

Recent progress of study on optical solitons in fiber lasers

Cite as: Appl. Phys. Rev. **6**, 021313 (2019); doi: [10.1063/1.5091811](https://doi.org/10.1063/1.5091811)

Submitted: 5 February 2019 · Accepted: 25 April 2019 ·

Published Online: 24 May 2019





View Online



Export Citation



CrossMark

Yufeng Song,¹  Xujie Shi,^{1,2} Chengfa Wu,² Dingyuan Tang,³ and Han Zhang^{1,a)} 

AFFILIATIONS

¹SZU-NUS Collaborative Innovation Center for Optoelectronic Science & Technology, International Collaborative Laboratory of 2D Materials for Optoelectronics Science and Technology of Ministry of Education, College of Physics and Optoelectronic Engineering, Shenzhen University, Shenzhen 518060, China

²Institute for Advanced Study, Shenzhen University, Shenzhen 518060, China

³School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore 639798

^{a)}Author to whom correspondence should be addressed: h Zhang@szu.edu.cn

ABSTRACT

Solitons are stable localized wave packets that can propagate long distance in dispersive media without changing their shapes. As particle-like nonlinear localized waves, solitons have been investigated in different physical systems. Owing to potential applications in optical communication and optical signal processing systems, optical solitons have attracted intense interest in the past three decades. To experimentally study the formation and dynamics of temporal optical solitons, fiber lasers are considered as a wonderful nonlinear system. During the last decade, several kinds of theoretically predicted solitons were observed experimentally in fiber lasers. In this review, we present a detailed overview of the experimentally verified optical solitons in fiber lasers, including bright solitons, dark solitons, vector solitons, dissipative solitons, dispersion-managed solitons, polarization domain wall solitons, and so on. An outlook for the development on the solitons in fiber lasers is also provided and discussed.

Published under license by AIP Publishing. <https://doi.org/10.1063/1.5091811>

I. INTRODUCTION

Solitons, also called solitary waves, are a specific kind of wave that exists in nonlinear systems.^{1,2} Solitons have been theoretically predicted for more than 50 years and have been observed in different physical systems. During the past decades, solitons have been intensively investigated in the field of nonlinear optics.^{3–13} Owing to the simplicity, easy, and precise control on the experimental parameters, ultrafast fiber lasers are considered a perfect platform to study the optical solitons. Various kinds of solitons have been experimentally investigated in the fiber laser systems. In the following text, we review recent progress on optical solitons in fiber lasers. Specifically, we emphasize on several types of solitons generated in fiber lasers and demonstrate their unique properties and characteristics. The review starts with a brief introduction to the historic development of solitons and basic theory on solitons in Sec. I. We then discuss the bright and dark solitons described by nonlinear Schrödinger equation (NLSE) in mode locked fiber lasers in Sec. II. In Sec. III, we review vector soliton formation in fiber lasers. In Sec. IV, we review dissipative solitons in fiber lasers. In Sec. V, breathers and dispersion-managed solitons are reviewed. In Sec. VI, multiple soliton interactions and dynamics are

reviewed. We briefly review polarization domain wall (DW) and its potential applications in Sec. VII. Finally, we discuss the challenges and future directions on this topic in Sec. VIII.

A. Brief history of solitons

The phenomenon of solitons was first observed by John Scott Russell in a canal of Scotland in 1834. In his report, a soliton was defined as a special kind of wave that can propagate over a long distance without distortion. In physics, a soliton is a self-reinforcing localized wave packet that maintains its shape while it propagates at a constant velocity. The study of solitons covers a large area of nonlinear physics scenarios, including fluid dynamics, plasma physics, optics, biological and atmospheric systems, nonlinear fiber optics, and so on. In 1965, soliton behavior in media described by the Korteweg–de Vries equation (KdV equation) was reported by Zabusky and Kruskal.¹⁴ In 1967, analytical solution of the KdV equation was developed by inverse scattering transformation.¹⁵ This solution was extended to soliton generation by Peter Lax who proposed Lax pairs and the Lax equation. Solitons are, by definition, unaltered in shape

and speed by a collision with other solitons. So, solitary waves on a water surface are not exactly solitons as after the interaction of two solitary waves, they change a bit in amplitude, and an oscillatory residual is left behind.¹⁶ Solitons are also studied in quantum mechanics, thanks to the fact that they could provide a new foundation through de Broglie's unfinished program, known as "double solution theory" or "nonlinear wave mechanics." This theory, developed by de Broglie in 1927 and revived in the 1950s, is the natural continuation of de Broglie's ideas developed between 1923 and 1926, which extended the wave-particle duality introduced by Einstein for the light quanta, to all the particles of matter.

B. Optical solitons

In optics, an optical soliton refers to an optical field that does not change during propagation because of a delicate balance between linear and nonlinear effects in the optical media.¹⁷ According to the formation mechanism, optical solitons can be classified as temporal solitons or spatial solitons. Temporal optical solitons are formed due to the combined effects of the refractive nonlinearity and the pulse dispersion, while spatial solitons are formed due to the combined effects of the nonlinearity and the beam diffraction.

The first theoretical prediction on the optical solitons shaping in optical fibers comes from Hasegawa and Tappert in 1973.^{18,19} In their prediction, optical solitons in fibers are mathematically described by the nonlinear Schrödinger equation (NLSE). NLSE and KdV equation have the same structure of Lax pair. The conventional temporal optical solitons governed by nonlinear Schrödinger equation can be basically classified as the bright solitons, which are characterized as intensity peaks in the time domain, and the dark solitons which are characterized as intensity dips embedded in a continuous wave background. Based on NLSE, optical bright solitons are supported when the group velocity dispersion (GVD) of the fiber is anomalous, while optical dark solitons are supported when the GVD of the fiber is normal.

Owing to bending and strains in single mode fibers, two orthogonal polarization modes are supported when light is propagating in it. Solitons with two coupled polarization components are named as vector solitons. Optical solitary wave propagation in birefringent optical fibers was first theoretically studied by Menyuk.²⁰ It is demonstrated that above a certain pulse intensity level, two orthogonally polarized solitons with different center wavelengths could couple together and propagate at the same group velocity. Coupled NLSEs have been employed to describe the solitons in the single mode fibers.

Based on the above-mentioned theory, it is natural to develop pulse fiber lasers as a platform to generate the solitons. Optical soliton formation in fiber lasers is a result of the mutual interaction among the cavity dispersion, fiber nonlinearity, laser gain saturation, and gain bandwidth filtering. Strictly speaking, the solitons in fiber lasers are dissipative solitons which are governed by the Ginzburg-Landau Equation (GLE). However, NLSE solitons could also be obtained in fiber lasers under certain conditions as long as the gain is balanced by losses and the gain bandwidth filtering effect could be ignored. Thus, both GLE and NLSE can be considered to describe the soliton generation in fiber lasers. Moreover, owing to the birefringence of the single mode fibers, vector solitons are also supported in single mode fiber lasers. Vector solitons in fiber lasers are mathematically described by the coupled GLEs or NLSEs.

Solitons can be experimentally generated by using passive mode locking technique in fiber lasers. Mode locking produces an initial optical pulse in the cavity. The pulse is then shaped into solitons under the balance between the dispersion and nonlinearity, gain, and losses. Based on the method bright solitons governed by the NLSE, dissipative solitons governed by the GLE, bright-bright solitons governed by the coupled NLSEs, and bright-bright vector dissipative solitons governed by the coupled GLEs were experimentally verified and studied. Moreover, in the nonmode locking regime, several other kinds of solitons that were theoretically predicted under certain conditions have also been experimentally observed, such as dark solitons, polarization domain wall solitons, vector dark-bright solitons, and so on. Besides, another intrinsic feature of solitons in fiber lasers is multiple soliton formation and soliton interactions. A typical pattern of multiple soliton interactions is bound solitons or also named as soliton molecules. In Secs. II–VIII, we will present a comprehensive review of theoretical and experimental investigations on the solitons formed in fiber lasers.

II. FUNDAMENTAL SOLITON GENERATION IN FIBER LASERS

When propagating in optical fibers, the properties of optical pulses are undergoing complicated changes due to chromatic dispersion and intermodal delay effects. Typical physical effects influencing pulses include fiber dispersion and nonlinearity.

Dispersion is an intrinsic feature of materials. In optical fibers, dispersion is defined as the dependence of the phase velocity on the optical frequency or the propagation mode. In single mode optical fibers (SMFs), there are mainly two types of dispersions that need to be considered, namely, the chromatic dispersion and the polarization mode dispersion. The chromatic dispersion of an optical medium is the phenomenon that the phase velocity and group velocity of light wave depend on the optical frequency. Generally, there are two sources of chromatic dispersion, namely, the material dispersion and the waveguide dispersion. Material dispersion depends on the materials used to make the optical fiber. For most fibers, material dispersion is the principle component of chromatic dispersion. The origin of chromatic dispersion is related to the characteristic resonance frequencies at which the medium absorbs the electromagnetic radiation through oscillations of bound electrons. In other words, the refractive index of optical media is a function of optical frequency of the light wave.

Nonlinear effects in optical fibers are originated from the nonlinear response of polarization to a strong optical field, which is another factor that must be taken into account in the study of pulse propagation in optical fibers. As optical fibers are made of silica, which has an amorphous microstructure, the lowest-order nonlinear effects in optical fibers are the third-order effects. In optical fibers, there are various types of third-order nonlinear effects, such as optical Kerr effect, third-harmonic generation, four-wave mixing, stimulated Raman scattering, and stimulated Brillouin scattering. The phase matching condition must be fulfilled for the occurrence of some third-order nonlinear effects. The optical Kerr effect is one of the most common nonlinear effects in optical fibers because the phase match condition for optical Kerr effect is automatically fulfilled. Thus, optical Kerr effect is always considered in the study of nonlinear pulse propagation in optical fibers. Optical Kerr effect originates from the intensity dependence of the refractive index which is given by

$$n(I) = n_0 + n_2 I, \tag{1}$$

where I is the optical intensity, n_0 is the linear index of refraction, and n_2 is the nonlinear refractive index of the medium. The value of n_2 can be measured with the z -scan technique.²¹ The optical Kerr effect has an instantaneously occurring nonlinear response and corresponds to the intensity variation of the light. Thus when we consider intense pulse propagation in optical fibers, both fiber dispersion and nonlinearity must be considered. Derived from the well-known Maxwell equation with several approximations, optical pulse propagation in single mode fibers is described by the nonlinear Schrodinger equation as follows:

$$\frac{\partial A}{\partial z} = -i\frac{\beta_2}{2}\frac{\partial^2 A}{\partial t^2} + i\gamma|A|^2 A, \tag{2}$$

where A is the slow varying envelope of the optical pulse, γ is the nonlinear parameter, β_2 is the second order dispersion coefficient which corresponds to the group velocity dispersion (GVD) effect. It is noted that the higher order dispersion effects are omitted. NLSE gives a simple description over the propagation of an optical pulse in optical fibers, which takes into account both the GVD and nonlinear effects. The nonlinear effects always generate a positive frequency chirp in the pulse and the GVD can introduce either positive or negative chirp, depending on the sign of GVD. If GVD introduces a negative chirp, i.e., the $\beta_2 < 0$, the frequency chirp can be eliminated by the balance between the nonlinear Kerr effect and the GVD. Under this condition, the soliton solution can be obtained by solving the NLSE using the inverse scattering method. Depends on the signature of β_2 , NLSE has different soliton solutions. The soliton solutions in the optical fibers will be discussed later.

Considering pulse propagation in fiber lasers, apart from the propagation in passive single mode fibers, pulse propagation in the gain fibers and the laser output loss must also be taken into account. Thus, the Ginzburg–Landau equation is used to describe the pulse propagation in fiber lasers

$$\frac{\partial A}{\partial z} = -\beta_1 \frac{\partial A}{\partial t} - \frac{i}{2}\beta_2 \frac{\partial^2 A}{\partial t^2} + i\gamma|A|^2 A + \frac{g-\alpha}{2}A + \frac{g}{2\Omega_g} \frac{\partial^2 A}{\partial t^2}, \tag{3}$$

where g represents the gain and α represents the loss, and Ω_g is the bandwidth of the laser gain. In a fiber laser system, if the effect of gain and gain bandwidth can be neglected, the equation will be simplified to NLSE.

A. Bright solitons and passive mode locking technique

Mathematically, according to the NLSE, a bright soliton is a solution of the NLSE when $\beta_2 < 0$, namely, the fiber dispersion is anomalous. The fundamental bright soliton solution is given by²²

$$A(z, t) = \sqrt{2\gamma}\eta \exp\{-4i(\xi^2 - \eta^2)z - 2i\xi\eta t + i\varphi\} \operatorname{sech}(2\eta(t - t_0) + 8\eta\xi z), \tag{4}$$

where η , ξ , φ , and t_0 are all constants and $\beta_2 = -2$.

As shown in Fig. 1, in the context of optical fibers, bright solitons are in the form of short pulses with high peak intensities. It is noted that the bright soliton solution of NLSE describes the pulse propagation in optical fibers that are a conservative system, where no gain and losses exist. In fiber lasers, pulse propagation experiences gain and

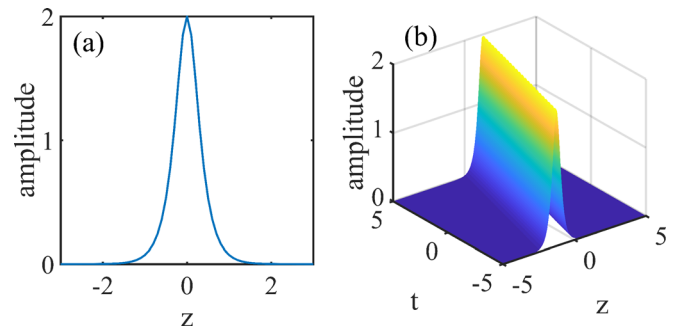


FIG. 1. A typical bright soliton solution of Eq. (4) with $\gamma = 2$, $\eta = 1/2$, $\xi = 1$, and $t_0 = 0$. (a) Bright soliton solution. (b) Evolution of a bright soliton.

losses; NLSE is thus extended to Ginzburg–Landau equation where gain and gain bandwidth effects have to be taken into account. However, in certain conditions, the solitons in fiber lasers can also be described by NLSE as long as the gain is balanced by the losses and the gain bandwidth effect can be ignored.

A typical method that can generate bright solitons in fiber lasers is based on the mode locking technique. A comprehensive review on the mode locking techniques in lasers can be found in Ref. 23. Depending on the mode locking methods used, laser mode locking can be classified as active mode locking and passive mode locking. Active mode-locking of a fiber laser is typically obtained by inserting a modulator into the cavity to modulate either the amplitude or the phase of the intracavity optical field at a frequency that equals to integer multiples of the cavity longitude mode spacing.²⁴ Active mode locking is a preferred choice for producing ultralow-jitter pulses at high repetition rates as high as tens of GHz with pulse duration in the picosecond range. However, as there are too many pulses in the lasers, which share the energy of the intra cavity laser beam, the energy of each pulse is weak and the pulse width is correspondingly broad. Moreover, a broad pulse width also means low peak power. Due to the low peak power of the active mode-locked pulses, nonlinear self-phase modulation (SPM) is so weak that soliton shaping is almost impossible in most actively mode locked lasers. Moreover, drop-out problem also exists in the actively mode-locked fiber lasers owing to the gain competition between the pulses.^{25,26} To overcome the shortcomings of active mode locking, passive mode locking of lasers is therefore proposed which may solve these problems.

Passive mode-locking is a widely used mode-locking technique which does not require any externally modulated media or devices^{27–29} but only employs a saturable absorber (SA) in the fiber laser cavity to realize the mode-locking. A typical passively mode locked fiber laser which can generate solitons is shown in Fig. 2. By inserting an SA into the laser cavity, passive mode locking is capable of producing ultra-short optical pulses, because SAs are able to modulate the resonator losses much faster than any electronic modulator. A SA usually refers to an optical device or medium that exhibits an intensity-dependent transmission,³⁰ which selectively absorbs low intensity light and transmits light with sufficiently high intensity. SAs in passively mode-locked lasers could be divided into two categories: Artificial SAs based on nonlinear light interference and real SAs based on material’s nonlinear optical absorption property. A typical artificial saturable absorber is the nonlinear polarization rotation (NPR) technique,

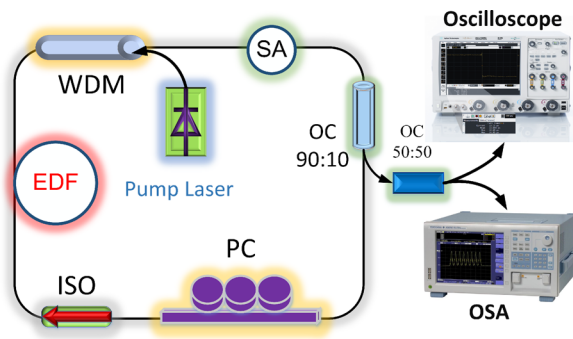


FIG. 2. A typical schematic diagram of a soliton fiber ring laser operating at 1550 nm based on passive mode locking technique. EDF: Erbium-doped fiber; WDM: Wavelength division multiplexer; PC: Polarization controller; OC: Optical coupler; ISO: Isolator; and OSA: Optical spectrum analyzer.

which is normally realized by inserting a polarizer in the fiber cavity. Thus, the solitons generated are linear polarized. These linear polarized bright solitons are defined as scalar solitons to distinguish with vector solitons. NPR technique was first successfully employed to mode lock fiber lasers in 1992. Since then solitons formed in fiber lasers mode-locked by the NPR technique have been intensively investigated.^{31–35} A great number of related interesting properties have been revealed and the corresponding physical mechanisms have been explained, such as Kelly sidebands,³⁶ multiple soliton formation,^{37–39} soliton energy quantization,^{39–41} and soliton interaction.^{42–44} For the real saturable absorbers, semiconductor saturable absorber mirrors (SESAMs) discovered by Keller *et al.* in the 1990s are widely used in fiber lasers.⁴⁵ Apart from the SESAMs, there are several other types of mode lockers discovered recently, like carbon nanotubes,^{46–66} graphene,^{67–105} black phosphorus,^{106–127} and other graphene-like two-dimensional materials.^{128–135} Discovery of the 2D materials based saturable absorbers significantly enhanced the study of the ultrashort pulse lasers and solitons in fiber lasers. Bright solitons have been so far observed in mode locked fiber lasers with various kinds of saturable absorbers.

Bright solitons governed by the NLSE are the first fundamental solitons verified experimentally in fiber laser systems. With different mode locking techniques, one can obtain the mode locked pulses with different pulse widths and stabilities in a fiber laser, but as long as solitons are formed, the influence of the specific mode locking technique on the general properties of solitons is subtle. In general, mode-locking can be considered as an effective approach to generate bright solitons in fiber lasers. As the mode-locking technique develops, bright solitons with new features and properties could be further discovered.

B. Dark solitons

Dark solitons are local pulses that appear as “holes” on a continuous wave background. A dark soliton is formed in optical fibers with normal dispersion and is also governed by the NLSE. In 1973, by using the inverse scattering method, Zakharov and Shabat obtained the dark soliton solutions with the boundary condition $|A(x, t)| \rightarrow 1$, $A_t \rightarrow 0$ (A_t refers to the first-order partial derivative of A with respect to t) as $t \rightarrow \pm\infty$ and they can be written as¹³⁶

$$A(z, t) = \sqrt{\frac{2(\lambda + iv)^2 + \exp\{2v(t - t_0 - 2\lambda z)\}}{\gamma}} \frac{1}{1 + \exp\{2v(t - t_0 - 2\lambda z)\}}, \quad (5)$$

where $\lambda^2 + v^2 = 1$. A typical dark soliton solution is shown in Fig. 3.

