

DESIGN AND OPTIMIZATION OF WIRELESS POWER TRANSFER SYSTEM

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Abstract

Wireless power transfer (WPT) refers to the transmission of electrical energy without a physical contact. WPT based on the resonance principle has been proposed for a wide range of applications, where implementation of physical connectors can be inconvenient, hazardous or impossible. The electromagnetic field generated by a transmitting resonator is captured by a receiving resonator which resonates at the excitation frequency of the power source. Although many researchers have been working on WPT technology, still there are numerous challenges to overcome in the process of wide adoption of commercialization.

Key performance indices of WPT technology are efficiency, transferred power, transfer distance and misalignment tolerance. There are several modelling methods for analyzing the WPT system, namely, equivalent circuit approach, two port network model and coupled mode theory. The performance of a WPT setup relies on number of design parameters such as WPT coils, power converters, transfer distance, frequency and load characteristics. A comprehensive review on the WPT technology and its applications are presented at the beginning of the thesis. WPT coils play an imperative role in WPT performance. With the assumption of optimum load condition, quality factor of the coils and coupling coefficient between coils are the main optimization objectives in improving WPT coils. Decreasing AC resistance of the coils while maintaining a high inductance is useful in high quality factor coils. An optimization guidelines for improving the WPT coils are investigated in this study. A novel toroidal shaped spiral coil has been proposed for high efficiency WPT.

Most WPT applications require wirelessly powered receiver to be moved freely. Therefore dynamic-WPT (D-WPT) which allows receiver movement, has become one of the attracting extensions of WPT technology. D-WPT can be realized either with an array of segmented transmitting coils or using long transmitter track. Misalignment problem becomes an inevitable design challenge for D-WPT. The study of this thesis is extended to investigate D-WPT system and its optimizations. It has been investigated that misalignment tolerance of the D-WPT system can be significantly improved with the use of repeater coils. In addition, the optimum current distribution among transmitter coils has been derived theoretically. A current modulation scheme is explored with the

use of an analogy of N-dimensional Cartesian coordinate equivalent of spherical coordinates. The optimal current distribution at arbitrary receiver position is a function of the mutual inductances between transmitter coils and receiver. This theoretical contribution can be equally applied to any kind of multi-transmitter WPT setups.

The study of this thesis is extended to address the misalignment problem. Receiver misalignment can be radial, axial or angular depending on the receiver's degrees of freedom (DoF). A figure-of-merit is proposed to optimize efficiency, transferred power and six-DoF misalignment tolerances simultaneously. Misalignments in six-DoF are segmented into sub regions, and performance within each region is prioritized based on probability of alignment in deriving the figure-of-merit. A WPT system with tri-spiral-repeater is introduced with the use of proposed figure-of-merit as the optimization objective.

Finally, a tuning method has been investigated to improve the performance against misalignment and load variations. Although, the investigation on the optimal load resistance is useful for the initial design stage, it is not always possible to operate at optimal value during the operational conditions. For example, equivalent load resistance of a battery charging application is dependent on the battery state-of-charge. Therefore, the repeater tuning approach investigated in this research can be applied to improve the performance in dynamic environments.

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List of Abbreviations

AC	Alternative Current
ADS	Advanced Design System
AIM	Adaptive Impedance Matching
CMT	Coupled Mode Theory
DC	Direct Current
DoF	Degrees of Freedom
DSR	Double Spiral Repeater
D-WPT	Dynamic Wireless Power Transfer
EV	Electric Vehicle
FOM	Figure-of-Merit
HF	High Frequency
IM	Impedance Matching
IoT	Internet of Things
MM	Metamaterial
PTE	Power Transfer Efficiency
Q	Quality Factor
RFID	Radio Frequency Identification
RMS	Root Mean Square
Rx	Receiver
SOC	State-Of-Charge
SSR	Single-Spiral-Repeater
TP	Transferred Power
TSR	Tri-Spiral-Repeater
Tx	Transmitter
VAD	Ventricular Assisted Device
WPT	Wireless Power Transfer

Chapter 1

Introduction

1.1 Background and motivation

The transmission of electrical power to various electronic loads has evolved since the discovery of electricity in 18th century. Wireless power transfer, one of the revolutionary concept in electricity transmission refers to the transmission of electrical energy without a physical connection. History of WPT goes to 19th century where Maxwell's equations have been introduced to formulate the phenomena of radio waves. Afterwards, Henry Poynting explained electromagnetic waves as an energy flow which is the basis for his Poynting theorem in 1884. The principle of WPT is investigated by Nikola Tesla at the end of the 19th century [1, 2]. Although Tesla's experiment was not successful at his time, with the advent of advanced semiconductor technologies, Tesla's proposition has now become a reality. WPT technology is appealing to a number of applications due to its convenience. This technology would revolutionize the way of living and create a whole new spectrum of electronics devices and usage behaviour.

There has been many types of wireless energy transfer technologies including laser, photoelectric, radio waves, microwaves, capacitive coupling and inductive coupling. Out of these, inductive coupling techniques based on the resonance theory has gain an increased attention as it is capable of delivering power with acceptable efficiency which is useful for wide range of applications. Therefore, this research will focus on addressing design challenges in the wireless power transmission based on the resonance inductive coupling technique. Therefore, the term wireless power transfer (WPT) refers to the resonance inductive power transfer in this thesis.

A typical WPT setup comprises of a transmitter coil (Tx) connected to a high frequency (HF) source and a receiver coil (Rx) connected to the electrical load. When HF current passes through the Tx, an electromagnetic field is generated and it is coupled with the Rx, which induces a voltage across it. In a practical application, there should be a substantial separation between Tx and Rx leading them to be loosely coupled. In addition, there can be a power converter, rectifier, or charging circuit connected to Tx

and Rx to manage the power flow. Utility power (AC grid or DC battery) is converted to HF using power converter connected to Tx, while the rectifier/charging circuit is connected in between Rx and the electrical load. The maximum energy can be transferred when Rx resonate at the same frequency as the frequency of HF source. If the operating frequency is well below the natural resonance frequencies of WPT coils, additional compensation capacitors are required to form the resonance in both Tx and Rx sides. The system components from Tx to Rx (including the compensation circuits) are referred to as the wireless link, which will be the major focus of this research.

If the WPT Rx is placed in a fixed position during the transmission of power, such a WPT scheme is termed as stationary WPT. This typical WPT scheme with a single Tx and a single Rx has progressed to variety of multi-Tx and multi-Rx schemes [3-7]. A revolutionary extension of stationary WPT is dynamic WPT (D-WPT) which seeks to power the moving receiver (Rx) wirelessly. Stationary WPT technique has been applied to a wide range of applications such as, biomedical implants [8-11], stationary charging of electric vehicles [12], consumer electronics [5, 13-15], and industrial applications [16]. On the other hand, promising applications of D-WPT include electric vehicles [17-19], biomedical applications [20], robotics [21] and manufacturing applications [22].

The key performances for applications include system level performances such as power transfer efficiency (PTE) and transferred power (TP), and application level requirements such as transfer distance, misalignment tolerance and load variation tolerance. PTE refers to the energy efficiency through WPT link whereas, TP refers to the normalized output power at the load. High PTE is essential to minimize system losses, while high TP allows higher output power delivered to the load. Therefore, it is important to consider both PTE and TP for the optimization of WPT systems. However, system performances (PTE and TP) are severely affected by the application specific performance requirements. For example, PTE deteriorates drastically with the increase of transfer distance and misalignment. A number of techniques have been proposed in literature to improve the performance. Such optimization approaches include compensation network and circuit design [23-28], coil design for high efficiency [29], repeaters [14, 30-33], tuning approaches [34, 35] and control methods [36, 37].

Although many researchers have been working on the WPT technology, there are still numerous challenges to overcome for wide adoption of commercialization. One of

the key design challenge is to increase the power transfer distance without affecting the performance. In addition, the improvement of misalignment tolerance and the improvement of load characteristics are also vital design challenges to be addressed. This research will focus on addressing the design challenges in the WPT technology.

1.2 Thesis objective and approach

As mentioned in the previous section, the performance improvement against transfer distance, misalignment and load variations is one of the major technical challenges in WPT. Therefore, this research focuses on the investigation of novel approaches and techniques to improve the performance of the wireless link of a WPT setup. The power source is assumed to be pure sinusoidal source considering only the fundamental frequency component, because the major focus of this study is not on the power converter design. In addition, the load impedance is assumed to be purely resistive. The equivalent circuit approach is used along with the two-port network theory for the analyses of the proposed WPT schemes.

1.3 Contributions of the thesis

The main contributions of the thesis are summarized below.

- *A coil optimization approach and coil design guidelines for high efficiency WPT* – An optimization of WPT coils considering the maximum power transfer efficiency is investigated in this study. Quality factor and coupling coefficient are identified as the key performance indices for WPT coil design. The effect of coil design parameters is investigated to achieve high quality factors and coupling, thereby empirical formulas are derived to obtain generalized guidelines for quality factor optimization. The design guidelines proposed in this study are extended to derive generalized coil design scheme for a WPT system with a miniaturized Rx.
- *Efficiency optimization scheme for D-WPT* – A comprehensive analysis is performed on the design of D-WPT system with a segmented Tx array. Efficiency characteristics against design parameters such as, operating frequency, load resistance, number of simultaneously powered Tx coils and Tx coil spacing are discussed in detail. In addition, a double spiral repeater

(DSR) is proposed for the PTE improvement of D-WPT system. The DSRs placed at the Rx coil was found to be a promising solution to obtain enhanced PTE with higher transfer distance and misalignment tolerance.

- *A theoretical analysis to derive the optimum Tx current distribution for D-WPT system with segmented Tx array* – The theoretical optimum current distribution within a Tx array is derived by applying an n-dimensional coordinate analogy onto the equivalent circuit. Both PTE and output power characteristics with respect to operating frequency and load resistance are investigated with the optimum current distribution. The theoretical upper bound of the PTE and TP derived for an arbitrary position of Rx within the Tx array.
- *A novel figure-of-merit (FOM) as an optimization objective function for the simultaneous improvement of the system performances (PTE and TP) and misalignment tolerance simultaneously* – An optimization of WPT system to improve the performances against misalignment is investigated. A novel FOM is introduced for the optimization of system level performances such as PTE and TP, and the application specific criteria such as misalignment tolerance in multiple directions and orientations. The proposed FOM is applied to the optimization of WPT system with repeaters. A novel repeater configuration, tri-spiral-repeater (TSR) is proposed to validate the proposed FOM. In addition, the variable frequency operation has been identified as a viable approach to improve the performance in multiple misalignment directions.
- *A dynamic optimization scheme for WPT with repeaters against load variations and Rx misalignment* – It is investigated that the self-resonance frequencies of Tx, Rx and repeater coils and the operating frequency can be optimized to improve the load characteristics and misalignment tolerance. An analytical approach is proposed to derive the optimal operating conditions for a given nominal load and Rx position.

1.4 Thesis organization

The thesis is organized into eight chapters, and as follows: Chapter 1 introduces the background, motivation and the direction of the research on wireless power transfer, Chapter 2 provides a detailed literature review and Chapters 3 to 7 describe the contributions of this study, followed by the conclusions in Chapter 8.

Chapter 2 provides a detailed literature review on the WPT technology including applications of WPT, modeling methods, performance evaluation and performance optimization. A range of WPT applications are briefly explained while giving a particular attention to WPT for electric vehicle applications, biomedical implants and consumer electronics applications. Equivalent circuit method, two port network method and coupling mode theory are presented as modeling methods, along with a comparison between them. The technical challenges and optimization approaches reported in literature are summarized.

Chapter 3 presents a coil optimization method for WPT considering the maximum PTE. Quality factor and coupling coefficient have been identified as the key performance indices for WPT coil design. The coil design parameters are investigated to achieve high quality factor, thereby empirical formulas are derived to obtain generalized guidelines for the design of WPT coils. High quality factor coils are experimentally realized using the proposed optimization, and these coils have been used for the subsequent studies in this thesis. In addition, an analysis on normalized distance which has been used for the comparison of dissimilar WPT schemes are performed. Merits of normalized distance and its limitations are investigated. Moving forward, the investigation is extended to derive generalized coil design guidelines for a WPT system with a miniaturized Rx. A design case is presented along with an experimental validation.

Chapter 4 presents numerical analysis on the design of dynamic wireless power transfer (D-WPT) system with a segmented Tx array. A detailed analysis is performed with regards to PTE characteristics against design parameters. A frequency splitting phenomena is observed in efficiency characteristic due to the reflected impedance variations within the Tx array. In addition, a double spiral repeater (DSR) for dynamic wireless power transfer is introduced in Chapter 4. The DSR placed at the receiver coil

is found to be a promising solution to obtain enhanced PTE with higher transfer distance and misalignment tolerance. Quasi static condition is analyzed for four types of Rx arrangement topologies, and performance of all the topologies are compared using the equivalent circuit model. Frequency characteristics, load variation characteristics and transient responses of the proposed topologies are investigated.

Chapter 5 investigates a theoretical optimum current distribution within a Tx array for D-WPT system powered with a set of current sources. An n-dimensional coordinate analogy has been applied along with the equivalent circuit to obtain the optimum Tx current distribution within the segmented Tx array. The optimization results are presented for an arbitrary number of Tx coils and further simplified for setups with two and three number of Tx coils. In addition, optimization design parameters such as operating frequency, load resistance and number of simultaneously powered Tx coils have been further explored analytically. Furthermore, the theoretical upper bound of the PTE and normalized output power is derived for optimal load and frequency conditions. Lastly, comparison between the conventional voltage source configuration studied in Chapter 4 and the current source with proposed optimization method in Chapter 5 are compared.

Chapter 6 explores the optimization of system level performances (PTE and TP) as well as the application specific requirements such as misalignment tolerance in multiple directions. A novel figure-of-merit (FOM) is introduced as an optimization objective function in order to improve system performance and misalignment tolerance simultaneously. It is investigated that the proposed FOM allows improved performance against misalignment with a slight performance reduction at the aligned position. Theoretical analysis is verified with a numerical case study of WPT scheme which comprises of a novel tri-spiral-repeater (TSR). The discussion is extended to achieve design freedom to select optimization parameters such as sub-regions in each of Rx's misalignment direction and probability of alignment in each sub region. Furthermore, it is pointed out that the proposed FOM can be applied not only to the WPT with repeaters, but also in a variety of other optimization approaches.

Moving forward, the study in Chapter 7 tackles the load variation characteristic improvement for WPT with repeaters. The study in Chapter 7 investigates the optimization of self-resonance frequencies of Tx, Rx and repeater coils and the

operating frequency in order to improve the load characteristics and misalignment tolerance. The chapter starts with an analysis of WPT with repeater by employing the two-port theory. The generalized theory was applied to two types of repeaters, single-spiral-repeater (SSR) and tri-spiral-repeater (TSR), and optimization steps are derived with regards to both types of repeaters. The proposed optimization of this chapter allows the improvement of both PTE and TP against load variations and misalignment tolerance.

Finally, Chapter 8 concludes the thesis and recommends a number of possible research areas that can be explored in the future.

Chapter 2

Literature Review

2.1 Introduction

Wireless power transfer (WPT) refers to the transmission of electrical energy from the power source to an electrical load without any physical connection. Wireless power transfer methods encompass technologies such as laser, photoelectric, radio waves, microwaves, capacitive coupling, inductive coupling and magnetic resonance coupling. These technologies can be broadly categorized based on underlying mechanism, transmission range, and power rating. Based on the power transfer distance, wireless energy transfer methods can be categorized into two types; near field and far field. If transfer distance is longer than the wavelength of electromagnetic wave, it is categorized into far field technique. Laser, photoelectric, RF, microwave can be considered as far field energy transfer methods. Inductive coupling and magnetic resonance coupling based methods are regarded as near field approaches. Even though far field techniques have transmission distance up to several kilometers, they suffer from the trade-off between directionality and efficiency. The operating frequencies of far field approaches are typically very high (GHz range) compared to near field (kHz to MHz). Inductively coupled near field approaches can be used to transmit high power efficiently in very near range (up to several centimeters). Efficiency of such systems deteriorates exponentially with the distance.

Traditional inductive power transfer (IPT) systems are based on inductive coupling and initial configurations of IPT systems used inductor in series with the coil [38]. This is very similar to a loosely coupled transformer. Later, IPT based topologies are adopted with compensation capacitors [39]. Conversely, magnetic resonance technology originally used with self-resonance coils which resonates with its self-inductance and parasitic capacitance [40]. An external lumped capacitor is added to build the resonance coils when parasitic capacitance is inadequate to make resonance at frequency of interest in later studies [41, 42]. Typical uncompensated IPT schemes are limited to a few centimeters whereas resonance based WPT can be used in larger transfer distances.

WPT technology has displayed significant improvement in the areas of RF technology, near-field energy transfer, energy conversion and management, energy storage elements, novel materials, fabrication techniques and EMC/EMI considerations. Design of low loss circuits, resonators, adaptive control strategies and magnetic shielding structures can be seen as key growth areas in WPT technology related to consumer electronics, industrial automation, electric vehicles and medical implants.

This chapter provides a literature review on WPT technology, which is organized as follows. Section 2.2 discuss on WPT applications highlighting some of the notable achievements from industry and academia. Next, Section 2.3 introduces various modeling techniques WPT system analysis and their merits. Section 2.4 provides a review on various optimization approaches reported in literature. Finally, operating frequency selection criteria is discussed in Section 2.5, which is followed by the chapter summary.

2.2 Applications of WPT

WPT technology has been applied in a wide range of applications such as consumer electronics [5, 13-15, 43, 44], biomedical implants [8, 45-47], sensor nodes [48, 49], internet of things (IoT) [50] and electric vehicles (EVs) [12, 51, 52]. Electrification impediments related to wired-electrification can be mitigated by adopting WPT techniques in each of these applications. However each application has its own specifications and requirements. Following subsections describe WPT systems applicable to EV charging, biomedical implants and consumer electronics applications.

2.2.1 WPT for EV charging

WPT for EV charging has many advantages compared to wired-EV charging. Inconveniences in wired EV charging process are major impediments in gaining interest of consumers. Due to industrial requirements and advancements in technology, power transfer distance in WPT systems has increased from several millimeters to several centimeters. Demonstrated research works show promises that the WPT can be brought into commercial level with a reasonable cost [53].

Stationary EV charging for EVs and dynamic EV charging for moving vehicles have been studied and demonstrated in the literature. Although both stationary and dynamic

WPT for EVs have been undergoing from laboratory demonstrations, to bring them to large scale commercialization, still there are significant works that need to be done on performance optimization, standardization, cost effectiveness and safety considerations. Charging process should be automated as much as possible.

Stationary WPT for EV charging

Stationary WPT can replace the charging cable for EVs. The WPT system is activated when a vehicle reaches to the charging area. Fig. 2-1 illustrates a typical stationary WPT platform for EV charging. High frequency power inverter converts low frequency utility power to high frequency AC power. Resonance electromagnetic field generated in the transmitting resonator transfers power to the receiving resonator. Received power at the secondary resonator is rectified to charge the battery pack. The charging system can be implemented in urban areas such as parking lots and bus stops.

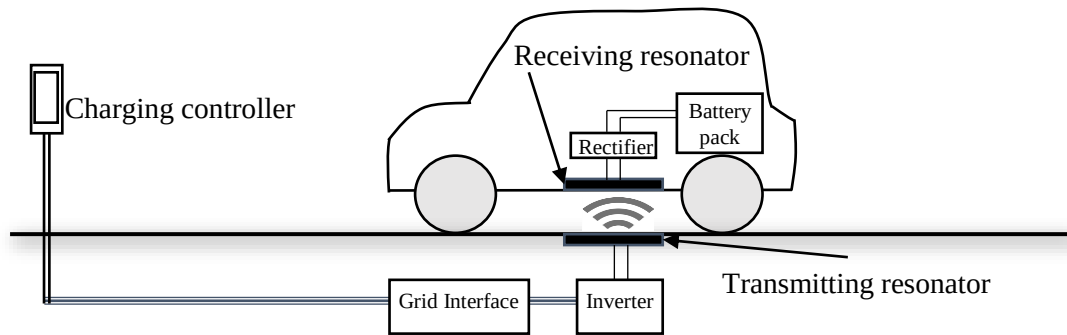


Fig. 2-1 Stationary WPT for EV

Stationary WPT for EV charging has better market acceptance and lower implementation cost compared to dynamic WPT. A few WPT enabled EVs have been introduced to market including “Leaf” by Nissan Motor Co., “2014Volt” by Chevrolet, “Qualcomm HaloIPT” by Qualcomm Co., and “PUGLESS” by Evatran Co.. Automotive manufacturers such as Delphi, Magna, Maxwell and Panasonic have been working on developing WPT systems. The WiT-3300 development kit by WiTricity Corporation is capable of delivering 3.3kW power over 18cm with 90% efficiency [54]. Size of WiT-3300 transmitter and receiver resonators of are 50cmX50cm [54]. University of Auckland has achieved 5kW stationary IPT system with 90% efficiency over 20cm distance using 75cm size coil design [55]. Stationary WPT solutions for EV charging has to be designed shifting the system complexity more towards the transmitting side infrastructure and keeping the vehicle component as simple as

possible. Although mechanical and electrical hazards with plugged-in charger can be eliminated using stationary WPT for EV charging, driving range and slower charging time will still be a dominant issue. Dynamic WPT approaches can be used to solve these issues.

Dynamic WPT for EV charging

Dynamic WPT for EV referred to the charging of EV continuously while the EV is in motion. From the vehicle viewpoint, dynamic WPT enabled infrastructure where EVs can be charged continuously while in motion, theoretically solve the EV battery problem with unlimited driving range. However, employment of such system is reliant on the infrastructure development, which in turn limited by its cost. In addition, amount of the energy gained through WPT depends on the power level of the system, vehicle speed and duration that vehicle travel within the WPT enabled zone.

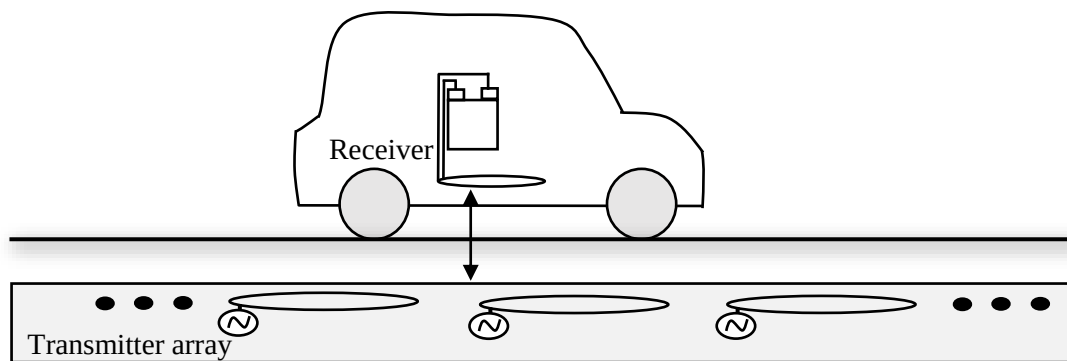


Fig. 2-2. Dynamic EV charging with segmented coil array

Dynamic EV charging approaches can be mainly categorized into two types based on transmitter array design: single transmitter track and segmented transmitter coil array. First type consists of a substantially long transmitter track connected to a power source. The receiver is noticeably smaller than the length of the track. Segmented coil array based designs have multiple coils connected to high frequency power sources as shown in Fig. 2-2. Some of the notable achievements in designing dynamic EV charging platforms can be identified as follows. UC Berkeley has conducted a test as a proof-of-concept of a dynamic WPT system for EV based on IPT in late 70s'[38]. They transferred a 60kW of power through 7.6 cm distance to a passenger bus along 213m long track. Due to limited semiconductor technologies, operating frequency of Berkeley system was 400 Hz and their efficiency was only 60%. From there, researchers and

industries have improved the performance of the dynamic EV charging systems. Korea Advanced Institute of Science and Technology (KAIST) is a leading research contributor in dynamic EV charging. The online electric vehicle (OLEV) center of KAIST has demonstrated a transmitter track based roadway powered EV system. The OLEV has achieved 20kW power transfer over 20cm at 83% efficiency with the use of Shaped Magnetic Field in Resonance (SMFIR) technology. SMFIR consists of current carrying transmitter rails of 5m to 60m long and pickup module of 80cm [56]. The relative transfer distance of KAIST-OLEV is still much smaller than unity. Oakridge National Lab (ORNL) has successfully demonstrated WPT system for moving EV using segmented coil array. ORNL has transferred 2kW over 10cm using 33cm diameter coils with a maximum efficiency about 85% [57]. But the efficiency of their setup tends to drop drastically with misalignment.

2.2.2 WPT for biomedical implants

Medical implants have a strong impact on future medicine. Healthcare affecting a patient's quality of life is becoming an increasingly attractive topic in research. Many implants are used to make life more comfortable. For example, Radio Frequency Identification (RFID) tags implanted under the skin to store information such as providing a key for doors or vehicles and patient identification. In addition, some implants used to enhance the quality of life dealing with physical restrictions and simplify medical care. These implants can be medical microelectronic devices, such as cochlear implants, retinal prostheses, drug delivery devices or more mechanical devices such as hip and knee implants. In certain circumstances, life-sustaining continuous treatment is necessary. For example, cardiac pacemaker implant and ventricular assisted device (VAD) include a battery and controlling units. Such life preserving implants need complicated medical interventions as well as service operations to change the battery. For instance, with current technology, once a patient becomes dependent on a VAD, he/she has to live for the duration of support with a wired connection exiting the body which connects the VAD to the external battery pack. This makes the VAD uncomfortable for the user and exposes the patient to a high risk of infection due to the percutaneous power line. In fact, infection is one of the most significant complications after VAD implantation in both the early and late phases of support [6]. Although

completely implantable VADs are proposed [8], they are not feasible at present due to the high power capacity requirement of the battery.

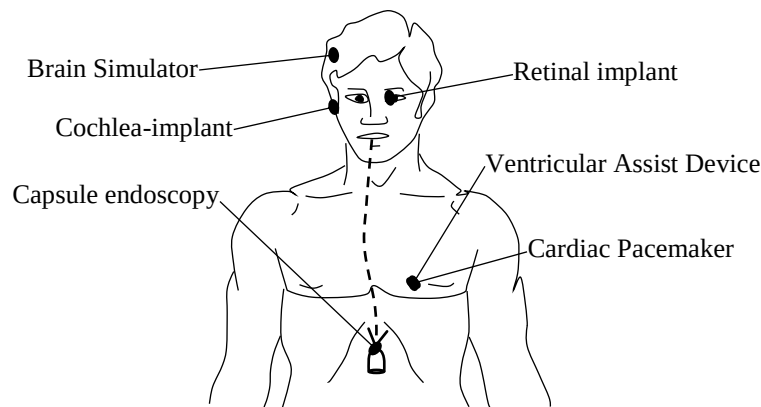


Fig. 2-3 Active biomedical implants with its own power supply that can be powered with WPT

WPT technology can be either used in direct powering of the implant or charging the implant battery. For example, direct powering capsule endoscopy can be implemented without on-board battery using D-WPT technology. On the other hand, stationary WPT technology can be employed in charging the implant batteries. This eliminates the requirement for battery replacement surgeries and reduces the capacity of the on-board battery. In the case of a WPT enabled VAD, energy can be transferred wirelessly using a wearable jacket containing a battery, power converters and transmitting coil arrangements. Transmitting coils connected to the body using a vest (or belt) like arrangement would give full freedom to the patient's movements and day to day activities. Uninstructive recharging of implants batteries using WPT technology has great advantage in usability. Some of the previous efforts suggest using WPT technology for the biomedical applications [8, 47, 58-61]. Fig. 2-3 illustrates possible biomedical implants that can be powered with the proposed technology in this thesis.

2.2.3 WPT for consumer electronics applications

The development of consumer electronics and battery powered portable devices since last few decades has uplifted the human life standard significantly. However, power supply and charging cables of these electronic components have become a major cause of electronic waste issue [62]. In addition, the power supply limits the device portability causing an inconvenient for the user. Therefore, the WPT technology has been proposed to replace the power cable of consumer electronics and charging of the battery powered portable devices. There has been a number of studies reported in

literature which aim to apply the WPT technology to consumer electronics applications [5, 13-15, 44]. Some commercial entities have been working on developing WPT technology into their products. For example, Wireless Power Consortium has come up with “Qi” standard for planar wireless charging technology applicable to portable electronics [15]. AirFuel Alliance has been working on developing WPT technology to enable charging through metal, which would be attracting solution for many portable electronics [63]. Sony corp., also working on integrating WPT technology into their products [64].

The contributions of thesis are mainly focus on the performance improvement techniques, therefore, useful in a wide spread range of applications such as in medicine towards the concept of power cable free biomedical sensing and monitoring system and EV charging applications. Modeling techniques of WPT system are discussed in next section.

2.3 Modelling methods

Modeling of a WPT system is crucial for the performance evaluation and optimization. Simplicity and the accuracy of the model are important. The modeling methodology needs to provide guidelines in the selection of system performance indices and design parameters. The block diagram in Fig. 2-4(a) shows typical two-coil WPT system powered using high frequency (HF) power source. HF source can either be an inverter or combination of a power amplifier (PA) and a signal generator (SG). PA and SG based setups are commonly used in laboratory prototyping, but PA and SG combination has to be replaced with an HF inverter in real implementation. Receiver resonator is connected to the load through a battery charging circuitry. Load impedance consists of the impedance of the charging circuit. Battery load can be approximated to a resistive AC load [65].

Equivalent source resistance represents output impedance of the power source. In case of PA and SG combination, source impedance is typically 50 Ω . However in case of power converter based designs, this value is much smaller than 50 Ω . Therefore, source resistance must be chosen with careful attention in the design process based on the type of the source.

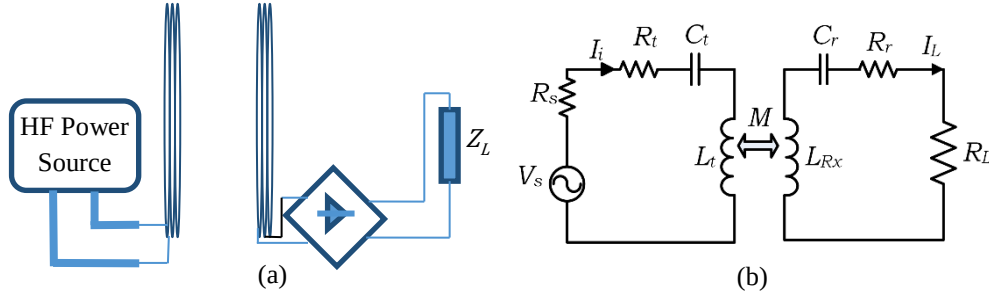


Fig. 2-4 (a) – Two resonator WPT system, (b) – The equivalent circuit.

2.3.1 Equivalent circuit theory

As electrical lengths (ratio of cable lengths to operating wavelength) of cables and coils are very short, the system can be analyzed in lumped parameter model. Lumped parameter circuit model is shown in Fig. 2-4(b). Two types of resonant coils have been used in WPT; self-resonators and LC resonators. Self-resonators resonate with the self-inductance of coil and parasitic capacitance between turns, whereas LC type has an externally added lumped capacitor. Self-resonators have the advantage of low loss, but realization at low frequencies is challenging because the typical parasitic capacitance is very low. LC type is more controllable during the design process. Compensation capacitor of the LC resonator can either be Parallel (P) or Series (S) to the coil. Series compensation inherits voltage source characteristic while parallel compensation has current source characteristic. Based on the transmitting and the receiving resonator capacitor compensation, four topologies; Series–Series (SS), Series–Parallel (SP), Parallel–Series (PS), and Parallel–Parallel (PP) can be realized. If parallel compensation is used at either side, the capacitance value depends on the coupling factor, which is a variable in a dynamic Rx behaviour. Therefore SS compensation is chosen for this study. Using Kirchhoff's voltage law, (2-1) can be obtained [15, 31].

$$\begin{bmatrix} Z_t + R_s & j\omega M \\ j\omega M & Z_r + R_L \end{bmatrix} \begin{bmatrix} I_t \\ I_L \end{bmatrix} = \begin{bmatrix} V_s \\ 0 \end{bmatrix} \quad (2-1)$$

$$Z_{t,r} = R_{t,r} + j\omega L_{t,r} + \frac{1}{j\omega C_{t,r}} \text{ and } M = k\sqrt{L_t L_r}$$

Where V_s is the RMS voltage of the HF source with a fundamental frequency ω , and Z_t and Z_r are the impedances of transmitter and receiver resonators respectively. These impedances are made up of $L_{t,r}$, $C_{t,r}$ and $R_{t,r}$ which are inductance, capacitance and resistance of the coils respectively. Capacitance includes both distributed capacitance of

the coil and externally added lumped capacitance. The mutual inductance (M) between the transmitter and the receiver can be expressed in terms of coupling coefficient (k) and coil inductances. Output power at the load and Power Transfer Efficiency (PTE) can be calculated as,

$$P_{out} = |I_L|^2 R_L$$

$$PTE = \frac{|I_L|^2 R_L}{|I_i|^2 (R_t) + |I_L|^2 (R_L + R_r)} \quad (2-2)$$

where $I_i = \frac{(R_L + Z_r)V_s}{(Z_t + R_s)(R_L + Z_r) + (\omega M)^2}$ and $I_L = \frac{(j\omega M)V_s}{(Z_t + R_s)(R_L + Z_r) + (\omega M)^2}$

At the resonance frequency condition, reactive impedance of the coil becomes zero. Resonance frequencies of the transmitter and receiver coil becomes,

$$X_{t,r} = j\omega_{t,r}L_{t,r} + \frac{1}{j\omega_{t,r}C_{t,r}} = 0 \rightarrow \omega_{t,r} = \frac{1}{\sqrt{L_{t,r}C_{t,r}}} \quad (2-3)$$

Equivalent circuit method allows to determine element parameters such as currents and voltage across coils, compensation capacitors. Therefore it is essential to use equivalent circuit based analysis to estimate component ratings in designing high power WPT system. Detailed analysis of design parameters and performance indices based on equivalent circuit is presented in Section 3.2.

