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ABSTRACT:

Many research works have been carried out to develop equations for shear strength of unsaturated soil. However, most of the equations are limited for the soil with unimodal SWCC. The objective of this study is to propose an equation for estimating the unsaturated shear strength of soils with bimodal SWCC. Several series of unsaturated consolidated drained triaxial tests were carried out on the statically compacted sand-kaolin specimens to evaluate the proposed equation. The experimental results indicated that the ϕ^b angle of the soil with bimodal SWCC at suctions between the first air-entry value (AEV1) and the second air-entry value (AEV2) was less than the ϕ^b angle at suctions below AEV1. However, the relationships between shear strength and suction at suctions between AEV1 and AEV2 were still linear. At suctions beyond AEV2, ϕ^b angle of the soil with bimodal SWCC started to decrease non-linearly. The evaluation results showed that the estimated bimodal shear strength using the proposed equation in this study had a good agreement with the shear strength data from the experimental works.

Keywords: bimodal, soil-water characteristic curve, unsaturated shear strength, air-entry value

1. INTRODUCTION

Majority of residual soils are called gap-graded soils (Satyanaga et al., 2017; Rahardjo et al., 2012a, b, c). Some gap-graded soils have soil-water characteristic curve (SWCC) with bimodal characteristics (Satyanaga et al., 2013). Satyanaga et al. (2013) proposed a new equation for best fitting a bimodal SWCC whose parameters represent the variables of SWCC. Therefore, Satyanaga et al. (2013) equation was used to best fit a bimodal SWCC in this study. Laboratory tests for measurement of the shear strength of unsaturated soils are time consuming and require extensive laboratory equipments (Fredlund et al, 2012; Fredlund and Rahardjo, 1993). Therefore, research works have been carried out to develop the equations to estimate the shear strength of unsaturated soil (Goh et al., 2010; Lee et al., 2005; Vanapalli et al., 1996a, b; Khalili and Khabbaz, 1998). The applicability of the existing published equations for unsaturated shear strength of soils is limited to soils with unimodal SWCC. Therefore, in this paper a new equation for estimating the unsaturated shear strength of soils with bimodal SWCC is proposed.

2. Proposed Theory and Equation

An appropriate mathematical equation is required to represent the unsaturated shear strength of soil since shear strength is the important property of unsaturated soil in geotechnical analyses. In this study, the proposed shear strength equation was developed with the consideration of a smooth transition of shear strength equation from bimodal characteristics to unimodal characteristics and a smooth transition from unsaturated shear strength to saturated shear strength. The proposed equation for estimating the shear strength of unsaturated soil with a bimodal characteristic was developed based on Fredlund et al. (1978) equation (Equation 1) to provide a smooth transition from unsaturated shear strength to saturated shear strength.

$$\tau = c' + (\sigma - u_a)\tan \phi' + (u_a - u_w)\tan \phi^b \quad (1)$$

In the proposed equation, at matric suctions less than air-entry value (AEV), soil is assumed to be in a saturated condition (Figure 1), the ϕ^b angle is assumed to be equal to ϕ' (Figure 1) and all matric suctions below the AEV contribute to the effective stress. This stage is called capillary zone. In order to describe the condition in the capillary zone, Equation 1 can be rewritten into Equation 2.

$$\tau = c' + (\sigma - u_a)\tan \phi' + (u_a - u_w)\tan \phi' \quad (2)$$

For soils with bimodal characteristics, the proposed physical model that identifies the four different stages during the desaturation of a soil, i.e. capillary zone, macro transition zone, micro transition zone and residual zone (Figure 1), were incorporated in the development of the new equation for estimating the unsaturated shear strength of soils with bimodal SWCC. At matric suctions higher than the first air-entry value (AEV1) and lower than the second air-entry value (AEV2), soil is in the macro transition stage. In the macro transition stage, the micro pores and macro pores of soil are assumed to be in the saturated and unsaturated conditions, respectively. Therefore, the relationship between shear strength and matric suction exhibits a linear behaviour similar to that in the capillary zone. However, the ϕ^b angle in the macro transition stage is less than the ϕ^b angle in the capillary zone (Figure 1). In order to describe the condition in the macro transition stage, Equation 2 can be modified into Equation 3.

