decisions in a cooperative way.

Effect of purchase discount λ on the optimal solution z^*

In Figure 33, we show the effect of the purchase discount λ on the optimal standardized MPC quantity z^* in channel coordination model. The four sets of parameters are [λ=0.3%, λ=0.5%, λ=0.8%, λ=*1*.0%,].

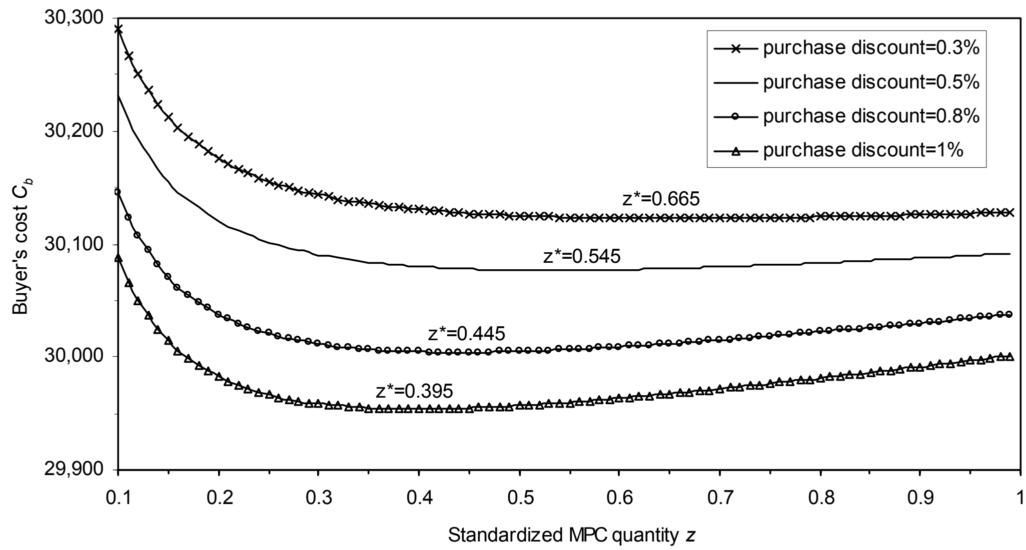

Figure 33. Effect of purchase discount λ on the optimal solution z^*

From Figure 33, we have the following observations,

(I). The optimal standardized MPC quantity z^* is sensitive to the change of the purchase discount λ and is increasing when the purchase discount λ decreases.

(II). In the channel coordination model, the optimal standardized MPC quantity z^* is less than the counterpart in a corresponding integrated coordination model. This is due to that the buyer decides the optimal standardized MPC quantity z^* without taking into account the vendor's cost.

Effect of purchase discount λ on the optimal buyer's cost optimal solution z^*

In Figure 34 and 35, we demonstrate the effect of the purchase discount λ on the optimal buyer's cost C_b^* and optimal standardized MPC quantity z^*.

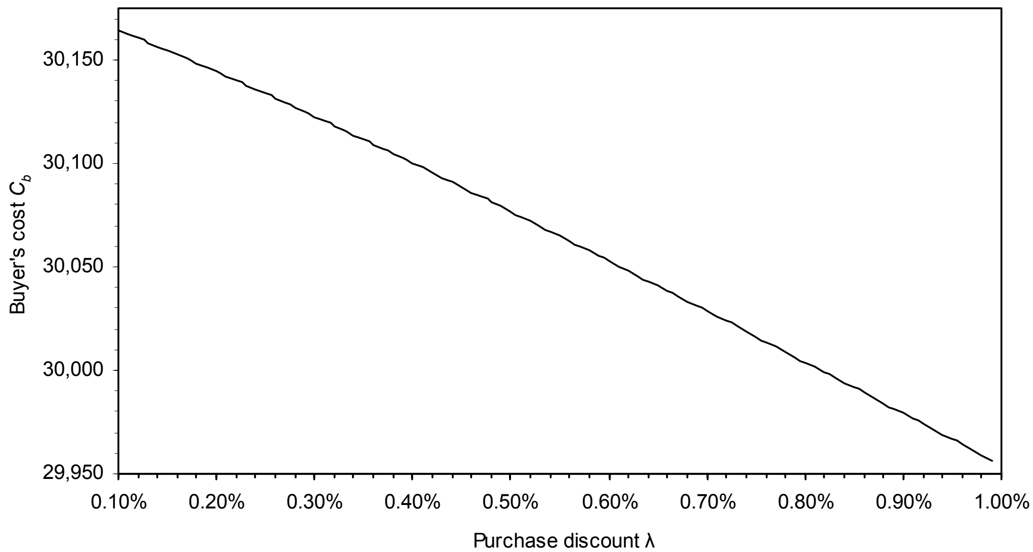

Figure 34. Effect of purchase discount λ on the optimal buyer's cost C_b^*

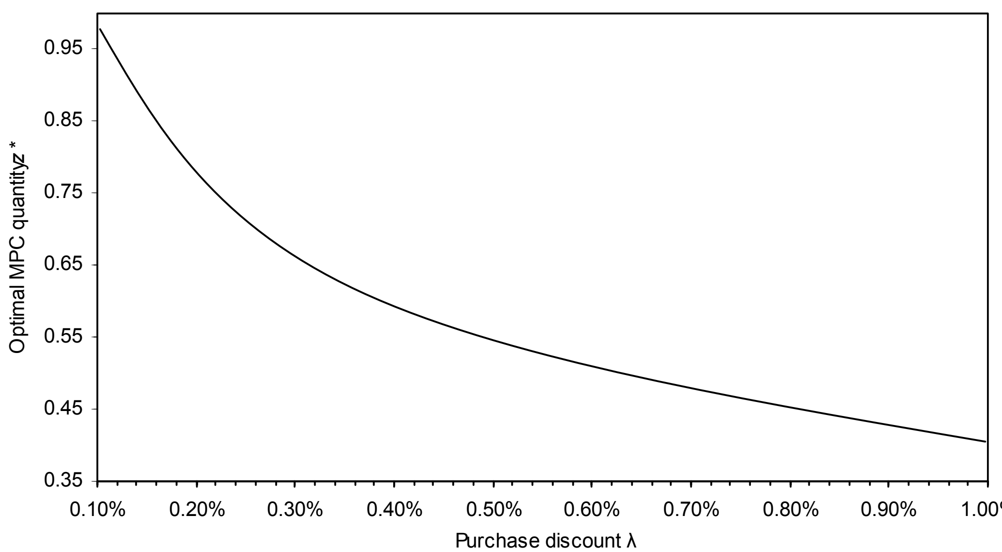

Figure 35. Effect of purchase discount λ on the optimal solution z^*

As we can see from the above two figures, the optimal buyer's cost and optimal MPC quantity decrease as the purchase discount decreases.

IV.8 Conclusions, Managerial Implications, and Future Research

IV.8.1 Conclusions

In this chapter, we studied the logistics coordination issue in a dual-channel vendor-buyer system that consists of a vendor-owned central DC, a vendor-owned regional DC and a buyer. Through an indirect channel, the single product is delivered from the central DC to the regional DC, and then to the buyer. Through a direct channel, the product is delivered directly from the distant central DC to the customer, bypassing the intermediate regional DC. In this dual-channel vendor-buyer system, the vendor and buyer can cooperate in making an MPC agreement to increase the system profitability as well as the individual profitability.

For the dual-channel vendor-buyer system, we developed both integrated coordination model and channel coordination model. We introduced a simulation-based method to quantitatively analyze each model. We found that the minimum purchase commitment results in surplus inventory buffered at the buyer and, in consequence, some amount of the burden of dealing with the demand variability is shifted away from the vendor to the buyer.

Our research contributes to the literature in the following two aspects.

(I). *Vendor-Buyer Coordination Context*

All the previous works study the dual-channel system from the individual perspective of the buyer. In this research, we investigate the coordination issue concerning how the vendor and buyer can cooperate in making decisions. In particular, we study an integrated coordination model in which the buyer and vendor can fully cooperate with each other to make decisions to minimize the total system cost, and a channel coordination model in which the vendor offers a purchase discount to entice the buyer to make decisions in a cooperative way.

(II). *Simulation-Based Method*

We develop a simulation-based approximation method to quantitatively analyze the dual-channel vendor-buyer system under the assumption that customer demand is *iid* normally distributed in each time period.

IV.8.2 Managerial Implications

This research can serve as a building block and an assisting tool for several aspects in vendor-buyer coordination, supply chain network design, supply strategy development, and supply contracts negotiation. It can also help a vendor to compete for a single sourcing agreement; that is, the vendor could design a dual-channel MPC supply contract to provide both economies of scale and substantial flexibility that make it unfavorable for the buyer to consider a second vendor.

IV.8.3 Future Research

The future research could be focusing on the following aspects.

(I). *Performance measure of fill rate*

For each vendor-buyer coordination model studied in this research, we assume that the safety stocks are carried to maintain a desired service level. A reasonable future research direction could be considering a fill rate as the performance measure in these models.

(II). *Dynamic, non-stationary, or price-sensitive demand*

In this research, we assume that the demand process is stationary and price-insensitive. This demand assumption could be limited in the seasonal products or short planning context. Especially, given the demand is price sensitive, the vendor and buyer can cooperate with each other to make decisions on the selling price to increase the demand so that the total system profit is increased. Thus, to consider dynamic, stochastic, or price-sensitive demand in dual-channel vendor-buyer coordination problem could be a good future research topic.

(III). *Vendor-buyer system with single vendor and multiple buyers*

In this research, we consider a vendor-buyer system including a single vendor and a single buyer. When multiple buyers replenish their inventories from a single vendor, the problem becomes complex in two aspects. First, we should evaluate the required safety stock at the vendor given the order processes from different buyers. Second, the discount scheme should be carefully designed to achieve the reasonable profit allocation between the vendor and multiple buyers.

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CHAPTER V

CONCLUSIONS

In this dissertation, we dealt with the logistics coordination issues in two distinct vendor- buyer systems, where the vendor and buyer may represent any two upstream-downstream logistics participants that are independently managed, whether they belong to different companies or simply behave as such. For each vendor-buyer system studied, we investigated how the two parties can cooperate with each other in making decisions to increase both the system and the individual profitability. We have

(I). *Single-Channel Vendor-Buyer System with Complex Transportation Schemes*

In the single-channel vendor-buyer system studied, multiple products are shipped from a vendor to a buyer through a single channel. At the buyer, each product has a deterministic and constant demand; at the vendor, each product is supplied at the same rate as its demand. In the previous literature, it has been shown that system cost savings can be obtained through coordinating the vendor and buyer in such a single-channel system.

The results and observations give insights as follows,

We found that utilizing the multi-cycle transportation policy may be an effective shipment schedule; given the products have the different ratio of h/v. our research also showed that shipping product in multiple transportation modes may result in cost savings. E.g., a company owning a private fleet could ship the products in full truckloads and outsource LTL transportation to an external freight company so that transportation cost is minimized and private vehicles are fully utilized.

(II). *Dual-Channel Vendor-Buyer system with Minimum Purchase Commitment*

In the dual-channel vendor-buyer system studied, a single product is shipped from a vendor, through two distinct channels, to satisfy the stochastic demand at a buyer. The buyer replenishes its inventory with a *minimum purchase commitment* (MPC) as follows. In each time period, the buyer regularly places an order for a predetermined and fixed quantity, which is delivered from a distant vendor-owned central facility. The buyer also has option of placing an additional order for flexible quantity, which is delivered from a nearby vendor-owned regional facility.

This research can serve as a building block and an assisting tool for several aspects in vendor- buyer coordination, supply chain network design, supply strategy development, and supply contracts negotiation. It can also help a vendor to compete for a single sourcing agreement; that is, the vendor could design a dual-channel MPC supply contract to provide both economies of scale and substantial flexibility that make it unfavorable for the buyer to consider a second vendor.

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APPENDICES

A.1 Surplus Inventory with Poisson Demand and Uniform Demand

In Section IV.4, we studied and analyzed the surplus inventory in dual-channel vendor-buyer system with the assumption that demand D is *iid* normally distributed in each time period. Thus, the stochastic demand D has a PDF of the form (see Figure 36-a)

$$f_D(d) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(d-\mu)^2/2\sigma^2} \tag{A.1}$$

In this section, we consider other two demand assumptions of Poisson distribution and uniform distribution, and discuss how to compute the surplus inventory level in each case. These demand distributions are presented in Figure 36 (b,c).

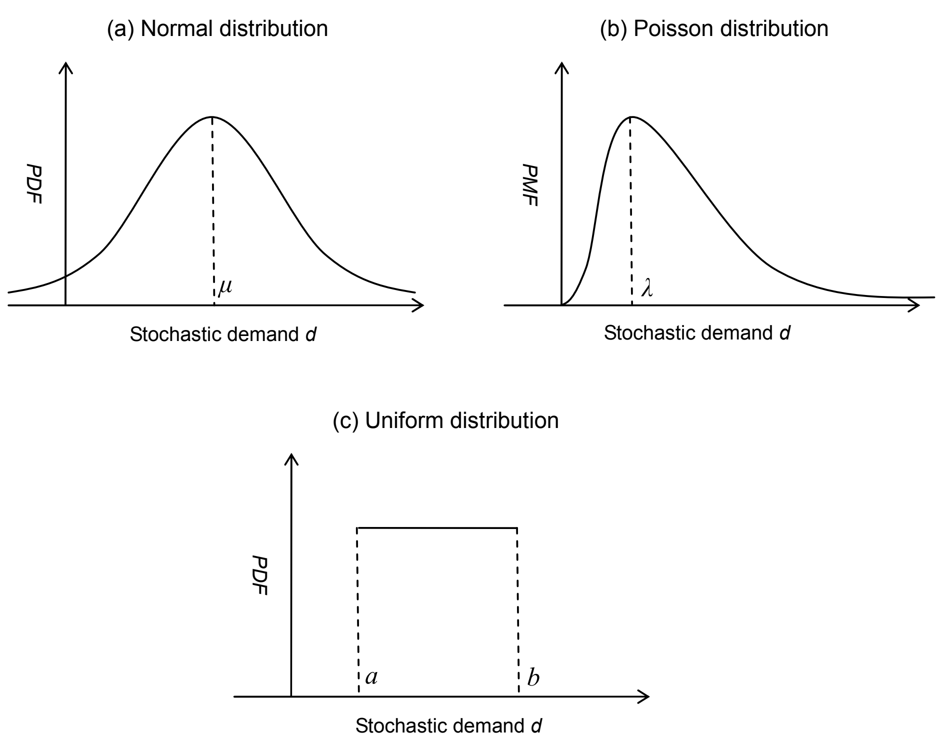

Figure 36. Normal distribution, Poisson distribution and uniform distribution

A.1.1 Poisson Distributed Demand

In the case of Poisson distributed demand, customers arrive according to a Poisson process with the given parameter λ and each customer places an order of unit product. The demand quantity d in a time interval of length t has a Poisson distribution as follows.

$$P_D(d) = \frac{(\lambda t)^d}{d!} e^{-\lambda t}, \quad d = 0,1,2,.... \qquad \text{(A.2)}$$

Where both the demand mean and the demand variance are equal to λt. When the Poisson demand mean λ is large enough, we can use the normal distribution to approximate the demand process. Based on our simulation results, the normal approximation can lead to a very small error in the computation of long term expected average surplus inventory level.

A.1.2 Uniform Distributed Demand

In the case of uniformly distributed demand, we have a close range [a, b] for the demand in each time period. The demand has PDF in the form

$$f_D(d) = \begin{cases} \dfrac{1}{(b-a)}, & \text{if } a \le d \le b \\ 0, & \text{otherwise} \end{cases} \qquad \text{(A.3)}$$

We have the following proposition.