2.3.2 Two port network theory

Measurement of scattering parameters (S-parameter) is easier compared to voltage and current measurements in laboratory prototyping. In particular, S-parameter measurement is suitable at the initial stage of the design. S-parameter analysis of WPT system can be done using two port network modeling as shown in Fig. 2-5. Using the definitions of two port network theory impedance parameter matrix (Z-matrix) and scattering parameter matrix (S-matrix) can be given as follows,

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ -I_2 \end{bmatrix} \text{ and } \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \quad (2-4)$$

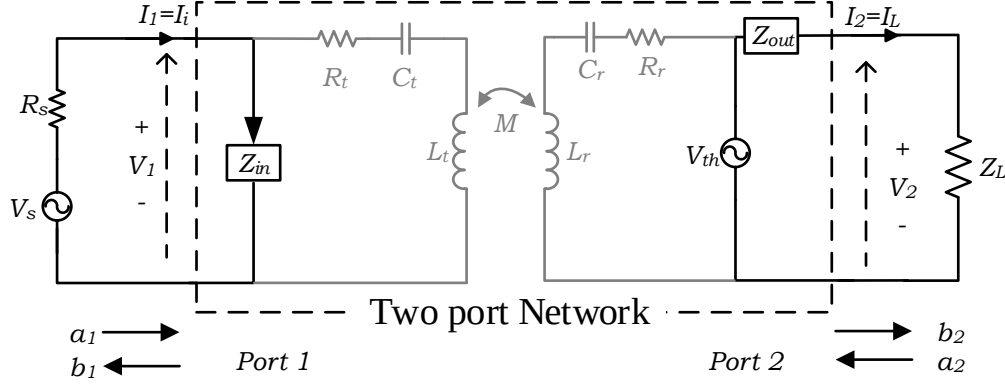


Fig. 2-5 Two port network modelling of the WPT system

Input and output impedances can be calculated using Z parameters;

$$Z_{in} = \frac{V_1}{I_1} = Z_{11} - \frac{Z_{21}Z_{12}}{Z_{22}+Z_L}; \quad V_{th} = \frac{Z_{21}}{Z_{22}+Z_L}V_{in}; \quad Z_{out} = \frac{V_{th}}{I_{sc}} = Z_{22} - \frac{Z_{12}Z_{21}}{Z_{11}+Z_s}; \quad (2-5)$$

$$R_{in} = \text{real}(Z_{in}), R_L = \text{real}(Z_L)$$

Input power (P_{in}), Output power (P_{out}), Transferred Power (TP) and the PTE become,

$$P_{in} = |I_1|^2 R_{in} = \frac{|V_s|^2}{|R_s+Z_{in}|^2} R_{in}$$

$$P_{out} = |I_2|^2 R_L = \frac{|V_{th}|^2}{|Z_{out}+R_L|^2} R_L = \frac{|Z_{21}|^2 |V_s|^2}{|(Z_{in}+Z_s)(Z_{22}+Z_L)|^2} R_L \quad (2-6)$$

$$PTE = \frac{P_{out}}{P_{in}} = \frac{|Z_{21}|^2}{|Z_L+Z_{22}|^2} \frac{R_L}{R_{in}}$$

S-parameters are convenient as they can be easily measured in laboratory experiments. Z parameters can be obtained by using the following conversion,

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \frac{Z_0}{\Delta} \begin{bmatrix} (1+S_{11})(1-S_{22})+S_{12}S_{21} & 2S_{12} \\ 2S_{21} & (1-S_{11})(1+S_{22})+S_{12}S_{21} \end{bmatrix} \quad (2-7)$$

$$\text{where } \Delta = (1-S_{11})(1-S_{22})-S_{12}S_{21}$$

Equation (2-6) can be rewritten as (2-8) with the use of S-parameters using (2-7),

$$\begin{aligned}
 P_{in} &= \frac{|V_s|^2}{4Z_0} \frac{(1-|\Gamma_{in}|^2)|1-\Gamma_s|^2}{|1-\Gamma_{in}\Gamma_s|^2} \\
 P_{out} &= \frac{|V_s|^2}{4R_s} \frac{(1-|\Gamma_L|^2)|1-\Gamma_s|^2|S_{21}|^2}{|(1-S_{11}\Gamma_s)(1-S_{22}\Gamma_L)-S_{12}S_{21}\Gamma_s\Gamma_L|^2} \\
 PTE &= \left(\frac{1}{1-|\Gamma_{in}|^2}\right) \left(\frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2}\right) |S_{21}|^2
 \end{aligned} \tag{2-8}$$

$$\text{Where } \Gamma_s = \frac{Z_s - Z_0}{Z_s + Z_0}, \Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}, \Gamma_{in} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1-S_{22}\Gamma_L}$$

When the impedances are matched ($R_s = Z_L = Z_0$), Transferred power (TP) and PTE becomes,

$$TP = \frac{P_{out}}{|V_s|^2/4R_s} = |S_{21}|^2; \quad PTE = \frac{|S_{21}|^2}{1-|S_{11}|^2} \tag{2-9}$$

S-parameters can be measured connecting network analyzer to Tx and Rx. Power electronic converter or power amplifier is not needed in these experiments. S-parameter based analysis and measurement are recommended at the initial stage of the design process. Frequency characteristic of performance indices can be determined using this method at lower power level. Experimental verifications presented in this thesis are based on the network analyzer measurements.

2.3.3 Coupled mode theory

Coupled mode theory (CMT) analyses energy exchange between resonant objects in time domain [66]. CMT analysis of WPT system is used to obtain more insight understanding of resonator behavior.

$$\begin{aligned}
 \frac{da_1(t)}{dt} &= -(j\omega + \Gamma_1)a_1(t) + jKa_2(t) + V_{in}(t) \\
 \frac{da_2(t)}{dt} &= -(j\omega + \Gamma_2 + \Gamma_L)a_2(t) + jKa_1(t)
 \end{aligned} \tag{2-10}$$

Where $a_{1,2}$ are time varying field amplitudes of two resonant objects (transmitting and receiving resonators) and ω is the eigen frequency. $\Gamma_{1,2}$ are intrinsic decay rates (or resonance widths) of sending and receiving coils due to losses, and Γ_L is the resonance width of the load. Resonance widths are inversely proportional to resistances in

equivalent circuit approach. $V_{in}(t)$ is the excitation signal applied to sending coil. K is the coupling rate between resonators.

In steady state and pure sinusoidal excitation signal ($F_s(t) = A_s e^{j\omega t}$), resulting field amplitudes are $a_1(t) = A_1 e^{j\omega t}$ and $a_2(t) = A_2 e^{j\omega t}$. Average power absorbed by the transmitting resonator and the power delivered to receiving resonator are $P_1 = 2\Gamma_1 |A_1|^2$ and $P_2 = 2\Gamma_2 |A_2|^2$ respectively. Input power from the source $P_{in} = P_1 + P_2 + P_{out}$ [67]. Output power delivered to the load and power transfer efficiency can be expressed as (2-11). CMT is applicable for designs with high quality factor and low coupling coefficient.

$$P_{out} = 2\Gamma_L |A_2|^2; PTE = \frac{1}{1 + \frac{\Gamma_2}{\Gamma_L} + \left[1 + \frac{\Gamma_1 \Gamma_2}{K^2} \left(1 + \frac{\Gamma_L}{\Gamma_2}\right)^2\right]} \quad (2-11)$$

2.3.4 Comparison of modeling methods

While all the modeling methods can be used to analyze the WPT systems, each approach has its own advantages and disadvantages. Therefore, merits of each modeling approach are briefly discussed here. The equivalent circuit method is useful for the derivation of electrical characteristics such as voltages across the components and current through the components. Therefore, the equivalent circuit approach becomes a powerful tool for the design of the power electronics. However, the transient analysis of WPT setups using equivalent circuit method becomes extremely complex, particularly when additional circuit components are included (e.g. complex compensation topologies, multi-coil telemetries). In such cases, CMT can be used for the transient characteristics exploration. On the other hand, two-port network theory can be easily adopted when the design focus is on the characterization of the wireless link. For example, two-port network theory is applied for the design of impedance matching circuitry. In addition, two-port method can be applied along with the equivalent circuit approach for the optimization of multi-coil WPT schemes such as WPT with repeaters.

2.4 Optimization of WPT systems

There have been various optimization efforts proposed in the literature. The use of relay (or repeater) resonators [31, 32], impedance matching techniques [68, 69], load transformation [45], frequency tuning approaches [34, 35], metamaterials [70] and

optimized resonator designs [55, 71-74] have been introduced in the literature to improve performance of WPT. This section presents some highlights of the optimization of WPT systems relevant to the study of this thesis.

2.4.1 Performance indices

PTE and TP are the key performance indices for WPT applications. PTE improvement is essential for the loss minimization, whereas, TP is important for effective power delivery to the load. TP is maximized when the maximum power is drawn from the source and the source impedance matches with the input impedance of the coil system. Both PTE and TP show different characteristics with respect to the design parameters. For example, maximum PTE occurs at self-resonance frequency, whereas, TP is maximized at two different frequencies away from self-resonance. On the other hand, at self-resonance frequency, the maximum TP occurs at a particular coupling value referred to as critical coupling point, whereas, PTE deteriorate drastically with the coupling reduction. Detailed analysis of the performance characteristics is provided in Chapter 3.

In addition to the system level performance indices such as PTE and TP, application specific criteria such as transfer distance, misalignment tolerance and nature of receiver positioning also affect the performance of the design. Typically, various optimizations presented in literature focus on improving the performance for a desired Rx position and orientation, and subsequently, the performance is evaluated against misalignment [75, 76]. However, such an approach gives rise to inferior performance with misalignment as Rx dynamics are not taken into account. Therefore, performance characteristics need to be analyzed with respect to design parameters and application constraints. A detailed review of the WPT optimization approaches such as coil optimization, the use of repeaters and metamaterials, impedance matching, and compensation circuits are discussed in following subsections.

2.4.2 WPT coil optimization

The coils play an imperative role in WPT design stages. For WPT coils, there are two performance indices: quality factor and electromagnetic coupling between transmitter and receiver. The quality factor can be improved by decreasing the AC resistance (skin and proximity losses) of the coil while maintaining high inductance.

But, enhancement of the electromagnetic coupling is severely limited by the misalignment and distance between coils which cannot be compromised due to most application constraints. Instead, coupling variation can be optimized by choosing appropriate coil structure for a particular position displacement profile.

There have been a number of studies which addressed the losses related to resonators. Several optimized resonator designs have been proposed. For example, reference [74] presents a detailed optimization process for double layer printed spiral coil. The authors have illustrated design procedure for optimal choice of number of turns, track width and turns separation. Reference [73] presents multi turn surface spiral for low AC resistance. However, the surface spiral can only be used in limited power levels as they are constructed in PCBs and substrate losses play considerable role in AC resistance. In order to address the issue of coupling deterioration with misalignment, inserting a negative current loop is considered in [71]. Although this approach reduces the coupling variation, it also reduces overall coupling factor due to its negative magnetic flux. WPT coil optimization is one of the main objectives of this thesis.

2.4.3 Repeaters for WPT

The use of repeater resonators has been reported in the literature to improve the performance [31-33, 77-80]. Repeater coils placed in-between Tx and Rx are found to be useful in improving PTE, improve TP and increase the transfer distance. The relay effect of the repeaters can be used to increase transfer distance from Tx to Rx significantly [77, 78]. However, it does not increase unobstructed distance between Tx to Rx as relay coils need to be placed in between Tx and Rx. Efficiency characteristics of WPT system with a single repeater placed near to Tx has been analyzed in [79]. The above study investigated that the optimized three-coil WPT offers higher efficiency than the classical two coil counterpart. The frequency characteristics of PTE and TP have been explored in [80]. However, these studies have not proposed a generalized design method for WPT repeaters. This thesis contributes a methodology to improve PTE and TP, unobstructed transfer distance and misalignment tolerance simultaneously using repeater coils.

2.4.4 Metamaterials for WPT

Metamaterials (MMs) are artificial materials which show extraordinary physical properties that cannot be seen in the natural materials. MMs make themselves enhance the evanescent near-field and eventually improve the power transfer efficiency in WPT applications. Urzhumov et al. [81] presented a rigorous theoretical study on the PTE improvement by using MMs. That work formulates the mutual inductance improvement that can be obtained by using MMs. This phenomenon can be used to improve PTE with respect to both the power transfer range and the misalignment tolerance. Usually MMs are composed of periodically placed structures. Dimensions of these composing unit cells are several times smaller than the exciting wave length of the MM. Therefore in high frequency applications such as optical and near infrared frequencies, the size of the unit cell becomes nanometers scale which intern becomes materials engineering optimization problem. But, at the low frequencies, the wavelength exceeds the dimension of the coils. Therefore designing MMs for WPT applications can be done using usual PCB technology.

The experiment presented by Wang et al., [70] has demonstrated PTE improvement using double sided spiral as the composing unit cell. A planar meta structure performs better than the 3D structure, which is advantageous for the design of a compacted system. A study by Fan et al. [82] proves efficiency improvement from an experimental study with a dual layer design, with planar spiral on one side and a Meander Line with narrow metallic strips on the other side, at 14.6MHz. Above study shows that the MM can be placed at the transmitter and receiver ends rather than at the middle which will effectively extend unobstructed distance.

2.4.5 Impedance matching and compensation networks

Adaptive impedance matching (AIM) networks has been proposed in literature to improve TP whereby input impedance of WPT network is matched to the source impedance [68, 69, 83]. Unlike Power amplifier based laboratory prototyping where source impedance is 50Ω , in power converter based designs the source impedance can be a very small value. Therefore, the matching network should be capable of bringing input impedance of the WPT system close to source impedance. AIM network between

source and the transmitter coils (Fig. 2-6) can be used for WPT schemes with varying coupling [68, 69, 83].

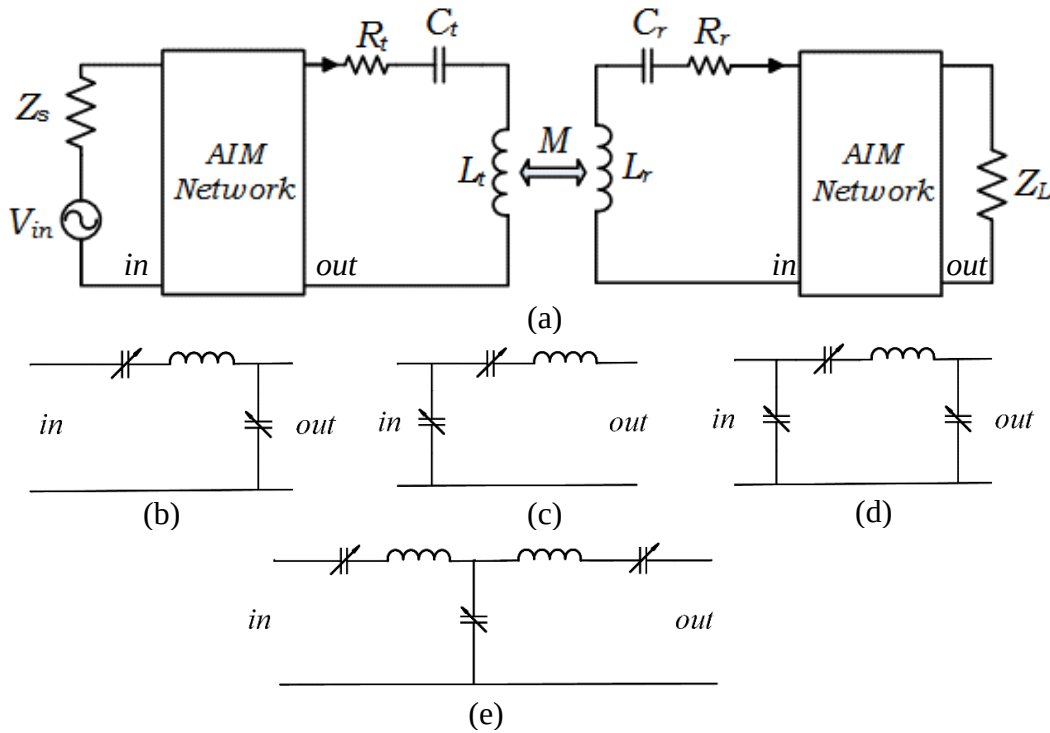


Fig. 2-6. (a) – Transmitter and receiver side Adaptive Impedance Matching (AIM), (b) – L-type, (c) – Inverted-L type, (d) – Π type and (e) – T-type matching networks

On the other hand, the equivalent load impedance is dependent on the application. For example, equivalent load resistance of a battery charging application is dependent on the battery state-of-charge (SOC), and equivalent load resistance of a VAD application varies according to heart pressure and pump speed. For such load varying situations, an AIM network can be used between receiver coil and battery charger to make equivalent load impedance to its optimal value.

Matching networks should be capable of adaptively changing the impedance of the network. Input impedance seen by the source has to be maintained at matched condition under dynamic changes in WPT system. Usually, a switched capacitor bank along with an inductor is used to change the impedance [68, 69, 83]. Current controlled inductor which has been used to control micro-inverter [84] can also be used as a variable inductor for the matching network. Based on the component arrangement of the matching network impedance, matching networks can be categorized into L-type, inverted L-type, T-type and Π -type. These matching networks are illustrated in Fig. 2-6.

L-type and inverted L-type matching networks are simple but design freedom is higher in complicated T and Π types. Resistive losses in the components also become significant in complex matching networks with number of switching devices. Therefore, choice of matching network has to be carried out based on the design constraints such as coupling variation, number of components and switching devices.

2.5 Operating frequency selection

Operating frequency selection of a WPT system has to be carried out with a number of technical and regulatory constraints which varies from country to country. Considering maximum achievable efficiency for a WPT system, highest possible quality factor is desirable. There are three options available in increasing quality factor; increasing coil inductance, increasing operating frequency, and decreasing coil resistance. However, AC resistance increases with the increase of frequency due to skin effect losses. On the other hand, inductance of a coil is determined by its shape and it is almost independent on the operating frequency. Therefore, there is an optimum frequency for a given design type for highest possible quality factor. Typically, this optimum frequency is in MHz range for most designs. However, design of high power WPT system in MHz range frequencies is still challenging with today's semiconductor technologies. Therefore, upper limit of the operating frequency is restricted by device ratings. There have been successful attempts to use frequencies ranges from 10 kHz to 150 kHz for high power applications such as EV [85, 86], whereas, operating frequencies up to 20MHz have been used for low power applications such as biomedical applications [20]. With the advent of modern semiconductor technologies such as wide band gap devices (e.g. GaN and SiC devices), frequency limit of the devices are increasing rapidly. Recent advancements in semiconductor technologies and research activities [53, 87-90] suggest high frequency high power devices will be available commercially in near future. Therefore, cost effective, high power near few MHz WPT for high power applications is not very far away, and is around the corner [53]. Therefore, operating frequency of this study is chosen to be around 1MHz.

2.6 Summary

This chapter provided a detailed literature review on the WPT, including applications of WPT, modeling methods, performance evaluation and optimization

approaches. A range of WPT applications are briefly explained giving a particular attention to WPT for electric vehicle applications, biomedical implants and consumer electronics applications. Equivalent circuit method, two port network method and coupling mode theory were presented as modeling methods along with a comparison of modeling methods. The optimization approaches of WPT were discussed highlighting significant research achievements.

Chapter 3

Coil Design for High Efficiency Wireless Power Transfer (WPT)

3.1 Introduction

WPT coil optimization is one of the key aspects to improve WPT performance. There has been a number of studies reported in the literature which sought to improve the WPT coils. Some studies focus on improving coupling enhancement, loss minimization and quality factor improvements in order to improve the efficiency [74, 91-95]. For example, [93] suggests that windings concentrated towards the edge of the coil provide higher coupling. On the other hand, [94] claims that a hybrid structure provides better performance for their application criteria. In addition, the study in [95] presents unequal pitch distribution for minimization of the coil AC resistance. Quality factor improvement is presented in [74] which utilizes a double layered printed spiral coil. While the improvement of the electromagnetic coupling, quality factor and AC resistance are all important aspects of performance improvement, it may not be the optimal condition for a given design scenario. Each of these studies focuses on particular design conditions, and disparities between design constraints and geometries lead to different conclusions. Therefore, there is a requirement for the derivation of a generalized coil design approach for WPT coil design.

This chapter analyses the WPT performance characteristics with regards to coil design, and provides a generalized design guideline for WPT coil optimization. Furthermore, empirical formulas are proposed to determine the optimal coil design for a given design scenario. The suggested empirical approach can be directly used for the design without using time consuming optimization algorithms. In addition, directly comparing dissimilar designs with respect to efficiency and transfer distance is misleading due to the differences in the design parameters. In general, higher efficiency can be achieved by using a larger coil size for a given distance. However it is not a good engineering approach. It would be appropriate if highest possible efficiency is achieved with minimum dimensions and cost. Therefore, power transfer distances need to be

analyzed by normalizing transfer distance to the coil dimensions. The term, normalized transfer distance has been proposed to compare diverse designs with different coil sizes and transfer distances. Furthermore, studying scalability of the WPT systems is beneficial for the performance estimation of scaled up versions, particularly, for the justification of results obtained from laboratory prototypes. Normalized transfer distance is an important parameter in studying scalability. Therefore, the study in this chapter is extended to analyze merits and limitations of the normalized distance. Moving forward, guidelines are investigated for the design of WPT systems with a constraint on Rx size. Theoretical analysis is validated using simulations and experimental results.

This chapter is organized as follows. Analysis of the 2-coil WPT scheme with regards to power transfer efficiency (PTE) and transferred power (TP) is discussed in Section 3.2. The equivalent circuit approach is used for the analysis. Next, Section 3.3 identifies the design parameters of WPT coil, and their relationship to the electrical parameters. Coil optimization for high quality factor and coupling are discussed in Section 3.4. Generalized empirical formulas are derived for high quality factor coils. Next, merits and limitations of the normalized distance for the comparison of dissimilar WPT schemes are evaluated in Section 3.5. Finally, a coil optimization approach for WPT with restrictions for Rx coil size is investigated in Section 3.6, followed by the summary of the chapter.

3.2 Analysis and performance characteristic of 2-coil WPT

3.2.1 Equivalent circuit analysis

This section analyses PTE and TP with respect to design parameters using the equivalent circuit approach. Both maximum power transfer condition and maximum efficiency condition are used as design goals in literature. The maximum TP refers to the condition where maximum power is drawn from the source. The maximum TP occurs when the source impedance and the input impedance seen at the source are matched. Conversely, objective of improvement of PTE is to optimize energy efficiency in the power transfer process. PTE and TP show different characteristics with respect to design parameters such as, source operating frequency (f_s), self-resonance frequency (f_0), coupling coefficient (k) and load resistance (R_L). Therefore, the characteristics of PTE and TP for the classical 2-coil WPT scheme is analyzed in this section.

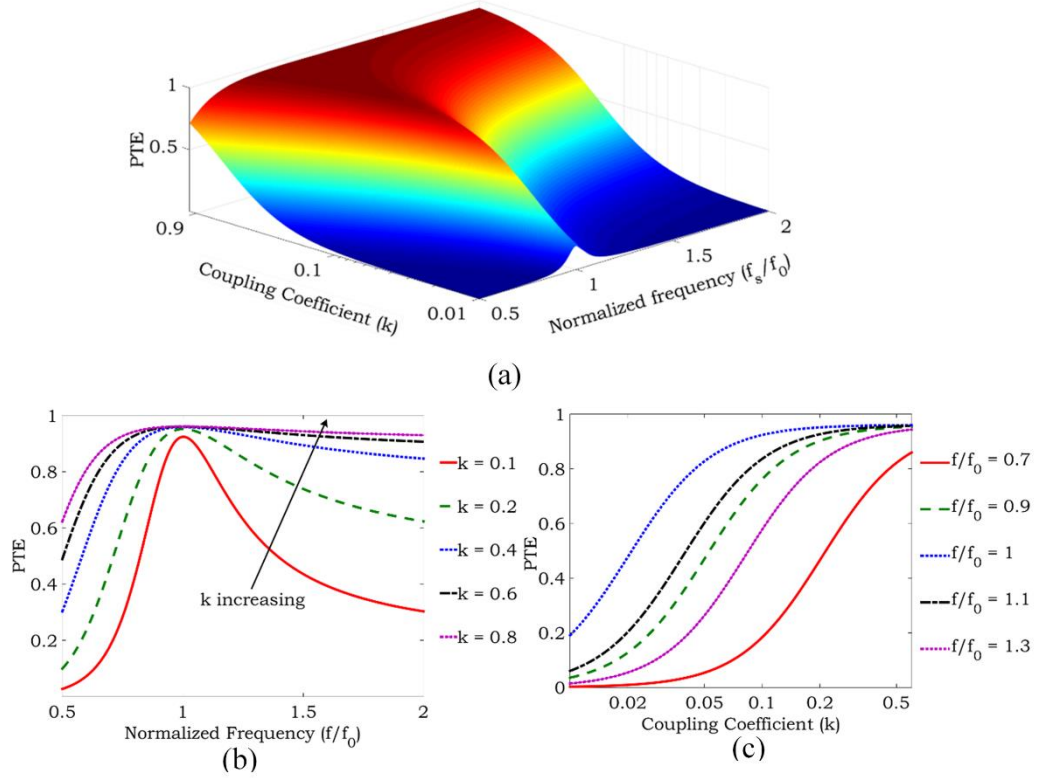


Fig. 3-1 (a) – Power transfer efficiency (PTE) variation with respect to frequency (f) and coupling coefficient (k), (b) – PTE Vs. frequency, (c) – PTE Vs. coupling coefficient. Frequency is normalized to resonance frequency (f_0)

Numerical results in Fig. 3-1 show PTE variation with respect to operating frequency and coupling coefficient. It can be seen from Fig. 3-1 that, the maximum PTE always occurs at the resonance frequency and PTE declines with the reduction of coupling coefficient. On the other hand, TP characteristic in Fig. 3-2 shows that TP is not always the maximum at the resonance frequency. When electromagnetic coupling is higher (strongly coupled), TP characteristic shows double peak behavior (the maximum TP occurs at two different frequencies away from resonance frequency) with respect to the operating frequency. This frequency splitting phenomenon has led to various adaptive controlling efforts to maximize TP, including frequency tuning [34, 35], impedance matching [68, 69, 83] and coupling manipulation [34, 96]. The point where the frequency splitting ends is referred as the critical coupling point. Beyond the critical coupling point the transferred power to the load drops exponentially with increasing coil separation.

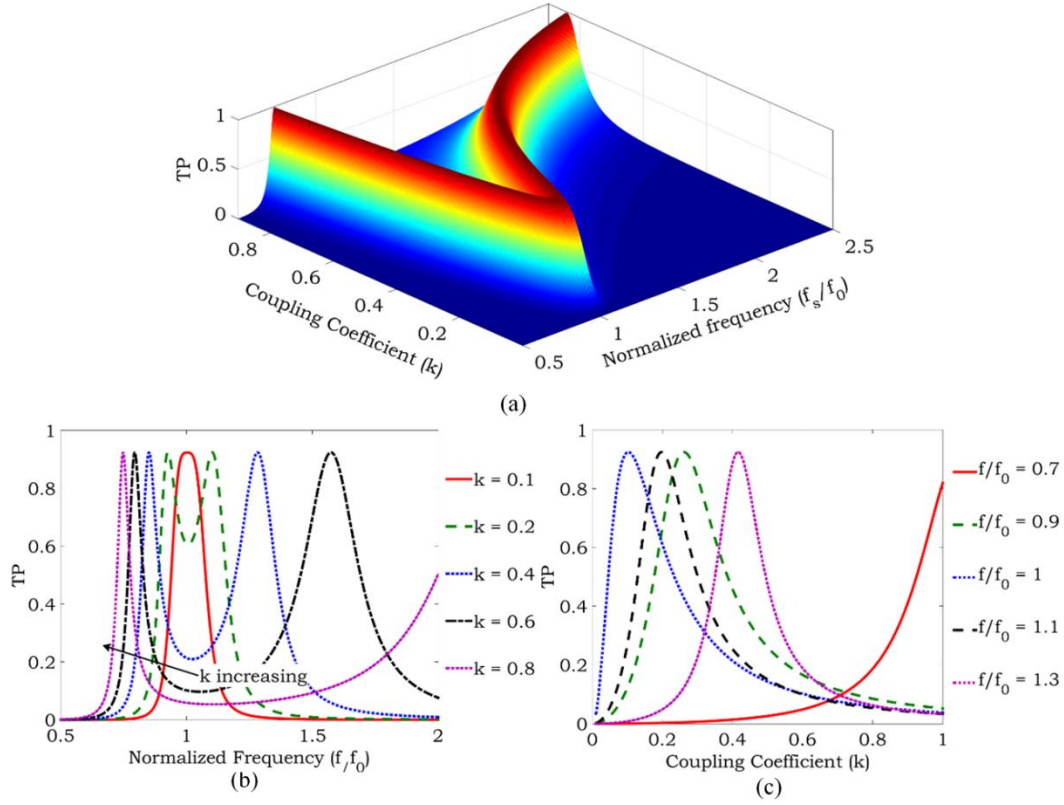


Fig. 3-2 (a) – Transferred power (TP) variation with respect to frequency (f) and coupling co-efficient (k), (b) – TP Vs. frequency, (c) – TP Vs. coupling coefficient. Frequency is normalized to resonance frequency (f_0)

At resonance frequency where $\omega_s = \omega_0 = 1/\sqrt{L_t C_t} = 1/\sqrt{L_r C_r}$ the PTE and output power defined in Chapter 2 can be expressed as,

$$P_{out, \omega_0} = \frac{V_s^2 R_L (\omega_0 M)^2}{((R_s + R_t)(R_L + R_r) + (\omega_0 M)^2)^2} \quad (3-1)$$

$$PTE_{\omega_0} = \frac{R_L (\omega_0 M)^2}{(R_t(R_L + R_r) + (\omega_0 M)^2)(R_L + R_r)}$$

Both PTE and TP at the resonance depend on the value of load resistance as depicted in Fig. 3-3. Equation (3-2) gives the optimum values of load resistance for both maximum PTE and maximum TP conditions (R_{L, PTE_max} and R_{L, TP_max} respectively). Fig. 3-4 shows the variation of optimal load resistance with respect to coupling coefficient. Therefore, it is evident that PTE and TP characteristics are different with respect to coupling variation, operating frequency and load resistance.

