J Syst Sci Complex (2018) 31: 1186–1205

Homogeneous Finite-Time Consensus Control for Higher-Order Multi-Agent Systems by Full Order Sliding Mode*

MONDAL Sanjoy · GHOMMAM Jawhar · SAAD Maarouf

DOI: 10.1007/s11424-018-6236-7
Received: 17 October 2016 / Revised: 26 October 2017

Abstract This paper investigates the distributed finite-time consensus tracking problem for higher-order nonlinear multi-agent systems ($\mathrm{MAS_s}$). The distributed finite-time consensus protocol is based on full order sliding surface and super twisting algorithm. The nominal consensus control for the $\mathrm{MAS_s}$ is designed based on the geometric homogeneous finite time control technique. The chattering is avoided by designing a full order sliding surface. The switching control is constructed by integrating super twisting algorithm, hence a chattering alleviation protocol is obtained to maintain a smooth control input. The finite time convergence analysis for the leader follower network is presented by using strict Lyapunov function. Finally, the numerical simulations validate the proposed homogeneous full-order sliding mode control for higher-order $\mathrm{MAS_s}$.

Keywords Finite time control, full order sliding mode, multi agent systems.

1 Introduction

In recent years, consensus problem of multi-agent systems ($\mathrm{MAS_s}$) has received significant attention from various scientific and research communities and emerged as a challenging new research area. It is due to the fact that many complex systems inherently represented by a network of interacting subsystems or agents. The consensus control is achieved by sharing the information between the group of agents in a connected network, which leads to its wide applications in many areas such as satellite formation flying[1, 2], air traffic control[3], cooperative control of unmanned air vehicles[4, 5], autonomous underwater vehicles[6], congestion control of communication networks[7], mobile robotic systems[8], robot synchronized control[9], and so on.

Among many wide spread formation control, the leader-following consensus technique for $\mathrm{MAS_s}$ is widely studied due to its variety of applications ranging from engineering[10–13] to

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MONDAL Sanjoy · GHOMMAM Jawhar · SAAD Maarouf
School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore; Department of Electrical Engineering, Ecole de technologie superieure, Montreal, Canada.
Email: mondals@ntu.edu.sg; jawhar.ghommam@etsmtl.ca; maarouf.saad@etsmtl.ca.

⋄ *This paper was recommended for publication by Editor JIA Yingmin.*

biology , ecology and social sciences. In the leader follower formation, the leader is generally independent from its followers, whereas it can easily influence the followers behaviour. The leaders characteristic can easily be controlled to obtain the desired formation[14–19]. The main advantage of leader follower technique is, its simplicity in implementation, system design. It is also cost effective and requires less control energy[20].

In the past decade asymptotic consensus problem for various $\mathrm{MAS_s}$ and graph topologies has been widely studied[13, 21, 22]. The main drawback of asymptotic consensus is that the convergence rate is rather slow, but the convergence rate is an important performance indicator for many consensus control. In case of asymptotic convergence the tracking errors exponentially converge to the origin. The convergence rate can be improved by increasing the control gain which also increases the control energy of the the system. Also in many cases, it is often required that the consensus be reached in finite time. Compared with the asymptotic control method, finite-time control approach posses faster convergence rate, higher accuracy and stronger robustness[16, 17, 23]. Therefore, several researchers invoke the concept of finite-time control schemes to guarantee the faster convergence and high robustness[14–19].

Among many other control schemes, the sliding mode control is an effective robust control strategy for systems affected by uncertainties and external disturbances. The main drawback of the sliding mode control technique is the chattering, which makes it extremely difficult to implement in real time applications as it can damage the actuator.

Finite-time consensus problem with sliding mode control method has been widely studied in many cases[14–19]. For finite time convergence terminal sliding mode control is discussed in network formation control[14, 16, 18]. As mentioned these methods also suffer significantly with chattering, as the control input is directly influenced by the switching function. Thus, it can easily damage the actuator.

Among many chattering mitigation techniques, the well-known high-order sliding mode (HOSM) control is developed with higher robustness and accuracy[24, 25]. As compared to traditional sliding mode technique which is mainly asymptotic HOSM provides the finite time convergence. However, several recent interesting results using the finite time and sliding mode approach for $\mathrm{MAS_s}$ have been reported in the literature [14–19]. In [26], an off-line fixed time convergence technique for a second-order $\mathrm{MAS_s}$ is developed for a group of interacting agents. For finite time convergence, an integral sliding mode control that employs super-twisting algorithm to neglect the effect of the uncertainties is recently reported for a double integrator $\mathrm{MAS_s}$[19]. All these new finite-time consensus protocol is designed for only second order agent dynamics, i.e., double integrators only. This assumption is very restrictive, as in many practical cases, the agent dynamics can be of any order and can have input dynamics also[22]. Therefore, designing a finite time controller for a higher order nonlinear agent dynamics is still a considerable challenge.

The main contribution is to develop a finite-time control technique for higher order nonlinear agent dynamics. A new full order sliding surface based on HOSM is designed to guarantee the finite time convergence and chattering mitigation. As compared to the conventional sliding-mode control, where the sliding-mode motion is of reduced order, but in case of full order sliding mode (FOSM) during the ideal sliding condition, the system behaves as a full order dynamics rather than a desirable reduced-order. In full order sliding mode, the switching control is obtained by integrating the signum function, hence the control is smooth. The use of FOSM guarantees the higher accuracy and robustness to uncertainties and external disturbances.

The main contributions are listed as follows:

1) In finite time consensus control, the agent dynamics are mostly considered as double integrators, i.e., only second order simple linear dynamics[9, 14, 15, 18, 19, 26]. The proposed technique can be used easily for a much more complex higher order nonlinear agent dynamics.

2) The consensus protocol is proposed for nonlinear agent dynamics with relative degree $r = n$.

3) The consensus sliding surface is designed using the higher-order sliding mode technique, once it is in the sliding motion, finite time convergence is guaranteed.

4) Full-order sliding surface with super twisting algorithm is proposed for the consensus protocol.

5) The control protocol is more suitable to alleviate the chattering problem. Hence, it is more effective for electromechanical applications.

6) The finite-time consensus issue is analyzed with the strict Lyapunov functions.

The reminder of this paper is organized as follows. Sections 2 and 3 include the mathematical preliminaries and problem formulation. The finite time consensus control are explained in Sections 4 and 5. Section 6 presents the simulation examples. The conclusions are drawn in Section 7.

2 Mathematical Preliminaries

In this section, we state the following definition, the two lemmas from the literature and some preliminary notations on graph theory which will be used in the subsequent analysis.

Definition 1[27] Consider the system

$$\dot{x} = f(x), \quad f(0) = 0, \quad x \in \Re^n, \tag{1}$$

where $f : U_0 \to \Re^n$ is continuous in an open neighborhood U_0 of the origin. Let $(r_1, r_2, \cdots, r_n) \in \Re^n$ with $r_i > 0, i = 1, 2, \cdots, n$ and $f(x) = [f_1(x), f_2(x), \cdots, f_n(x)]^{\mathrm{T}}$ be a continuous vector field. Vector function $f(x)$ is said to be homegeneous of degree $\kappa \in \Re$ with respect to $(r_1, r_2, \cdots, r_n)$ if for a given $\varepsilon > 0$, $f_i(\varepsilon^{r_1} x_1, \varepsilon^{r_2} x_2, \cdots, \varepsilon^{r_n} x_n) = \varepsilon^{\kappa + r_i} f_i(x), i = 1, 2, \cdots, n, \forall x = [x_1, x_2, \cdots, x_n]^{\mathrm{T}} \in \Re^n$. System (1) is said to be homogeneous if $f(x)$ is homogeneous.