PROPOSITION A.1. When stochastic demand D is *iid* uniformly distributed in [a, b]. The expected average surplus inventory level SI equals to the product of (b-a) and a coefficient function u of time horizon N, and the standardized MPC quantity $w = \dfrac{0.5(b+a)-Q}{0.5(b-a)}$. That is

$$SI(D,Q,N) = (b-a)\cdot u\left(N,w\right) \qquad \text{(A.4)}$$

PROOF. To prove Proposition A.1, it is sufficient to prove that any two sets of uniformly distributed demand parameters $D'\left(a',b'\right)$ and $D"\left(a",b"\right)$ can lead to the same probability density function (PDF) of $\left(SI_n/(b-a)\right)$, given the same values of w. That is

$$g_n^{'}\left[\left(\frac{SI_n^{'}}{b'-a'}\right)=x\right] = g_n^{"}\left[\left(\frac{SI_n^{"}}{b"-a"}\right)=x\right], \;\; given \;\; \frac{0.5(b'+a')-Q'}{0.5(b'-a')} = \frac{0.5(b'+a')-Q"}{0.5(b"-a")} \qquad \text{(A.5)}$$

The statement (A.5) can be proved by induction as follows.

When n = 0, (A.5) holds according to the assumption of $SI_0^{'} = SI_0^{"} = 0$.

When n = 1, we have

$$g_1^{'}\left[\left(\frac{SI_1^{'}}{b'-a'}\right)=x\right]=\begin{cases}1, & \text{when } 0<x\leq\dfrac{Q'-a'}{b'-a'}\\ 0, & \text{when } x<0 \text{ and } x>\dfrac{Q'-a'}{b'-a'}\end{cases}$$
$$G_1^{'}\left[\left(\frac{SI_1^{'}}{b'-a'}\right)=0\right]=\frac{b'-Q'}{b'-a'} \qquad \text{when } x=0 \tag{A.6}$$

$$g_1^{''}\left[\left(\frac{SI_1^{''}}{b''-a''}\right)=x\right]=\begin{cases}1, & \text{when } 0<x\leq\dfrac{Q''-a''}{b''-a''}\\ 0, & \text{when } x<0 \text{ and } x>\dfrac{Q''-a''}{b''-a''}\end{cases}$$
$$G_1^{''}\left[\left(\frac{SI_1^{''}}{b''-a''}\right)=0\right]=\frac{b''-Q''}{b''-a''} \qquad \text{when } x=0 \tag{A.7}$$

Where G(·) denotes the PMF function. Thus, statement (A.5) holds when n = 1. Now, suppose that (A.5) also holds when $n = m$ ($1 < m < N$-1); that is

$$g_m^{'}\left[\left(\frac{SI_m^{'}}{b'-a'}\right)=x\right]=g_m^{''}\left[\left(\frac{SI_m^{''}}{b''-a''}\right)=x\right], \qquad m\in[2,N-2] \tag{A.8}$$

When $n = m$+1, we have

$$\left.\begin{aligned}&g_{m+1}^{'}\left[\left(\frac{SI_{m+1}^{'}}{b'-a'}\right)=x\right]=\begin{cases}1, & \text{when } 0<x\leq\dfrac{SI_m^{'}+Q'-a'}{b'-a'}\\ 0, & \text{when } x<0 \text{ and } x>\dfrac{SI_m^{'}+Q'-a'}{b'-a'}\end{cases}\\ &G_{m+1}^{'}\left[\left(\frac{SI_{m+1}^{'}}{b'-a'}\right)=0\right]=\frac{b'-Q'-SI_m^{'}}{b'-a'}, \quad \text{when } x=0\end{aligned}\right\} \text{ when } \frac{SI_m^{'}}{b'-a'}\leq\frac{b'-Q'}{b'-a'}$$

$$g_{m+1}^{'}\left[\left(\frac{SI_{m+1}^{'}}{b'-a'}\right)=x\right]=\begin{cases}1, & \text{when } \dfrac{SI_m^{'}+Q'-b'}{b'-a'}<x\leq\dfrac{SI_m^{'}+Q'-a'}{b'-a'}\\ 0, & \text{otherwise}\end{cases}, \quad \text{when } \frac{SI_m^{'}}{b'-a'}>\frac{b'-Q'}{b'-a'} \tag{A.9}$$

$$\left.\begin{aligned}&g_{m+1}^{''}\left[\left(\frac{SI_{m+1}^{''}}{b''-a''}\right)=x\right]=\begin{cases}1, & \text{when } 0<x\leq\dfrac{SI_m^{''}+Q''-a''}{b''-a''}\\ 0, & \text{when } x<0 \text{ and } x>\dfrac{SI_m^{''}+Q''-a''}{b''-a''}\end{cases}\\ &G_{m+1}^{''}\left[\left(\frac{SI_{m+1}^{''}}{b''-a''}\right)=0\right]=\frac{b''-Q''-SI_m^{''}}{b''-a''}, \quad \text{when } x=0\end{aligned}\right\} \text{ when } \frac{SI_m^{''}}{b''-a''}\leq\frac{b''-Q''}{b''-a''}$$

$$g_{m+1}^{''}\left[\left(\frac{SI_{m+1}^{''}}{b''-a''}\right)=x\right]=\begin{cases}1, & \text{when } \dfrac{SI_m^{''}+Q''-b''}{b''-a''}<x\leq\dfrac{SI_m^{''}+Q''-a''}{b''-a''}\\ 0, & \text{otherwise}\end{cases}, \quad \text{when } \frac{SI_m^{''}}{b''-a''}>\frac{b''-Q''}{b''-a''} \tag{A.10}$$

Thus, statement (A.5) holds when $n = m+1$. By induction, statement (A.5) holds for any value n. Therefore, Proposition A.1 is true.□

We can compute the long term expected average surplus inventory SI as follows.

$$SI = (b-a)\cdot u(w), \qquad w = \frac{0.5(b+a)-Q}{0.5(b-a)} \tag{A.11}$$

The equation (A.11) implies that there is a unique coefficient function u(w) as long as the demand D is uniformly distributed in each time period. Thus, to decide any long term expected average surplus inventory level SI, it is sufficient to know the value of the coefficient function $u(w)$, whether analytically or quantitatively.

Simulation Results on Surplus Inventory Coefficient Function u(w)

We conducted simulations in Simul8™ to get the values of long term expected average surplus inventory level SI with the following simulation parameters:

- The demand process D is unidormally distributed with a= 100 and b=200.
- The MPC quantity Q varies between 100 and 150 with an incremental of 0.5; that is, the standard MPC quantity w varies between 0 and 1 with an increment of 0.01.
- The starting surplus inventory SI_0 is zero, and the time horizon N is 10,000.
- Each simulation trial includes 1,000 random runs.

Each trial result has a 99.5% confidence interval that is at most 1% of its mean. Based on the trial results, we get the values of coefficient function $u(w)$ as presented in Table 11.

Table 11. The simulation results of surplus inventory coefficient function $u(w)$

$u(w)$	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.00	10.357	7.291	4.225	3.116	2.007	1.659	1.311	1.120	0.929	0.818
.10	0.707	0.635	0.563	0.512	0.461	0.425	0.389	0.360	0.331	0.310
.20	0.289	0.267	0.245	0.231	0.216	0.204	0.192	0.181	0.170	0.161
.30	0.152	0.143	0.133	0.127	0.120	0.114	0.108	0.102	0.097	0.092
.40	0.087	0.082	0.077	0.074	0.070	0.066	0.063	0.060	0.057	0.054
.50	0.051	0.048	0.045	0.043	0.041	0.039	0.036	0.034	0.032	0.030
.60	0.028	0.027	0.025	0.024	0.022	0.021	0.019	0.018	0.017	0.016
.70	0.014	0.013	0.012	0.011	0.010	0.010	0.009	0.008	0.007	0.006
.80	0.006	0.005	0.005	0.004	0.004	0.003	0.003	0.002	0.001	0.001
.90	0.001	0.001	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000

For any value of standardized MPC quantity w, the surplus inventory coefficient function $u(w)$ can be estimated by assuming a linear function of $u(w)$ between any two neighboring points in Table 11. Thus, we have

$$u(w) \approx u(w_i) + \frac{w - w_i}{w_j - w_i}\left[k(w_j) - k(w_i)\right], \quad given \;\; w_i < w < w_j, \tag{A.12}$$

where w_i and w_j are two consecutive value w in Table 11.The simulation results of the coefficient function $u(w)$ are shown in Figure 37.

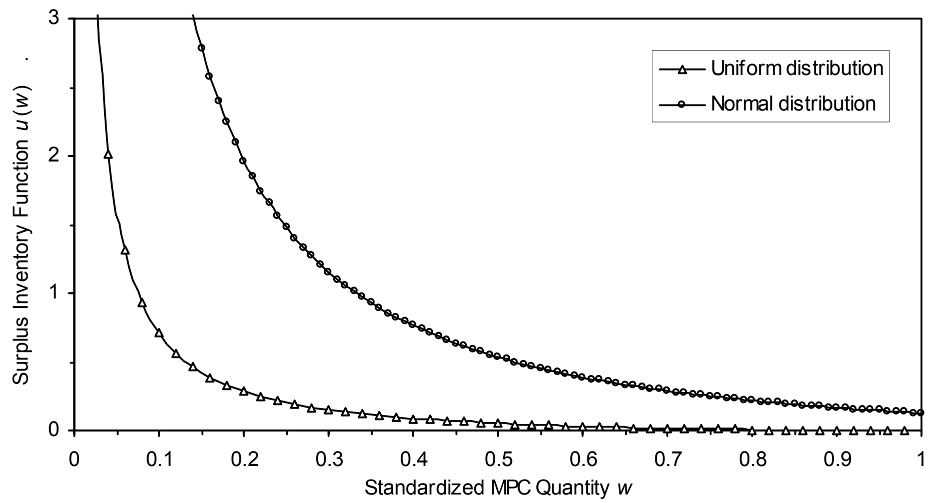

Figure 37. Surplus inventory coefficient function $u(w)$

From our simulation results, we have two observations as follows.

(I). *Exponential trend of long term expected average surplus inventory level SI*

The long term expected average surplus inventory SI is exponentially increasing when standardized MPC quantity w is decreasing. This exponential trend be interpreted by the cumulation of surplus inventories across consecutive time periods. When the standardized MPC quantity w decreases, the probability of demand D being less than MPC quantity Q increases and, in consequence, the probability of the surplus inventories being built up over consecutive time periods also increases.

(II). *The optimal standardized MPC quantity w takes value between 0 and 1.*

Since realized demand $d \in [a,b]$, the MPC quantity Q should be set to be larger than the lower demand limit a; that is $w \leq 1$. Since the demand mean = $0.5(b+a)$, the MPC quantity Q should be set to be smaller than the mean to ensure a stable system; that is $w>0$.

A.2 Simulation Results for Safety Stock Coefficient Functions $\psi_{\alpha,L}(z)$ and $\varphi_{\alpha,L}(z)$

We conducted simulations with VBA programming in Microsoft Excel™ to get the long-term safety stock coefficient functions $\psi_{\alpha,L}(z)$ and $\varphi_{\alpha,L}(z)$ with the following parameters:

- The demand process D is *iid* normally distributed with the mean of 400 and the standard deviation of 100 in each time period.
- The MPC quantity Q varies between 300 and 390 with an incremental of 1; that is, the standard MPC quantity z varies between 0.1 and 1 with an increment of 0.01.
- The starting surplus inventory SI_0 is zero, and the time horizon N is 20,000.
- For each safety stock coefficient function, we consider the service levels of [98%, 95%, 90%] and the lead time of [1, 3, 5, 7, 15, 25].
- Each simulation trial includes 2,000 random runs.

Each trial result has a 99.5% confidence interval that is at most 1% of its mean. From the simulation results, we have the following observations.

(I). Given the same lead time L_b, service level a and standardized MPC quantity z, the required safety stock level at the buyer is always less than the one at the vendor; that is

$$\varphi(\alpha, z, L_b, N) \geq \psi(\alpha, z, L_v, N), \qquad \forall L_b + 1 = L_v \tag{A.12}$$

This observation confirms our analysis in Section IV.5.3.

(II). For certain safety stock coefficient function $\psi_{\alpha,L}(z)$ or $\varphi_{\alpha,L}(z)$, it is increasing when the standardized MPC quantity z increase and converging to constant level $\chi(\alpha)$, which is the required safety factor in the case where no MPC is applied.

(III). Given the service level a and standardized MPC quantity z, the required safety stock level is increasing when the lead time L increases and converging to the constant level $\chi(\alpha)$, which is the required safety factor in the case where no MPC is applied

(I). The safety stock coefficient functions at the buyer in the cases where the service level α = 98% and the lead time L_b+1 = [1, 3, 5, 7, 15, 25].

Table 12. The simulation results of safety stock coefficient function $\psi_{98\%,1}(z)$

$\psi_{98\%,1}(z)$	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.10	1.273	1.317	1.358	1.394	1.426	1.455	1.482	1.507	1.531	1.552
.20	1.572	1.591	1.608	1.624	1.639	1.654	1.668	1.681	1.693	1.705
.30	1.716	1.726	1.737	1.746	1.756	1.765	1.773	1.782	1.789	1.797
.40	1.804	1.811	1.818	1.824	1.831	1.837	1.842	1.848	1.854	1.859
.50	1.864	1.869	1.874	1.878	1.883	1.887	1.891	1.895	1.899	1.903
.60	1.907	1.910	1.914	1.918	1.921	1.924	1.927	1.930	1.933	1.936
.70	1.939	1.942	1.944	1.947	1.950	1.952	1.954	1.957	1.959	1.961
.80	1.963	1.965	1.967	1.969	1.971	1.973	1.975	1.977	1.979	1.981
.90	1.982	1.984	1.986	1.987	1.989	1.990	1.992	1.993	1.994	1.996

Table 13. The simulation results of safety stock coefficient function $\psi_{98\%,3}(z)$

$\psi_{98\%,3}(z)$	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.10	1.384	1.424	1.459	1.492	1.520	1.546	1.571	1.593	1.613	1.632
.20	1.650	1.666	1.681	1.695	1.709	1.721	1.733	1.744	1.755	1.765
.30	1.775	1.784	1.793	1.801	1.809	1.817	1.824	1.831	1.838	1.844
.40	1.850	1.856	1.862	1.867	1.873	1.878	1.883	1.888	1.892	1.897
.50	1.901	1.905	1.909	1.913	1.917	1.920	1.924	1.927	1.931	1.934
.60	1.937	1.940	1.943	1.946	1.949	1.951	1.954	1.956	1.959	1.961
.70	1.963	1.966	1.968	1.970	1.972	1.974	1.976	1.978	1.980	1.982
.80	1.984	1.985	1.987	1.989	1.990	1.992	1.993	1.995	1.996	1.997
.90	1.999	2.000	2.001	2.002	2.004	2.005	2.006	2.007	2.008	2.009

Table 14. The simulation results of safety stock coefficient function $\psi_{98\%,5}(z)$

$\psi_{98\%,5}(z)$	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.10	1.443	1.480	1.514	1.544	1.571	1.595	1.617	1.638	1.657	1.674
.20	1.691	1.706	1.720	1.733	1.745	1.757	1.769	1.779	1.789	1.798
.30	1.807	1.816	1.824	1.832	1.839	1.846	1.853	1.859	1.865	1.871
.40	1.877	1.882	1.887	1.892	1.897	1.902	1.906	1.910	1.914	1.918
.50	1.922	1.926	1.929	1.933	1.936	1.939	1.942	1.945	1.948	1.951
.60	1.954	1.957	1.959	1.962	1.964	1.966	1.969	1.971	1.973	1.975
.70	1.977	1.979	1.981	1.983	1.985	1.986	1.988	1.990	1.991	1.993
.80	1.995	1.996	1.998	1.999	2.000	2.001	2.003	2.004	2.005	2.006
.90	2.008	2.009	2.010	2.011	2.012	2.013	2.014	2.015	2.016	2.017