According to the minimum pulse intensity, dark solitons can be divided into black solitons and gray solitons. The soliton shown above is known as a gray soliton. The black solitons with the boundary condition were given by Menza and Gallo in 2007.¹³⁷ In 2007, Takhtajan and Faddeev got the dark N-soliton solutions.¹³⁸ Gredeskul and Kivshar theoretically predicted that different from the NLSE type of bright solitons, whose formation requires a certain fixed pulse intensity threshold, the formation of the NLSE type of dark solitons could have no threshold.¹³⁹ A comprehensive review on the dark solitons, which includes the physical origin and properties of dark solitons in optical fibers, was finished by Kivshar in 1998.¹⁴⁰

In addition of the above-mentioned theoretical studies, there are numerical studies on the dark solitons in optical fibers^{141–143} and fiber lasers.^{144–146} However, experimental observation of dark solitons in optical fibers and fiber lasers are still challenging. In optical fibers, dark solitons were first experimentally observed by utilizing specially shaped antisymmetric input pulses at a wavelength of ~ 617 nm in 1988.¹⁴⁷ In fiber lasers, it is well known that mode-locking technique can be used to generate “bright pulses.” However, there is no similar technique to generate dark pulses. Nevertheless, according to Kivshar’s prediction, in a normal dispersion cavity fiber laser a small intensity dip embedded in a high intensity continuous wave background could evolve into dark solitons. In 2009, a scalar dark pulse train was first observed in a continuous wave fiber laser by Zhang *et al.*¹⁴⁸ Although the measured optical spectrum of the laser emission exhibited clear characteristics of the dark solitons numerically simulated for the fiber laser, the measured dark pulses had a broad pulse width which could not be verified as dark solitons, as pointed out by Coen and Sylvestre.¹⁴⁹ The dark solitons related to the measured optical spectra would have several picosecond pulse width, which cannot be detected by the low speed detecting system used in the experiment. Thanks to the rapid advance of the ultrafast optoelectronics technology, Tang *et al.* repeated the experiment using an ultrahigh-speed real-time electronic detection system in 2013 and revealed the detailed properties of the dark solitons formed in fiber lasers.¹⁵⁰ According to Tang *et al.*, a small initial intensity dip in the laser cavity could be evolved into dark solitons, and with a high speed detection system these dark solitons can be experimentally observed.¹⁵⁰ The dark

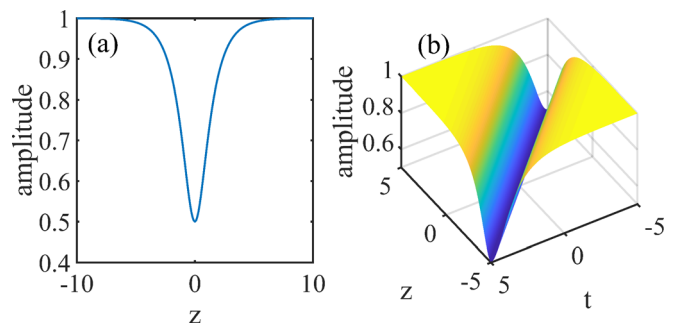


FIG. 3. A typical dark soliton described by Eq. (5) with $v = \frac{\sqrt{3}}{2}$, $\lambda = -\frac{1}{2}$, and $\gamma = -2$. (a) Dark soliton profile. (b) Evolution plot of the dark soliton.

solitons were always automatically formed in the cavity once the intracavity laser intensity was sufficiently strong, which is different from the bright solitons formation in a fiber laser, where in order to form the bright solitons a mode locked pulse needs to be formed in prior. As the dark solitons can be easily formed, and in a fiber ring laser there are plenty sources, e.g., the environmental noise and the mode beating, that could cause weak intensity dips, many dark solitons were simultaneously formed in the laser cavity. The dark solitons are randomly embedded in the laser emission. However, due to their narrow pulse width they are undetectable with even a moderate (<15 GHz) speed detection system. Under sufficiently strong pumping where dark solitons have bunched together and formed giant dark pulses, their existence would also become detectable even with 1 GHz bandwidth detection systems, as reported previously.¹⁴⁸ Tang further study the dark soliton formation in fiber lasers without antisaturable absorber and verified that dark soliton formation is a general feature of fiber laser.¹⁵¹ In 2014, a 280 GHz dark soliton fiber laser was reported which opens the study of high repetition rate dark solitons.¹⁵² The modulation instability induced in a fiber loop cavity was adopted to increase the repetition rate of the dark solitons. The repetition rate of the dark solitons was tunable by adjusting the power of the pump laser. This finding may enhance the dark soliton's potential for optical communication applications in the future.

In general, study of dark solitons in the fiber lasers is still at its early stage. Comparing to the bright solitons, dark soliton pulses show more stable behavior under perturbations, including amplifier noise, fiber losses, and so on. The dark solitons are also less sensitive to the backgrounds and lower pulse shape distortion. Study of the dark solitons would be a promising field of the optical solitons and may find applications in optical signal processing, optical computing and optical communications, and so on.

III. VECTOR SOLITONS IN FIBER LASERS

Vector solitons generally refer to solitons that have multiple mutually coupled components. In fiber optics, vector solitons can be formed in weakly birefringent single mode optical fibers (SMFs), where the light propagation is mathematically described by the coupled NLSEs. Theoretically, different forms of vector solitons, such as the bright-bright solitons, dark-bright solitons, and dark-dark solitons, have been predicted for the coupled NLSEs. Mathematically, coupled NLSEs are integrable if the nonlinear terms would have the same magnitude of coefficients. When both of the nonlinear terms are focusing, the integrable case is known as the focusing Manakov model, where the bright-bright soliton solutions can be found.¹⁵³ When both of the nonlinear terms are defocusing, the integrable case is called the defocusing Manakov model, where the dark-bright solitons¹⁵⁴ and dark-dark solitons^{154–156} can be found. If the nonlinear terms are a mixture of focusing and defocusing, then bright-bright solitons,^{157,158} dark-bright solitons,¹⁵⁹ and dark-dark solitons^{154,156,160–163} can be found.

The light propagation in weakly birefringent single mode fiber is described by the coupled NLSEs as follows:

$$\begin{aligned}\frac{\partial u}{\partial z} &= i\beta u - \delta \frac{\partial u}{\partial t} - \frac{i\beta_2}{2} \frac{\partial^2 u}{\partial t^2} + i\gamma \left(|u|^2 + \frac{2}{3} |v|^2 \right) u + i\frac{\gamma}{3} v^2 u^*, \\ \frac{\partial v}{\partial z} &= i\beta v + \delta \frac{\partial v}{\partial t} - \frac{i\beta_2}{2} \frac{\partial^2 v}{\partial t^2} + i\gamma \left(|v|^2 + \frac{2}{3} |u|^2 \right) v + i\frac{\gamma}{3} u^2 v^*,\end{aligned}\quad (6)$$

where u and v are the two normalized slowly varying pulse envelopes along the slow and the fast axes, u^* and v^* represent their conjugates, $2\beta = 2\Delta n/\lambda$ is the wave-number difference, and $2\delta = 2\beta\lambda/2\pi c$ is the group velocity difference. The additional terms v of the first term on the right hand side refers to the cross phase modulation effect. The second term of the right hand side refers to the effect of four-wave mixing.

Furthermore, if a light pulse is propagating in a weakly birefringent cavity fiber laser where the laser gain and losses must also be considered, the NLSE can be further extended to the coupled GLEs as follows:

$$\begin{aligned}\frac{\partial u}{\partial z} &= i\beta u - \delta \frac{\partial u}{\partial t} - \frac{i\beta_2}{2} \frac{\partial^2 u}{\partial t^2} + \frac{\beta_3}{6} \frac{\partial^3 u}{\partial t^3} + i\gamma \left(|u|^2 + \frac{2}{3} |v|^2 \right) u \\ &\quad + i\frac{\gamma}{3} v^2 u^* + \frac{g}{2} u + \frac{g}{2\Omega_g^2} \frac{\partial^2 u}{\partial t^2}, \\ \frac{\partial v}{\partial z} &= -i\beta v + \delta \frac{\partial v}{\partial t} - \frac{i\beta_2}{2} \frac{\partial^2 v}{\partial t^2} + \frac{\beta_3}{6} \frac{\partial^3 v}{\partial t^3} + i\gamma \left(|v|^2 + \frac{2}{3} |u|^2 \right) v \\ &\quad + i\frac{\gamma}{3} u^2 v^* + \frac{g}{2} v + \frac{g}{2\Omega_g^2} \frac{\partial^2 v}{\partial t^2}.\end{aligned}\quad (7)$$

The coupled GLEs are traditionally used to model the vector soliton formation and dynamics in dissipative nonlinear systems. The equations can also be used to study the vector soliton formation in fiber lasers. However, it is to note that a soliton circulating in a laser cavity is also subjected to the actions of other cavity components and feedback. These effects are not considered in the coupled GLEs above.

A. Bright-bright vector solitons

Bright-bright vector solitons refer to the vector solitons with bright pulse in both of the two polarization components. In 1973, Manakov studied the Manakov system

$$\begin{aligned}iu_t + u_{xx} + (|u|^2 + |v|^2)u &= 0 \\ iv_t + v_{xx} + (|u|^2 + |v|^2)v &= 0\end{aligned}\quad (8)$$

and obtained the bright-bright soliton solution as

$$\begin{aligned}(u, v)^T(x, t) &= 2\eta \operatorname{sech}[2\eta(x + 4\zeta t - x_0)] e^{-2i\zeta x - 4i(\zeta^2 - \eta^2)t} \\ &\quad \times \frac{1}{\sqrt{|\alpha_1|^2 + |\alpha_2|^2}} (\alpha_1, \alpha_2)^T,\end{aligned}\quad (9)$$

where the two real parameters ζ and η determine the velocity and amplitude, respectively, x_0 is the coordinate at $t=0$, and α_1 and α_2 are two complex constants. A typical bright-bright vector soliton is shown in Fig. 4.

Experimentally, bright-bright vector solitons can be simply generated in passively mode-locked fiber lasers, which has been intensively studied.^{71,84,89,93,98–100,109,144,164–219} To generate the vector solitons, all the fibers and passive components of the mode locked fiber lasers have to be polarization insensitive. The main challenge for achieving vector soliton operation of a fiber laser is to find an appropriate saturable absorber that has polarization insensitive saturable absorption. In fiber lasers, SESAM is the earliest proposed SA which has a polarization independent saturable absorption; vector soliton

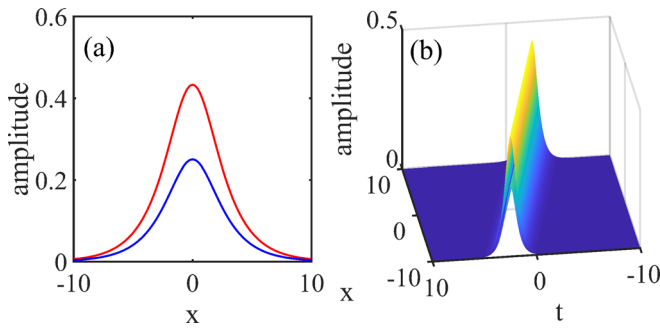


FIG. 4. The bright-bright soliton described by Eq. (9) with $\alpha_1 = \sqrt{3}$, $\alpha_2 = 1$, $\eta = 0.5$, $\xi = 1$, $x_0 = 0$. (a) Bright-bright soliton profiles. (b) Evolution plot of the bright-bright vector soliton.

properties in SESAM fiber lasers have been verified and intensively investigated.^{20–24}

In the last decade, carbon based SAs, represented by carbon nanotubes, with low cost and ultrafast recovery time were proposed and applied in the ultrafast lasers. Since 2009, graphene and graphene-like 2D materials as a group of novel wideband saturable absorbers with low cost, high damage threshold, large modulation depth, and easy fabrication were proposed and widely studied in the mode-locked fiber lasers. Similar to SESAM, these SAs are also polarization independent which supports vector soliton formation in fiber lasers. So far, vector solitons have been observed in ultrafast fiber lasers passively mode locked by carbon nanotubes,^{181,187} graphene,^{71,84,89,93,98–100,198} PbS quantum dots,¹⁸³ black phosphorus,¹⁰⁹ and so on. Vector solitons are not only observed in ytterbium-doped fiber lasers and erbium-doped fiber lasers but also in mid-infrared mode locked fiber lasers.^{183,204,208,217}

It is noted that bright-bright vector solitons produced by the coupled NLSEs may be asymmetric, with different energies and propagation constants in the two components. Bright-bright vector solitons with internal dynamics were studied by Kaup *et al.*²²⁰ Indeed, depending on the net cavity birefringence and coupling strength between the two components, experimentally the two orthogonal polarization components of the formed vector solitons could have different phases and intensities group velocities.

In a weak linear birefringence cavity, vector solitons will experience coherent energy exchange caused by the four wave mixing that was observed by Zhang *et al.*¹⁷³ In a birefringent fiber laser cavity induced solitons could also be formed.²²¹

Besides, depending on the cavity birefringence and cross-polarization coupling strength, vector solitons formed can be classified as polarization locked vector solitons, polarization rotation vector solitons, and group-velocity locked vector solitons and so on. Vector solitons with a uniform, nonevolving polarization state during the propagation is referred to as polarization locked vector solitons or phase locked vector solitons.^{167,168,172,181,222–224} Polarization locked vector solitons were first experimentally reported and investigated by Cundiff *et al.* in a SESAM mode locked fiber laser.²²² They presented a comprehensive study on polarization locked vector solitons in fiber lasers in 2000,^{167,168} which reveals that the two orthogonal polarization components of the polarization locked vector solitons have $\pm\pi/2$ relative phases. In 2008, high order polarization locked vector solitons in a SESAM mode locked fiber laser were reported.¹⁷²

Polarization rotation vector solitons are theoretically predicted by Afanasjev.²²⁵ The polarization rotation period of a polarization rotation vector soliton formed in a fiber laser is equal to (or a multiple of) the period of the cavity round trip time. Although the total soliton intensity is still uniform, the pulse intensity along two orthogonal polarization components varies with a certain period. The intensity of the pulses typical alters among two or several values, which is different from that of polarization locked vector solitons. Polarization rotation is a general feature of the vector solitons in fiber lasers, which has been observed in erbium-doped fiber lasers mode locked by SESAM,^{174,178} graphene,⁹⁹ carbon nanotube,²¹³ and so on. Sergeev *et al.* investigated the polarization dynamics of vector solitons in a carbon nanotube saturable absorber mode locked fiber laser and it was found that the solitons have a locked and processing polarization states.²²⁶

Another type of vector solitons is the group velocity locked vector solitons. Formed in a highly birefringent fiber, the two orthogonally polarized components of the vector soliton can trap and overcome the group-velocity difference through the nonlinear cross coupling.²²⁷ This type of vector solitons has been observed experimentally in fiber transmission systems. Group velocity locked vector solitons (GVLVSs) are reported as high order GVLVS,^{228,229} dissipative GVLVS,^{200,230,231} and bound state of GVLVS²¹⁴ and so on.

B. Dark-bright vector solitons

In 1997, Sheppard and Kivshar¹⁵⁴ investigated the NLSEs

$$i \frac{\partial \vec{e}}{\partial z} + \frac{1}{2} \frac{\partial^2 \vec{e}}{\partial t^2} - |\vec{e}|^2 \vec{e} = \vec{0}, \tag{10}$$

where z and t are distance and time coordinates, respectively, and $\vec{e} = (e_+, e_-)^T$ refers to the transversely polarized light. By Hirota's bilinear method, the authors derived the dark-bright soliton solutions as the form of

$$\begin{aligned} e_+ &= \tau \{ i \sin \phi + \cos \phi \tanh[a(t - bz)] \} e^{i c t + i [\frac{c^2}{2} + \tau^2] z}, \\ e_- &= \sqrt{\tau^2 \cos^2 \phi - a^2} \operatorname{sech}[a(t - bz)] e^{i b t + i [\frac{c^2}{2} - \tau^2] z}, \end{aligned} \tag{11}$$

where a , b , c , and τ are constants satisfying $a^2 + (c^2 - b^2) \leq \tau^2$ and $\phi = \arctan \frac{c-b}{a}$. A typical dark-bright vector soliton is shown in Fig. 5.

For the mixed case, Vijayajayanthi, Kanna, and Lakshmanan got the dark-bright soliton solutions in 2008.¹⁵⁹ The existence of dark-bright solitons in the single mode fibers is first theoretically predicted

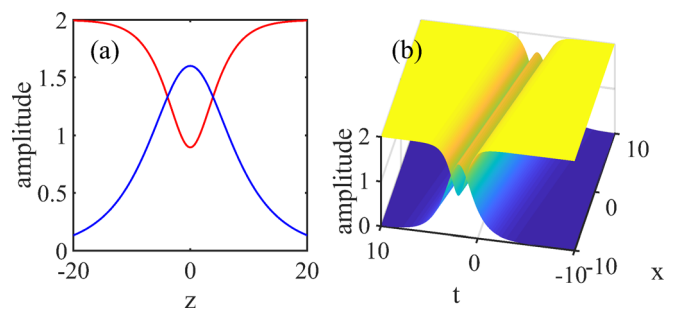


FIG. 5. The dark-bright soliton solution of Eq. (11) with $a = 0.8$, $b = -0.2$, $c = 0.2$, and $\tau = 2$. (a) Bright-bright soliton solution. (b) Evolution plot of bright soliton.

by Christodoulides.²³² Based on the coherently coupled NLSEs, he pointed out that under a weak birefringence, the dark-bright vector solitons can exist in different dispersion regimes with different properties. In normal dispersion regime, the dark soliton plays a dominant role, and in the anomalous dispersion regime the bright soliton dominates. Their predictions provide a guideline for the experimental study on the dark-bright solitons in fiber lasers.

According to the theoretical prediction of Christodoulides, there are two crucial conditions for the dark-bright soliton formation in fiber lasers. First, the birefringence of the fiber cavity must be sufficiently weak so that the two polarization components of the light can be strongly coupled, and second, the energy of the intra cavity should be sufficiently high to support the formation of dark solitons. Similar to the case of scalar dark soliton, the main challenge in the dark-bright soliton experiment study is how to detect the existence of the dark solitons as they are difficult to be observed in the time domain. Although there are several experimental observations in fiber lasers,^{233–244} in which they claim that the “dark-bright pulse” was obtained. However, these reported pulses were not verified as dark-bright solitons. Owing to the limitation of the detection system, there are still no systematic experimental studies on the formation of the dark-bright solitons in fiber lasers, to the best of the authors’ knowledge.

To generate the dark-bright vector solitons, both dark pulse and bright pulse must be automatically obtained in the cavity. Therefore, proper cavity dispersion must be chosen. Based on the previous experiments, it was found that in all anomalous dispersion cavities, dark solitons were difficult to be achieved, while bright pulses cannot be shaped into solitons without mode locking in all normal dispersion cavities. A dispersion-managed cavity is thus necessary in order to achieve the dark-bright soliton operation of the fiber lasers. The authors have experimentally investigated dark-bright solitons in the fiber lasers. We found that stable dark-bright solitons could be formed in fiber lasers when the net cavity dispersion is set near the net zero point. Technically, this was achieved with a dispersion-managed laser cavity. Another essential condition for dark-bright soliton formation is low birefringence of the cavity so that the coherent coupling could occur between the two polarizations of the light in the fiber. A near zero dispersion regime is preferred to keep the balance between the dark component and the bright component. Fulfilling the above conditions, a stable dark-bright soliton pulse train can be generated. Through the above experimental studies, the mechanism of the dark-soliton generation is also revealed. The dark-bright soliton formation is due to the strong cross polarization coupling of light in the fiber lasers, which coincides with the theoretical prediction of Christodoulides.²³² The generation of stable dark-bright soliton pulses provides a potential on the study of the vector dark-bright solitons and other vector solitons for optics communications.

C. Dark-dark vector solitons

Dark-dark soliton solutions appear in the case of defocusing Manakov model and the mixed focusing and defocusing nonlinearity. For the first case, Radhakrishnan and Lakshmanan studied the coupled equations as follows:

$$\begin{aligned} iu_z - u_{tt} + 2\mu(|u|^2 + |v|^2)u &= 0, \\ iv_z - v_{tt} + 2\mu(|u|^2 + |v|^2)v &= 0, \end{aligned} \tag{12}$$

by Hirota’s bilinear method in 1995.¹⁵⁵ They derived the dark-dark solitons

$$\begin{aligned} u(z, t) &= -\frac{\tau_1}{2} \left[(1 + Z_g) - (1 - Z_g) \tanh\left(\frac{P_1 t - \Omega_1 z + \xi_1^0}{2}\right) \right] \\ &\quad \times e^{i[l_1 t - (\lambda - l_1^2)z + \psi_1^0]}, \\ v(z, t) &= -\frac{\tau_2}{2} \left[(1 + Z_h) - (1 - Z_h) \tanh\left(\frac{P_2 t - \Omega_2 z + \xi_2^0}{2}\right) \right] \\ &\quad \times e^{i[l_2 t - (\lambda - l_2^2)z + \psi_2^0]}, \end{aligned} \tag{13}$$

where $l_1, l_2, \psi_1^0, \psi_2^0, P_1, \Omega_1,$ and ξ_1^0 are all real constants, $\tau_1, \tau_2, Z_g,$ and Z_h are all complex constants, and $|Z_g| = |Z_h| = 1$. A typical dark-dark vector soliton is shown in Fig. 6.

For the second case, Ohta, Wang, and Yang solved the N-dark-dark soliton solutions by Hirota’s bilinear method in 2011.¹⁶⁰ Vector dark solitons in optical fibers have been theoretically predicted for two decades.²⁴⁵ Vector dark pulses were reported by Zhang *et al.*²⁴⁶ In 2015, Guo predicted the energy exchange of dark solitons in fiber lasers.¹⁴⁴ However, the soliton properties of the pulses have not been confirmed owing to the low speed detecting system. Dark vector solitons so far have not been experimentally verified in the fiber laser systems. It would be a potential study direction on the soliton fiber lasers.

IV. DISSIPATIVE SOLITONS IN FIBER LASERS

Dissipative solitons are localized structures of an electromagnetic field that are balanced through an energy exchange with the environment in the presence of nonlinearity, dispersion, or diffraction.²⁴⁷ The concept of dissipative solitons is not restricted to optics, which can be extended to physics, biology, and medicine and so on. The most prominent feature of dissipative solitons is that they exist only when there is a continuous energy supply from an external source. In the fiber lasers, conventional solitons are formed due to the balance between anomalous fiber dispersion and self-phase modulation (SPM), whose properties are determined by the nonlinear Schrödinger equation, while dissipative solitons are formed as a result of the mutual nonlinear interactions among the normal cavity dispersion, SPM, effective gain bandwidth filtering, and gain saturation. Properties of the dissipative solitons are determined by the Ginzburg–Landau equation. As a result of different soliton formation mechanism, the dissipative solitons

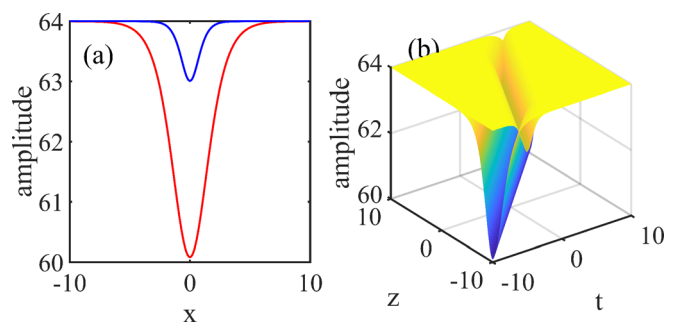


FIG. 6. The dark-dark soliton solution of Eq. (13) with $\tau_1 = 4\sqrt{2} + 4\sqrt{2}i, \tau_2 = 4\sqrt{2} - 4\sqrt{2}i, Z_g = \cos\frac{1}{2} + i\sin\frac{1}{2}, Z_h = \cos\frac{1}{4} + i\sin\frac{1}{4}, P_1 = \Omega_1 = \Omega_2 = 1, \Omega_2 = 2,$ and $\xi_1^0 = \xi_2^0 = 0$. (a) Dark-dark vector soliton profiles. (b) Evolution plot of the dark-dark vector soliton.

exhibit remarkably distinct properties from those of the conventional solitons.