Lemma 1 *Consider a system described with $\dot{x} = f(x)$ with $f(0) = 0$ and there exists a continuous differential positive-definite function $V(x) : D \to R$, and $\beta > 0$, $0 < \eta < 1$, $p, \eta \in R$ such that*

$$\dot{V}(x) + \beta V^{\eta}(x) \leq 0, \quad \forall x \in D. \tag{2}$$

Then the origin of the system is a locally finite-time stable, with the settling time depending on *the initial state* $x(0) = x_0$*, satisfies*[23]

$$T(x_0) \leq \frac{V^{1-\eta}(x_0)}{\beta(1-\eta)}. \tag{3}$$

Lemma 2[28] *Suppose* $\dot{x} = f(x)$ *is homogeneous of degree* κ*. The origin of the system is finite-time stable if the origin is asymptotically stable and the system is with a negative homogeneity, i.e.,* $\kappa < 0$*.*

Graph Theory A directed graph $\mathcal{G} = (V, E)$ consists of a non empty node set $V = \{0, 1, \cdots, N\}$, and the edge set $E \subseteq V \times V\{(i,i)|i \in V\}$ represents the set of directed edges without any self-loops. Without loss of generality, assume node 0 is the leader, and $(i,j) \in E$ denotes that agent j can send message to agent i. Let $\overline{\mathcal{A}} \in \Re^{(N+1)\times(N+1)}$ be the adjacent matrix, where $(i,j) \in E \Rightarrow \rho_{ij} = 1$ and $(i,j) \notin E \Rightarrow \rho_{ij} = 0$. The pinning gain from the leader is denoted by $\overline{\mathcal{B}} = [\sigma_1, \sigma_2, \cdots, \sigma_N]^{\mathrm{T}} \in \Re^N$ with the adjacency element $\sigma_i > 0$ if agent i is a neighbor of the leader, i.e., $(i, 0) \in E$, otherwise $\sigma_i = 0$. The absolute in degree matrix $\overline{\mathcal{D}}$ is defined as $\overline{\mathcal{D}} = \mathrm{diag}\{\sum_{j=1}^{N} \rho_{ij}\}$. The Laplacian is defined as $\overline{\mathcal{L}} = \overline{\mathcal{D}} - \overline{\mathcal{A}}$. The followers can receive information from the leader, but cannot send information to the leader[9, 14, 18], i.e., $(0, i) \notin E$ for each follower i.

Assumption 1 For the leader-follower multi agent system, the communication topology of the directed graph $\mathcal{G}$ contains at least one spanning tree.

3 Problem Formulation

For the leader following MAS$_\mathrm{s}$, the dynamics of the ith follower is given by

$$\begin{aligned} &\dot{x}_i = F_i(x_i, t) + G_i(x_i, t)u_i \quad i = 1, 2, \cdots, N, \\ &y_i = h_i(x_i), \end{aligned} \tag{4}$$

where $x_i \in \Re^n$ is the state vector, $u_i \in \Re$ is the control input, and $F_i(x_i, t)$ and $G_i(x_i, t) \neq 0$ are uncertain smooth functions. It is n dimensional vector fields available for feedback.

The Lie derivative of the output function $h_i(x_i)$ with respect to the vector field $F_i(x_i, t)$ can be obtained as follows:

$$L_{F_i} h_i(x_i) = \frac{\partial h}{\partial x} F_i(x_i, t). \tag{5}$$

Also the Lie derivative of $L_{F_i} h_i(x_i)$ with respect to the vector field $G_i(x_i, t)$ can be defined as

$$L_{G_i} L_{F_i} h_i(x_i) = \frac{\partial}{\partial x}(L_{F_i} h_i(x_i)) G_i(x_i, t). \tag{6}$$

Since the follower (4) has a relative degree, $r = n$, therefore, one can easily obtain:

$$\begin{aligned} &L_{G_i} L_{F_i}^{k-1} h_i(x_i) = 0, \quad \forall k = 1, 2, \cdots, n-1, \\ &L_{G_i} L_{F_i}^{n-1} h_i(x_i) \neq 0. \end{aligned} \tag{7}$$

Using above, the nth derivative of the output can be obtained as

$$y_i^n = L_{F_i}^n h_i(x_i) + L_{G_i} L_{F_i}^{n-1} h_i(x_i) u_i. \tag{8}$$

So the ith follower (4) can be transformed as:

$$\begin{aligned}
\dot{x}_{i1} &= x_{i2},\\
\dot{x}_{i2} &= x_{i3},\\
&\vdots\\
\dot{x}_{in} &= f_i(x_i) + g_i(x_i)u_i, \quad i = 1, 2, \cdots, N,\\
&= f_{i0}(x_i) + g_{i0}(x_i)u_i + \underbrace{\Delta f_i(x_i) + \Delta g_i(x_i)u_i}_{\delta_i},\\
y_i &= x_{i1},
\end{aligned} \tag{9}$$

where $f_i = f_{i0} + \Delta f_i = L_{F_i}^n h_i$ and $g_i = g_{i0} + \Delta g_i = L_{G_i} L_{F_i}^{n-1} h_i$ are the Lie derivatives. Δf_i and Δg_i are modeling uncertainties. It is also assumed that, $f_{i0}(x_i, t)$ and $g_{i0}(x_i, t) \neq 0$ are smooth nonlinear functions available for feedback. $g_{i0}(x_i, t)$ is invertible.

Now $x_{i1} \in \Re$ is the output of the ith agent, $x_i = [x_{i1}, x_{i2}, \cdots, x_{in}]^{\mathrm{T}} \in \Re^n$ are the state variables, $u_i \in \Re$ is the control input for the ith agent. δ_i is the bounded unknown disturbances and uncertainties of the ith agent.

Assumption 2 In the followers-dynamics (9), there exists a constant $k_{id} > 0$, such that $||\delta_i|| > k_{id}$.

The leader dynamics can be modeled as

$$\begin{aligned}
\dot{x}_{01} &= x_{02},\\
\dot{x}_{02} &= x_{03},\\
&\vdots\\
\dot{x}_{0n} &= u_0,
\end{aligned} \tag{10}$$

where $x_0 = [x_{01}, x_{02}, \cdots, x_{0n}]^{\mathrm{T}} \in \Re^n$ is the state vector of the leader, $y_0 = x_{01} \in \Re$ is the output. $u_0 \in \Re$ is the control input. Here the leader dynamics is considered as a chain of integrators only, as the leader will act as a signal generators for the followers.

The homogenous MAS$_{\mathrm{s}}$ (9) is said to reach consensus in finite time, if for any initial condition, there exists a finite-time such that $\lim_{t \to T_0}(x_{i1} - x_{01}) = 0$.

