Distributed Adaptive Control for Consensus Tracking with Application to Formation Control of Nonholonomic Mobile Robots

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Abstract

In this paper, we investigate the output consensus problem of tracking a desired trajectory for a class of systems consisting of multiple nonlinear subsystems with intrinsic mismatched unknown parameters. The subsystems are allowed to have non-identical dynamics, whereas with similar structures and the same yet arbitrary system order. Suppose that the communications among the subsystems can be represented by a directed graph. Different from the traditional centralized tracking control problem, only a subset of the subsystems can obtain the desired trajectory information directly. A distributed adaptive control approach based on backstepping technique is proposed. By introducing the estimates to account for the parametric uncertainties of the desired trajectory and its neighbors' dynamics into the local controller of each subsystem, the information exchanges of online parameter estimates and local synchronization errors among linked subsystems can be avoided. It is proved that the boundedness of all closed-loop signals and the asymptotically consensus tracking for all the subsystems' outputs are ensured. In simulation studies, a numerical example is illustrated to show the effectiveness of the proposed control scheme. Moreover, the design strategy is successfully applied to solve a formation control problem for multiple nonholonomic mobile robots.

Key words: Distributed coordination; Adaptive control; Consensus tracking; Nonlinear systems; Nonholonomic mobile robots.

1 Introduction

Because of its widespread potential applications in various fields such as mobile robot networks, intelligent transportation management, surveillance and monitoring, distributed coordination of multiple dynamic subsystems (also known as multi-agent systems) has achieved rapid development during the past decades. Consensus is one of the most popular topics in this area, which has received significant attention by numerous researchers. It is often aimed to achieve an agreement for certain variables of the subsystems in a group. A large number of effective control approaches have been proposed to solve the consensus problems (see Jadbabaie, Lin & Morse, 2003; Ren and Beard, 2005; Moreau, 2005; Hong, Hu & Gao, 2006; Ren, 2007; Arcak, 2007; Bai, Arcak & Wen, 2008, 2009, for instance). According to whether the desired consensus values are determined by exogenous inputs, which are sometimes regarded as virtual leaders, these approaches are often classified as leaderless consensus and leader-following consensus solutions (see Song, Cao & Yu, 2010; Kaizuka & T-sumura, 2011; Ni and Cheng, 2010, and the references therein). Besides, many of the early works were established for systems with first-order dynamics, whereas more results have been reported in recent years such as Ren, Moore & Chen (2007); Seo, Shim & Back (2009); Ni and Cheng (2010); Yu, Chen, Ren, Kurths & Zheng (2011) for systems with second or higher-order dynamics. A comprehensive overview of the state of the art in consensus control can be found in Ren & Cao (2010), in which the results on some other interesting topics including finite-time consensus and consensus under limited communication conditions including time delays,

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asynchronization and quantization are also discussed.

It is worth mentioning that except for Kaizuka & Tsumura (2011), all the aforementioned results are developed based on the assumptions that the considered model precisely represents the actual system and is exactly known. However, such assumptions are rather restrictive since model uncertainties, regardless of their forms, inevitably exist in almost all the control problems. Motivated by this fact, the intrinsic model uncertainty has become a new hot-spot issue in the area of consensus control. In Liu and Jia (2011); Hu (2011); Yang et al. (2011), robust control techniques are adopted in consensus protocols to address the intrinsic uncertainties including unknown parameters, unmodeled dynamics and exogenous disturbances. In addition, adaptive control has also been proved as a promising tool in dealing with such an issue. In Kaizuka & Tsumura (2011), a group of linear subsystems with unknown parameters are considered and a distributed model reference adaptive control (MRAC) strategy is proposed. Different from Liu and Jia (2011) where H_∞ control is investigated, the bounds of the unknown parameters are not required *a* priori by using adaptive control. However, the result is only applicable to the case that the control coefficient vectors of all the subsystems are the same and known. In Nuño, Ortega, Basañez & Hill (2011), adaptive consensus tracking controllers are designed for Euler-Lagrange swarm systems with non-identical dynamics, unknown parameters and communication delays. However, it is assumed that the exact knowledge of the desired trajectory is accessible for all the subsystems. In Das & Lewis (2010), a distributed neural adaptive control protocol is proposed for multiple first-order nonlinear subsystems with unknown nonlinear dynamics and disturbances. The state of the reference system is only available to a subset of the subsystems. Based on the condition that the basis neural network (NN) activation functions and the reference system dynamics are bounded, the convergence of the consensus errors to a bound can be ensured if the local control gains are chosen to be sufficiently large. The results are extended to more general class of systems with second and higher-order dynamics in Das & Lewis (2011) and Zhang & Lewis (2012). In Yu & Xia (2012), distributed adaptive control on first-order systems with similar structures to those in Das & Lewis (2010) is investigated. By introducing extra information exchange of local consensus errors among the linked agents, the assumptions on boundedness of inherent nonlinear functions can be relaxed. Apart from these, there are also some other results on distributed adaptive control of multi-agent systems, for instance Hou, Cheng & Tan (2009); Su, Chen, Wang & Lin (2011); Mei, Ren & Ma (2011); Zhao, Zhou, Li & Zhu (2011). Nevertheless, to the best of our knowledge, results on distributed adaptive consensus control of more general multiple high-order nonlinear systems are still limited. In Wang, Zhang & Guo (2011), output consensus tracking problem for nonlinear subsystems in the presence of mismatched unknown parameters is investigated. By designing an estimator whose dynamics is governed by a chain of n integrators for the desired trajectory in each subsystem, a bounded output consensus tracking for the overall system can be achieved. However, it is not easy to check whether the derived sufficient condition in the form of LMI is satisfied by choosing the design parameters properly. Moreover, transmissions of online parameter estimates among the neighbors are required, which may increase communication burden and also cause some other potential problems such as those related to network security.

In this paper, we shall present a backstepping based distributed adaptive consensus tracking control scheme for a class of nonlinear systems with mismatched uncertainties as similar to Wang et al. (2011). Suppose that only part of subsystems can acquire the exact information of the desired trajectory. Inspired by Bai et al. (2008, 2009); Yu and Xia (2012), the time-varying reference is assumed to be linearly parameterized. The main differences between our proposed scheme and the existing representative approaches can be summarized as follows. (i) We consider the case with directed graph representing the communication status among subsystems, thus the control protocols in Bai et al. (2008, 2009) by employing the graph symmetry property is not applicable to solve our problem. More discussions about this issue are given in later parts of the paper. (ii) The nonlinearities accompanied with unknown parameters in each subsystem's dynamics cannot be assumed bounded beforehand as those activation functions in Das and Lewis (2010, 2011); Zhang and Lewis (2012). To bypass this difficulty, an error variable by introducing local estimates of the reference' uncertainties is defined in each subsystem. Based on this, an alternative form of Lyapunov function is constructed. Then the coupling terms related to local consensus errors and the neighbors' parameter estimation errors can be eliminated in computing the derivative of the Lyapunov function. Moreover, the parameter update laws can be designed locally without information exchange of synchronization errors among subsystems as required in Yu and Xia (2012). (iii) By introducing additional estimates to account for the uncertainties involved in its neighbors' dynamics, the extra transmissions of online parameter estimates required in Wang et al. (2011) among linked subsystems can be avoided. It is shown that with the proposed distributed control scheme, not only the boundedness of all closed-loop signals is ensured, but also asymptotically consensus tracking of all the subsystems' outputs can be achieved with the proposed control scheme.

Apart from these, the proposed design strategy is successfully applied to solve a formation control problem for multiple nonholonomic mobile robots. Such a challenging problem can be regarded as a generalized problem of one-dimensional output consensus tracking by considering demanding distances on 2-D plane. Note that the considered robots are uncertain underactuated mechanical systems with both dynamic and kinematic models, which brings new difficulties in designing distributed adaptive controllers. Therefore, only a few results have been reported in this area so far. In Do and Pan (2007), formation control of multiple unicycle-type mobile robots at the dynamic model level is investigated. A path-following approach by combining the virtual structure technique is presented to derive the formation architecture. In Do (2008), a formation control scheme is proposed for multiple mobile robots and no collision between any two robots is guaranteed. In the two schemes, all the robots require the exact information of the reference trajectory. In Dong (2011), the flocking control of a collection of nonholonomic mobile robots is proposed, where only part of the robots can obtain the exact knowledge of the reference directly. However, the system model considered is limited at the kinematic level. Motivated by these, we investigate the formation control problem for multiple nonholonomic mobile robots at dynamic model level with unknown parameters under the assumption that only part of the robots can access the exact information of the reference directly. It is proved that the formation errors of the overall system can be made as small as desired by adjusting the design parameters properly with the combination of our proposed distributed control strategy and the transverse function technique Morin & Samson (2003).

2 Problem Formulation

Similar to Wang et al. (2011), we consider a group of N nonlinear subsystems which can be modeled as follows.

$$\begin{aligned}\dot{x}_{i,q} &= x_{i,q+1} + \varphi_{i,q}(x_{i,1},\ldots,x_{i,q})^T\theta_i, \quad q=1,\ldots,n-1\\ \dot{x}_{i,n} &= b_i\beta_i(x_i)u_i + \varphi_{i,n}(x_i)^T\theta_i\\ y_i &= x_{i,1}, \quad \text{for } i=1,2,\ldots,N \end{aligned}\tag{1}$$

where $x_i = [x_{i,1},\ldots,x_{i,n}]^T \in \Re^n$, $u_i \in \Re$, $y_i \in \Re$ are the state, control input and output of the ith subsystem, respectively. $\theta_i \in \Re^{p_i}$ is a vector of unknown constants and the high frequency gain $b_i \in \Re$ is an unknown nonzero constant. $\varphi_{i,j} : \Re^j \to \Re^{p_i}$ for $j = 1,\ldots,n$ and $\beta_i : \Re^n \to \Re^1$ are known smooth nonlinear functions.