A. Ginzburg–Landau equation and dissipative soliton solution

The Ginzburg–Landau equation, which is a nonintegrable system, can be seen as the nonlinear Schrödinger equation with gain and loss. In 2016, the bright solitons and dark solitons were demonstrated by Mirzazadeh *et al.*²⁴⁸ The earlier study of the bright-soliton-like solutions was given by Pereira and Stenflo in 1977.²⁴⁹ The works of Liu, Li, and Tian demonstrated the dark-soliton-like solutions in 2009.²⁵⁰

Considering the Ginzburg–Landau equation, If $g = 0$ and $\alpha = 0$, the equation returns back to the NLSE. According to the study of Pereira and Stenflo,²⁴⁹ an exact solution of GLE in the anomalous dispersion regime is a chirped hyper-secant pulse which is given by

$$\mu(\zeta, \tau) = N_s \operatorname{sech}(p\tau) \exp[iK_s \zeta - iq \ln(\cosh(p\tau))], \quad (14)$$

where $\zeta = \frac{z|\beta_2|}{T_0}$, $\tau = \frac{T}{T_0}$, $u = \left(\frac{\gamma T_0^2}{|\beta_2|}\right)^{1/2}$. N_s , p , q , and K_s are constants depending on the parameter $d = \frac{gT_0^2}{|\beta_2|}$. Equation (14) is a dissipative soliton solution. Solitons whose dynamics are described by one-dimensional GLE are known as the scalar dissipative solitons. Vector dissipative solitons are governed by a coupled GLE.

Strictly speaking, solitons in the fiber lasers are all dissipative owing to the existence of the gain fiber and losses. As NLSE based bright solitons are not supported in all normal dispersion regime, NLSE type solitons were experimentally studied in mode locked fiber lasers with anomalous dispersion. While dissipative solitons are intensively studied in the mode-locked fiber lasers with all normal dispersion.²⁵¹ Dissipative solitons in the fiber lasers are initially named as gain-guided solitons when they were first observed in fiber lasers with net normal dispersion.^{252–255} In 2012, Grelu and Akhmediev reviewed the recent development of dissipative solitons in lasers.²⁴⁷ Therefore, we will not discuss it further here.

B. Flat-top dissipative soliton

Dissipative solitons are closely related to the high-energy pulses in the fiber lasers owing to its intrinsic feature.^{256–267} As the development of the dissipative soliton research, an interesting pattern called dissipative soliton resonance (DSR) was reported. A DSR pulse is a complex of two interacting dissipative fronts. Under small gain conditions, two fronts are closely connected, forming a plain dissipative soliton. With increasing gain, the resonance effect limits the growth of peak power, while allowing for two fronts moving apart from each other. The pulse generates a plane wave in the center, which strongly binds two fronts together. The central plane wave extends, and the distance between the two fronts grows linearly and infinitely with the energy supply, whereas the fronts themselves do not change. The central plane wave and the fronts were also found to feature different chirps: A moderately low linear chirp throughout the extended central plane wave and large linear chirps across both fronts.

Dissipative soliton resonance is an interesting phenomenon in fiber lasers with dissipative solitons and has been frequently observed and reported.^{90,188,218,257,268–328} The concept of dissipative soliton resonance was first proposed by Chang in 2008.^{269,270} They theoretically predicted existence of dissipative soliton resonance in a mode-locked

fiber laser, which enables an almost infinite boost in the pulse energy without wave breaking.²⁷⁸ They also predicted that DSR operation could be obtained in both anomalous and normal dispersion regions.^{269,272} Experimentally in the fiber lasers, DSR phenomenon leads to the formation of flat-top dissipative solitons. In 2009, Wu *et al.* observed DSR operation in a 1.56 μm normal-dispersion erbium-doped fiber laser mode-locked by nonlinear polarization rotation.²⁷³ The pulse width of the as observed flat top solitons increased as the pump power increased. It was claimed that the DSR occurs when the pulse peak is clamped in the laser cavity. In 2012, Luo *et al.* reported the pulse dynamics operating in the DSR region of a ring laser, which showed that DSR phenomenon transits a sech-like pulse to flat top dissipative soliton as the input power increased.²⁷⁶ In 2014, Mei *et al.* demonstrated the generation of width- and amplitude-tunable DSR pulses by a dual-pump passive mode-locked erbium-doped fiber laser.³²⁵ A numerical study on the DSR in the fiber laser showed that DSR-type flat-top dissipative soliton generation is attributed to the peak power clamping effect by inducing spectral filtering into the laser cavity.²⁸⁸

DSR type flat-top pulses were continuously studied because of their high pulse energy. Krzempek *et al.* successfully realized high-power pulse generation in figure-8 Er:Yb double-clad fiber laser with a 2.3 μJ pulse energy and a 455 ns pulse width.²⁹³ Semaan *et al.* further increased the DSR pulse energy to as high as $\sim 10 \mu\text{J}$ with a pulse width of 416 ns from figure-8 double-clad Er:Yb fiber laser.²⁹⁷ Du *et al.* proposed a short-length nonlinear optical loop mirror to significantly boost the peak power of the DSR pulse and achieved 100 ps DSR pulses at a wavelength of 1.56 μm with an average power of 1.2 W and a peak power of 700 W.³⁰⁸ DSR pulses in 2 μm all-fiber laser were recently reported by Xu *et al.* with 6.19 ns pulse width and 19.51 nJ pulse energy²⁹⁰ and they further obtained 1.96 μm DSR pulses with a tunable pulse width of 3.74–72.19 ns and a peak power of only 0.56 W.³⁰¹ Du *et al.* reported direct generation of a high power and large energy dissipative soliton resonance in a thulium-doped double-clad fiber laser with pulse energy of 353 nJ.³²⁴ Recently, DSR is reported in a holmium doped fiber laser,³²⁹ which further proves that DSR is an intrinsic feature of the mode locked fiber lasers.

In the fiber lasers, another flat-top solitons based on GLE, which was named as kink-antikink bound states, was theoretical predicted by Malomed.³³⁰ Experimentally, this type of soliton was observed together with polarization domain walls in the fiber lasers, which will be further discussed in Sec. VII.

V. BREATHERS: DISPERSION-MANAGED SOLITONS

Breather as an essential extension of the concept of fundamental single-soliton state is an important class of solutions in nonlinear Schrodinger equation. Breather is the periodic solution with two types: Standing ones and traveling ones. The first was obtained by Kuznetsov.³³¹ In 1979, Ma studied the focusing NLS equation (2) and derived the regular breather³³² as follows:

$$A(z, t) = e^{i\omega z} \left(d + \frac{4\nu b}{d^2} - \frac{4i\zeta}{d^2} (\mu e^{i\omega z} + \mu^* e^{-i\omega z}) \right) \left(\frac{4}{d^2} e^{2i\nu t} + \frac{\zeta^2 |\mu|^2}{d^2} e^{-2i\nu t} + \frac{4b}{d^2 \nu} \right), \quad (15)$$

where

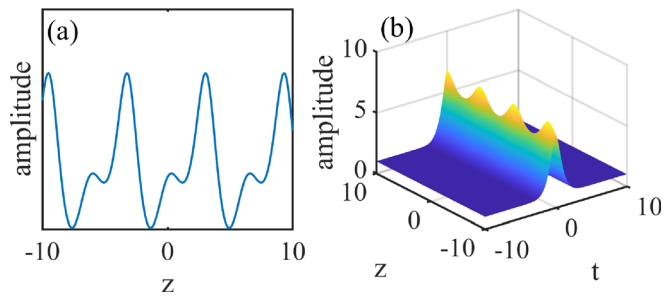


FIG. 7. The breather solution of Eq. (15) with $\omega = 1$, $d = 1$, $\mu = 1$, $\xi = 4$, $\nu = 1$, and $z_0 = 0$. (a) Breather solution. (b) Evolution plot of breather.

$$\mu(t) = \mu(0)e^{(-4iv\xi - 2id^2)z}, \quad \frac{\xi|\mu|^2}{4\nu^2} = e^{4\nu z_0}, \quad b = i(\mu e^{i\omega z} - \mu^* e^{-i\omega z}).$$

Later, Akhmediev also derived the breather solution in 1987.³³³ Recently, dynamics of nonlinear Schrödinger breathers in a potential trap was reported by Malomed *et al.*³³⁴ A typical breather solution is shown in Fig. 7.

Experimentally, two-soliton or N-soliton breathers are intrinsically unstable in a fiber laser due to the perturbations of the cavity components and gain and losses, so far they have not been observed in fiber lasers; therefore, we did not specifically discuss the case. In the mode locked fiber lasers, breathers were generated in the oscillator with dispersion-management technique. As discussed in Sec. II, conventional solitons governed by NLSE are generated and studied in the fiber laser with all anomalous dispersion. While the dissipative solitons are studied in all-normal dispersion cavity. Breathers are studied in a dispersion-managed cavity, in which fibers with negative dispersion and positive dispersion are distributed. The generated pulses are compressed and stretched in different dispersion regime, which is named as stretch pulse generation in fiber lasers. Stretch pulse technique, first proposed by Tamura *et al.*,³³⁵ by using the dispersion management, is capable to support the generation of ultrashort pulse with pulse duration of <100 fs or large single pulse energy >1 nJ in the fiber laser cavity. Dispersion managed soliton is intensively studied in mode locked fiber lasers.^{87,177,196,336–358} A detailed review on the dispersion-managed solitons in fiber lasers was reported by Turitsyn *et al.* at 2012.³⁵⁹

VI. SOLITON DYNAMICS: MULTIPLE SOLITON INTERACTIONS AND BOUND STATES OF SOLITONS

Soliton dynamics has been a subject of intensive research in fiber lasers.^{38,360,361} Harmonic mode-locking,³⁶² period doubling,³⁶³ and soliton pairs^{364,365} are examples of phenomena that deserved substantial theoretical and experimental efforts. In the case of vector soliton, Menyuk *et al.* showed that two orthogonally polarized components of pulse propagating in a birefringent nonlinear environment can be coupled and propagate with equal group velocity.²⁰ This phenomenon underlies the formation of vector solitons in the laser cavity. In multiple-pulse regime typical for cavities with anomalous dispersion, the vector soliton interaction could lead to a bunch development propagating as an entity at fundamental repetition rate. It was found that contrary to scalar soliton bunch behavior, the vector solitons exhibit periodic-like contractive and repulsive motion within the bunch. This

slow response of absorption from mode locker induces an attractive force between vector solitons resulting in a tight bunch formation with temporal separation between pulses at a picoseconds scale.¹⁷⁹ Therefore, it was established that details of absorption recovery could be an instrumental for vector soliton control.

In this section, we will briefly review the multiple soliton formation, and interactions and formation of bound solitons, which is a typical pattern owing to the effect of soliton interactions.

A. Multiple soliton interactions

Multiple soliton formation in mode locked fiber lasers is a well-known phenomenon and has been extensively investigated in the past.^{43,44,366–371} It has been shown that various mechanisms could lead to the formation of multiple solitons. These mechanisms include the wave-breaking effect, the effective spectral filter effect, the soliton peak clamping effect, and the soliton shaping of dispersive waves. Indeed, in previous experimental studies on the multiple solitons formed in the fiber lasers, people have observed various modes of multiple soliton operation, such as soliton bunches, soliton collisions,³⁷² vibration of soliton pairs,³⁶⁵ restless solitons,¹⁷⁹ bound state of solitons,^{373,374} and so on. Some of these effects can be traced back as a result of the direct soliton interaction of the dissipative solitons, or the dispersive waves mediated NLSE type of soliton interaction. Therefore, based on the different features of the multiple soliton operation of a fiber laser, one can get an insight into the properties of the formed solitons. Recently, a novel form of multiple soliton operation named as “soliton rain” was observed by Chouli *et al.*^{375–377} It was shown that the multiple soliton formation in a fiber laser could even manifest the process of the rain-drop formation in the nature, or in another word, the multiple soliton interaction in a fiber laser follows the universal statistics of the many body systems.

To study the multiple vector soliton formation, SESAM was initially employed in the fiber laser cavity. In an experiment with a SESAM as the passive mode locker, Zhao *et al.* have observed a state of so-called bunched restless vector solitons.¹⁷⁹ Vector soliton bunching controlled by SESAMs with different recovery times was also experimentally investigated by Gumenyuk *et al.*³⁷⁸ Recently, the mode locking of fiber lasers with atomic-layer graphene based saturable absorbers has attracted considerable attention of research.^{84,89,93,98–100,191,198}

B. Bound states of solitons

Bound states of solitons, also named as bound soliton or soliton molecules, are referred to that two or more fundamental solitons bind tightly together in the temporal or spatial domain through the direct soliton interaction. The solitons in the state have not only fixed separations but also fixed phase differences, and the assembly of the solitons behaves like a new super-soliton. The formation of bound solitons can be attributed to the multiple soliton interactions.

In 1994, Haelterman and Sheppard studied the coupled NLSEs and showed the existence of bound vector solitary waves which had the possibility to increase the bandwidth of transmission lines.³⁷⁹ Theoretically, formation of bound states of conventional solitons in the extended nonlinear Schrödinger equation systems was first predicted by Malomed.^{380,381} Afanasjev *et al.* theoretically studied the stability of bound state of dissipative solitons in the Ginzburg–Landau equations.³⁸² Akhmediev *et al.* also studied the formation of bound

states of solitons in the complex Ginzburg–Landau equation system.^{383,384} Formation of bound states of solitons in fiber lasers has attracted considerable interest experimentally. It is anticipated that the soliton fiber lasers could serve as an ideal testbed for the study on bound solitons. So far, there have been plenty of reports on bound solitons^{72,75,80,86,100,110,130,134,187,348,371,374,385–442} or claimed as soliton molecules^{70,74,78,96,131,189,209,214,354,443–463} in mode locked fiber lasers. The bound states of solitons were first experimentally observed in a fiber laser mode locked with the nonlinear polarization rotation technique.^{374,385} Later, with the development of novel material based real saturable absorber (SA) mode locking techniques, such as the carbon nanotube mode locking and 2D-nano-materials mode locking, formation of bound states of solitons has also been observed in fiber lasers mode locked with the carbon nanotubes,^{410,421,423,464,465} graphene,^{70,72,74,75,78,80,86,96,100} MoS₂,^{130,131,134} black phosphorus,^{109,110} and so on. As these saturable absorbers may be polarization independent, bound states of vector solitons were also reported in fiber lasers.^{100,187,189,209,214,464} Mou *et al.* reported the existence of bound vector solitons in a carbon nanotube mode locked fiber laser.⁴⁶⁴ Luo *et al.* observed the group velocity locked vector soliton molecules in a SESAM mode locked fiber laser.²¹⁴ Bound vector solitons were also found with properties of soliton interaction in the laser cavity. In 2016, Song reported the experimental observation of coexistence and interactions between vector and bound vector solitons in a fiber laser passively mode locked by graphene.¹⁰⁰ Owing to the bound soliton interactions, bound-bound soliton was also found experimentally.⁴⁶⁵

VII. POLARIZATION DOMAIN WALL AND DOMAIN WALL SOLITONS

Domain wall (DW) refers to a topological defect that connects two stable static states of a physical system. Formation of the DWs is recognized as a spontaneous symmetry-breaking phase transition in a variety of contexts, which are ubiquitous and generic in the entire field of nonlinear physics ranging from ferromagnetism theory to optics and Bose-Einstein condensate, DNA fluctuations, deoxyribonucleic acid, and string theory. In mathematics, domain wall also appears as kinks, in close analogy with the celebrated kink solutions of the Sine-Gordon equation. In 1982, Boiti, Laddomada, and Pempinelli solved kink solitons by applying the Bäcklund transformation to the NLSE.⁴⁶⁶ The single-kink soliton solution is given by

$$A(z, t) = 2\sigma_0 \tanh \left[\sqrt{2}\sigma_0(t - t_0 + 4\zeta_0 z) \right] e^{i[-2\zeta_0 t - 4(\sigma_0^2 + \zeta_0^2)z - \theta_0]}, \quad (16)$$

where σ_0 and ζ_0 are two real parameters, the real constants θ_0 , t_0 fix the location of the traveling wave at the initial time, $\beta = -2$, and $\gamma = -1$. A kink soliton solution is shown in Fig. 8. It is noted that the kink solitons are different from the dark solitons shown in Fig. 3. As shown in Figs. 3 and 8, kink soliton has different boundary conditions as $x \rightarrow \pm\infty$ or $t \rightarrow \pm\infty$. While the dark soliton has the same boundary conditions as $x \rightarrow \pm\infty$ or $t \rightarrow \pm\infty$. By a method based on the association of the Painlevé test theory and Hirota's bilinear technique, Pelap and Faye obtain the kink solitons of Ginzburg–Landau equation in 2004.⁴⁶⁷

In the field of nonlinear optics, Zakharov and Mikhailov first theoretically predicted the existence of stable optical domain walls.⁴⁶⁸ Formation of optical domain walls is independent of dispersion, purely through the interaction between the polarization states of two counter-

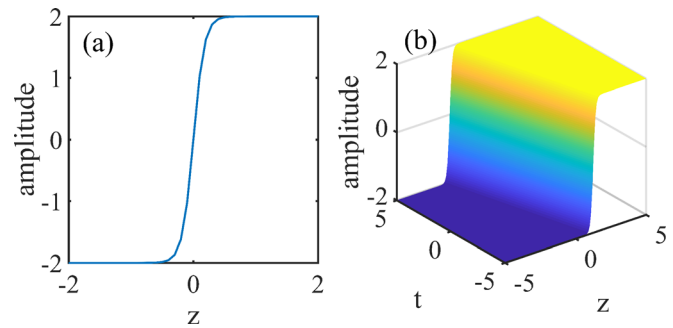


FIG. 8. The kink soliton solution of Eq. (16) with $\sigma_0 = \zeta_0 = 1$, $\theta_0 = t_0 = 1$. (a) Kink soliton solution. (b) Evolution plot of kink soliton.

propagating electromagnetic waves.⁴⁶⁸ They pointed out that in analog to the formation of the magnetic domains in the ferromagnetic materials, the cross-interaction between two counter-propagating optical beams in a third-order nonlinear medium could lead to the formation of optical polarization domains. Physically, the formation of polarization domains is due to that a Hamiltonian system in the stable state must have the minimum energy. Associated with the formation of polarization domains, it turns out that the domain walls that separate the polarization domains are a stable localized structure. They constitute a fundamentally new type of solitary waves, known as the polarization domain wall solitons.⁴⁶⁹ In optical fibers, Wabnitz and Daino analytically predicted the possibility of generating domain walls as a result of polarization switching,⁴⁷⁰ and Pitois experimentally confirmed the prediction.⁴⁷¹ Later, Haelterman, Sheppard, and Malomed found that the dynamics of domain wall soliton could be well encapsulated by the incoherently coupled NLSEs where both the nonlinear polarization coupling and normal dispersion were taken into account.^{173,174} In 1990, the domain wall of coupled Ginzburg–Landau equation was obtained by Malomed, Nepomnyashchy, and Tribelsky.⁴⁷² In 1994, Haelterman and Sheppard obtained the domain walls by studying the coupled NLSEs.⁴⁷³ Malomed also theoretically studied the formation of domain walls between two traveling waves,⁴⁷⁴ and the dynamical properties of domain walls formed in twist single mode fibers.⁴⁷⁵ It was shown that domain walls between linearly polarized beams could also be possible.

Experimental observation of kink solitons in fiber lasers does not rely on the dispersion of the fiber cavity. And the kink solitons observed have good long-term stability. It is different from the NLSE dark solitons, which were only supported in normal dispersion regime and difficult to be observed. Experimental demonstration of the polarization domains is as early as 1995 when Kockaert *et al.* experimentally demonstrated the generation of polarization domains using a spun fiber.⁴⁷⁶

So far, a few experimental observations on the polarization domain formation and polarization domain wall solitons in fiber lasers are reported.^{180,246,320,477–483} Zhang *et al.* reported the experimental observation of a kind of periodic fast polarization switching between the two orthogonal linear eigen-polarizations of an erbium-doped fiber ring laser.²⁴⁶ The experimental studies on the phenomenon suggested that it could be due to the polarization domain formation in the laser. In their experiment, the detection system is of 1 GHz bandwidth to monitor the polarization resolved laser emission. Limited by the

bandwidth of the detection system, some important features of the polarization dynamics of the laser could not be observed. In 2014, Tang *et al.* experimentally and theoretically studied the polarization domain formation and domain dynamics in a quasi-isotropic cavity fiber laser.⁴⁸³ It is shown experimentally that the polarization domain formation is a general feature of the quasi-isotropic cavity fiber lasers; it can occur either under the incoherent or coherent coupling between the two orthogonal linear polarization modes of the lasers. Lecaplain *et al.* present a simple theoretical model that explains polarization switching in fiber ring lasers operating with a normal path-averaged dispersion and a typical intermediate level of birefringence. The proposed polarization dynamics is based on a type of polarization domain wall structures and was named as polarization domain wall complex in fiber lasers.⁴⁷⁹

Recently, Gilles *et al.* experimentally demonstrated the existence of a universal class of polarization domain walls in the form of localized polarization knots in conventional optical fibers.⁴⁸⁴ It is demonstrated that how trapping energy in a well-defined train of polarization domain walls allows undistorted propagation of polarization knots at a rate of 28 GHz along a 10 km length of normally dispersive optical fiber. These results constitute the first experimental observation of kink-antikink solitary wave propagation in nonlinear fiber optics, which may find important applications for the polarization domain walls in the future.