In this section, the finite-time consensus algorithm for nth order MAS$_{\mathrm{s}}$ with one leader under directed network topology is considered. The dynamics of the followers and the single leader are given by (9) and (10). Let $\chi_1 = [e_1^1, e_2^1, \cdots, e_N^1]^{\mathrm{T}}$, $\chi_2 = [e_1^2, e_2^2, \cdots, e_N^2]^{\mathrm{T}}$ and $\chi_n = [e_1^n, e_2^n, \cdots, e_N^n]^{\mathrm{T}}$. Here for each follower, define the consensus tracking errors

$$\begin{aligned}
e_i^1 &= \sum_{j=1, j\neq i}^{N} \rho_{ij}(x_{i1} - x_{j1}) + \sigma_i(x_{i1} - x_{01}),\\
e_i^2 &= \sum_{j=1, j\neq i}^{N} \rho_{ij}(x_{i2} - x_{j2}) + \sigma_i(x_{i2} - x_{02}),\\
&\vdots
\end{aligned}$$

$$e_i^n = \sum_{j=1,j\neq i}^{N} \rho_{ij}(x_{in} - x_{jn}) + \sigma_i(x_{in} - x_{0n}). \tag{11}$$

The error dynamics can be written as

$$\begin{aligned}
\dot{\chi}_1 &= \chi_2, \\
\dot{\chi}_2 &= \chi_3, \\
&\vdots \\
\dot{\chi}_n &= (\overline{\mathcal{L}} + \overline{\mathcal{B}})f(\chi) + (\overline{\mathcal{L}} + \overline{\mathcal{B}})g(\chi)u + \underbrace{(\overline{\mathcal{L}} + \overline{\mathcal{B}})\delta}_{d_o} - \overline{\mathcal{B}}\overline{1}u_0,
\end{aligned} \tag{12}$$

where $f(\chi) = [f_{10}(x_1), f_{20}(x_2), \cdots, f_{N0}(x_N)]^{\mathrm{T}}$, $g(\chi) = \mathrm{diag}[g_{10}(x_1), g_{20}(x_2), \cdots, g_{N0}(x_N)]$, $u = [u_1, u_2, \cdots, u_N]^{\mathrm{T}}$, $\overline{1} = [1, 1, \cdots, 1]^{\mathrm{T}} \in \Re^N$ and $\delta = [\delta_1, \delta_2, \cdots, \delta_N]^{\mathrm{T}}$ and d_o is the over all uncertainty. Now assume that the uncertainty d_o is bounded and known such that the condition $||d_o|| < k_d$ satisfies[19, 29].

Consider the leader-follower system given by (9) and (10) where the directed graph $\mathcal{G}$ has a directed spanning tree and $\chi_1 = 0, \chi_2 = 0$ and $\chi_n = 0$, then

$$[x_{11} x_{21} \cdots x_{N1}]^{\mathrm{T}} = \overline{1}x_{01}, \tag{13}$$

$$[x_{1n} x_{2n} \cdots x_{Nn}]^{\mathrm{T}} = \overline{1}x_{0n}. \tag{14}$$

Now $e_i^1 = \sum_{j=1,j\neq i}^{N} \rho_{ij}(x_{i1} - x_{j1}) + \sigma_i(x_{i1} - x_{01})$, when $\chi_1 = 0$ it is easy to see that

$$(\overline{\mathcal{L}} + \overline{\mathcal{B}}) \begin{pmatrix} x_{11} \\ x_{21} \\ \vdots \\ x_{N1} \end{pmatrix} = \overline{\mathcal{B}}\overline{1}x_{01}. \tag{15}$$

Since $\overline{\mathcal{L}}\overline{1} = 0$, we have

$$(\overline{\mathcal{L}} + \overline{\mathcal{B}}) \begin{pmatrix} x_{11} \\ x_{21} \\ \vdots \\ x_{N1} \end{pmatrix} = (\overline{\mathcal{L}}\overline{1} + \overline{\mathcal{B}}\overline{1})x_{01} = (\overline{\mathcal{L}} + \overline{\mathcal{B}})\overline{1}x_{01}. \tag{16}$$

We know the matrix $(\overline{\mathcal{L}} + \overline{\mathcal{B}})$ is invertible[9, 14, 18]. Hence, one can easily obtain

$$\begin{pmatrix} x_{11} \\ x_{21} \\ \vdots \\ x_{N1} \end{pmatrix} = \overline{1}x_{01}. \tag{17}$$

Remark 1 Sometime fixed distances are expected to be maintained between the leader and the followers. Then the consensus error can be defined by following way:

$$e_i^1 = \sum_{j=1,j\neq i}^{N} \rho_{ij}(x_{i1} + \Delta_i - x_{j1} - \Delta_j) + \sigma_i(x_{i1} + \Delta_i - x_{01} - \Delta_0), \tag{18}$$

where Δ_i represents the distance between the agent i and the leader in different directions, $i = 1, 2, \cdots, N$[9, 14].

4 Homogeneous Finite-Time Consensus Control with Integral Sliding Mode (ISM)

The ISM design procedure can be divided into two phases $u = u^{nom} + u^{sw}$: The nominal control (u^{nom}) for the nominal system, i.e., without disturbances, generates the desired trajectory, and then the switching control action (u^{sw}) keeps the system on the trajectory all the time in spite of uncertainties and disturbances[29].

To converge the error dynamics (12) in finite-time, the integral sliding surface can be considered as

$$s = \chi_n + \int_0^t \{k_n|\chi_n|^{\alpha_n}\text{sign}(\chi_n) + k_{n-1}|\chi_{n-1}|^{\alpha_{n-1}}\text{sign}(\chi_{n-1}) + \cdots + k_1|\chi_1|^{\alpha_1}\text{sign}(\chi_1)\}d\tau, \tag{19}$$

where k_l and α_l $(l = 1, 2, \cdots, n)$ are constants. k_l can be found such $\psi \in \Re$ the polynomial

$$\psi^n + k_n\psi^{n-1} + \cdots + k_2\psi + k_1 \tag{20}$$

is Hurwitz, α_l can be obtained satisfying the following conditions

$$\begin{aligned} &\alpha_1 = \alpha, \quad n = 1, \\ &\alpha_{l-1} = \frac{\alpha_l\alpha_{l+1}}{2\alpha_{l+1} - \alpha_l}, \quad l = 2, \cdots, n, \quad \forall n \geq 2, \end{aligned} \tag{21}$$

where $\alpha_{n+1} = 1$, $\alpha_n = \alpha, \alpha \in (1 - \varepsilon, 1), \varepsilon \in (0, 1)$[28].

Theorem 1 *The error dynamics* (12) *will converge to equilibrium in finite-time along* $s = 0$*, if the sliding surface* s *is selected as* (19) *and the control is designed as follows:*

$$\begin{aligned} u &= [g(\chi)(\overline{\mathcal{L}} + \overline{\mathcal{B}})]^{-1}[u^{nom} + u^{sw}], \\ u^{nom} &= \overline{\mathcal{B}}\overline{1}u_0 - (\overline{\mathcal{L}} + \overline{\mathcal{B}})f(\chi) - k_n|\chi_n|^{\alpha_n}\text{sign}(\chi_n) - k_{n-1}|\chi_{n-1}|^{\alpha_{n-1}}\text{sign}(\chi_{n-1}) \\ &\quad - \cdots - k_1|\chi_1|^{\alpha_1}\text{sign}(\chi_1), \\ u^{sw} &= -G\text{sign}(s), \end{aligned} \tag{22}$$

where G *is a positive constant and* $[g(\chi)(\overline{\mathcal{L}} + \overline{\mathcal{B}})]^{-1}$ *is nonsingular.*

Proof Let us consider a Lyapunov function $V = \frac{1}{2}s^{\mathrm{T}}s$. Taking the derivative and using the

ISM control law (22), we have:

$$\begin{aligned}\dot{V} &= s^{\mathrm{T}}[\dot{\chi}_n + k_n|\chi_n|^{\alpha_n}\mathrm{sign}(\chi_n) + k_{n-1}|\chi_{n-1}|^{\alpha_{n-1}}\mathrm{sign}(\chi_{n-1}) \\ &\quad + \cdots + k_1|\chi_1|^{\alpha_1}\mathrm{sign}(\chi_1)] \\ &= s^{\mathrm{T}}[(\overline{\mathcal{L}}+\overline{\mathcal{B}})f(\chi) + (\overline{\mathcal{L}}+\overline{\mathcal{B}})g(\chi)u + \underbrace{(\overline{\mathcal{L}}+\overline{\mathcal{B}})\delta}_{d_o} - \overline{\mathcal{B}}\overline{1}u_0 + k_n|\chi_n|^{\alpha_n}\mathrm{sign}(\chi_n) \\ &\quad + k_{n-1}|\chi_{n-1}|^{\alpha_{n-1}}\mathrm{sign}(\chi_{n-1}) + \cdots + k_1|\chi_1|^{\alpha_1}\mathrm{sign}(\chi_1)] \\ \dot{V} &\le -(G - \underbrace{||d_o||}_{k_d})||s|| \\ \dot{V} &\le -\zeta V^{1/2}, \end{aligned} \tag{23}$$

where $\zeta = G - k_d$ is a positive constant. Hence, on the basis of Theorem 1, the ISM(19) for the $\mathrm{MAS_s}$ with the protocol (22) is finite-time stable. ∎

Remark 2 From (23), of the ISM (22), $s = \dot{s} = 0$ for $t \ge 0$ means the disturbance is compensated by the equivalent value $G\mathrm{sign}(s)$ of discontinuous control action, larger the value of G more switching in the control signal.