Table 15. The simulation results of safety stock coefficient function $\psi_{98\%,7}(z)$

$\psi_{98\%,7}(z)$	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.10	1.477	1.513	1.545	1.574	1.599	1.622	1.644	1.663	1.681	1.698
.20	1.713	1.727	1.740	1.753	1.765	1.776	1.787	1.797	1.806	1.815
.30	1.823	1.831	1.839	1.846	1.853	1.859	1.866	1.872	1.877	1.883
.40	1.888	1.893	1.898	1.903	1.907	1.912	1.916	1.920	1.923	1.927
.50	1.931	1.934	1.937	1.941	1.944	1.947	1.950	1.953	1.955	1.958
.60	1.961	1.963	1.966	1.968	1.970	1.972	1.974	1.976	1.978	1.980
.70	1.982	1.984	1.986	1.988	1.989	1.991	1.992	1.994	1.995	1.997
.80	1.998	1.999	2.001	2.002	2.003	2.004	2.006	2.007	2.008	2.009
.90	2.010	2.011	2.012	2.013	2.014	2.015	2.016	2.017	2.018	2.018

Table 16. The simulation results of safety stock coefficient function $\psi_{98\%,15}(z)$

$\psi_{98\%,15}(z)$	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.10	1.563	1.595	1.622	1.647	1.670	1.690	1.710	1.727	1.742	1.757
.20	1.770	1.783	1.794	1.805	1.815	1.825	1.834	1.842	1.850	1.857
.30	1.865	1.871	1.878	1.884	1.890	1.895	1.900	1.905	1.910	1.915
.40	1.919	1.923	1.927	1.931	1.934	1.938	1.941	1.945	1.948	1.951
.50	1.954	1.957	1.960	1.962	1.965	1.967	1.970	1.972	1.974	1.976
.60	1.979	1.981	1.983	1.984	1.986	1.988	1.990	1.991	1.993	1.995
.70	1.996	1.997	1.999	2.000	2.002	2.003	2.004	2.005	2.006	2.008
.80	2.009	2.010	2.011	2.012	2.013	2.014	2.015	2.015	2.016	2.017
.90	2.018	2.019	2.020	2.020	2.021	2.022	2.023	2.023	2.024	2.025

Table 17. The simulation results of safety stock coefficient function $\psi_{98\%,25}(z)$

$\psi_{98\%,25}(z)$	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.10	1.632	1.662	1.687	1.710	1.731	1.749	1.766	1.781	1.795	1.808
.20	1.820	1.831	1.841	1.851	1.859	1.868	1.876	1.883	1.890	1.896
.30	1.902	1.908	1.914	1.919	1.924	1.928	1.933	1.937	1.941	1.945
.40	1.949	1.952	1.956	1.959	1.962	1.965	1.968	1.971	1.973	1.976
.50	1.979	1.981	1.983	1.986	1.988	1.990	1.992	1.994	1.995	1.997
.60	1.999	2.001	2.002	2.004	2.005	2.006	2.008	2.009	2.010	2.012
.70	2.013	2.014	2.015	2.016	2.018	2.019	2.020	2.021	2.022	2.023
.80	2.024	2.024	2.025	2.026	2.027	2.028	2.029	2.029	2.030	2.031
.90	2.031	2.032	2.033	2.033	2.034	2.035	2.035	2.036	2.036	2.037

(II). The safety stock coefficient functions at the buyer in the cases where the service level α = 95% and the lead time L_b+1 = [1, 3, 5, 7, 15, 25].

Table 18. The simulation results of safety stock coefficient function $\psi_{95\%,1}(z)$

$\psi_{95\%,1}(z)$	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.10	0.722	0.778	0.828	0.872	0.912	0.947	0.980	1.010	1.038	1.064
.20	1.087	1.110	1.130	1.149	1.168	1.185	1.201	1.217	1.231	1.245
.30	1.258	1.271	1.283	1.294	1.305	1.315	1.325	1.335	1.344	1.353
.40	1.361	1.369	1.377	1.385	1.392	1.399	1.406	1.412	1.419	1.425
.50	1.431	1.436	1.442	1.447	1.452	1.457	1.462	1.467	1.471	1.476
.60	1.480	1.484	1.488	1.492	1.496	1.499	1.503	1.506	1.510	1.513
.70	1.516	1.519	1.522	1.525	1.528	1.531	1.534	1.536	1.539	1.541
.80	1.544	1.546	1.548	1.551	1.553	1.555	1.557	1.559	1.561	1.563
.90	1.565	1.567	1.568	1.570	1.572	1.574	1.575	1.577	1.579	1.580

Table 19. The simulation results of safety stock coefficient function $\psi_{95\%,3}(z)$

$\psi_{95\%,3}(z)$	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.10	0.874	0.922	0.965	1.003	1.037	1.068	1.096	1.122	1.146	1.168
.20	1.189	1.208	1.225	1.242	1.258	1.272	1.286	1.299	1.312	1.324
.30	1.335	1.345	1.355	1.365	1.374	1.383	1.391	1.399	1.406	1.414
.40	1.421	1.428	1.434	1.440	1.446	1.452	1.458	1.463	1.468	1.473
.50	1.478	1.482	1.487	1.491	1.495	1.500	1.504	1.507	1.511	1.515
.60	1.518	1.521	1.525	1.528	1.531	1.534	1.536	1.539	1.542	1.544
.70	1.547	1.550	1.552	1.554	1.557	1.559	1.561	1.563	1.565	1.567
.80	1.569	1.571	1.573	1.574	1.576	1.578	1.579	1.581	1.583	1.584
.90	1.586	1.587	1.589	1.590	1.591	1.592	1.594	1.595	1.596	1.597

Table 20. The simulation results of safety stock coefficient function $\psi_{95\%,5}(z)$

$\psi_{95\%,5}(z)$	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.10	0.946	0.991	1.030	1.066	1.097	1.126	1.153	1.176	1.198	1.218
.20	1.237	1.255	1.271	1.286	1.300	1.314	1.327	1.339	1.350	1.361
.30	1.371	1.380	1.389	1.398	1.406	1.414	1.421	1.428	1.435	1.442
.40	1.448	1.454	1.460	1.465	1.471	1.476	1.481	1.485	1.490	1.494
.50	1.498	1.503	1.507	1.510	1.514	1.518	1.521	1.525	1.528	1.531
.60	1.534	1.537	1.540	1.543	1.545	1.548	1.550	1.553	1.555	1.557
.70	1.560	1.562	1.564	1.566	1.568	1.570	1.572	1.574	1.575	1.577
.80	1.579	1.581	1.582	1.584	1.585	1.587	1.588	1.589	1.591	1.592
.90	1.593	1.595	1.596	1.597	1.598	1.599	1.600	1.601	1.602	1.603

Table 21. The simulation results of safety stock coefficient function $\psi_{95\%,7}(z)$

$\psi_{95\%,7}(z)$	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.10	1.013	1.054	1.091	1.125	1.155	1.182	1.206	1.229	1.250	1.269
.20	1.286	1.302	1.318	1.332	1.345	1.357	1.369	1.380	1.391	1.401
.30	1.410	1.419	1.427	1.435	1.443	1.450	1.457	1.464	1.470	1.476
.40	1.482	1.488	1.493	1.498	1.503	1.508	1.512	1.517	1.521	1.525
.50	1.529	1.533	1.537	1.540	1.543	1.547	1.550	1.553	1.556	1.559
.60	1.561	1.564	1.567	1.569	1.572	1.574	1.576	1.579	1.581	1.583
.70	1.585	1.587	1.589	1.591	1.592	1.594	1.596	1.598	1.599	1.601
.80	1.602	1.604	1.605	1.606	1.608	1.609	1.611	1.612	1.613	1.614
.90	1.615	1.616	1.618	1.619	1.620	1.621	1.622	1.623	1.624	1.625

Table 22. The simulation results of safety stock coefficient function $\psi_{95\%,15}(z)$

$\psi_{95\%,15}(z)$	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.10	1.106	1.143	1.176	1.205	1.231	1.254	1.276	1.295	1.312	1.328
.20	1.343	1.357	1.370	1.382	1.394	1.404	1.414	1.423	1.432	1.440
.30	1.448	1.455	1.462	1.469	1.475	1.481	1.487	1.492	1.497	1.502
.40	1.507	1.511	1.515	1.520	1.524	1.527	1.531	1.535	1.538	1.541
.50	1.544	1.547	1.550	1.553	1.556	1.558	1.561	1.563	1.565	1.568
.60	1.570	1.572	1.574	1.576	1.578	1.580	1.582	1.583	1.585	1.587
.70	1.588	1.590	1.591	1.593	1.594	1.595	1.597	1.598	1.599	1.600
.80	1.602	1.603	1.604	1.605	1.606	1.607	1.608	1.609	1.610	1.611
.90	1.612	1.613	1.613	1.614	1.615	1.616	1.617	1.617	1.618	1.619

Table 23. The simulation results of safety stock coefficient function $\psi_{95\%,25}(z)$

$\psi_{95\%,25}(z)$	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.10	1.173	1.207	1.237	1.263	1.286	1.307	1.326	1.343	1.359	1.373
.20	1.386	1.399	1.410	1.421	1.430	1.439	1.448	1.456	1.464	1.471
.30	1.478	1.484	1.490	1.496	1.501	1.506	1.511	1.516	1.521	1.525
.40	1.529	1.533	1.536	1.540	1.543	1.547	1.550	1.553	1.555	1.558
.50	1.561	1.563	1.566	1.568	1.570	1.572	1.575	1.577	1.578	1.580
.60	1.582	1.584	1.585	1.587	1.589	1.590	1.592	1.593	1.594	1.596
.70	1.597	1.598	1.600	1.601	1.602	1.603	1.604	1.605	1.606	1.607
.80	1.608	1.609	1.610	1.611	1.612	1.613	1.614	1.614	1.615	1.616
.90	1.617	1.617	1.618	1.619	1.619	1.620	1.621	1.621	1.622	1.622

(III). The safety stock coefficient functions at the buyer in the cases where the service level α = 90 and the lead time L_b+1 = [1, 3, 5, 7, 15, 25].

Table 24. The simulation results of safety stock coefficient function $\psi_{90\%,1}(z)$

$\psi_{90\%,1}(z)$	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.10	0.184	0.256	0.319	0.374	0.424	0.468	0.509	0.546	0.580	0.611
.20	0.639	0.666	0.691	0.714	0.736	0.757	0.776	0.794	0.811	0.827
.30	0.843	0.858	0.872	0.885	0.897	0.910	0.921	0.932	0.943	0.953
.40	0.962	0.972	0.981	0.989	0.998	1.006	1.013	1.021	1.028	1.035
.50	1.042	1.048	1.054	1.060	1.066	1.072	1.077	1.083	1.088	1.093
.60	1.098	1.102	1.107	1.111	1.116	1.120	1.124	1.128	1.132	1.135
.70	1.139	1.142	1.146	1.149	1.152	1.155	1.159	1.162	1.164	1.167
.80	1.170	1.173	1.175	1.178	1.180	1.183	1.185	1.187	1.190	1.192
.90	1.194	1.196	1.198	1.200	1.202	1.204	1.206	1.207	1.209	1.211

Table 25. The simulation results of safety stock coefficient function $\psi_{90\%,3}(z)$

$\psi_{90\%,3}(z)$	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.10	0.390	0.449	0.501	0.547	0.588	0.625	0.659	0.689	0.718	0.743
.20	0.768	0.790	0.810	0.830	0.848	0.865	0.881	0.896	0.910	0.923
.30	0.936	0.948	0.959	0.970	0.981	0.991	1.000	1.009	1.018	1.026
.40	1.034	1.041	1.049	1.056	1.062	1.069	1.075	1.081	1.087	1.092
.50	1.098	1.103	1.108	1.112	1.117	1.122	1.126	1.130	1.134	1.138
.60	1.142	1.146	1.149	1.153	1.156	1.159	1.163	1.166	1.169	1.171
.70	1.174	1.177	1.180	1.182	1.185	1.187	1.190	1.192	1.194	1.196
.80	1.198	1.200	1.202	1.204	1.206	1.208	1.210	1.212	1.213	1.215
.90	1.217	1.218	1.220	1.221	1.223	1.224	1.225	1.227	1.228	1.229

Table 26. The simulation results of safety stock coefficient function $\psi_{90\%,5}(z)$

$\psi_{90\%,5}(z)$	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.10	0.489	0.542	0.589	0.631	0.669	0.702	0.732	0.760	0.785	0.808
.20	0.830	0.850	0.868	0.886	0.902	0.917	0.932	0.945	0.958	0.970
.30	0.981	0.992	1.002	1.012	1.021	1.030	1.038	1.046	1.054	1.061
.40	1.068	1.075	1.081	1.088	1.093	1.099	1.105	1.110	1.115	1.120
.50	1.125	1.129	1.133	1.138	1.142	1.146	1.149	1.153	1.157	1.160
.60	1.163	1.167	1.170	1.173	1.176	1.179	1.181	1.184	1.187	1.189
.70	1.191	1.194	1.196	1.198	1.201	1.203	1.205	1.207	1.209	1.210
.80	1.212	1.214	1.216	1.217	1.219	1.221	1.222	1.224	1.225	1.227
.90	1.228	1.229	1.231	1.232	1.233	1.234	1.236	1.237	1.238	1.239

Table 27. The simulation results of safety stock coefficient function $\psi_{90\%,7}(z)$

$\psi_{90\%,7}(z)$	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.10	0.551	0.601	0.645	0.684	0.719	0.750	0.779	0.804	0.828	0.850
.20	0.870	0.888	0.906	0.922	0.937	0.951	0.964	0.977	0.988	0.999
.30	1.010	1.019	1.029	1.038	1.046	1.054	1.062	1.069	1.076	1.083
.40	1.089	1.095	1.101	1.107	1.112	1.117	1.122	1.127	1.132	1.136
.50	1.141	1.145	1.149	1.152	1.156	1.160	1.163	1.166	1.170	1.173
.60	1.176	1.179	1.181	1.184	1.187	1.189	1.192	1.194	1.196	1.199
.70	1.201	1.203	1.205	1.207	1.209	1.211	1.213	1.214	1.216	1.218
.80	1.219	1.221	1.223	1.224	1.226	1.227	1.228	1.230	1.231	1.232
.90	1.234	1.235	1.236	1.237	1.238	1.239	1.240	1.241	1.242	1.243

Table 28. The simulation results of safety stock coefficient function $\psi_{90\%,15}(z)$

$\psi_{90\%,15}(z)$	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.10	0.683	0.726	0.763	0.797	0.826	0.853	0.877	0.899	0.919	0.937
.20	0.954	0.969	0.984	0.997	1.009	1.021	1.032	1.042	1.051	1.060
.30	1.069	1.077	1.085	1.092	1.099	1.105	1.111	1.117	1.123	1.128
.40	1.133	1.138	1.143	1.147	1.152	1.156	1.160	1.164	1.167	1.171
.50	1.174	1.177	1.180	1.183	1.186	1.189	1.192	1.194	1.197	1.199
.60	1.201	1.204	1.206	1.208	1.210	1.212	1.214	1.216	1.218	1.219
.70	1.221	1.223	1.224	1.226	1.227	1.229	1.230	1.231	1.233	1.234
.80	1.235	1.236	1.238	1.239	1.240	1.241	1.242	1.243	1.244	1.245
.90	1.246	1.247	1.248	1.248	1.249	1.250	1.251	1.252	1.252	1.253

Table 29. The simulation results of safety stock coefficient function $\psi_{90\%,25}(z)$

$\psi_{90\%,25}(z)$	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.10	0.765	0.804	0.837	0.867	0.893	0.917	0.938	0.957	0.974	0.990
.20	1.005	1.018	1.031	1.042	1.053	1.063	1.073	1.082	1.090	1.097
.30	1.104	1.111	1.118	1.124	1.130	1.135	1.140	1.145	1.150	1.154
.40	1.159	1.163	1.167	1.171	1.174	1.178	1.181	1.184	1.187	1.190
.50	1.193	1.195	1.198	1.200	1.203	1.205	1.207	1.209	1.212	1.214
.60	1.215	1.217	1.219	1.221	1.222	1.224	1.226	1.227	1.229	1.230
.70	1.231	1.233	1.234	1.235	1.237	1.238	1.239	1.240	1.241	1.242
.80	1.243	1.244	1.245	1.246	1.247	1.248	1.249	1.249	1.250	1.251
.90	1.252	1.252	1.253	1.254	1.255	1.255	1.256	1.257	1.257	1.258

(IV). The safety stock coefficient functions at the vendor (central DC and regional DC) in the cases where the service level $\alpha = 98\%$ and the lead time $L_v = [1, 3, 5, 7, 15, 25]$.