$$\tau = c' + (\sigma - u_a)\tan \phi' + AEV1 \tan \phi' + (u_a - u_w - AEV1)\tan(b\phi') \quad (3)$$

where:

b = parameter representing the ratio of ϕ^b angle to ϕ' in the macro transition

At matric suctions higher than AEV2, soil is in the micro transition stage where the relationship between shear strength and matric suction exhibits a non-linear behavior. Therefore, the ϕ^b angle at matric suctions higher than AEV2 varies with matric suction. In order to describe the condition in the micro transition stage, Equation 3 can be modified into Equation 4.

$$\tau = c' + (\sigma - u_a)\tan \phi' + AEV1 \tan \phi' + ((u_a - u_w) - AEV1)\tan(b\phi') + (AEV2 - (u_a - u_w))\tan(k\phi') + ((u_a - u_w) - AEV2)\tan(b\phi') \quad (4)$$

where:

k = parameter representing the ratio of ϕ^b angle to ϕ' at the micro transition

One advantage of the proposed shear strength equation for unsaturated soil developed in this study is a smooth transition of soil shear strength from bimodal characteristics into unimodal characteristics and a

smooth transition from unsaturated shear strength into saturated shear strength. Therefore, Equation 4 can be rearranged into Equation 5 in order to accommodate this smooth transition.

$$\begin{aligned} \tau = & c' + (\sigma - u_a) \tan \phi' + AEV1 \tan \phi' + (u_a - u_w) \tan(b\phi') - AEV1 \tan(b\phi') \\ & + AEV2 \tan(k\phi') + (u_a - u_w) \tan(b\phi') - AEV2 \tan(b\phi') - (u_a - u_w) \tan(k\phi') \end{aligned} \quad (5)$$

Equation 5 can be rearranged into Equation 6 which is the general equation for the bimodal unsaturated shear strength.

$$\begin{aligned} \tau = & c' + [(\sigma - u_a) + AEV1] \tan \phi' + [2(u_a - u_w) - AEV1 - AEV2] \tan(b\phi') \\ & + [AEV2 - (u_a - u_w)] \tan(k\phi') \end{aligned} \quad (6)$$

where:

b and k = 1 for $u_a - u_w < AEV1$

k = b for $AEV1 < u_a - u_w < AEV2$

3. Laboratory Tests

In this study, soil mixtures with different percentages of medium sand and coarse kaolin were used. Index property tests were conducted in accordance with the ASTM soil testing standard. Specific gravity, grain-size distribution and Atterberg limit tests were performed following ASTM D854-06E01, ASTM D0422-63R07 and ASTM D4318-10, respectively. The soil classification was conducted based on the Unified Soil Classification System (ASTM D2487-10). The SWCC tests were carried out following ASTM D6838-02 (2008). In this study, drying SWCCs were determined by combining the results from Tempe cell tests and pressure plate tests. Tempe cell was used for measurements of SWCC up to 100 kPa matric suction whereas the 5-bar and 15-bar pressure plates were used for measurements of SWCC up to 500 kPa and 1500 kPa matric suction, respectively. Consolidated undrained saturated triaxial tests with pore-water pressure measurement were performed using a triaxial cell in accordance with ASTM D4767-04. The shear strength tests of soil under an unsaturated condition were carried out using a modified triaxial apparatus as described by Fredlund and Rahardjo (1993).