VIII. SUMMARY AND OUTLOOK

In summary, we have presented divergent aspects of the physics of optical solitons in fiber lasers, including analytical, numerical, and experimental results. They demonstrate a number of interesting properties of these nonlinear waves which can exist on a background being characterized by a nontrivial phase distribution of the field. In many cases discussed in this review, these solitary waves not only can be described analytically and numerically for a variety of nonlinear models but also, and this is the most amazing fact, be verified, with relatively good accuracy, by employing the fiber lasers as a platform.

In the last decade, development of two-dimensional materials has enhanced the investigation on solitons in fiber lasers. Searching for and employing new materials with strong nonlinear properties may sufficiently speed up this process. Recently, a novel technique named time-stretch spectroscopy was applied in the analysis of the ultrafast lasers.^{485,486} This technique has been applied for the analysis of pulse growth dynamics,⁴⁸⁷ soliton molecules,⁴⁸⁸ soliton burst,⁴⁸⁹ dissipative solitons,⁴⁹⁰ dissipative soliton molecules and interactions,⁴⁹¹ rogue waves,⁴⁹² and so on. It can be anticipated that time-stretch dispersive Fourier transform technique will be a powerful tool for the study of soliton in fiber lasers in the future.

ACKNOWLEDGMENTS

National Natural Science Foundation of China (61705140).

REFERENCES

- ¹R. F. Gwyther, "The general motion of long waves, with an examination of the direct reflexion of the solitary wave," *Philos. Mag.* **50**(304), 349–352 (1900).
- ²A. Boulanger, "Theory of the solitary wave which propagates itself the length of a horizontal flexible tube," *C. R. Hebd. Seances Acad. Sci.* **141**, 1001–1004 (1905).
- ³J. D. Gibbon and J. C. Eilbeck, "Possible N soliton solution for a nonlinear optics equation," *J. Phys. A: Gen. Phys.* **5**(11), L122 (1972).
- ⁴P. J. Caudrey *et al.*, "Multiple soliton and bisoliton bound-state solutions of sine-Gordon equation and related equations in nonlinear optics," *J. Phys. A: Math. Gen.* **6**(8), L112–L115 (1973).
- ⁵J. C. Eilbeck *et al.*, "Solitons in nonlinear optics.1. More accurate description of 2pi pulse in self-induced transparency," *J. Phys. A: Math. Gen.* **6**(9), 1337–1347 (1973).
- ⁶A. E. Kaplan, "Bistable solitons of highly-nonlinear Schrodinger-equations in nonlinear optics," *Math. Comput. Modell.* **11**, 106–111 (1988).
- ⁷Y. S. Kivshar, "Dark solitons in nonlinear optics," *IEEE J. Quantum Electron.* **29**(1), 250–264 (1993).
- ⁸A. Boardman, "Nonlinear optics - Solitons light the way," *Nature* **387**(6636), 854–855 (1997).
- ⁹K. Porsezian, "Soliton propagation in nonlinear optics with higher-order effects," *J. Mod. Opt.* **44**(2), 387–394 (1997).
- ¹⁰U. Brinkmann, "Nonlinear optics - Spatial solitons of moderate power interact," *Laser Focus World* **40**(4), 32 (2004).
- ¹¹A. I. Maimistov, "Solitons in nonlinear optics," *Quantum Electron.* **40**(9), 756–781 (2010).
- ¹²X. G. Lin, W. J. Liu, and M. Lei, "Oscillating solitons in nonlinear optics," *Pramana* **86**(3), 575–580 (2016).
- ¹³M. Inc *et al.*, "Optical solitons for complex Ginzburg-Landau model in nonlinear optics," *Optik* **158**, 368–375 (2018).
- ¹⁴N. J. Zabusky and M. D. Kruskal, "Interaction of solitons in a collisionless plasma and recurrence of initial states," *Phys. Rev. Lett.* **15**(6), 240 (1965).
- ¹⁵C. S. Gardner *et al.*, "Method for solving Korteweg-Devries equation," *Phys. Rev. Lett.* **19**(19), 1095 (1967).
- ¹⁶T. Maxworthy, "Experiments on collisions between solitary waves," *J. Fluid Mech.* **76**, 177–185 (1976).
- ¹⁷G. P. Agrawal, "Nonlinear fiber optics," in *Quantum Electronics-Principles and Applications*, 4th ed. (Elsevier/Academic Press, Amsterdam, Boston, 2007), Vol. xvi, p. 529.
- ¹⁸A. Hasegawa and F. Tappert, "Transmission of stationary nonlinear optical pulses in dispersive dielectric fibers.1. Anomalous dispersion," *Appl. Phys. Lett.* **23**(3), 142–144 (1973).
- ¹⁹A. Hasegawa and F. Tappert, "Transmission of stationary nonlinear optical pulses in dispersive dielectric fibers.2. Normal dispersion," *Appl. Phys. Lett.* **23**(4), 171–172 (1973).
- ²⁰C. R. Menyuk, "Stability of solitons in birefringent optical fibers.1. Equal propagation amplitudes," *Opt. Lett.* **12**(8), 614–616 (1987).
- ²¹M. Sheikbaha, A. A. Said, and E. W. Vanstryland, "High-sensitivity, single-beam N2 measurements," *Opt. Lett.* **14**(17), 955–957 (1989).
- ²²V. E. Zakharov and A. B. Shabat, "Exact theory of 2-dimensional self-focusing and one-dimensional self-modulation of waves in nonlinear media," *J. Exp. Theor. Phys.* **34**(1), 62 (1972).
- ²³H. A. Haus, "Mode-locking of lasers," *IEEE J. Sel. Top. Quantum Electron.* **6**(6), 1173–1185 (2000).
- ²⁴L. E. Hargrove, R. L. Fork, and M. A. Pollack, "Locking of He-Ne laser modes induced by synchronous intracavity modulation," *Appl. Phys. Lett.* **5**(1), 4 (1964).
- ²⁵M. Horowitz *et al.*, "Pulse dropout in harmonically mode-locked fiber lasers," *IEEE Photonics Technol. Lett.* **12**(3), 266–268 (2000).
- ²⁶M. Horowitz and C. R. Menyuk, "Analysis of pulse dropout in harmonically mode-locked fiber lasers by use of the Lyapunov method," *Opt. Lett.* **25**(1), 40–42 (2000).
- ²⁷A. R. Clobs and M. J. Brienza, "Passive mode locking of a pulsed Nd-Yag laser," *Appl. Phys. Lett.* **14**(9), 287 (1969).
- ²⁸O. R. Wood and S. E. Schwarz, "Passive mode locking of a Co2 laser," *Appl. Phys. Lett.* **12**(8), 263 (1968).
- ²⁹A. G. Bulushev, E. M. Dianov, and O. G. Okhotnikov, "Passive-mode locking of a laser with a nonlinear fiber reflector," *Opt. Lett.* **15**(17), 968–970 (1990).
- ³⁰T. L. Paoli, "Saturable absorption effects in the self-pulsing (Alga)as junction laser," *Appl. Phys. Lett.* **34**(10), 652–655 (1979).
- ³¹C. W. Chang and S. Chi, "Mode-locked erbium-doped fibre ring laser using nonlinear polarization rotation," *J. Mod. Opt.* **45**(2), 355–362 (1998).

- ³²E. Yoshida, Y. Kimura, and M. Nakazawa, "Femtosecond erbium-doped fiber laser with nonlinear polarization rotation and its soliton compression," *Jpn. J. Appl. Phys., Part 1* **33**(10), 5779–5783 (1994).
- ³³N. Pandit, D. U. Noske, and J. R. Taylor, "350-fs pulse generation from an erbium fiber ring laser mode-locked using nonlinear polarization rotation," *J. Mod. Opt.* **41**(1), 11–14 (1994).
- ³⁴V. J. Matsas *et al.*, "Self-starting passively mode-locked fiber ring soliton laser exploiting nonlinear polarization rotation," *Electron. Lett.* **28**(15), 1391–1393 (1992).
- ³⁵A. Komarov, H. Leblond, and F. Sanchez, "Passive harmonic mode-locking in a fiber laser with nonlinear polarization rotation," *Opt. Commun.* **267**(1), 162–169 (2006).
- ³⁶S. M. J. Kelly, "Characteristic side-band instability of periodically amplified average soliton," *Electron. Lett.* **28**(8), 806–807 (1992).
- ³⁷A. Komarov, H. Leblond, and F. Sanchez, "Modelling of multiple soliton operation of fiber lasers with anomalous frequency dispersion," in *Proceedings of 2007 Icton Mediterranean Winter Conference* (2007), pp. 316–319.
- ³⁸P. Grellu and J. M. Soto-Crespo, "Multisoliton states and pulse fragmentation in a passively mode-locked fibre laser," *J. Opt. B: Quantum Semiclassical Opt.* **6**(5), S271–S278 (2004).
- ³⁹D. Y. Tang *et al.*, "Mechanism of multisoliton formation and soliton energy quantization in passively mode-locked fiber lasers," *Phys. Rev. A* **72**(4), 043816 (2005).
- ⁴⁰L. R. Wang *et al.*, "Energy quantisation for dissipative solitons," *Electron. Lett.* **46**(6), 436–U80 (2010).
- ⁴¹W. H. Renninger, A. Chong, and F. W. Wise, "Area theorem and energy quantization for dissipative optical solitons," in *Proceedings of 2009 Conference on Lasers and Electro-Optics and Quantum Electronics and Laser Science Conference* (Cleo/QELS 2009), Vol. 1–5, pp. 1899–1900.
- ⁴²R. Weill *et al.*, "Long range soliton interaction related to sidebands generation in mode-locked lasers," in *Proceedings of 2007 Conference on Lasers & Electro-Optics/Quantum Electronics and Laser Science Conference* (Cleo/QELS 2007), Vol. 1–5, pp. 1265–1266.
- ⁴³D. Y. Tang *et al.*, "Soliton interaction in a fiber ring laser," *Phys. Rev. E* **72**(1), 016616 (2005).
- ⁴⁴N. Akhmediev *et al.*, "Dissipative soliton interactions inside a fiber laser cavity," *Opt. Fiber Technol.* **11**(3), 209–228 (2005).
- ⁴⁵U. Keller *et al.*, "Semiconductor saturable absorber mirrors (SESAM's) for femtosecond to nanosecond pulse generation in solid-state lasers," *IEEE J. Sel. Top. Quantum Electron.* **2**(3), 435–453 (1996).
- ⁴⁶S. Yamashita, "Carbon nanotube based mode-locked fiber lasers," in *Proceedings of Aoe 2008: Asia Optical Fiber Communication and Optoelectronic Exposition and Conference* (2009).
- ⁴⁷Z. Sun *et al.*, "L-band ultrafast fiber laser mode locked by carbon nanotubes," *Appl. Phys. Lett.* **93**(6), 061114 (2008).
- ⁴⁸A. V. Tausenev *et al.*, "177 fs erbium-doped fiber laser mode locked with a cellulose polymer film containing single-wall carbon nanotubes," *Appl. Phys. Lett.* **92**(17), 171113 (2008).
- ⁴⁹S. Yamashita *et al.*, "Ultrafast saturable absorbers based on carbon nanotubes and their applications to passively mode-locked fiber lasers," *Electron. Commun. Jpn. Part II-Electron.* **90**(2), 17–24 (2007).
- ⁵⁰S. Yamashita *et al.*, "Passively mode-locked short-cavity 10 GHz Er: Yb-codoped phosphate-fiber laser using carbon nanotubes," in *Proceedings of Fiber Lasers IV: Technology, Systems, and Applications* (2007), Vol. 6453.
- ⁵¹K. Jiang *et al.*, "A wavelength-switchable passively harmonically mode-locked fiber laser with low pumping threshold using single-walled carbon nanotubes," *IEEE Photonics Technol. Lett.* **22**(11), 754–756 (2010).
- ⁵²K. Kieu, R. J. Jones, and N. Peyghambarian, "Generation of few-cycle pulses from an amplified carbon nanotube mode-locked fiber laser system," *IEEE Photonics Technol. Lett.* **22**(20), 1521–1523 (2010).
- ⁵³J. W. Nicholson and D. J. DiGiovanni, "High-repetition-frequency low-noise fiber ring lasers mode-locked with carbon nanotubes," *IEEE Photonics Technol. Lett.* **20**(21–24), 2123–2125 (2008).
- ⁵⁴Y. W. Song *et al.*, "1300-nm pulsed fiber lasers mode-locked by purified carbon nanotubes," *IEEE Photonics Technol. Lett.* **17**(8), 1623–1625 (2005).
- ⁵⁵S. Yamashita *et al.*, "5-GHz pulsed fiber Fabry-Perot laser mode-locked using carbon nanotubes," *IEEE Photonics Technol. Lett.* **17**(4), 750–752 (2005).
- ⁵⁶S. Yamashita *et al.*, "Mode-locked fiber lasers using adjustable saturable absorption in vertically aligned carbon nanotubes," *Jpn. J. Appl. Phys., Part 2* **45**(1–3), L17–L19 (2006).
- ⁵⁷E. J. R. Kelleher *et al.*, "Bismuth fiber integrated laser mode-locked by carbon nanotubes," *Laser Phys. Lett.* **7**(11), 790–794 (2010).
- ⁵⁸G. S. Qin, T. Suzuki, and Y. Ohishi, "Widely tunable passively mode-locked fiber laser with carbon nanotube films," *Opt. Rev.* **17**(3), 97–99 (2010).
- ⁵⁹S. Y. Choi *et al.*, "Femtosecond mode-locked fiber laser employing a hollow optical fiber filled with carbon nanotube dispersion as saturable absorber," *Opt. Express* **17**(24), 21788–21793 (2009).
- ⁶⁰C. M. Ouyang *et al.*, "Observation of timing jitter reduction induced by spectral filtering in a fiber laser mode locked with a carbon nanotube-based saturable absorber," *Opt. Lett.* **35**(14), 2320–2322 (2010).
- ⁶¹M. A. Solodyankin *et al.*, "Mode-locked 1.93 μm thulium fiber laser with a carbon nanotube absorber," *Opt. Lett.* **33**(12), 1336–1338 (2008).
- ⁶²Y. W. Song *et al.*, "All-fiber pulsed lasers passively mode locked by transferable vertically aligned carbon nanotube film," *Opt. Lett.* **32**(11), 1399–1401 (2007).
- ⁶³S. Yamashita *et al.*, "Saturable absorbers incorporating carbon nanotubes directly synthesized onto substrates and fibers and their application to mode-locked fiber lasers," *Opt. Lett.* **29**(14), 1581–1583 (2004).
- ⁶⁴F. Wang *et al.*, "Soliton fiber laser mode-locked by a single-wall carbon nanotube-polymer composite," *Phys. Status Solidi B* **245**(10), 2319–2322 (2008).
- ⁶⁵A. V. Tausenev *et al.*, "Self-mode-locking in erbium-doped fibre lasers with saturable polymer film absorbers containing single-wall carbon nanotubes synthesised by the arc discharge method," *Quantum Electron.* **37**(3), 205–208 (2007).
- ⁶⁶Q. Wang *et al.*, "Mode-locked fiber/waveguide lasers based on a fiber taper embedded in carbon nanotubes/polymer composite," in *Proceedings of Silicon Photonics and Photonic Integrated Circuits* (2008), Vol. 6996.
- ⁶⁷H. Zhang *et al.*, "Large energy soliton erbium-doped fiber laser with a graphene-polymer composite mode locker," *Appl. Phys. Lett.* **95**(14), 141103 (2009).
- ⁶⁸H. Zhang *et al.*, "Graphene mode locked, wavelength-tunable, dissipative soliton fiber laser," *Appl. Phys. Lett.* **96**(11), 111112 (2010).
- ⁶⁹M. A. Ismail, H. Ahmad, and S. W. Harun, "Soliton mode-locked erbium-doped fiber laser using non-conductive graphene oxide paper," *IEEE J. Quantum Electron.* **50**(2), 85–87 (2014).
- ⁷⁰L. L. Gui and C. X. Yang, "Soliton molecules with $\pm\pi/2$, 0, and π phase differences in a graphene-based mode-locked erbium-doped fiber laser," *IEEE Photonics J.* **10**(3), 1502609 (2018).
- ⁷¹Y. F. Song *et al.*, "Period-doubling and quadrupling bifurcation of vector soliton bunches in a graphene mode locked fiber laser," *IEEE Photonics J.* **9**(5), 4502308 (2017).
- ⁷²L. L. Gui *et al.*, "Widely spaced bound states in a soliton fiber laser with graphene saturable absorber," *IEEE Photonics Technol. Lett.* **25**(12), 1184–1187 (2013).
- ⁷³Z. Q. Luo *et al.*, "Multiwavelength dissipative-soliton generation in Yb-fiber laser using graphene-deposited fiber-taper," *IEEE Photonics Technol. Lett.* **24**(17), 1539–1542 (2012).
- ⁷⁴N. Zhao *et al.*, "Trapping of soliton molecule in a graphene-based mode-locked ytterbium-doped fiber laser," *IEEE Photonics Technol. Lett.* **26**(24), 2450–2453 (2014).
- ⁷⁵H. Haris *et al.*, "Generation of soliton and bound soliton pulses in mode-locked erbium-doped fiber laser using graphene film as saturable absorber," *J. Mod. Opt.* **63**(8), 777–782 (2016).
- ⁷⁶W. J. Liu *et al.*, "Analytic study on soliton amplification in graphene oxide mode-locked Er-doped fiber lasers," *J. Mod. Opt.* **61**(9), 773–777 (2014).
- ⁷⁷L. Q. Zhang *et al.*, "Wavelength tunable passively Q-switched Yb-doped double-clad fiber laser with graphene grown on SiC," *Chin. Opt. Lett.* **12**(2), 021405 (2014).
- ⁷⁸B. Gao *et al.*, "Soliton molecules in a fiber laser mode-locked by a graphene-based saturable absorber," *Laser Phys.* **25**(7), 075103 (2015).
- ⁷⁹X. Y. He, T. Chen, and D. N. Wang, "Ultra-fast solitons in a long cavity multi-mode-fiber-based graphene mode-locked fiber laser with high slope efficiency," *Laser Phys.* **24**(8), 085109 (2014).