Table 30. The simulation results of safety stock coefficient function $\varphi_{98\%,1}(z)$

$\varphi_{98\%,1}(z)$	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.10	1.274	1.319	1.359	1.394	1.427	1.456	1.483	1.508	1.531	1.552
.20	1.572	1.590	1.608	1.624	1.639	1.654	1.667	1.680	1.693	1.704
.30	1.715	1.726	1.736	1.746	1.755	1.764	1.773	1.781	1.788	1.796
.40	1.803	1.810	1.817	1.823	1.830	1.836	1.842	1.847	1.853	1.858
.50	1.863	1.868	1.873	1.878	1.882	1.886	1.891	1.895	1.898	1.902
.60	1.906	1.910	1.913	1.917	1.920	1.923	1.926	1.929	1.932	1.935
.70	1.938	1.941	1.943	1.946	1.949	1.951	1.953	1.956	1.958	1.960
.80	1.962	1.965	1.967	1.969	1.971	1.972	1.974	1.976	1.978	1.979
.90	1.981	1.983	1.985	1.986	1.988	1.989	1.991	1.992	1.993	1.995

Table 31. The simulation results of safety stock coefficient function $\varphi_{98\%,3}(z)$

$\varphi_{98\%,3}(z)$	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.10	1.440	1.476	1.508	1.537	1.563	1.587	1.609	1.629	1.648	1.665
.20	1.681	1.695	1.709	1.722	1.734	1.746	1.756	1.767	1.777	1.786
.30	1.795	1.803	1.811	1.819	1.826	1.833	1.840	1.846	1.852	1.858
.40	1.864	1.870	1.875	1.880	1.885	1.889	1.893	1.898	1.902	1.906
.50	1.910	1.914	1.918	1.921	1.924	1.928	1.931	1.934	1.937	1.940
.60	1.943	1.946	1.948	1.951	1.953	1.956	1.958	1.961	1.963	1.965
.70	1.968	1.970	1.972	1.973	1.975	1.977	1.979	1.981	1.982	1.984
.80	1.986	1.987	1.989	1.991	1.992	1.993	1.995	1.996	1.998	1.999
.90	2.000	2.001	2.002	2.004	2.005	2.006	2.007	2.008	2.009	2.010

Table 32. The simulation results of safety stock coefficient function $\varphi_{98\%,5}(z)$

$\varphi_{98\%,5}(z)$	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.10	1.521	1.555	1.585	1.610	1.634	1.656	1.676	1.694	1.711	1.725
.20	1.740	1.753	1.765	1.777	1.788	1.798	1.807	1.816	1.825	1.833
.30	1.840	1.848	1.854	1.861	1.867	1.873	1.879	1.885	1.890	1.895
.40	1.900	1.904	1.909	1.913	1.917	1.921	1.924	1.928	1.932	1.935
.50	1.938	1.941	1.944	1.947	1.950	1.953	1.955	1.958	1.960	1.962
.60	1.965	1.967	1.969	1.971	1.973	1.975	1.977	1.979	1.981	1.983
.70	1.984	1.986	1.988	1.989	1.990	1.992	1.993	1.995	1.996	1.998
.80	1.999	2.000	2.001	2.003	2.004	2.005	2.006	2.007	2.008	2.009
.90	2.010	2.011	2.012	2.013	2.014	2.015	2.016	2.017	2.017	2.018

Table 33. The simulation results of safety stock coefficient function $\varphi_{98\%,7}(z)$

$\varphi_{98\%,7}(z)$	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.10	1.573	1.604	1.632	1.657	1.679	1.700	1.718	1.734	1.750	1.763
.20	1.777	1.789	1.800	1.811	1.821	1.830	1.839	1.847	1.854	1.862
.30	1.869	1.875	1.882	1.887	1.893	1.898	1.904	1.908	1.913	1.917
.40	1.921	1.925	1.929	1.933	1.936	1.940	1.943	1.946	1.949	1.952
.50	1.955	1.957	1.960	1.963	1.965	1.967	1.970	1.972	1.974	1.976
.60	1.978	1.980	1.982	1.984	1.985	1.987	1.989	1.990	1.992	1.993
.70	1.994	1.996	1.997	1.998	2.000	2.001	2.002	2.003	2.004	2.006
.80	2.007	2.008	2.009	2.010	2.011	2.012	2.013	2.014	2.015	2.015
.90	2.016	2.017	2.018	2.019	2.019	2.020	2.021	2.022	2.022	2.023

Table 34. The simulation results of safety stock coefficient function $\varphi_{98\%,15}(z)$

$\varphi_{98\%,15}(z)$	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.10	1.690	1.716	1.740	1.760	1.780	1.797	1.813	1.826	1.839	1.850
.20	1.860	1.870	1.879	1.887	1.895	1.902	1.909	1.915	1.921	1.927
.30	1.932	1.937	1.941	1.946	1.950	1.954	1.958	1.961	1.965	1.968
.40	1.971	1.974	1.977	1.979	1.982	1.984	1.987	1.989	1.991	1.993
.50	1.995	1.997	1.998	2.000	2.002	2.003	2.005	2.006	2.008	2.009
.60	2.010	2.012	2.013	2.014	2.015	2.016	2.017	2.018	2.019	2.020
.70	2.021	2.022	2.023	2.024	2.025	2.026	2.026	2.027	2.028	2.028
.80	2.029	2.030	2.030	2.031	2.032	2.032	2.033	2.033	2.034	2.035
.90	2.035	2.036	2.036	2.036	2.037	2.037	2.038	2.038	2.039	2.039

Table 35. The simulation results of safety stock coefficient function $\varphi_{98\%,25}(z)$

$\varphi_{98\%,25}(z)$	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.10	1.755	1.778	1.800	1.819	1.835	1.850	1.863	1.875	1.885	1.895
.20	1.904	1.912	1.919	1.926	1.933	1.938	1.944	1.949	1.954	1.959
.30	1.963	1.967	1.971	1.974	1.978	1.981	1.984	1.986	1.989	1.991
.40	1.994	1.996	1.998	2.000	2.002	2.004	2.006	2.007	2.009	2.010
.50	2.012	2.013	2.015	2.016	2.017	2.018	2.020	2.021	2.022	2.023
.60	2.024	2.025	2.025	2.026	2.027	2.028	2.029	2.029	2.030	2.031
.70	2.031	2.032	2.032	2.033	2.034	2.034	2.035	2.035	2.036	2.036
.80	2.037	2.037	2.037	2.038	2.038	2.039	2.039	2.039	2.040	2.040
.90	2.040	2.041	2.041	2.041	2.042	2.042	2.042	2.043	2.043	2.043

(V). The safety stock coefficient functions at the vendor (central DC and regional DC) in the cases where the service level α = 95% and the lead time L_v = [1, 3, 5, 7, 15, 25].

Table 36. The simulation results of safety stock coefficient function $\varphi_{95\%,1}(z)$

$\varphi_{95\%,1}(z)$	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.10	0.722	0.778	0.828	0.872	0.912	0.948	0.981	1.011	1.038	1.064
.20	1.088	1.110	1.131	1.150	1.168	1.186	1.202	1.217	1.232	1.245
.30	1.259	1.271	1.283	1.294	1.305	1.315	1.326	1.335	1.344	1.353
.40	1.361	1.369	1.377	1.385	1.392	1.399	1.406	1.412	1.418	1.424
.50	1.430	1.436	1.441	1.446	1.452	1.456	1.461	1.466	1.470	1.475
.60	1.479	1.483	1.487	1.491	1.495	1.499	1.502	1.505	1.509	1.512
.70	1.515	1.518	1.521	1.524	1.527	1.530	1.533	1.535	1.538	1.541
.80	1.543	1.545	1.548	1.550	1.552	1.554	1.557	1.559	1.561	1.563
.90	1.565	1.566	1.568	1.570	1.572	1.573	1.575	1.576	1.578	1.580

Table 37. The simulation results of safety stock coefficient function $\varphi_{95\%,3}(z)$

$\varphi_{95\%,3}(z)$	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.10	0.974	1.015	1.052	1.085	1.114	1.141	1.166	1.188	1.208	1.228
.20	1.245	1.262	1.278	1.292	1.306	1.319	1.331	1.342	1.353	1.363
.30	1.372	1.382	1.390	1.399	1.407	1.414	1.422	1.428	1.435	1.441
.40	1.447	1.453	1.459	1.464	1.469	1.474	1.479	1.484	1.488	1.492
.50	1.497	1.501	1.505	1.508	1.512	1.515	1.519	1.522	1.525	1.528
.60	1.531	1.534	1.537	1.540	1.542	1.545	1.547	1.550	1.552	1.554
.70	1.556	1.559	1.561	1.563	1.565	1.567	1.569	1.570	1.572	1.574
.80	1.576	1.577	1.579	1.581	1.582	1.584	1.585	1.587	1.588	1.589
.90										

Table 38. The simulation results of safety stock coefficient function $\varphi_{95\%,5}(z)$

$\varphi_{95\%,5}(z)$	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.10	1.071	1.109	1.143	1.173	1.199	1.223	1.245	1.265	1.284	1.301
.20	1.316	1.331	1.344	1.357	1.368	1.379	1.390	1.400	1.409	1.418
.30	1.426	1.434	1.441	1.448	1.455	1.461	1.468	1.473	1.479	1.484
.40	1.489	1.494	1.499	1.503	1.507	1.511	1.515	1.519	1.523	1.526
.50	1.529	1.533	1.536	1.539	1.542	1.544	1.547	1.550	1.552	1.555
.60	1.557	1.560	1.562	1.564	1.566	1.568	1.570	1.572	1.574	1.575
.70	1.577	1.579	1.581	1.582	1.584	1.585	1.587	1.588	1.590	1.591
.80	1.592	1.594	1.595	1.596	1.597	1.598	1.600	1.601	1.602	1.603
.90	1.604	1.605	1.606	1.606	1.607	1.608	1.609	1.610	1.611	1.611

Table 39. The simulation results of safety stock coefficient function $\varphi_{95\%,7}(z)$

$\varphi_{95\%,7}(z)$	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.10	1.137	1.172	1.204	1.231	1.255	1.277	1.298	1.316	1.333	1.348
.20	1.362	1.376	1.388	1.399	1.410	1.420	1.429	1.438	1.446	1.454
.30	1.461	1.468	1.475	1.481	1.487	1.492	1.498	1.503	1.508	1.512
.40	1.517	1.521	1.525	1.529	1.532	1.536	1.539	1.542	1.545	1.548
.50	1.551	1.554	1.557	1.559	1.562	1.564	1.566	1.568	1.570	1.573
.60	1.574	1.576	1.578	1.580	1.582	1.584	1.585	1.587	1.588	1.590
.70	1.591	1.593	1.594	1.595	1.597	1.598	1.599	1.600	1.601	1.602
.80	1.603	1.604	1.605	1.606	1.607	1.608	1.609	1.610	1.611	1.612
.90	1.612	1.613	1.614	1.615	1.615	1.616	1.617	1.617	1.618	1.619

Table 40. The simulation results of safety stock coefficient function $\varphi_{95\%,15}(z)$

$\varphi_{95\%,15}(z)$	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.10	1.267	1.297	1.323	1.345	1.366	1.384	1.401	1.415	1.429	1.441
.20	1.453	1.463	1.472	1.481	1.489	1.497	1.504	1.510	1.516	1.522
.30	1.527	1.532	1.537	1.542	1.546	1.550	1.553	1.557	1.560	1.563
.40	1.566	1.569	1.572	1.575	1.577	1.579	1.582	1.584	1.586	1.588
.50	1.590	1.591	1.593	1.595	1.596	1.598	1.599	1.601	1.602	1.604
.60	1.605	1.606	1.607	1.608	1.609	1.611	1.612	1.613	1.613	1.614
.70	1.615	1.616	1.617	1.618	1.618	1.619	1.620	1.621	1.621	1.622
.80	1.623	1.623	1.624	1.625	1.625	1.626	1.626	1.627	1.627	1.628
.90	1.628	1.629	1.629	1.629	1.630	1.630	1.631	1.631	1.632	1.632

Table 41. The simulation results of safety stock coefficient function $\varphi_{95\%,25}(z)$

$\varphi_{95\%,25}(z)$	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.10	1.339	1.365	1.388	1.408	1.426	1.441	1.454	1.467	1.478	1.488
.20	1.497	1.506	1.513	1.520	1.527	1.533	1.538	1.544	1.549	1.553
.30	1.557	1.561	1.565	1.568	1.571	1.575	1.577	1.580	1.583	1.585
.40	1.587	1.590	1.592	1.594	1.596	1.597	1.599	1.600	1.602	1.603
.50	1.605	1.606	1.607	1.609	1.610	1.611	1.612	1.613	1.614	1.615
.60	1.615	1.616	1.617	1.618	1.619	1.620	1.620	1.621	1.622	1.622
.70	1.623	1.624	1.624	1.625	1.625	1.626	1.626	1.627	1.627	1.627
.80	1.628	1.628	1.629	1.629	1.630	1.630	1.630	1.631	1.631	1.631
.90	1.632	1.632	1.632	1.633	1.633	1.633	1.633	1.634	1.634	1.634

(VI). The safety stock coefficient functions at the vendor (central DC and regional DC) in the cases where the service level $\alpha = 90\%$ and the lead time $L_v = [1, 3, 5, 7, 15, 25]$.