5 Homogeneous Finite-Time Consensus Control with Full Order Sliding Mode (FOSM) Control

With the notations above, the control objectives can be summarized as: To develop a higher-order sliding mode control approach for nth order $\mathrm{MAS_s}$ (9) with corresponding stability analysis, and to make the leader-follower system finite-time consensus, that is, $\chi_l = 0$ $\forall l = 1, 2, \cdots, n$ in finite-time. The choice of FOSM is to reduce the chattering developed by ISM. Now to converge the error dynamics (12) in finite-time, consider a full order sliding surface[30] for the ith agent as:

$$\begin{aligned} s = \dot{\chi}_n &+ k_n|\chi_n|^{\alpha_n}\mathrm{sign}(\chi_n) + k_{n-1}|\chi_{n-1}|^{\alpha_{n-1}}\mathrm{sign}(\chi_{n-1}) \\ &+ \cdots + k_1|\chi_1|^{\alpha_1}\mathrm{sign}(\chi_1), \end{aligned} \tag{24}$$

where k_l and α_l $(l = 1, 2, \cdots, n)$ are constants. k_l can be found such $\psi \in \Re$ the polynomial

$$\psi^n + k_n\psi^{n-1} + \cdots + k_2\psi + k_1 \tag{25}$$

is Hurwitz, α_l can be obtained satisfying the following conditions

$$\begin{aligned} &\alpha_1 = \alpha, \quad n = 1, \\ &\alpha_{l-1} = \frac{\alpha_l\alpha_{l+1}}{2\alpha_{l+1} - \alpha_l}, \quad l = 2, 3, \cdots, n, \quad \forall n \ge 2, \end{aligned} \tag{26}$$

where $\alpha_{n+1} = 1$, $\alpha_n = \alpha, \alpha \in (1-\varepsilon, 1), \varepsilon \in (0, 1)$[31].

When the ideal sliding-mode $s = 0$ is reached, it is easy to obtain

$$\begin{aligned} &\dot{\chi}_n + k_n|\chi_n|^{\alpha_n}\mathrm{sign}(\chi_n) + k_{n-1}|\chi_{n-1}|^{\alpha_{n-1}}\mathrm{sign}(\chi_{n-1}) + \cdots + k_1|\chi_1|^{\alpha_1}\mathrm{sign}(\chi_1) = 0, \\ &\dot{\chi}_n = -k_n|\chi_n|^{\alpha_n}\mathrm{sign}(\chi_n) - k_{n-1}|\chi_{n-1}|^{\alpha_{n-1}}\mathrm{sign}(\chi_{n-1}) - \cdots - k_1|\chi_1|^{\alpha_1}\mathrm{sign}(\chi_1). \end{aligned} \tag{27}$$

Hence, the error dynamics (12) can be expressed as

$$\begin{aligned} \dot{\chi}_1 &= \chi_2, \\ \dot{\chi}_2 &= \chi_3, \\ &\vdots \\ \dot{\chi}_n &= -k_n|\chi_n|^{\alpha_n}\mathrm{sign}(\chi_n) - k_{n-1}|\chi_{n-1}|^{\alpha_{n-1}}\mathrm{sign}(\chi_{n-1}) - \cdots - k_1|\chi_1|^{\alpha_1}\mathrm{sign}(\chi_1), \end{aligned} \tag{28}$$

which represents the establishment of the ideal sliding-mode $s = 0$, for System (12). It will converge to the equilibrium point from any initial condition in finite-time[28].

Theorem 2 *For the error network* (12)*, the finite-time convergence can be obtained, if the sliding variable s is chosen as* (24) *and the formation control is developed as:*

$$\begin{aligned} u &= [g(\chi)(\overline{\mathcal{L}} + \overline{\mathcal{B}})]^{-1}[u^{nom} + u^{sw}], \\ u^{nom} &= \overline{\mathcal{B}}\mathbf{1}u_0 - (\overline{\mathcal{L}} + \overline{\mathcal{B}})f(\chi) - k_n|\chi_n|^{\alpha_n}\mathrm{sign}(\chi_n) - k_{n-1}|\chi_{n-1}|^{\alpha_{n-1}}\mathrm{sign}(\chi_{n-1}) \\ &\quad - \cdots - k_1|\chi_1|^{\alpha_1}\mathrm{sign}(\chi_1), \\ \dot{u}^{sw} &= -\lambda|s|^{1/2}\mathrm{sign}(s) - \theta\int_0^t \mathrm{sign}(s)d\tau, \end{aligned} \tag{29}$$

where λ and θ are positive constants and the inverse of $g(\chi)(\overline{\mathcal{L}} + \overline{\mathcal{B}})$ exists.

Distributed format of the control law The main advantage of distributed control is its simplicity to use, fast in computing and easy for operation. Now for the leader following network, the ith sliding variable can be expressed as

$$\begin{aligned} s_i = \dot{e}_i^n &+ k_{in}|e_i^n|^{\alpha_{in}}\mathrm{sign}(e_i^n) + k_{i(n-1)}|e_i^{n-1}|^{\alpha_{i(n-1)}}\mathrm{sign}(e_i^{n-1}) \\ &+ \cdots + k_{i1}|e_i^1|^{\alpha_{i1}}\mathrm{sign}(e_i^1), \end{aligned} \tag{30}$$

where k_{il} and α_{il} ($\forall i = 1, 2, \cdots, N$ and $l = 1, 2, \cdots, n$) are constants. k_{il} can be found such $\psi \in \Re$ the polynomial

$$\psi^n + k_{in}\psi^{n-1} + \cdots + k_{i2}\psi + k_{i1} \tag{31}$$

is Hurwitz, α_{il} $\forall$ $l = 1, 2, \cdots n$ can be obtained satisfying the following conditions[29]

$$\begin{aligned} &\alpha_{il} = \alpha, \quad n = 1, \quad i = 1, 2, \cdots, N, \\ &\alpha_{i(l-1)} = \frac{\alpha_{il}\alpha_{i(l+1)}}{2\alpha_{i(l+1)} - \alpha_{il}}, \quad l = 2, 3, \cdots, n, \quad \forall n \geq 2, \quad i = 1, 2, \cdots, N, \end{aligned} \tag{32}$$

where $\alpha_{i(n+1)} = 1$, $\alpha_{il} = \alpha, \alpha \in (1 - \varepsilon, 1), \varepsilon \in (0, 1)$.