- ⁸⁰X. L. Li *et al.*, “Observation of soliton bound states in a graphene mode locked erbium-doped fiber laser,” *Laser Phys.* **22**(4), 774–777 (2012).
- ⁸¹S. Y. Choi *et al.*, “All-fiber dissipative soliton laser with 10.2 nJ pulse energy using an evanescent field interaction with graphene saturable absorber,” *Laser Phys. Lett.* **11**(1), 015101 (2014).
- ⁸²Y. D. Cui, X. M. Liu, and C. Zeng, “Conventional and dissipative solitons in a CFBG-based fiber laser mode-locked with a graphene-nanotube mixture,” *Laser Phys. Lett.* **11**(5), 055106 (2014).
- ⁸³S. S. Huang *et al.*, “Soliton rains in a graphene-oxide passively mode-locked ytterbium-doped fiber laser with all-normal dispersion,” *Laser Phys. Lett.* **11**(2), 025102 (2014).
- ⁸⁴Y. F. Song *et al.*, “Quasi-periodicity of vector solitons in a graphene mode-locked fiber laser,” *Laser Phys. Lett.* **10**(12), 125103 (2013).
- ⁸⁵Z. H. Wang *et al.*, “The simultaneous generation of soliton bunches and Q-switched-like pulses in a partially mode-locked fiber laser with a graphene saturable absorber,” *Laser Phys. Lett.* **15**(5), 055101 (2018).
- ⁸⁶J. Boguslawski *et al.*, “Bound soliton state in all-polarization maintaining fiber laser mode-locked by graphene,” in *Proceedings of Laser Technology 2016: Progress and Applications of Lasers* (2016), p. 10159.
- ⁸⁷W. B. Wang *et al.*, “Delivering dispersion-managed soliton and Q-switched pulse in fiber laser based on graphene and nonlinear optical loop mirror,” *Opt. Laser Technol.* **85**, 41–47 (2016).
- ⁸⁸G. Yang *et al.*, “Dual-wavelength mode-locked Tm³⁺-doped fiber laser at 2 μm region with soliton pulse number by employing graphene on microfiber,” *Optics Laser Technol.* **105**, 76–79 (2018).
- ⁸⁹H. Zhang *et al.*, “Vector dissipative solitons in graphene mode locked fiber lasers,” *Opt. Commun.* **283**(17), 3334–3338 (2010).
- ⁹⁰Z. C. Cheng *et al.*, “Dissipative soliton resonance and reverse saturable absorption in graphene oxide mode-locked all-normal-dispersion Yb-doped fiber laser,” *Opt. Express* **23**(6), 7000–7006 (2015).
- ⁹¹S. Y. Choi *et al.*, “Graphene-filled hollow optical fiber saturable absorber for efficient soliton fiber laser mode-locking,” *Opt. Express* **20**(5), 5652–5657 (2012).
- ⁹²Y. D. Cui and X. M. Liu, “Graphene and nanotube mode-locked fiber laser emitting dissipative and conventional solitons,” *Opt. Express* **21**(16), 18969–18974 (2013).
- ⁹³M. M. Han *et al.*, “Polarization dynamic patterns of vector solitons in a graphene mode-locked fiber laser,” *Opt. Express* **23**(3), 2424–2435 (2015).
- ⁹⁴S. S. Huang *et al.*, “Tunable and switchable multi-wavelength dissipative soliton generation in a graphene oxide mode-locked Yb-doped fiber laser,” *Opt. Express* **22**(10), 11417–11426 (2014).
- ⁹⁵H. Jeong *et al.*, “All-fiber Tm-doped soliton laser oscillator with 6 nJ pulse energy based on evanescent field interaction with monolayer graphene saturable absorber,” *Opt. Express* **24**(13), 14152–14158 (2016).
- ⁹⁶A. P. Luo *et al.*, “Microfiber-based, highly nonlinear graphene saturable absorber for formation of versatile structural soliton molecules in a fiber laser,” *Opt. Express* **22**(22), 27019–27025 (2014).
- ⁹⁷Y. C. Meng *et al.*, “Multiple-soliton dynamic patterns in a graphene mode-locked fiber laser,” *Opt. Express* **20**(6), 6685–6692 (2012).
- ⁹⁸Y. F. Song *et al.*, “Vector multi-soliton operation and interaction in a graphene mode-locked fiber laser,” *Opt. Express* **21**(8), 10010–10018 (2013).
- ⁹⁹Y. F. Song *et al.*, “Polarization rotation vector solitons in a graphene mode-locked fiber laser,” *Opt. Express* **20**(24), 27283–27289 (2012).
- ¹⁰⁰Y. F. Song *et al.*, “Coexistence and interaction of vector and bound vector solitons in a dispersion-managed fiber laser mode locked by graphene,” *Opt. Express* **24**(2), 1814–1822 (2016).
- ¹⁰¹W. Xin *et al.*, “Flexible graphene saturable absorber on two-layer structure for tunable mode-locked soliton fiber laser,” *Opt. Express* **22**(9), 10239–10247 (2014).
- ¹⁰²J. Xu *et al.*, “Dissipative soliton generation from a graphene oxide mode-locked Er-doped fiber laser,” *Opt. Express* **20**(21), 23653–23658 (2012).
- ¹⁰³J. Sotor, G. Sobon, and K. M. Abramski, “Scalar soliton generation in all-polarization-maintaining, graphene mode-locked fiber laser,” *Opt. Lett.* **37**(11), 2166–2168 (2012).
- ¹⁰⁴L. M. Zhao *et al.*, “Dissipative soliton operation of an ytterbium-doped fiber laser mode locked with atomic multilayer graphene,” *Opt. Lett.* **35**(21), 3622–3624 (2010).
- ¹⁰⁵L. Yun, “Switchable dual-wavelength conventional soliton delivered from a graphene-mode-locked fiber laser,” *Optik* **145**, 549–554 (2017).
- ¹⁰⁶Y. Chen *et al.*, “Sub-300 femtosecond soliton tunable fiber laser with all-anomalous dispersion passively mode locked by black phosphorus,” *Opt. Express* **24**(12), 13316–13324 (2016).
- ¹⁰⁷B. Gao *et al.*, “Dissipative solitons characteristics in passively mode-locked Er-doped fiber laser based on black phosphorus as a new saturable absorber,” *Opt. Commun.* **406**, 192–198 (2018).
- ¹⁰⁸C. Y. Ma *et al.*, “Dynamic evolution of the dissipative soliton in passively mode-locked fiber laser based on black phosphorus as a new saturable absorber,” *Opt. Commun.* **406**, 177–182 (2018).
- ¹⁰⁹Y. F. Song *et al.*, “Vector soliton fiber laser passively mode locked by few layer black phosphorus-based optical saturable absorber,” *Opt. Express* **24**(23), 25933–25942 (2016).
- ¹¹⁰Z. T. Wang *et al.*, “Black phosphorus quantum dots as an efficient saturable absorber for bound soliton operation in an erbium doped fiber laser,” *IEEE Photonics J.* **8**(5), 1503310 (2016).
- ¹¹¹J. Sotor *et al.*, “Ultrafast thulium-doped fiber laser mode locked with black phosphorus,” *Opt. Lett.* **40**(16), 3885–3888 (2015).
- ¹¹²H. Yu *et al.*, “Thulium/holmium-doped fiber laser passively mode locked by black phosphorus nanoplatelets-based saturable absorber,” *Appl. Opt.* **54**(34), 10290–10294 (2015).
- ¹¹³M. H. M. Ahmed *et al.*, “Ultrafast erbium-doped fiber laser mode-locked with a black phosphorus saturable absorber,” *Laser Phys. Lett.* **13**(9), 095104 (2016).
- ¹¹⁴J. Du *et al.*, “Microfiber-based few-layer black phosphorus quantum dots saturable absorber for mode-locked fiber laser,” in *Proceedings of 2016 Conference on Lasers and Electro-Optics (CLEO)* (2016).
- ¹¹⁵E. I. Ismail *et al.*, “Black phosphorus crystal as a saturable absorber for both a Q-switched and mode-locked erbium-doped fiber laser,” *RSC Adv.* **6**(76), 72692–72697 (2016).
- ¹¹⁶A. A. Latiff *et al.*, “Black phosphorus as a saturable absorber for generating mode-locked fiber laser in normal dispersion regime,” in *Proceedings of Second International Seminar on Photonics, Optics, and Its Applications* (Isphoa 2016) (2016), p. 10150.
- ¹¹⁷J. F. Li *et al.*, “Black phosphorus: A two-dimension saturable absorption material for mid-infrared Q-switched and mode-locked fiber lasers,” *Sci. Rep.* **6**, 30361 (2016).
- ¹¹⁸Z. T. Wang *et al.*, “Black phosphorus quantum dots (BPQDs) saturable absorber for the passive mode-locking of an Er-doped fiber laser,” in *Proceedings of 2016 Conference on Lasers and Electro-Optics (CLEO)* (2016).
- ¹¹⁹R. W. Zhao *et al.*, “Triwavelength synchronously mode-locked fiber laser based on few-layered black phosphorus,” *Appl. Phys. Express* **9**(9), 092701 (2016).
- ¹²⁰A. H. H. Al-Masoodi *et al.*, “Mode-locked ytterbium-doped fiber laser using mechanically exfoliated black phosphorus as saturable absorber,” *Optik* **147**, 52–58 (2017).
- ¹²¹M. B. Hisyam *et al.*, “Generation of mode-locked ytterbium doped fiber ring laser using few-layer black phosphorus as a saturable absorber,” *IEEE J. Sel. Top. Quantum Electron.* **23**(1), 1100205 (2017).
- ¹²²X. Jin *et al.*, “Long term stable black phosphorus saturable absorber for mode-locked fiber laser,” in *Proceedings of 2017 Conference on Lasers and Electro-Optics (CLEO)* (2017).
- ¹²³X. X. Jin *et al.*, “Observation of tunable dual-wavelength in a fiber laser mode-locked by black phosphorus,” in *Proceedings of 2017 Opto-Electronics and Communications Conference (OeCC) and Photonics Global Conference (Pgc)* (2017).
- ¹²⁴H. Q. Song *et al.*, “Mode-locked ytterbium-doped all-fiber lasers based on few-layer black phosphorus saturable absorbers,” *Opt. Commun.* **394**, 157–160 (2017).
- ¹²⁵B. Gao *et al.*, “Influence of gain fiber on dissipative soliton pairs in passively mode-locked fiber laser based on BP as a saturable absorber,” *Opt. Commun.* **410**, 191–196 (2018).
- ¹²⁶X. X. Jin *et al.*, “102 fs pulse generation from a long-term stable, inkjet-printed black phosphorus-mode-locked fiber laser,” *Opt. Express* **26**(10), 12506–12513 (2018).
- ¹²⁷K. Wu *et al.*, “High-performance mode-locked and Q-switched fiber lasers based on novel 2D materials of topological insulators, transition metal

- dichalcogenides and black phosphorus: Review and perspective (invited)," *Opt. Commun.* **406**, 214–229 (2018).
- ¹²⁸Z. K. Jiang *et al.*, "256 fs, 2 nJ soliton pulse generation from MoS₂ mode-locked fiber laser," *Appl. Phys. Express* **10**(12), 122702 (2017).
- ¹²⁹Z. H. Wang *et al.*, "Generation of trapezoidal envelope pulses and soliton rains from passively mode-locked fiber laser with MoS₂ saturable absorber on microfiber," *Appl. Phys. Express* **11**(7), 072504 (2018).
- ¹³⁰M. Liu *et al.*, "Coexistence of bound soliton and harmonic mode-locking soliton in an ultrafast fiber laser based on MoS₂-deposited microfiber photonic device," *Chin. Opt. Lett.* **16**(2), 020008 (2018).
- ¹³¹P. Wang *et al.*, "Self-organized structures of soliton molecules in 2- μ m fiber laser based on MoS₂ saturable absorber," *IEEE Photonics Technol. Lett.* **30**(13), 1210–1213 (2018).
- ¹³²T. H. Chen *et al.*, "MoS₂ nano-flake doped polyvinyl alcohol enabling polarized soliton mode-locking of a fiber laser," *J. Mater. Chem. C* **4**(40), 9454–9459 (2016).
- ¹³³F. F. Lu, "MoS₂-wrapped microfiber-based multi-wavelength soliton fiber laser," *Mod. Phys. Lett. B* **31**(32), 1750303 (2017).
- ¹³⁴Y. D. Wang *et al.*, "Harmonic mode locking of bound-state solitons fiber laser based on MoS₂ saturable absorber," *Opt. Express* **23**(1), 205–210 (2015).
- ¹³⁵Y. F. Song *et al.*, "Few-layer antimonene decorated microfiber: Ultra-short pulse generation and all-optical thresholding with enhanced long term stability," *2D Mater.* **4**(4), 045010 (2017).
- ¹³⁶V. E. Zakharov and A. B. Shabat, "Interaction between solutions in a stable medium," *Zh. Eksp. Teor. Fiz.* **64**(5), 1627–1639 (1973).
- ¹³⁷L. Di Menza and C. Gallo, "The black solitons of one-dimensional NLS equations," *Nonlinearity* **20**(2), 461–496 (2007).
- ¹³⁸L. Faddeev and L. Takhtajan, *Hamiltonian Methods in the Theory of Solitons* (Springer Science & Business Media, 2007).
- ¹³⁹S. A. Gredeskul and Y. S. Kivshar, "Generation of dark solitons in optical fibers," *Phys. Rev. Lett.* **62**(8), 977–977 (1989).
- ¹⁴⁰Y. S. Kivshar and B. Luther-Davies, "Dark optical solitons: Physics and applications," *Phys. Rep.* **298**(2–3), 81–197 (1998).
- ¹⁴¹D. X. Huang and H. Li, "Dark soliton transmission twin-core fiber," *Int. J. Infrared Millimeter Waves* **22**(1), 93–100 (2001).
- ¹⁴²M. Stratmann and F. Mitschke, "Chains of temporal dark solitons in dispersion-managed fiber," *Phys. Rev. E* **72**(6), 066616 (2005).
- ¹⁴³S. M. Ao and J. R. Yan, "Effect of higher-order terms on nonlinear Schrödinger dark solitons in optical fibres," *Chin. Phys. Lett.* **23**(10), 2774–2777 (2006).
- ¹⁴⁴J. Guo *et al.*, "Formation and energy exchange of vector dark solitons in fiber lasers," *IEEE Photonics J.* **7**(1), 1500509 (2015).
- ¹⁴⁵J. Guo *et al.*, "Controlled generation of bright or dark solitons in a fiber laser by intracavity nonlinear absorber," *IEEE Photonics J.* **8**(3), 1502112 (2016).
- ¹⁴⁶Y. Q. Ge *et al.*, "Characterization of dark soliton sidebands in all-normal-dispersion fiber lasers," *IEEE J. Sel. Top. Quantum Electron.* **24**(3), 0903407 (2018).
- ¹⁴⁷A. M. Weiner *et al.*, "Experimental-observation of the fundamental dark soliton in optical fibers," *Phys. Rev. Lett.* **61**(21), 2445–2448 (1988).
- ¹⁴⁸H. Zhang *et al.*, "Dark pulse emission of a fiber laser," *Phys. Rev. A* **80**(4), 045803 (2009).
- ¹⁴⁹S. Coen and T. Sylvestre, "Comment on 'Dark pulse emission of a fiber laser'," *Phys. Rev. A* **82**(4), 047801 (2010).
- ¹⁵⁰D. Y. Tang *et al.*, "Evidence of dark solitons in all-normal-dispersion-fiber lasers," *Phys. Rev. A* **88**(1), 013849 (2013).
- ¹⁵¹D. Y. Tang *et al.*, "Dark soliton fiber lasers," *Optics Express* **22**(16), 19831 (2014).
- ¹⁵²Y. F. Song *et al.*, "280 GHz dark soliton fiber laser," *Opt. Lett.* **39**(12), 3484–3487 (2014).
- ¹⁵³S. V. Manakov, "On the theory of two-dimensional stationary self-focusing of electromagnetic waves," *Sov. Phys. - JETP* **38**(2), 248–253 (1974).
- ¹⁵⁴A. P. Sheppard and Y. S. Kivshar, "Polarized dark solitons in isotropic Kerr media," *Phys. Rev. E* **55**(4), 4773–4782 (1997).
- ¹⁵⁵R. Radhakrishnan and M. Lakshmanan, "Bright and dark soliton-solutions to coupled nonlinear Schrödinger-equations," *J. Phys. A: Math. Gen.* **28**(9), 2683–2692 (1995).
- ¹⁵⁶B. Prinari, M. J. Ablowitz, and G. Biondini, "Inverse scattering transform for the vector nonlinear Schrödinger equation with nonvanishing boundary conditions," *J. Math. Phys.* **47**(6), 063508 (2006).
- ¹⁵⁷D. S. Wang, D. J. Zhang, and J. K. Yang, "Integrable properties of the general coupled nonlinear Schrödinger equations," *J. Math. Phys.* **51**(2), 023510 (2010).
- ¹⁵⁸T. Kanna *et al.*, "Soliton collisions with shape change by intensity redistribution in mixed coupled nonlinear Schrödinger equations," *Phys. Rev. E* **73**(2), 026604 (2006).
- ¹⁵⁹M. Vijayajayanthi, T. Kanna, and M. Lakshmanan, "Bright-dark solitons and their collisions in mixed N-coupled nonlinear Schrödinger equations," *Phys. Rev. A* **77**(1), 013820 (2008).
- ¹⁶⁰Y. Ohta, D. S. Wang, and J. K. Yang, "General N-dark-dark solitons in the coupled nonlinear Schrödinger equations," *Stud. Appl. Math.* **127**(4), 345–371 (2011).
- ¹⁶¹H. Cai, F. M. Liu, and N. N. Huang, "Dark multi-soliton solution of the nonlinear Schrödinger equation with non-vanishing boundary," *Int. J. Theor. Phys.* **44**(2), 255–265 (2005).
- ¹⁶²L. M. Ling, L. C. Zhao, and B. L. Guo, "Darboux transformation and multi-dark soliton for N-component nonlinear Schrödinger equations," *Nonlinearity* **28**(9), 3243–3261 (2015).
- ¹⁶³H. Q. Zhang and Y. Wang, "Multi-dark soliton solutions for the higher-order nonlinear Schrödinger equation in optical fibers," *Nonlinear Dyn.* **91**(3), 1921–1930 (2018).
- ¹⁶⁴V. V. Bryksin, M. P. Petrov, and R. V. Kiyay, "Generation of vector solitons in fiber laser," *Pis'ma Zh. Tekh. Fiz.* **20**(10), 6–10 (1994).
- ¹⁶⁵C. Deangelis, M. Santagiustina, and S. Wabnitz, "Stability of vector solitons in fiber laser and transmission-systems," *Opt. Commun.* **122**(1–3), 23–27 (1995).
- ¹⁶⁶J. W. Haus *et al.*, "Vector soliton fiber lasers," *Opt. Lett.* **24**(6), 376–378 (1999).
- ¹⁶⁷B. C. Collings *et al.*, "Polarization-locked temporal vector solitons in a fiber laser: Experiment," *J. Opt. Soc. Am. B* **17**(3), 354–365 (2000).
- ¹⁶⁸J. M. Soto-Crespo *et al.*, "Polarization-locked temporal vector solitons in a fiber laser: Theory," *J. Opt. Soc. Am. B* **17**(3), 366–372 (2000).
- ¹⁶⁹W. C. Chen *et al.*, "Multiple polarization states of vector soliton in fiber laser," in *Proceedings of Passive Components and Fiber-Based Devices Iv, Pts 1 and 2* (2007), p. 6781.
- ¹⁷⁰W. C. Chen *et al.*, "Vector solitons in femtosecond fibre lasers (vol 48, pg 255, 2008)," *Eur. Phys. J. D* **50**(1), 123–123 (2008).
- ¹⁷¹W. C. Chen *et al.*, "Vector solitons in femtosecond fibre lasers," *Eur. Phys. J. D* **48**(2), 255–260 (2008).
- ¹⁷²D. Y. Tang *et al.*, "Observation of high-order polarization-locked vector solitons in a fiber laser," *Phys. Rev. Lett.* **101**(15), 0153904 (2008).
- ¹⁷³H. Zhang *et al.*, "Coherent energy exchange between components of a vector soliton in fiber lasers," *Opt. Express* **16**(17), 12618–12623 (2008).
- ¹⁷⁴L. M. Zhao *et al.*, "Polarization rotation locking of vector solitons in a fiber ring laser," *Opt. Express* **16**(14), 10053–10058 (2008).
- ¹⁷⁵L. M. Zhao *et al.*, "Period-doubling of vector solitons in a ring fiber laser," *Opt. Commun.* **281**(22), 5614–5617 (2008).
- ¹⁷⁶D. Y. Tang *et al.*, "Vector soliton fiber lasers," in *Proceedings of 14th Optoelectronics and Communications Conference (Oecc 2009)* (2009), pp. 84–85.
- ¹⁷⁷H. Zhang *et al.*, "Dissipative vector solitons in a dispersion-managed cavity fiber laser with net positive cavity dispersion," *Opt. Express* **17**(2), 455–460 (2009).
- ¹⁷⁸L. M. Zhao *et al.*, "Coexistence of polarization-locked and polarization-rotating vector solitons in a fiber laser with SESAM," *Opt. Lett.* **34**(20), 3059–3061 (2009).
- ¹⁷⁹L. M. Zhao *et al.*, "Bunch of restless vector solitons in a fiber laser with SESAM," *Opt. Express* **17**(10), 8103–8108 (2009).
- ¹⁸⁰H. Zhang *et al.*, "Vector dark domain wall solitons in a fiber ring laser," *Opt. Express* **18**(5), 4428–4433 (2010).
- ¹⁸¹C. Mou *et al.*, "All-fiber polarization locked vector soliton laser using carbon nanotubes," *Opt. Lett.* **36**(19), 3831–3833 (2011).
- ¹⁸²C. M. Ouyang *et al.*, "Properties of a vector soliton laser passively mode-locked by a fiber-based semiconductor saturable absorber operating in transmission," *Opt. Commun.* **284**(2), 619–624 (2011).
- ¹⁸³R. Gumenyuk *et al.*, "Vector soliton bunching in thulium-holmium fiber laser mode-locked with PbS quantum-dot-doped glass absorber," *IEEE J. Quantum Electron.* **48**(7), 903–907 (2012).