Table 42. The simulation results of safety stock coefficient function $\varphi_{90\%,1}(z)$

$\varphi_{90\%,1}(z)$	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.10	0.181	0.253	0.317	0.373	0.422	0.467	0.507	0.544	0.578	0.609
.20	0.638	0.665	0.689	0.713	0.735	0.755	0.774	0.793	0.810	0.826
.30	0.842	0.856	0.870	0.884	0.896	0.908	0.920	0.931	0.941	0.952
.40	0.961	0.971	0.980	0.989	0.997	1.005	1.013	1.020	1.027	1.034
.50	1.041	1.047	1.054	1.060	1.066	1.071	1.077	1.082	1.087	1.092
.60	1.097	1.102	1.106	1.111	1.115	1.119	1.123	1.127	1.131	1.135
.70	1.138	1.142	1.145	1.149	1.152	1.155	1.158	1.161	1.164	1.167
.80	1.170	1.172	1.175	1.177	1.180	1.182	1.185	1.187	1.189	1.191
.90	1.193	1.196	1.198	1.200	1.201	1.203	1.205	1.207	1.209	1.211

Table 43. The simulation results of safety stock coefficient function $\varphi_{90\%,3}(z)$

$\varphi_{90\%,3}(z)$	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.10	0.544	0.593	0.635	0.673	0.707	0.738	0.766	0.791	0.814	0.835
.20	0.855	0.873	0.890	0.907	0.922	0.936	0.949	0.961	0.973	0.984
.30	0.994	1.004	1.014	1.023	1.031	1.039	1.047	1.054	1.062	1.068
.40	1.075	1.081	1.087	1.093	1.098	1.103	1.108	1.113	1.118	1.122
.50	1.127	1.131	1.135	1.139	1.143	1.146	1.150	1.153	1.157	1.160
.60	1.163	1.166	1.169	1.172	1.175	1.177	1.180	1.183	1.185	1.187
.70	1.190	1.192	1.194	1.196	1.198	1.200	1.202	1.204	1.206	1.207
.80	1.209	1.211	1.213	1.214	1.216	1.217	1.219	1.220	1.222	1.223
.90	1.224	1.226	1.227	1.228	1.229	1.231	1.232	1.233	1.234	1.235

Table 44. The simulation results of safety stock coefficient function $\varphi_{90\%,5}(z)$

$\varphi_{90\%,5}(z)$	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.10	0.667	0.710	0.747	0.781	0.811	0.838	0.862	0.885	0.905	0.924
.20	0.941	0.957	0.972	0.985	0.998	1.010	1.021	1.032	1.042	1.051
.30	1.060	1.068	1.076	1.083	1.091	1.097	1.104	1.110	1.115	1.121
.40	1.126	1.131	1.136	1.141	1.145	1.149	1.153	1.157	1.161	1.164
.50	1.168	1.171	1.174	1.177	1.180	1.183	1.186	1.189	1.191	1.194
.60	1.196	1.198	1.201	1.203	1.205	1.207	1.209	1.211	1.213	1.214
.70	1.216	1.218	1.220	1.221	1.223	1.224	1.226	1.227	1.228	1.230
.80	1.231	1.232	1.233	1.235	1.236	1.237	1.238	1.239	1.240	1.241
.90	1.242	1.243	1.244	1.245	1.246	1.247	1.248	1.248	1.249	1.250

Table 45. The simulation results of safety stock coefficient function $\varphi_{90\%,7}(z)$

$\varphi_{90\%,7}(z)$	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.10	0.736	0.776	0.811	0.842	0.869	0.894	0.916	0.937	0.955	0.972
.20	0.988	1.002	1.016	1.028	1.040	1.050	1.060	1.070	1.078	1.086
.30	1.094	1.101	1.108	1.115	1.121	1.127	1.132	1.138	1.143	1.147
.40	1.152	1.156	1.160	1.164	1.168	1.172	1.175	1.178	1.181	1.185
.50	1.187	1.190	1.193	1.196	1.198	1.200	1.203	1.205	1.207	1.209
.60	1.211	1.213	1.215	1.216	1.218	1.220	1.221	1.223	1.225	1.226
.70	1.227	1.229	1.230	1.231	1.233	1.234	1.235	1.236	1.237	1.239
.80	1.240	1.241	1.242	1.243	1.243	1.244	1.245	1.246	1.247	1.248
.90	1.249	1.249	1.250	1.251	1.251	1.252	1.253	1.253	1.254	1.255

Table 46. The simulation results of safety stock coefficient function $\varphi_{90\%,15}(z)$

$\varphi_{90\%,15}(z)$	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.10	0.888	0.920	0.949	0.974	0.996	1.015	1.033	1.049	1.063	1.076
.20	1.087	1.098	1.108	1.117	1.125	1.133	1.141	1.147	1.154	1.160
.30	1.165	1.170	1.175	1.180	1.184	1.188	1.192	1.195	1.199	1.202
.40	1.205	1.208	1.211	1.213	1.216	1.218	1.221	1.223	1.225	1.227
.50	1.228	1.230	1.232	1.233	1.235	1.236	1.238	1.239	1.240	1.242
.60	1.243	1.244	1.245	1.246	1.247	1.248	1.249	1.250	1.251	1.252
.70	1.253	1.254	1.254	1.255	1.256	1.257	1.257	1.258	1.259	1.259
.80	1.260	1.261	1.261	1.262	1.262	1.263	1.263	1.264	1.264	1.265
.90	1.265	1.266	1.266	1.266	1.267	1.267	1.268	1.268	1.268	1.269

Table 47. The simulation results of safety stock coefficient function $\varphi_{90\%,25}(z)$

$\varphi_{90\%,25}(z)$	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.10	0.969	0.998	1.022	1.043	1.062	1.079	1.093	1.106	1.118	1.128
.20	1.138	1.146	1.154	1.162	1.168	1.174	1.180	1.185	1.190	1.195
.30	1.199	1.203	1.207	1.210	1.214	1.217	1.219	1.222	1.224	1.227
.40	1.229	1.231	1.233	1.235	1.237	1.238	1.240	1.241	1.243	1.244
.50	1.245	1.247	1.248	1.249	1.250	1.251	1.252	1.253	1.254	1.255
.60	1.256	1.256	1.257	1.258	1.259	1.259	1.260	1.261	1.261	1.262
.70	1.262	1.263	1.263	1.264	1.264	1.265	1.265	1.266	1.266	1.267
.80	1.267	1.267	1.268	1.268	1.268	1.269	1.269	1.269	1.270	1.270
.90	1.270	1.271	1.271	1.271	1.271	1.272	1.272	1.272	1.272	1.273

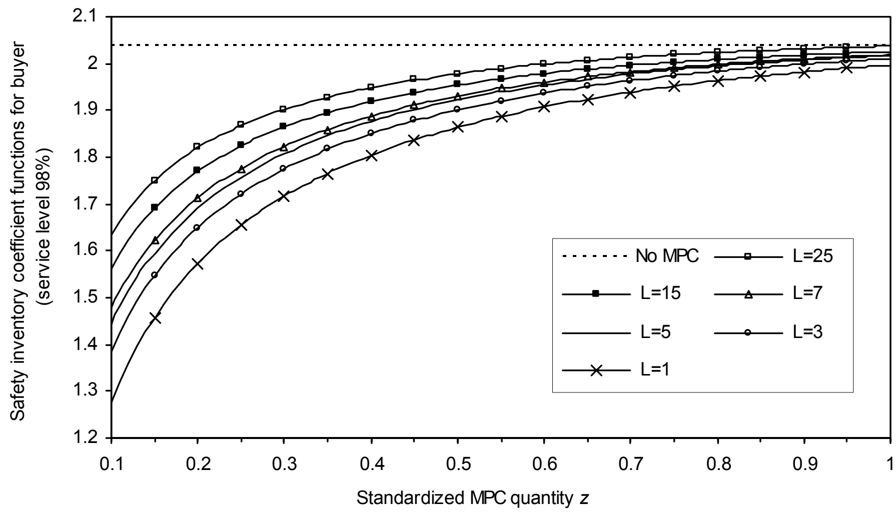

Figure 38. Safety stock coefficient functions of the buyer (α=98%)

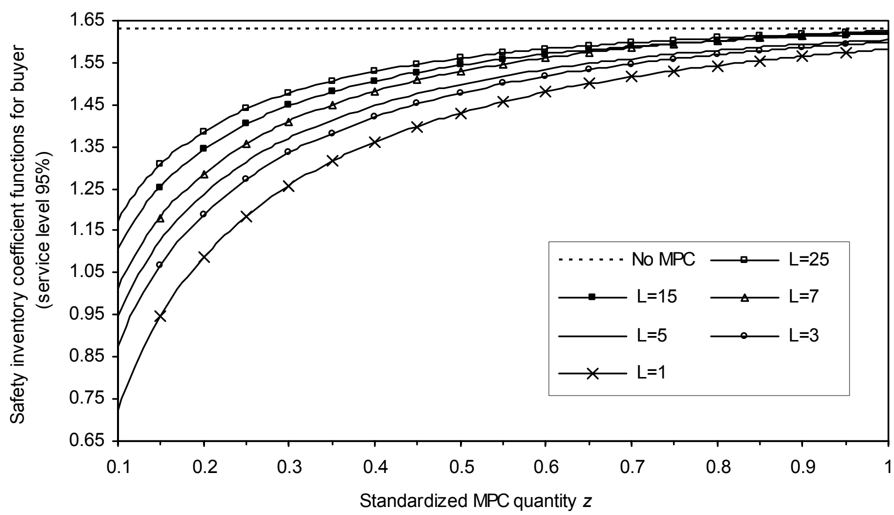

Figure 39. Safety stock coefficient functions of the buyer (α=95%)

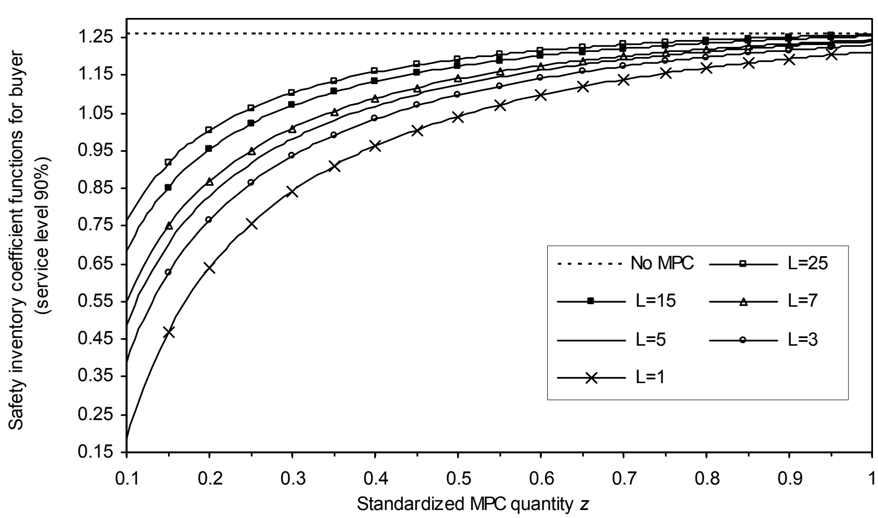

Figure 40. Safety stock coefficient functions of the buyer (α=90%)

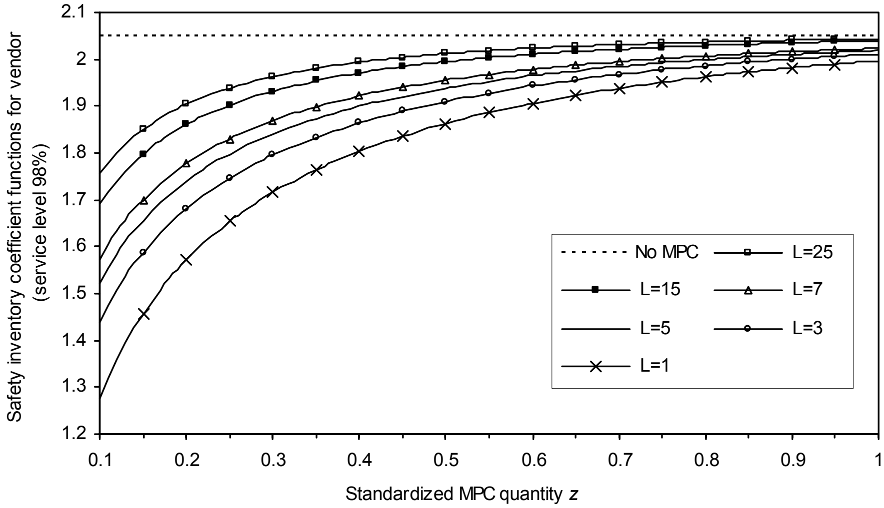

Figure 41. Safety stock coefficient functions of the vendor (α=98%)

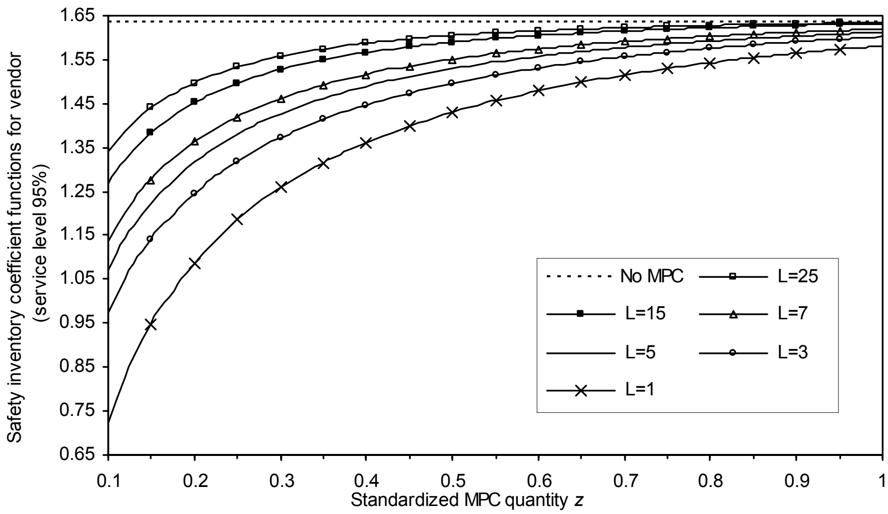

Figure 42. Safety stock coefficient functions of the vendor (α=95%)

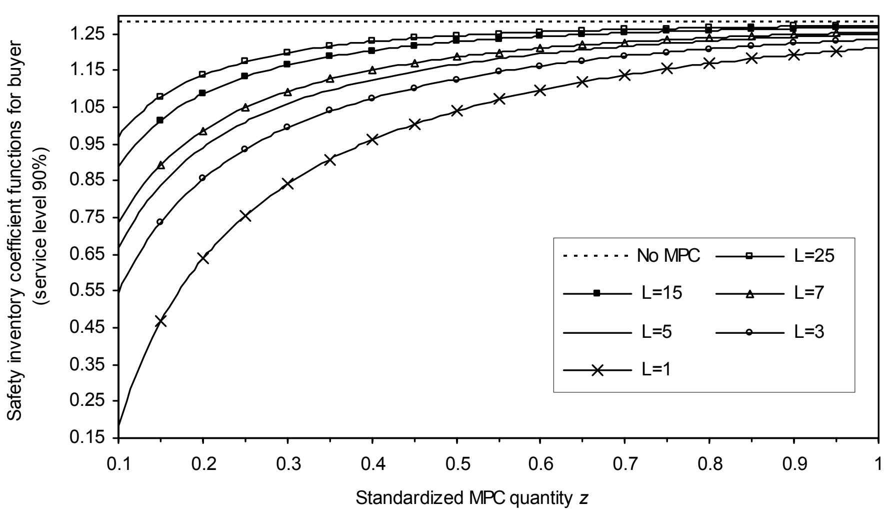

Figure 43. Safety stock coefficient functions of the vendor (α=90%)

A.3 Simulation-Based Quadratic Approximation Method

In Chapter IV, we showed that when the stochastic demand D is *iid* normally distributed in each time period, the surplus inventory at the buyer and the safety stocks at the buyer and the vendor can be estimated with three coefficient functions $k(z)$, $\psi_{\alpha,L}(z)$ and $\varphi_{\alpha,L}(z)$ as follows

$$SI = \sigma \cdot k(z) \tag{A.13}$$

$$SS_b = \sigma \cdot \psi_{a,L}(z) \tag{A.14}$$

$$SS_v = \sigma \cdot \sqrt{L} \cdot \varphi_{a,L}(z) \tag{A.15}$$

Using the simulation results of these coefficient functions, we introduced a method to quantitatively analyze the dual-channel vendor-buyer coordination model. In some circumstances, however, an analytical method might be still desirable or necessary.