When the ideal sliding-mode $s_i = 0$ is reached, one can easily get

$$\begin{aligned} &\dot{e}_i^n + k_{in}|e_i^n|^{\alpha_{in}}\mathrm{sign}(e_i^n) + k_{i(n-1)}|e_i^{n-1}|^{\alpha_{i(n-1)}}\mathrm{sign}(e_i^{n-1}) \\ &+ \cdots + k_{i1}|e_i^1|^{\alpha_{i1}}\mathrm{sign}(e_i^1) = 0. \end{aligned} \tag{33}$$

Hence the error dynamics (12) can be expressed as

$$\begin{aligned}
\dot{e}_i^1 &= e_i^2,\\
\dot{e}_i^2 &= e_i^3,\\
&\ \ \vdots\\
\dot{e}_i^n &= -k_{in}|e_i^n|^{\alpha_{in}}\mathrm{sign}(e_i^n) - k_{i(n-1)}|e_i^{n-1}|^{\alpha_{i(n-1)}}\mathrm{sign}(e_i^{n-1})\\
&\quad - \cdots - k_{i1}|e_i^1|^{\alpha_{i1}}\mathrm{sign}(e_i^1), \qquad (34)
\end{aligned}$$

which represents the establishment of the ideal sliding-mode $s_i = 0$, for System (12). It can converge to its equilibrium point from any initial condition in finite-time. Matrix $\overline{\mathcal{L}} + \overline{\mathcal{B}}$ has been proved invertible[9, 14]. Note that $\overline{\mathcal{L}} + \overline{\mathcal{B}} = \overline{\mathcal{D}} + \overline{\mathcal{B}} - \overline{\mathcal{A}}$, therefore, control law (22) can be written as

$$\begin{aligned}
\dot{\chi}_1 &= \chi_2,\\
\dot{\chi}_2 &= \chi_3,\\
&\ \ \vdots\\
\dot{\chi}_n &= (\overline{\mathcal{D}} + \overline{\mathcal{B}})f(\chi) - \overline{\mathcal{A}}f(\chi) + (\overline{\mathcal{D}} + \overline{\mathcal{B}})g(\chi)u - \overline{\mathcal{A}}g(\chi)u + (\overline{\mathcal{D}} + \overline{\mathcal{B}} - \overline{\mathcal{A}})\delta - \overline{\mathcal{B}}\overline{1}u_0. \qquad (35)
\end{aligned}$$

The error dynamics (12) of ith agent will converge to equilibrium in finite-time along $s_i = 0$ within, if the sliding-mode surface s_i is selected as (24) and the control is designed as follows:

$$\begin{aligned}
u &= [g(\chi)(\overline{\mathcal{D}} + \overline{\mathcal{B}})]^{-1}[u^{nom} + u^{sw}],\\
u^{nom} &= \overline{\mathcal{B}}\overline{1}u_0 + \overline{\mathcal{A}}g(\chi)u + \overline{\mathcal{A}}f(\chi) - (\overline{\mathcal{D}} + \overline{\mathcal{B}})f(\chi) - k_n|\chi_n|^{\alpha_n}\mathrm{sign}(\chi_n)\\
&\quad -k_{n-1}|\chi_{n-1}|^{\alpha_{n-1}}\mathrm{sign}(\chi_{n-1}) - \cdots - k_1|\chi_1|^{\alpha_1}\mathrm{sign}(\chi_1),\\
\dot{u}^{sw} &= -\lambda|s|^{1/2}\mathrm{sign}(s) - \theta\int_0^t \mathrm{sign}(s)d\tau, \qquad (36)
\end{aligned}$$

where the inverse of $[g(\chi)(\overline{\mathcal{D}} + \overline{\mathcal{B}})]$ exists.

So in distributed format the control law can be written as

$$\begin{aligned}
u_i &= \Big(\sum_{j=1,j\neq i}^{N}\rho_{ij} + \sigma_i\Big)^{-1}[u_i^{nom} + u_i^{sw}],\\
u_i^{nom} &= \sigma_i u_0 + \sum_{j=1,j\neq i}^{N}\rho_{ij}(f_j(x_j) + g_j(x_j)u_j) - \Big(\sum_{j=1,j\neq i}^{N}\rho_{ij} + \sigma_i\Big)f_i(x_i) - k_{in}|e_i^n|^{\alpha_{in}}\mathrm{sign}(e_i^n)\\
&\quad -k_{i(n-1)}|e_i^{n-1}|^{\alpha_{i(n-1)}}\mathrm{sign}(e_i^{n-1}) - \cdots - k_{i1}|e_i^1|^{\alpha_{i1}}\mathrm{sign}(e_i^1),\\
\dot{u}_i^{sw} &= -\lambda_i|s_i|^{1/2}\mathrm{sign}(s_i) - \theta_i\int_0^t \mathrm{sign}(s_i)d\tau, \qquad (37)
\end{aligned}$$

where λ_i and θ_i are positive constants and $(\sum_{j=1,j\neq i}^{N}\rho_{ij} + \sigma_i)$ is invertible. u_j is the control information of the jth agent.

Proof For the error system (12) the sliding surface can be written as

$$\begin{aligned}s &= \dot{\chi}_n + k_n|\chi_n|^{\alpha_n}\mathrm{sign}(\chi_n) + k_{n-1}|\chi_{n-1}|^{\alpha_{n-1}}\mathrm{sign}(\chi_{n-1}) + \cdots + k_1|\chi_1|^{\alpha_1}\mathrm{sign}(\chi_1)\\ &= (\overline{\mathcal{L}}+\overline{\mathcal{B}})f(\chi) + (\overline{\mathcal{L}}+\overline{\mathcal{B}})g(\chi)u + \underbrace{(\overline{\mathcal{L}}+\overline{\mathcal{B}})\delta}_{d_o} - \overline{\mathcal{B}}\overline{1}u_0 + k_n|\chi_n|^{\alpha_n}\mathrm{sign}(\chi_n)\\ &\quad + k_{n-1}|\chi_{n-1}|^{\alpha_{n-1}}\mathrm{sign}(\chi_{n-1}) + \cdots + k_1|\chi_1|^{\alpha_1}\mathrm{sign}(\chi_1).\end{aligned} \tag{38}$$

Using (29) one can easily obtain

$$s = u^{sw} + d_o. \tag{39}$$

Taking the derivative of (39) and using (29) we obtain

$$\begin{aligned}\dot{s} &= \dot{u}^{sw} + \dot{d}_o\\ &= -\lambda|s|^{1/2}\mathrm{sign}(s) - \theta\int_0^t \mathrm{sign}(s)d\tau + \dot{d}_o.\end{aligned} \tag{40}$$

Thus, (40) can be simplified as

$$\begin{aligned}\dot{s} &= -\lambda|s|^{1/2}\mathrm{sign}(s) + \rho + \dot{d}_o,\\ \dot{\rho} &= -\theta\mathrm{sign}(s).\end{aligned} \tag{41}$$

Let $z = \rho + \dot{d}_o$, then (41) can be re written as

$$\begin{aligned}\dot{s} &= -\lambda|s|^{1/2}\mathrm{sign}(s) + z,\\ \dot{z} &= \dot{\rho} + \ddot{d}_o.\end{aligned} \tag{42}$$