The desired trajectory for the outputs of the overall system can be expressed by a linear combination of q_r basis functions, that is

$$y_r(t) = \sum_{l=1}^{q_r} f_{r,k}(t)w_{r,l} + c_r = f_r(t)^T w_r + c_r \tag{2}$$

where $f_r(t) = [f_{r,1}(t), f_{r,2}(t),\ldots,f_{r,q_r}(t)]^T \in \Re^{q_r}$ is the vector of basis functions which is available to all the N subsystems. However, $w_r = [w_{r,1}, w_{r,2},\ldots,w_{r,q_r}]^T \in \Re^{q_r}$ and $c_r \in \Re$ are constant parameters which are known only to part of N subsystems.

***Remark* 1** *It is worth mentioning that the trajectory given in (2) is a commonly employed expression which has appeared in many relevant literature such as Bai et al. (2008, 2009); Yu and Xia (2012). As we know, a function can be represented or approximated as a linear combination of a set of prescribed basis functions in a function space. For example, if a desired trajectory $y_r(t)$ is periodic with period T, then $y_r(t)$ can be written as $y_r(t) = a_0 + \sum_{k=1}^{\infty}\left(a_k \cos\frac{2\pi kt}{T} + b_k \sin\frac{2\pi kt}{T}\right)$. This is known as trigonometric form of the Fourier series, in which a_0, a_k and b_k are constants called Fourier coefficients (Keryszig, 2006). If there are only finite dominant frequency components in $y_r(t)$, in other words, the contributions of $a_k \cos\frac{2\pi kt}{T}$ and $b_k \sin\frac{2\pi kt}{T}$ are negligible for $k > K$, then $y_r(t)$ can be approximated well by $\hat{y}_r(t) = a_0 + \sum_{k=1}^{K}\left(a_k \cos\frac{2\pi kt}{T} + b_k \sin\frac{2\pi kt}{T}\right)$ which has a similar form as (2).*

***Remark* 2** *In contrast to the traditional centralized trajectory tracking control problem, not all subsystems in the group can obtain the exact knowledge of the trajectory y_r directly. However, the consensus tracking to y_r for all the N subsystems' outputs can still be expected if the subsystems are able to share information with some others in their neighboring areas via communication networks.*

Suppose that the communications among the N subsystems can be represented by a directed graph $\mathcal{G} \triangleq (\mathcal{V},\mathcal{E})$ where $\mathcal{V} = \{1,\ldots,N\}$ denotes the set of indexes (or vertices) corresponding to each subsystem, $\mathcal{E} \subseteq \mathcal{V}\times\mathcal{V}$ is the set of edges between two distinct subsystems. An edge $(i,j) \in \mathcal{E}$ indicates that subsystem j can obtain information from subsystem i, but not necessarily vice versa (Ren and Cao, 2010). In this case, subsystem i is called a neighbor of subsystem j. We denote the set of neighbors for subsystem i as $\mathcal{N}_i$. Self edges (i,i) is not allowed in this paper, thus $(i,i) \notin \mathcal{E}$ and $i \notin \mathcal{N}_i$. The connectivity matrix $A = [a_{ij}] \in \Re^{N\times N}$ is defined such that $a_{ij} = 1$ if $(j,i) \in \mathcal{E}$ and $a_{ij} = 0$ if $(j,i) \notin \mathcal{E}$. Clearly, the diagonal elements $a_{ii} = 0$. We introduce an in-degree matrix $\triangle$ such that $\triangle = \text{diag}(\triangle_i) \in \Re^{N\times N}$ with $\triangle_i = \sum_{j\in\mathcal{N}_i} a_{ij}$ being the ith row sum of A. Then, the Laplacian matrix of $\mathcal{G}$ is defined as $\mathcal{L} = \triangle - A$.

We now use $\mu_i = 1$ to indicate the case that y_r is accessible directly to subsystem i; otherwise, μ_i is set as $\mu_i = 0$. Based on this, the *control objective* is to design distributed adaptive controllers u_i for each subsystem by utilizing only locally available information obtained from the intrinsic subsystem and its neighbors such that:

(i) all the signals in the closed-loop system are globally uniformly bounded;

(ii) the outputs of all the overall system can still track the desired trajectory $y_r(t)$ asymptotically, i.e. $\lim_{t\to\infty}[y_i(t) - y_r(t)] = 0$, $\forall i$, though $\mu_i = 1$ only for some $i \in \{1,2,\ldots,N\}$.

To achieve the objective, the following assumptions are imposed.

Assumption 1 *The first nth-order derivatives of $f_r(t)$ are bounded, piecewise continuous and known to all subsystems in the group.*

Assumption 2 *The sign of b_i is available in constructing u_i for subsystem i and $\beta_i(x_i) \neq 0$.*

***Remark* 3** *Observing (1), the system model considered in this paper is similar to that in Wang et al. (2011). Such a model is more general than those in most of the currently available results on distributed consensus control including Hong et al. (2006); Arcak (2007); Bai et al. (2008, 2009); Ren (2007); Ren et al. (2007); Seo et al. (2009); Ni and Cheng (2010); Yu et al. (2011) by combining the following features: i) the subsystems are nonlinear and allowed to have non-identical dynamics; ii) intrinsic mismatched unknown parameters are involved. Moreover, (1) is in the parametric strict feedback form, which can be commonly encountered in many nonlinear control problems. In Krstic, Kanellakopoulos and Kokotovic (1995), the conditions that a class of general nonlinear systems $\dot{\chi} = f_0(\chi) + \sum_{l=1}^{p} \theta_l f_l(x) + g(\chi)u$, $y = h(\chi)$ are transformable into such form have been provided. It should be noted that Assumptions 1 and 2 are also required for standard backstepping tracking control of single input systems.*

3 Controller Design and Stability Analysis

3.1 Design of Distributed Adaptive Controllers

In this part, a distributed adaptive control scheme is proposed to achieve the control objective presented in Section 2. An additional assumption is needed.

Assumption 3 *The directed graph $\mathcal{G}$ contains a spanning tree and the root node i_l has direct access to y_r, i.e. $\mu_{i_l} = 1$.*

The following lemma brought from Zhang & Lewis (2012); Qu (2009) is then introduced, which will be useful in our design and analysis of the distributed adaptive controllers.

Lemma 1 *Based on Assumption 3, the matrix $(\mathcal{L} + \mathcal{B})$ is nonsingular where $\mathcal{B} = diag\{\mu_1, \ldots, \mu_N\}$. Define*

$$\begin{aligned} \bar{q} &= [\bar{q}_1, \ldots, \bar{q}_N]^T = (\mathcal{L} + \mathcal{B})^{-1}[1, \ldots, 1]^T \\ P &= diag\{P_1, \ldots, P_N\} = diag\left\{\frac{1}{\bar{q}_1}, \ldots, \frac{1}{\bar{q}_N}\right\} \\ Q &= P(\mathcal{L} + \mathcal{B}) + (\mathcal{L} + \mathcal{B})^T P, \end{aligned} \tag{3}$$

then $\bar{q}_i > 0$ for $i = 1, \ldots, N$ and Q is positive definite.

To achieve the output consensus tracking objective for multiple subsystems with arbitrary relative degrees, backstepping technique (Krstic et al., 1995) is adopted. Thus our design follows a step-by-step procedure. Due to page limits, only the first two steps are elaborated in details.

• *Step 1.* For subsystem i with $\mu_i = 0$, we introduce $\hat{\bar{w}}_{ri} = [\hat{w}_{ri}^T, \hat{c}_{ri}]^T$ to estimate the unknown parameters w_r and c_r. Then the following error variables are defined

$$e_{i,1} = y_i - \mu_i y_r - (1 - \mu_i)\bar{f}_r^T \hat{\bar{w}}_{ri} \tag{4}$$

$$e_{i,2} = x_{i,2} - \alpha_{i,1} \tag{5}$$

$$z_i = \sum_{j=1}^{N} a_{ij}(y_i - y_j) + \mu_i(y_i - y_r) \tag{6}$$

where $\bar{f}_r = [f_r^T, 1]^T$. Note that $\alpha_{i,1}$ is a virtual control to be chosen.

The actual tracking errors between each subsystems' outputs and y_r are defined as $\delta_i = y_i - y_r$, for $i = 1, \ldots, N$. Clearly, the *control objective* is to ensure that $\lim_{t\to\infty} \delta_i(t) = 0$ for all subsystems in the group.

Eqn. (6) is a standard definition of the local neighborhood consensus error for the ith subsystem (Das and Lewis, 2010). By defining $z = [z_1, \ldots, z_N]^T$, we have

$$z = (\mathcal{L} + \mathcal{B})\,\delta \tag{7}$$

where $\delta = [\delta_1, \ldots, \delta_N]^T$.

From (2) and (4), there is

$$\begin{aligned} e_{i,1} &= y_i - y_r + (1 - \mu_i)\left(y_r - \bar{f}_r^T \hat{\bar{w}}_{ri}\right) \\ &= \delta_i + (1 - \mu_i)\bar{f}_r^T \tilde{\bar{w}}_{ri} \end{aligned} \tag{8}$$

where $\tilde{\bar{w}}_{ri}$ denotes the estimation error for subsystems with $\mu_i = 0$ such that $\tilde{\bar{w}}_{ri} = [w_r^T, c_r]^T - \hat{\bar{w}}_{ri}$.