In this section, we introduce a method to approximate the three coefficient functions $k(z)$, $\psi_{\alpha,L}(z)$ and $\varphi_{\alpha,L}(z)$ as piece-wise quadratic functions, which are in the structure follows.

$$k(z) \approx A_k^r z^2 + B_k^r z + C_k^r, \qquad \text{when} \quad z \in (Z_r, Z_{r+1}] \tag{A.16}$$

$$\psi_{\alpha,L}(z) \approx A_{\psi,\alpha,L}^r z^2 + B_{\psi,\alpha,L}^r z + C_{\psi,\alpha,L}^r, \qquad \text{when} \quad z \in (Z_r, Z_{r+1}] \tag{A.17}$$

$$\varphi_{\alpha,L}(z) \approx A_{\varphi,\alpha,L}^r z^2 + B_{\varphi,\alpha,L}^r z + C_{\varphi,\alpha,L}^r, \qquad \text{when} \quad z \in (Z_r, Z_{r+1}] \tag{A.18}$$

In (A.13)-(A.15), all the values of A, B, C are constant in the three ranges $(Z_r, Z_{r+1}]$: (0.1, 0.2], (0.2, 0.3] and (0.3, 1]. Given the three quadratic approximation functions, we can develop analytical integrated coordination model and channel coordination model for the dual-channel vendor-buyer system studied in Chapter IV.

Dual-Channel Integrated Coordination Model

In an *integrated coordination model*, the buyer and vendor can fully cooperate with each other to make decision on the standardized MPC quantity z that minimizes the total system cost C_{sys}. We can estimate the total system cost $C_{sys}(z)$ in a piece-wise quadratic function of the standardized MPC quantity z as follows.

$$\begin{aligned} C_{sys}(z) \approx\ & \sigma\left(A_k^r h_b + \sqrt{L_b+1} A_{\psi,\alpha,L_b+1}^r h_b + \sqrt{L_{RDC}}\, A_{\varphi,\alpha,L_{RDC}}^r h_{RDC} + \sqrt{L_{CDC}}\, A_{\varphi,\alpha,L_{CDC}}^r h_{CDC} \right) z^2 \\ & + \sigma\left[(c_2 - c_1) + B_k^r h_b + B_{\psi,\alpha,L_b+1}^r h_b + B_{\varphi,\alpha,L_{RDC}}^r h_{RDC} + B_{\varphi,\alpha,L_{CDC}}^r h_{CDC} \right] z \\ & + \left(c_1 \mu + 0.5 \mu h_b + C_k^r h_b + C_{\psi,\alpha,L_b+1}^r h_b + C_{\varphi,\alpha,L_{RDC}}^r h_{RDC} + C_{\varphi,\alpha,L_{CDC}}^r h_{CDC} \right) \end{aligned} \tag{A.19}$$

We notice that the system cost function (A.19) is in the quadratic structure of

$$C_{sys}(z) \approx A_{sys}^r z^2 + B_{sys}^r z + C_{sys}^r, \qquad \text{when} \quad z \in (Z_r, Z_{r+1}] \qquad \text{(A.20)}$$

According to the property of piece-wise quadratic function, the optimal standardized MPC quantity z^* should take the value of $-B_{sys}^r \big/ 2A_{sys}^r$ that falls into the range of $(Z_r, Z_{r+1}]$.

$$z^* = \frac{-\left[(c_2 - c_1) + B_k^r h_b + B_{\psi,\alpha,L_b+1}^r h_b + B_{\varphi,\alpha,L_{RDC}}^r h_{RDC} + B_{\varphi,\alpha,L_{CDC}}^r h_{CDC}\right]}{2\left(A_k^r h_b + \sqrt{L_b+1}A_{\psi,\alpha,L_b+1}^r h_b + \sqrt{L_{RDC}}A_{\varphi,\alpha,L_{RDC}}^r h_{RDC} + \sqrt{L_{CDC}}A_{\varphi,\alpha,L_{CDC}}^r h_{CDC}\right)}, \; z^* \in (Z_r, Z_{r+1}] \quad \text{(A.21)}$$

From (A.21), we have the similar observations as the ones stated in Section IV.6.1.

(I). The optimal solution z^* is independent of the demand parameters μ *and* σ.

(II). The optimal solution z^* decreases supply cost rates difference (c_2-c_1) increases, the optimal MPC quantity Q^* should also increases and, in consequence,

<u>*Numerical Case Study*</u>

Now we use the quadratic approximation method to analyze the same numerical case as given in the Figure 29, we have

- Demand *D* is *iid* normally distributed ~ *Norm* (1000, 400) per week.
- The product has the cumulated product costs of $27, $23, and $22 at the retailer, the regional DC, and the regional PCC, respectively.
- Annual interest rate = 25% and 50 weeks in a year.
- Supply cost rates are c_1=$0.8 and c_2=$1.2 for the direct and indirect channels.
- Each location replenishes its inventory every week.
- At the retailer, the target service level α is 98%.
- The replenishment lead times are L_{CDC}=5 and L_{RDC}= 3 (weeks).

We conducted regression on the simulation results of the coefficient functions $k(z)$, $\psi_{98\%,1}(z)$, $\varphi_{98\%,3}(z)$ and $\varphi_{98\%,5}(z)$. The quadratic approximation functions are presented in Figure 44 and 45.The parameters are given in Table 48.

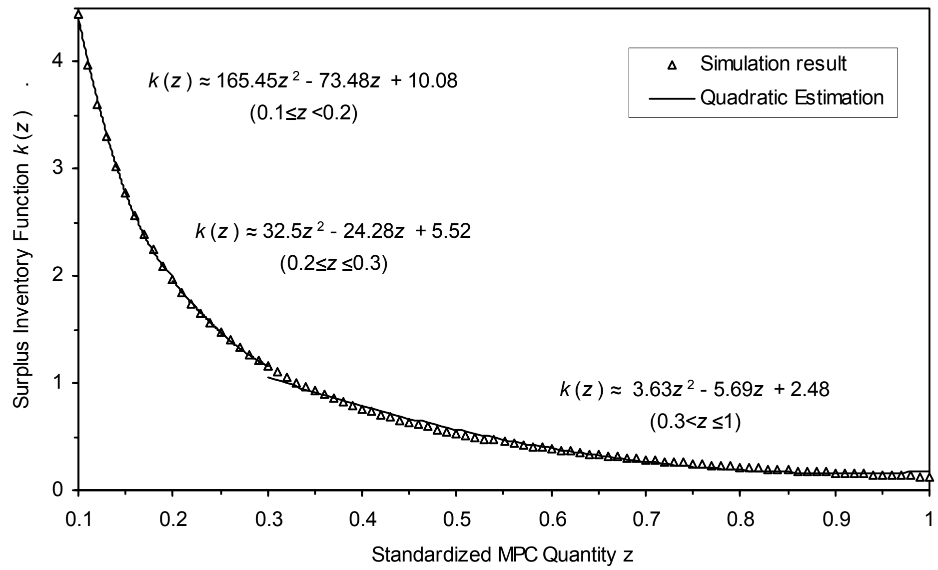

Figure 44. Quadratic approximation functions of $k(z)$

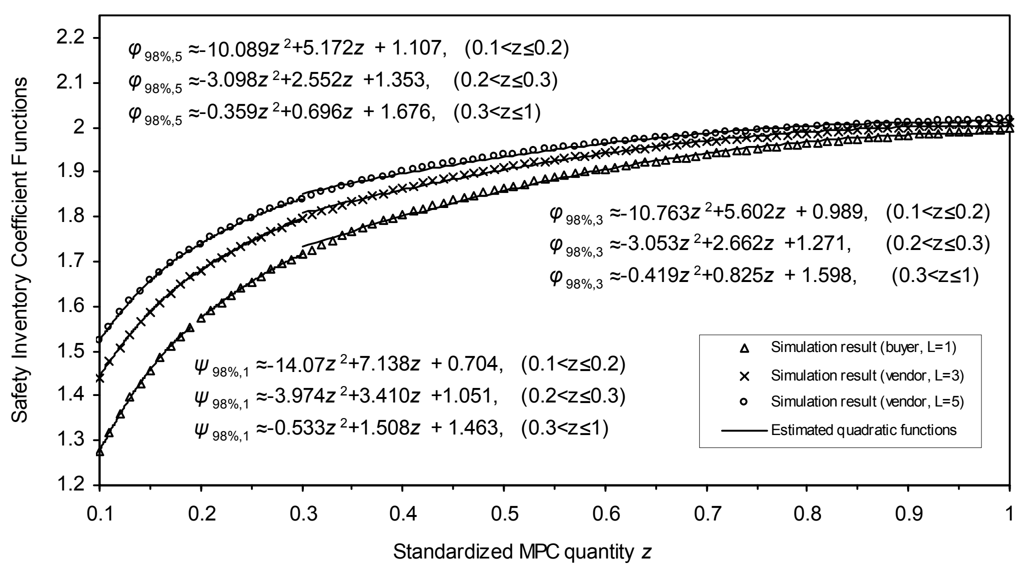

Figure 45. Quadratic approximation functions of $\psi_{98\%,1}(z)$, $\varphi_{98\%,3}(z)$ and $\varphi_{98\%,5}(z)$

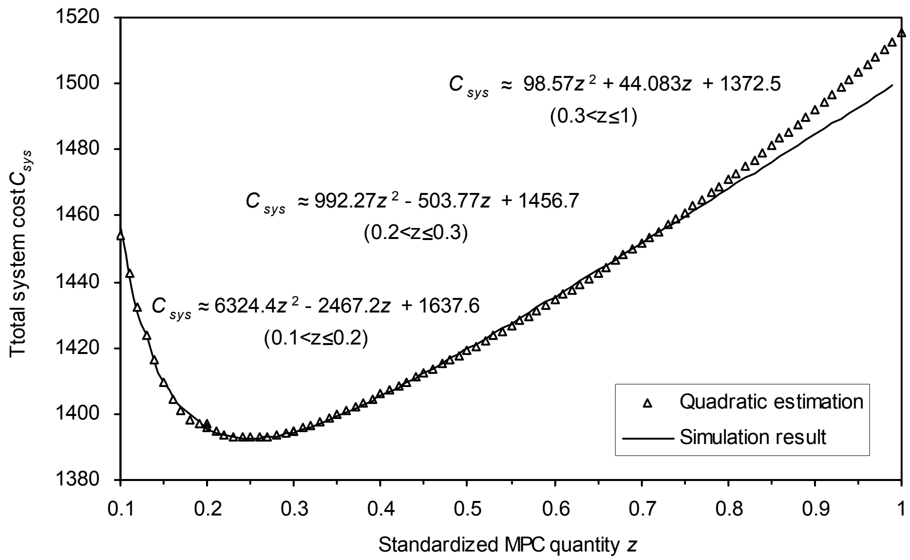

Figure 46. Quadratic approximation functions of total system cost $C_{sys}(z)$

From the above figure, we can see that the quadratic approximation function fits the simulation results well within the range of [0.1, 0.8] and the error is getting larger when the standardized MPC quantity z is getting close to 1. The minimum total system cost happens during the range of $z \in (0.2, 0.3)$, and the optimal standardized MPC quantity z^* should take the value of $-B_{sys}^2 / 2A_{sys}^2 = 0.254$ and the estimated optimal system cost C_{sys} is \$1392 week.

Compared to the optimal solution in simulation of [z^*=0.248, C_{sys}=1392], the quadratic approximation method results in a very close optimal solution.

Table 48. Estimation parameters for quadratic estimation functions

		$k(z)$	$\psi_{98\%,1}(z)$	$\varphi_{98\%,3}(z)$	$\varphi_{98\%,5}(z)$	$C_{sys}(z)$
$0.1 < z \le 0.2$	A^1	165.45	-14.07	-10.763	-10.089	6324.4
	B^1	-73.48	7.138	5.602	5.172	-2467.2
	C^1	10.08	0.704	0.989	1.107	1637.6
$0.2 < z \le 0.3$	A^2	32.50	-3.976	-3.053	-3.098	992.27
	B^2	-24.28	3.410	2.662	2.552	-503.77
	C^2	5.52	1.051	1.271	1.353	1456.7
$0.3 < z \le 1.0$	A^3	3.63	-0.533	-0.419	-0.359	98.57
	B^3	-5.69	1.058	0.825	0.696	44.083
	C^3	2.48	1.463	1.598	1.676	1372.5

Dual-Channel Channel Coordination Model

In the *channel coordination model*, the vendor offers purchase discount λ to entice the buyer to make decisions in a cooperative way. Both the buyer's cost C_b and vendor' C_v can be estimated in piece-wise quadratic functions of the standardized MPC quantity z as follows.

$$\begin{aligned} C_b(z) \approx \sigma\left(A_k^r h_b + \sqrt{L_b+1}A_{\psi,\alpha,L_b+1}^r h_b\right)z^2 + \sigma\left[\lambda P + B_k^r h_b + B_{\psi,\alpha,L_b+1}^r h_b\right]z \\ +\left(P\mu - \lambda P\mu + \mu h_b + C_k^r h_b + C_{\psi,\alpha,L_b+1}^r h_b\right) \end{aligned} \quad \text{(A.22)}$$

The quadratic system cost function (A.22) can be expressed as

$$C_b(z) \approx A_b^r z^2 + B_b^r z + C_b^r, \qquad \text{when } z \in (Z_r, Z_{r+1}] \quad \text{(A.23)}$$

According to the property of piece-wise quadratic function, the optimal standardized MPC quantity z_b^* should be chosen from the values Z_r of (0.1, 0.2, 0.3 and 1) and the possible values $-B_b^r / 2A_b^r$, if they fall into the corresponding range of (Z_r, Z_{r+1}].

Simulation Results

The following six tables present the quadratic approximation parameters for the coefficient functions $\psi_{\alpha,L}(z)$ and $\varphi_{\alpha,L}(z)$. For each coefficient function, we consider the service levels of [98%, 95%, 90%] and the lead time of [1, 3, 5, 7, 15, 25]. These parameters are estimated based on the simulation results in Section A.2.