4 Results and Discussions

Soil mixtures M3, M4, M6 and M8 were classified as gap-graded soils (soils with bimodal grain-size distributions) their soil particle size can be divided into two distinct groups (Figure 2). The index properties of soil mixtures M3, M4, M6 and M8 are shown in Table 1. Soils M3 and M4 were classified as SM (silty sand), whereas soils M6 and M8 were classified as MH (sandy silt with high plasticity). The SWCCs of soil specimens M3, M4, M6 and M8 (Figure 2) were classified as a bimodal SWCC since they had two slopes with different gradients. The parameters from Satyanaga et al. (2013) equation for best fitting the bimodal SWCC in Figure 2 are summarized in Table 2.

The results of the consolidated undrained saturated triaxial tests on specimens M3, M4, M6 and M8 are summarized in Table 3. The c' varied from 5 to 16 kPa whereas the ϕ' varied from 25° to 34°. The lowest c' was observed in the specimen with the highest percentage of sand while the lowest ϕ' was observed in the

specimen with the lowest percentage of sand. The ϕ^b angles of specimens M3, M4, M6 and M8 are summarized in Table 3. Figure 3 shows that ϕ^b angle obtained from the unsaturated triaxial test was equal to ϕ' of each specimen at matric suctions less than or equal to the first air-entry value of the soil specimen with bimodal SWCC.

ϕ^b angle of soil specimen with bimodal SWCC started to decrease linearly at matric suctions higher than the first air-entry value and less than the second air-entry value. At suctions higher than the second air-entry value, ϕ^b angle of soil specimen with bimodal SWCC started to decrease nonlinearly with suction. In other words, ϕ^b angle at suctions higher than the second air-entry value is much lower than ϕ^b angle at suctions between the first and the second air-entry value. This could be attributed to the reduction of area of water (water menisci area which were in contact with soil particles) within the macro pores of soil specimen with bimodal characteristics at suctions higher than the first air-entry value. However, the micro pores within the soil specimen with bimodal characteristics were still in a saturated condition. As a result, the rate of decrease of ϕ^b angle was constant at suctions between the first and the second air-entry values. As suctions went beyond the second air-entry value, the area of water menisci within the micro pores started to decrease due to the desaturation of the micro pores of soil specimen with bimodal characteristics. Hence, the ϕ^b angle decreased non-linearly at suctions beyond the second air-entry value.

The shear strengths of soil with bimodal SWCC obtained from laboratory tests carried out in this study were used to evaluate the new shear strength equations proposed in this study (Equation 8). The parameters of the proposed shear strength equation are summarized in Table 4. The performance of the proposed equation in estimating the bimodal unsaturated shear strength was evaluated using average relative error (ARE) (Goh et al., 2010). In this study, the performance of the proposed equation was considered good if ARE was less than or equal to 10%.

It was observed that the parameter b of the proposed shear strength equation for soils M3, M4, M6 and M8 were 0.89, 0.72, 0.57 and 0.23, respectively, which corresponded to the ratio of ϕ^b/ϕ' of these soils for suctions between the first air-entry value and the second air-entry value as shown in Figure 3 and Table 5. Table 5 also shows that the ARE values of the proposed shear strength equation developed in this study (Equation 6) were less than 10%, indicating good performance of the proposed shear strength equation developed in this study in estimating the bimodal unsaturated shear strength.

The new shear strength equations proposed in this study were also evaluated using the published data from literatures. The soils from the published data include undisturbed soils and residual soils with different percentages of sand, silt and clay (Table 4). The parameters of the proposed shear strength equation developed in this study are summarized in Table 5. It indicates that the parameter b of the proposed shear strength equation for soils from Coutinho et al. (2011), Jotisankasa et al. (2009) and Rahardjo et al. (2011) were 0.57, 0.32 and 0.69, respectively, corresponding to the ratio of ϕ^b/ϕ' of these soils for suctions between AEV1 and AEV2. Table 5 shows that the ARE values of the proposed shear strength equation developed in this study were less than 10% indicating a good performance of the proposed shear strength equation developed in this study to estimate the bimodal unsaturated shear strength of soil.