From (1) and (4), the derivative of $e_{i,1}$ is computed as

$$\begin{aligned} \dot{e}_{i,1} = {} & \alpha_{i,1} + e_{i,2} + \varphi_{i,1}^T \theta_i - \mu_i \dot{f}_r^T w_{ri} \\ & - (1 - \mu_i)\left(\dot{f}_r^T \hat{w}_{ri} - \bar{f}_r^T \dot{\hat{\bar{w}}}_{ri}\right). \end{aligned} \tag{9}$$

We design $\alpha_{i,1}$ as

$$\begin{aligned} \alpha_{i,1} = {} & -k_1 P_i z_i - \varphi_{i,1}^T \hat{\theta}_i + \mu_i \dot{f}_r^T w_{ri} \\ & + (1 - \mu_i)\left(\dot{f}_r^T \hat{w}_{ri} - \bar{f}_r^T \dot{\hat{\bar{w}}}_{ri}\right) \end{aligned} \tag{10}$$

where k_1 is a positive constant, P_i is defined in (3) and $\hat{\theta}_i$ is the parameter estimate of θ_i. Substituting (10) into (9) yields

$$\dot{e}_{i,1} = -k_1 P_i z_i + e_{i,2} + \varphi_{i,1}^T \tilde{\theta}_i. \tag{11}$$

We define a Lyapunov function candidate at this step as

$$V_1 = \frac{1}{2} e_1^T e_1 + \frac{1}{2} \sum_{i=1}^{N} \tilde{\theta}_i^T \Gamma_i^{-1} \tilde{\theta}_i + \frac{k_1}{2} \sum_{i=1}^{N} (1-\mu_i) P_i \tilde{\bar{w}}_{ri}^T \Gamma_{ri}^{-1} \tilde{\bar{w}}_{ri} \tag{12}$$

where $e_1 = [e_{1,1}, \ldots, e_{N,1}]^T$, $\tilde{\theta}_i = \theta_i - \hat{\theta}_i$. Γ_i and Γ_{ri} are positive definite matrices with appropriate dimensions. From (5,)(7) and (11), the derivative of V_1 is computed as

$$\begin{aligned}
\dot{V}_1 = & -k_1 \delta^T P \left(\mathcal{L} + \mathcal{B}\right) \delta + \sum_{i=1}^{N} \Big[e_{i,1} e_{i,2} + \tilde{\theta}_i^T \Gamma_i^{-1} \\
& \times \left(\Gamma_i \varphi_{i,1} e_{i,1} - \dot{\hat{\theta}}_i \right) \Big] + k_1 \sum_{i=1}^{N} (1-\mu_i) p_i \tilde{\bar{w}}_{ri}^T \Gamma_{ri}^{-1} \\
& \times \left(-\Gamma_{ri} \bar{f}_r z_i - \dot{\hat{\bar{w}}}_{ri} \right).
\end{aligned} \tag{13}$$

We then choose the parameter update law for $\dot{\hat{\bar{w}}}_{ri}$ if $\mu_i = 0$ as

$$\dot{\hat{\bar{w}}}_{ri} = -\Gamma_{ri} \bar{f}_r z_i. \tag{14}$$

Defining $\tau_{i,1} = \varphi_{i,1} e_{i,1}$ and substituting (14) into (13), we have

$$\begin{aligned}
\dot{V}_1 = & -\frac{k_1}{2} \delta^T \left[P(\mathcal{L} + \mathcal{B}) + (\mathcal{L} + \mathcal{B})^T P \right] \delta \\
& + \sum_{i=1}^{N} \left[e_{i,1} e_{i,2} + \tilde{\theta}_i^T \Gamma_i^{-1} \left(\Gamma_i \tau_{i,1} - \dot{\hat{\theta}}_i \right) \right] \\
= & -\frac{k_1}{2} \delta^T Q \delta + \sum_{i=1}^{N} \left[e_{i,1} e_{i,2} + \tilde{\theta}_i^T \Gamma_i^{-1} \left(\Gamma_i \tau_{i,1} - \dot{\hat{\theta}}_i \right) \right].
\end{aligned} \tag{15}$$

where Q is defined in Lemma 1.

• *Step 2.* We now clarify the arguments of $\alpha_{i,1}$. By examining (10) along with (6) and (14), it can be seen that $\alpha_{i,1}$ is a function of y_i, $\hat{\theta}_i$, f_r, $\dot{f}_r$, y_j (if $a_{ij} = 1$) and $\hat{\bar{w}}_{ri}$ (if $\mu_i = 0$). Introduce a new error variable as

$$e_{i,3} = x_{i,3} - \alpha_{i,2} \tag{16}$$

where $\alpha_{i,2}$ is chosen as

$$\begin{aligned}
\alpha_{i,2} = & -e_{i,1} - k_{i,2} e_{i,2} + \frac{\partial \alpha_{i,1}}{\partial x_{i,1}} x_{i,2} + \frac{\partial \alpha_{i,1}}{\partial \hat{\theta}_i} \Gamma_i \tau_{i,2} \\
& - \left(\varphi_{i,2} - \frac{\partial \alpha_{i,1}}{\partial x_{i,1}} \varphi_{i,1} \right)^T \hat{\theta}_i \\
& + \sum_{j=1}^{N} a_{ij} \frac{\partial \alpha_{i,1}}{\partial x_{j,1}} \left(x_{j,2} + \varphi_{j,1}^T \hat{\theta}_{ij} \right) + \frac{\partial \alpha_{i,1}}{\partial f_r} \dot{f}_r \\
& + \frac{\partial \alpha_{i,1}}{\partial \dot{f}_r} \ddot{f}_r + (1-\mu_i) \frac{\partial \alpha_{i,1}}{\partial \hat{\bar{w}}_{ri}} \dot{\hat{\bar{w}}}_{ri}.
\end{aligned} \tag{17}$$

with $k_{i,2}$ a positive constant. $\tau_{i,2}$ is the tuning function defined as follows for generating $\dot{\hat{\theta}}_i$ that

$$\tau_{i,2} = \tau_{i,1} + \left(\varphi_{i,2} - \frac{\partial \alpha_{i,1}}{\partial x_{i,1}} \varphi_{i,1} \right) e_{i,2}. \tag{18}$$

$\hat{\theta}_{ij}$ is an estimator introduced in subsystem i to account for the unknown parameter vector contained in its neighbors' dynamics (i.e. θ_j if $a_{ij} = 1$).

From (5), (16)-(18), the derivative of $e_{i,2}$ is computed as

$$\begin{aligned}
\dot{e}_{i,2} = & -e_{i,1} - k_{i,2} e_{i,2} + e_{i,3} + \left(\varphi_{i,2} - \frac{\partial \alpha_{i,1}}{\partial x_{i,1}} \varphi_{i,1} \right)^T \tilde{\theta}_i \\
& + \frac{\partial \alpha_{i,1}}{\partial \hat{\theta}_i} \left(\Gamma_i \tau_{i,2} - \dot{\hat{\theta}}_i \right) - \sum_{j=1}^{N} a_{ij} \frac{\partial \alpha_{i,1}}{\partial x_{j,1}} \varphi_{j,1}^T \tilde{\theta}_{ij}
\end{aligned} \tag{19}$$

Define a Lyapunov function candidate V_2 at this step as

$$V_2 = V_1 + \sum_{i=1}^{N} e_{i,2}^2 + \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} a_{ij} \tilde{\theta}_{ij}^T \Gamma_{ij}^{-1} \tilde{\theta}_{ij} \tag{20}$$

where $\tilde{\theta}_{ij} = \theta_j - \hat{\theta}_{ij}$ and Γ_{ij} is a positive definite matrix. From (15) and (19), we obtain that

$$\begin{aligned}
\dot{V}_2 = & -\frac{k}{2} \delta^T Q \delta + \sum_{i=1}^{N} \Bigg[-k_{i,2} e_{i,2}^2 + e_{i,2} e_{i,3} \\
& + \tilde{\theta}_i^T \Gamma_i^{-1} \left(\Gamma_i \tau_{i,2} - \dot{\hat{\theta}}_i \right) + e_{i,2} \frac{\partial \alpha_{i,1}}{\partial \hat{\theta}_i} \left(\Gamma_i \tau_{i,2} - \dot{\hat{\theta}}_i \right) \\
& + \sum_{j=1}^{N} a_{ij} \tilde{\theta}_{ij}^T \Gamma_{ij}^{-1} \left(\Gamma_{ij} \bar{\tau}_{ij,1} - \dot{\hat{\theta}}_{ij} \right) \Bigg]
\end{aligned} \tag{21}$$

where $\bar{\tau}_{ij,1}$ is defined as

$$\bar{\tau}_{ij,1} = -\frac{\partial \alpha_{i,1}}{\partial x_{j,1}} \varphi_{j,1} e_{i,2}, \quad \text{if} \quad a_{ij} = 1. \tag{22}$$

• *Step q* ($q = 3, \ldots, n$). For easier reading, the design details of the remaining steps are summarized in Table 1. Note that $k_{i,q}$ and γ_i are positive constants. $\hat{\varrho}_i$ is the estimate of $\varrho = 1/b_i$,.

Remark 4 *In Table 1, the fact that $\alpha_{i,q}$ for $q = 3, \ldots, n$ is a function of $x_{i,1}, \ldots, x_{i,q}$, $\hat{\theta}_i$, f_r, $f_r^{(1)}, \ldots, f_r^{(q-1)}$ and $\hat{\theta}_{ij}$, $x_{j,1}, \ldots, x_{j,q}$ (if $a_{ij} = 1$), $\hat{\bar{w}}_{ri}$ (if $\mu_i = 0$) has been used.*

3.2 Stability Analysis

The main results of our distributed adaptive control design scheme can be formally stated in the following theorem.

Table 1: The design of distributed adaptive controllers for Step q ($q = 3, \ldots, n$).