Table 49. Quadratic estimation parameters for safety stock coefficient function $\psi_{98\%,L}(z)$

		$\psi_{98\%,1}(z)$	$\psi_{98\%,3}(z)$	$\psi_{98\%,5}(z)$	$\psi_{98\%,7}(z)$	$\psi_{98\%,15}(z)$	$\psi_{98\%,25}(z)$
$0.1 < z \le 0.2$	A^1	-14.07	-12.002	-11.352	-11.029	-9.6553	-9.1593
	B^1	7.138	6.2148	5.8448	5.6257	4.9364	4.5904
	C^1	0.704	0.8847	0.9741	1.027	1.1675	1.2668
$0.2 < z \le 0.3$	A^2	-3.976	-3.6341	-3.3344	-3.2176	-2.893	-2.6018
	B^2	3.410	3.0602	2.8209	2.7072	2.3862	2.1175
	C^2	1.051	1.1832	1.2606	1.3003	1.4088	1.501
$0.3 < z \le 1.0$	A^3	-0.533	-0.4663	-0.4266	-0.4032	-0.3384	-0.2896
	B^3	1.058	0.9123	0.8254	0.7764	0.6457	0.5481
	C^3	1.463	1.5564	1.6113	1.6389	1.7119	1.7733

Table 50. Quadratic estimation parameters for safety stock coefficient function $\psi_{95\%,L}(z)$

		$\psi_{95\%,1}(z)$	$\psi_{95\%,3}(z)$	$\psi_{95\%,5}(z)$	$\psi_{95\%,7}(z)$	$\psi_{95\%,15}(z)$	$\psi_{95\%,25}(z)$
$0.1<z\le 0.2$	A^1	-17.167	-14.765	-13.83	-12.868	-11.853	-10.886
	B^1	8.7376	7.5173	7.0087	6.5506	5.8826	5.3546
	C^1	0.0236	0.2732	0.3864	0.4888	0.6388	0.749
$0.2<z\le 0.3$	A^2	-4.8176	-4.2076	-4.0676	-3.6848	-3.448	-2.9366
	B^2	4.1059	3.558	3.3598	3.0714	2.7572	2.3722
	C^2	0.4595	0.6458	0.7285	0.8198	0.9303	1.03
$0.3<z\le 1.0$	A^3	-0.6306	-0.5299	-0.4771	-0.4409	-0.3646	-0.3139
	B^3	1.2394	1.0292	0.9211	0.8497	0.692	0.5909
	C^3	0.9616	1.0898	1.1518	1.2091	1.285	1.3397

Table 51. Quadratic estimation parameters for safety stock coefficient function $\psi_{90\%,L}(z)$

		$\psi_{90\%,1}(z)$	$\psi_{90\%,3}(z)$	$\psi_{90\%,5}(z)$	$\psi_{90\%,7}(z)$	$\psi_{90\%,15}(z)$	$\psi_{90\%,25}(z)$
$0.1<z\le 0.2$	A^1	-22.68	-18.668	-17.084	-15.821	-13.751	-12.726
	B^1	11.262	9.3017	8.4662	7.8671	6.7753	6.1608
	C^1	- 0.7098	- 0.3496	- 0.1833	- 0.0735	0.1462	0.2792
$0.2<z\le 0.3$	A^2	-6.1032	-5.1848	-4.7971	-4.495	-3.8464	-3.4742
	B^2	5.0689	4.2629	3.9019	3.6368	3.0668	2.727
	C^2	- 0.1293	0.123	0.2418	0.3226	0.4946	0.5987
$0.3<z\le 1.0$	A^3	-0.7282	-0.6011	-0.5365	-0.4917	-0.3974	-0.3359
	B^3	1.4245	1.16	1.0285	0.9383	0.7505	0.6307
	C^3	0.5033	0.6609	0.7381	0.7884	0.8932	0.9571

Table 52. Quadratic estimation parameters for safety stock coefficient function $\varphi_{98\%,L}(z)$

		$\varphi_{98\%,1}(z)$	$\varphi_{98\%,3}(z)$	$\varphi_{98\%,5}(z)$	$\varphi_{98\%,7}(z)$	$\varphi_{98\%,15}(z)$	$\varphi_{98\%,25}(z)$
$0.1<z\le 0.2$	A^1	-13.16	-10.763	-10.089	-9.8075	-8.6753	-8.2011
	B^1	6.8756	5.6021	5.172	4.9408	4.2844	3.9235
	C^1	0.721	0.9892	1.107	1.1791	1.3495	1.4459
$0.2<z\le 0.3$	A^2	-3.9663	-3.053	-3.098	-2.8604	-2.405	-1.9805
	B^2	3.4092	2.6623	2.552	2.343	1.9091	1.5749
	C^2	1.0493	1.2709	1.353	1.4229	1.5752	1.6684
$0.3<z\le 1.0$	A^3	-0.535	-0.4185	-0.359	-0.32	-0.2416	-0.1915
	B^3	1.0617	0.8251	0.696	0.613	0.4483	0.3473
	C^3	1.4603	1.5975	1.676	0.7245	1.8278	1.8835

Table 53. Quadratic estimation parameters for safety stock coefficient function $\varphi_{95\%,L}(z)$

		$\varphi_{95\%,1}(z)$	$\varphi_{95\%,3}(z)$	$\varphi_{95\%,5}(z)$	$\varphi_{95\%,7}(z)$	$\varphi_{95\%,15}(z)$	$\varphi_{95\%,25}(z)$
$0.1 < z \leq 0.2$	A^1	-17.303	-12.694	-11.876	-11.198	-9.6798	-8.9922
	B^1	8.7818	6.4803	5.9702	5.5724	4.7218	4.2412
	C^1	0.0203	0.4549	0.5953	0.6939	0.8939	1.007
$0.2 < z \leq 0.3$	A^2	-4.9672	-3.8792	-3.2797	-3.1951	-2.6572	-2.2147
	B^2	4.1822	3.2036	2.7297	2.5778	2.068	1.7029
	C^2	0.4504	0.7601	0.902	0.975	1.1456	1.2455
$0.3 < z \leq 1.0$	A^3	-0.622	-0.4549	-0.3839	-0.3402	-0.2396	-0.1845
	B^3	1.2273	0.8873	0.7367	0.6421	0.4411	0.3338
	C^3	0.9651	1.1614	1.2524	1.3109	1.4258	1.4813

Table 54. Quadratic estimation parameters for safety stock coefficient function $\varphi_{90\%,L}(z)$

		$\varphi_{90\%,1}(z)$	$\varphi_{90\%,3}(z)$	$\varphi_{90\%,5}(z)$	$\varphi_{90\%,7}(z)$	$\varphi_{90\%,15}(z)$	$\varphi_{90\%,25}(z)$
$0.1 < z \leq 0.2$	A^1	-22.859	-15.426	-13.653	-12.903	-11.116	-10.054
	B^1	11.329	7.6815	6.7862	6.3418	5.2892	4.6589
	C^1	- 0.7177	- 0.0668	0.1275	0.233	0.4723	0.6063
$0.2 < z \leq 0.3$	A^2	-6.0886	-4.3507	-3.8522	-3.733	-2.8092	-2.3131
	B^2	5.0692	3.5566	3.1063	2.9208	2.1761	1.7656
	C^2	- 0.1316	0.3183	0.4742	0.5533	0.7649	0.8776
$0.3 < z \leq 1.0$	A^3	-0.7299	-0.4894	-0.406	-0.3543	-0.244	-0.179
	B^3	1.4279	0.9462	0.7694	0.6624	0.4437	0.3206
	C^3	0.5012	0.7704	0.8796	0.9402	1.064	1.1272

A.4 Safety Stock Placement Model with Minimum Purchase Commitment

In Chapter IV, we studied the impact of the dual-channel MPC supply strategy on the Hewlett-Packard ink cartridge supplies system, where the replenishment lead time at each stage is treated as a given parameter. In practice, however, the Hewlett-Packard logistics manager determines these replenishment lead times using a *guaranteed-service safety placement model*, which is firstly introduced in the 1955 manuscripts and later reprinted in Kimball (1988). Kimball introduces the mechanics for a single stage that operates an order-up-to policy and faces bounded demand. Beyond the deterministic production time assumed at the stage, there is an incoming service time that represents the delivery time quoted from the stage's supplier and an outgoing service time representing the delivery time the stage quotes to its customer. Kimball further assumes that demand over any interval of time is bounded. Given this characterization, the order-up-to level at the stage is set equal to the maximum demand over the net replenishment time, which is defined as the incoming service time plus the production time minus the outgoing service time.

Simpson (1958) develops a safety stock placement model to determine the optimal safety stocks in a serial supply chain. Simpson uses Kimball's work as the building block, coupling adjacent stages together through the use of service time. Simpson also provides a rich interpretation for the bounded demand process. Rather than saying that bounded demand reflects the maximum demand the stage will face, the bound can instead reflect the maximum amount of demand the company wants to satisfy from safety stock. Graves (1988) observes that the serial-line problem, as formulated by Simpson (1958), can be solved as a dynamic program. Inderfurth (1991), Inderfurth and Minner (1998), Graves and Willems (1996, 2000) extend Simpson's work to supply chains modeled as assembly networks, distribution networks, and spanning trees. In each case, the optimization problem is still to determine the service times that minimize the total cost for safety stock in the supply chain.

In this section, we incorporate the MPC issue into the traditional guaranteed-service safety stock placement problem. In Subsection A.4.1, we give the notation, assumptions and formulations. We study a simple safety stock placement model with minimum purchase commitment in Subsection A.4.2. Finally, we give a numerical case study in Subsection A.4.3.

A.4.1 Notation, Assumptions, and Formulations

In this section, we introduce the notation, assumptions and formulations of the vendor-buyer system with safety stock placement and minimum purchase commitment (MPC). The notation is as follows.

Table 55. Notation for safety placement model with minimum purchase commitment

j	Index of stage in the supply chain network, $j=1,2,\ldots,J$.
μ_j, σ_j	Mean and standard deviation of the demand at stage j.
h_j	Holding cost rate at stage j.
L_j	Production lead time at stage j.
s_j^{in}, s_j^{out}	Inbound/outbound service time at stage j.
Q_i	Minimum purchase quantity at stage j.
z_i	Standardized minimum purchase commitment quantity at stage j.

(I). *Multi-Stage Serial Supply Chain Network*

We consider a multi-stage serial supply chain network where stages are indexed from 1 to J. The starting stage 1 is supplied by an external upstream stage, and the end stage J supplies an external downstream stage.

(II). *Production Lead Time L*

Each stage j has a deterministic and fixed product lead time L_j, which is defined as the total time from when all the inputs are available to serve the downstream stage j+1. The production lead time may include the waiting and processing time at the stage, plus any transportation time to put the item into inventory.

(III). *Minimum Purchase Commitment (MPC)*

In the serial supply chain network, minimum purchase commitment can be implemented at a single stage, which is called MPC stage. All the downstream stages to the MPC stages are called Down-MPC stages, and all the upstream stages to the MPC stages are called Up-MPC stages. We use z_j to denote the standardized MPC quantity at the stage j.

(IV). *Periodic Order-Up-To Replenishment Policy*

Each stage *i* replenishes its inventory with a periodic order-up-to policy at a common time period. We assume there is no time delay in placing orders. For the MPC stage and all the Down-MPC stages, the order process is identical to the external customer demand process. For all the Up-MPC stages, the order process has a lower limit of MPC quantity *Q*.

(V). *Inbound/Outbound Service Time*

The inbound service time s_j^{in} is the time for stage *j* to get supplies from its immediate upstream stage (*j*-1). The outbound service time s_j^{out} is the time for stage *j* to supply its immediate downstream stage (*j*+1). Note that, the inbound service time s_0^{in} at the beginning stage *0* and the outbound service time s_N^{out} at the end stage *N* are given parameters rather than decision variables.

A.4.2 Safety Stock Placement Model with Minimum Purchase Commitment

With consideratio of minimum purchase commitment, the safety stock placement model can be formulated by using the variables as follows.

e_j = binary variable to denote if the stage *j* is an MPC stage.

θ_j = binary variable to denote if the stage *j* is an Up-MPC stage.

ϑ_j = binary variable to denote if the stage *j* is a Down-MPC stage.

t_j^{χ} = net replenishment lead time if the stage *j* is a Down-MPC stage.

t_j^{ψ} = net replenishment lead time if the stage *j* is an MPC stage.

t_j^{φ} = net replenishment lead time if the stage *j* is an Up-MPC stage.

z_j = standardized MPC quantity at the stage *j*.

$$\text{Min} \quad \sum_{j=1}^{N} k(z_j)\sigma_j h_j + \sum_{j=1}^{N} \chi_\alpha \sigma_j \sqrt{t_j^\chi} + \sum_{j=1}^{N} \psi_{\alpha,z,L} \sigma_j \sqrt{t_j^\psi} + \sum_{j=1}^{N} \varphi_{\alpha,z,L} \sigma_j \sqrt{t_j^\varphi} \tag{A.24}$$

$$\text{s.t.} \quad s_j^{out} - s_j^{in} \le L_j, \qquad \forall j = 1,2,....,N \tag{A.25}$$

$$s_j^{out} = s_{j+1}^{in}, \qquad \forall j = 1,2,....,N-1 \tag{A.26}$$

$$\sum_{j=1}^{N} e_j = 1 \tag{A.27}$$

$$\theta_j = \sum_{i=j+1}^{N} e_j, \qquad j = 1,2,...,N-1 \tag{A.28}$$

$$e_j + \theta_j + \vartheta_j = 1, \qquad j = 1,2,....,N \tag{A.29}$$

$$t_j^\chi + t_j^\psi + t_j^\varphi = s_j^{in} + L_j - s_j^{out}, \qquad \forall j = 1,2,....,N \tag{A.30}$$

$$t_j^\chi \le T\vartheta_j, \qquad j = 1,2,....,N \tag{A.31}$$

$$t_j^\psi \le Te_j, \qquad j = 1,2,....,N \tag{A.32}$$

$$t_j^\varphi \le T\theta_j, \qquad j = 1,2,....,N \tag{A.33}$$

$$\sum_{j=1}^{N} z_j \in [0,1] \tag{A.34}$$

$$0 \le z_j \le e_j, \qquad \forall j = 1,2,....,N \tag{A.35}$$

$$e_j, \theta_j, \vartheta_j \quad \text{binary} \tag{A.36}$$

$$t_j^\chi, t_j^\psi, t_j^\varphi, s_j^{in}, s_j^{out} \quad \ge 0 \text{ and integer} \tag{A.37}$$

In the objective (A.24), the four terms represent the surplus inventory cost at the MPC stage, the safety stock costs at the Down-MPC stages, the safety stock cost at the MPC stage and the safety stock cost at the Up-MPC stages. (A.25) and (A.26) set the feasible value of inbound/outbound service times Constraint (A.27) indicates that there is only one MPC stage in the supply chain. Constraints (A.28) set the stage *j* as Up-MPC stage if MPC is implemented at any downstream stage. Constraints (A.29) set the stage *j* as Down-MPC stage if MPC is implemented at any upstream stage. Constraints (A.30)-(A.33) set the net replenishment time for each stage.

A.4.3 Numerical Case study

In this section, we study a numerical case of battery manufacturing and distribution, cited from Graves and Willems (2003). We consider an additional assumption that minimum purchase commitment can be applied at certain stage. The battery supply chain is presented in Figure 47 and parameters are presented in Table 56.