To analyze finite time stability of the system (42), the non smooth Lyapunov candidate is taken as, $V = \gamma^{\mathrm{T}}P\gamma$, where $\gamma = [|s|^{1/2}\mathrm{sign}(s) \quad s]^{\mathrm{T}}$, where P is a positive definite matrix. The Lyapunov function is absolutely continuous but not Lipschitz on the set $s, z \in R^2|s = 0$ due to the terms including $|s|^{1/2}\mathrm{sign}(s)$. Thus, the Lyapunov function is not continuously differentiable nor locally Lispchitz. The work shown in [19, 32] states, with certain initial conditions, the only requirement of a Lyapunov function is to be absolutely continuous for a stability analysis. The derivative of γ

$$\begin{aligned}\dot{\gamma} &= \begin{bmatrix}\frac{1}{2}|s|^{-1/2}\dot{s}\\ \dot{z}\end{bmatrix}\\ &= \begin{bmatrix}\frac{1}{2}|s|^{-1/2}[-\lambda|s|^{1/2}\mathrm{sign}(s) + z]\\ -\theta\mathrm{sign}(s) + \ddot{d}_o\end{bmatrix}\\ &= \frac{1}{2}|s|^{-1/2}\begin{bmatrix}[-\lambda|s|^{1/2}\mathrm{sign}(s) + z]\\ -2[\theta - \ddot{d}_o\mathrm{sign}(s)]|s|^{1/2}\mathrm{sign}(s)\end{bmatrix}\\ &= |s|^{-1/2}A\gamma,\end{aligned} \tag{43}$$

where $A = \begin{bmatrix} -\frac{1}{2}\lambda & \frac{1}{2} \\ -[\theta - \ddot{d}_o \mathrm{sign}(s)] & 0 \end{bmatrix}$. Now assume that $|\ddot{d}_o| < L$. Then with the condition $\theta - \ddot{d}_o \mathrm{sign}(s) \leq \theta - L > 0$. The matrix A becomes Hurwitz. Then the first time derivative of $V = \gamma^{\mathrm{T}} P \gamma$

$$\dot{V} = |s|^{-1/2} \gamma^{\mathrm{T}} (A^{\mathrm{T}} P + PA) \gamma, \tag{44}$$

since A matrix is Hurwitz as $\lambda > 0, \theta > 0$ and P is positive definite. Thus, it can be simplified as

$$\dot{V} = -|s|^{-1/2} \gamma^{\mathrm{T}} Q \gamma < 0, \tag{45}$$

where P and Q are related by

$$A^{\mathrm{T}} P + PA = -Q. \tag{46}$$

Now for every $Q > 0$, there exists a unique solution $P > 0$. Since V is absolutely continuous and $\dot{V}$ is negative definite. Now if $|s|^{1/2} = ||s|^{1/2} \mathrm{sign}(s)| \leq ||\gamma||_2 < \xi_{\min}^{1/2} P V^{1/2}$ then the inequality (45)

$$\begin{aligned} \dot{V} &\leq -\xi_{\min}^{1/2}(P) V^{-1/2} \frac{\xi_{\min}(Q)}{\xi_{\max}(P)} V \\ &\leq -\beta V^{1/2}, \end{aligned} \tag{47}$$

where $\beta = \min\{\frac{\xi_{\min}^{1/2}(P)\xi_{\min}(Q)}{\xi_{\max}(P)}\} > 0$. Based on the equality (47) we have,

$$\dot{V} \leq -\beta V^{1/2}. \tag{48}$$

It follows that $\dot{V}(t, s(0), z(0)) \leq -\beta V^{1/2}(t, s(0), z(0))$. Thus, an accurate tracking of s and z is achieved in finite time in spite of uncertainties and disturbances. The proof of this can be followed from [32]. ∎

Another advantage is that, since the time derivative of discontinuous function, i.e., the super twisting algorithm is hidden by the integral operation, the control law can be much smooth as it attenuates considerable amount of chattering from the control signal.

Remark 3 The convergence of distributed control law (37) can be found easily by similar way, where the dynamics of the distributed sliding surface (s_i) can be obtained by using the control law u_i, which will be similar to (39). Now it is very easy to follow the finite time stability proof in case of distributed system.

Remark 4 The higher derivatives of $d_o(x, t)$ (39) is bounded: $|\ddot{d}_o| < L$ where $L > 0$ is a constant. Note that this assumption is realistic in practical applications. For example, when a cutting tool or an end mill of a CNC machine tool cuts a work-piece, the load torque may change as the cutting thickness changes, but the change rate of the load torque is always limited. For design purpose one can choose a sufficiently high value of L for convergence.

6 Results

Example 1: Second order MAS$_{\rm s}$ In this section, we give different examples to show the effectiveness of the proposed algorithm in the consensus framework.

Consider a leader-follower system composed of five agents[9, 14], where the leader is indexed by 0, and the four followers are indexed by 1, 2, 3 and 4, respectively. The communication graph is shown in Figure 1.

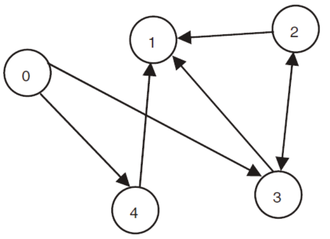

Figure 1 The communication graph

We consider the a dynamics of the agents are of second order. As compared to [9, 14], where agents dynamics were considered to be simple double integrators only, here we added some input dynamics for the followers as

$$\begin{aligned}\dot{x}_{i1} &= x_{i2},\\ \dot{x}_{i2} &= x_{i1} + \cos(x_{i2}) + u_i + 0.01\sin(x_{i1}),\end{aligned} \tag{49}$$

where $i = 1, 2, 3, 4$. The term $d_i = 0.01\sin(x_{i1})$ represents the unmodelled dynamics and external disturbances. The initial conditions are chosen as $[x^0_{11}\ \ x^0_{12}]^{\rm T} = [1\ \ 0]^{\rm T}$, $[x^0_{21}\ \ x^0_{22}]^{\rm T} = [1.2\ \ 0]^{\rm T}$, $[x^0_{31}\ \ x^0_{32}]^{\rm T} = [2\ \ 0]^{\rm T}$, $[x^0_{41}\ \ x^0_{41}]^{\rm T} = [-1.2\ \ 0]^{\rm T}$.

Assume the followers are required to track the leader, is given by[9, 14]

$$\begin{aligned}\dot{x}_{01} &= x_{02},\\ \dot{x}_{02} &= -\sin(x_{01})/(1 + {\rm e}^{-t}).\end{aligned} \tag{50}$$

The initial conditions for the leader is $[x^0_{01}\ \ x^0_{02}]^{\rm T} = [\pi/2\ \ 0]^{\rm T}$.

Case 1 In this part, ISM controller (22) is used for tracking. The sliding surface is chosen as,

$$s = \chi_2 + \int_0^t \{k_2|\chi_2|^{\alpha_2}{\rm sign}(\chi_2) + k_1|\chi_1|^{\alpha_1}{\rm sign}(\chi_1)\}d\tau, \tag{51}$$

where $\alpha_2 = 2/3, \alpha_1 = 1/2$, $k_1 = {\rm diag}([0.3,\ 0.3,\ 0.3,\ 0.3])$, $k_2 = {\rm diag}([0.45, 0.45, 0.45, 0.45])$. The followers track the leader's trajectory in finite-time. The evolution of output trajectory and the control signal using the ISM control law (22) with $G = {\rm diag}([4.5, 4.5, 4.5, 4.5])$ are shown in Figures 2 and 3. It is evident that, although the tracking is satisfactory the control signal is not smooth, it suffers with undesirable chattering.