Introduce error variables:

$$e_{i,q+1} = x_{i,q+1} - \alpha_{i,q} \tag{23}$$

Control Laws:

$$u_i = \frac{\hat{\varrho}_i}{\beta_i(x_i)}\alpha_{i,n} \tag{24}$$

with

$$\begin{aligned}\alpha_{i,q} &= -e_{i,q-1} - k_{i,q}e_{i,q} - \zeta_{i,q}^T\hat{\theta}_i + \frac{\partial\alpha_{i,q-1}}{\partial\hat{\theta}_i}\Gamma_i\tau_{i,q} \\ &+ \sum_{l=1}^{q-1}\frac{\partial\alpha_{i,q-1}}{\partial x_{i,l}}x_{i,l+1} + \left(\sum_{l=2}^{q-1}\frac{\partial\alpha_{i,l-1}}{\partial\hat{\theta}_i}\right)\Gamma_i\zeta_{i,q}e_{i,l} \\ &+ \sum_{j=1}^{N}a_{ij}\left[\sum_{l=1}^{q-1}\frac{\partial\alpha_{i,q-1}}{\partial x_{j,l}}x_{j,l+1} + \bar{\zeta}_{ij,q-1}^T\hat{\theta}_{ij}\right. \\ &\left.+ \frac{\partial\alpha_{i,q-1}}{\partial\hat{\theta}_{ij}}\Gamma_{ij}\bar{\tau}_{ij,q-1} - \sum_{l=3}^{q-1}\frac{\partial\alpha_{i,l-1}}{\partial\hat{\theta}_{ij}}\Gamma_{ij}\bar{\zeta}_{ij,q-1}e_{i,l}\right] \\ &+ \sum_{l=1}^{q}\frac{\partial\alpha_{i,l}}{\partial f_r^{(l-1)}}f_r^{(l)} + (1-\mu_i)\frac{\partial\alpha_{i,q-1}}{\partial\hat{\bar{w}}_{ri}}\dot{\hat{\bar{w}}}_{ri}\end{aligned} \tag{25}$$

$$\zeta_{i,q} = \varphi_{i,q} - \sum_{l=1}^{q-1}\frac{\partial\alpha_{i,q-1}}{\partial x_{i,l}}\varphi_{i,l} \tag{26}$$

$$\bar{\zeta}_{ij,q-1} = \sum_{l=1}^{q-1}\frac{\partial\alpha_{i,q-1}}{\partial x_{j,l}}\varphi_{j,l} \tag{27}$$

$$\tau_{i,q} = \tau_{i,q-1} + \zeta_{i,q}e_{i,q} \tag{28}$$

$$\bar{\tau}_{ij,q-1} = \bar{\tau}_{ij,q-2} - \bar{\zeta}_{ij,q-1}e_{i,q} \tag{29}$$

Parameter Estimators:

$$\dot{\hat{\varrho}}_i = -\gamma_i\mathrm{sgn}(b_i)\alpha_{i,n}e_{i,n} \tag{30}$$

$$\dot{\hat{\theta}}_i = \Gamma_i\tau_{i,n} \tag{31}$$

$$\dot{\hat{\theta}}_{ij} = \Gamma_{ij}\bar{\tau}_{i,n-1} \tag{32}$$

Theorem 1 *Consider the closed-loop adaptive system consisting of N uncertain nonlinear subsystems (1) satisfying Assumptions 1-3, the local controllers (24) and the parameter estimators (14), (30)-(32). All the signals in the closed-loop system are globally uniformly bounded and asymptotic consensus tracking of all the subsystems' outputs to $y_r(t)$ is achieved, i.e. $\lim_{t\to\infty}[y_i(t) - y_r(t)] = 0$ for $i = 1, \ldots, N$.*

Proof: We define the Lyapunov function for the overall system as

$$V_n = V_2 + \frac{1}{2}\sum_{i=1}^{N}\left(\sum_{q=3}^{n}e_{i,q}^2 + \frac{|b_i|}{\gamma_i}\tilde{\varrho}_i^2\right) \tag{33}$$

where $\tilde{\varrho}_i = \varrho_i - \hat{\varrho}_i$. From (21)-(29), the derivative of V_n is computed as

$$\begin{aligned}\dot{V}_n = &-\frac{k}{2}\delta^TQ\delta + \sum_{i=1}^{N}\left[-\sum_{l=2}^{n}k_{i,l}e_{i,l}^2 + \tilde{\theta}_i^T\Gamma_i^{-1}\left(\Gamma_i\tau_{i,n}\right.\right. \\ &\left.-\dot{\hat{\theta}}_i\right) + \left(\sum_{l=1}^{n}e_{i,l}\frac{\partial\alpha_{i,l-1}}{\partial\hat{\theta}_i}\right)\left(\Gamma_i\tau_{i,n} - \dot{\hat{\theta}}_i\right) \\ &+ \sum_{j=1}^{N}a_{ij}\tilde{\theta}_{ij}^T\Gamma_{ij}^{-1}\left(\Gamma_{ij}\bar{\tau}_{ij,n-1} - \dot{\hat{\theta}}_{ij}\right) \\ &+ \sum_{j=1}^{N}a_{ij}\left(\sum_{l=1}^{n}e_{i,l}\frac{\partial\alpha_{i,l-1}}{\partial\hat{\theta}_{ij}}\right)\left(\Gamma_{ij}\bar{\tau}_{ij,n-1} - \dot{\hat{\theta}}_{ij}\right) \\ &\left.+ \frac{|b_i|}{\gamma_i}\tilde{\varrho}_i\left(-\dot{\hat{\varrho}}_i - \gamma_i\mathrm{sgn}(b_i)\alpha_{i,n}\right)\right]\end{aligned} \tag{34}$$

Based on Assumption 3 and Lemma 1, Q is positive definite. Thus by choosing the parameter update laws as (30)-(32), $\dot{V}_n$ can be rendered negative definite such that

$$\dot{V}_n = -\frac{k}{2}\delta^TQ\delta - \sum_{i=1}^{N}\sum_{k=2}^{n}k_{i,l}e_{i,l}^2 \tag{35}$$

From the definition of V_n in (33) along with (12) and (20), we establish that $e_{i,l}$ for $l = 1, \ldots, n$, $\hat{\theta}_i$, $\hat{\theta}_{ij}$, $\hat{\varrho}_i$ and $\hat{\bar{w}}_{ri}$ are bounded for all subsystem i. From (4) and the boundedness of f_r, we obtain that y_i, i.e. $x_{i,1}$ for $i = 1, \ldots, N$ are bounded. From (7) and $\delta = y - \underline{y}_r$, z is also bounded. From (10) and the smoothness of φ_i, $\alpha_{i,1}$ for $i = 1, \ldots, N$ are bounded. From the definition of $e_{i,2}$ in (5), it follows that $e_{i,2}$ is bounded. By following similar procedure, the boundedness of $\alpha_{i,q}$ and $x_{i,q}$ for $q = 3, \ldots, n$ is ensured. From (24), we can conclude that the control signal u_i is bounded. Thus the boundedness of all the signals in the closed-loop adaptive systems is guaranteed. By applying the LaSalle-Yoshizawa theorem, it further follows that $\lim_{t\to\infty}\delta_i(t) = 0$ for $i = 1, \ldots, N$ and $q = 1, \ldots, n$. This implies that asymptotic consensus tracking of all the N subsystems' outputs to a desired trajectory $y_r(t)$ is also achieved, i.e. $\lim_{t\to\infty}[y_i(t) - y_r(t)] = 0$ for $i = 1, \ldots, N$. □

Remark 5 *The distributed adaptive control scheme presented in this section is also applicable to the following two cases if there is at least one subsystem in $\mathcal{G}$ has direct access to y_r. i) The graph $\mathcal{G}$ is undirected and connected; ii) The graph $\mathcal{G}$ is directed, balanced and strongly connected. This is because under these two cases, matrices $(\mathcal{L}+\mathcal{B})$ and $(\mathcal{L} + \mathcal{L}^T + 2\mathcal{B})$ are symmetric positive definite, respectively (Ren and Cao, 2010; Zhang and Lewis, 2012).*

Thus by modifying $\alpha_{i,1}$ in (10) with P_i chosen as $P_i = 1$, it can be shown that the control objective is achieved by following similar analysis in the proof of Theorem 1.

***Remark* 6** *Similar to Wang et al. (2011), in constructing the local controller u_i with our proposed method as presented in Steps 1, 2 and Table 1, $\varphi_{j,l}$ involving the structural knowledge of its neighbors' intrinsic dynamics needs to be collected if $a_{ij} = 1$. However, in contrast to Wang et al. (2011), by introducing $\hat{\theta}_{ij}$ in subsystem i to estimate the uncertain parameters (i.e. θ_j) contained in its neighbors's dynamics, the information exchange of local parameter estimates among linked subsystems is avoided and the communication burden can thus be reduced. Moreover, no other conditions such as LMI are required to achieve the main results than Assumptions 1-3 which are checkable.*

***Remark* 7** *The idea of introducing local estimators for unknown trajectory parameters is analogous to Bai et al. (2009), in which the adaptive controllers are designed based on passivity framework Arcak (2007). Similar to Bai et al. (2009), the consensus tracking is achieved directly no matter whether $\hat{\varpi}_{ri}$ will converge to $[w_r^T, c_r]^T$. Nevertheless, the coordination method presented in Bai et al. (2009) is not applicable to directed graph as in our paper. This can be observed from (40) in Bai et al. (2009). Since the symmetry property of undirected graphs cannot hold for directed graphs even when the graphs are balanced and strongly connected, the first term in the local control law u_i in Bai et al. (2009) actually involves information from the subsystems $j \notin \mathcal{N}_i$.*

3.3 An Illustrative Example

We now consider an example to illustrate the proposed design schemes and verify the established theoretical results. Suppose that there are a group of 4 nonlinear subsystems with the following dynamics

$$\begin{aligned} \dot{x}_{i,1} &= x_{i,2} + \varphi_{i,1}(x_{i,1})\theta_i \\ \dot{x}_{i,2} &= b_i u_i + \varphi_{i,2}(x_{i,1}, x_{i,2})\theta_i, \qquad i = 1, \ldots, 4 \end{aligned} \tag{36}$$