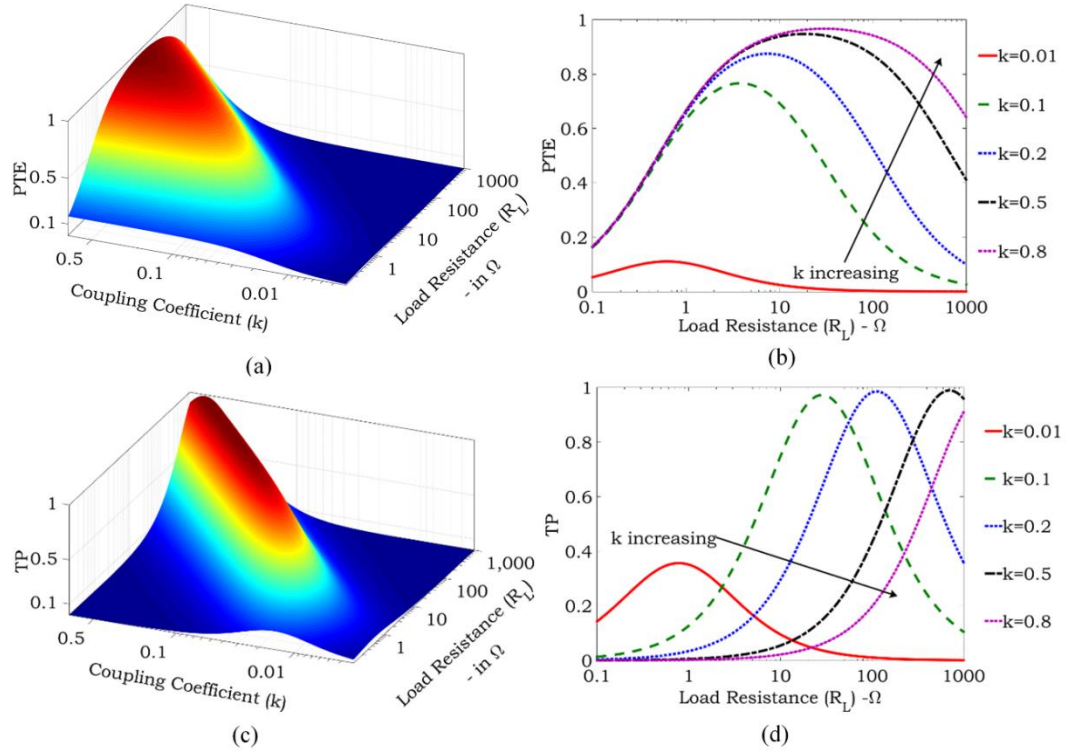


Fig. 3-3 (a) and (b) – Power transfer efficiency (PTE), and (c) and (d) –Transferred Power (TP) variation with respect to Load Resistance (R_L) and coupling co-efficient (k)

$$R_{L,PTE_max} = \sqrt{\frac{(\omega_0 M)^2 R_r}{(R_t)} + R_r^2} \quad (3-2)$$

$$R_{L,TP_max} = \frac{(\omega M)^2}{(R_s + R_t)} + R_r \quad (3-3)$$

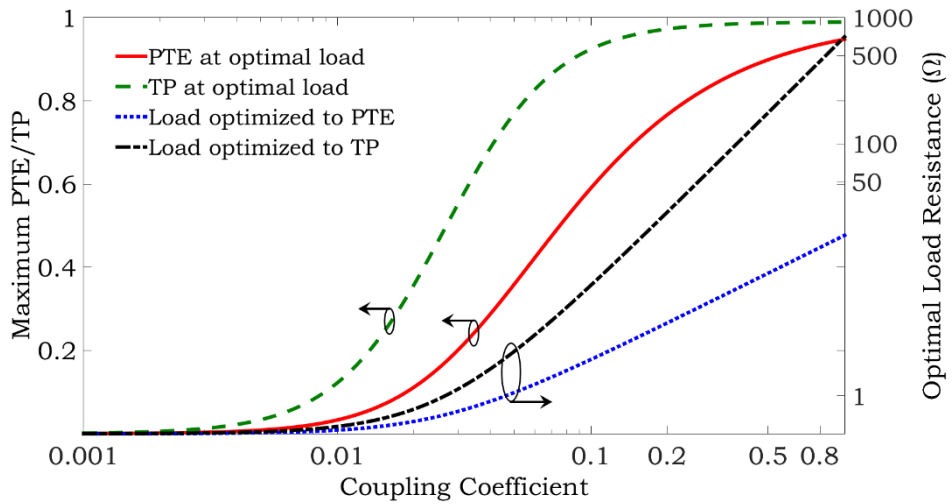


Fig. 3-4. The effect of load resistance on Power transfer efficiency (PTE) and Transferred power (TP)

3.2.2 Maximum efficiency

Due to the different characteristics, it is difficult to optimize both PTE and TP simultaneously in classical 2-coil WPT scheme. If TP is chosen as the first priority in the optimization process, the maximum achievable PTE can be limited. This is not desirable for most WPT applications, particularly for high power applications. On the other hand, if the first focus is on improving PTE, other techniques such as impedance matching can be used to improve TP. Therefore, it is recommended to focus on improving the PTE in initial design stages, in particular for the WPT coil designs.

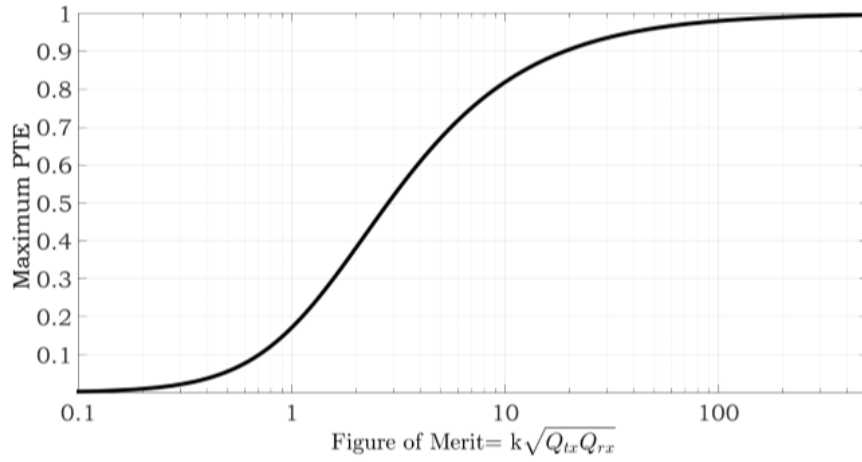


Fig. 3-5. Maximum achievable Efficiency as a function of the term $k\sqrt{Q_{Tx}Q_{Rx}}$

Maximum achievable efficiency (PTE_{max}) when load resistance is optimized to $R_{L,PTE_{max}}$ can be expressed in (3-4) [42, 45].

$$PTE_{max} = \frac{k^2 Q_{Tx} Q_{Rx}}{(1 + \sqrt{1 + k^2 Q_{Tx} Q_{Rx}})^2} \quad (3-4)$$

Where $Q_{Tx,Rx} = \frac{\omega_0 L_{t,r}}{R_{t,r}}$ are unloaded quality factors of the transmitter and the receiver coils. The maximum efficiency is dependent only on the coil design (determining Q) and the relative geometry of Tx and Rx (determining k). Hence, (3-4) can be directly used for comparison of the coil designs and geometry. Once the coils are optimized for the desired geometric constraints, a load transformation technique can be used to achieve the optimum load condition derived in (3-2). Therefore, this chapter focuses on optimizing the WPT coils to achieve maximum PTE. Fig. 3-5 illustrates the maximum PTE variation with respect to the term $k\sqrt{Q_{Tx}Q_{Rx}}$. In order to achieve acceptable

efficiency in a larger relative distance, (i.e. low coupling coefficient value) it is required to have coils with high quality factor. Therefore, quality factor and electromagnetic coupling are key performance indices for WPT coils. The following section analyses WPT coil optimization with regards to quality factor, coupling and maximum efficiency.

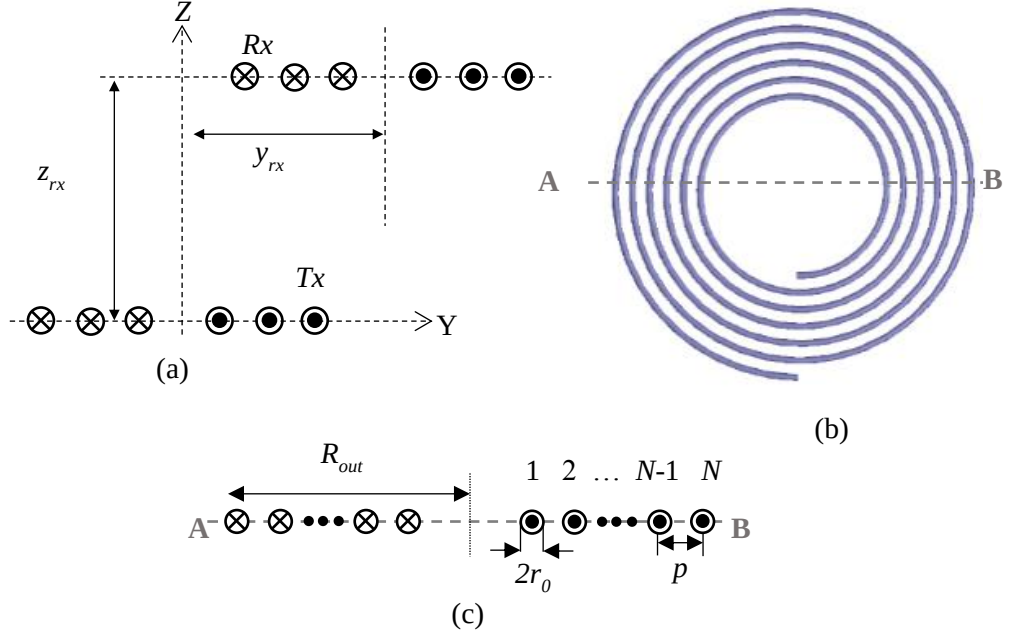


Fig. 3-6. WPT with spiral shaped coils (a) – Positioning transmitter (Tx) and receiver (Rx) coils, (b) – The spiral shaped coil, (c) – 2-D approximation of the spiral coil

3.3 Coil parameter calculations

Coil parameter calculations for circular shaped coils is presented in this section sought to get an understanding on the coil optimization. Typical circular coils can be either helical or spiral shaped. For example, Fig. 3-6 (a) shows a geometry of a WPT system with circular spiral shaped Tx and Rx coils. Spiral shaped coil shown in Fig. 3-6(b) can be approximated as a set of circular loops for the coil parameter calculations. Such a 2-D circular loop approximation is illustrated in Fig. 3-6(c). The geometric design parameters such as outer radius (R_{out}), pitch (p), wire radius (r_o) and number of turns (N) are defined in Fig. 3-6. Axial and radial separation between Tx and Rx are z_{rx} and y_{rx} respectively. Electrical parameters of the coils such as AC resistance, coupling coefficient, self-inductance and quality factor can be computed using geometric design parameters.

3.3.1 AC resistance

Skin effect and proximity effect are the major sources of losses contributed to the AC resistance in air-core coils. The current density within a conductor becomes non-uniform due to the eddy currents caused by the skin effect and proximity effect. Fig. 3-7(a-b) show simulated current distributions of a single turn loop wire at 100 kHz and 1 MHz frequencies. Simulations are carried out in Ansoft[®] Maxwell finite element analysis (FEA) software. At higher frequencies, current is confined into the surface due to the skin effect. Fig. 3-7(c) and (d) show simulation results for current distributions of two neighboring turns. Magnetic field generated by one wire influences the current distribution of the other. It can be seen that the proximity effect becomes more significant when two wires are close to each other. Therefore, pitch between turns is an important parameter for the design of low loss coils.

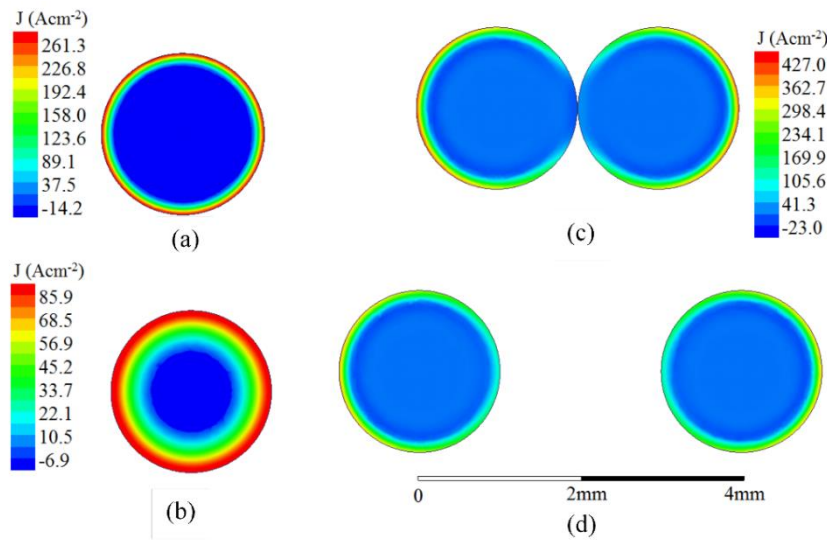


Fig. 3-7. Simulation results for the current distribution of circular loops (a) – Single turn loop at 1 MHz, (b) – Single turn loop at 100 kHz, (c) – tightly wound two turns with 2mm pitch, and (d) – two turns with 4mm pitch (wire radius is kept at 1mm).

AC resistance of circular coils can be calculated as in (3-5) where R_{skin} and R_{prox} refer to skin effect resistance and proximity effect resistance respectively [95]. R_{skin} is mainly dependent on skin depth (δ), wire radius (r_0) and the DC resistance of the coil. On the other hand, R_{prox} is determined by the magnetic field applied on each turn (H_m). The ratio of R_{prox} to R_{skin} is called proximity factor (G_p). The magnetic field applied on each turn can be determined by using the simplified method proposed in [95]. AC resistance calculations with respect to pitch at different frequencies are illustrated in Fig. 3-8. A

steep increase in the resistance seen at lower pitch values is attributed by the proximity effect resistance.

$$R_{ac} = R_{skin} + R_{prox} = R_{skin}(1 + G_p) \quad (3-5)$$

$$\text{where } G_p = \frac{8\pi^2 x^3(x-1)}{(2x+1)^2+2} \left(\frac{1}{N} \sum_{m=1}^N \frac{H_m^2}{I_0^2} \right) \text{ and } x = \frac{2r_0}{\delta}$$

Litz wires have been proposed to reduce the skin effect losses for frequencies less than 2MHz. For circular coils constructed using litz wire winding, AC resistance can be calculated as in (3-6) [97].

$$R_{ac} = R_{dc} \left(1 + \frac{f_s^2}{f_h^2} \right) \text{ where } f_h = \frac{2\sqrt{2}}{\pi r_s^2 \mu_0 \sigma \sqrt{N N_s \eta_a \beta}} \quad (3-6)$$

R_{dc} , r_s , N_s and β are the DC resistance of the coil, radius of each single strand, number of strands per bunch and the area efficiency of the bunch. DC resistance is dependent on the manufacturing methods and the type of the litz wire, and it is usually available in the datasheet. Value of η_a is the area efficiency of the coil and is dependent on the height and width of the coil (Fig. 2. of [97]).

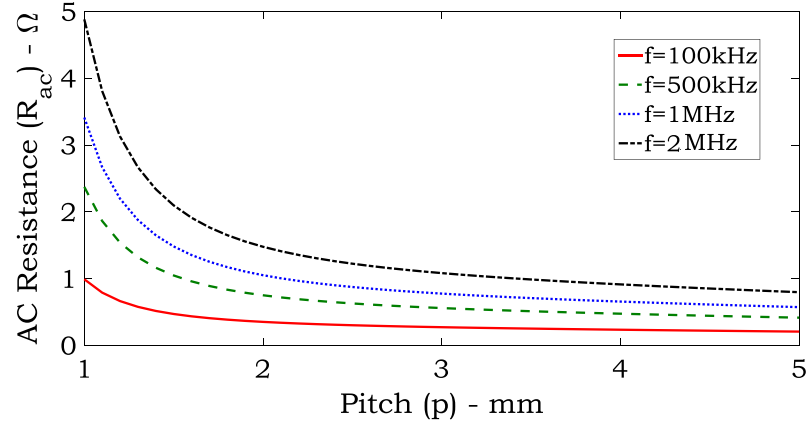


Fig. 3-8. AC resistance variation against pitch between turns at different frequencies (Other coil parameters: $r_0=0.5\text{mm}$, $R_{out}=10\text{cm}$, $N=15$).

3.3.2 Coupling coefficient

Mutual inductance between circular coils can be calculated using filament method by modelling the coil as a set of filamentary circular loops. Mutual inductance between circular coils with parallel axes can be computed using (3-7)[98].

$$M = \sum_{i=1}^{N_{Tx}} \sum_{j=1}^{N_{Rx}} M_{ij}$$

$$\text{where } M_{ij} = \frac{\mu_0}{\pi} \sqrt{R_{txi} R_{rxj}} \int_0^\pi \frac{\left(1 - \frac{y_{rx}}{R_{rxj}} \cos \phi\right) \Phi(m)}{\sqrt{V^3}} d\phi, \quad a = \frac{R_{rxj}}{R_{txi}},$$

$$b = \frac{z_{rx}}{R_{txi}}, \quad m^2 = \frac{4aV}{((1+aV)^2 + b^2)}, \quad V = \sqrt{1 + \frac{y_{rx}^2}{R_{rxj}^2} - 2 \frac{y_{rx}}{R_{rxj}} \cos \phi} \quad (3-7)$$

$$\text{and } \Phi(k) = \left(\frac{2}{m} - m\right) K(m) - \frac{2}{m} E(m)$$

$$\text{Coupling coefficient is defined as, } k = \frac{M}{\sqrt{L_t L_r}}$$

M_{ij} is the mutual inductance between i^{th} Tx turn with R_{txi} radius and j^{th} Rx loop with R_{rxj} radius. $K(m)$ and $E(m)$ are complete elliptic integrals of first kind and second kind respectively. Coupling coefficient can be determined by total mutual inductance (M), Tx coil inductance (L_t) and Rx coil inductance (L_r). Mutual inductance for circular elementary coils with 89cm radius is depicted in Fig. 3-9(b). Mutual inductance reduces steeply with the increase of radial distance. For a given displacement, higher mutual inductance can be achieved by using larger coil size. Therefore, by increasing the dimensions higher efficiency can be achieved for a given distance. However this is not a good engineering approach. It would be appropriate if highest possible efficiency is achieved with minimum dimensions and cost. Therefore, power transfer distance has been analyzed by normalizing it to the coil dimension. This thesis analyses the merits and limitations of normalized distance based analysis.

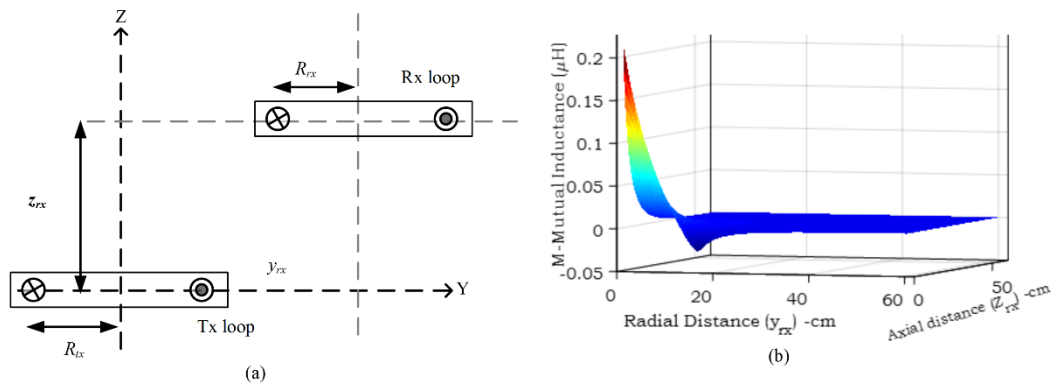


Fig. 3-9 (a) – Filamentary circular coil arrangement, (b) – Mutual inductance with respect to radial and lateral misalignment for elementary circular loops

3.3.3 Self-inductance

Self-inductance of a circular coil with N number of turns can be calculated using circular loop approximation as in equation (3-8)[93]. Inductance of coil comprises of self-inductance of each turn and mutual inductance between turns.

$$L = \sum_{i=1}^N L_i(R_i, r_0) + \sum_{i=1}^N \sum_{j=1}^N M_{ij}(1 - \delta_{ij}) \quad (3-8)$$

$$\text{where } L_i(R_i, r_0) = \mu_0 R_i \left(\ln \frac{8R_i}{r_0} - 2 \right)$$

δ_{ij} is kronecker delta function ($\delta_{ij}=1$ for $i=j$ and $\delta_{ij}=0$ otherwise). $L_i(R_i, r_i)$ is inductance of a circular loop with radius R_i . M_{ij} is the mutual inductance between i^{th} and j^{th} circular turns.

3.3.4 Quality factor

Quality factor, Q , is determined by the coil inductance (L), AC resistance (R_{ac}) and operating angular frequency (ω) as in (3-9) [15].

$$Q = \frac{\omega L}{R_{ac}} \quad (3-9)$$

3.3.5 Experimental validation

The expressions for resistance, inductance and mutual inductance are validated using simulations and experiments. Fig. 3-10 shows resistance comparison between calculations and simulations for a single turn loop constructed using 0.5mm radius wire. Simulations are carried out in Ansoft® Maxwell FEA software. It can be seen that calculations agree well with the simulations. Next, six sets of single turn loops are constructed using 0.5mm wires for experimental validation as shown in Fig. 3-11(a), and Fig. 3-11(b) compares calculated and measured values of resistance and inductance at 1MHz. Comparison between calculations and simulations in Fig. 3-12 shows mutual inductance variation with respect to receiver radius (R_{rx}) and z_{rx} for single turn loops.

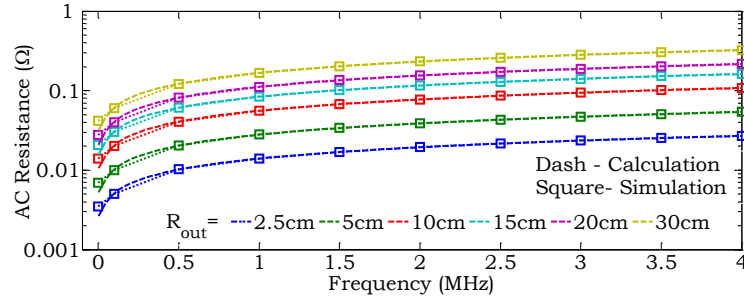


Fig. 3-10. AC resistance of single turn loop: simulations and calculations.

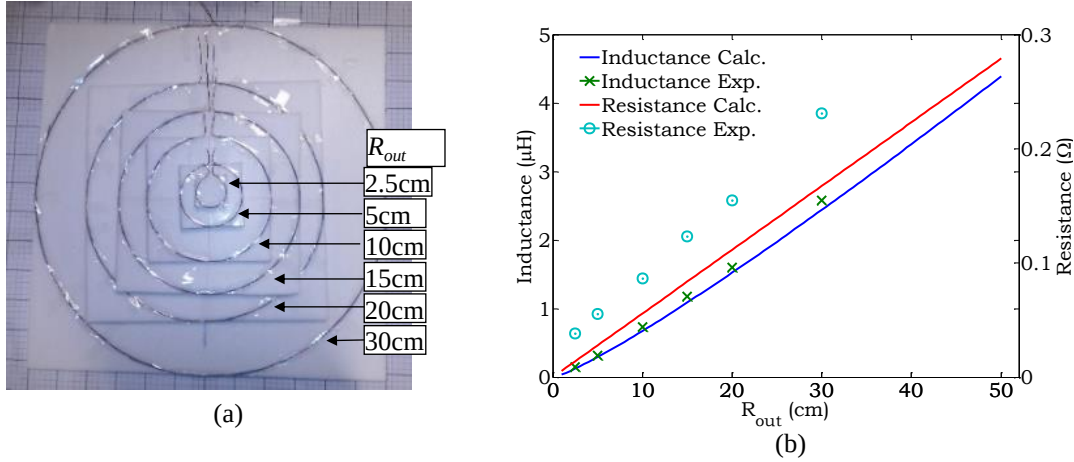


Fig. 3-11. (a) – Experimental realization of single turn loops and (b) – comparison of calculations and experimental results: Inductance and resistance of single turn loops vs. loop radius

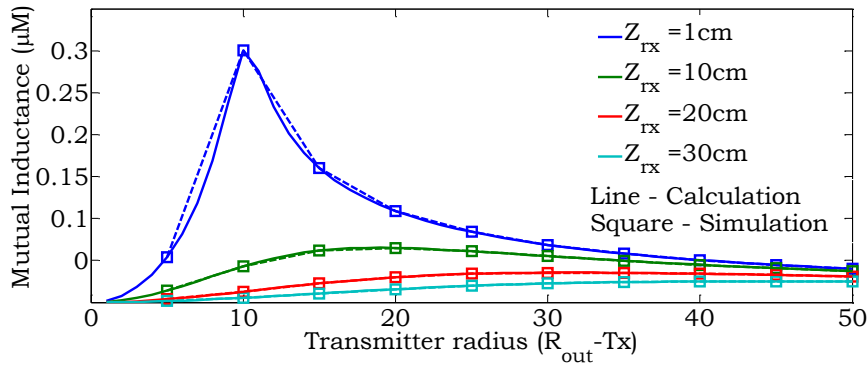


Fig. 3-12. Mutual inductance comparison with respect to transmitter radius for single turn loops, $N=1$, $R_{rx}=10\text{cm}$.

Experiments are extended with seven sets of different coils as illustrated in Fig. 3-13. Design parameters illustrated in Table 3-1 are chosen to analyze effect of the each parameter. Coils with a range of design parameters such as; outer radius varying from 2.5cm to 15cm and pitch between turns varying from 1mm to 8mm are constructed for the comparison. Fig. 3-14 compares experimentally measured inductances and AC resistances with the calculations. Measurement results agree well with the theoretical calculations. Therefore, equations for parameter calculations can be directly used for the subsequent coil optimizations.

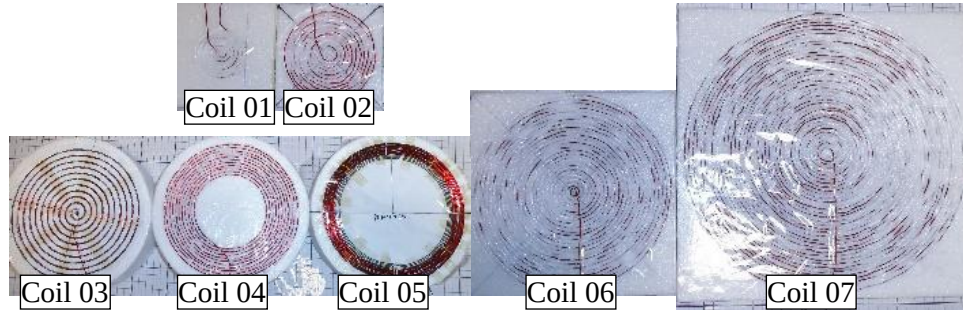


Fig. 3-13. Calculations and experimental results: Different types of coil designs

Table 3-1. Design parameters of different coils

	Outer radius (R_{out}) - cm	Pitch (p) - mm	Number of turns (N)
Coil 01	2.5	4	5
Coil 02	5	4	11
Coil 03	8.5	8	11
Coil 04	8.5	4	11
Coil 05	8.5	1	11
Coil 06	10	4	25
Coil 07	15	4	37

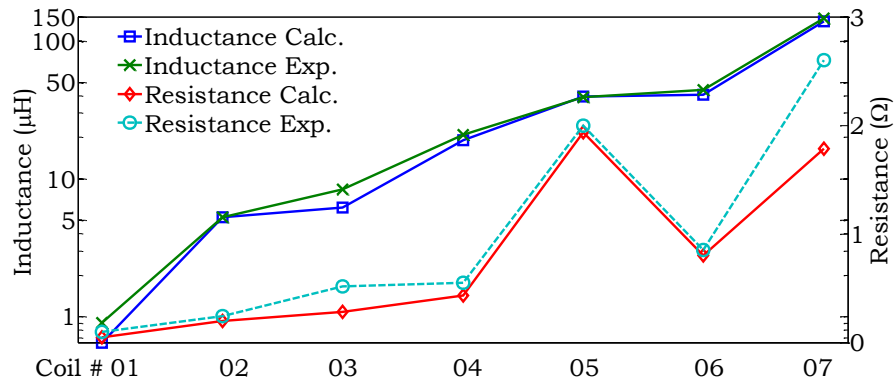


Fig. 3-14. Inductance and AC resistance of different types of coil designs

3.4 Coil optimization

3.4.1 Quality factor optimization

Quality factor variation of spiral coil with respect to the design parameters is analyzed in this section. Fig. 3-15 shows quality factor variation against number of turns (N) and pitch (p) at different coil radii. With increasing N , coil inductance increases which results in incremental impact to the quality factor. In contrast, the physical effect of higher turns increases the coil length which lead to the increase in the resistance, causing detrimental impact on the quality factor. On the other hand, lower pitch

increases coil inductance as turns are confined to outer radius. However, when turns are very close to each other with low pitch, proximity losses increase, resulting in high AC resistance. Therefore, both N and p have to be optimized for the realization of high quality factor coils.

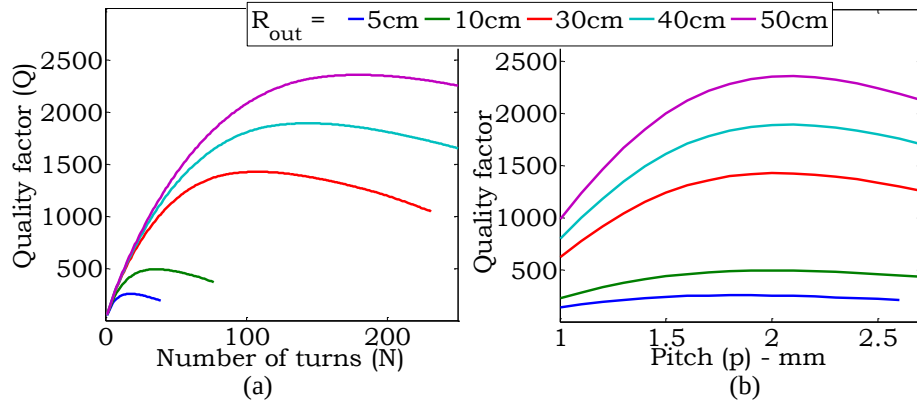


Fig. 3-15. Quality factor variation with respect to (a) – Number of turns and (b) – pitch.

Fig. 3-16(a) and Fig. 3-16 (b) illustrate maximum Q for 1mm wire radius and 0.5mm wire radius respectively. Few observations can be made upon the optimum Q characteristics. Firstly, the quality factor improvement seen from larger wire radius is not very significant. Therefore, the use of larger wire is not a good approach to improve the quality factor. Secondly, the optimum N proportionally increases with the coil radius. However, the optimum N is less than its maximum possible value, because the most inner turns with small radii doesn't contribute the inductance effectively. Lastly, the optimum pitch increases at low coil radii and reaches to a value close to four times of wire radii ($4r_0$) at higher coil radii.

With these observations, curve fitting technique is used to obtain a generalized design guideline for high Q coils. Wire radius is varied from 0.5mm to 2mm and coil radius is varied from 1cm to 50cm for the curve fitting approximation. The optimum pitch is approximated for given coil radius and wire radius using the curve fitting technique as derived in (3-10). The optimum pitch is approximated as a function of R_{out} and r_0 . The value of optimum pitch is near to its minimum value (resulting in a tightly wound coil) at lower R_{out} and it reaches ($4a_0r_0$) as R_{out} increases. The approximation agrees well with the numerical results as compared for different wire radii in Fig. 3-17(a). The optimum pitch becomes four times the wire radius when outer radius is roughly 150 times ($a_1/a_0 - 1$) wire radius, after which increment of the optimum pitch is

insignificant. Therefore, as a rule of thumb for the coil design, optimum pitch can be chosen to be four times of the wire radius if the coil radius is higher than 150 times of the wire radius.