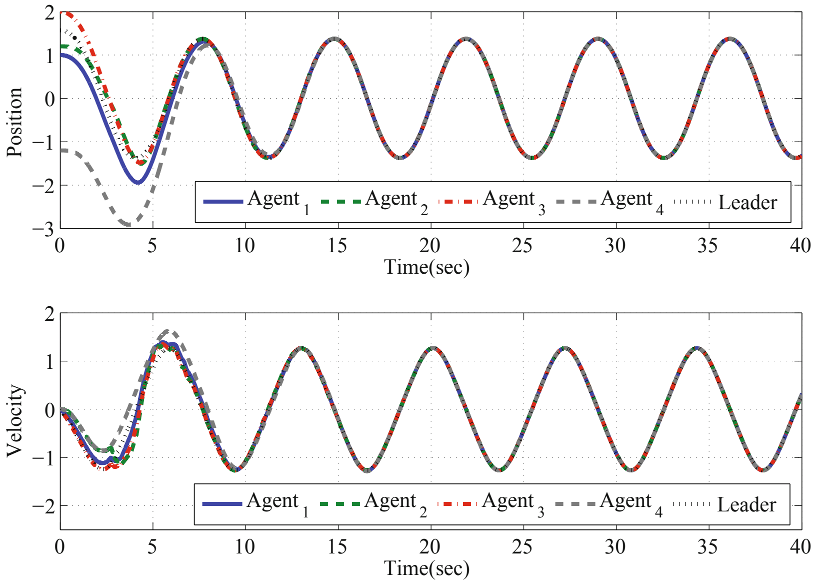

Figure 2 Consensus tracking

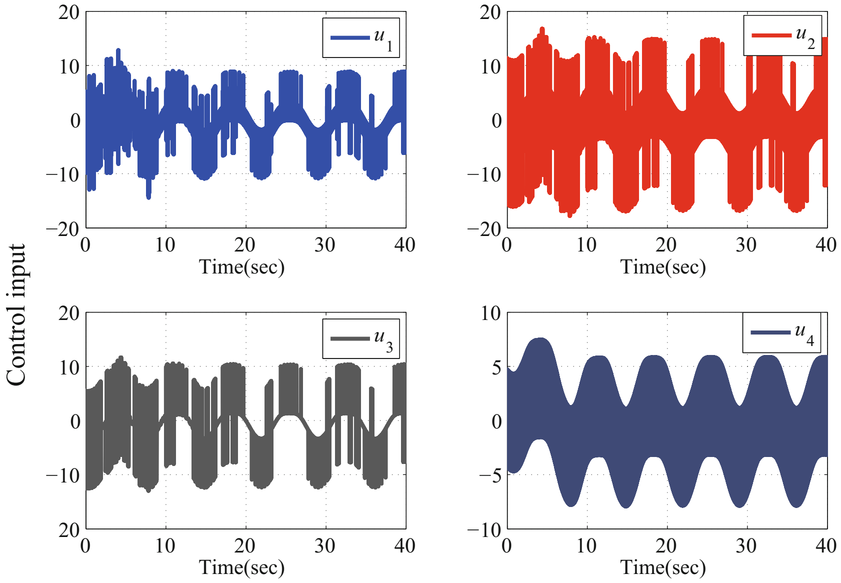

Figure 3 Control signal

Case 2 To overcome the chattering problem, we design a full order sliding surface using super twisting algorithm. The sliding surface for the followers, can be written as

$$s = \dot{\chi}_2 + k_2|\chi_2|^{\alpha_2}\mathrm{sign}(\chi_2) + k_1|\chi_1|^{\alpha_1}\mathrm{sign}(\chi_1), \tag{52}$$

$\alpha_2 = 2/3, \alpha_1 = 1/2,\ k_1 = \mathrm{diag}([0.3,\ 0.3,\ 0.3,\ 0.3]),\ k_2 = \mathrm{diag}([0.45, 0.45, 0.45, 0.45])$. The control law for each agent can be obtained from (29), where $\lambda = \mathrm{diag}([1,1,1,1])$ and $\theta = \mathrm{diag}([4.5,4.5,4.5,4.5])$.

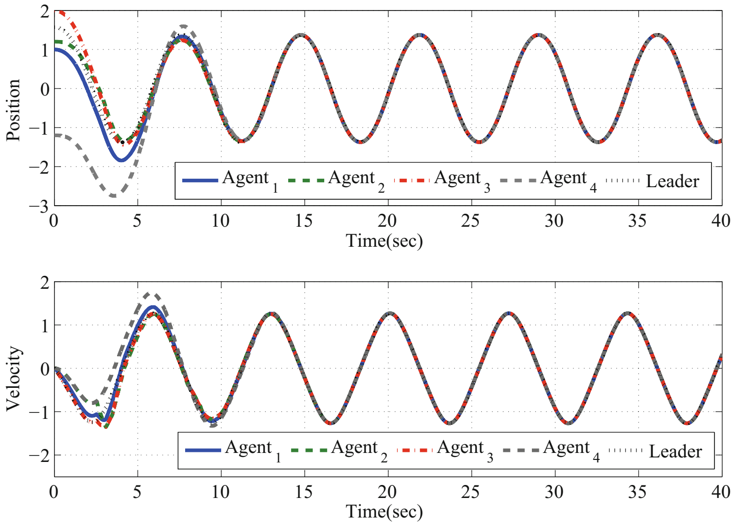

Figure 4 Consensus tracking

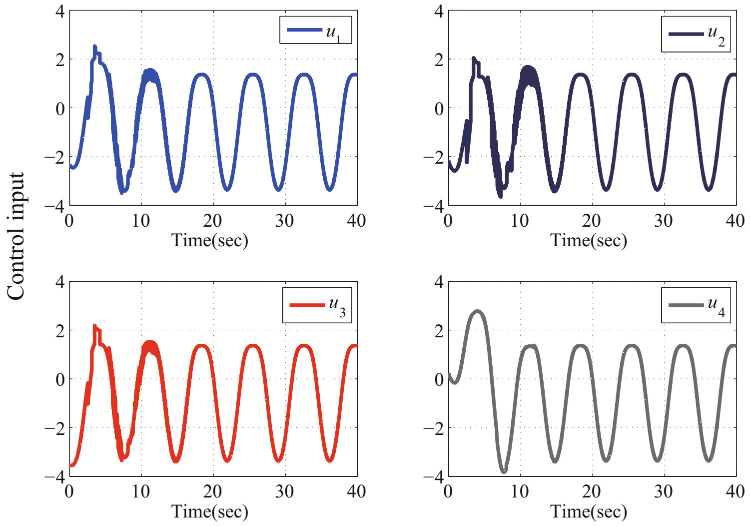

Figure 5 Control signal

The output and the control signal using the proposed method (29) are shown in Figures 4 and 5. It is clear that, the output of the followers track accurately that of the leader. The control input of each follower is provided in Figure 5. Since the switching control is obtained by integrating the super twisting algorithm, hence the control signal is much more smooth and it is more superior in eliminating chattering.

Example 2: Third order MAS$_\mathrm{s}$ To show the effectiveness of the proposed control technique for higher-order MAS$_\mathrm{s}$, the following directed topologies Figure 6, with five followers and one leader are considered for simulation.

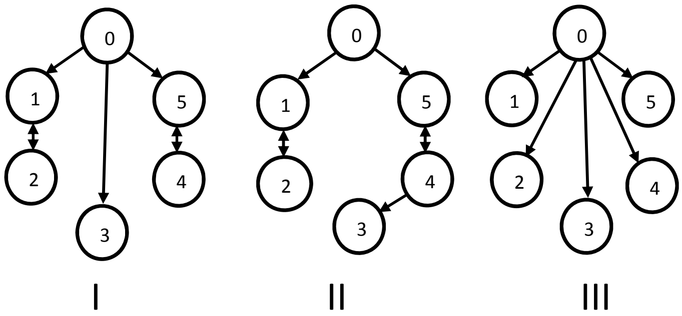

Figure 6 The communication graph

Now we consider the agent dynamics are of third order

$$\begin{aligned}\dot{x}_{i1} &= x_{i2},\\ \dot{x}_{i2} &= x_{i3},\\ \dot{x}_{i3} &= x_{i2}^2 + x_{i2}x_{i3} + u_i + d_i, \quad i = 1, 2, \cdots, n,\end{aligned} \tag{53}$$