$\varphi_{i,1} = \sin(x_{i,1})$, $\varphi_{1,2} = x_{1,2}^3$, $\varphi_{2,2} = x_{2,2}^2$, $\varphi_{3,2} = x_{3,2}$, $\varphi_{4,2} = x_{4,1}x_{4,2}$. $\theta_1 = 1$, $\theta_2 = 0.5$, $\theta_3 = -2$, $\theta_4 = -3$. $b_1 = 1$, $b_2 = -2$, $b_3 = 0.5$, $b_4 = 3$. The communication topology for these 4 subsystems is given in Fig. 1. The desired trajectory is given by $y_r(t) = \sin(t)$. In

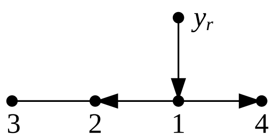

Fig. 1. Communication topology for a group of 4 nonlinear subsystems.

simulation, all the state initials are set as zero except that $x_{1,1}(0) = 1$, $x_{2,1}(0) = 0.5$, $x_{4,1}(0) = -0.9$ and $x_{5.1}(0) = -1.2$. Besides, the design parameters are chosen as $k_1 = 2$, $k_{i,2} = 1$, $\gamma_i = \Gamma_i = \Gamma_{ri} = 1$ for $1 \le i \le 4$. The adaptive gains $\Gamma_{21} = \Gamma_{23} = \Gamma_{32} = \Gamma_{41} = 1$. The tracking errors ($\delta_i$) and control inputs ($u_i$) for all the subsystems are shown in Fig. 2 and Fig. 3, respectively. It can be seen that asymptotically consensus tracking is achieved and all the input signals are bounded with the proposed distributed adaptive control scheme.

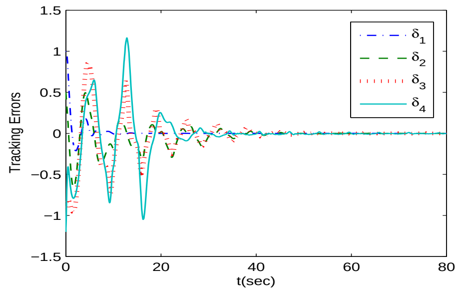

Fig. 2. Tracking errors $\delta_i = y_i - y_r$ for $1 \le i \le 4$.

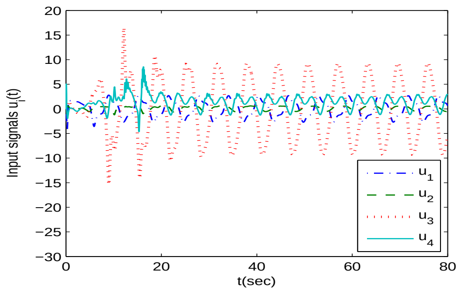

Fig. 3. Input signals $u_i(t)$ for $1 \le i \le 4$.

4 Application to Formation Control of Nonholonomic Mobile Robots

In this section, we shall apply the distributed adaptive tracking control strategy in Section 3 to solve a formation control problem for multiple nonholonomic mobile robots at dynamic model level with unknown parameters.

4.1 Robot Dynamics

We consider a group of N two-wheeled mobile robots as shown in Fig. 4. For the ith robot, point P_{ic} represents the center of the mass, P_{io} is the middle point located on the straight line connecting the left and right wheels and ι_i denotes the distance between P_{ic} and P_{io}. b_i is the half width of the robot, r_i is the radius of the wheel. OXY is the earth-fixed coordinate system. $(\bar{x}_i, \bar{y}_i, \bar{\phi}_i)$ denotes the position and orientation of the ith robot.

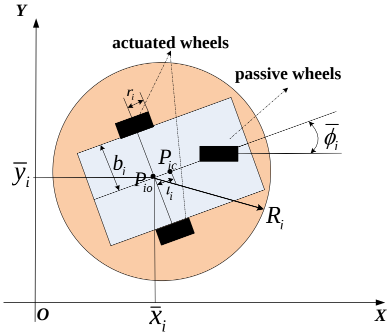

Fig. 4. A schematic diagram of the ith mobile robot.

According to Do and Pan (2007), the ith mobile robot can be described by the following dynamic model

$$\dot{\eta}_i = J(\eta_i)\omega_i \tag{37}$$

$$M_i\dot{\omega}_i + C_i(\dot{\eta}_i)\omega_i + D_i\omega_i = \tau_i, \quad \text{for } i = 1, \ldots, N \tag{38}$$

where $\eta_i = [\bar{x}_i, \bar{y}_i, \bar{\phi}_i]^T$, $\omega_i = [\omega_{i1}, \omega_{i2}]^T$ denotes the angular velocities of the left and right wheels, $\tau_i = [\tau_{i1}, \tau_{i2}]^T$ represents the control torques applied to the wheels. M_i is a symmetric, positive definite inertia matrix, $C_i(\dot{\eta}_i)$ is the centripetal and coriolis matrix, D_i denotes the surface friction. These matrices have the same form as those in Do and Pan (2007), which are given below for completeness.

$$J(\eta_i) = \frac{r_i}{2}\begin{bmatrix} \cos\bar{\phi}_i & \cos\bar{\phi}_i \\ \sin\bar{\phi}_i & \sin\bar{\phi}_i \\ b_i^{-1} & -b_i^{-1} \end{bmatrix}, \quad M_i = \begin{bmatrix} m_{i1} & m_{i2} \\ m_{i2} & m_{i1} \end{bmatrix}$$

$$C_i(\dot{\eta}_i) = \begin{bmatrix} 0 & c_i\dot{\bar{\phi}}_i \\ -c_i\dot{\bar{\phi}}_i & 0 \end{bmatrix}, \quad D_i = \begin{bmatrix} d_{i1} & 0 \\ 0 & d_{i2} \end{bmatrix}$$

$$\begin{aligned} m_{i1} &= \frac{1}{4}b_i^{-2}r_i^2(m_i b_i^2 + I_i) + I_{wi} \\ m_{i2} &= \frac{1}{4}b_i^{-2}r_i^2(m_i b_i^2 - I_i) \\ I_i &= m_{ci}\iota_i^2 + 2m_{wi}b_i^2 + I_{ci} + 2I_{mi} \\ c_i &= \frac{1}{2}b_i^{-1}r_i^2 m_{ci}\iota_i, \quad m_i = m_{ci} + 2m_{wi} \end{aligned} \tag{39}$$

In (39), m_{ci}, m_{wi}, I_{ci}, I_{wi}, I_{mi} and d_{ik} are unknown system parameters of which the physical meanings can be found in Do and Pan (2007).

***Remark* 8** *Observing (37) with $J(\eta_i)$ in (39), it can be seen that the number of inputs (i.e. ω_{i1} and ω_{i2}) is less than the number of configurations (i.e. $\bar{x}_i$, $\bar{y}_i$ and $\bar{\phi}_i$). Thus the considered mobile robot is an underactuated mechanical system. To achieve the tracking objectives of $\bar{x}_i$, $\bar{y}_i$ and $\bar{\phi}_i$ separately, transverse function approach (Morin and Samson, 2003) will be employed. An auxiliary manipulated variable will be introduced with which the underactuated problem can be transformed to a fully-actuated one.*

4.1.1 Change of Coordinates

We change the original coordinates of the ith robot as follows:

$$\begin{bmatrix} x_i \\ y_i \end{bmatrix} = \begin{bmatrix} \bar{x}_i \\ \bar{y}_i \end{bmatrix} + R(\phi_i)\begin{bmatrix} f_{1i}(\xi_i) \\ f_{2i}(\xi_i) \end{bmatrix} \tag{40}$$

$$\phi_i = \bar{\phi}_i - f_{3i}(\xi_i) \tag{41}$$

where

$$R(\phi_i) = \begin{bmatrix} \cos(\phi_i) & -\sin(\phi_i) \\ \sin(\phi_i) & \cos(\phi_i) \end{bmatrix} \tag{42}$$

and $f_{li}(\xi_i)$ for $l = 1, 2, 3$ are functions of ξ_i designed as

$$\begin{aligned} f_{1i}(\xi_i) &= \varepsilon_{1i}\sin(\xi_i)\frac{\sin(f_{3i})}{f_{3i}} \\ f_{2i}(\xi_i) &= \varepsilon_{1i}\sin(\xi_i)\frac{1 - \cos(f_{3i})}{f_{3i}} \\ f_{3i}(\xi_i) &= \varepsilon_{2i}\cos(\xi_i) \end{aligned} \tag{43}$$

where ε_{1i} and ε_{2i} are positive constants and ε_{2i} satisfies $0 < \varepsilon_{2i} < \frac{\pi}{2}$. The following properties can be shown.

$$|f_{1i}| < \varepsilon_{1i}, |f_{2i}| < \varepsilon_{1i}, |f_{3i}| < \varepsilon_{2i}. \tag{44}$$

Computing the derivatives of x_i, y_i and ϕ_i yields that

$$\begin{aligned} \begin{bmatrix} \dot{x}_i \\ \dot{y}_i \end{bmatrix} &= Q_i\begin{bmatrix} r_i u_{i1} \\ \dot{\xi}_i \end{bmatrix} + \frac{\partial R(\phi_i)}{\partial \phi_i}\begin{bmatrix} f_{1i}(\xi_i) \\ f_{2i}(\xi_i) \end{bmatrix} \\ &\quad \times \left(r_i b_i^{-1}u_{i2} - \frac{\partial f_{3i}(\xi_i)}{\partial \xi_i}\dot{\xi}_i \right) \end{aligned} \tag{45}$$

$$\dot{\phi}_i = r_i b_i^{-1}u_{i2} - \frac{\partial f_{3i}(\xi_i)}{\partial \xi_i}\dot{\xi}_i \tag{46}$$

where $u_{i1} = 0.5(\omega_{i1} + \omega_{i2})$ and $u_{i2} = 0.5(\omega_{i1} - \omega_{i2})$.

$$Q_i = \left[\begin{pmatrix} \cos(\bar{\phi}_i) \\ \sin(\bar{\phi}_i) \end{pmatrix} \quad R(\phi_i)\begin{pmatrix} \frac{\partial f_{1i}(\xi_i)}{\partial \xi_i} \\ \frac{\partial f_{2i}(\xi_i)}{\partial \xi_i} \end{pmatrix} \right] \tag{47}$$

is ensured to be invertible (Morin and Samson, 2003). Different from $(\bar{x}_i, \bar{y}_i, \bar{\phi}_i)$, the transformed coordinates (x_i, y_i and ϕ_i) can be controlled separately by tuning u_{i1}, u_{i2} and $\dot{\xi}_i$ which is deemed as an auxiliary manipulated variable.