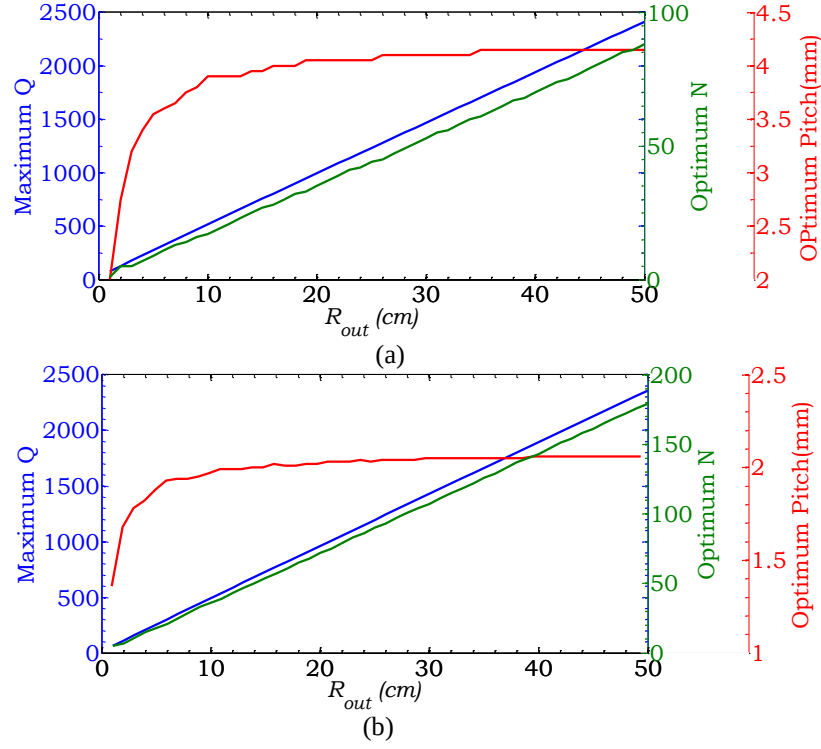


Fig. 3-16. Maximum achievable quality factor with respective optimum number of turns and optimum pitch. (a) –Wire radius of 1mm, (b) –wire radius of 0.5mm

Similarly, the optimum N is approximated as shown in (3-11) and compared with numerical results in Fig. 3-17(b). It can be seen that the optimum N is directly proportional to ratio between outer radius (in cm) to wire radius (in mm) with a slope of b_0 . Equations (3-10) and (3-11) can be used as the generalized design equations for realizing high Q spiral coils.

$$p = \frac{a_0 4r_0}{\left(\frac{a_1 r_0}{R_{out}} + 1\right)} \quad (3-10)$$

where $a_0 = 1.066$ and $a_1 = 10.23$ units: all in millimetre (mm)

$$N_{opt} = b_0 \left(\frac{R_{out}}{r_0}\right) \text{ where } b_0 = 0.1775, \text{ units: all in millimetre (mm)} \quad (3-11)$$

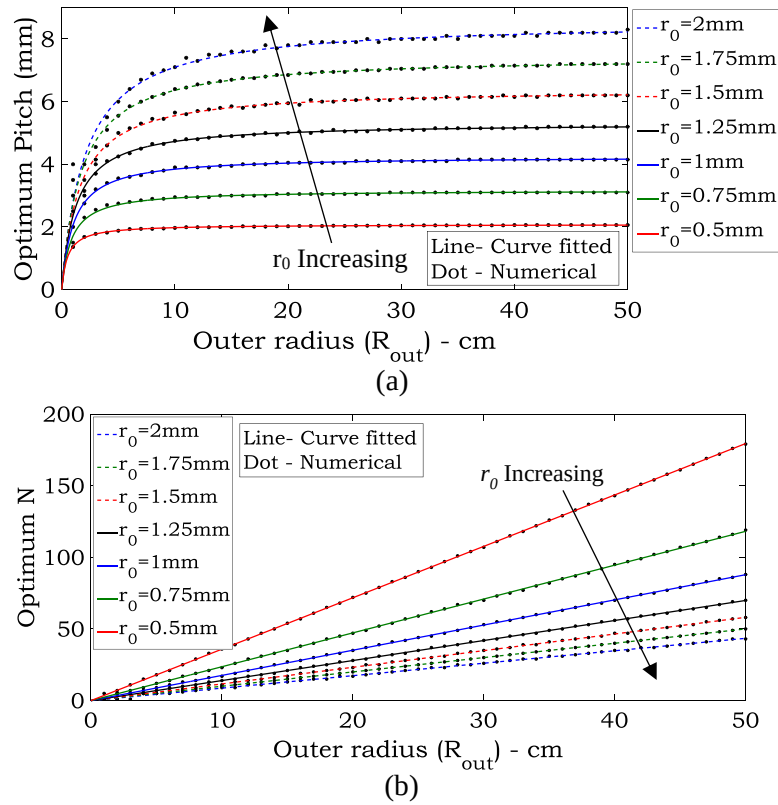


Fig. 3-17. Comparison of numerical optimal and optimal derived from curve fitting technique for (a) – optimum pitch and (b) – optimum number of turns.

3.4.2 Coupling variation

The enhancement of the electromagnetic coupling is severely limited by the misalignment and distance between coils which cannot be compromised due to most application constraints. Therefore coupling optimization needs to be considered along with the design constraints. In addition, coupling variation can be optimized by choosing appropriate coil shape for a particular position displacement profile. For example, the usability of typical spiral and helical coil shapes are compared for WPT applications. Design parameters of coils depicted in Table 3-2 are chosen to obtain highest quality factor for the selected footprint area. Fig. 3-18 compares magnitude of coupling coefficient between different coil shapes with respect to axial and radial displacements between coils. Coupling coefficient deteriorates with radial distance and becomes negative after a particular distance. The dip seen in the graph corresponds to the zero crossing. It can be seen from Fig. 3-18 that coupling between helical shaped coils is higher than that of the spiral shaped coils in higher axial and radial distances. Therefore, helical shaped coils are preferred for WPT applications. However, helical shaped coils occupy larger space, making them unsuitable when space restrictions are

imposed. For example, WPT repeaters need to be realized in a planar form, whereby spiral shape is the preferred choice.

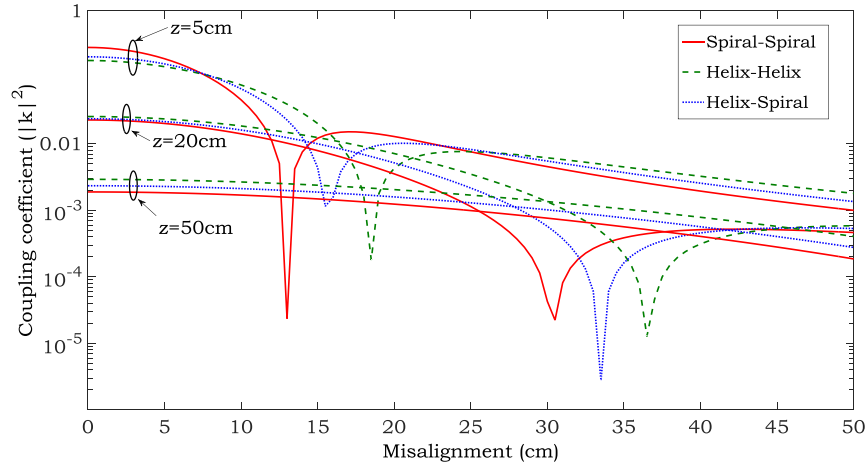


Fig. 3-18. Magnitude of coupling coefficient between different coil shapes

3.4.3 Experimental results

An experimental study is conducted to validate theoretical analysis. Helical and spiral shaped coils are constructed with the design parameters depicted in Table 3-2, using litz wire (46/1162, type 2) windings. The experimental quality factor characteristics of the constructed coils are shown in Fig. 3-19. It can be seen that the quality factor of helical and spiral shaped coils at 1MHz are more than 1000 and 800 respectively. Afterwards, each type of coils are tuned using series lumped capacitors in order to test series compensated WPT scheme. All the experimental resonance frequencies are close to 0.99MHz as shown in Table 3-3.



Fig. 3-19 Experimental quality factor characteristics (a) – Helical, (b) – Spiral

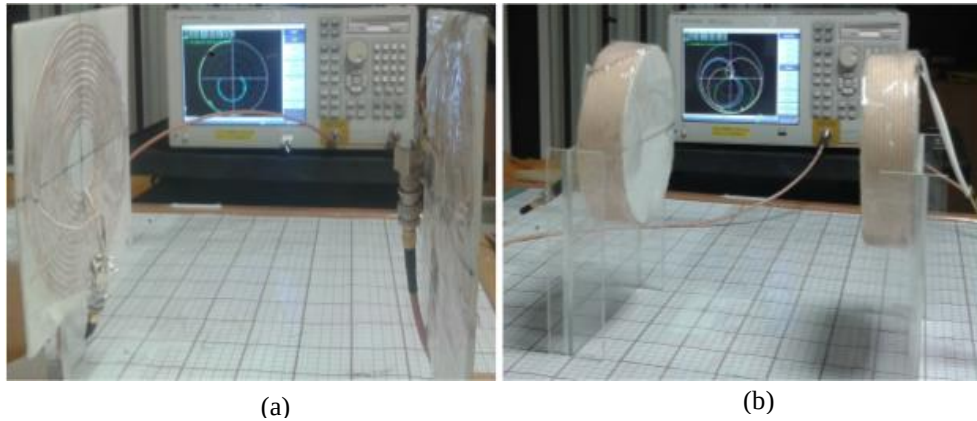


Fig. 3-20. The experimental setup (a) – Spiral coils and (b) Helical coils

Next, laboratory prototype of a WPT setup is evaluated using network analyzer measurements. The experimental setup is shown in Fig. 3-20 where Tx coil is connected to network analyzer port-1 while Rx is connected to port-2 of the network analyzer. Measured S-parameters and Agilent Advanced Design System (ADS) tool is used for performance evaluation [99]. The experimental result comparison illustrated in Fig. 3-21 shows that the experimental results agree well with the theoretical analysis with a slight deviation. It can also be seen that the helical shaped coils outperform the spiral shape, therefore, the constructed helical shaped coils are used as Tx and Rx for the subsequent optimization in this thesis.

Table 3-2. Design parameters of helical and spiral coils

Parameter	Helical	Spiral
Outer radius	89mm	89mm
Pitch	4.2mm	5mm
Number of turns	11	14
Inductance	30 μ H	20 μ H

Table 3-3. Experimental self-resonance frequencies

Type	Coil	Self-resonance (MHz)
Helical	Tx	0.990
	Rx	0.991
Spiral	Tx	0.989
	Rx	0.991

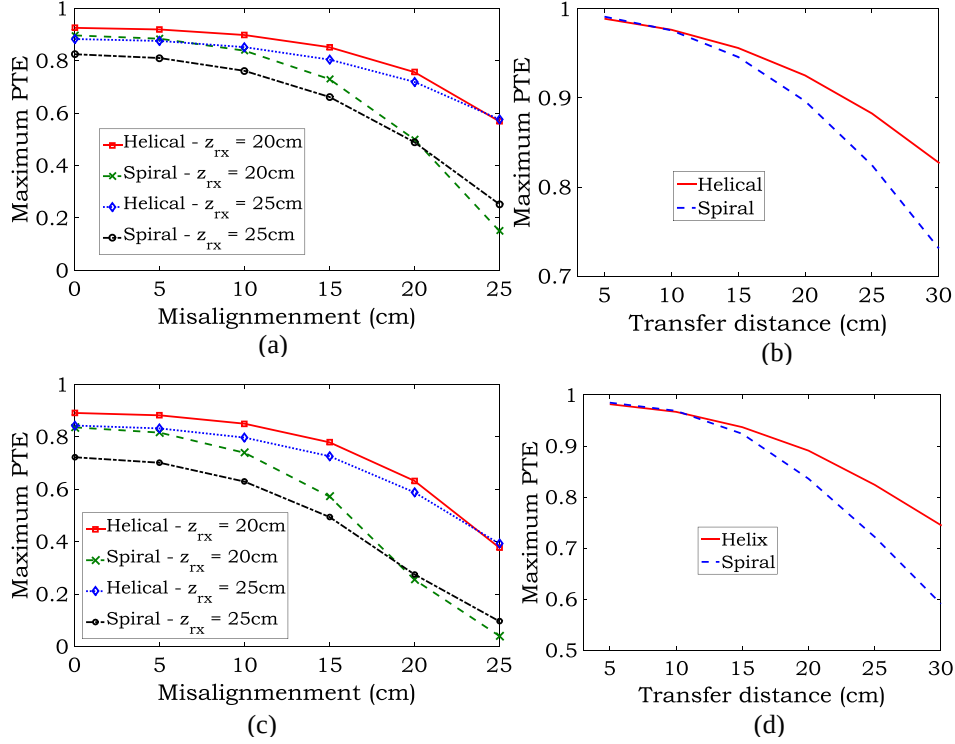


Fig. 3-21. Experimental results comparison (a) – Theory: PTE_{max} vs. misalignment, (b) – Theory: PTE_{max} vs. transfer distance at aligned position, (c) – Experimental: PTE_{max} vs. misalignment and (d) – Experimental: PTE_{max} vs. transfer distance at aligned position.

3.5 Normalized distance for comparison of dissimilar WPT schemes

Directly comparing dissimilar designs with different sizes relating to performance and transfer distance is misleading due to the differences in the design parameters. In general, higher efficiency can be achieved by using a larger coil size for a given distance. But this is not a good engineering approach. It would be appropriate if highest possible efficiency is achieved with minimum dimensions and cost. Therefore, transfer distances need to be analyzed by normalizing to the coil dimensions. The term, normalized distance has been proposed to compare diverse designs with different coil sizes and transfer distances. Furthermore, studying scalability of the WPT systems is beneficial in estimating performance of scaled up versions, particularly, for the justification of results obtained from laboratory prototypes. Normalized transfer distance is an important parameter in studying scalability. This section highlights merits and limits of the normalized distance.

Normalized distance (Z_{norm}) has been used as the ratio between transfer distance (z_{rx}) and geometric mean of Tx and Rx coil sizes as defined in (3-12) [8]. D_{Tx} and D_{Rx} are maximum dimension of Tx and Rx respectively. However, Z_{norm} may not be suitable as

a generalized term to compare different types of WPT designs. For example, design conditions: $R_{tx} = R_{rx} = 10\text{cm}$, results unity Z_{norm} at $z_{rx} = 10\text{cm}$, and design conditions: $R_{tx} = 100\text{cm}$, $R_{rx} = 1\text{cm}$ and $z_{rx} = 10\text{cm}$ also results unity Z_{norm} . However, it is apparent that performance in two cases are different. Therefore, studying limitations of Z_{norm} is a crucial aspect for the scalability analysis of WPT systems.

$$Z_{norm} = \frac{Z_{rx}}{\sqrt{\frac{D_{Tx}}{2} \frac{D_{Rx}}{2}}} \quad (3-12)$$

3.5.1 Identical Tx and Rx

Identical Tx and Rx coils with the same outer radius is studied ($R_{tx} = R_{rx} = R_{out}$). With identical coils, Z_{norm} becomes ratio between z_{rx} and R_{out} . Fig. 3-22(a) illustrates maximum PTE variation with respect to Z_{norm} with single turn loop Tx and Rx ($N=1$). Fig. 3-22 (b) shows respective optimum load values. z_{rx} is varied with each R_{out} value in generating Fig. 3-22. It can be seen that maximum PTE resulted from identical single turn loops is solely dependent on Z_{norm} . Therefore, Z_{norm} completely defines the maximum PTE in such a scenario. However, it should be noted that the optimum load varies with the coil radius and Z_{norm} . Furthermore, the maximum PTE deteriorates beyond unity Z_{norm} .

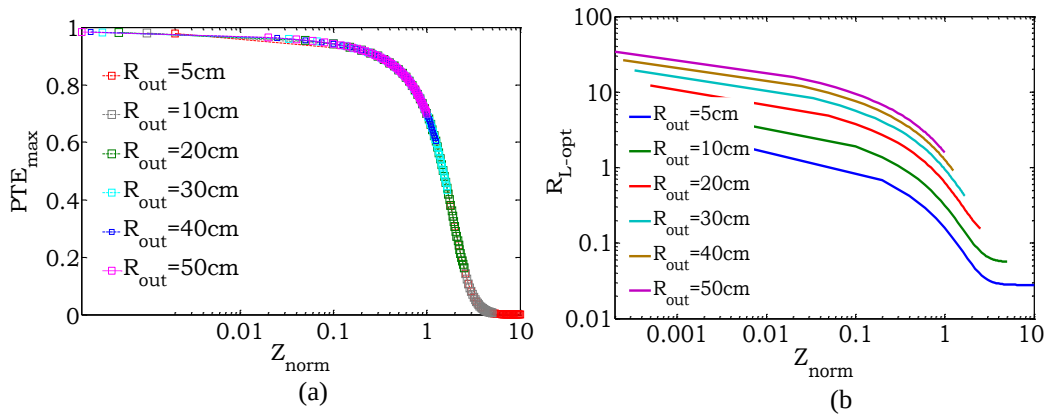


Fig. 3-22. (a) – Maximum PTE and (b) – Optimum load resistance with respect to normalized distance for single loop coils ($N=1$, $R_{tx}=R_{rx}=R_{out}$).

Fig. 3-23(a) shows the maximum PTE variation with respect to Z_{norm} when coils are optimized to the maximum Q. Unlike in single turn case, PTE variation for coils with optimum Q is not solely dependent on Z_{norm} . It can be seen that PTE_{max} profiles for different R_{out} are similar up to unity Z_{norm} . Beyond that, higher radius shows higher

efficiency at a given Z_{norm} . For example, $R_{out}=5\text{cm}$ gives the maximum efficiency of 23% at $Z_{norm}=4$ whereas, $R_{out}=25\text{cm}$ gives the maximum efficiency of 70% at same Z_{norm} . Absolute distances of above two cases are 20cm and 100cm respectively. Therefore, it can be concluded that higher R_{out} offers higher efficiencies in same Z_{norm} if Q optimized identical coils are used. However, load characteristics in Fig. 3-23(b) show that optimum load for higher R_{out} is much higher ($\text{k}\Omega$ range) in value.

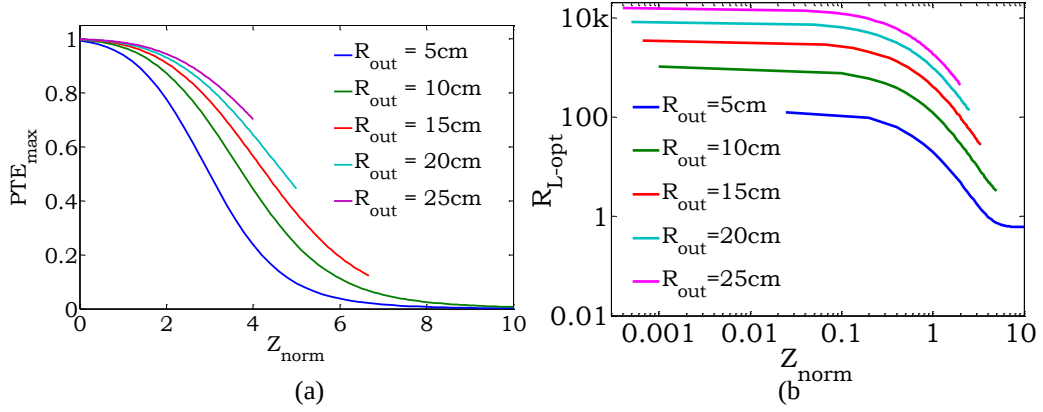


Fig. 3-23. (a) – Maximum PTE and (b) – Optimum load resistance with respect to normalized distance for coils optimized to highest quality factor ($R_{tx}=R_{rx}=R_{out}$).

3.5.2 Non identical Tx and Rx

In this section, non-identical Tx and Rx coils are analyzed because most applications required to be designed with non-identical coils. First, PTE_{max} with single turn loops are analyzed with respect to normalized distance, coil sizes and transfer distance. Fig. 3-24(a) shows the maximum PTE variation with respect to Tx and Rx coil radii for single loop coils at $z_{rx}=10\text{cm}$. It can be seen that PTE_{max} is highest when Tx and Rx loop radii are equal. Fig. 3-24(b) shows maximum PTE with respect to Z_{norm} at different Tx and Rx coil radii for single loop coils. PTE_{max} in Fig. 3-24(b) is dependent on ratio of Tx and Rx loop radii as well. Higher ratios result in lower PTE at a given Z_{norm} . This result reveals that the increase of Tx or Rx radii without proper optimization degrades the performance of WPT systems.

Next, coils optimized for the maximum Q at each outer radius are analyzed. Fig. 3-25 shows the maximum PTE variation with respect to Tx and Rx coil radii at 10cm transfer distance for the coils with maximum Q. In contrast to the single loop case, coils optimized to maximum Q shows better performance. Nevertheless, if Rx (or Tx) coil is too small, PTE_{max} cannot be improved with larger Tx (or Rx) coil. For example, if Rx

radius is less than 3cm, PTE_{max} at $z_{rx}=10\text{cm}$ is less than 40% irrespective of the Tx coils size. On the other hand, if Rx radius is higher than 5cm and Tx radius is higher than 10cm, PTE_{max} at $z_{rx}=10\text{cm}$ is higher than 90%. However, increase in coil radii beyond that does not increase the maximum efficiency significantly. Therefore, choosing proper sizes of Tx and Rx will be required by considering the performance effects.

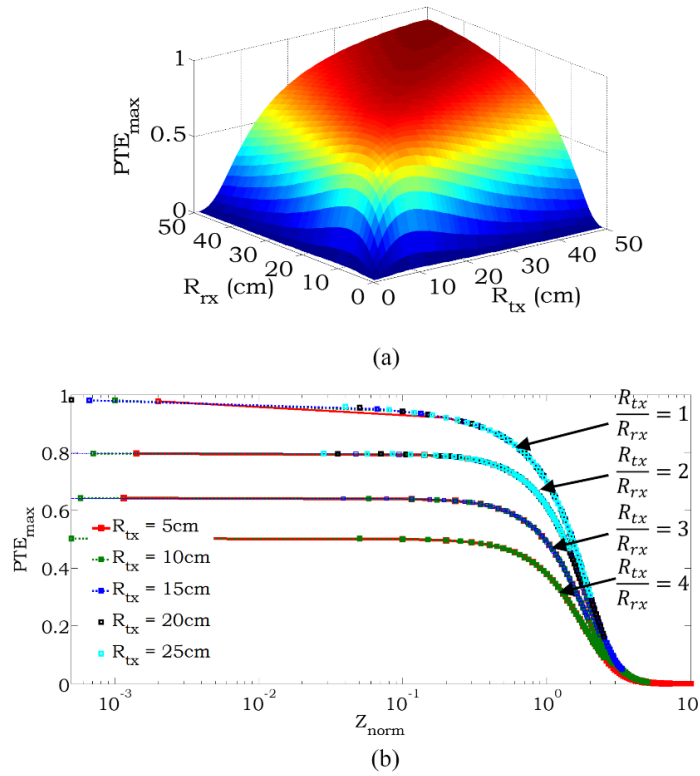


Fig. 3-24. (a) – Maximum PTE variation with respect to Tx and Rx coil radii for single loop coils at $z_{rx}=10\text{cm}$, and (b) – Maximum PTE variation with respect to normalized distances with different R_{tx}/R_{rx} ratios for single loop coils.

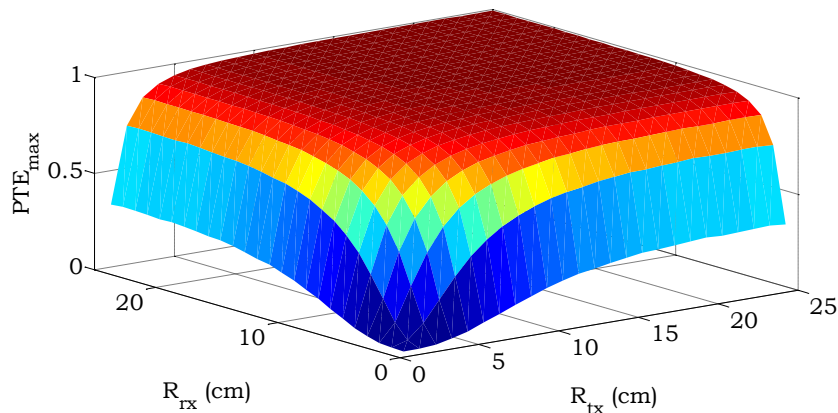


Fig. 3-25 Maximum PTE variation with respect to Tx and Rx coil radii for coils optimized to maximum quality factor at $z_{rx}=10\text{cm}$.

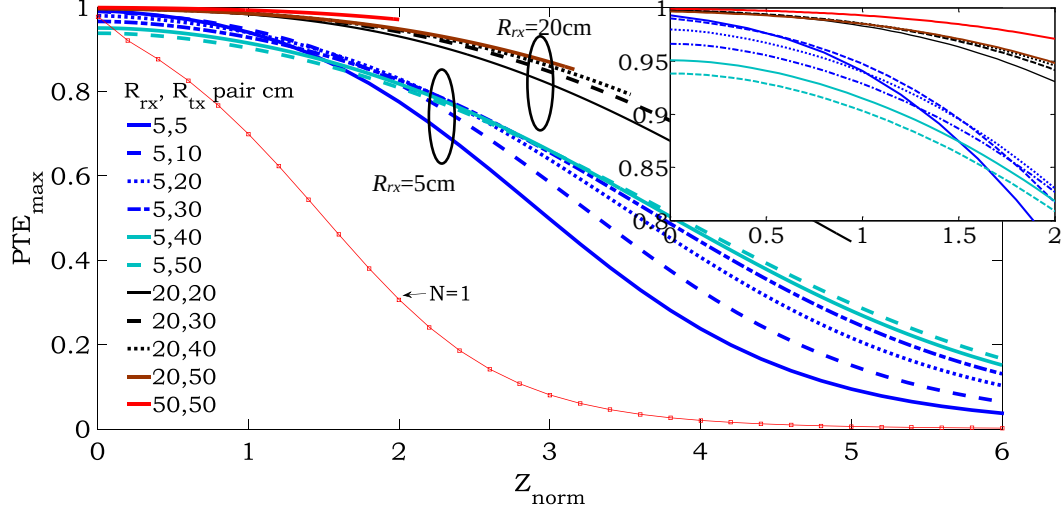


Fig. 3-26. Maximum PTE variation with respect to normalized distance (Z_{norm}) at different Tx and Rx coil radii, when coils are optimized to achieve maximum quality factor at each radii.

Fig. 3-26 illustrates PTE_{max} variation with respect to Z_{norm} at different Tx and Rx coil radii, when coils are optimized to achieve maximum Q at each radii. Despite being non-identical coils, PTE_{max} can be improved significantly by Q optimization compared to the single loop scenario (labelled as $N=1$ in Fig. 3-26). However, non-identical coils result in lower PTE at Z_{norm} value less than unity. For example, $R_{tx}=R_{rx}=5\text{cm}$ results in 94% PTE_{max} at unity Z_{norm} whereas, with $R_{tx}=50\text{cm}$ and $R_{rx}=5\text{cm}$ PTE_{max} is reduced to 90% at unity Z_{norm} . On the other hand, larger Tx coils shows better PTE at higher Z_{norm} values. For example, comparing above two scenarios at $Z_{norm}=4$ shows better PTE with larger Tx coil (R_{tx}, R_{rx} pairs 5cm,5cm and 50cm,5cm show PTEs of 30% and 50% at $Z_{norm}=4$ respectively). Similar PTE profile can be observed for 20cm Rx coil as well. Therefore, it is apparent that the use of larger coil sizes may not be suitable for all the design cases, particularly when Rx is small, an additional consideration should be given based on R_{rx} and z_{rx} . Therefore, next section presents an optimization scheme for WPT with a miniaturized Rx.

3.6 Optimization with miniature Rx coil

Many WPT applications need to be realized with a space restriction for receiving coil. For example, WPT for biomedical implants need to be realized with a miniaturized receiving coil because the available space for the implantable Rx is extremely limited. On the other hand, Tx coil can be substantially larger in size for such an application. It was identified in the previous section that Tx size needs to be carefully optimized, particularly, with a miniaturized Rx. Therefore, there is a requirement for the derivation

of a coil design approach when coil size constraints are imposed. This section investigates design parameter optimization for WPT with miniaturized Rx coil. Similar to the analysis on normalized distance, spiral shape is chosen for this study as illustrated in Fig. 3-27.

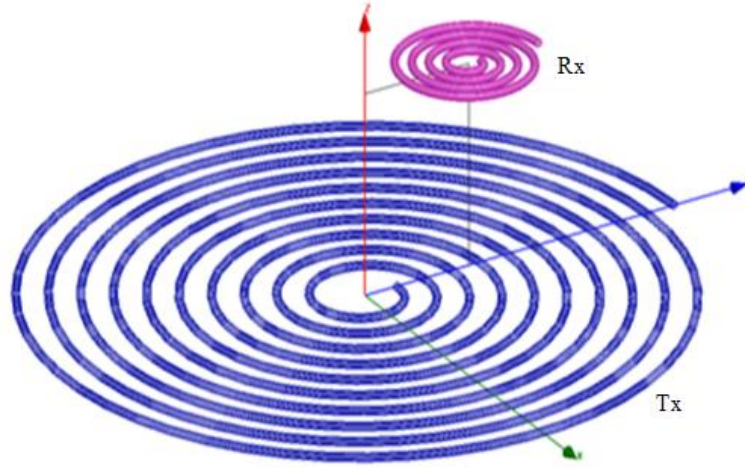


Fig. 3-27. WPT with miniaturized receiver.

When the design area for Rx coil is restricted, coupling coefficient variation against Tx size is crucial design aspect. For example, Fig. 3-28 illustrates coupling variation with respect to Tx coil radius at different transfer distances when coil design parameters are optimized for maximum Q. It can be seen that higher Tx coil size is not always good choice for achieving high coupling. Instead, Tx coil radii which gives maximum coupling is dependent on both Rx coil radius and transfer distance. Therefore, Tx coil radius needs to be determined based on Rx size and transfer distance.

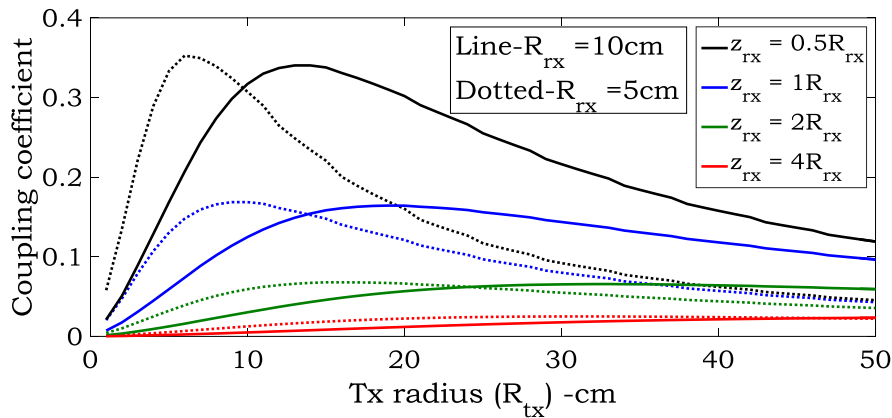


Fig. 3-28. Coupling coefficient variation with respect to Tx radius at different transfer distances (number of turns and pitch are optimized for optimal quality factor at respective coil radii).

A numerical analysis is performed to study the effect of Tx coil size with regards to maximum efficiency. Fig. 3-29 illustrates optimal Tx coil radii with respect to transfer distance and Rx coil radii when the other design parameters (N and p) are optimized for maximum quality factor. Transfer distance and Rx radius is varied up to 30cm for wire radii of 0.5mm, 1mm, 1.5mm and 2mm. The optimal Tx radius is increasing with both Rx radius and transfer distance. It is interesting to note that the optimal Tx radius variation with respect to constructing wire radius is negligible. This is because the optimum Tx radius is mainly determined by the coupling between Tx and Rx, which is independent of the wire radius because a filamentary current is assumed for mutual inductance calculations.

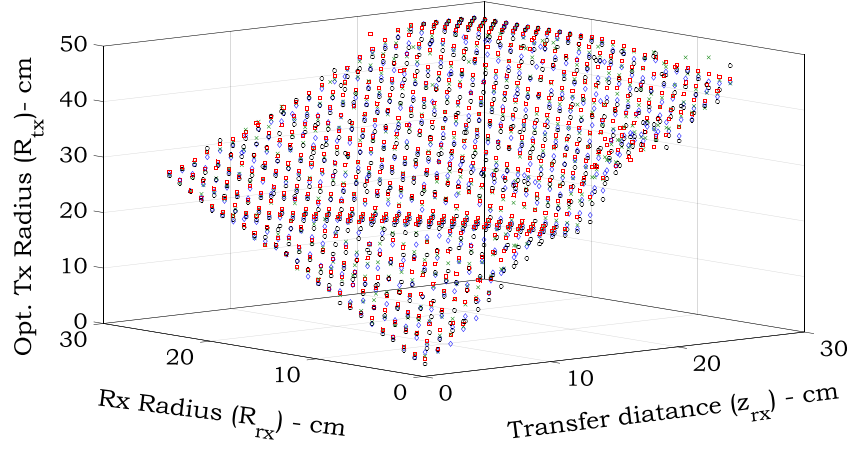


Fig. 3-29. Numerical results: Optimum Tx radius vs. transfer distance and Rx radius for different wire radii.

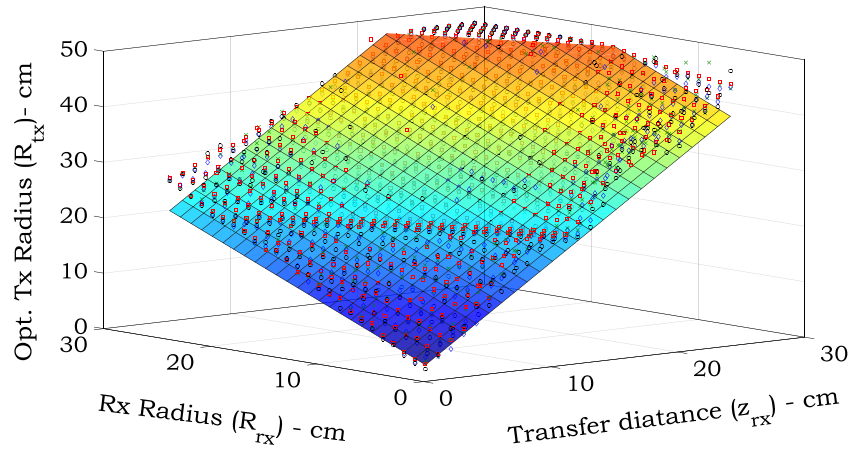


Fig. 3-30. Comparison of optimum Numerical optimum vs. curve fitting (3.8)

Therefore, curve fitting approach can be adopted to derive optimal Tx coil radius as a function of Rx coil radius and transfer distance. Single order polynomial fit derived in (3-13) shows a fairly close fit with the numerical results as compared in Fig. 3-30. The term Δ_{PTE} is defined as difference between maximum PTE for numerical optimum Tx coil radii (PTE_{max}) and maximum PTE for Tx coil radii calculated using curve fitting technique (PTE_{cf}). It can be seen from Fig. 3-31 that Δ_{PTE} is less than 2% for z_{rx} less than 5cm, and it is close to zero for z_{rx} higher than 5cm. Therefore the proposed empirical relationship for optimal Tx coil radius provides a good guideline for the design of WPT system with Rx size restrictions. However, if the optimal Tx radius is out of the size constraint for Tx coil, the maximum possible Tx radius is recommended for the Tx coil design. Therefore, the overall design process can be summarized as illustrated in the flow chart in Fig. 3-32.