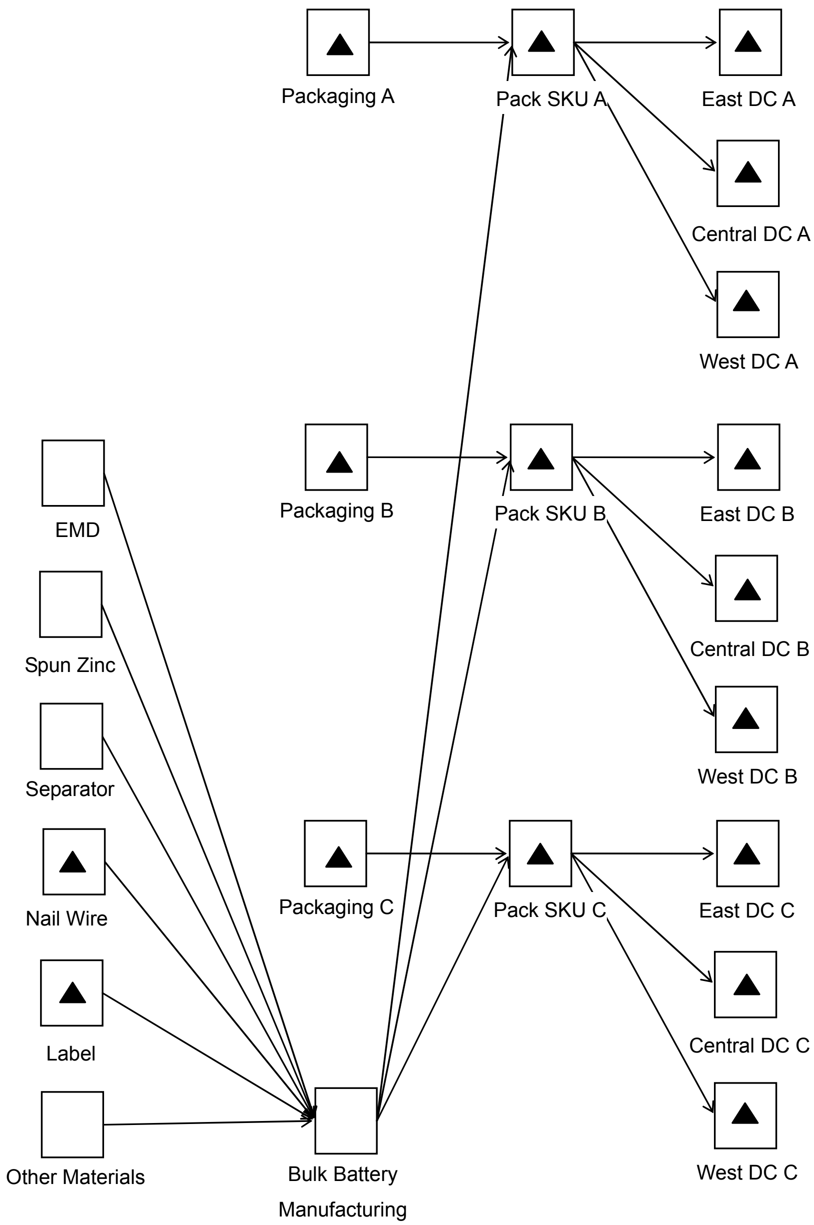

Figure 47. Graphical representation of the battery supply chain

Table 56. Parameters for battery supply chain

Stage Name	Nominal time	Cumulated cost ($)	Mean demand	STD of demand
East DC A	4	0.82	67,226	109,308
Central DC A	6	0.84	43,422	67,236
West DC A	5	0.83	65,638	119,901
East DC B	4	0.96	15,765	34,079
Central DC B	6	0.96	16,350	39,552
West DC B	8	0.98	10,597	23,277
East DC C	4	1.20	6,416	14,125
Central DC C	4	1.20	5,536	11,213
West DC C	6	1.25	3,519	6,576
Pack SKU A	11	0.82		
Pack SKU B	11	0.95		
Pack SKU C	9	1.19		
Packaging A	28	0.16		
Packaging B	28	0.24		
Packaging C	28	0.36		
Bulk battery mfg.	5	0.59		
EMD	2	0.13		
Spun zinc	2	0.05		
Separator	2	0.02		
Nail wire	24	0.02		
Label	28	0.06		
Other Materials	1	0.24		

The company's holding cost rate is 25%. The daily demand parameters are given in Table 56. Assuming 260 days per year, the expected cost of goods sold per year is $53,569,000. On a daily basis the demand is highly variable with the coefficient of variation. In addition to the case assumptions in Graves and Willems (2003), we have MPC cost savings assumption

(I). At the DC stage, regularly ordering per unit will generate the system cost savings of 0.2% of the product price; that is, $0.002 per regularly ordered quantity. This cost savings may be due to the smooth production and regular transportation schedule etc.

(II). At the Pack SKU stage, regularly ordering per unit will generate the system cost savings of 0.15% of the product price; that is, $0.0015 per regularly ordered quantity.

Numerical Results

Surprisingly, we find that implementing minimum purchase commitment does not affect the safety stock placement solution. The optimal standardized MPC quantity z^* is 0.26 at the DC stage. As a consequence, we have the annual MPC cost savings of $64,427. From the Table 57, we have the annual system cost savings = 854,278-(886,755-64,427)=$31,950.

Table 57. Optimal solution for the case of Implementing MPC at DC stage

Stage Name	Net Repl. Time	Cumulated Product Cost	No MPC	MPC (z=0.26)	
			Safety Stock Cost	Safety Inv Cost	Surplus Inv Cost
East DC A	4	0.82	73,723	60,962	31,358
Central DC A	6	0.84	56,894	48,750	19,759
West DC A	5	0.83	91,515	73,816	34,816
East DC B	4	0.96	26,909	22,251	11,446
Central DC B	6	0.96	38,249	32,774	13,284
West DC B	8	0.98	26,534	22,083	7,981
East DC C	4	1.20	13,941	11,528	5,930
Central DC C	4	1.20	11,067	9,152	4,707
West DC C	6	1.25	8,280	7,095	2,876
Pack SKU A	18	0.82	251,275	229,679	0
Pack SKU B	18	0.95	94,750	86,606	0
Pack SKU C	16	1.19	37,577	34,348	0
Packaging A	21	0.16	52,958	49,525	0
Packaging B	21	0.24	25,855	24,179	0
Packaging C	21	0.36	13,024	12,179	0
Bulk battery mfg.	0	0.59	0	0	0
EMD	0	0.13	0	0	0
Spun zinc	0	0.05	0	0	0
Separator	0	0.02	0	0	0
Nail wire	22	0.02	7,164	6,699	0
Label	26	0.06	24,564	22,972	0
Other Materials	0	0.24	0	0	0
Total Inventory Cost			**854,278**	**754,599**	**132,156**

REFERENCES

Abad, P. L. 1994. Supplier pricing and lot sizing when demand is price sensitive. *Euro. J. Oper. Res*. **78** 334-354.

Alfredsson, P., J. Verrijdt. 1999. Modeling emergency supply flexibility in a two-echelon inventory system. *Management Sci*. **45** (10) 1416-1431.

Andel, T. 1996. Manage inventory, own information. *Transportation and Distribution* **37** (5) 54-58.

Anupindi, R., R. Akella. 1993. An inventory model with commitments. Unpublished manuscript, October 4, 1993.

Balakrishnan, A., S. C. Graves. 1989. A composite algorithm for a concave-cost network flow problem. *Networks*. **19**, 175-202.

Balintfy, J. L. 1964. On a basic class of multi-cycle inventory problems. *Management Sci.* **10**. 287-297

Banerjee, A. 1986. On a quantity discount pricing model to increase vendor profit. *Management Sci*. **32** 1513–1517.

Banerjee, A., J. Burton. S. Banerjee. 2003. A simulation study of lateral shipment in single supplier, multiple buyers supply chain networks. *Inernat. J. Production Economics.* **81-82** 103-114.

Barnhart, C., E. L. Johnson, G. L. Nemhauser, G. Sigismondi, P. Vance. 1993. Formulating a mixed integer programming problem to improve solvability. *Oper. Res*. **41** 1013-1019.

Bassok, Y., R. Anupindi. 1997. Supply chain management with minimum purchasing commitment. *IIE Transactions* **29** 373 – 381.

Ben-Daya, A., M. Hariga. 2004. Integrated single vendor single buyer model with stochastic demand and variable lead time. *Interna. J. Production Economics*. **92** 75-80.

Bertazzi, L., M. G. Speranza. W. Ukovich. 1994. Minimization of logistic costs with given frequencies. *Transportation Sci*. 19 361-380.

Bertazzi, L., M. G. Speranza. 1999. Models and algorithms for the minimization of inventory and transportation costs: A survey. *Lecture Notes in Economics and Mathematical Systems*, **480** 137-57.

Bertazzi, L., M. G. Speranza, W. Ukovich. 2000. Exact and heuristic solutions for a shipment problem with given frequencies. *Management Sci*. **46** 973-988.

Bertazzi, L., M. G. Speranza. 2002. Continuous and discrete shipping strategies for the single-link problem. *Transportation Sci*. **36** 314-325.

Blumenfeld, D. E., L. D. Burns, C. F. Daganzo, M. C. Frick, R. W. Hall. 1987. Reducing logistics costs at General Motors. *Interfaces*. **17** 26-47.

Billington, C., G. Callioni, B. Crane, J. D. Ruark, J. U. Rapp, T. White, S. P. Willems. 2004. Accelerating the profitability of Hewlett-Packard's supply chain. *Interfaces*, **34** 59-72.

Blumenfeld, D. E., L. D. Burns, J. D. Diltz, C.F. Daganzo. 1985. Analyzing tradeoffs between transportation, inventory and production costs on freight networks. *Transportation Res*. **19B** 361-380.

Burke, M. 1996. It's time for vendor managed inventory. *Industrial Distribution* **85**(2) 90.

Buxbaum, P. A. 1995. Taking control. *Distribution* 32.

Cachon, G. 2003. Supply chain coordination with contracts. A .G. de Kok, S.C. Graves, eds., Handbook in Operatios Research and Management Science, Vol. 11: *Supply Chain Management: Design, Coordination and Operatio.* North Holland, Amsterdam. 229-339.

Chan, L. M. A., A. Muriel, Z. J. Shen, D. Simchi-Levi, C. P. Teo. 2000. Effective zero inventory ordering policies for the single-warehouse and multi-retailer problem with piecewise linear cost structures. Technical report, University of Massachusetts, Amherst, MA.

Chen, F., Z. Drezner, J. K. Ryan, D. Simchi-Levi. 2000. Quantifying the bullwhip effect in a simple supply chain: the impact of forecasting, lead times and information. *Management Sci* **46** (3) 436-443.

Cottrill, K. 1997. Selling on consignment: another tool in the credit arsenal. *Business Credit* **98**(9) 6-8.

Dada, M. 1992. A two-echelon system with priority shipments. *Management Sci*. **38** (8) 1140-1153.

De Kok, A. G. 1989. A moment-iteratio method for approximating the waiting-time characteristics of the GI/G/1 queue, *Probability and Engineering Information Science*. **3** 273-287.

Elhedhli, S., O. S. Benli. 2004. Optimal lot sizing under carload discount schedules. Working dissertation, University of Waterloo, Waterloo, Canada.

Fleischmann, B. 1993. Designing distribution systems with transportation economies of scale. *Eur. J. Oper. Res*. **70** 31-42.

Gentry, J. J. 1991. Purchasing's involvement in transportation decision making. Center for advanced purchasing studies. Tempe, Arizona.

Goyal, S. K. 1976. An integrated inventory model for a single supplier-single customer problem. *International Journal of Production Research*. **15** 107–111.

Graves, S. C. 1988. Safety stocks in manufacturing systems. *Journal of Manufacturing and Operatios Management*. **1** 67-101.

Graves, S. C., S. P. Willems. 1996. Strategic safety stock placement in supply chains, *Proceeding of the 1996 MSOM conference*, Hanover, NH.

Graves, S. C., S. P. Willems. 2000. Optimizing strategic safety stock placement in supply chains. *Manufacturing & Service Operatios Management*. **2** 68-83.

Graves, S. C., S. P. Willems. 2003. Supply chain design. G. de Kok, S.C. Graves, eds., Handbook in Operatios Research and Management Science, Vol. 11: *Supply Chain Management: Design, Coordination and Operatio.* North Holland, Amsterdam. 95-132.

Hall, R. W. 1985a. Determining vehicle dispatch frequency when shipping frequency differs among suppliers. *Transportation Res*. **19B** 421-431.

Hall, R. W. 1985b. Dependence between shipment size and mode in freight transportation. *Transportation Res*. **19B** 436-444.

Hall, R. M. 1991. Route selection on fright networks with weight and volume constraints. *Transportation Res*. **25B** 175-189.

Inderfurth, K. 1991. Safety stock optimization in multi-stage inventory systems. *International journal of Production Economics*. **24** 103-113.

Inderfurth, K., S. Minner. 1988. Safety stocks in multi-stage inventory systems under different service levels. *European Journal of Operatioal Research*. **106** 57-73.

Janssen, F., T. de Kok. 1999. A two-supplier inventory model. *Int.J.Production Economics* **59** 395-403.

Kimball, G.E. 1988. General principles of inventory control. *Journal of Manufacturing and Operatios Management*. **1** 119-130.

Lal, R., R. Staelin. 1984. An approach for developing an optimal discount pricing policy. *Management Sci*. **32** 1177-1185.

Lapierre, S. D., A. B. Ruiz, P. Soriano. 2004. Designing distribution networks: formulations and solution heuristic. *Transportation Sci*. **38** 174-187.

Lee, H. 1987. A multi-echelon inventory model for repairable items with emergency lateral transshipments. *Management Sci.* **33** 1302-1316.

Lee, H., P. Padmanabhan, S. Whang. 1997. Information distortion in a supply chain: the bullwhip effect. *Management Sci*. **34** 1391-1398.

Li, C. L., V. N. Hsu, W. Q. Xiao. 2004. Dynamic lot sizing with batch ordering and truckload discounts. *Operatios Res*. **52** 639-654.

Moinzadeh, K., S. Nahmias. 2000. Adjustment strategies for a fixed delivery contract. *Operatios Res*. **48** (3) 408-423.

Monahan, J. P. 1984. A quantity discount pricing model to increase vendor profit. *Management Sci*. **30** 720–726.

Muckstadt, J., R. O. Roundy. 1993. Analysis of multistage production systems. S. C. Graves, A. H. G. Rinnooy Kan, P. H. Zipkin, eds., Handbook in Operatios Research and Management Science, Vol. 4: *logistics of Production and Inventory*. North Holland, Amsterdam. 59-131.

Nahmias, S. 2001. *Production and Operatios Analysis*, 4th ed. McGraw-Hill, Boston, MA.

Rosenshine, M., D. Obee. 1976. analysis of a standing order inventory system with emergency orders. *Operatios Res*. **24** (6) 1143-1155.

Rubin, P. A., J. R. Carter. 1990. Joint optimality in buyer-supplier negotiations. *Journal of Purchasing and Materials Management* **26** 20-26.

Simchi-Levi, D., P. Kaminsky, E. Simchi-Levi. 2003. Designing and Managing the Supply Chain: Concepts, Strategies, and Case Studies. McGraw-Hill, New York.

Simpson, K. F. 1958. In-process inventories. *Operatios Res*. **6**, 863-873.

Speranza, M. G., W. Ukovich. 1994. Minimizing transportation and inventory costs for several products on a single-link. *Operatios Res*. **42** 879-894.

Speranza, M. G., W. Ukovich. 1996. An algorithm for optimal shipments with given frequencies. *Naval Res. Logist*. **43** 655-671.

Thomas, D. J., S. T. Hackman. 2003. A committed delivery strategy with fixed frequency and quantity. *Eur. J. Oper. Res.* **148** 363-373.

Toptal, A., S. Cetinkaya, C. Y. Lee. 2003. The buyer-vendor coordination problem: modeling inbound and outbound cargo capacity and costs. *IIE Transactions* **35** 987-1002.

Tsay, A., S. Nahmias, N. Agrawal. 2000. Modeling supply chain contracts: A review. S. Tayur, R. Ganeshan, M. Magazine, eds. *Quantitative Models For Supply Chain Management*. Kluwer Academic Publishers, Norwell, MA, 299-330.

Waller, M., M. E. Johnson, T. Davis. 1999. Vendor-managed inventory in the retail supply chain. *Journal of Business Logistics* **20** (1) 183-203.

Weng, Z. K. 1995. Modeling quantity discounts under general price-sensitive demand functions: Optimal policies and relationships. *Euro. J. Oper. Res*. **86** 300-314.

Xu, K., Y. Dong, P. T. Evers. 2001. Towards better coordination of the supply chain. *Transportation Res. Part E: Logistics and Transportation Review* **37**(1) 35-54.

Yao, Y., P. T. Evers, M. E. Dresner. 2005. Supply chain integratio in vendor-managed inventory. *Decision Support Systems*. In press.