4.1.2 Formation Control Objective

The components of the desired trajectory in X and Y directions can be expressed as

$$x_r(t) = w_r f_{rx}(t) + c_{rx} \quad y_r(t) = w_r f_{ry}(t) + c_{ry}. \tag{48}$$

Similar to Section 3, it is assumed that $f_{rx}(t)$ and $f_{ry}(t)$ are known by all the robots, whereas the parameters w_r,

c_{rx} and c_{ry} are only available to part of the robots. Besides, $\phi_r(t) \triangleq \arctan\left(\frac{\dot{y}_r}{\dot{x}_r}\right)$ denotes the reference trajectory for the orientation of each robot.

The *control objective* in this section is to design distributed adaptive formation controllers such that all the robots can follow a desired trajectory in X-Y plane by maintaining certain prescribed demanding distances from the desired trajectory, i.e.

$$\lim_{t\to\infty} x_i(t) - x_r(t) = -\rho_{ix} \tag{49}$$

$$\lim_{t\to\infty} y_i(t) - y_r(t) = -\rho_{iy} \tag{50}$$

$$\lim_{t\to\infty} \phi_i(t) - \phi_r(t) = 0. \tag{51}$$

Similar to Section 2, we suppose that the communication status among the N robots can be represented by a directed graph $\mathcal{G}$ and Assumption 3 holds. To achieve the formation control objective, the following assumptions are also needed.

Assumption 4 *f_{rx}, f_{ry}, $\dot{f}_{rx}$, $\dot{f}_{ry}$ and $\ddot{f}_{rx}$, $\ddot{f}_{ry}$ are bounded, piece-wise continuous bounded and known to all the robots.*

Assumption 5 *The parameters r_i and b_i fall in known compact sets, i.e. there exist some known positive constants $\bar{r}_i$, $\underline{r}_i$, $\bar{b}_i$ and $\underline{b}_i$ such that $\underline{r}_i < r_i < \bar{r}_i$ and $\underline{b}_i < b_i < \bar{b}_i$.*

Assumption 6 *The demanding distances ρ_{ix} and ρ_{iy} for robot i are available to its neighbors.*

***Remark* 9**

1) *It can be seen that the consensus tracking objective (ii) stated in Section 2, i.e. $\lim_{t\to\infty}[y_i(t) - y_r(t)] = 0$, is actually a special case of the formation objectives in (49) or (50) with $\rho_{ix} = 0$ or $\rho_{iy} = 0$. In contrast to the fact that exact information about $x_r(t)$ and $y_r(t)$ are only accessible to a subset of the robots, the desired orientation $\phi_r(t) = \arctan\left(\frac{\dot{y}_r}{\dot{x}_r}\right)$ is available to all the robots since $f_{rx}(t)$ and $f_{ry}(t)$ are available to all the robots.*

2) *Note that (45), (46) and (38) constitute the new system to be controlled. In (45)-(46), u_{i1}, $\dot{\xi}_i$ and u_{i2} act as the control inputs while x_i, y_i and ϕ_i are the outputs. Thus different from the traditional underactuated kinematic model for mobile robots, the new MIMO kinematic model can be treated as three separate SISO systems with the aid of transverse function technique. Moreover, since τ_{i1} and τ_{i2} in (38) are the actual control inputs of each robot system, the relative degree of the entire model at dynamic level is two. This indicates that the backstepping based adaptive control scheme proposed for one-dimensional output consensus tracking problem in Section 3 can be extended to solve the formation control problem in this section.*

3) *From (40), (41) and the properties of f_{li} in (44), it is clear that the transformation errors $x_i - \bar{x}_i$, $y_i - \bar{y}_i$, $\phi_i - \bar{\phi}_i$ are bounded by ε_{1i} and ε_{2i}. It will be shown that the designed distributed adaptive controllers can guarantee the convergence of the formation control errors with respect to x_i, y_i and ϕ_i. Therefore, the formation control errors with respect to the true position and orientation, i.e. $\bar{x}_i$, $\bar{y}_i$ and $\bar{\phi}_i$, can be made as small as desired by adjusting ε_{1i} and ε_{2i} properly.*

4.2 Control Design

As discussed in Remark 9, the control design procedure in this part involves two steps by adopting the backstepping technique. In the first step, the virtual controls for u_{i1}, u_{i2} and the auxiliary manipulated variable $\dot{\xi}_i$ will be chosen. In the second step, the actual control inputs τ_i will be derived.

- *Step 1*: Define local error variables as

$$\begin{aligned}
z_{ix,1} &= \sum_{j=1}^{N} a_{ij}(x_i + \rho_{ix} - x_j - \rho_{jx}) + \mu_i(x_i + \rho_{ix} - x_r) \\
z_{iy,1} &= \sum_{j=1}^{N} a_{ij}(y_i + \rho_{iy} - y_j - \rho_{jy}) + \mu_i(y_i + \rho_{iy} - y_r) \\
e_{ix,1} &= x_i - \mu_i x_r - (1-\mu_i)\left(f_{rx}\hat{w}_{rx,i} - \hat{c}_{rx,i}\right) + \rho_{ix} \\
e_{iy,1} &= y_i - \mu_i y_r - (1-\mu_i)\left(f_{ry}\hat{w}_{ry,i} - \hat{c}_{ry,i}\right) + \rho_{iy} \\
\delta_{i\phi} &= \phi_i - \phi_r \\
e_{ix,2} &= u_{i1} - \alpha_{i1},\ e_{i\phi,2} = u_{i2} - \alpha_{i2}
\end{aligned} \tag{52}$$

where $\hat{w}_{rx,i}$ ($\hat{w}_{ry,i}$), $\hat{c}_{rx,i}$ and $\hat{c}_{ry,i}$ are the estimates introduced in the ith robot for the unknown trajectory parameters if $\mu_i = 0$. We choose the virtual controls (α_{i1}, α_{i2}) and $\dot{\xi}$ in transverse function technique as

$$\begin{bmatrix} \alpha_{i1} \\ \dot{\xi}_i \end{bmatrix} = \begin{bmatrix} \hat{\theta}_{i1}^{-1} & 0 \\ 0 & 1 \end{bmatrix} Q_i^{-1}\Omega_i \tag{53}$$

$$\alpha_{i2} = \hat{\theta}_{i2}^{-1}\left(-k_2\delta_{i\phi} + \frac{\partial f_{3i}(\xi_i)}{\partial \xi_i}\dot{\xi}_i + \dot{\phi}_r\right) \tag{54}$$

where $\hat{\theta}_{i1}$ and $\hat{\theta}_{i2}$ are the estimates of r_i and $r_i b_i^{-1}$, respectively.

$$\begin{aligned}
\Omega_i = &-k_1 P_i \begin{bmatrix} z_{ix,1} \\ z_{iy,1} \end{bmatrix} - \frac{\partial R(\phi_i)}{\partial \phi_i}\begin{bmatrix} f_{1i}(\xi_i) \\ f_{2i}(\xi_i) \end{bmatrix}\Big(-k_2\delta_{i\phi} \\
&+\dot{\phi}_r\Big) - \begin{bmatrix} \dot{\rho}_{ix} \\ \dot{\rho}_{iy} \end{bmatrix} + \mu_i \begin{bmatrix} \dot{f}_{rx} w_{rx} \\ \dot{f}_{ry} w_{ry} \end{bmatrix} \\
&+(1-\mu_i)\begin{bmatrix} \dot{f}_{rx}\hat{w}_{rx,i} + f_{rx}\dot{\hat{w}}_{rx,i} + \dot{\hat{c}}_{rx,i} \\ \dot{f}_{ry}\hat{w}_{ry,i} + f_{ry}\dot{\hat{w}}_{ry,i} + \dot{\hat{c}}_{ry,i} \end{bmatrix}
\end{aligned}$$

where k_1, k_2 being positive constants. P_i is defined in (3). The above design delivers the following results.

$$\begin{bmatrix}\dot{e}_{ix,1}\\ \dot{e}_{iy,1}\end{bmatrix} = -k_1 P_i \begin{bmatrix} z_{ix,1}\\ z_{iy,1}\end{bmatrix} + \frac{\partial R(\phi_i)}{\partial \phi_i}\begin{bmatrix} f_{1i}(\xi_i)\\ f_{2i}(\xi_i)\end{bmatrix}\Big(\tilde{\theta}_{i2}u_{i2} + \hat{\theta}_{i2}e_{i\phi,2}\Big) + Q_i \begin{bmatrix}\tilde{\theta}_{i1}u_{i1} + \hat{\theta}_{i1}e_{ix,2}\\ 0\end{bmatrix}$$

$$\dot{\delta}_{i\phi} = -k_2\delta_{i\phi} + \tilde{\theta}_{i2}u_{i2} + \hat{\theta}_{i2}e_{i\phi,2} \tag{55}$$