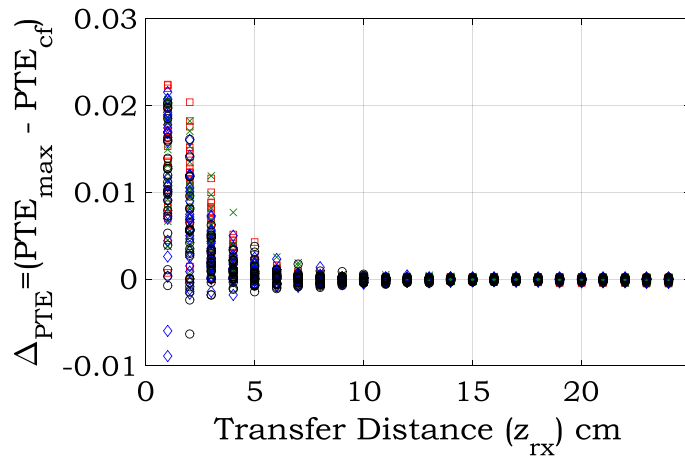


Fig. 3-31. Comparison of Δ_{PTE} ($\Delta_{PTE} = PTE_{cf} - PTE_{max}$: difference between maximum PTE for numerical optimum Tx coil radii and maximum PTE for Tx coil radii calculated using curve fitting technique).

$$R_{tx} = p_0 z_{rx} + p_1 R_{rx} \quad (3-13)$$

where $p_1 = 1.6$ and $p_1 = 0.84$

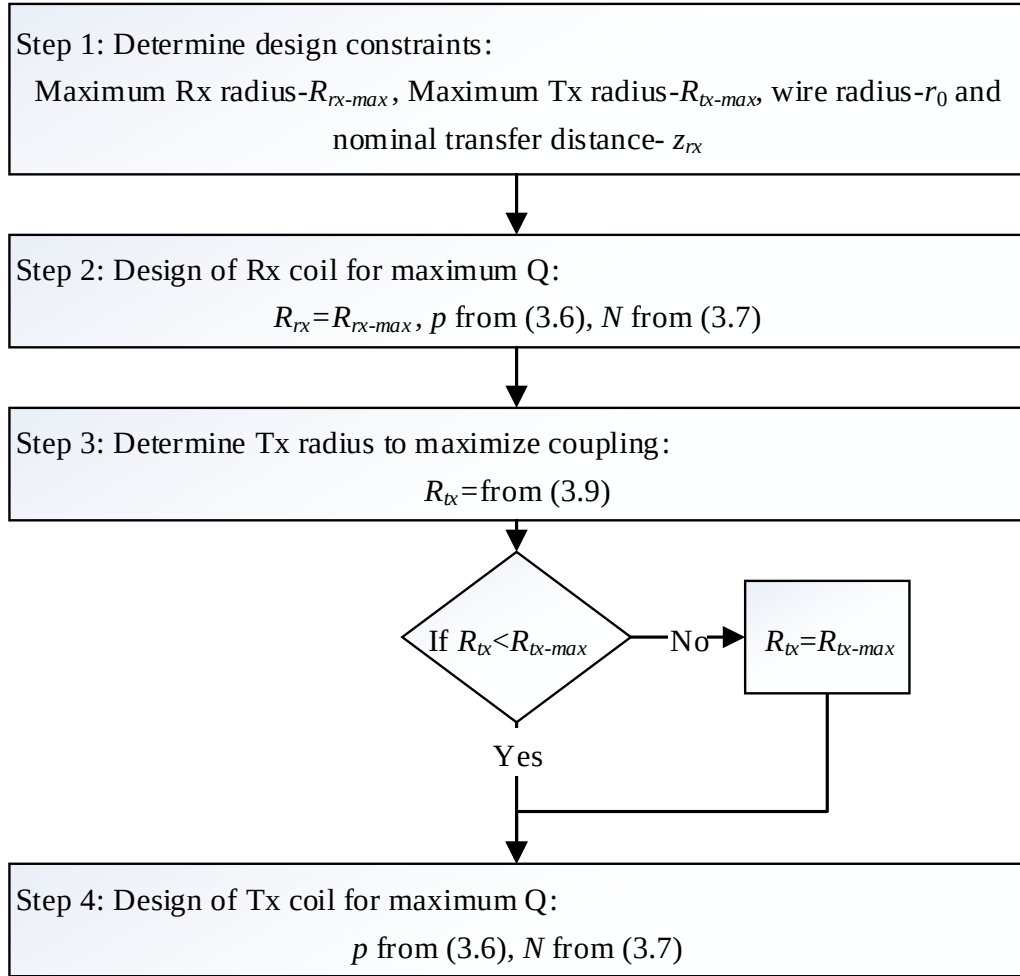


Fig. 3-32. Design steps for WPT setup with a miniaturized Rx

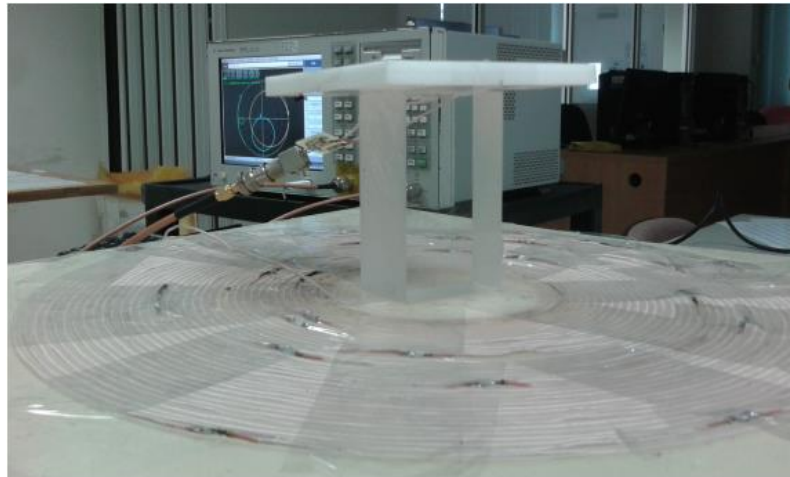


Fig. 3-33. The experimental setup ($R_{tx}=20\text{cm}$, $R_{rx}=5\text{cm}$ and $z_{rx}=10\text{cm}$)

3.6.1 Experimental verification

An experimental study is performed in order to validate the theoretical analysis. Design constraint for Rx coil radius is set to be less than 5cm, and nominal transfer distance of 10cm is chosen. Similar to the previous sections, network analyzer measurements along with ADS is used to evaluate maximum PTE. The experimental setup is illustrated in Fig. 3-33.

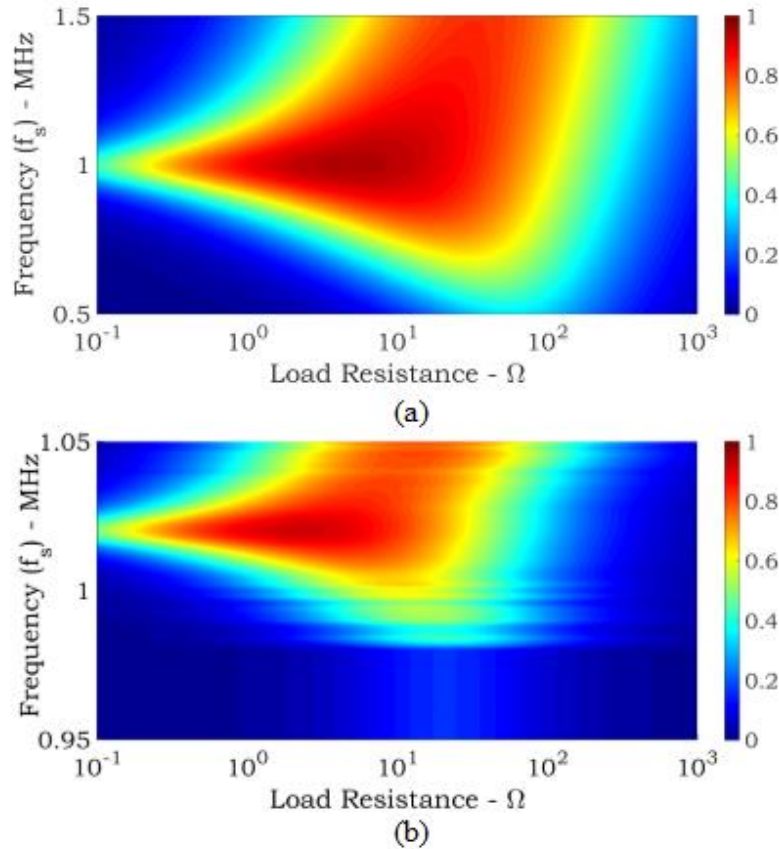


Fig. 3-34. Efficiency comparison with respect to frequency and load at 10cm transfer distance (a) – theoretical results and (b) – experimental results.

The comparison of experimental results and theoretical calculations for frequency and load characteristics of PTE is shown in Fig. 3-34. It can be seen that the experimental PTE profile with respect to operating frequency and load resistance closely match with the theoretical results. However, frequency bandwidth of experimental results is much narrow compared to theoretical counterpart. This can be attributed to the disparities in experimental self-resonance frequencies and the losses introduced by connectors and cables. Load characteristic shown in Fig. 3-35 compares theoretical and experimental PTE at optimal operating frequency. Experimental results show maximum

PTE of 91.5% for the particular case study, while the theoretical calculations show 94.5% maximum efficiency. These differences can be justified by experimental disparities and additional losses contributed from cables, connectors and capacitors. Overall, experimental results agree well with the theoretical analysis, proving the efficacy of the proposed optimization approach.

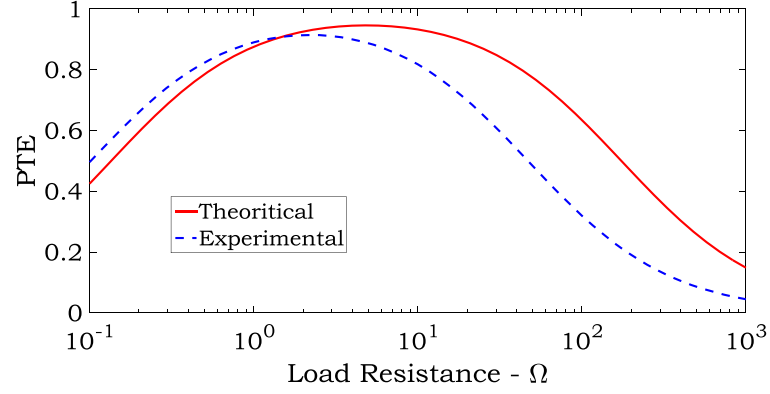


Fig. 3-35. Efficiency characteristics against load resistance.

3.7 Summary

This chapter presented methods for the optimization of wireless power transfer (WPT) coils considering the maximum power transfer efficiency (PTE_{max}). Quality factor and coupling coefficient were identified as the key performance indices for WPT coil design. The effect of design parameters is investigated to achieve high quality factors, thereby empirical formulas were derived to obtain generalized design guidelines for coil quality factor optimization. High quality factor coils were experimentally realized using the proposed optimization, and these coils will be used for the subsequent studies in this thesis. In addition, an analysis on normalized distance which has been used for the comparison of dissimilar WPT schemes was performed. Merits of normalized distance and its limitations were investigated as well. It was investigated that transmitter coil size needs to be carefully optimized when space restrictions are imposed on the receiver circuitry. Therefore, the design guidelines proposed in this chapter were extended to derive generalized coil design scheme for a WPT system with a miniaturized Rx. A design case was presented along with an experimental validation. Next chapter presents an efficiency enhancement approach for dynamic wireless power transfer system.

Chapter 4

Efficiency Enhancement for Dynamic Wireless Power Transfer (D-WPT)

4.1 Introduction

Dynamic WPT (D-WPT) technology has been built upon the fundamentals of WPT and aims to power the moving receiver (Rx) wirelessly. Promising applications of D-WPT include electric vehicles (EV) [12, 17-19], biomedical applications [20], robotics [21] and manufacturing applications [22]. For example, D-WPT for EV charging has many advantages compared to wired-EV charging. Inconveniences in wired-EV charging process are a major impediment in their widespread adoption. From the receiver viewpoint, D-WPT theoretically solves the battery problem as it can provide an unlimited driving range. On the other hand, in case of manufacturing applications, power supplies of overhead conveyers and moving robots can be implemented using D-WPT technology. However, the employment of such systems is reliant on the infrastructure development, which in turn, is limited by its cost. D-WPT can be broadly classified into track based [18] and segmented transmitter (Tx) array types based on the Tx coil configuration [19]. Track-based solutions offer continuous power transfer to the moving receiver by using a long transmitter track whereas, segmented approaches consist of a set of segmented Tx coils placed in an array. Transmitter track based systems are easier to control as the track is powered from a single source. Coupling coefficient along the track is nearly constant as the receiver (Rx) moves along the track. The transmitter track can be a few meters to several tens of meters long depending on the nature of application [18]. However, this type of design suffers from a fairly low coupling coefficient because of the smaller transmitter region covered by the Rx thus giving rise to a lower efficiency. Moreover, electromagnetic field emitted in the uncoupled region has to be suppressed to mitigate harmful exposure.

On the other hand, segmented Tx coils give higher coupling coefficient and efficiency compared to Tx track based methods. Nevertheless, misalignment tolerance, position tracking, transmitter separation and controlling the power flow are design

challenges that need to be solved in segmented Tx designs. There are two methods in connecting the power source to segmented Tx coils. Several Tx coils can be connected to a single power source in parallel, or dedicated sources can be used to supply power to each coil. Furthermore, the connection of dedicated power source for each coil is not cost effective. In contrast, connecting multiple Tx coils to a single source is relatively simpler and cost effective. Another approach of dedicated source configuration is to use single source connected to multiple coils, and switch between each coil when the receiver moves along the Tx array using a switch box [17]. In addition to source configuration, separation between transmitters needs to be carefully optimized. If coils are placed far apart, efficiency reduces drastically when the Rx moves away from Tx and the power transfer will not be continuous. However, coils cannot be kept too close to each other because the system cost will be increased with the increased number of transmitters placed in a given length of the track. In addition, placing Tx coils too close to each other may experience lower performance due to the cross coupling effect.

Misalignment problem dominates the technical challenges in designing D-WPT systems with segmented Tx arrays [57]. In [100], the authors present a method for segmentation and compensation of Tx coils for D-WPT. However, literature still lacks a comprehensive analysis on design parameters such as number of Tx coils powered simultaneously, separation between Tx coils and, frequency and load characteristics. Therefore, the requirement of such an analysis on segmented D-WPT system is a vital aspect in optimal design process. Therefore, this study investigates a detailed analysis on the designing of D-WPT system using segmented Tx array. The frequency and load characteristics of D-WPT system with segmented Tx array are explored. In an n -Tx coil array, there are n number of simultaneously powered Tx coils. Frequency splitting phenomena that is investigated for $n > 1$ has not been studied previously. A novel double spiral repeater (DSR) scheme is proposed for efficiency and misalignment tolerance improvement.

The chapter is organized as follows. Section 4.1 introduces the analysis of D-WPT system using equivalent circuit theory. D-WPT using an array of Tx resonators is discussed in Section 4.3. Frequency and load characteristic of the system is also evaluated. Subsequently, the system with focusing DSR is introduced and its performance is analyzed in Section 4.4 followed by the chapter summary.

4.2 Equivalent circuit analysis

For the system analysis, the equivalent circuit method discussed in Chapter 2 is used. Fig. 4-1(a) shows the equivalent circuit of dedicated power source configuration and Fig. 4-1(b) illustrates the single power source configuration for 3-Tx setup. Power sources are modelled as voltage sources and all excitation voltages are chosen to be equal in the analysis ($V_{t1} = V_{t2} = V_{t3} = V_s$). Equivalent source resistance (R_s) represents the output impedance of the power source. The notion of source resistance is sometimes misleading in literature, particularly studies focus on impedance matching (IM) or two port network model [101] use 50Ω source impedance. However, source impedance is much smaller in value when power converters are used as the source [102]. The equivalent circuits with either source connection types become identical with the assumption of 0Ω source. Therefore, the system equation in (4-1) can be used to study the system performance for both parallel single-source and dedicated multi-source configurations with 0Ω source impedance. PTE is defined as Tx coils-to-Rx coil efficiency to study coil configurations for improved efficiency in (4-2).

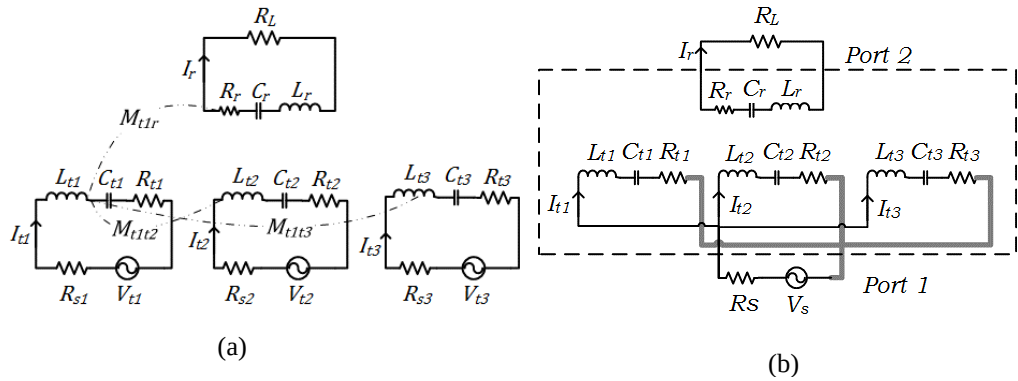


Fig. 4-1. Equivalent circuit for three adjacent transmitters (3-Tx setup) and a receiver: (a) –Dedicated power source configuration where separate sources connected to each transmitter, and (b) – Single power source configuration where transmitter coils are connected in parallel with a single source.

$$\begin{bmatrix} Z_{t1} & X_{t1t2} & X_{t1t3} & X_{t1r} \\ X_{t2t1} & Z_{t2} & X_{t2t3} & X_{t2r} \\ X_{t3t1} & X_{t3t2} & Z_{t3} & X_{t3r} \\ X_{rt1} & X_{rt2} & X_{rt3} & (Z_r + R_L) \end{bmatrix} \begin{bmatrix} I_{t1} \\ I_{t2} \\ I_{t3} \\ I_r \end{bmatrix} = \begin{bmatrix} V_{t1} \\ V_{t2} \\ V_{t3} \\ 0 \end{bmatrix} \quad (4-1)$$

$$P_{out} = |I_r|^2 R_L$$

$$PTE = \frac{P_{out}}{P_{in}} = \frac{|I_r|^2 R_L}{|I_{t_1}|^2 R_{t_1} + |I_{t_2}|^2 R_{t_2} + |I_{t_3}|^2 R_{t_3} + |I_r|^2 (R_L + R_r)} \quad (4-2)$$

$Z_m = R_m + j(\omega L_m - 1/\omega C_m)$, ($m = t_{1,2,3,r}$) are the self-impedances of Tx and Rx coils where L_m , C_m , and R_m represent inductance, capacitance and parasitic resistance of the coil. Capacitance includes both distributed capacitance and externally added capacitance. $X_{mn} = X_{nm} = j\omega M_{mn} = j\omega M_{nm}$ ($m = t_{1,2,3,r}$) where M_{mn} ($= M_{nm}$) is mutual inductance between m^{th} and n^{th} coils, and ω is the operating frequency of the source. Subscripts $t_{1,2,3}$ represent three nearest Tx coils to the Rx coil concerned (represented by subscript r). Solving for currents and PTE explicitly from (4-2) results in a very complex expression. Therefore, system performance is numerically analyzed and validated with the experiments. Equations (4-1) and (4-2) will be used extensively for the analyses in the rest of this chapter. The optimized helical shaped coil in Chapter 3 is chosen as Tx coils. Resonance frequency (f_0) of Tx and Rx resonators is chosen to be 0.99MHz ($f_0 = 1/(2\pi\sqrt{LC})$).

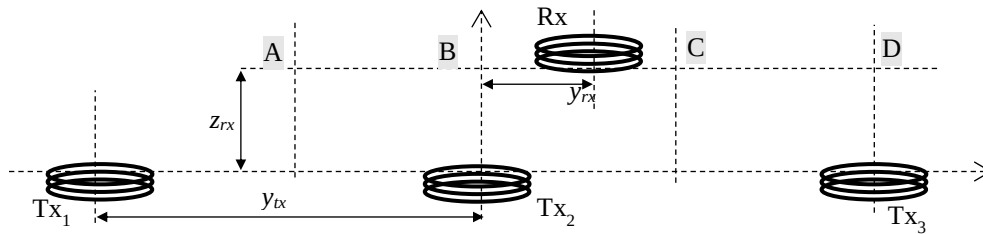


Fig. 4-2. D-WPT system with segmented transmitter array (3-Tx setup)

4.3 Analysis of the classical transmitter array

Before moving into the analysis of electrical characteristics, the implementation of an n -Tx D-WPT setup is discussed. Fig. 4-2 illustrates a section of a segmented Tx array. System performance is analyzed for different number of Tx coils powered at a time. In case of 1-Tx or 3-Tx setup, the segment from A to C is repeated when Rx moves along the Tx array, and segment B to D is repeated for 2-Tx setup. Next Tx coil has to be powered on and last Tx coil has to be disconnected once the Rx moves away from respective segment. Therefore, in all three cases ($n=1,2$, and 3), length of a segment is equal to the separation between Tx coils (y_{tx}). Position B is considered as the reference

position ($y_{rx}=0$), while position C is considered as the maximum misaligned position ($y_{rx}=y_{tx}/2$) in subsequent analyses.

4.3.1 Frequency and load characteristic

Fig. 4-3 illustrates PTE variation calculated at the aligned position and 20cm transfer distance ($y_{rx} = 0, z_{rx} = 20\text{cm}$) with respect to operating frequency and load resistance for 1-, 2- and 3-Tx setups. As expected, PTE is always maximum at self-resonance frequency for 1-Tx setup. In contrast, when two or three Tx coils are simultaneously powered, a dip can be observed at a frequency close to self-resonance whereas, the maximum PTE is achieved when operating at frequencies away from f_0 . A frequency splitting phenomena, where the maximum PTE occurs at two different frequencies, can be observed. These two frequencies can be considered as the resonance frequencies of the overall system, lower resonance frequency (LRF), and higher resonance frequency (HRF). It should be noted that this frequency splitting phenomena for PTE characteristic is different from the frequency splitting discussed in Chapter 3 regarding the TP characteristics of two coil WPT.

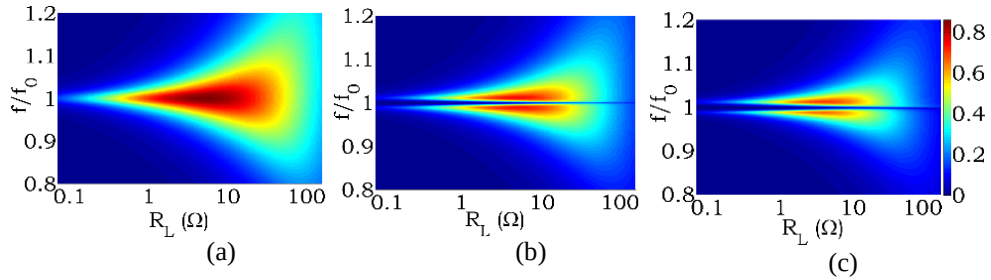


Fig. 4-3. PTE with respect to operating frequency and load resistance, (a) – 1-Tx setup, (b) 2-Tx setup and (c) – 3-Tx setup; ($y_{rx} = 0, y_{tx} = 50 \text{ cm}, z_{rx} = 20\text{cm}$)

The frequency splitting phenomena can be explained by investigating the current ratios and the input impedances seen by the source at each Tx coil. The PTE expression in (4-2) can be simplified to (4-3) assuming identical Tx and Rx coils. Transfer current ratio (χ) is defined as the ratio between summation of absolute squares of Tx currents and absolute square of Rx current. It can be seen from (4-3) that PTE decreases with the increase of χ . The input impedance characteristics shown in Fig. 4-4(a) is evaluated at aligned position for 3-Tx setup. The input impedance is determined by the resonator impedance and the reflected impedance from the surrounding coils. At self-resonance frequency ($f_s=f_0$), all resonator impedances are equal to their respective coil resistances. As the middle transmitter (Tx₂) is nearest to the Rx, reflected impedance is higher

compared to that of the other two resonators (Tx_1 & Tx_3 for 3-Tx setup and Tx_3 for 2-Tx setup). This explains higher input impedance seen at Tx_2 near resonance frequency. Consequently, there is a significantly lower current in Tx_2 compared to the transmitters farther away (Tx_1 & Tx_3) leading to high χ near self-resonance frequency as seen in Fig. 4-4(b). On the other hand, at LRF and HRF, input impedance of Tx_2 becomes lower than that of the Tx_1 and Tx_3 (Fig. 4-4(a)). This regulates the currents in Tx_1 and Tx_3 , causing χ to be minimized at HRF and LRF.

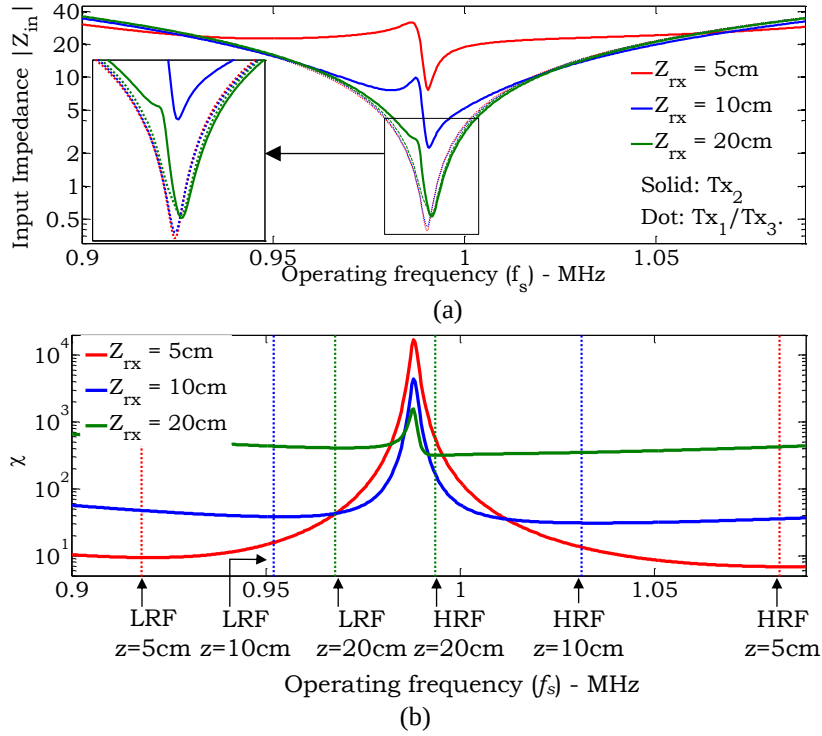


Fig. 4-4. (a) – Input impedance of each transmitter coil at aligned position and (b) – Transfer current ratio variation with respect to operating frequency. (3-Tx setup).

$$PTE = \frac{\xi}{\chi + \xi + 1}$$

$$\chi = \left(\frac{|I_{t_1}|^2 + |I_{t_2}|^2 + |I_{t_3}|^2}{|I_r|^2} \right); \xi = \frac{R_L}{R} \quad (4-3)$$

$$R = R_{t_1} = R_{t_2} = R_{t_3} = R_r$$

Furthermore, it is observed in Fig. 4-3 that highest PTE occurs when the load resistance is lower than 10Ω . However, equivalent load is dependent on the application specific criteria. Therefore, D-WPT system should be capable of delivering power with an acceptable PTE in a wide load range. It is clear that the classical segmented Tx array

experiences lower efficiency at higher load. However, the proposed receiver structure with DSR allows higher PTE in a wide range of load variation.

4.3.2 Transmitter separation and number of transmitter coils

The effect of number of Tx coils (n) powered at a time and separation between Tx coils (y_{tx}) is analyzed in this section. Numerically calculated results in Fig. 4-5 illustrate the maximum PTE at y_{tx} values of 50cm and 20cm when load resistance and operating frequency are at their optimum values. If Tx coils are far apart, power transfer contribution from Tx coils placed farther away is not very significant at lower axial distances. This is because coupling between Rx and nearer Tx is much higher than the coupling between Rx and Tx coils farther away. This attributes to the highest PTE observed for 1-Tx setup at Z_{rx} =20cm and for 2-Tx at Z_{rx} =50cm as illustrated in Fig. 4-5(a). In contrast, when Tx coils are placed very close to each other (y_{tx} =20cm in Fig. 4-5(b)) coils farther away also contribute to the power transfer. Results in Fig. 4-5(b) show that PTE is almost identical for all setups (1-, 2- or 3-Tx) for Z_{rx} =20cm whereas, 3-Tx setup shows the best performance for Z_{rx} =50cm. However, Tx coils that are too close result in higher implementation cost as the number of Tx coils to be placed in a given length of Tx array is higher. Therefore, it would be favorable if PTE and misalignment tolerance can be improved with a larger separation between Tx coils. The proposed receiver structure of this study allows enhanced PTE and misalignment tolerance even with higher separation (y_{tx} =50cm) between Tx coils.

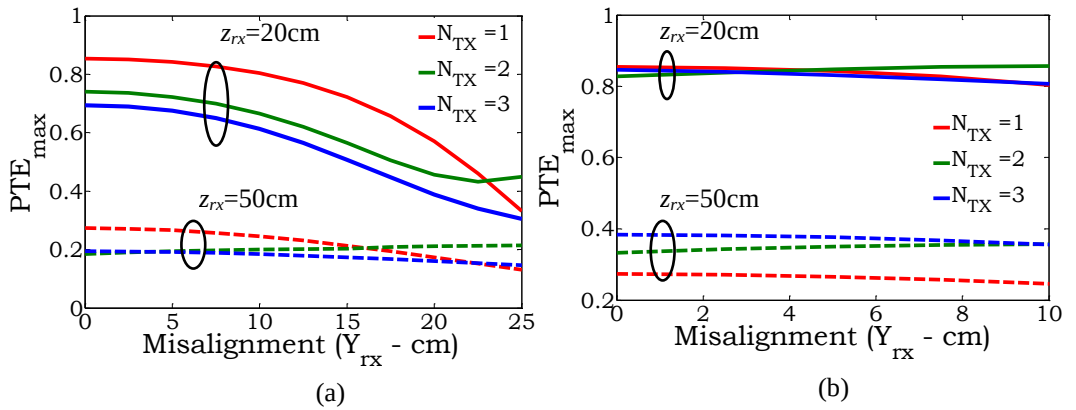


Fig. 4-5. Maximum PTE (when load resistance and operating frequency is optimum) with respect to misalignment. (a) – y_{tx} =50cm and (b) – y_{tx} =20cm

4.3.3 Experimental results

Network analyzer measurements are used for the experimental verification. Tx coils are connected in parallel with the network analyzer (Agilent E5062A) Port 1, while Rx is connected to the Port 2. Measured S-parameters are used to evaluate PTE. The load resistance is chosen to be $50\ \Omega$ for the subsequent comparisons with experimental results as the characteristic impedance of the network analyzer is $50\ \Omega$.

4.3.3.1 Frequency Response

Fig. 4-6(a) shows frequency characteristic at the aligned position for different axial distances. Both LRF and HRF are dependent on the position of the Rx (coupling among coils). The maximum PTEs (at LRF and HRF) and respective frequencies are illustrated in Fig. 4-6(b). PTE at HRF is higher than PTE at LRF for 3-Tx setup, while both PTEs (at LRF and HRF) are almost same for 2-Tx setup. It can be seen that both LRF and HRF are close to self-resonance frequency for higher axial distances (i.e. lower coupling) whereas, LRF and HRF are further apart for low axial distances (i.e. higher coupling). This is because the input impedance seen at Tx_2 is reduced at higher axial distances (Fig. 4-4(a)), and becomes comparable with the input impedances seen at Tx_1 and Tx_3 . This reduction in input impedance is due to the decrease in the reflected impedance at Tx_2 with the deterioration in coupling coefficient. Therefore, the input impedance of Tx_2 is lower at higher axial distances and it results in a lower frequency separation between HRF and LRF at higher axial distances. Frequency splitting disappears when axial distance is further away, due to insignificant reflected impedance between Tx coils.