- ¹⁸⁴D. K. Tang, J. G. Zhang, and Y. S. Liu, "Vector solitons with polarization instability and locked polarization in a fiber laser," *Opt. Eng.* **51**(7), 074202 (2012).
- ¹⁸⁵D. B. Zeng *et al.*, "Numerical investigation of polarization rotation locking of vector solitons in a fiber ring laser," in Proceedings of 2012 Asia Communications and Photonics Conference (Acp) (2012).
- ¹⁸⁶W. C. Chen *et al.*, "Vector solitons with a uniform polarisation state induced by polarisation filtering in a fibre laser," *Quantum Electron.* **43**(6), 526–530 (2013).
- ¹⁸⁷Z. Gong *et al.*, "Observation of stable, polarization-locked, vector bound states of solitons from a carbon-nanotube mode-locked fiber laser," in *Proceedings of 2013 Ieee Photonics Conference (Ipc)* (2013), pp. 386–387.
- ¹⁸⁸X. C. Luo *et al.*, "Vector dissipative soliton resonance in a fiber laser," *Opt. Express* **21**(8), 10199–10204 (2013).
- ¹⁸⁹V. Tsaturian *et al.*, "Polarisation dynamics of vector soliton molecules in mode locked fibre laser," *Sci. Rep.* **3**, 3154 (2013).
- ¹⁹⁰X. Z. Yuan *et al.*, "Experimental observation of vector solitons in a highly birefringent cavity of ytterbium-doped fiber laser," *Opt. Express* **21**(20), 23866–23872 (2013).
- ¹⁹¹T. Chen *et al.*, "Polarization-locked vector solitons in a mode-locked fiber laser using polarization-sensitive few-layer graphene deposited D-shaped fiber saturable absorber," *J. Opt. Soc. Am. B* **31**(6), 1377–1382 (2014).
- ¹⁹²T. Habruseva *et al.*, "Vector solitons in harmonic mode-locked erbium-doped fiber lasers," in *Proceedings of Nonlinear Optics and Its Applications VIII; and Quantum Optics Iii* (2014), p. 9136.
- ¹⁹³Q. Y. Ning *et al.*, "Vector nature of multi-soliton patterns in a passively mode-locked figure-eight fiber laser," *Opt. Express* **22**(10), 11900–11911 (2014).
- ¹⁹⁴S. V. Sergeev, "Fast and slowly evolving vector solitons in mode-locked fibre lasers," *Philos. Trans. R. Soc., A* **372**(2027), 20140006 (2014).
- ¹⁹⁵S. V. Sergeev *et al.*, "Vector solitons in mode locked fibre lasers," in Proceedings of 16th International Conference on Transparent Optical Networks (Icton) (2014).
- ¹⁹⁶S. M. Wang *et al.*, "Dissipative vector soliton in a dispersion-managed fiber laser with normal dispersion," *Appl. Opt.* **53**(35), 8216–8221 (2014).
- ¹⁹⁷Y. Wang *et al.*, "Vector soliton generation in a Tm fiber laser," *IEEE Photonics Technol. Lett.* **26**(8), 769–772 (2014).
- ¹⁹⁸N. Zhao *et al.*, "Multiple vector solitons in an ytterbium-doped fiber laser based on evanescent field interaction with graphene saturable absorber," in Proceedings of Opto-Electronics and Communications Conference (Oecc) (2015).
- ¹⁹⁹X. X. Jin *et al.*, "Manipulation of group-velocity-locked vector solitons from fiber lasers," *IEEE Photonics J.* **8**(2), 1501206 (2016).
- ²⁰⁰Y. Y. Luo *et al.*, "Group velocity locked vector dissipative solitons in a high repetition rate fiber laser," *Opt. Express* **24**(16), 18718–18726 (2016).
- ²⁰¹G. D. Shao *et al.*, "Vector gain-guided dissipative solitons in a net normal dispersive fiber laser," *IEEE Photonics Technol. Lett.* **28**(9), 975–978 (2016).
- ²⁰²G. D. Shao *et al.*, "Temporal vector cavity solitons in a net anomalous dispersion fiber laser," *Laser Phys. Lett.* **13**(2), 025103 (2016).
- ²⁰³G. D. Shao *et al.*, "Black-white vector solitons in a fiber ring laser," in Proceedings of 2016 Conference on Lasers and Electro-Optics (Cleo) (2016).
- ²⁰⁴P. Wang, C. Y. Bao, and C. X. Yang, "Vector solitons in a mode-locked Tm-doped fiber laser," in Proceedings of 2016 Conference on Lasers and Electro-Optics (Cleo) (2016).
- ²⁰⁵Z. C. Wu *et al.*, "Switchable thulium-doped fiber laser from polarization rotation vector to scalar soliton," *Sci. Rep.* **6**, 34844 (2016).
- ²⁰⁶Z. C. Wu *et al.*, "Scalar-vector soliton fiber laser mode-locked by nonlinear polarization rotation," *Opt. Express* **24**(16), 18764–18771 (2016).
- ²⁰⁷A. E. Akosman and M. Y. Sander, "Frequency-halved orthogonally polarized vector soliton states from a single fiber laser source," in Proceedings of 2017 Conference on Lasers and Electro-Optics (Cleo) (2017).
- ²⁰⁸A. E. Akosman *et al.*, "Vector solitons in harmonically mode-locked Tm/Ho doped fiber laser," in Proceedings of 2017 Conference on Lasers and Electro-Optics (Cleo) (2017).
- ²⁰⁹Y. Q. Du and X. W. Shu, "Molecular and vectorial properties of the vector soliton molecules in anomalous-dispersion fiber lasers," *Opt. Express* **25**(23), 28035–28052 (2017).
- ²¹⁰Y. Q. Du, X. W. Shu, and P. Y. Cheng, "Numerical simulations of fast-axis instability of vector solitons in mode-locked fiber lasers," *Opt. Express* **25**(2), 1131–1141 (2017).
- ²¹¹Y. Q. Du, X. W. Shu, and P. Y. Cheng, "Vector solitons in mode-locked fiber lasers by fast-axis instability," in Proceedings of 2017 Conference on Lasers and Electro-Optics Pacific Rim (Cleo-Pr) (2017).
- ²¹²M. M. Han and S. M. Zhang, "Polarization domains and polarization locked vector solitons in a fiber laser," *IEEE Photonics Technol. Lett.* **29**(24), 2230–2233 (2017).
- ²¹³M. Liu *et al.*, "Dynamic trapping of a polarization rotation vector soliton in a fiber laser," *Opt. Lett.* **42**(2), 330–333 (2017).
- ²¹⁴Y. Y. Luo *et al.*, "Group-velocity-locked vector soliton molecules in fiber lasers," *Sci. Rep.* **7**, 2369 (2017).
- ²¹⁵S. M. Zhang *et al.*, "Coexistence of polarization domains and polarization vector solitons in fiber lasers," in Proceedings of 16th International Conference on Optical Communications & Networks (Iocn) (2017).
- ²¹⁶A. E. Akosman *et al.*, "Polarization rotation dynamics in harmonically mode-locked vector soliton fiber lasers," *IEEE J. Sel. Top. Quantum Electron.* **24**(3), 1101107 (2018).
- ²¹⁷X. L. Fan *et al.*, "Generation of polarization-locked vector solitons in mode-locked thulium fiber laser," *IEEE Photonics J.* **10**(1), 1500308 (2018).
- ²¹⁸D. J. Li *et al.*, "Internal polarization dynamics of vector dissipative-soliton-resonance pulses in normal dispersion fiber lasers," *Opt. Lett.* **43**(6), 1222–1225 (2018).
- ²¹⁹Y. X. Yan *et al.*, "Wavelength tunable L band polarization-locked vector soliton fiber laser based on SWCNT-SA and CFBG," *Opt. Commun.* **412**, 55–59 (2018).
- ²²⁰D. J. Kaup, B. A. Malomed, and R. S. Tasgal, "Internal dynamics of a vector soliton in a nonlinear-optical fiber," *Phys. Rev. E* **48**(4), 3049–3053 (1993).
- ²²¹H. Zhang *et al.*, "Induced solitons formed by cross-polarization coupling in a birefringent cavity fiber laser," *Opt. Lett.* **33**(20), 2317–2319 (2008).
- ²²²S. Cundiff, B. Collings, and W. Knox, "Polarization locking in an isotropic, modelocked soliton Er/Yb fiber laser," *Opt. Express* **1**(1), 12–21 (1997).
- ²²³S. T. Cundiff *et al.*, "Observation of polarization-locked vector solitons in an optical fiber," *Phys. Rev. Lett.* **82**(20), 3988–3991 (1999).
- ²²⁴S. T. Cundiff, B. C. Collings, and K. Bergman, "Polarization locked vector solitons and axis instability in optical fiber," *Chaos* **10**(3), 613–624 (2000).
- ²²⁵V. V. Afanasjev, "Soliton polarization rotation in fiber lasers," *Opt. Lett.* **20**(3), 270–272 (1995).
- ²²⁶S. V. Sergeev *et al.*, "Vector solitons with locked and precessing states of polarization," *Opt. Express* **20**(24), 27434–27440 (2012).
- ²²⁷M. N. Islam, C. D. Poole, and J. P. Gordon, "Soliton trapping in birefringent optical fibers," *Opt. Lett.* **14**(18), 1011–1013 (1989).
- ²²⁸X. X. Jin *et al.*, "Generation of high-order group-velocity-locked vector solitons," *IEEE Photonics J.* **7**(5), 7102206 (2015).
- ²²⁹S. N. Zhu *et al.*, "Evidence of pseudo-high-order group-velocity-locked vector dissipative solitons," in *Proceedings of Ieee Photonics Conference (Ipc)* (2016), pp. 172–173.
- ²³⁰X. Wang *et al.*, "Decomposition of group-velocity-locked-vector-dissipative solitons and formation of the high-order soliton structure by the product of their recombination," *Appl. Opt.* **57**(4), 746–751 (2018).
- ²³¹S. N. Zhu *et al.*, "Manipulation of group-velocity-locked vector dissipative solitons and properties of the generated high-order vector soliton structure," *Appl. Opt.* **57**(9), 2064–2068 (2018).
- ²³²D. N. Christodoulides, "Black and white vector solitons in weakly birefringent optical fibers," *Phys. Lett. A* **132**(8–9), 451–452 (1988).
- ²³³Y. C. Meng *et al.*, "Bright-dark soliton pairs emission of a fiber laser," in *Proceedings of Passive Components and Fiber-Based Devices VIII* (2011), p. 8307.
- ²³⁴Y. C. Meng *et al.*, "Bright-dark soliton pairs in a self-mode locking fiber laser," *Opt. Eng.* **51**(6), 064302 (2012).
- ²³⁵Y. C. Meng *et al.*, "Bright-dark soliton pairs emission of a fiber laser," in Proceedings of 2011 Asia Communications and Photonics Conference and Exhibition (Acp) (2012).
- ²³⁶Q. Y. Ning *et al.*, "Bright-dark pulse pair in a figure-eight dispersion-managed passively mode-locked fiber laser," *IEEE Photonics J.* **4**(5), 1647 (2012).

- ²³⁷H. Y. Wang *et al.*, “Experimental observation of bright-dark pulse emitting in an all-fiber ring cavity laser,” *Laser Phys.* **22**(1), 282–285 (2012).
- ²³⁸L. R. Wang, “Coexistence and evolution of bright pulses and dark solitons in a fiber laser,” *Opt. Commun.* **297**, 129–132 (2013).
- ²³⁹J. Gao *et al.*, “Bright-dark pair in passively mode-locked fiber laser based on graphene,” *Laser Phys.* **24**(8), 085104 (2014).
- ²⁴⁰R. Y. Lin *et al.*, “Bright and dark square pulses generated from a graphene-oxide mode-locked ytterbium-doped fiber laser,” *IEEE Photonics J.* **6**(3), 1500908 (2014).
- ²⁴¹S. S. Huang *et al.*, “Observation of multipulse bunches in a graphene oxide passively mode-locked ytterbium-doped fiber laser with all-normal dispersion,” *Appl. Phys. B: Lasers Opt.* **116**(4), 939–946 (2014).
- ²⁴²B. Guo *et al.*, “Observation of bright-dark soliton pair in a fiber laser with topological insulator,” *IEEE Photonics Technol. Lett.* **27**(7), 701–704 (2015).
- ²⁴³Z. X. Zhang *et al.*, “Orthogonally polarized bright-dark pulse pair generation in mode-locked fiber laser with a large-angle tilted fiber grating,” *Appl. Phys. B: Lasers Opt.* **122**(6), 161 (2016).
- ²⁴⁴R. W. Zhao *et al.*, “Multi-wavelength bright-dark pulse pair fiber laser based on rhenium disulfide,” *Opt. Express* **26**(5), 5819–5826 (2018).
- ²⁴⁵Y. S. Kivshar and S. K. Turitsyn, “Vector dark solitons,” *Opt. Lett.* **18**(5), 337–339 (1993).
- ²⁴⁶H. Zhang *et al.*, “Observation of polarization domain wall solitons in weakly birefringent cavity fiber lasers,” *Phys. Rev. B* **80**(5), 052302 (2009).
- ²⁴⁷P. Grelu and N. Akhmediev, “Dissipative solitons for mode-locked lasers,” *Nat. Photonics* **6**(2), 84–92 (2012).
- ²⁴⁸M. Mirzazadeh *et al.*, “Optical solitons with complex Ginzburg-Landau equation,” *Nonlinear Dyn.* **85**(3), 1979–2016 (2016).
- ²⁴⁹N. R. Pereira and L. Stenflo, “Nonlinear Schrödinger equation including growth and damping,” *Phys. Fluids* **20**(10), 1733–1734 (1977).
- ²⁵⁰J. G. Liu, Y. Z. Li, and B. Tian, “Soliton-like solutions for the modified variable-coefficient Ginzburg-Landau equation,” *Commun. Nonlinear Sci. Numer. Simul.* **14**(4), 1214–1226 (2009).
- ²⁵¹A. Chong *et al.*, “All-normal-dispersion femtosecond fiber laser,” *Opt. Express* **14**(21), 10095–10100 (2006).
- ²⁵²L. M. Zhao *et al.*, “Gain-guided solitons in dispersion-managed fiber lasers with large net cavity dispersion,” *Opt. Lett.* **31**(20), 2957–2959 (2006).
- ²⁵³L. M. Zhao *et al.*, “Gain-guided and dispersion-managed soliton fiber lasers,” in *Proceedings of Tencon 2006 – 2006 IEEE Region 10 Conference* (2006), Vol. 1–4, p. 66–+.
- ²⁵⁴L. M. Zhao, D. Y. Tang, and J. Wu, “Gain-guided soliton in a positive group-dispersion fiber laser,” *Opt. Lett.* **31**(12), 1788–1790 (2006).
- ²⁵⁵H. H. Lin *et al.*, “Gain-guided solitons in positive dispersion lasers,” *Acta Phys. Sin.* **57**(9), 5646–5650 (2008).
- ²⁵⁶C. Lecaplain, B. Ortac, and A. Hideur, “High-energy femtosecond pulses from a dissipative soliton fiber laser,” *Opt. Lett.* **34**(23), 3731–3733 (2009).
- ²⁵⁷P. Grelu *et al.*, “Dissipative soliton resonance as a guideline for high-energy pulse laser oscillators,” *J. Opt. Soc. Am. B* **27**(11), 2336–2341 (2010).
- ²⁵⁸L. R. Wang, X. M. Liu, and Y. K. Gong, “Experimental research on high-energy dissipative solitons in an erbium-doped fiber laser,” *Acta Phys. Sin.* **59**(9), 6200–6204 (2010).
- ²⁵⁹K. Jiang *et al.*, “High-energy dissipative soliton with MHz repetition rate from an all-fiber passively mode-locked laser,” *Opt. Commun.* **285**(9), 2422–2425 (2012).
- ²⁶⁰C. M. Ouyang, P. Shum, and K. Wu, “High pulse energy all-fiber F-P cavity dissipative soliton laser,” *IEEE Photonics Technol. Lett.* **25**(3), 303–305 (2013).
- ²⁶¹C. Xie *et al.*, “High energy dissipative soliton mode-locked fiber oscillator based on a multipass cell,” *Acta Phys. Sin.* **62**(5), 054203 (2013).
- ²⁶²J. H. Yang *et al.*, “High-energy rectangular pulse dissipative soliton generation in a long-cavity sigma-shaped configuration mode-locked fiber laser,” *Chin. Phys. Lett.* **31**(2), 024208 (2014).
- ²⁶³C. Y. Huang *et al.*, “Developing high energy dissipative soliton fiber lasers at 2 micron,” *Sci. Rep.* **5**, 13680 (2015).
- ²⁶⁴M. Tang *et al.*, “High-energy dissipative solitons generation from a large normal dispersion Er-fiber laser,” *Opt. Lett.* **40**(7), 1414–1417 (2015).
- ²⁶⁵N. Yang, Y. L. Tang, and J. Q. Xu, “High-energy harmonic mode-locked 2 μ m dissipative soliton fiber lasers,” *Laser Phys. Lett.* **12**(8), 085102 (2015).
- ²⁶⁶H. X. Zhang *et al.*, “Optimal design of higher energy dissipative-soliton fiber lasers,” *Opt. Commun.* **335**, 212–217 (2015).
- ²⁶⁷R. Becheker *et al.*, “High-energy dissipative soliton-driven fiber optical parametric oscillator emitting at 1.7 μ m,” *Laser Phys. Lett.* **15**(11), 115103 (2018).
- ²⁶⁸M. L. Y. Francisco, J. H. Lee, and O. K. Pashaev, “Dissipative hierarchies and resonance solitons for KP-II and MKP-II,” *Math. Comp. Simul.* **74**(4–5), 323–332 (2007).
- ²⁶⁹W. Chang *et al.*, “Dissipative soliton resonances in laser models with parameter management,” *J. Opt. Soc. Am. B* **25**(12), 1972–1977 (2008).
- ²⁷⁰W. Chang *et al.*, “Dissipative soliton resonances,” *Phys. Rev. A* **78**(2), 023830 (2008).
- ²⁷¹A. S. Kiselev, A. S. Kiselev, and N. N. Rozanov, “Dissipative discrete spatial optical solitons in a system of coupled optical fibers with the Kerr and resonance nonlinearities,” *Opt. Spectrosc.* **105**(4), 547–556 (2008).
- ²⁷²W. Chang *et al.*, “Dissipative soliton resonances in the anomalous dispersion regime,” *Phys. Rev. A* **79**(3), 033840 (2009).
- ²⁷³X. Wu *et al.*, “Dissipative soliton resonance in an all-normal-dispersion erbium-doped fiber laser,” *Opt. Express* **17**(7), 5580–5584 (2009).
- ²⁷⁴E. Ding, P. Grelu, and J. N. Kutz, “Dissipative soliton resonance in a passively mode-locked fiber laser,” *Opt. Lett.* **36**(7), 1146–1148 (2011).
- ²⁷⁵L. N. Duan *et al.*, “Experimental observation of dissipative soliton resonance in an anomalous-dispersion fiber laser,” *Opt. Express* **20**(1), 265–270 (2012).
- ²⁷⁶Z. C. Luo *et al.*, “Pulse dynamics of dissipative soliton resonance with large duration-tuning range in a fiber ring laser,” *Opt. Lett.* **37**(22), 4777–4779 (2012).
- ²⁷⁷Z. C. Cheng *et al.*, “Dissipative soliton resonance in an all-normal-dispersion graphene oxide mode-locked Yb-doped fiber laser,” in *Proceedings of 2013 Conference on Lasers and Electro-Optics (CLEO)* (2013).
- ²⁷⁸A. Komarov *et al.*, “Competition and coexistence of ultrashort pulses in passive mode-locked lasers under dissipative-soliton-resonance conditions,” *Phys. Rev. A* **87**(2), 023838 (2013).
- ²⁷⁹L. Liu *et al.*, “Wave-breaking-free pulse in an all-fiber normal-dispersion Yb-doped fiber laser under dissipative soliton resonance condition,” *Opt. Express* **21**(22), 27087–27092 (2013).
- ²⁸⁰Z. C. Luo *et al.*, “Dissipative soliton resonance in an anomalous-dispersion figure-eight fiber laser,” in *Proceedings of 2013 Conference on Lasers and Electro-Optics (CLEO)* (2013).
- ²⁸¹S. K. Wang *et al.*, “Dissipative soliton resonance in a passively mode-locked figure-eight fiber laser,” *Opt. Express* **21**(2), 2402–2407 (2013).
- ²⁸²Z. W. Xu and Z. X. Zhang, “Diverse output states from an all-normal dispersion ytterbium-doped fiber laser: Q-switch, dissipative soliton resonance, and noise-like pulse,” *Opt. Laser Technol.* **48**, 67–71 (2013).
- ²⁸³J. H. Yang *et al.*, “Observation of dissipative soliton resonance in a net-normal dispersion figure-of-eight fiber laser,” *IEEE Photonics J.* **5**(3), 1500806 (2013).
- ²⁸⁴H. Q. Lin *et al.*, “Dissipative soliton resonance in an all-normal-dispersion Yb-doped figure-eight fibre laser with tunable output,” *Laser Phys. Lett.* **11**(8), 085102 (2014).
- ²⁸⁵N. A. Veretenov *et al.*, “Modulational instability, switching waves, bistability and dissipative solitons at resonance excitation of molecular J-aggregates,” in *Proceedings of 2014 International Conference Laser Optics* (2014).
- ²⁸⁶Z. C. Cheng, H. H. Li, and P. Wang, “Simulation of generation of dissipative soliton, dissipative soliton resonance and noise-like pulse in Yb-doped mode-locked fiber lasers,” *Opt. Express* **23**(5), 5972–5981 (2015).
- ²⁸⁷K. Krzempek, “Dissipative soliton resonances in all-fiber Er-Yb double clad figure-8 laser,” *Opt. Express* **23**(24), 30651–30656 (2015).
- ²⁸⁸D. J. Li *et al.*, “Mechanism of dissipative-soliton-resonance generation in passively mode-locked all-normal-dispersion fiber lasers,” *J. Lightwave Technol.* **33**(18), 3781–3787 (2015).
- ²⁸⁹W. Lin *et al.*, “Analytical identification of soliton dynamics in normal-dispersion passively mode-locked fiber lasers: From dissipative soliton to dissipative soliton resonance,” *Opt. Express* **23**(11), 14860–14875 (2015).
- ²⁹⁰Y. Xu *et al.*, “Dissipative soliton resonance in a wavelength-tunable thulium-doped fiber laser with net-normal dispersion,” *IEEE Photonics J.* **7**(3), 1502007 (2015).