5. Correlations between The Proposed Shear Strength Parameters and Index Properties

Trial and errors as well as best-fit analyses were performed to relate parameters b , q and f in the equation 7 with the index properties.

$$k = b + q \left[\log \left(\frac{u_a - u_w}{AEV2} \right) \right]^f \quad \text{for } u_a - u_w > AEV2 \quad (7)$$

b , q and f = fitting parameters.

The parameter b was found to be closely related to liquid limit (LL) of soil using the following relationship:

$$b = 5 \exp(-0.047LL) \quad (8)$$

The parameter q was found to be closely related to total density of soil (ρ) using the following relationship:

$$q = 0.752\rho - 1.12 \quad (9)$$

The parameter f was found to be exponentially related to the standard deviation of the second sub-curve of SWCC (parameter s_2) using the following relationship:

$$f = 0.088 \exp(0.83s_2) \quad (10)$$

where:

- b = the parameter in the proposed unsaturated shear strength equation (Equation 6)
- LL = Liquid limit of soil (%)
- q = the parameter in the proposed unsaturated shear strength equation (Equation 6)
- ρ = total density of soil (Mg/m^3)
- f = the parameter in the proposed unsaturated shear strength equation (Equation 6)
- s_2 = parameter in SWCC represents standard deviation of second subcurve

In summary, parameter b is one of the fitting parameters in the proposed unsaturated shear strength equation which can be estimated based on its relationship to Liquid Limit of soil. There is no influence of soil structure in the b parameter. Parameter q is one of the fitting parameters in the proposed unsaturated shear strength equation which can be estimated based on its relationship to total density of soil. Parameter f is one of the fitting parameters in the proposed unsaturated shear strength equation which can be estimated based on its relationship to fitting parameter s_2 in the bimodal SWCC equation. Hence, the variations in the fitting parameter f in the proposed shear strength equation reflects the variations in the standard deviation of the pore-size distribution within micropores of soil. The standard deviation of the pore-size distribution is a function of the slope of SWCC which is affected by volume change of soil. In other words, the fitting parameter f incorporates the effect of volume change in the proposed shear strength equation.

The evaluation of the correlations between the parameters b , q and f and the index properties was based on four (4) compacted soils used in the experimental works of this study and three (3) residual soils from the literature. Further evaluation on the proposed correlations should be carried out using a larger soil database.

6. Conclusions

Results of series of unsaturated consolidated drained (CD) triaxial tests showed that ϕ^b angle of the soils with bimodal SWCC was equal to ϕ' at matric suctions less than or equal to the first air-entry value. ϕ^b angle of the soils with bimodal SWCC was constant, but less than ϕ' at suctions higher than the first air-entry value and less than the second air-entry value. As suctions beyond the second air-entry value, ϕ^b angle of the soils with bimodal SWCC started to vary with suction. The equations for estimating the unsaturated shear strength of soils with bimodal SWCC have been proposed and developed in this study. The proposed shear strength equation was evaluated using the soil data from the laboratory tests carried out in this study and published in the literature. The proposed equations were able to estimate the bimodal unsaturated shear strength successfully since ARE of the proposed equations were less than 10%. In addition, the parameter b of the proposed shear strength equation can be used to represent the ratio of ϕ^b/ϕ' for suctions between the first air-entry value and the second air-entry value.

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Table 1. Summary of index properties of soil mixtures used in the laboratory tests

Index Properties	M3	M4	M6	M8
Specific gravity, G _s	2.65	2.62	2.60	2.62
Liquid Limit, LL (%)	36.8	41.2	48.9	57.8
Plastic Limit, PL (%)	21.6	24.8	28.2	33.6
Dry density, ρ _d (Mg/m ³)	1.85	1.84	1.59	1.45
Water content, w (%)	5.0	6.0	11.5	18.0
Sand (%)	70.0	60.0	40.0	20.0
Silt (%)	25.3	32.8	52.4	72.8
Clay (%)	4.7	7.2	7.6	7.2
Soil classification according to USCS	SM	SM	MH	MH