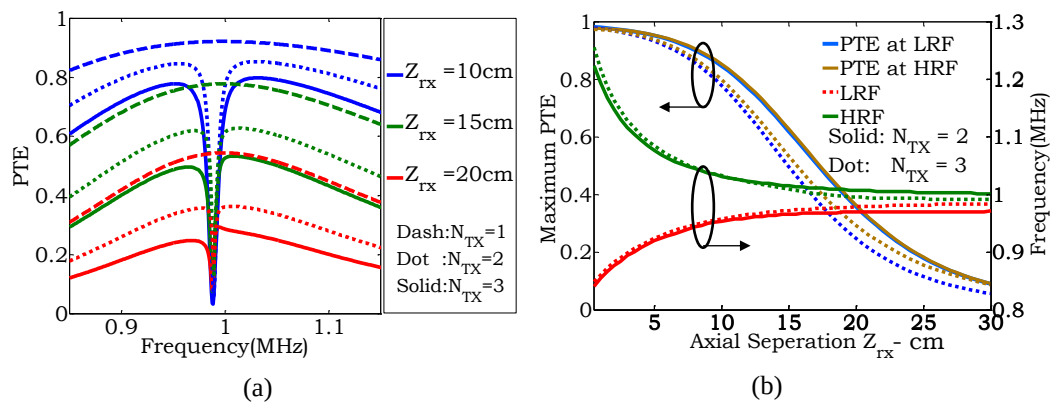


Fig. 4-6. (a) – PTE variation with respect to operating frequency at different axial distances at the aligned position, (b) – Maximum achievable PTE

Fig. 4-7 shows frequency response observed in the experiment. Due to manufacturing tolerances, coil resonances are slightly higher than 0.99MHz as depicted in Table 4-1. Frequency splitting phenomena can be clearly observed in the experimental results as well. Such a frequency characteristic has not been reported in previous research efforts on D-WPT with a moving load. This could be attributed to low frequency (in kHz range) operation where resultant lower reflected impedance is not significant in the realization of splitting phenomena. D-WPT system tested for EV applications in [57] has reported a slight dip in efficiency at self-resonance frequency (Fig 4. of [57]). Therefore, careful frequency selection must be carried out in designing a D-WPT system in high frequency.

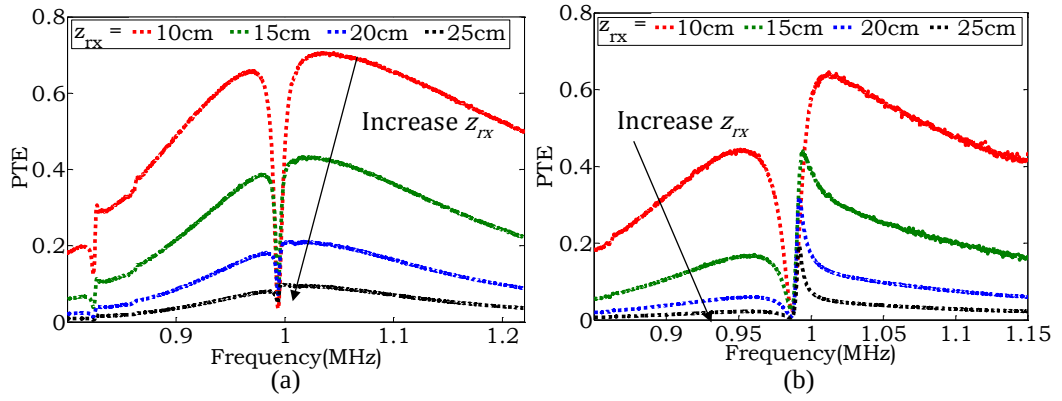


Fig. 4-7. Experimental results: PTE variation with respect to operating frequency at the aligned position.
(a) – 2-Tx setup, (b) – 3-Tx setup.

4.3.3.2 Misalignment tolerance

Misalignment tolerance is crucial in designing D-WPT systems in order to maintain continuous power availability while receiver moves along the Tx array. Fig. 4-8 compares the calculated maximum PTE and experimental results with respect to misalignment when operating at the optimum resonant frequency. Fig. 4-8 shows the maximum PTE at HRF with respect to misalignment and respective resonance frequencies. Experimental results follow theoretical model well with exceptions at low axial distances and low misalignments. This is because the system performance is sensitive to coupling changes at shorter distances (i.e. high coupling). Even with very low axial separation, PTE drops to less than 2% with the maximum misalignment. A similar efficiency deterioration has been reported in previous research efforts [57]. Therefore, designing D-WPT system with classical receiver arrangement is not desirable

as power transfer is not continuous. An Rx coil arrangement to improve the PTE and misalignment tolerance is introduced in the following section.

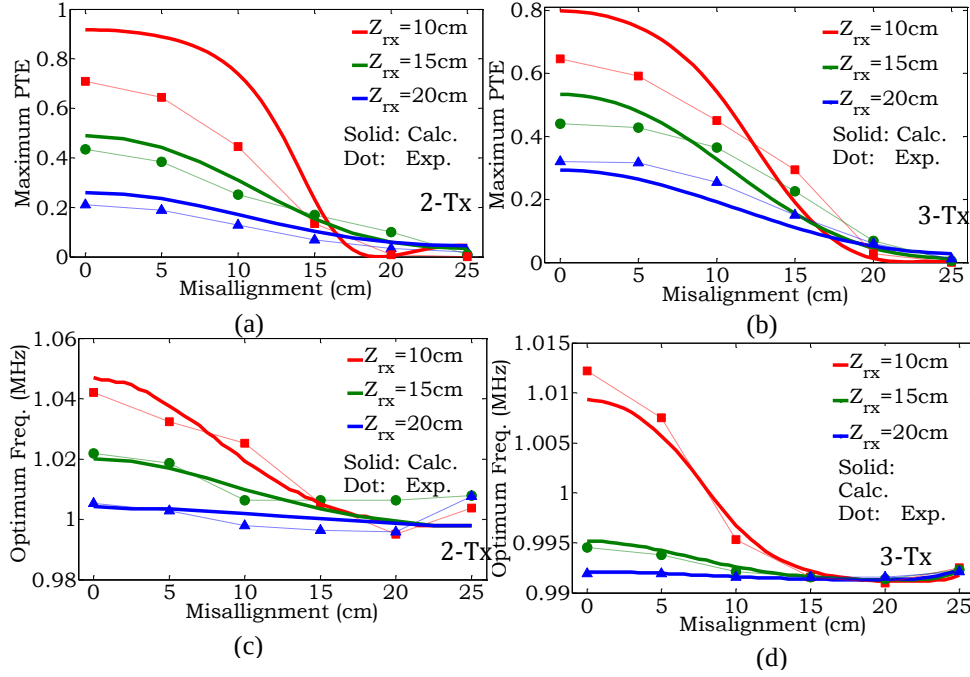


Fig. 4-8. Comparison of calculations and experimental results: (a), (b) – The maximum PTE with respect to misalignment, (c), (d) – Optimum resonance frequencies (HRFs) with respect to misalignment. (a) & (c) – 2-Tx setup, (b) & (d) – 3-Tx setup.

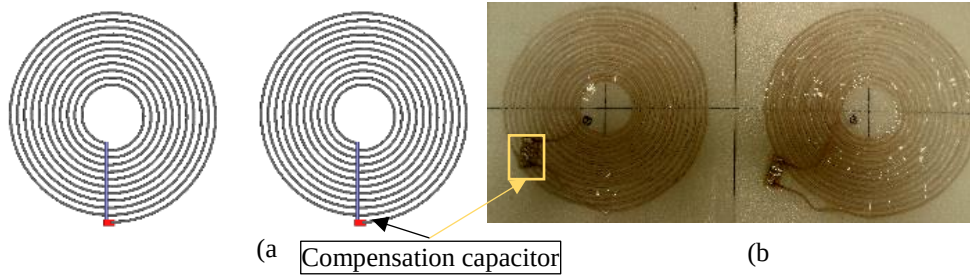


Fig. 4-9. The proposed DSR (a) – Drawing of DSR, (b) – Experimental realization of DSR

4.4 Receiver structure with double spiral repeater

In this section, DSR with two spiral resonators is proposed to enhance the PTE against axial distance and misalignment. DSR is composed of two spiral coils placed side by side as shown in Fig. 4-9. Two spiral coils are not connected to each other. Four resonator arrangement topologies are proposed and compared. Topologies with the proposed DSR are shown in Fig. 4-10. Topology ① is the classical receiver arrangement as shown in Fig. 4-10(a) without any repeater structures, and is used as a reference arrangement to compare the performance of the proposed arrangements. Topology ②

consists of a DSR placed in front of the Rx at the Tx side. DSR placed behind the Rx is considered as topology ③. The topology ④ is a combination of both topologies ② and ③. Axial distance (z_{rx}) refers to unobstructed air gap between the Tx array and the receiver structure (Fig. 4-10).

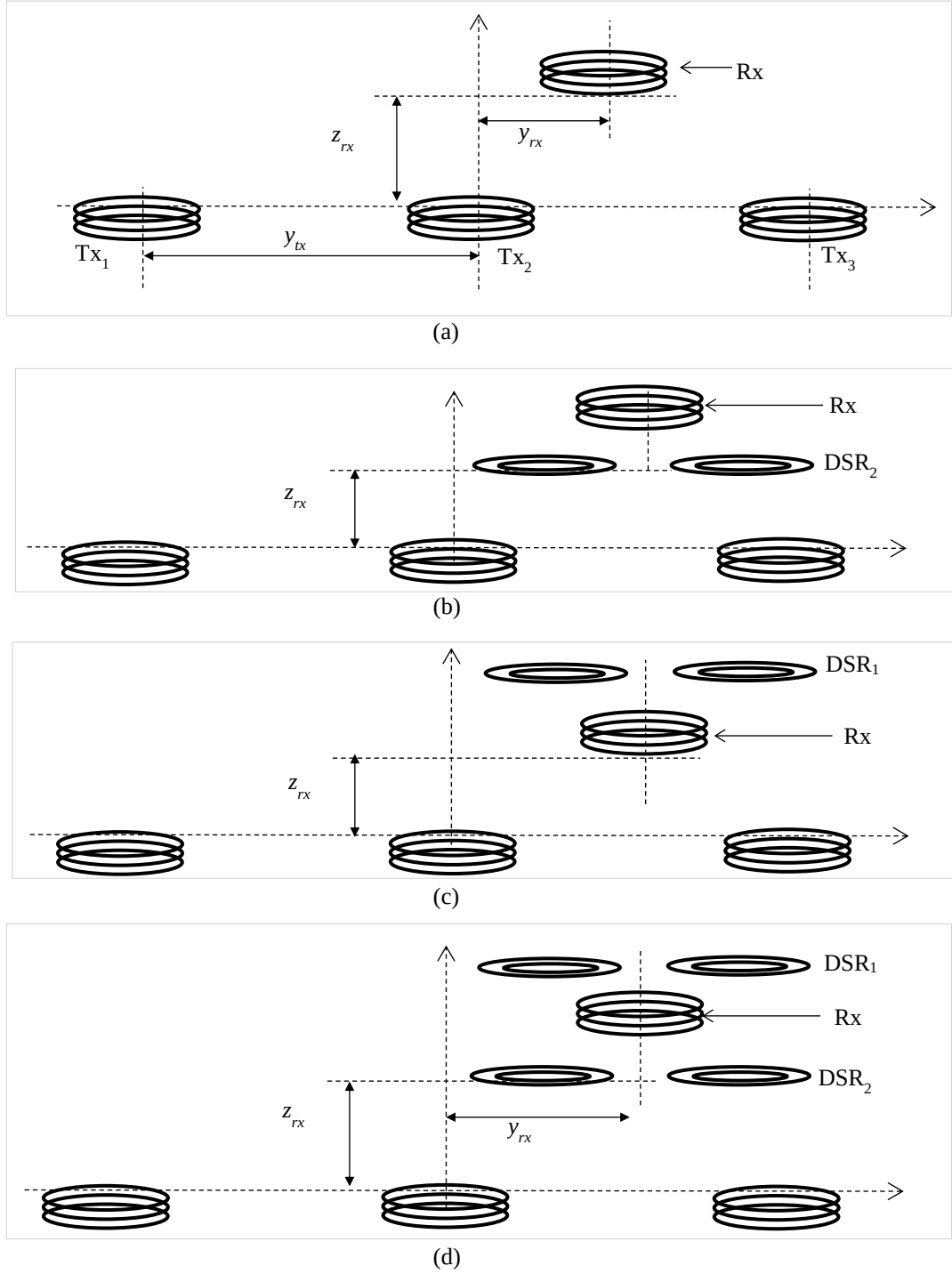


Fig. 4-10. Proposed topologies with DSR. (a) – Topology ①, (b) – Topology ②, (c) – Topology ③ and (d) – Topology ④.

Fig. 4-11 shows the proposed Rx arrangement with DSR for topology ④. The DSR placed behind the Rx coil is named as DSR1 (RxR1 & RxR2) and the DSR placed in front of Rx coil is named as DSR2 (RxR3 & RxR4) (refer Fig. 4-11). DSR1 can be viewed as a magnetic reflector whereas, DSR2 can be viewed as a magnetic lens which congregates magnetic flux towards Rx. Therefore, performance improvements from topologies ② and ③ can be analyzed as lensing and reflecting effects respectively. Topology ④ exhibits both lensing and reflecting effects. Y_{DSR1} (or Y_{DSR2}) is lateral separation between Rx and DSR1 (or DSR2) while Z_{DSR1} (or Z_{DSR2}) is axial separation between Rx and DSR1 (or DSR2) (refer Fig. 4-11).

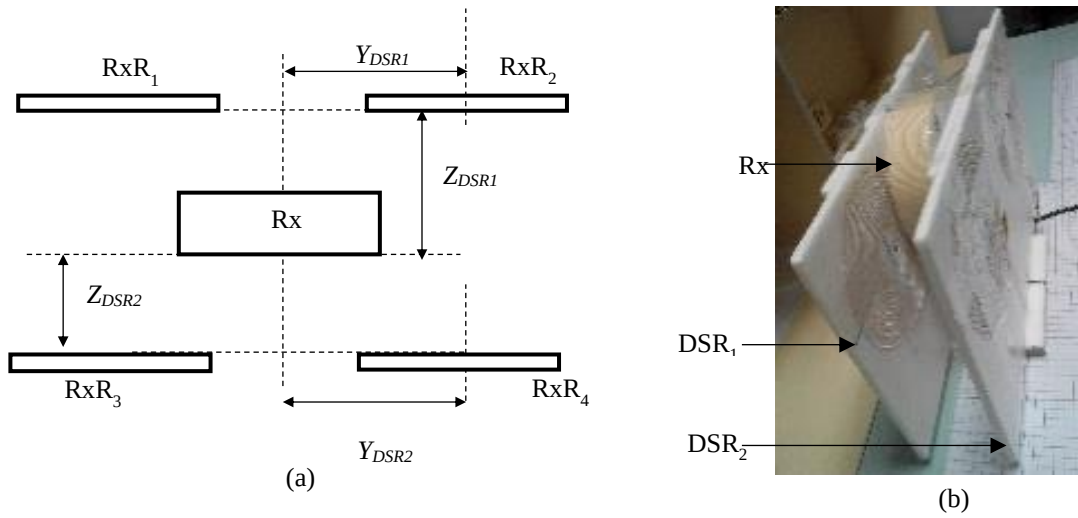


Fig. 4-11. (a) – 2-D view and dimensions of the proposed receiver structure with double spiral repeater, (b) – experimental realization of receiver for topology ④.

4.4.1 Optimization of DSR

In this section, enhanced PTE and misalignment tolerance of the proposed receiver topologies with DSRs are analyzed. The system equation in (4-1) is extended to analyze the proposed topologies. Rank of the impedance matrix and the equation is equal to the number of coils present in each topology. As identified in the previous section, frequency characteristic of the system is dependent on the position of the Rx and the self-resonance of each coil. The optimum resonance changes with coupling between coils and thus, with the position of the Rx and arrangement of DSRs. Matlab[®] optimization toolbox is used to find the optimum design parameters. PTE is implemented as a function of resonance frequency of each coil, operating frequency, dimensions of the DSR and receiver position. PTE is employed as the maximization

objective function in Matlab[®] using interior point algorithm. Optimum design variables can be executed for each receiver position.

A set of switched capacitor banks can be used to tune each coil's self-resonance at a particular position to obtain the maximum PTE. However, it would not be practically feasible due to the complex nature of the D-WPT system. Therefore, a nominal axial distance of 20cm is chosen for the experimental validation. The optimum values for $Y_{DSR1,2}$ and $Z_{DSR1,2}$ aligned position are less than few centimeters in all three topologies. This results in a compact version of DSR. However, misalignment tolerance is poor with low $Y_{DSR1,2}$. The optimum $Y_{DSR1,2}$ for the maximum misalignment (25cm for $y_{tx}=50\text{cm}$) is found to be higher than 10cm. Therefore, $Y_{DSR1,2}$ is chosen to be higher than 9cm in the optimization process.

Table 4-1. Theoretical and experimental optimum self-resonance frequencies (in MHz) at aligned position for 20cm axial separation.

Topology		①	②	③	④
f_{Tx1}	Theoretical	0.99	0.4	0.4	0.4002
	Experimental	0.9925	0.403	0.403	0.403
f_{Tx2}	Theoretical	0.99	0.4	0.4	0.4002
	Experimental	0.9905	0.407	0.407	0.407
f_{Tx3}	Theoretical	0.99	0.4	0.4	0.4002
	Experimental	0.9925	0.407	0.407	0.407
f_{Rx}	Theoretical	0.99	1.079	1.175	1.1104
	Experimental	0.9915	1.071	1.6875	1.1003
f_{RxR1}	Theoretical	NA	NA	1.0871	1.0028
	Experimental			1.0869	1.0013
f_{RxR2}	Theoretical	NA	NA	1.0871	1.0028
	Experimental			1.0875	1.0025
f_{RxR3}	Theoretical	NA	1.0150	NA	1.0084
	Experimental		1.0159		1.0103
f_{RxR4}	Theoretical	NA	1.0150	NA	1.0084
	Experimental		1.0155		1.0035

4.4.2 Efficiency enhancement with DSR

PTE can be improved up to 76% using the proposed topologies whereas PTE is around 35% in the classical arrangement at nominal transfer distance of 20cm. Table

4-1 shows both theoretical and experimentally tuned optimum self-resonance frequencies which produce maximum PTE for each topology. Each coil is separately tuned with high quality factor lumped capacitors. It can be seen from Table 4-1 that the optimum transmitter resonance (f_{Tx}) is much lower than that of Rx (f_{Rx}) and DSRs (f_{RxR1} - f_{RxR4}). Optimum operating frequency (f_s) is slightly higher than DSR resonances and lower than f_{Rx} . Therefore, it is apparent that the reactive impedance of Tx coils farther away limits the currents, thus improving the system efficiency at the optimum operating frequency. The optimum frequency characteristics of all topologies follow similar behaviour in all Rx positions.

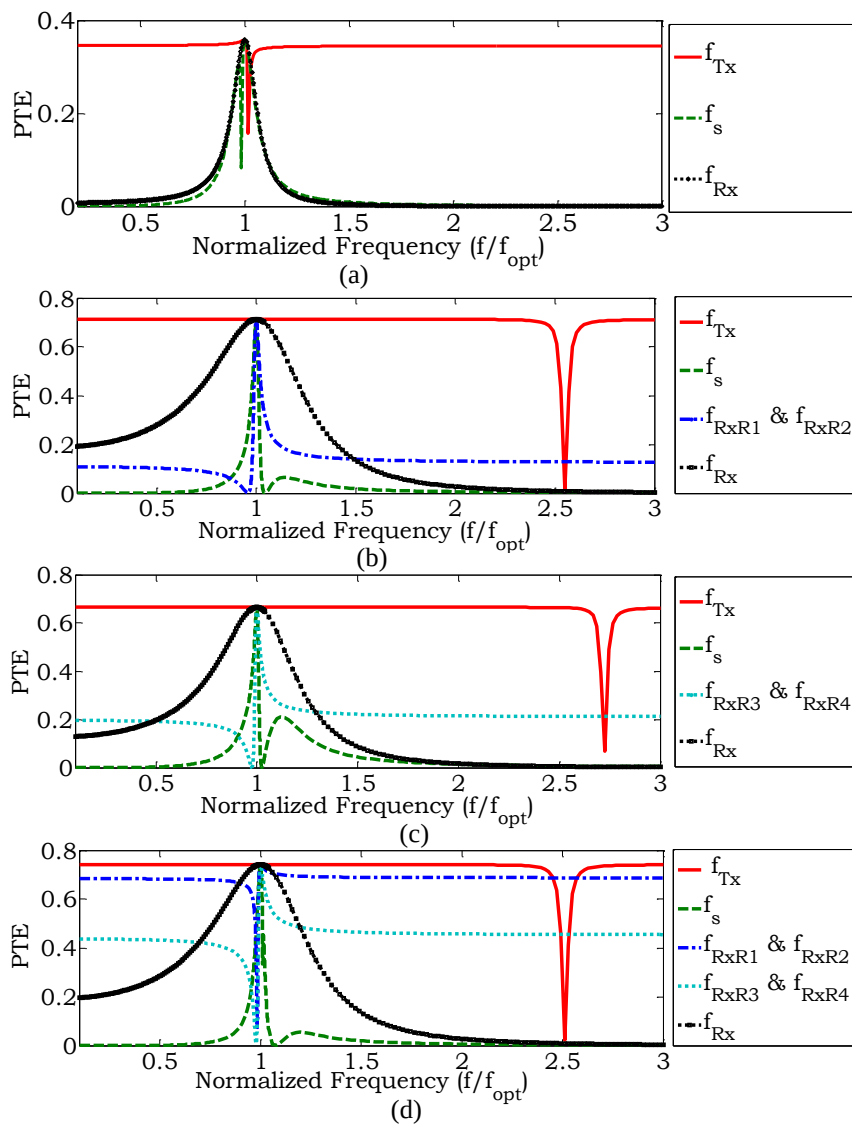


Fig. 4-12. Frequency characteristic of proposed topologies at aligned position at 20cm axial distance.

Each frequency is normalized to its optimum value. (a) – Topology ①, (b) – Topology ②, (c) – Topology ③ and (d) – Topology ④.

Simulation results illustrated in Fig. 4-12 show dependence of PTE on each coil's self-resonance. In this analysis, individually selected frequency is varied while other frequencies are kept at their optimal values. It can be seen that the bandwidth of the operating frequency (f_s) is very narrow in all the topologies. Therefore, the selection of proper operating frequency is very important in designing D-WPT system with the proposed topologies. Conversely, bandwidths of f_{Tx} and f_{Rx} are wider than f_s . A larger bandwidth allows relatively easier tuning during the experiment. Tx coils are connected in parallel with the network analyzer (Agilent E5062A) to experimentally validate the proposed topologies. The experimental setup is illustrated in Fig. 4-13. Double peak phenomena can be observed in the experimental results shown in Fig. 4-14 as well. It should be noted that the optimum experimental operating frequency is slightly different from that of the theoretical analysis. This is because the assumption of identical coils in the analysis is not valid due to the experimental disparities as seen in Table 4-1. However, maximum efficiencies and frequency responses still follow the theoretical analysis.

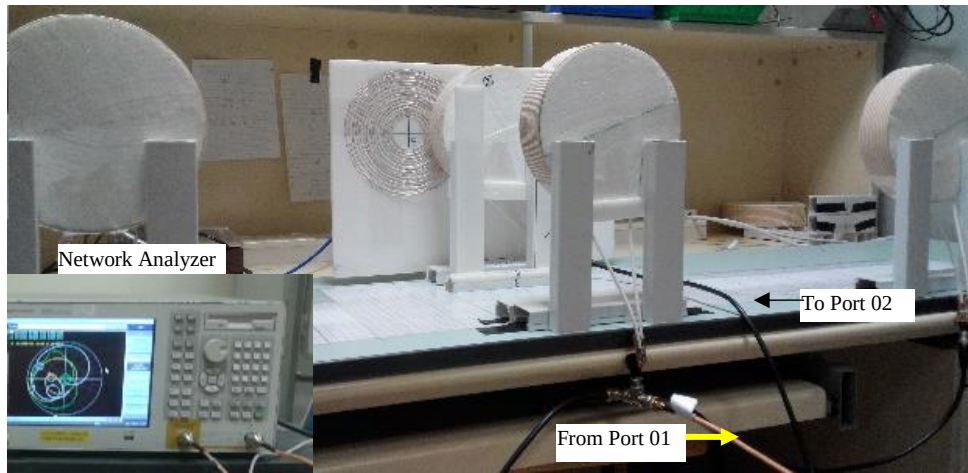


Fig. 4-13. The experimental setup – topology ③

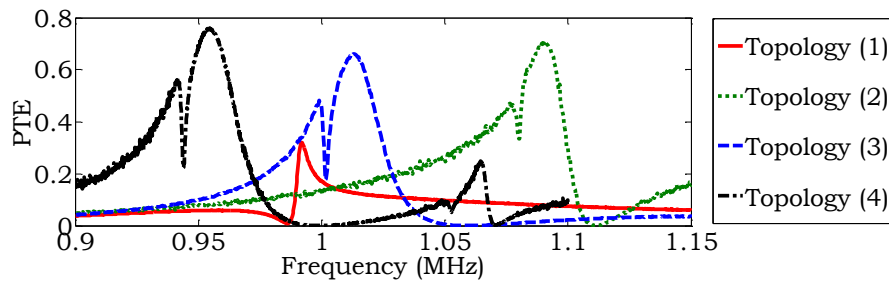


Fig. 4-14. Experimental results: PTE vs. operating frequency at aligned position at 20cm axial separation.

Next, PTEs of the proposed topologies, optimized at 20cm nominal distance are compared with the classical arrangement in terms of transfer distance and misalignment. It can be seen from Fig. 4-15(a) that the proposed topologies show greater efficiency improvement against transfer distance. Topology ④ with two DSRs in front and behind Rx shows 81% and 60% efficiency at 10cm and 35cm, respectively. It should be noted that the Rx size of the proposed topologies are larger than that of the classical topology. The performance improvement is further compared using the concept of normalized distance (z_{norm}) introduced in Chapter 3, to incorporate the size difference.

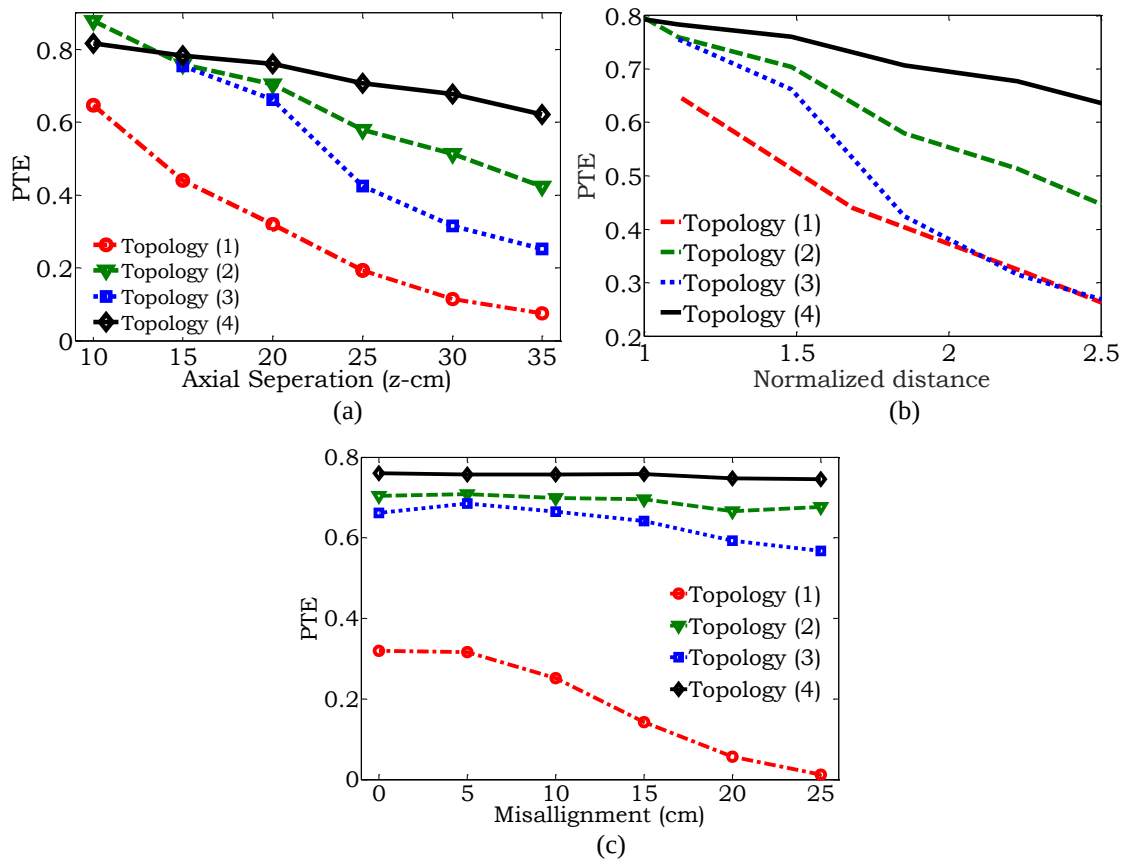


Fig. 4-15. Experimental results: (a) – PTE with respect to axial separation at the aligned position, (b) – PTE variation with respect to normalized distance, (c) – PTE with respect to misalignment when optimized to aligned position at $z_{rx}=20$ cm.

Fig. 4-15 (b) compares experimental results with respect to the normalized distance. It can be seen that, proposed topologies allow higher PTE with respect to normalized distance except for the topology ③. PTE profile for Topology ③ is similar to the classical topology when $z_{norm} > 1.7$. Apart from Topology ③ at high normalized distances, all other cases with proposed topologies show higher PTE than classical topology. For example, when distances are normalized to Tx and Rx sizes, 20cm

nominal distance for proposed topologies is equivalent to 13.2cm for classical topology (respective z_{norm} is around 1.5). PTEs obtained for normalized distance of 1.5 in topologies ①, ②, ③ and ④ are 51%, 70%, 66% and 76% respectively.

Fig. 4-15(c) illustrates the variation of experimental PTE with respect to misalignment at 20cm axial distance. All three topologies with DSR show significant efficiency improvement against misalignment. PTE of Topology ④ drops only 1.5% from 76% with the maximum misalignment even with fixed frequency operation. Maximum and minimum PTE pairs for topologies ①, ② and ③ against misalignments are 32%-1.2%, 70%-66.7% and 68.5%-56.7% respectively. It is apparent that PTE improvement that can be obtained by the lensing effect (Topology ②) is higher than that from the reflecting effect (Topology ③). The highest PTE improvement results in Topology ④ can be viewed as combination of both lensing and reflecting effects. In addition to y-direction misalignment which is unavoidable in D-WPT system, lateral displacement along x-direction is also examined. The numerical results in Fig. 4-16 compare performance variation against lateral displacement at nominal transfer distance of 20cm ($z_{norm} \sim 1.5$ for topologies ②-④). It can be seen that the proposed topology allows higher misalignment tolerance in lateral displacement compared to the classical topology. Nevertheless, PTE declines in a faster rate against lateral displacement in all topologies compared to y-direction misalignment. It should be noted that, the lateral displacement along x-direction can be minimized using techniques such as tracking and autonomous driving.

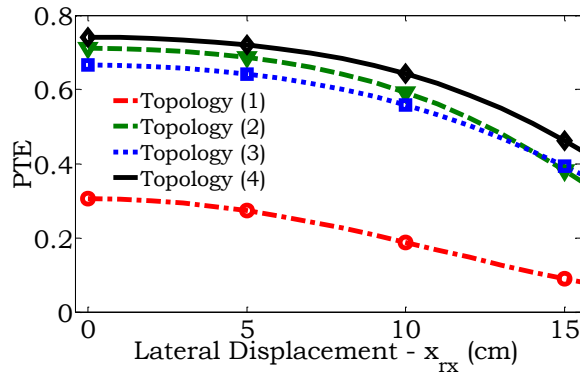


Fig. 4-16. PTE variation with respect to lateral displacement (x-direction) for $z_{rx}=20$ cm.

Therefore, it is apparent that misalignment problem of the D-WPT technology can be solved to a greater extent by using the proposed topologies with DSR. The system

resonance frequency changes with the position of the Rx. Frequency tuning has been understood as a promising approach to provide enhanced misalignment tolerance. Fig. 4-17 illustrates the maximum achievable PTEs with variable frequency operation. It can be seen from the experimental results in Fig. 4-17(b) that the efficiency improvements in topologies ②, ③ and ④ with the variable frequency operation are around 4%, 1% and 2% respectively. The cost effectiveness of frequency tuning has to be decided based on the application as the communication, control and location needed for tracking makes the design more complex. Other than the operating frequency tuning, resonance of the DSR resonators can also be tuned using switched capacitor bank to improve the PTE.