where $d_i \neq 0$ are the uncertainties, chosen as $d_1 = 1.15\sin(0.3t + \pi/3)$, $d_2 = 1.6\sin(0.1t + \pi/6)$, $d_3 = 1.25\sin(0.4t + \pi/3)$, $d_4 = \sin(0.5t + \pi/2)$, $d_5 = 1.5\sin(0.6t + \pi/2)$. The design objective is to develop a consensus algorithm in such a way that the leader following formation control is achieved in finite time. The output of the leader is taken as $y_0 = t$. The initial conditions are chosen as $[x_{11}^0\ x_{12}^0\ x_{13}^0]^\mathrm{T} = [-2\ 0\ 0]^\mathrm{T}$, $[x_{21}^0\ x_{22}^0\ x_{23}^0]^\mathrm{T} = [6\ 3\ -3]^\mathrm{T}$, $[x_{31}^0\ x_{32}^0\ x_{33}^0]^\mathrm{T} = [3\ 0\ 3]^\mathrm{T}$, $[x_{41}^0\ x_{42}^0\ x_{43}^0]^\mathrm{T} = [-5\ -5\ 0]^\mathrm{T}$, $[x_{51}^0\ x_{52}^0\ x_{53}^0]^\mathrm{T} = [5\ 0\ 0]^\mathrm{T}$. The sliding surface chosen from (24) for the agents are taken as

$$s = \dot{\chi}_3 + k_3|\chi_3|^{\alpha_3}\mathrm{sign}(\chi_3) + k_2|\chi_2|^{\alpha_2}\mathrm{sign}(\chi_2) + k_1|\chi_1|^{\alpha_1}\mathrm{sign}(\chi_{i1}), \tag{54}$$

where $\alpha_3 = 3/4, \alpha_2 = 3/5, \alpha_1 = 1/2$, $k_1 = 1 \times \mathrm{diag}([2,2,2,2,2])$, $k_2 = 1 \times \mathrm{diag}([3,3,3,3,3])$, $k_3 = 1 \times \mathrm{diag}([4,4,4,4,4])$. The control law for each agent can be obtained from (29) where $\lambda = 2.2 \times \mathrm{diag}([2,2,2,2,2])$ and $\theta = 2.2 \times \mathrm{diag}([3,3,3,3,3])$.

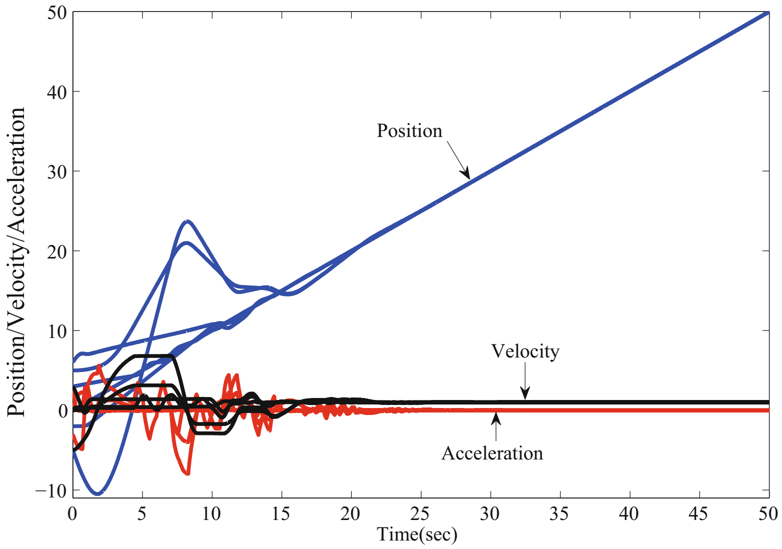

Figure 7 Consensus tracking using the topology I

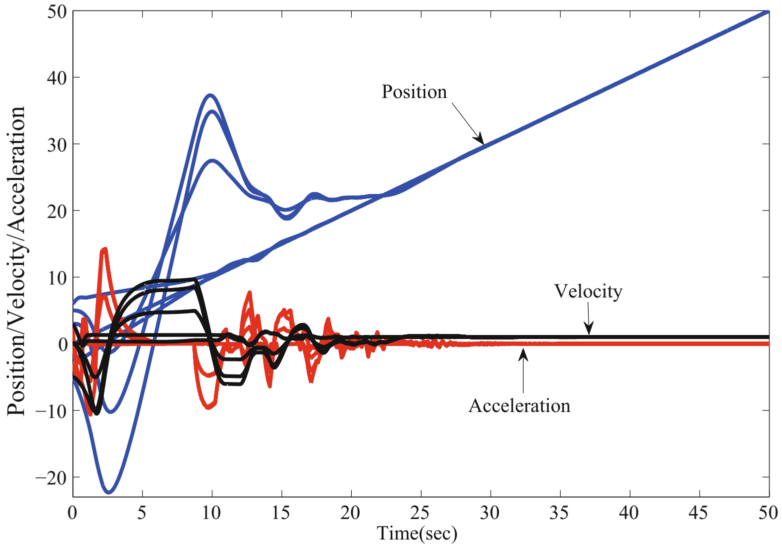

Figure 8 Consensus tracking using the topology II

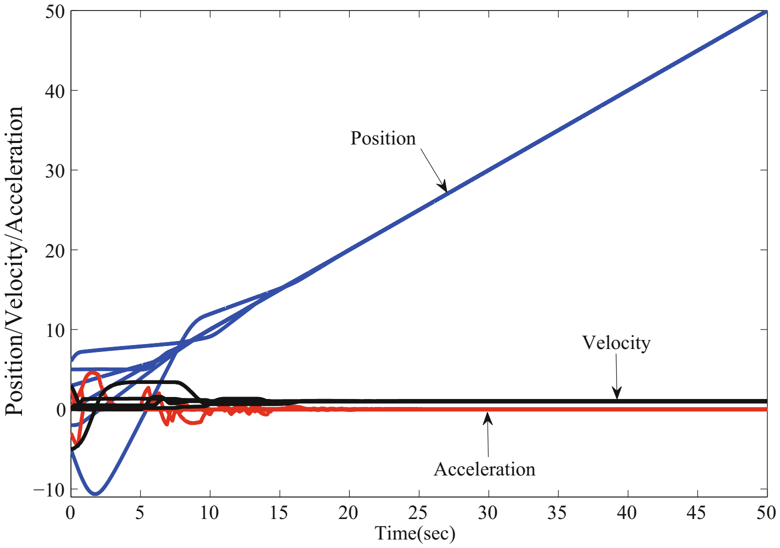

Figure 9 Consensus tracking using the topology III

Simulation results are shown in the Figures 7–9. The output tracking performance of the MAS$_\mathrm{s}$ using the proposed consensus protocol with different topologies are shown in Figures 7–9, from which it can be observed that the leader is accurately followed by the followers. In all the simulations, the curves in blue, black and red denote the position and velocity and acceleration states of the leader and followers, respectively. From the above two examples, it can be observed that the proposed method can realize the globally finite-time stability of the nth-order MAS$_\mathrm{s}$ using a higher-order sliding surface. In addition, the control signals are smooth, as shown in Figure 5, which means that the chattering problem in sliding mode control has been further attenuated.

7 Conclusion

A full order sliding surface is designed for higher-order multi-agent systems (MAS$_\mathrm{s}$) using higher-order sliding mode. The proposed method shows the finite-time tracking of the agents. The consensus control law generalize the results for the higher order nonlinear systems. The switching control is obtained by integrating the super twisting algorithm, thus it alleviates the chattering heavily. The finite-time convergence analysis of higher-order MAS$_\mathrm{s}$ is developed by using strict Lyapunov function. Finally, numerical simulations has been presented to demonstrate the effectiveness of the proposed consensus control scheme for higher order nonlinear multi-agent system.

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