Table 2. Fitting parameters of Satyanaga et al. (2013) equations for best fitting SWCC of soil specimens M3, M4, M6 and M8

Parameters	M3	M4	M6	M8
θ_{s1}	0.339	0.282	0.207	0.595
ψ_{a1} (kPa)	2	3	10	1
ψ_{m1} (kPa)	7	4	7.5	1.1
s_1	1.75	0.36	0.36	0.36
θ_{s2}	0.28	0.234	0.174	0.475
ψ_{a2} (kPa)	60	35	25	6
ψ_{m2} (kPa)	120	100	150	40
s_2	2	1	1	2.5
ψ_r (kPa)	600	500	500	400
θ_r	0.06	0.03	0.03	0.05

Table 3. Shear strength properties of specimens M3, M4, M6 and M8

Properties	M3	M4	M6	M8
c' (kPa)	5	8	12	16
ϕ' (°)	34	31	26	25
ϕ^b (°) for $u_a - u_w \leq AEV_1$	34	31	26	25
ϕ^b (°) for $AEV_1 < u_a - u_w \leq AEV_2$	29	26	13	6
ϕ^b (°) for $u_a - u_w > AEV_2$	< 20	< 17	< 7	< 5

Table 4. Soil data sets from published literatures used for evaluation of the proposed shear strength equation in this study (Equation 8)

Soil name	Saturated water content (%)	ρ (Mg/m ³)	LL (%)	Gravel (%)	Sand (%)	Silt (%)	Clay (%)	Source
Sandy Clay	25.7	1.79	46	0	35	26	39	Coutinho et al. (2011)
Clayey Silt	25.9	1.76	58	0	22	52	26	Jotisankasa et al. (2009)
Sandy Silt	34.7	1.68	42	0	40	57	3	Rahardjo et al. (2011)

Table 5. Best fitting parameters of the proposed shear strength developed in this study for soils M3, M4, M6, M8 and soils from published literatures

Parameters	M3	M4	M6	M8	Coutinho et al. (2011)	Jotisankasa et al. (2009)	Rahardjo et al. (2011)
AEV1	6	3	4	0.8	0.75	0.80	5
AEV2	50	25	30	9	100	50	200
b	0.89	0.72	0.57	0.23	0.57	0.32	0.69
q	0.42	0.34	0.43	0.10	0.23	0.21	0.14
f	0.23	0.21	0.19	0.66	0.46	0.31	0.46
ϕ^b/ϕ' ($AEV_1 < u_a - u_w < AEV_2$)	0.86	0.83	0.51	0.25	0.57	0.32	0.69
Average Relative Error (%)	3.4	4.8	3.9	3.7	3.5	8.0	1.5