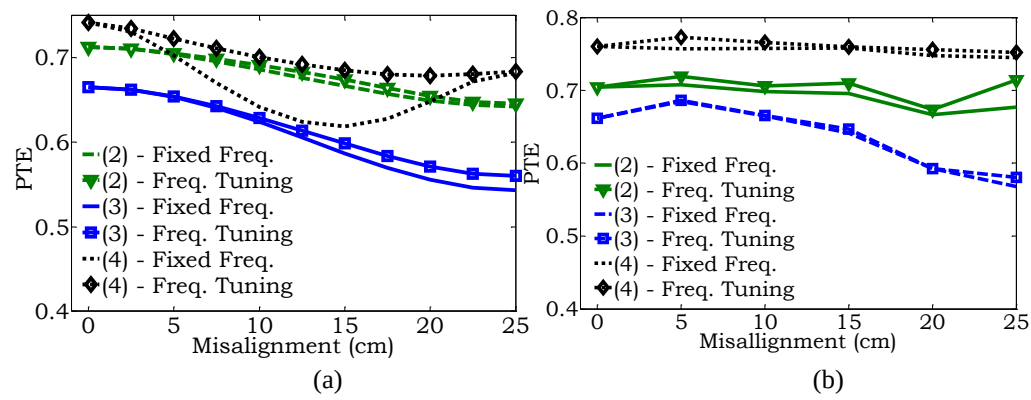


Fig. 4-17. PTE comparison, operating frequency tuning vs. fixed frequency operation. (a) – Theoretical analysis, (b) - Experimental results

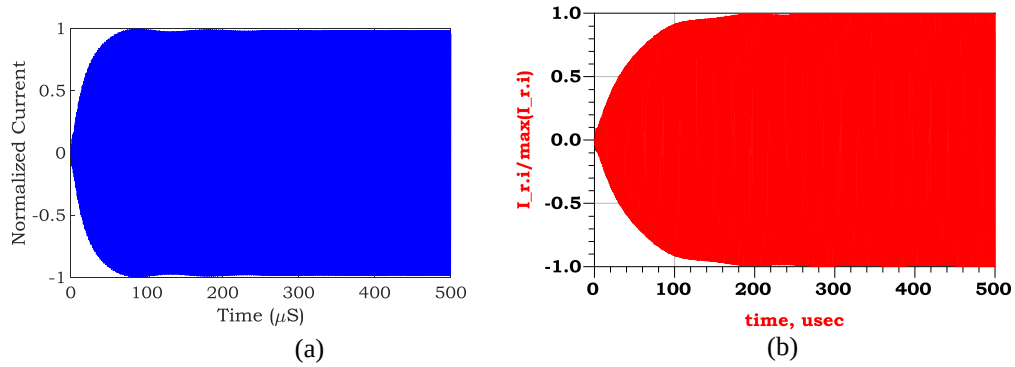


Fig. 4-18. Normalized output current - transient response for topology ④. (a) – Simulation results, (b) – Experimental results

4.4.3 Transient response and load variation

The transient response of the 2-coil WPT system is characterized with an exponential decay constant of ω/Q [103]. A similar behaviour is also observed with the proposed topologies. For example, Fig. 4-18 shows simulation and experimental results of the transient response for normalized receiver current in topology ④. (The simulation is

performed in PLECS using multi-terminal mutual inductance model, while the measured S-parameters are used to generate experimental transient response along with Agilent ADS tool [99]) It can be seen that steady state condition is reached before 200 μ s time. Therefore, when EV charging application is considered, an adaptive tuning technique can be implemented for an EV moving in highway speed.

Next, the load variation characteristics is analyzed in proposed topologies optimized to nominal load of 50 Ω . Numerical results in Fig. 4-19(a) show that proposed topologies allow higher PTE in a wider load variations compared to the classical topology. Theoretical results are verified with experiments using S-parameter based technique [99] as shown in Fig. 4-19(b).

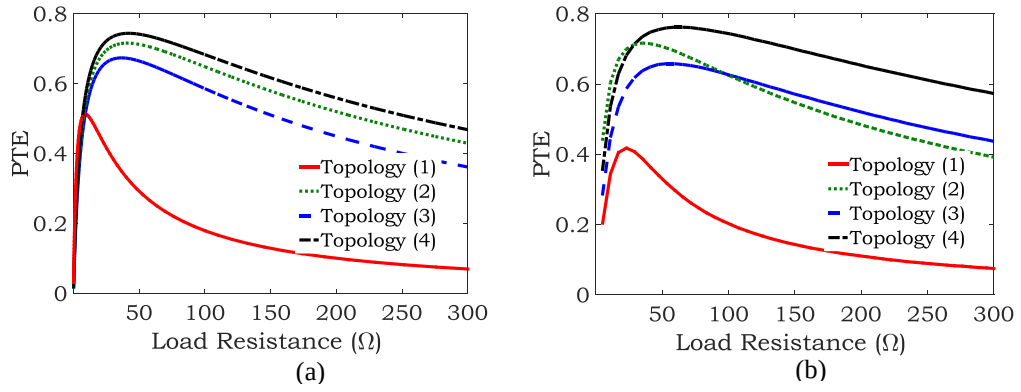


Fig. 4-19. PTE variation against load resistance. (a) – Theoretical analysis, (b) - Experimental results ($z_{rx}=20\text{cm}$, $y_{rx}=0$)

4.5 Summary

This chapter presented numerical analysis on the design of dynamic wireless power transfer (D-WPT) system with segmented transmitter (Tx) array. Efficiency characteristics against design parameters such as, operating frequency, load resistance, number of simultaneously powered Tx coils (n) and Tx coil spacing were presented. A frequency splitting phenomena was observed in efficiency characteristic for $n>1$. In addition, a double spiral repeater (DSR) for dynamic wireless power transfer was introduced. The DSR placed at the receiver coil was found to be a promising solution to obtain enhanced PTE with higher transfer distance and misalignment tolerance. The performance deterioration effect that occurs when the load is moving away from a source resonator can be compensated by congregating magnetic flux towards the load. Efficiency enhancement due to lensing DSR placed in front of the Rx was found to be

higher than that of the reflecting DSR placed behind Rx. Quasi static condition was analyzed using equivalent circuit model and validated with the experimental results. Furthermore, proposed topologies showed high efficiency in a wide range of loads. It was also found that the optimal operating frequency is not identical to the self-resonance of the individual resonators, and frequency splitting phenomenon was investigated in all resonator arrangements.

Chapter 5

Optimum Transmitter Current Distribution for Dynamic Wireless Power Transfer (D- WPT) with segmented Array

5.1 Introduction

By using Tx arrays with voltage sources, the D-WPT system performances degrade because the Tx currents cannot be controlled with Rx misalignment, as investigated in Chapter 4. Tx coils farther away from the Rx coil experience higher currents as compared to Tx coils nearer to Rx coil. This is due to lower reflected impedance induced in the Tx coils that are farther away from Rx. Consequently, this causes lower transferred power and higher losses, thus leading to efficiency degradation. In order to address this issue, an compensation network along with an auxiliary current controlled circuitry was proposed in [104] and a reflexive field containment method was proposed in [100]. These approaches regulate the Tx coil (that are farther from the Rx coil) currents by increasing the impedances using compensation networks. However, theoretical optimum Tx currents were not considered in these approaches as D-WPT systems experience varying coupling levels between its moving Rx and respective Tx coils. On the other hand, optimum currents are numerically evaluated for a case study in Chapter 4 and a variable operating frequency approach is proposed to increase the input impedance of Tx coils that are farther away. However, the investigation in Chapter 4 did not provide generalized analytical solution for the optimum current distribution.

This chapter investigates an analytical solution for the theoretical optimum Tx currents using a current modulation scheme based on the analogy of N-dimensional Cartesian coordinates equivalent of spherical coordinates. The inspiration of this current modulation scheme comes from [105] where a 3-dimensional current modulation scheme is used to achieve omnidirectional WPT. This is achieved by arranging three Tx coils orthogonal to each other. Therefore, their theoretical analysis is limited to three Tx coils setup. In addition, frequency and load characteristics are not explored. Instead of

using orthogonal coils used in [105], this study proposes current modulation scheme for arbitrary number of Tx coils in D-WPT system. Taking it further, this study defines optimization goals and proposes an optimization for the design parameters such as operating frequency, load resistance and number of simultaneously powered Tx coils.

This chapter is organized as follows. Section 5.2 introduces the principle of current modulation scheme which employs the analogy of N-dimensional Cartesian coordinate equivalent of spherical coordinates. Section 5.3 discusses the optimization of design parameters. The theoretical analysis is verified by the experimental results in Section 5.4 which is followed by the summary.

5.2 Current modulated D-WPT

The typical arrangement of segmented D-WPT system, similar to the classical topology described in Chapter 4 is shown in Fig. 4-2. The Tx coils are spaced uniformly apart (i.e. y_{tx}), and the Rx moves along the Tx array while having an axial separation (i.e. z_{rx}) from the Tx array. Radial separation, y_{rx} is changed dynamically while the Rx moves along the Tx array. In an n -Tx coil array, there are n number of simultaneously powered Tx coils. When the Rx moves away from a particular segment, the farthest active Tx coil will be disconnected from source and upcoming inactive Tx coil will be activated. It is important to note that the origin ($y_{rx}=0$) is defined as the Rx aligned position to the middle Tx in a segment. For example, the reference position will be aligned to the Tx_1 in a 2-Tx coil array setup, whereas it will be aligned to Tx_2 in a 3-Tx coil array setup. On the other hand, performance within a segment is symmetric from its middle, and length of a segment spans y_{tx} . Therefore, performance evaluation in this study is performed for displacement (y_{rx}) ranging from 0 to $y_{tx}/2$.

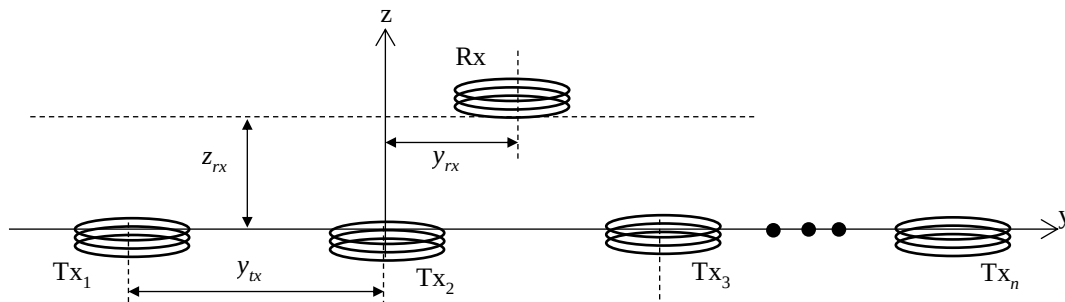


Fig. 5-1. Segmented D-WPT system with n number of transmitter coils

5.2.1 Equivalent circuit

The equivalent circuit for a segmented D-WPT system is shown in Fig. 5-2. The number of simultaneously powered Tx coils are denoted as n . The Tx coils are modelled to be powered by current sources, and then system equation is characterized in (5-1).

$$[Z]_{(n+1) \times (n+1)} [I]_{(n+1) \times 1} = [V]_{(n+1) \times 1}$$

$$\text{where } [Z_{kl}] = \begin{cases} Z_k = R_k + jX_k, & (k = l) \\ Z_{n+1} = R_r + R_L + jX_r & (k, l = n + 1) \\ X_{kl} = X_{lk} = j\omega M_{k,l} = j\omega M_{l,k}, & (k \neq l) \end{cases}$$

$$X_k = \omega L_k - 1/\omega C_k \quad (5-1)$$

$$[I]_{(n+1) \times 1} = [I_1 \ I_2 \ \dots \ I_n \ I_r]^T$$

$$[V]_{(n+1) \times 1} = [V_1 \ V_2 \ \dots \ V_n \ 0]^T$$

Diagonal elements of the impedance matrix, Z_k ($k=1,2,\dots, n$ and r) are the self-impedances of transmitters and receiver. L_k , C_k and R_k represent inductance, capacitance and parasitic resistance of the coil. Subscripts 1, 2 ... n represent the respective Tx coil and subscript r represents the Rx coil. $M_{k,l}$ ($=M_{l,k}$) is mutual inductance between k^{th} coil and l^{th} coil. ω is the operating angular frequency of the source. RMS voltages across Tx coils are defined as V_k ($k=1,2,\dots, n$), and RMS currents in each coil is I_k ($k=1,2,\dots, n$ and r). After defining the system equation, output power and PTE can be calculated as in (5-2). Input power can be seen in the PTE equation to be the accumulation of the losses in the coils and the output power.

$$P_{out} = |I_r|^2 R_L$$

$$PTE = \frac{P_{out}}{P_{in}} = \frac{|I_r|^2 R_L}{\sum (|I_k|^2 R_k) + |I_r|^2 (R_r + R_L)} \quad (5-2)$$

5.2.2 Current modulation scheme

Currents in Tx coils are modelled using the analogy of N-dimensional Cartesian coordinate equivalent of spherical coordinates as shown in (5-3). The current

modulation angles $(\theta_1, \theta_2 \dots \theta_{n-1})$ determine the magnitude of each Tx current. Accordingly, square summation of each RMS currents is equal to square of the nominal current (I^2).

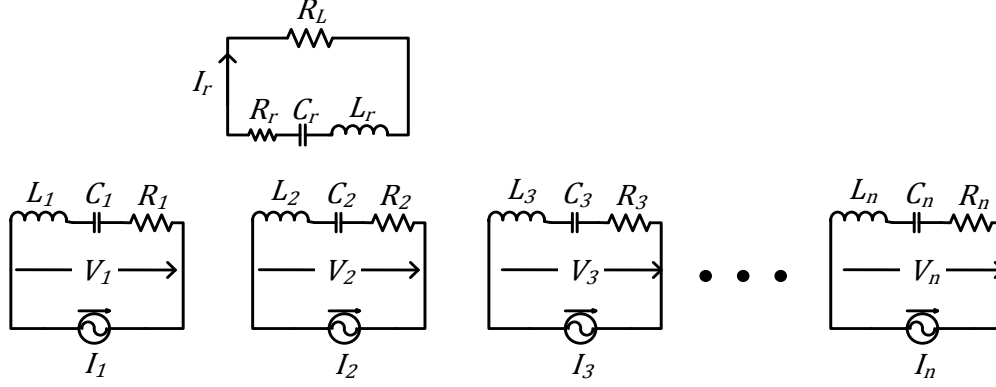


Fig. 5-2. The equivalent circuit of a segmented D-WPT system with n number of transmitter coils

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ \vdots \\ I_{n-1} \\ I_n \end{bmatrix} = \begin{bmatrix} \cos(\theta_1) \\ \sin(\theta_1)\cos(\theta_2) \\ \sin(\theta_1)\sin(\theta_2)\cos(\theta_3) \\ \vdots \\ \sin(\theta_1) \dots \sin(\theta_{n-2})\cos(\theta_{n-1}) \\ \sin(\theta_1) \dots \sin(\theta_{n-2})\sin(\theta_{n-1}) \end{bmatrix} I \quad (5-3)$$

Using (5-1) and (5-3), the Rx current can be written as in (5-4). After which, the output power defined in (5-5) can be simplified as in (5-6) by using trigonometric manipulations (derivation of (5-6) is provided in Appendix). Output power is a function of transfer impedance ratio, K_n (defined in (5-6)), and load resistance. The transfer impedance ratio, K_n is defined as the ratio between induced impedances (comprise of mutual inductance between each Tx and Rx and current modulation angles) and Rx loop impedance. Therefore, the value of K_n is a function of current modulation angles.

$$I_r = \frac{-j\omega I}{R_L + R_r + jX_r} (M_{1,r}\cos(\theta_1) + \dots + M_{n,r}\sin(\theta_1) \dots \sin(\theta_{n-2})\sin(\theta_{n-1})) \quad (5-4)$$

$$P_{out} = |I_r|^2 R_L = \frac{\omega^2 I^2 R_L}{(R_r + R_L)^2 + X_r^2} [M_{1,r}\cos(\theta_1) + \dots + M_{n,r}\sin(\theta_1) \dots \sin(\theta_{n-2})\sin(\theta_{n-1})]^2 \quad (5-5)$$

$$P_{out} = I^2 K_n R_L \quad (5-6)$$

where $K_n = \frac{\omega^2}{(R_r + R_L)^2 + X_r^2} \times \left\{ \left[\left((M_{n,r}^2 + M_{n-1,r}^2) \sin^2(\theta_{n-1} + \alpha_{n-1}) + M_{n-2,r}^2 \right) \sin^2(\theta_{n-2} + \alpha_{n-2}) + M_{n-3,r}^2 \right] \dots + M_{2,r}^2 \right] \sin^2(\theta_2 + \alpha_2) + M_{1,r}^2 \} \sin^2(\theta_1 + \alpha_1)$,

$$\alpha_{n-1} = \tan^{-1} \left\{ \frac{M_{n-1,r}}{M_{n,r}} \right\},$$

$$\alpha_{n-2} = \tan^{-1} \left\{ \frac{M_{n-2,r}}{\sqrt{M_{n-1,r}^2 + M_{n,r}^2} \sin(\theta_{n-1} + \alpha_{n-1})} \right\},$$

$$\alpha_{n-3} = \tan^{-1} \left\{ \frac{M_{n-3,r}}{\sqrt{[(M_{n-1,r}^2 + M_{n,r}^2) \sin^2(\theta_{n-1} + \alpha_{n-1}) + M_{n-2,r}^2] \sin(\theta_{n-2} + \alpha_{n-2})}} \right\},$$

$$\alpha_1 =$$

$$\tan^{-1} \left\{ \frac{M_{1r}}{\sqrt{[(M_{n-1,r}^2 + M_{n,r}^2) \sin^2(\theta_{n-1} + \alpha_{n-1}) + M_{n-2,r}^2] \sin^2(\theta_{n-2} + \alpha_{n-2}) + M_{n-3,r}^2 \dots + M_{2,r}^2} \sin(\theta_2 + \alpha_2)}} \right\}$$

$$P_{in} = I^2 R_1 \cos^2(\theta_1) + I^2 R_2 \sin^2(\theta_1) \cos^2(\theta_2) + \dots + I^2 R_n \sin^2(\theta_1) \dots \sin^2(\theta_{n-2}) \sin^2(\theta_{n-1}) + I_r^2 (R_r + R_L) \quad (5-7)$$

Input power can be calculated as in (5-7) using the approach in (5-2). Moreover, with the identical Tx coil assumption, (5-7) can be simplified into (5-8). It is important to note that the total loss contribution from the Tx coils is not dependent on Rx coil position and limited to $I^2 R$. PTE can be calculated as in (5-9) using (5-2). The normalized output power, NOP as defined by normalizing the output power to the square of nominal current (I^2) as shown in (5-10). It is clear to see that both PTE and NOP are functions of current modulation angles, $\theta_1, \theta_2 \dots \theta_{n-1}$. Most importantly, both matrices become optimum when K_n maximizes. The value of K_n is optimized at the maximum induced impedance components whereby sinusoidal terms become unity according to (5-11). These optimum current modulation angles are functions of mutual inductances between each

Tx coil and the Rx coil. For an arbitrary Rx position, optimum Tx current distribution can be achieved by applying (5-11) into (5-3). Noteworthy, this defines the theoretical optimum current distribution for a WPT with multiple Tx coils. After achieving optimum Tx current distribution, the performance metrics PTE and NOP are derived in (5-12). Next, to apply the optimized current modulation scheme, 2- and 3-Tx coils setup are chosen in the following subsections.

$$\text{If Tx coils are assumed to be identical, } R_1 = R_2 \dots R_n = R_t \quad (5-8)$$

$$P_{in} = I^2 R_t + I_r^2 (R_r + R_L)$$

$$PTE = \frac{P_{out}}{P_{in}} = \frac{R_L}{\frac{R_t}{K_n} + (R_r + R_L)} \quad (5-9)$$

$$NOP = \frac{P_{out}}{I^2} = R_L K_n \quad (5-10)$$

$$\sin^2(\theta_1 + \alpha_1) = 1 \Rightarrow \theta_{1-opt} = \tan^{-1} \left(\frac{\sqrt{\sum_{k=2}^n M_{kr}^2}}{M_{1r}} \right) \quad (5-11)$$

$$\sin^2(\theta_2 + \alpha_2) = 1 \Rightarrow \theta_{2-opt} = \tan^{-1} \left(\frac{\sqrt{\sum_{k=3}^n M_{kr}^2}}{M_{2r}} \right)$$

$$\sin^2(\theta_{n-1} + \alpha_{n-1}) = 1 \Rightarrow \theta_{n-1-opt} = \tan^{-1} \left(\frac{M_{nr}}{M_{n-1r}} \right)$$

$$NOP = R_L K_{n-opt} \quad (5-12)$$

$$PTE = \frac{R_L}{\frac{R_t}{K_{n-opt}} + (R_r + R_L)}$$

$$\text{where, } K_{n-opt} = \frac{\omega^2 M_{sum}^2}{(R_r + R_L)^2 + X_r^2} \text{ and } M_{sum}^2 = \sum_{k=1}^n M_{kr}^2$$

2-Tx D-WPT setup

2-Tx coil setup is the minimum requirement for a D-WPT scheme. The Tx current magnitudes can be defined as in (5-13), and the current modulation angle, θ_1 determines the percentage currents in each Tx coil. By applying similar principle derived in n-Tx coil setup, output power, input power, PTE and NOP are defined in (5-13)–(5-16) respectively.

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \cos(\theta_1) \\ \sin(\theta_1) \end{bmatrix} I \quad (5-13)$$

$$P_{out} = |I_r|^2 R_L = I^2 K_2 R_L \quad (5-14)$$

where $\alpha_1 = \tan^{-1} \left(\frac{M_{1r}}{M_{2r}} \right)$ and $K_2 = \frac{\omega^2 (M_{1r}^2 + M_{2r}^2)}{(R_r + R_L)^2 + X_r^2} \sin^2(\theta_1 + \alpha_1)$

$$P_{in} = I^2 R_1 \cos^2(\theta_1) + I^2 R_2 \sin^2(\theta_1) + I_r^2 (R_r + R_L) \quad (5-15)$$

If Tx coils are assumed to be identical, $R_1 = R_2 = R$:

$$P_{in} = I^2 R + I_r^2 (R_r + R_L)$$

$$PTE = \frac{P_{out}}{P_{in}} = \frac{R_L}{\frac{R_t}{K_2} + (R_r + R_L)} \quad (5-16)$$

$$NOP = \frac{P_{out}}{I^2} = R_L K_2$$

The optimum current modulation angle which maximize K_2 can be obtained from (5-17) and respective optimized metrics are derived in (5-18).

$$\sin^2(\alpha_1 + \theta_1) = 1 \Rightarrow \theta_{1-opt} = \tan^{-1} \left(\frac{M_{2r}}{M_{1r}} \right) \quad (5-17)$$

$$P_{out} = I^2 R_L K_{2-opt} \quad (5-18)$$

$$PTE = \frac{R_L}{\frac{R_t}{K_{2-opt}} + (R_r + R_L)}$$

where $K_{2-opt} = \frac{\omega^2 (M_{1r}^2 + M_{2r}^2)}{(R_r + R_L)^2 + X_r^2}$

In order to validate the theoretical analysis, numerical analysis is performed with same helical shaped Tx and Rx coils used in Chapter 4, and with the parameters depicted in Table 5-1. Numerical results in Fig. 5-3 show PTE and NOP variations with respect to the current modulation angle (θ_1) at various Rx positions which corresponds to different mutual inductance (i.e. M_{1r} and M_{2r}). The polar form of PTE and NOP are illustrated in Fig. 5-4 where the magnitude of each position vector corresponds to the respective performance index. This dynamic behavior yields a corresponding optimum angle for θ_1 (illustrated in straight lines) which is in line with (5-17).

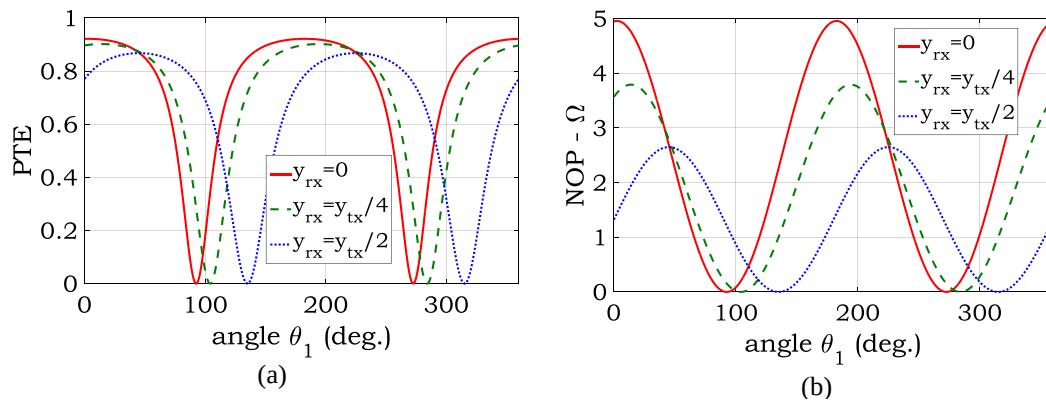


Fig. 5-3. Performance variation with respect to current modulation angle in Cartesian form ($z_{rx}=100\text{mm}$, $R_L=50\Omega$), (a) – PTE, (b) – NOP.

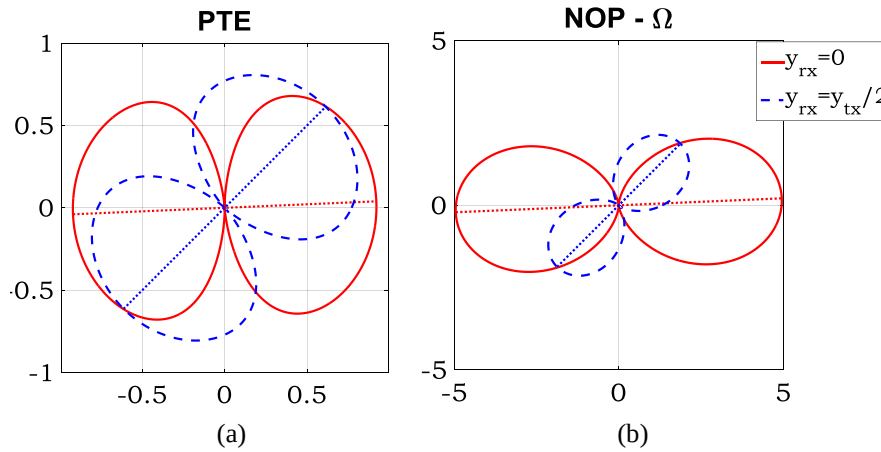


Fig. 5-4. Performance variation with respect to current modulation angle in polar form ($z_{rx}=100\text{mm}$, $R_L=50\Omega$), (a) – PTE, (b) – NOP.

Fig. 5-5 shows mutual inductance and the optimum Tx current variations with respect to the Rx position. When Rx is at reference position (Rx is aligned to Tx₁), mutual inductance M_{1r} is higher than M_{2r} , resulting in higher optimum current in Tx₁ than Tx₂. On the other hand, when $y_{rx} = y_{tx}/2$ (equi-radial displacement between Tx₁ and Tx₂), mutual inductances, M_{1r} and M_{2r} become identical, and that results in equal

optimum current magnitudes ($\theta_1=45^\circ$). At reference position ($y_{rx}=0$), power transfer contribution from Tx₁ is higher than Tx₂ whereas, at $y_{rx}=y_{tx}/2$ position, power transfer contributions from Tx₁ and Tx₂ are uniform. Therefore, it is evident that the proposed current modulation scheme allows optimum power transfer contributions.

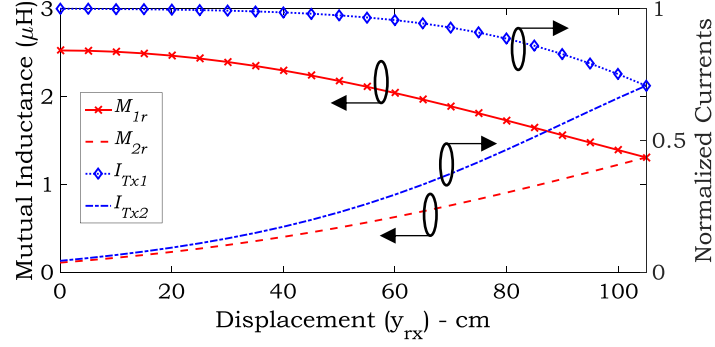


Fig. 5-5. Mutual inductance variation and normalized optimal Tx currents ($z_{rx}=100\text{mm}$).

Table 5-1. Design parameters of coils.

Parameter	Value
Outer radius	89mm
Turns separation	4.2mm
Number of turns	11
Tx separation (y_{tx})	210mm
Inductance Calc.(Exp.)	30μH(~32μH)

3-Tx D-WPT setup

3-Tx coil setup is similar to the 2-Tx coil setup studied in previous subsection. The Tx currents, output power, input power, PTE and NOP for 3-Tx setup are defined in (5-19)–(5-21) respectively. In order to modulate the currents in the 3-Tx coil setup, a pair of current angles are required. (Note that the θ_1 in the 2-Tx setup is not the same with this setup)

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} \cos(\theta_1) \\ \sin(\theta_1)\cos(\theta_2) \\ \sin(\theta_1)\sin(\theta_2) \end{bmatrix} I \quad (5-19)$$

$$I_r = \frac{\omega I}{(R_r + R_L) + jX_r} (M_{1r}\cos(\theta_1) + M_{2r}\sin(\theta_1)\cos(\theta_2) + M_{3r}\sin(\theta_1)\sin(\theta_2))$$

$$P_{out} = |I_r|^2 R_L = \frac{\omega^2 I^2 R_L}{(R_r + R_L)^2 + X_r^2} (M_{1r}\cos(\theta_1) + M_{2r}\sin(\theta_1)\cos(\theta_2) + M_{3r}\sin(\theta_1)\sin(\theta_2))^2$$

$$P_{out} = I^2 K_3 R_L$$

$$\text{where } K_3 = \frac{\omega^2}{(R_r + R_L)^2 + X_r^2} [(M_{2r}^2 + M_{3r}^2) \sin^2(\theta_2 + \alpha_2) + M_{1r}^2] \sin^2(\theta_1 + \alpha_1)$$

$$\alpha_1 = \tan^{-1} \left\{ \frac{M_{1r}}{\sqrt{(M_{2r}^2 + M_{3r}^2)} \sin(\theta_2 + \alpha_2)} \right\}$$

$$\alpha_2 = \tan^{-1} \left\{ \frac{M_{2r}}{M_{3r}} \right\}$$

$$P_{in} = I^2 R_1 \cos^2(\theta_1) + I^2 R_2 \sin^2(\theta_1) \cos^2(\theta_2) + I^2 R_3 \sin^2(\theta_1) \sin^2(\theta_2) + I_r^2 (R_r + R_r) \quad (5-20)$$

If identical Tx coils $R_1 = R_2 = R_3 = R$:

$$P_{in} = I^2 R + I_r^2 (R_r + R_r)$$

$$PTE = \frac{P_{out}}{P_{in}} = \frac{R_L}{\frac{R}{K_3} + (R_r + R_L)} \quad (5-21)$$

NOP can be defined as,

$$NOP = \frac{P_{out}}{I^2} = R_L K_3$$

The optimum current modulation angles, θ_1 and θ_2 can be easily obtained from (5-22) and its optimized metrics are derived in (5-23).

$$\sin^2(\theta_1 + \alpha_1) = 1 \Rightarrow \theta_1 = \tan^{-1} \left(\frac{\sqrt{M_{2r}^2 + M_{3r}^2}}{M_{1r}} \right) \quad (5-22)$$

$$\sin^2(\theta_2 + \alpha_2) = 1 \Rightarrow \theta_2 = \tan^{-1} \left(\frac{M_{3r}}{M_{2r}} \right)$$

$$P_{out} = I^2 R_L K_{3-opt} \quad (5-23)$$

$$PTE = \frac{R_L}{\frac{R}{K_{3-opt}} + (R_r + R_L)}$$

$$K_{3-opt} = \frac{\omega^2 (M_{1r}^2 + M_{2r}^2 + M_{3r}^2)}{(R_r + R_L)^2 + X_r^2}$$

Similar to the 2-Tx coil setup, numerical analysis is performed to substantiate the theoretical analysis. Numerical results in Fig. 5-6 show PTE and NOP in spherical coordinates with respect to both current modulation angle, θ_1 and θ_2 at two Rx positions. The magnitude in each position vector corresponds to respective performance index. Optimal θ_1 and θ_2 can be visualized from the projected contours and indicated by the respective markers. Clearly in both metrics, the optimum current angles θ_1 and θ_2 differ at different positions.