- ²⁹¹I. Armas-Rivera *et al.*, “Dissipative soliton resonance in a full polarization-maintaining fiber ring laser at different values of dispersion,” *Opt. Express* **24**(9), 9966–9974 (2016).
- ²⁹²K. Krzempek and K. Abramski, “Dissipative soliton resonance mode-locked double clad Er:Yb laser at different values of anomalous dispersion,” *Opt. Express* **24**(20), 22379–22386 (2016).
- ²⁹³K. Krzempek, J. Sotor, and K. Abramski, “Compact all-fiber figure-9 dissipative soliton resonance mode-locked double-clad Er:Yb laser,” *Opt. Lett.* **41**(21), 4995–4998 (2016).
- ²⁹⁴J. S. Lee, J. H. Koo, and J. H. Lee, “A pulse-width-tunable, mode-locked fiber laser based on dissipative soliton resonance using a bulk-structured Bi2Te3 topological insulator,” *Opt. Eng.* **55**(8), 081309 (2016).
- ²⁹⁵D. J. Li *et al.*, “Compression of dissipative-soliton-resonance pulses in a mode-locked fiber laser with a nonlinear optical loop mirror,” in Proceedings of 2016 Conference on Lasers and Electro-Optics (CLEO) (2016).
- ²⁹⁶D. J. Li *et al.*, “Characterization and compression of dissipative-soliton-resonance pulses in fiber lasers,” *Sci. Rep.* **6**, 23631 (2016).
- ²⁹⁷G. Semaan *et al.*, “10 mJ dissipative soliton resonance square pulse in a dual amplifier figure-of-eight double-clad Er: Yb mode-locked fiber laser,” *Opt. Lett.* **41**(20), 4767–4770 (2016).
- ²⁹⁸S. S. Tan *et al.*, “Dissipative soliton resonance in thulium-doped fiber laser and its application for microscopy,” in Proceedings of 2016 Conference on Lasers and Electro-Optics (CLEO) (2016).
- ²⁹⁹Z. K. Wang *et al.*, “All fiber tunable- or dual-wavelength Yb-doped fiber laser covering from dissipative soliton to dissipative soliton resonance,” *Chin. Opt. Lett.* **14**(4), 041401 (2016).
- ³⁰⁰B. K. Yang *et al.*, “Dissipative soliton resonance pulse generation from an all-fiber mode-locked dumbbell-shaped fiber laser,” in Proceedings of 15th International Conference on Optical Communications and Networks (Icofn) (2016).
- ³⁰¹J. Q. Zhao *et al.*, “100 W dissipative soliton resonances from a thulium-doped double-clad all-fiber-format MOPA system,” *Opt. Express* **24**(11), 2072–2081 (2016).
- ³⁰²F. Ben Braham *et al.*, “Experimental optimization of dissipative soliton resonance square pulses in all anomalous passively mode-locked fiber laser,” *J. Opt.* **19**(10), 105501 (2017).
- ³⁰³F. Ben Braham *et al.*, “Exhaustive study of dissipative soliton resonance in a dual amplifier passively mode-locked fiber laser,” in Proceedings of 19th International Conference on Transparent Optical Networks (ICTON) (2017).
- ³⁰⁴J. H. Cai, S. P. Chen, and J. Hou, “1.1-kW peak-power dissipative soliton resonance in a mode-locked Yb-fiber laser,” *IEEE Photonics Technol. Lett.* **29**(24), 2191–2194 (2017).
- ³⁰⁵S. Das Chowdhury *et al.*, “High repetition rate gain-switched 1.94 μm fiber laser pumped by 1.56 μm dissipative soliton resonance fiber laser,” *Opt. Lett.* **42**(13), 2471–2474 (2017).
- ³⁰⁶Z. S. Deng *et al.*, “Switchable generation of rectangular noise-like pulse and dissipative soliton resonance in a fiber laser,” *Opt. Lett.* **42**(21), 4517–4520 (2017).
- ³⁰⁷Z. Y. Dou *et al.*, “The generation of dissipative soliton resonance from dumbbell-shaped Er-doped fiber laser,” in Proceedings of 16th International Conference on Optical Communications & Networks (Icofn 2017) (2017).
- ³⁰⁸T. J. Du *et al.*, “1.2-W average-power, 700-W peak-power, 100-ps dissipative soliton resonance in a compact Er:Yb co-doped double-clad fiber laser,” *Opt. Lett.* **42**(3), 462–465 (2017).
- ³⁰⁹W. X. Du *et al.*, “Simulation of dissipative-soliton-resonance generation in a passively mode-locked Yb-doped fiber laser,” in Proceedings of AOPC 2017: Laser Components, Systems, and Applications (2017), p. 10457.
- ³¹⁰A. Ferrando, “Nonlinear plasmonic amplification via dissipative soliton-plasmon resonances,” *Phys. Rev. A* **95**(1), 013816 (2017).
- ³¹¹S. Kharitonov and C. S. Bres, “All-fiber dissipative soliton resonance mode-locked Figure-9 thulium-doped fiber laser,” in Proceedings of Conference on Lasers and Electro-Optics Europe & European Quantum Electronics Conference (CLEO/Europe-EQEC) (2017).
- ³¹²K. Krzempek and K. Abramski, “6.5 mJ pulses from a compact dissipative soliton resonance mode-locked erbium-ytterbium double clad (DC) laser,” *Laser Phys. Lett.* **14**(1), 015101 (2017).
- ³¹³K. Krzempek, D. Tomaszewska, and K. M. Abramski, “Dissipative soliton resonance mode-locked all-polarization-maintaining double clad Er:Yb fiber laser,” *Opt. Express* **25**(21), 24853–24860 (2017).
- ³¹⁴J. L. Zheng *et al.*, “Black phosphorus based all-optical-signal-processing: Toward high performances and enhanced stability,” *ACS Photonics* **4**(6), 1466–1476 (2017).
- ³¹⁵Y. J. Lyu *et al.*, “Harmonic dissipative soliton resonance pulses in a fiber ring laser at different values of anomalous dispersion,” *Photonics Res.* **5**(6), 612–616 (2017).
- ³¹⁶Y. J. Lyu *et al.*, “Multipulse dynamics under dissipative soliton resonance conditions,” *Opt. Express* **25**(12), 13286–13295 (2017).
- ³¹⁷A. Niang *et al.*, “Dynamics of dissipative soliton resonance square pulses in fiber lasers,” in Proceedings of 19th International Conference on Transparent Optical Networks (ICTON) (2017).
- ³¹⁸M. Salhi *et al.*, “Route to high energy dissipative soliton resonance pulse in a dual amplifier figure-of-eight fiber laser,” in Proceedings of Nonlinear Optics and Applications X (2017), p. 10228.
- ³¹⁹G. Semaan *et al.*, “Harmonic dissipative soliton resonance square pulses in an anomalous dispersion passively mode-locked fiber ring laser,” *Laser Phys. Lett.* **14**(5), 055401 (2017).
- ³²⁰P. Wang *et al.*, “Pulse dynamics of dual-wavelength dissipative soliton resonances and domain wall solitons in a Tm fiber laser with fiber-based Lyot filter,” *Opt. Express* **25**(24), 30708–30719 (2017).
- ³²¹Z. Xu *et al.*, “Ultra-long duration and ultra-high duty cycle dissipative soliton resonance in a mode-locked thulium-doped fiber laser,” in Proceedings of AOPC 2017: Laser Components, Systems, and Applications (2017), p. 10457.
- ³²²G. K. Zhao *et al.*, “Dissipative soliton resonance in bismuth-doped fiber laser,” *Opt. Express* **25**(17), 20923–20931 (2017).
- ³²³J. Q. Zhao *et al.*, “1.04 km ultra-long cladding-pumped thulium-doped fiber laser with large energy noise-like-topped dissipative soliton resonances,” in Proceedings of 2017 Conference on Lasers and Electro-Optics Pacific Rim (CLEO-PR) (2017).
- ³²⁴T. J. Du *et al.*, “2 mJ high-power dissipative soliton resonance in a compact sigma-shaped Tm-doped double-clad fiber laser,” *Appl. Phys. Express* **11**(5), 052701 (2018).
- ³²⁵L. Mei *et al.*, “Width and amplitude tunable square-wave pulse in dual-pump passively mode-locked fiber laser,” *Opt. Lett.* **39**(11), 3235–3237 (2014).
- ³²⁶M. Salhi *et al.*, “Broadly peak power and pulse width tunable dissipative soliton resonance generation in figure of eight fiber laser,” *Rom. Rep. Phys.* **70**(1), 402 (2018).
- ³²⁷N. Wang *et al.*, “Ultraviolet-enhanced supercontinuum generation with a mode-locked Yb-doped fiber laser operating in dissipative-soliton-resonance region,” *Opt. Express* **26**(2), 1689–1696 (2018).
- ³²⁸X. H. Li *et al.*, “cavity passively mode-locked fiber ring laser with high-energy rectangular-shape pulses in anomalous dispersion regime,” *Opt. Lett.* **35**(19), 3249–3251 (2010).
- ³²⁹J. Q. Zhao *et al.*, “Dissipative soliton resonances in a mode-locked holmium-doped fiber laser,” *IEEE Photonics Technol. Lett.* **30**(19), 1699–1702 (2018).
- ³³⁰B. A. Malomed and A. A. Nepomnyashchy, “Kinks and solitons in the generalized Ginzburg-Landau equation,” *Phys. Rev. A* **42**(10), 6009–6014 (1990).
- ³³¹E. A. Kuznetsov, “Solitons in parametrically unstable plasma,” *Dokl. Akad. Nauk SSSR* **236**(3), 575–577 (1977).
- ³³²Y. C. Ma, “Perturbed plane-wave solutions of the cubic Schrödinger equation,” *Stud. Appl. Math.* **60**(1), 43–58 (1979).
- ³³³N. N. Akhmediev, V. M. Eleonskii, and N. E. Kulagin, “Exact 1st-order solutions of the nonlinear Schrödinger-equation,” *Theor. Math. Phys.* **72**(2), 809–818 (1987).
- ³³⁴B. A. Malomed, N. N. Rosanov, and S. V. Fedorov, “Dynamics of nonlinear Schrödinger breathers in a potential trap,” *Phys. Rev. E* **97**(5), 052204 (2018).
- ³³⁵K. Tamura *et al.*, “77-fs pulse generation from a stretched-pulse mode-locked all-fiber ring laser,” *Opt. Lett.* **18**(13), 1080–1082 (1993).
- ³³⁶L. M. Zhao *et al.*, “Multi-pulse dispersion-managed solitons in a fiber laser at near zero dispersion,” in Proceedings of 2007 Pacific Rim Conference on Lasers and Electro-Optics (2007), Vol. 1–4, p. 790–+.
- ³³⁷L. M. Zhao *et al.*, “Period-doubling of dispersion-managed solitons in an erbium-doped fiber laser at around zero dispersion,” *Opt. Commun.* **278**(2), 428–433 (2007).

- ³³⁸M. J. Ablowitz, T. P. Horikis, and B. Ilan, "Solitons in dispersion-managed mode-locked lasers," *Phys. Rev. A* **77**(3), 033814 (2008).
- ³³⁹A. Chong, W. H. Renninger, and F. W. Wise, "Observation of antisymmetric dispersion-managed solitons in a mode-locked laser," *Opt. Lett.* **33**(15), 1717–1719 (2008).
- ³⁴⁰L. M. Zhao *et al.*, "Dynamics of gain-guided solitons in a dispersion-managed fiber laser with large normal cavity dispersion," *Opt. Commun.* **281**(12), 3324–3326 (2008).
- ³⁴¹B. G. Bale, S. Boscolo, and S. K. Turitsyn, "Dissipative dispersion-managed solitons in mode-locked lasers," *Opt. Lett.* **34**(21), 3286–3288 (2009).
- ³⁴²B. G. Bale, S. Boscolo, and S. K. Turitsyn, "Dissipative dispersion-managed solitons in mode-locked fibre lasers," in *Proceedings of 2009 35th European Conference on Optical Communication (Ecoc)* (2009).
- ³⁴³Z. C. Luo *et al.*, "Modulation instability induced by cross-phase modulation in a dual-wavelength dispersion-managed soliton fiber ring laser," *Appl. Phys. B: Lasers Opt.* **100**(4), 811–820 (2010).
- ³⁴⁴R. Gumenyuk *et al.*, "Dissipative dispersion-managed soliton 2 μ m thulium/holmium fiber laser," *Opt. Lett.* **36**(5), 609–611 (2011).
- ³⁴⁵D. J. Lei and H. Dong, "Generalized characteristics of soliton in dispersion-managed fiber lasers," *Optik* **124**(16), 2544–2548 (2013).
- ³⁴⁶B. H. Yue *et al.*, "Evolution of dark-dark soliton pairs in a dispersion managed erbium-doped fiber ring laser," *Laser Phys.* **23**(7), 075106 (2013).
- ³⁴⁷X. X. Han, "Nanotube-mode-locked fiber laser delivering dispersion-managed or dissipative solitons," *J. Lightwave Technol.* **32**(8), 1472–1476 (2014).
- ³⁴⁸J. H. Lin *et al.*, "Bound states of dispersion-managed solitons from single-mode Yb-doped fiber laser at net-normal dispersion," *IEEE Photonics J.* **7**(5), 1 (2015).
- ³⁴⁹Z. Zhang *et al.*, "All-fiber nonlinearity- and dispersion-managed dissipative soliton nanotube mode-locked laser," *Appl. Phys. Lett.* **107**(24), 241107 (2015).
- ³⁵⁰D. A. Dvoretzkiy *et al.*, "Dispersion-managed soliton generation in the hybrid mode-locked erbium-doped all-fiber ring laser," in *Proceedings of 2016 International Conference Laser Optics (Lo)* (2016).
- ³⁵¹Q. Yu *et al.*, "A versatile fiber laser delivering dispersion-managed soliton and Q-switched pulse," in *Proceedings of 2016 Progress in Electromagnetics Research Symposium (Piers)* (2016), pp. 1606–1610.
- ³⁵²D. A. Dvoretzkiy *et al.*, "Comb peculiarities of dispersion-managed solitons in a hybrid mode-locked all-fiber ring laser," *IEEE Photonics Technol. Lett.* **29**(18), 1588–1591 (2017).
- ³⁵³J. K. Shi *et al.*, "Femtosecond pulse coupling dynamics between a dispersion-managed soliton oscillator and a nonlinear amplifier in an all-PCF-based laser system," *Optik* **145**, 569–575 (2017).
- ³⁵⁴P. Wang, X. S. Xiao, and C. X. Yang, "Quantized pulse separations of phase-locked soliton molecules in a dispersion-managed mode-locked Tm fiber laser at 2 μ m," *Opt. Lett.* **42**(1), 29–32 (2017).
- ³⁵⁵L. Hou *et al.*, "Sub-200 femtosecond dispersion-managed soliton ytterbium-doped fiber laser based on carbon nanotubes saturable absorber," *Opt. Express* **26**(7), 9063–9070 (2018).
- ³⁵⁶L. Hou *et al.*, "Stable dispersion-managed soliton molecules in Yb-doped polarization-maintaining fiber laser with chirped fiber Bragg grating," *Opt. Eng.* **57**(8), 086102 (2018).
- ³⁵⁷Y. Y. Luo *et al.*, "Dispersion-managed soliton molecules in a near zero-dispersion fiber laser," *IEEE Photonics J.* **10**(6), 7105210 (2018).
- ³⁵⁸G. S. Parmar *et al.*, "Dispersion-managed soliton fiber laser with random dispersion, multiphoton absorption and gain dispersion," *J. Opt.* **20**(10), 105501 (2018).
- ³⁵⁹S. K. Turitsyn, B. G. Bale, and M. P. Fedoruk, "Dispersion-managed solitons in fibre systems and lasers," *Phys. Rep.* **521**(4), 135–203 (2012).
- ³⁶⁰S. M. J. Kelly *et al.*, "Average soliton dynamics of a high-gain erbium fiber laser," *Opt. Lett.* **16**(17), 1337–1339 (1991).
- ³⁶¹L. M. Zhao *et al.*, "Dynamics of gain-guided solitons in an all-normal-dispersion fiber laser," *Opt. Lett.* **32**(13), 1806–1808 (2007).
- ³⁶²A. B. Grudinin and S. Gray, "Passive harmonic mode locking in soliton fiber lasers," *J. Opt. Soc. Am. B* **14**(1), 144–154 (1997).
- ³⁶³L. M. Zhao *et al.*, "Observation of period-doubling bifurcations in a femtosecond fiber soliton laser with dispersion management cavity," *Opt. Express* **12**(19), 4573–4578 (2004).
- ³⁶⁴J. M. Soto-Crespo *et al.*, "Soliton complexes in dissipative systems: Vibrating, shaking, and mixed soliton pairs," *Phys. Rev. E* **75**(1), 016613 (2007).
- ³⁶⁵M. Grapinet and P. Grelu, "Vibrating soliton pairs in a mode-locked laser cavity," *Opt. Lett.* **31**(14), 2115–2117 (2006).
- ³⁶⁶Y. Kodama, M. Romagnoli, and S. Wabnitz, "Soliton stability and interactions in fiber lasers," *Electron. Lett.* **28**(21), 1981–1983 (1992).
- ³⁶⁷B. Zhao *et al.*, "Soliton interaction in a fiber ring laser," *Passive Compon. Fiber-Based Devices, Pts 1 2* **5623**, 652–662 (2005).
- ³⁶⁸A. Komarov *et al.*, "Spectral management of solitons interaction and generation regimes of fiber laser," in *Proceedings of the 9th International Conference on Transparent Optical Networks (Icton)*, Vol. 1, p. 217–+.
- ³⁶⁹A. Komarov *et al.*, "Interaction of dissipative solitons under spectral and amplitude control of pulse wings in fiber lasers," *Proc. SPIE* **6612**, 661209 (2007).
- ³⁷⁰W. C. Chen, Z. C. Luo, and W. C. Xu, "The interaction of dual wavelength solitons in fiber laser," *Laser Phys. Lett.* **6**(11), 816–820 (2009).
- ³⁷¹N. D. Nguyen and L. N. Binh, "Solitonic interactions in actively multi-bound soliton fiber lasers," in *Proceedings of 2009 Conference on Lasers and Electro-Optics and Quantum Electronics and Laser Science Conference (CLEO/QELS 2009)*, Vol. 1–5, pp. 2613–2614.
- ³⁷²M. Olivier *et al.*, "Pulse collisions in the stretched-pulse fiber laser," *Opt. Lett.* **29**(13), 1461–1463 (2004).
- ³⁷³M. Stratmann, T. Pagel, and F. Mitschke, "Experimental observation of temporal soliton molecules," *Phys. Rev. Lett.* **95**(14), 143902 (2005).
- ³⁷⁴D. Y. Tang *et al.*, "Observation of bound states of solitons in a passively mode-locked fiber laser," *Phys. Rev. A* **64**(3), 033814 (2001).
- ³⁷⁵S. Chouli and P. Grelu, "Rains of solitons in a fiber laser," *Opt. Express* **17**(14), 11776–11781 (2009).
- ³⁷⁶S. Chouli and P. Grelu, "Soliton rains in a fiber laser: An experimental study," *Phys. Rev. A* **81**(6), 063829 (2010).
- ³⁷⁷S. Chouli and P. Grelu, "Complex self-organized multi-pulse dynamics in a fiber laser: The rain of solitons," in *Proceedings of 2011 Progress in Electromagnetics Research Symposium, Piers*, Marrakesh (2011), pp. 12–16.
- ³⁷⁸R. Gumenyuk and O. G. Okhotnikov, "Temporal control of vector soliton bunching by slow/fast saturable absorption," *J. Opt. Soc. Am. B* **29**(1), 1–7 (2012).
- ³⁷⁹M. Haelterman and A. Sheppard, "Bifurcation phenomena and multiple soliton-bound states in isotropic Kerr media," *Phys. Rev. E* **49**(4), 3376–3381 (1994).
- ³⁸⁰B. A. Malomed, "Bound solitons in coupled nonlinear Schrödinger equations," *Phys. Rev. A* **45**(12), R8321–R8323 (1992).
- ³⁸¹B. A. Malomed, "Bound solitons in the nonlinear Schrödinger-Ginzburg-Landau equation," *Phys. Rev. A* **44**(10), 6954–6957 (1991).
- ³⁸²V. V. Afanasjev, B. A. Malomed, and P. L. Chu, "Stability of bound states of pulses in the Ginzburg-Landau equations," *Phys. Rev. E* **56**(5), 6020–6025 (1997).
- ³⁸³N. N. Akhmediev, A. Ankiewicz, and J. M. Soto-Crespo, "Multisoliton solutions of the complex Ginzburg-Landau equation," *Phys. Rev. Lett.* **79**(21), 4047–4051 (1997).
- ³⁸⁴N. N. Akhmediev, A. Ankiewicz, and J. M. Soto-Crespo, "Stable soliton pairs in optical transmission lines and fiber lasers," *J. Opt. Soc. Am. B* **15**(2), 515–523 (1998).
- ³⁸⁵Y. D. Gong *et al.*, "Bound soliton pulses in passively mode-locked fiber laser," *Opt. Commun.* **200**(1–6), 389–399 (2001).
- ³⁸⁶Y. D. Gong *et al.*, "Mechanism of bound soliton pulse formation in a passively mode locked fiber ring laser," *Opt. Eng.* **41**(11), 2778–2782 (2002).
- ³⁸⁷N. H. Seong and D. Y. Kim, "Experimental observation of stable bound solitons in a figure-eight fiber laser," *Opt. Lett.* **27**(15), 1321–1323 (2002).
- ³⁸⁸P. Shum *et al.*, "Closely spaced bound solitons with FWHM duration of 326 fs and separation of 938 fs from a passively mode locked fiber ring laser," in *Proceedings of 3rd International Conference on Microwave and Millimeter Wave Technology* (2002), pp. 1083–1086.
- ³⁸⁹D. Y. Tang *et al.*, "Bound-soliton fiber laser," *Phys. Rev. A* **66**(3), 033806 (2002).
- ³⁹⁰Y. D. Gong *et al.*, "Bound solitons with 103-fs pulse width and 585.5-fs separation from Di-nolm figure-8 fiber laser," *Microwave Opt. Technol. Lett.* **39**(2), 163–164 (2003).