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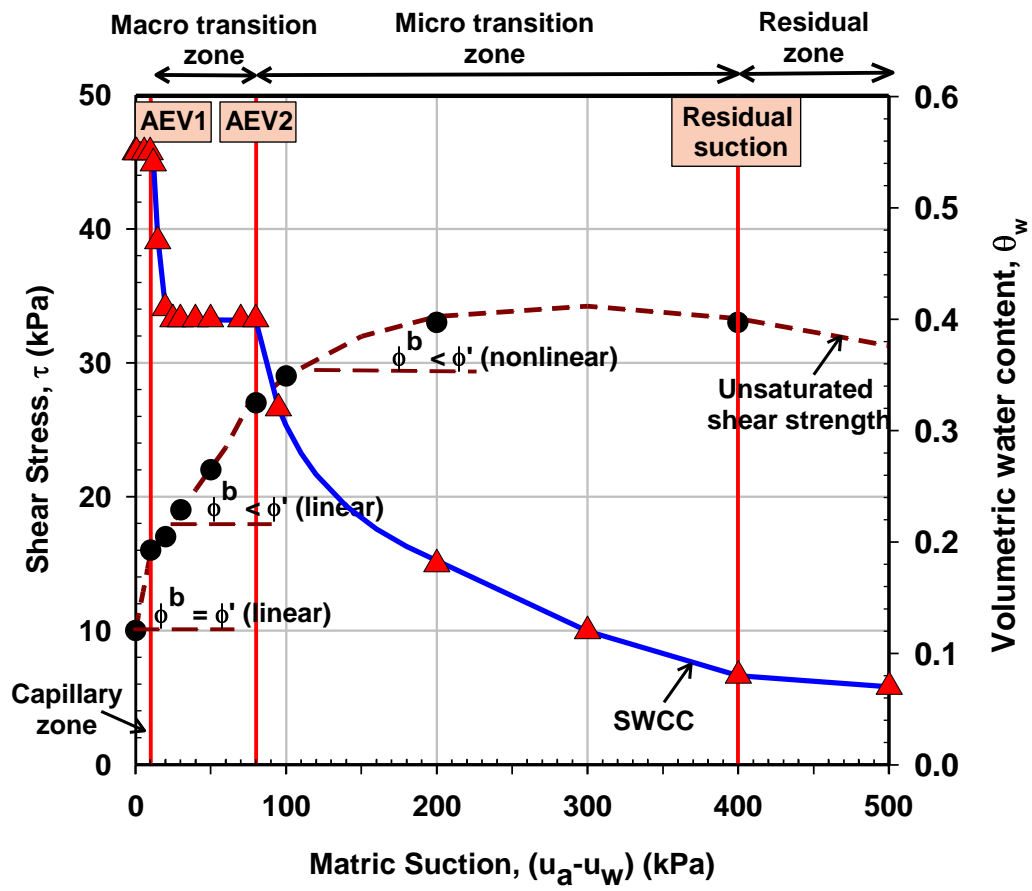