The parameter estimators at this step are designed as

$$\begin{aligned}
&\dot{\hat{w}}_{rx,i} = -\gamma_{ri} f_{rx} e_{ix,1}, \quad \dot{\hat{w}}_{ry,i} = -\gamma_{ri} f_{ry} e_{iy,1}\\
&\dot{\hat{c}}_{rx,i} = -\gamma_{ri} e_{ix,1}, \quad \dot{\hat{c}}_{ry,i} = -\gamma_{ri} e_{iy,1}\\
&\dot{\hat{\theta}}_{i1} = \text{Proj}\left(\hat{\theta}_{i1}, \gamma_{\theta_{i1}}\pi_{i1}u_{i1}\right)\\
&\dot{\hat{\theta}}_{i2} = \text{Proj}\left(\hat{\theta}_{i2}, \gamma_{\theta_{i1}}\pi_{i2}u_{i2}\right)
\end{aligned} \tag{56}$$

with

$$\begin{aligned}
\pi_{i1} &= e_{ix,1}\cos(\bar{\phi}_i) + e_{iy,1}\sin(\bar{\phi}_i)\\
\pi_{i2} &= [e_{ix,1}, e_{iy,1}]\frac{\partial R(\phi_i)}{\partial \phi_i}\begin{bmatrix} f_{1i}(\xi_i)\\ f_{2i}(\xi_i)\end{bmatrix} + \delta_{i\phi}.
\end{aligned} \tag{57}$$

Note $\text{Proj}(\cdot,\cdot)$ denotes a Lipschitz continuous projection operator about which the design details and properties can be found in Krstic et al. (1995). It is adopted here to ensure that $\hat{\theta}_{i1} > 0$ and $\hat{\theta}_{i2} > 0$. Thus $\hat{\theta}_{i1}^{-1}$ and $\hat{\theta}_{i2}^{-1}$ in (53) and (54) are well defined.

We choose a Lyapunov function candidate at this step as

$$V_1 = \frac{1}{2}\sum_{i=1}^{N}\left(e_{ix,1}^2 + e_{iy,1}^2 + \delta_{i\phi}^2 + \frac{1}{\gamma_{\theta_{i1}}}\tilde{\theta}_{i1}^2 + \frac{1}{\gamma_{\theta_{i2}}}\tilde{\theta}_{i2}^2\right) + \frac{k_1}{2}\sum_{i=1}^{N}(1-\mu_i)\frac{P_i}{\gamma_{ri}}\left(\tilde{w}_{rx,i}^2 + \tilde{w}_{ry,i}^2 + \tilde{c}_{rx,i}^2 + \tilde{c}_{ry,i}^2\right) \tag{58}$$

From (55) and (56), the derivative of V_1 in (58) can be computed as

$$\begin{aligned}
\dot{V}_1 =& -\frac{k_1}{2}\left(\delta_x^T Q\delta_x + \delta_y^T Q\delta_y\right) - k_2\delta_{i\phi}^T\delta_{i\phi} + \sum_{i=1}^{N}\Bigg[\frac{\tilde{\theta}_{i1}}{\gamma_{\theta_{i1}}}\\
&\times\left(-\dot{\hat{\theta}}_{i1} + \gamma_{\theta_{i1}}\pi_{i1}u_{i1}\right) + \frac{\tilde{\theta}_{i2}}{\gamma_{\theta_{i2}}}\left(-\dot{\hat{\theta}}_{i2} + \gamma_{\theta_{i2}}\pi_{i2}u_{i2}\right)\Bigg]\\
&+ k_1\sum_{i=1}^{N}(1-\mu_i)\frac{P_i}{\gamma_{ri}}\Big[\tilde{w}_{rx,i}\left(-\dot{\hat{w}}_{rx,i} - \gamma_{ri}e_{ix,1}f_{rx}\right)\\
&+ \tilde{w}_{ry,i}\left(-\dot{\hat{w}}_{ry,i} - \gamma_{ri}e_{iy,1}f_{ry}\right) + \tilde{c}_{rx,i}\\
&\times\left(-\dot{\hat{c}}_{rx,i} - \gamma_{ri}e_{ix,1}\right) + \tilde{c}_{ry,i}\left(-\dot{\hat{c}}_{ry,i} - \gamma_{ri}e_{iy,1}\right)\Big]\\
&+ \sum_{i=1}^{N}\left(\pi_{i1}\hat{\theta}_{i1}e_{ix,2} + \pi_{i2}\hat{\theta}_{i2}e_{i\phi,2}\right)\\
\le& -\frac{k_1}{2}\left(\delta_x^T Q\delta_x + \delta_y^T Q\delta_y\right) - k_2\delta_{i\phi}^T\delta_{i\phi}\\
&+ \sum_{i=1}^{N}\left(\pi_{i1}\hat{\theta}_{i1}e_{ix,2} + \pi_{i2}\hat{\theta}_{i2}e_{i\phi,2}\right)
\end{aligned} \tag{59}$$

where $\delta_x = [\delta_{1x}, \ldots, \delta_{Nx}]$ with $\delta_{ix} = x_i - x_r + \rho_{ix}$ and $\delta_y = [\delta_{1y}, \ldots, \delta_{Ny}]$ with $\delta_{iy} = y_i - y_r + \rho_{iy}$. Q is defined in (3). The property of projection such that $\tilde{b}\text{Proj}(\hat{b}, a) \ge \tilde{b}a$ for $\tilde{b} = b - \hat{b}$ (Krstic et al., 1995) has been used.

• *Step 2*: We now at the position to derive the actual control torque τ_i. Define $\omega_{i1d} = \alpha_{i1} + \alpha_{i2}$, $\omega_{i2d} = \alpha_{i1} - \alpha_{i2}$. $z_{i,1} = \omega_{i1} - \omega_{i1d}$, $z_{i,2} = \omega_{i2} - \omega_{i2d}$. From (52) and the fact that $\omega_{i1} = u_{i1} + u_{i2}$ and $\omega_{i2} = u_{i1} - u_{i2}$, there is $e_{ix,2} = 0.5(z_{i,1} + z_{i,2})$ and $e_{i\phi,2} = 0.5(z_{i,1} - z_{i,2})$. Let $z_i = [z_{i,1}, z_{i,2}]^T$. Thus we have

$$z_i = \omega_i - \begin{bmatrix}\omega_{i1d}\\ \omega_{i2d}\end{bmatrix} \tag{60}$$

Multiplying the derivatives of both sides of (60) by M_i and combining it with (53) and (54), we obtain that

$$M_i\dot{z}_i = -D_i z_i + \Phi_i^T\Theta_i + \tau_i \tag{61}$$

where matrix Φ_i and Θ_i are defined as

$$\Phi_i = [\chi_i, \chi_{i,j_1}, \chi_{i,j_2}, ..., \chi_{i,j_{n_i}}]^T \tag{62}$$

$$\Theta_i = [\vartheta_i^T, \vartheta_{i,j_1}^T, \vartheta_{i,j_2}^T, ..., \vartheta_{i,j_{n_i}}^T]^T \tag{63}$$

Note j_p for $p = 1, \ldots, n_i$ are the indexes of robot i's neighboring robots (i.e. $j_p \in \mathcal{N}_i$) of which the total number is n_i. The elements in Φ_i and Θ_i are given in (64).

***Remark* 10** *Θ_i in (63) is a vector of unknown parameters involved in the ith robot dynamic subsystem, in which ϑ_i is the local unknown parameters, while ϑ_{i,j_p} is the coupled uncertainties related to the unknown parameters in robot j_p's dynamics if $a_{ij_p} = 1$. Thus online estimates of ϑ_{i,j_p}, i.e. $\hat{\vartheta}_{i,j_p}$, will be introduced in designing the torques for robot i.*

Introduce the estimate $\hat{\Theta}_i$ for unknown parameter vector Θ_i. Then the local control torque and adaptive law are designed as

$$\tau_i = -K_i z_i - \Phi_i^T\hat{\Theta}_i - 0.5\Xi_i \tag{65}$$

$$\dot{\hat{\Theta}}_i = \Gamma_i\Phi_i z_i \tag{66}$$

where $\Xi_i = [\Xi_{i,1}, \Xi_{i,2}]^T$ with

$$\begin{aligned}
\Xi_{i,1} &= \pi_{i1}\hat{\theta}_{i1} + \pi_{i2}\hat{\theta}_{i2}\\
\Xi_{i,2} &= \pi_{i1}\hat{\theta}_{i1} - \pi_{i2}\hat{\theta}_{i2}.
\end{aligned} \tag{67}$$

Choose the Lyapunov function for the overall system as

$$\vartheta_i = [c_i r_i b_i^{-1} \;\; d_{i1} \;\; d_{i2} \;\; m_{i1} \;\; m_{i2} \;\; m_{i1} r_i \;\; m_{i2} r_i \;\; m_{i1} r_i b_i^{-1} \;\; m_{i2} r_i b_i^{-1}]^T$$

$$\vartheta_{i,j_{ni}} = [m_{i1} r_j \;\; m_{i2} r_j \;\; m_{i1} r_j b_j^{-1} \;\; m_{i2} r_j b_j^{-1}]^T$$

$$\chi_i = \begin{bmatrix} -\omega_{i2} u_{i2} & -\omega_{i1d} & 0 & -\Delta_{i11} & -\Delta_{i12} & -\Delta_{i21} & -\Delta_{i22} & -\Delta_{i31} & -\Delta_{i32} \\ \omega_{i1} u_{i2} & 0 & -\omega_{i2d} & -\Delta_{i12} & -\Delta_{i11} & -\Delta_{i22} & -\Delta_{i21} & -\Delta_{i32} & -\Delta_{i31} \end{bmatrix}$$

$$\chi_{ij} = \begin{bmatrix} -\Delta_{ij11} & -\Delta_{ij12} & -\Delta_{ij21} & -\Delta_{ij22} \\ -\Delta_{ij12} & -\Delta_{ij11} & -\Delta_{ij22} & -\Delta_{ij21} \end{bmatrix}$$