- ³⁹¹Y. D. Gong *et al.*, “Close spaced ultra-short bound solitons from DI-NOLM figure-8 fiber laser,” *Opt. Commun.* **220**(4–6), 297–302 (2003).
- ³⁹²R. K. Lee, Y. Lai, and B. A. Malomed, “Quantum correlations in bound-soliton pairs and trains in fiber lasers,” *Phys. Rev. A* **70**(6), 063817 (2004).
- ³⁹³B. Zhao *et al.*, “Bound twin-pulse solitons in a fiber ring laser,” *Phys. Rev. E* **70**(6), 067602 (2004).
- ³⁹⁴R. K. Lee, Y. C. Lai, and B. A. Malomed, “Photon-number fluctuation and correlation of bound soliton pairs in mode-locked fiber lasers,” *Opt. Lett.* **30**(22), 3084–3086 (2005).
- ³⁹⁵S. M. Zhang *et al.*, “Bound soliton pulses in a passively mode-locked fiber ring laser,” *Chin. Phys.* **14**(9), 1839–1843 (2005).
- ³⁹⁶L. M. Zhao, D. Y. Tang, and B. Zhao, “Period-doubling and quadrupling of bound solitons in a passively mode-locked fiber laser,” *Opt. Commun.* **252**(1–3), 167–172 (2005).
- ³⁹⁷W. W. Hsiang, C. Y. Lin, and Y. C. Lai, “Stable new bound soliton pairs in a 10 GHz hybrid frequency modulation mode-locked Er-fiber laser,” *Opt. Lett.* **31**(11), 1627–1629 (2006).
- ³⁹⁸G. Martel *et al.*, “On the possibility of observing bound soliton pairs in a wave-breaking-free mode-locked fiber laser,” *Opt. Lett.* **32**(4), 343–345 (2007).
- ³⁹⁹L. M. Zhao *et al.*, “Bound states of dispersion-managed solitons in a fiber laser at near zero dispersion,” *Appl. Opt.* **46**(21), 4768–4773 (2007).
- ⁴⁰⁰L. M. Zhao *et al.*, “Bound states of gain-guided solitons in a passively mode-locked fiber laser,” *Opt. Lett.* **32**(21), 3191–3193 (2007).
- ⁴⁰¹L. N. Binh *et al.*, “Multi-bound solitons in a FM mode-locked fiber laser,” in *Proceedings of 2008 Conference on Optical Fiber Communication/National Fiber Optic Engineers Conference* (2008), Vol. 1–8, p. 1737–+.
- ⁴⁰²A. Komarov, A. Haboucha, and F. Sanchez, “Ultrahigh-repetition-rate bound-soliton harmonic passive mode-locked fiber lasers,” *Opt. Lett.* **33**(19), 2254–2256 (2008).
- ⁴⁰³W. W. Hsiang *et al.*, “Passive synchronization between a self-similar pulse and a bound-soliton bunch in a two-color mode-locked fiber laser,” *Opt. Lett.* **34**(13), 1967–1969 (2009).
- ⁴⁰⁴A. Komarov, K. Komarov, and F. Sanchez, “Quantization of binding energy of structural solitons in passive mode-locked fiber lasers,” *Phys. Rev. A* **79**(3), 033807 (2009).
- ⁴⁰⁵L. M. Zhao, D. Y. Tang, and D. Liu, “Ultrahigh-repetition-rate bound-soliton fiber laser,” *Appl. Phys. B: Lasers Opt.* **99**(3), 441–447 (2010).
- ⁴⁰⁶X. Wu *et al.*, “Bound states of solitons in a fiber laser mode locked with carbon nanotube saturable absorber,” *Opt. Commun.* **284**(14), 3615–3618 (2011).
- ⁴⁰⁷X. M. Wei *et al.*, “All fiber ring bound-soliton laser with a round trip time of 5.7 ns,” *Opt. Commun.* **285**(24), 5449–5451 (2012).
- ⁴⁰⁸J. Du *et al.*, “Observation of bound states of solitons in an L-band passive mode-locking ring fiber laser,” *Opt. Laser Technol.* **46**, 61–66 (2013).
- ⁴⁰⁹Z. Gong *et al.*, “Observation of continuously tuning of the phase-difference and separation of bound solitons from a carbon-nanotube mode-locked fiber laser,” in *Proceedings of Conference on Lasers and Electro-Optics Pacific Rim (Cleo-Pr)* (2013).
- ⁴¹⁰L. L. Gui, X. S. Xiao, and C. X. Yang, “Observation of various bound solitons in a carbon-nanotube-based erbium fiber laser,” *J. Opt. Soc. Am. B* **30**(1), 158–164 (2013).
- ⁴¹¹R. Gumenyuk and O. G. Okhotnikov, “Impact of gain medium dispersion on stability of soliton bound states in fiber laser,” *IEEE Photonics Technol. Lett.* **25**(2), 133–135 (2013).
- ⁴¹²C. Mou *et al.*, “Polarization dynamics of bound state solitons in a carbon nanotubes mode locked erbium doped fiber laser,” in *Proceedings of 2013 Conference on International Quantum Electronics Conference Lasers and Electro-Optics Europe (Cleo Europe/Iqec)* (2013).
- ⁴¹³X. Zhao *et al.*, “Generation of higher-order bound solitons in a carbon nanotube mode-locked fiber laser,” in *Proceedings of 2013 Conference on Lasers and Electro-Optics (Cleo)* (2013).
- ⁴¹⁴F. Bahloul *et al.*, “Numerical demonstration of generation of bound solitons in figure of eight microstructured fiber laser in normal dispersion regime,” *Opt. Commun.* **311**, 282–287 (2014).
- ⁴¹⁵J. Guo, “Bound-state solitons in a linear-cavity fiber laser mode-locked by single-walled carbon nanotubes,” *J. Mod. Opt.* **61**(12), 980–985 (2014).
- ⁴¹⁶Y. Liu *et al.*, “Generating ultra-long bound soliton sequences from a mode-locked fiber laser through intracavity spectral shaping,” in *Proceedings of 2014 Conference on Lasers and Electro-Optics (Cleo)* (2014).
- ⁴¹⁷A. P. Luo *et al.*, “Observation of three bound states from a topological insulator mode-locked soliton fiber laser,” *IEEE Photonics J.* **6**(4), 1501508 (2014).
- ⁴¹⁸C. J. Luo, S. M. Wang, and Y. Lai, “10 GHz bound soliton mode-locking in an environmentally stable FM mode-locked Er-doped fiber soliton laser,” in *Proceedings of 2014 Conference on Lasers and Electro-Optics (Cleo)* (2014).
- ⁴¹⁹L. Yun and D. Han, “Bound state of dissipative solitons in a nanotube-mode-locked fiber laser,” *Opt. Commun.* **313**, 70–73 (2014).
- ⁴²⁰A. Komarov, K. Komarov, and F. Sanchez, “Harmonic passive mode locking lasers of bound-soliton structures in fiber,” *Opt. Commun.* **354**, 158–162 (2015).
- ⁴²¹H. H. Liu and K. K. Chow, “High fundamental-repetition-rate bound solitons in carbon nanotube-based fiber lasers,” *IEEE Photonics Technol. Lett.* **27**(8), 867–870 (2015).
- ⁴²²S. Sugavanam *et al.*, “Pulse-to-pulse spectral evolution of breathing bound solitons in a mode-locked fiber laser,” in *Proceedings of 2015 Conference on Lasers and Electro-Optics (Cleo)* (2015).
- ⁴²³C. Zeng, Y. D. Cui, and J. Guo, “Observation of dual-wavelength solitons and bound states in a nanotube/microfiber mode-locking fiber laser,” *Opt. Commun.* **347**, 44–49 (2015).
- ⁴²⁴X. He *et al.*, “Bound states of dissipative solitons in the single-mode Yb-doped fiber laser,” *IEEE Photonics J.* **8**(2), 1500706 (2016).
- ⁴²⁵D. A. Korobko *et al.*, “Analysis of steady bound soliton-state attributes in hybrid mode-locked fiber laser,” *Laser Phys. Lett.* **13**(10), 105103 (2016).
- ⁴²⁶L. Li *et al.*, “Bidirectional operation of 100 fs bound solitons in an ultra-compact mode-locked fiber laser,” *Opt. Express* **24**(18), 21020–21026 (2016).
- ⁴²⁷C. J. Luo and Y. C. Lai, “RIN noise reduction effect of a 10 GHz hybrid bound soliton mode-locked fiber laser,” in *Proceedings of 2016 IEEE Photonics Conference (Ipc)* (2016), pp. 170–171.
- ⁴²⁸C. J. Luo, S. M. Wang, and Y. Lai, “Bound soliton fiber laser mode-locking without saturable absorption effect,” *IEEE Photonics J.* **8**(4), 1502609 (2016).
- ⁴²⁹Y. Y. Luo *et al.*, “Wavelength tuning and bound states of dissipative solitons in fiber lasers,” in *Proceedings of 2016 15th International Conference on Optical Communications and Networks (Icofn)* (2016).
- ⁴³⁰M. Pang *et al.*, “All-optical bit storage in a fibre laser by optomechanically bound states of solitons (vol 10, pg 454, 2016),” *Nat. Photonics* **10**(12), 814–814 (2016).
- ⁴³¹M. Pang *et al.*, “All-optical bit storage in a fibre laser by optomechanically bound states of solitons,” *Nat. Photonics* **10**(7), 454 (2016).
- ⁴³²X. Q. Wang and Y. Yao, “Switchable repetition-rate bound solitons passively mode-locked fiber laser,” *Adv. Laser Processes Manuf.* **10018**, 1001801 (2016).
- ⁴³³X. Zou *et al.*, “Versatile mode-locked fiber laser with switchable operation states of bound solitons,” *Appl. Opt.* **55**(16), 4323–4327 (2016).
- ⁴³⁴M. Chernysheva *et al.*, “Double-wall carbon nanotube hybrid mode-locker in Tm-doped fibre laser: A novel mechanism for robust bound-state solitons generation,” *Sci. Rep.* **7**, 44314 (2017).
- ⁴³⁵Y. L. Gu *et al.*, “Observation of bound soliton in mode locked fiber laser exploiting simplified nonlinear polarization rotation,” in *Proceedings of 2017 Conference on Lasers and Electro-Optics Pacific Rim (Cleo-Pr)* (2017).
- ⁴³⁶W. B. He, M. Pang, and P. S. Russell, “Multi-soliton bound states in fibre laser harmonically mode-locked at GHz-rates by optoacoustic effects in PCF,” in *Proceedings of 2017 Conference on Lasers and Electro-Optics Europe & European Quantum Electronics Conference (Cleo/Europe-Eqec)* (2017).
- ⁴³⁷K. X. Li *et al.*, “Analysis of bound-soliton states in a dual-wavelength mode-locked fiber laser based on Bi₂Se₃,” *IEEE Photonics J.* **9**(3), 1400209 (2017).
- ⁴³⁸X. Li *et al.*, “Bound states of solitons in a fiber laser with a microfiber-based WS₂ saturable absorber,” *IEEE Photonics Technol. Lett.* **29**(23), 2071–2074 (2017).
- ⁴³⁹B. L. Lu *et al.*, “Observation of bound state solitons in tunable all-polarization-maintaining Yb-doped fiber laser,” *Laser Phys.* **27**(7), 075102 (2017).
- ⁴⁴⁰H. X. Wang, L. Li, and L. M. Zhao, “Numerical study of bound states solitons in a dispersion-managed fiber laser,” in *Proceedings of 16th International Conference on Optical Communications & Networks (Icofn 2017)* (2017).

- 441 Y. Zheng *et al.*, "Observation of stable bound soliton with dual-wavelength in a passively mode-locked Er-doped fiber laser," *Chin. Phys. B* **26**(7), 074212 (2017).
- 442 H. Liang *et al.*, "Evolution of complex pulse-bunches in a bound-state soliton fiber laser," *IEEE Photonics Technol. Lett.* **30**(16), 1475–1478 (2018).
- 443 A. Zavyalov *et al.*, "Dissipative soliton molecules with independently evolving or flipping phases in mode-locked fiber lasers," *Phys. Rev. A* **80**(4), 043829 (2009).
- 444 A. Zavyalov *et al.*, "Hysteresis of dissipative soliton molecules in mode-locked fiber lasers," *Opt. Lett.* **34**(24), 3827–3829 (2009).
- 445 W. C. Chen *et al.*, "Soliton molecule in fiber laser," *Laser Phys.* **20**(9), 1818–1823 (2010).
- 446 X. M. Liu, "Dynamic evolution of temporal dissipative-soliton molecules in large normal path-averaged dispersion fiber lasers," *Phys. Rev. A* **82**(6), 063834 (2010).
- 447 B. Ortac *et al.*, "Observation of soliton molecules with independently evolving phase in a mode-locked fiber laser," *Opt. Lett.* **35**(10), 1578–1580 (2010).
- 448 W. C. Chen *et al.*, "Effect of gain media characteristics on the formation of soliton molecules in fiber laser," *Laser Phys.* **21**(11), 1919–1924 (2011).
- 449 X. H. Li *et al.*, "Numerical investigation of soliton molecules with variable separation in passively mode-locked fiber lasers," *Opt. Commun.* **285**(6), 1356–1361 (2012).
- 450 A. Zavyalov, P. Grelu, and F. Lederer, "Impact of slow gain dynamics on soliton molecules in mode-locked fiber lasers," *Opt. Lett.* **37**(2), 175–177 (2012).
- 451 J. S. Peng *et al.*, "Generation of soliton molecules in a normal-dispersion fiber laser," *IEEE Photonics Technol. Lett.* **25**(10), 948–951 (2013).
- 452 X. H. Li *et al.*, "Experimental observation of soliton molecule evolution in Yb-doped passively mode-locked fiber lasers," *Laser Phys. Lett.* **11**(7), 075103 (2014).
- 453 A. P. Luo *et al.*, "Generation of a noiselike soliton molecule induced by a comb filter in a figure-eight fiber laser," *Appl. Phys. Express* **8**(4), 042702 (2015).
- 454 Y. Q. Huang *et al.*, "Coexistence of harmonic soliton molecules and rectangular noise-like pulses in a figure-eight fiber laser," *Opt. Lett.* **41**(17), 4056–4059 (2016).
- 455 P. Wang *et al.*, "Generation of wavelength-tunable soliton molecules in a 2- μ m ultrafast all-fiber laser based on nonlinear polarization evolution," *Opt. Lett.* **41**(10), 2254–2257 (2016).
- 456 P. Wang, C. Y. Bao, and C. X. Yang, "Soliton molecules in a 2- μ m Thulium-doped fiber laser," in Proceedings of 2016 Conference on Lasers and Electro-Optics (CLEO) (2016).
- 457 C. Y. Ma *et al.*, "Dynamic evolution of the soliton molecules in an all-normal dispersion fiber laser," *Laser Phys.* **27**(6), 065102 (2017).
- 458 L. L. Gui *et al.*, "Soliton molecules and multisoliton states in ultrafast fibre lasers: Intrinsic complexes in dissipative systems," *Appl. Sci.* **8**(2), 201 (2018).
- 459 R. H. Li *et al.*, "Ultrawide-space and controllable soliton molecules in a narrow-linewidth mode-locked fiber laser," *IEEE Photonics Technol. Lett.* **30**(16), 1423–1426 (2018).
- 460 B. W. Liu *et al.*, "Soliton molecules in a fiber laser based on optic evanescent field interaction with WS₂," *Appl. Phys. B: Lasers Opt.* **124**(7), 151 (2018).
- 461 M. Liu *et al.*, "Real-time visualization of soliton molecules with evolving behavior in an ultrafast fiber laser," *J. Opt.* **20**(3), 034010 (2018).
- 462 A. Niknafs, H. Rooholaminnejad, and A. Bahrapour, "Generation of two-soliton and three-soliton molecules in a circular fiber array laser," *Laser Phys.* **28**(4), 045406 (2018).
- 463 H. Q. Qin *et al.*, "Observation of soliton molecules in a spatiotemporal mode-locked multimode fiber laser," *Opt. Lett.* **43**(9), 1982–1985 (2018).
- 464 C. Mou *et al.*, "Bound state vector solitons with locked and precessing states of polarization," *Opt. Express* **21**(22), 26868–26875 (2013).
- 465 L. L. Gui *et al.*, "Self-assembled graphene membrane as an ultrafast mode-locker in an erbium fiber laser," *IEEE Photonics Technol. Lett.* **23**(23), 1790–1792 (2011).
- 466 M. Boiti, C. Laddomada, and F. Pempinelli, "Non-linear Schrödinger-equation, potential non-linear Schrödinger-equation and soliton-solutions," *Nuovo Cimento Soc. Ital. Fis., A* **68**(3), 236–248 (1982).
- 467 F. B. Pelap and M. M. Faye, "Modulational instability and exact solutions of the modified quintic complex Ginzburg-Landau equation," *J. Phys. A: Math. Gen.* **37**(5), 1727–1735 (2004).
- 468 V. E. Zakharov and A. V. Mikhailov, "Polarization domains in nonlinear optics," *JETP Lett.* **45**(6), 349–352 (1987).
- 469 M. Haelterman, "Polarization domain-wall solitary waves for optical-fiber transmission," *Electron. Lett.* **30**(18), 1510–1511 (1994).
- 470 S. Wabnitz and B. Daino, "Polarization domains and instabilities in nonlinear-optical fibers," *Phys. Lett. A* **182**(2–3), 289–293 (1993).
- 471 S. Pitois, G. Millot, and S. Wabnitz, "Polarization domain wall solitons with counterpropagating laser beams," *Phys. Rev. Lett.* **81**(7), 1409–1412 (1998).
- 472 B. A. Malomed, A. A. Nepomnyashchy, and M. I. Tribelsky, "Domain boundaries in convection patterns," *Phys. Rev. A* **42**(12), 7244–7263 (1990).
- 473 M. Haelterman and A. P. Sheppard, "Polarization domain-walls in diffractive or dispersive Kerr media," *Opt. Lett.* **19**(2), 96–98 (1994).
- 474 B. A. Malomed, "Domain-wall between traveling waves," *Phys. Rev. E* **50**(5), R3310–R3313 (1994).
- 475 B. A. Malomed, "Optical domain-walls," *Phys. Rev. E* **50**(2), 1565–1571 (1994).
- 476 P. Kockaert *et al.*, "Isotropic polarization modulational instability and domain walls in spun fibers," *Appl. Phys. Lett.* **75**(19), 2873–2875 (1999).
- 477 H. Zhang *et al.*, "Dual-wavelength domain wall solitons in a fiber ring laser," *Opt. Express* **19**(4), 3525–3530 (2011).
- 478 Z. B. Lin *et al.*, "Generation of dual-wavelength domain-wall rectangular-shape pulses in HNLF-based fiber ring laser," *Opt. Laser Technol.* **44**(7), 2260–2264 (2012).
- 479 C. Lecaplain, P. Grelu, and S. Wabnitz, "Polarization-domain-wall complexes in fiber lasers," *J. Opt. Soc. Am. B* **30**(1), 211–218 (2013).
- 480 Z. C. Luo *et al.*, "Generation of high-energy dual-wavelength domain wall pulse with low repetition rate in an HNLF-based fiber ring laser," *Chin. Phys. B* **23**(6), 064203 (2014).
- 481 H. Ahmad *et al.*, "Domain-wall dark pulse generation in fiber laser incorporating MoS₂," *Appl. Phys. B: Lasers Opt.* **122**(4), 69 (2016).
- 482 J. M. Liu *et al.*, "Polarization domain wall pulses in a microfiber-based topological insulator fiber laser," *Sci. Rep.* **6**, 29128 (2016).
- 483 D. Y. Tang *et al.*, "Polarization domain formation and domain dynamics in a quasi-isotropic cavity fiber laser," *IEEE J. Sel. Top. Quantum Electron.* **20**(5), 0901309 (2014).
- 484 M. Gilles *et al.*, "Polarization domain walls in optical fibres as topological bits for data transmission," *Nat. Photonics* **11**(2), 102–107 (2017).
- 485 G. Herink *et al.*, "Resolving the build-up of femtosecond mode-locking with single-shot spectroscopy at 90 MHz frame rate," *Nat. Photonics* **10**(5), 321 (2016).
- 486 A. Mahjoubfar *et al.*, "Time stretch and its applications," *Nat. Photonics* **11**(6), 341–351 (2017).
- 487 M. Popov and O. Gat, "Pulse growth dynamics in laser mode locking," *Phys. Rev. A* **97**(1), 011801(R) (2018).
- 488 X. M. Liu, X. K. Yao, and Y. D. Cui, "Real-time observation of the buildup of soliton molecules," *Phys. Rev. Lett.* **121**(2), 023905 (2018).
- 489 M. M. Han *et al.*, "Generation of soliton bursts with flexibly controlled pulse intervals based on the dispersive Fourier-transform technique," *IEEE J. Sel. Top. Quantum Electron.* **25**(4), 7500106 (2019).
- 490 P. Ryczkowski *et al.*, "Real-time full-field characterization of transient dissipative soliton dynamics in a mode-locked laser," *Nat. Photonics* **12**(4), 221 (2018).
- 491 J. S. Peng and H. P. Zeng, "Build-up of dissipative optical soliton molecules via diverse soliton interactions," *Laser Photonics Rev.* **12**(8), 1800009 (2018).
- 492 A. Klein *et al.*, "Ultrafast rogue wave patterns in fiber lasers," *Optica* **5**(7), 774–778 (2018).