Figure 1. Proposed shear strength with bimodal characteristics of SWCC

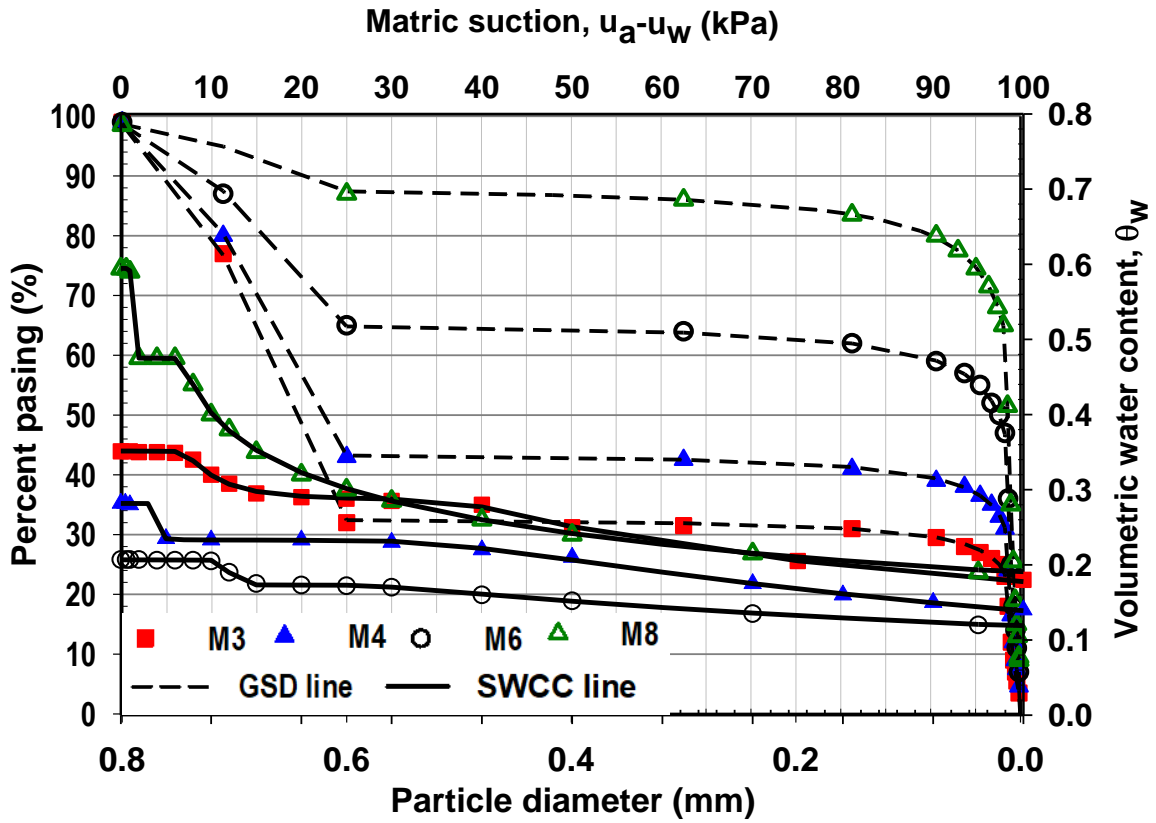


Figure 2. Grain-size distribution and soil-water characteristic curve of soil mixtures used in the laboratory tests

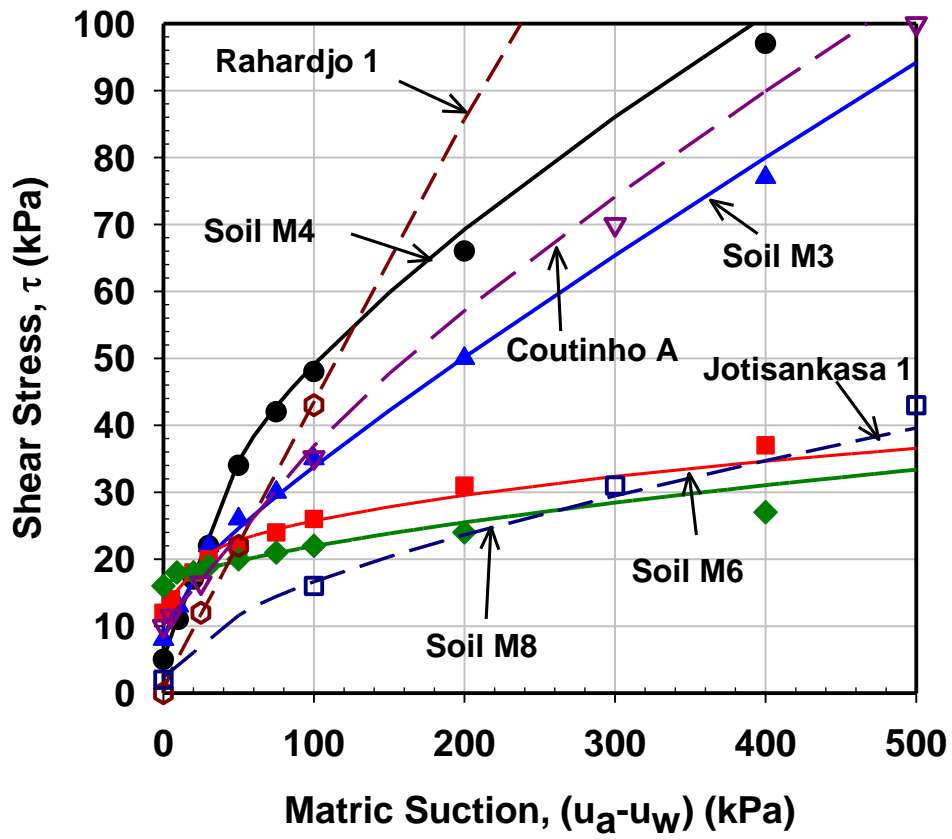


Figure 3. Comparison between the unsaturated bimodal shear strength estimated using the equation developed in this study and the unsaturated bimodal shear strength data of soil mixtures M3, M4, M6, M8 and soils from published literatures