$$\begin{aligned} \Delta_{i1k} &= \frac{\partial \omega_{ikd}}{\partial \rho_{ix}} \dot{\rho}_{ix} + \frac{\partial \omega_{ikd}}{\partial \dot{\rho}_{ix}} \ddot{\rho}_{ix} + \frac{\partial \omega_{ikd}}{\partial \rho_{iy}} \dot{\rho}_{iy} + \frac{\partial \omega_{ikd}}{\partial \dot{\rho}_{iy}} \ddot{\rho}_{iy} + \frac{\partial \omega_{ikd}}{\partial \dot{f}_{rx}} \ddot{f}_{rx} + \frac{\partial \omega_{ikd}}{\partial \dot{f}_{ry}} \ddot{f}_{ry} \\ &\quad + \frac{\partial \omega_{ikd}}{\partial \dot{\phi}_r} \ddot{\phi}_r + \frac{\partial \omega_{ikd}}{\partial \hat{\theta}_{i1}} \dot{\hat{\theta}}_{i1} + \frac{\partial \omega_{ikd}}{\partial \hat{\theta}_{i2}} \dot{\hat{\theta}}_{i2} + \frac{\partial \omega_{ikd}}{\partial \hat{w}_{rx,i}} \dot{\hat{w}}_{rx,i} + \frac{\partial \omega_{ikd}}{\partial \hat{w}_{ry,i}} \dot{\hat{w}}_{ry,i} \\ \Delta_{i2k} &= \frac{\partial \omega_{ikd}}{\partial \bar{x}_i} (\cos(\bar{\phi}_i) u_{i1}) + \frac{\partial \omega_{ikd}}{\partial \bar{y}_i} (\sin(\bar{\phi}_i) u_{i1}), \quad \Delta_{i3k} = \frac{\partial \omega_{ikd}}{\partial \bar{\phi}_i} u_{i2} \\ \Delta_{ij1k} &= \frac{\partial \omega_{ikd}}{\partial \bar{x}_j} (\cos(\bar{\phi}_j) u_{j1}) + \frac{\partial \omega_{ikd}}{\partial \bar{y}_j} (\sin(\bar{\phi}_j) u_{j1}), \quad \Delta_{ij2k} = \frac{\partial \omega_{ikd}}{\partial \bar{\phi}_j} u_{j2}, \qquad \text{for } \; k = 1, 2 \end{aligned} \tag{64}$$

$$V_2 = V_1 + \frac{1}{2} \left(z_i^T M_i z_i + \tilde{\Theta}_i^T \Gamma_i^{-1} \tilde{\Theta}_i \right) \tag{68}$$

where Γ_i is a symmetric and positive definite matrix and $\tilde{\Theta}_i = \Theta_i - \hat{\Theta}_i$. From (59) and (65), (66), we obtain that

$$\begin{aligned} \dot{V}_1(t) \leq & -\frac{k_1}{2} \left(\delta_x^T Q \delta_x + \delta_y^T Q \delta_y \right) - k_2 \delta_{i\phi}^T \delta_{i\phi} \\ & - z_i^T (K_i + D_i) z_i \end{aligned} \tag{69}$$

The main results in this section are formally presented in the following theorem.

Theorem 2 *Consider the closed-loop adaptive system consisting of N nonholonomic mobile robots (37)-(38), the control torques (65) and parameter estimators (56) and (66) under Assumptions 3-6. The formation errors for each robot are ensured to satisfy that*

$$\lim_{t \to \infty} \bar{x}_i(t) + \rho_{ix} - x_r(t) \leq \sqrt{2} \varepsilon_{i1} \tag{70}$$

$$\lim_{t \to \infty} \bar{y}_i(t) + \rho_{iy} - y_r(t) \leq \sqrt{2} \varepsilon_{i1} \tag{71}$$

$$\lim_{t \to \infty} \bar{\phi}_i(t) - \phi_r(t) \leq \varepsilon_{i2}. \tag{72}$$

Proof: By following similar analysis in the proof of Theorem 1 and from (69), it can be shown that δ_{ix}, δ_{iy} and $\delta_{i\phi}$ will converge to zero asymptotically. This indicates that $\lim_{t \to \infty} [x_i(t) - x_r(t)] = -\rho_{ix}$, $\lim_{t \to \infty} [y_i(t) - y_r(t)] = -\rho_{iy}$ and $\lim_{t \to \infty} [\phi_i(t) - \phi_r(t)] = 0$.

From (40), (41) and (43), we obtain that

$$\|(x_i - \bar{x}_i, y_i - \bar{y}_i)\| \leq \sqrt{2\varepsilon_{i1}^2}, \quad |\phi_i - \bar{\phi}_i| \leq \varepsilon_{i2}. \tag{73}$$

It then follows that

$$\begin{aligned} |\bar{x}_i + \rho_{ix} - x_r| &\leq |\bar{x}_i - x_i| + |x_i + \rho_{ix} - x_r| \\ |\bar{y}_i + \rho_{iy} - y_r| &\leq |\bar{y}_i - y_i| + |y_i + \rho_{iy} - y_r| \\ |\bar{\phi}_i - \phi_r| &\leq |\bar{\phi}_i - \phi_i| + |\phi_i - \phi_r|. \end{aligned} \tag{74}$$

Since $x_i + \rho_{ix} - x_r$, $y_i + \rho_{iy} - y_r$ and $\phi_i - \phi_r$ will converge to zero asymptotically, (70)-(72) hold. As discussed in Remark 9, by properly adjusting ε_{i1} and ε_{i2}, the formation errors of the overall system can be made as small as desired. □

4.3 Simulation Results

We now use 4 mobile robots to demonstrate the effectiveness of the controllers. The communication topology is given in Fig. 5. The reference trajectory is given as follows. $x_r(t) = t$, $y_r(t) = 10\sin(0.1t)$. $\rho_{1x} = 3$, $\rho_{2x} = 3$, $\rho_{3x} = 6$, $\rho_{4x} = 6$, $\rho_{1y} = 0$, $\rho_{2y} = 3$, $\rho_{3y} = 0$, $\rho_{1y} = 3$. The parameters of the robots are chosen as: $b_i = 0.75$, $d_i = 0.3$, $r_i = 0.25$, $m_{ci} = 10$, $m_{wi} = 1$, $I_{ci} = 5.6$, $I_{wi} = 0.005$, $I_{mi} = 0.0025$, $d_{i1} = d_{i2} = 5$. The control parameters are chosen as: $\varepsilon_{i1} = 0.1$, $\varepsilon_{i2} = 0.1$, $k_1 = 2$, $k_2 = 2$, $\gamma_{\theta_{i1}} = \gamma_{\theta_{i2}} = 5$, $\gamma_{r_i} = 4$, $K_i = 2I$, $\Gamma_i = 4I$, parameter ϵ for projection is chosen as $\epsilon = 0.1$. θ_{i1} and θ_{i2} are assumed to be in $[0.15, 0.4]$ and $[0.1, 0.3]$ respectively. The initial values are chosen as: $\hat{\theta}_{i1}(0) = 0.16$, $\hat{\theta}_{i2}(0) = 0.12$, $\hat{\vartheta}_i(0) = [0.05, 3, 3, 0.1, 0, 0.1, 0.01, 0.1, 0.01]^T$, $\hat{w}_{rx,i}(0) = 0.8$, $\hat{c}_{rx} = 0.5$, $\hat{w}_{ry,i}(0) = 1.2$ and $\hat{c}_{ry} = 0.5$. The robot position is shown in Fig. 6 and the orientation error $\delta_{i\phi}$ are shown in Fig. 7. Clearly, these results are consistent with those stated in Theorem 2 and therefore illustrate our theoretical findings.

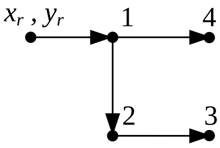

Fig. 5. Communication topology of the 4 mobile robots and the reference x_r and y_r are only available to robot 1.

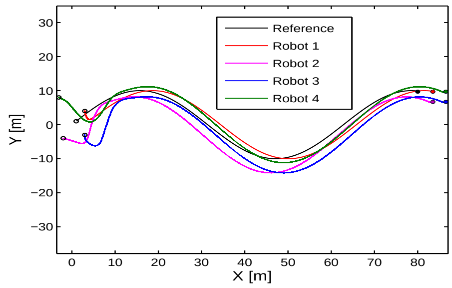

Fig. 6. The positions of the 4 mobile robots in X-Y plane.

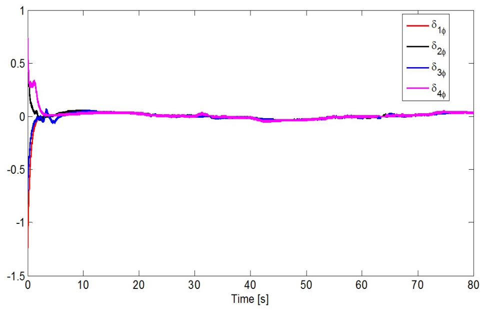

Fig. 7. Orientation errors $\delta_{i\phi}$

5 Conclusion

In this paper, we have investigated the output consensus tracking problem for a collection of nonlinear subsystems with intrinsic mismatched unknown parameters. We assume that only part of the subsystems can obtain the exact information of the desired trajectory directly. By adopting backstepping technique, distributed adaptive control laws are designed based on the information collected within neighboring areas. It is shown that all signals in the closed-loop system are bounded and the asymptotically consensus tracking for all the subsystems' outputs can be ensured. The simulation results show the effectiveness of the proposed control approach. Moreover, in contrast to currently available scheme in which the information exchange of online parameter estimates among linked subsystems is required, the condition can be relaxed by introducing additional estimates to account for the uncertainties in the neighbors' dynamics. Thus the desired results are achieved with less transmission burden. We also apply the distributed control strategy to successfully solve the formation control problem for multiple nonhonolomic mobile robots.

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