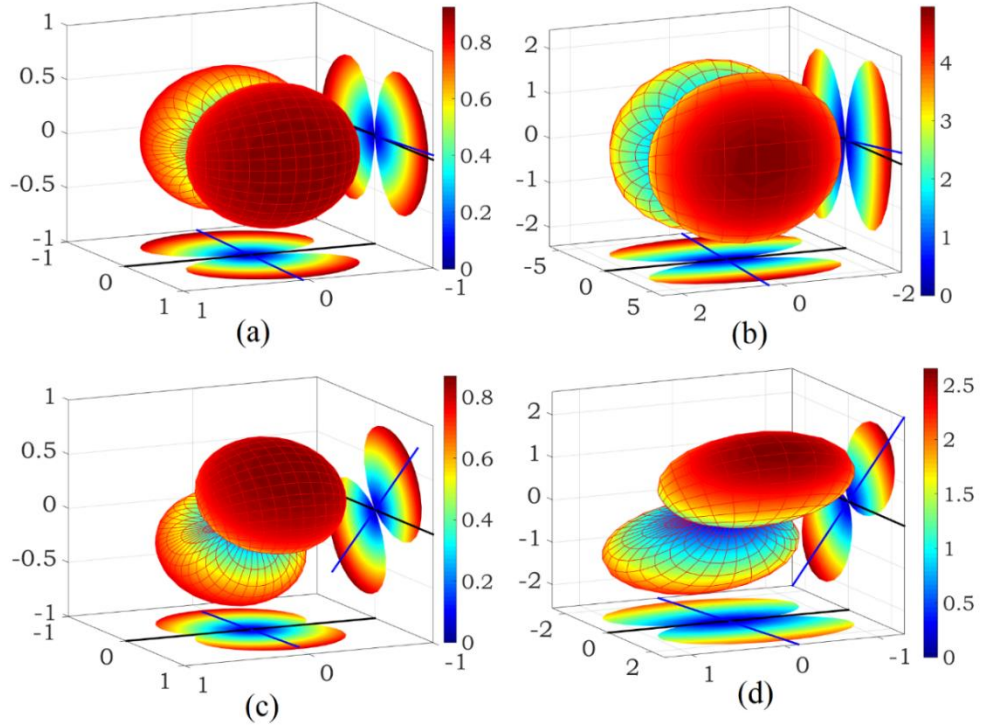


Fig. 5-6. PTE and NOP variation against current modulation angles, θ_1 and θ_2 . (a) – PTE for $y_{rx}=0$, (b) – NOP for $y_{rx}=0$, (c) – PTE for $y_{rx}=y_{tx}/2$ and (d) – NOP for $y_{rx}=y_{tx}/2$ ($z_{rx}=100\text{mm}$ $R_L=50\Omega$).

Similar to Fig. 5-5 in 2-Tx setup, Fig. 5-7 shows optimum Tx current variations with respect to the Rx position. When $y_{rx} = 0$ (Rx is aligned to Tx₂), the optimal current in Tx₂ is much higher than Tx₁ and Tx₃, and the optimal currents in Tx₁ and Tx₃ are identical. Moving along, when $y_{rx} = y_{tx}/2$ (Rx farthest away from Tx₁ while being equi-radial displaced between Tx₂ and Tx₃), current in Tx₁ is much lower than Tx₂ and Tx₃, and currents in Tx₂ and Tx₃ are identical.

According to (5-12), higher n gives better performance. However, comparison of 2-Tx and 3-Tx coil setups did not show any significant difference. Therefore the effectiveness of higher number of Tx coils needs to be further investigated. In addition, the parameters used for the above numerical analysis do not provide comprehensive

operating guidelines such as frequency and load characteristics. In the next section, further analyses will be carried out so as to provide a robust and rigorous optimization scheme.

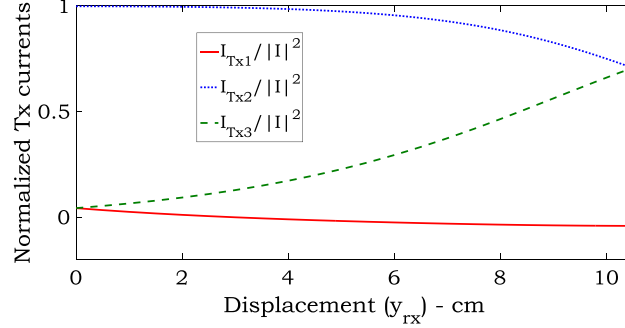


Fig. 5-7. Normalized optimal Tx currents vs. lateral displacement (y_{rx}) ($z_{rx}=100\text{mm}$).

5.3 Performance optimization

5.3.1 Frequency optimization

Frequency optimization can be done by using (5-12) for both PTE and NOP. It is important to note that coil resistance is a function of frequency as shown in (5-24), and should be given considerable attention during frequency optimization. $R_{dc,t}$ and $R_{dc,r}$ are the DC resistances of the Tx and Rx coils respectively, while ω_k and ω_l are the frequencies at which AC resistances double their respective DC counterparts [97]. By incorporating (5-24) into (5-12), the optimum frequencies for PTE and NOP can be obtained using differential techniques, and solving the polynomials (for real and positive roots) shown in (5-25) and (5-26) respectively.

$$R_t = R_{dc,t} \left(1 + \frac{\omega^2}{\omega_k^2} \right) \text{ and } R_r = R_{dc,r} \left(1 + \frac{\omega^2}{\omega_l^2} \right) \quad (5-24)$$

$$\omega_{PTE-opt} = x \text{ s.t. } a_1 x^8 + a_2 x^6 + a_3 x^2 + a_4 = 0 \quad (5-25)$$

$$\text{where } a_1 = 2R_{dc,t}R_{dc,r}^2,$$

$$a_2 = R_{dc,t}L_r^2\omega_k^4 + M_{sum}^2R_{dc,r}\omega_k^2\omega_l^2 + R_{dc,t}R_{dc,r}^2\omega_k^2 + 2R_{dc,t}R_{dc,r}^2\omega_l^2 + 2R_LR_{dc,r}R_{dc,t}\omega_l^2,$$

$$a_3 = -R_{dc,t}\omega_l^4(L_r^2\omega_r^4 - 2L_r^2\omega_r^2\omega_k^2 + 2R_L^2\omega_k^2 + 2R_LR_{dc,r}\omega_k^2 + R_{dc,r}^2\omega_k^2)$$

$$\text{and } a_4 = -2R_{dc,t}L_r^2\omega_r^4\omega_k^2\omega_l^4$$

$$\omega_{NOP-opt} = x \text{ s.t. } b_1 x^6 + b_2 x^2 + b_3 = 0 \quad (5-26)$$

$$\text{where } b_1 = R_{dc,r}^2, \quad b_2 = \left(2L_r^2 \omega_r^2 - (R_L + R_{dc,r})^2 \right) \omega_l^4, \quad \text{and } b_3 = -2L_r^2 \omega_r^4 \omega_l^4$$

With the numerical results from (5-25) and (5-26) as illustrated in Fig. 5-8 and Fig. 5-9 respectively, optimum frequencies are observed to be fairly constant during low load resistance region. Therefore, coil resistances are assumed to be frequency independent in order to further simplify optimum frequencies. With the assumption of constant coil resistances, both PTE and NOP are optimized when K_{n-opt} becomes maximum. By differential technique, the optimum frequencies for PTE and NOP are derived in (5-27). The condition for (5-27) that needs to be valid is the load resistance has to satisfy the load boundary in (5-28).

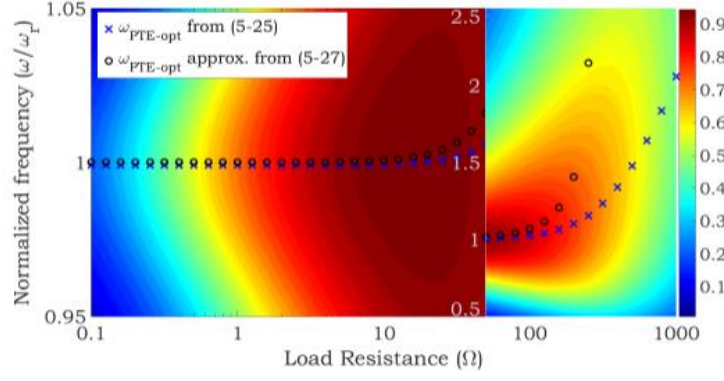
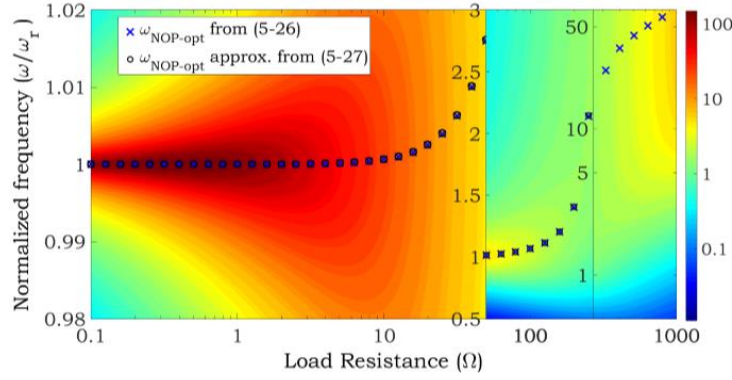
$$\omega_{PTE-opt} = \omega_{NOP-opt} = \omega_r \sqrt{\frac{1}{1 - \frac{(R_r + R_L)^2}{2(\omega_r L_r)^2}}} \quad (5-27)$$

$$\left(1 - \frac{(R_r + R_L)^2}{2(\omega_r L_r)^2} \right) > 0 \rightarrow R_L < (\sqrt{2} \omega_r L_r - R_r) \quad (5-28)$$

When the load resistance is much less than the condition in (5-28), (5-27) can be approximated to be the resonance frequency of the receiver as given in (5-29).

$$\text{When } R_L \ll (\sqrt{2} \omega_r L_r - R_r) \rightarrow \frac{(R_r + R_L)^2}{2(\omega_r L_r)^2} \ll 1 \rightarrow \omega_{PTE-opt} = \omega_{NOP-opt} = \omega_r \quad (5-29)$$

Numerical results are shown in Fig. 5-8 and Fig. 5-9 when contours are derived from (12). For PTE, it can be seen that the approximated frequency from (5-27) is only valid at low load resistance region which agrees with (5-29). In contrast, for NOP, approximated frequency from (5-27) is valid within the load resistance boundary defined in (5-28). This difference is attributed from the additional Tx resistance, R , factor (which is the parasitic resistance of Tx coils) in the PTE equation of (5-12). Outside the boundary for NOP, the optimum frequency experiences a drastic increment which renders it operationally unfeasible for most setups.


 Fig. 5-8. PTE variation vs. operating frequency and load resistance ($z_{rx}=100\text{mm}$, $y_{rx}=0$ $n=3$).

 Fig. 5-9. NOP variation vs. operating frequency and load resistance ($z_{rx}=100\text{mm}$, $y_{rx}=0$ $n=3$).

It is apparent from the frequency optimization that designers can choose their operating frequency to be same as the Rx self-resonance if their load requirements are suitable. Nonetheless, if load requirements are not met, equivalent load resistances can be controlled using load transformation techniques [106]. This would form the prerequisite for load optimization presented in the next sub-section.

5.3.2 Load variation

Load optimization can be done by using (5-12) for both PTE and NOP in a similar manner. By using differential techniques, the optimum load resistance can be derived in (5-30) and (5-31) for PTE and NOP respectively. If the condition stipulated in (5-29) is satisfied (i.e. operating frequency equals to Rx resonance frequency), further simplification can be done and it is shown in (5-32) and (5-33). With (5-33), it is clear that the optimum load resistance for NOP is equal to the Rx coil resistance, R_r which fulfils the impedance matching principle within the receiver circuitry.

$$R_{L(PTE-opt)} = \sqrt{\frac{\omega_r^2 M_{sum}^2 R_r + R(R_r^2 + X_r^2)}{R}} \quad (5-30)$$

$$R_{L(NOP-opt)} = \sqrt{R_r^2 + X_r^2} \quad (5-31)$$

$$\omega = \omega_r \rightarrow X_r = 0 \quad (5-32)$$

$$R_{L(PTE-opt)} = \sqrt{\frac{\omega_r^2 M_{sum}^2 R_r + R R_r^2}{R}} \quad (5-33)$$

$$R_{L(NOP-opt)} = R_r$$

Analytical results from (5-30) and approximated results from (5-32) are compared in Fig. 5-10. In evaluating analytical results, the operating frequency is chosen to be at its optimal value (from (5-25)). In contrast, when evaluating approximated results, operating frequency is maintained at ω_r (as assumed in (5-29)). It can be clearly seen that the approximation matches the analysis, leading to the validation of the assumption of (5-29) which states that optimum frequency is equal to Rx resonance frequency. It is interesting to note that as axial displacement z_{rx} increases, optimal load resistance within the segment decreases.

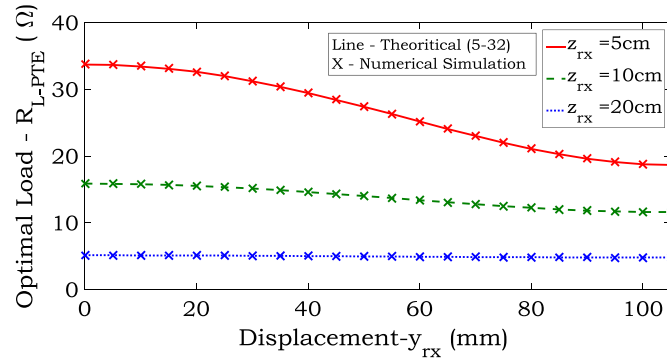


Fig. 5-10. Variation of the optimum load resistance with respect to lateral displacement (y_{rx}).

5.3.3 Number of simultaneously powered Tx coils

According to (5-12), the higher the number of Tx coils, better the performance, in terms of NOP and PTE. However, high n increases control complexity. On the other hand, by adding additional Tx coils might provide diminishing marginal performance. Hence, optimum n has to be determined based on the performance increment and

additional control complexity. For NOP optimization, the percentage condition in (5-34) can be defined to determine the optimum n . The threshold margin, $\delta_{NOP-thld}$, needs to be determined based on the application criteria. Moving forward, (5-34) can be simplified to (5-35) using (5-12). If the mutual inductance between the additional Tx and Rx is higher than desired threshold margin as stated in (5-35), the addition of coil is deemed to be effective. Similarly for PTE optimization, in order to evaluate the effectiveness of adding Tx coils, the condition in (5-36) can be defined. The addition of a Tx coil is effective only if the PTE increment is greater than desired threshold margin, $\delta_{PTE-thld}$. Equation (5-36) can be extended to (5-37) using (5-12).

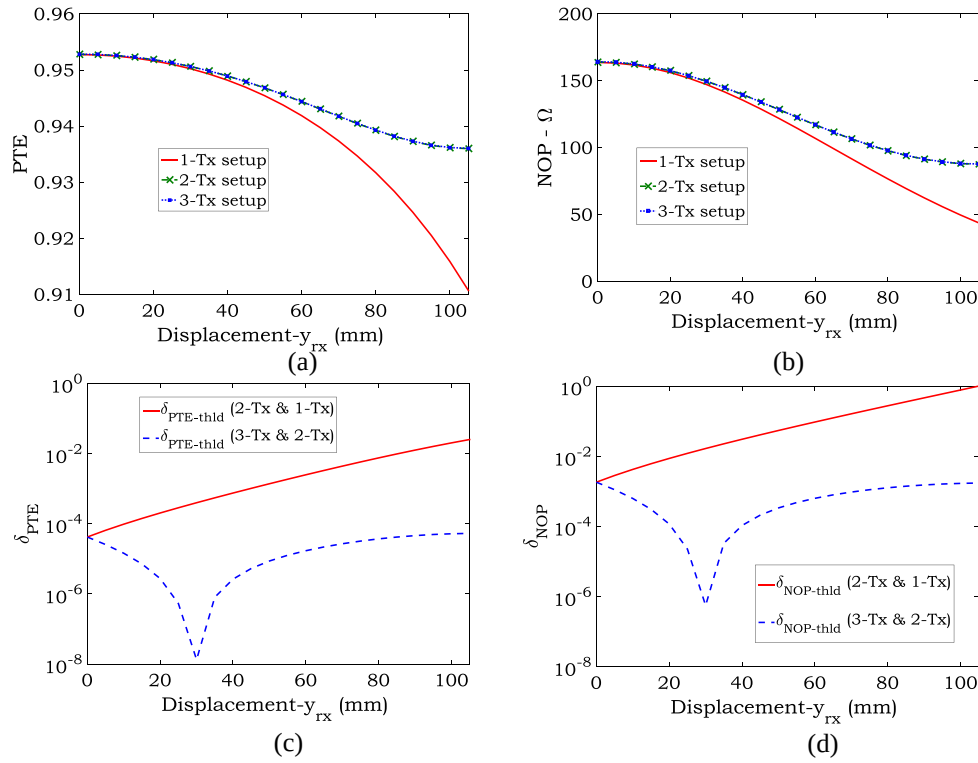


Fig. 5-11. Optimal performance comparison for different number of simultaneously powered Tx coils (n) at axial distance (z_{rx}) of 100mm (PTE and NOP at each position is illustrated for respective optimized load resistance). (a) – PTE, (b) – NOP, (c) – $\delta_{PTE-thld}$ and (d) – $\delta_{NOP-thld}$.

$$\frac{NOP_{n+1} - NOP_n}{NOP_n} > \delta_{NOP-thld} \quad (5-34)$$

$$\frac{M_{n+1,r}^2}{M_{sum}^2} > \delta_{NOP-thld} \quad (5-35)$$

$$PTE_{n+1} - PTE_n > \delta_{PTE-thld} \quad (5-36)$$

$$\frac{\frac{R_L}{\frac{2Rn}{K_{n+1}} + (R_r + R_L)}}{\frac{R_L}{\frac{2Rn}{K_n} + (R_r + R_L)}} > \delta_{PTE-thld} \quad (5-37)$$

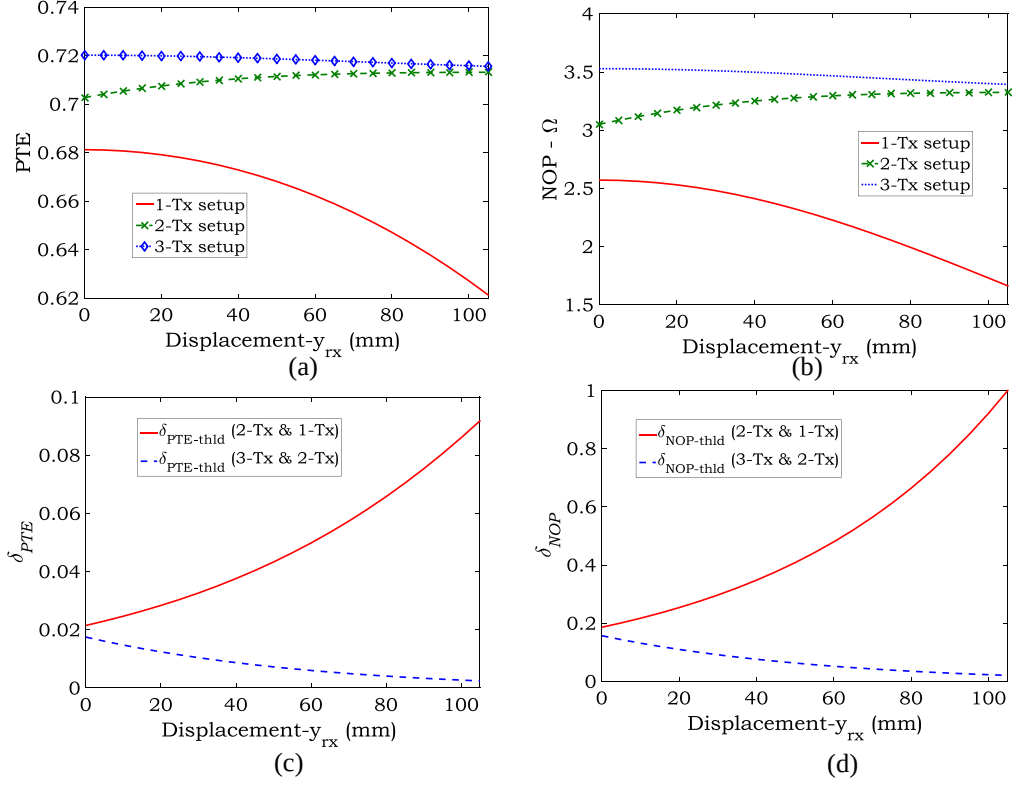


Fig. 5-12. Optimal performance comparison for different number of simultaneously powered Tx coils (n) at axial distance (z_{rx}) of 300mm (PTE and NOP at each position is illustrated for respective optimized load resistance). (a) – PTE, (b) – NOP, (c) – $\delta_{PTE-thld}$ and (d) – $\delta_{NOP-thld}$.

Varying the number of Tx coils, numerical results for PTE and NOP are illustrated in Fig. 5-11 (a) and (b) respectively. Load resistances are chosen to be their respective (PTE and NOP) optimal values as of (5-32) and (5-33) and, operating frequency is selected to be at Rx self-resonance (ω_r) as of (5-29). Then, Fig. 5-11 (c) and (d) show the difference in PTE and NOP, $\delta_{PTE-thld}$ and $\delta_{NOP-thld}$ respectively, for different number of Tx coils. It is evident from this setup, that increasing n from 1 to 2 significantly improves the PTE and NOP performances. However, increasing n from 2 to 3 provides negligible enhancements.

Next, for results shown in Fig. 5-12, the operating conditions are similar to that of Fig. 5-11, with the only exception of axial distance (z_{rx}) changing 100mm to 300mm. As shown in Fig. 5-12 (c) and (d), increasing n from 1 to 2 and, from 2 to 3, provided considerable improvements. Comparing these two case studies with different z_{rx} , it is

clear that z_{rx} has an impact on the effectiveness of the optimum number of Tx coils. It should be re-emphasized that the coils used in this study are identical as size difference in coils can affect the optimum number of Tx coils. For example, if Rx coil is relatively larger than Tx coil [107], the optimum n would be higher than that of similar size Rx coil.

5.3.4 Maximum performance

With the parameter optimization provided in previous sub-sections, the maximum PTE and NOP for a given D-WPT setup can be derived. For this purpose, an equivalent coupling coefficient is defined as in (5-38), where L_t and L_r are the inductances of Tx and Rx coils respectively. Unloaded quality factors of Tx coils and Rx coil is defined in (5-39).

$$k_{eq} = \frac{\sqrt{M_{sum}^2}}{\sqrt{L_t L_r}} \quad (5-38)$$

$$Q_{Tx} = \frac{\omega L_t}{R_t}, Q_{Rx} = \frac{\omega L_r}{R_r} \quad (5-39)$$

The maximum PTE can be obtained as in (5-40) by applying optimum frequency (in (5-29)) and optimum load (in (5-32)) conditions to (5-12). It is interesting to note that the maximum PTE (PTE_{max}) is a function of equivalent coupling and coil quality factors which is identical to the well-established maximum efficiency formula discussed in Chapter 3. This provides a concise design goal for D-WPT systems which is universally applicable to any number of Tx coils. Similar to the maximum PTE, the maximum NOP (NOP_{max}) can be computed in (5-41) by applying optimum frequency (in (5-29)) and load conditions (in (5-33)) to (5-12). Similar to PTE, it is important to note that the maximum NOP for a D-WPT is restricted by the coil design (determine Q_{Tx} , Q_{Rx} and R_t) and the system parameters such as transfer distance and misalignment (determines k_{eq}).

$$PTE_{max} = \frac{k_{eq}^2 Q_{Tx} Q_{Rx}}{(\sqrt{1 + k_{eq}^2 Q_{Tx} Q_{Rx}} + 1)^2} \quad (5-40)$$

$$NOP_{max} = \frac{R}{4} k_{eq}^2 Q_{Tx} Q_{Rx} \quad (5-41)$$

5.4 Experimental results

A laboratory prototype of a D-WPT system is built to validate the theoretical analysis. In this experimental setup, a 4-port network analyzer is used to measure S-parameters and Advanced Design System (ADS) is used for performance evaluation. The experimental setup is illustrated in Fig. 5-13. The coils are constructed based on the design guidelines of Chapter 3, and the specifications of D-WPT setup are shown in Table 5-1. The experimental self-resonances of coils are listed in Table 5-2 when connected to series lumped capacitors. Throughout the experiment, the modulations of Tx currents are manually controlled in an open loop manner with the use of ADS. Closed loop implementation of similar current modulation scheme can be found in [108]. Experimental validation for 2-Tx and 3-Tx D-WPT setups are presented in following subsections.

Table 5-2. Experimental self-resonances

Coil	Resonance frequency
Tx ₁	0.990MHz
Tx ₂	0.990MHz
Tx ₃	0.988MHz
Rx	0.993MHz

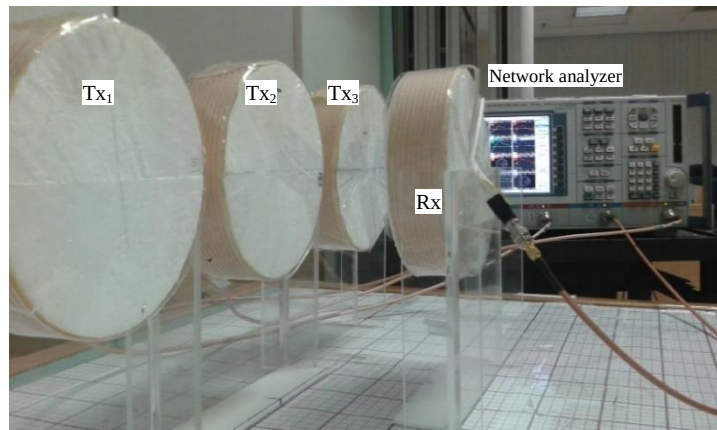


Fig. 5-13. The experimental setup (3-Tx setup)

First, experimental validation is performed with a 2-Tx D-WPT setup. The polar form illustration in Fig. 5-14 shows experimental PTE and NOP variations with respect to current modulation angle at two different Rx positions. It can be seen that the experimental results for both PTE and NOP agree well with the theoretical results presented in Fig. 5-4. Furthermore, the experimental optimal current modulation angles

within a segment are compared with their theoretical counterparts in Fig. 5-15 for two axial distances.

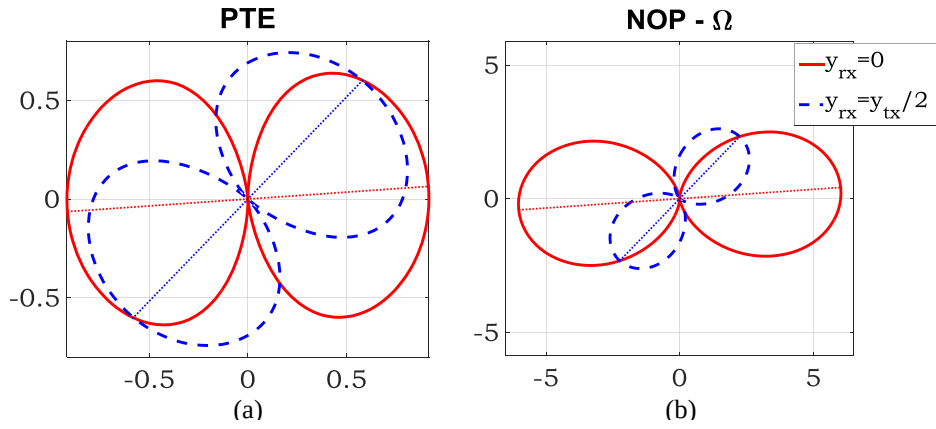


Fig. 5-14. Experimental performance variation with respect to current modulation angle in polar form ($z_{rx}=100\text{mm}$ $R_L=50\Omega$), (a) – PTE, (b) – NOP.

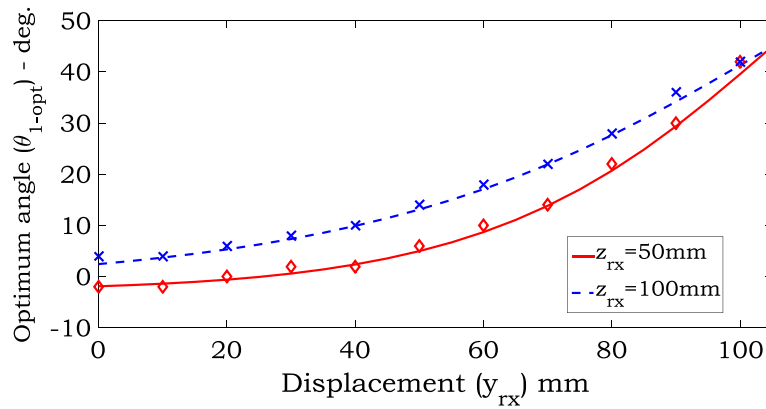


Fig. 5-15. Comparison between theoretical and experimental optimal current modulation angles: lines represent theoretical results and points represent experimental results.

Next, experimental verification is performed for 3-Tx coil array. Fig. 5-16 shows PTE and NOP variations for two different y_{rx} positions, and its shape agrees with the theoretical results presented in Fig. 5-6. Furthermore, the experimental optimal current modulation angles within a segment are compared with their theoretical counterparts in Fig. 5-17 for two axial distances. Moving on, the experimental frequency characteristic of PTE and NOP are compared with theoretical values in Fig. 5-18. Then, the optimal load resistances within a segment are compared with theoretical values in Fig. 5-19. Lastly, experimental and theoretical results for different axial distances against lateral displacements are shown in Fig. 5-20. While the experimental trend largely follows theoretical analysis, it should be noted that the discrepancies could be due to experimental tolerances and different self-resonances of the coils.

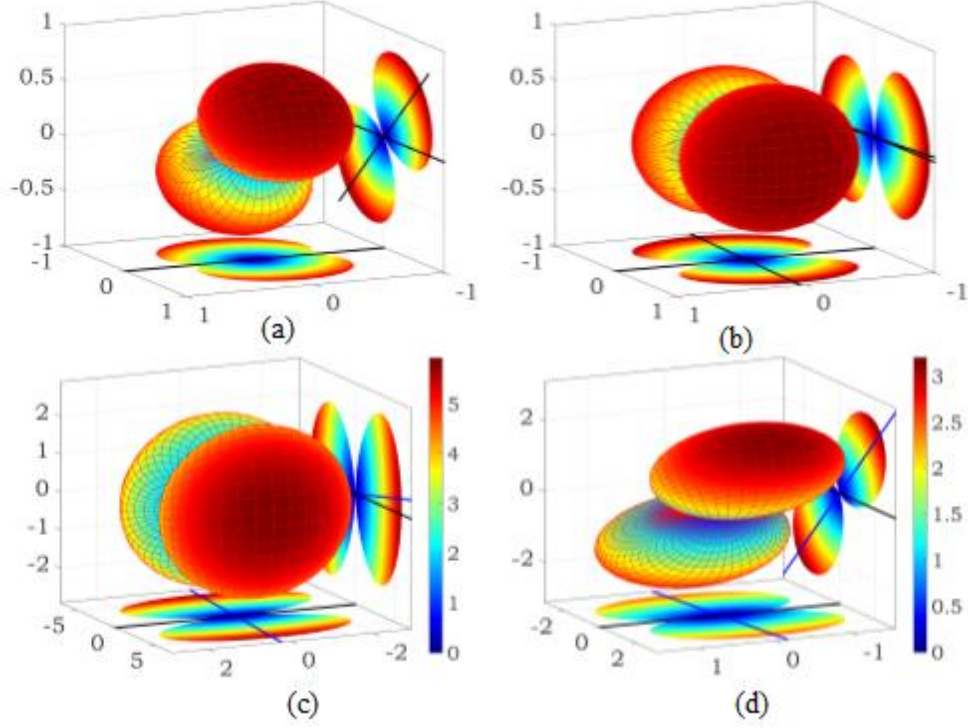


Fig. 5-16. Experimental performance variation against current modulation angles, θ_1 and θ_2 . (a) – PTE for $y_{rx}=0$, (b) – NOP for $y_{rx}=0$, (c) – PTE for $y_{rx}= y_{tx}/2$ and (d) – NOP for $y_{rx}= y_{tx}/2$ ($n=3$, $z_{rx}=100\text{mm}$ $R_L=50\Omega$).

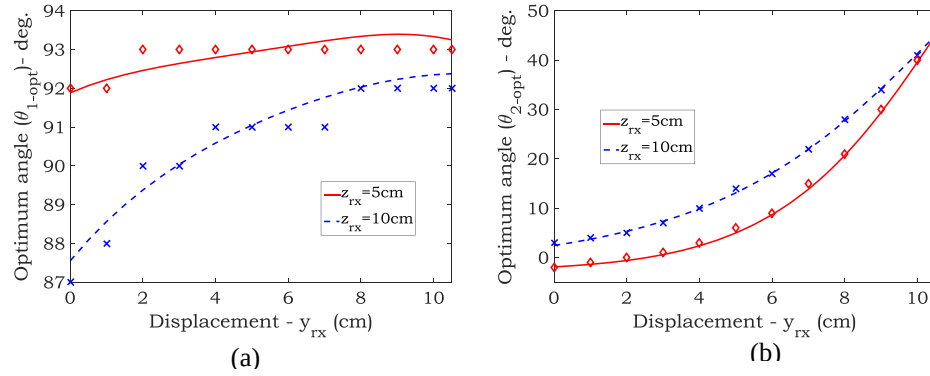


Fig. 5-17. Comparison between theoretical and experimental optimal current modulation angles: line: theoretical results and points – experimental results ($n=3$, $z_{rx}=100\text{mm}$ $R_L=50\Omega$) (a) – θ_1 and (b) – θ_2 .

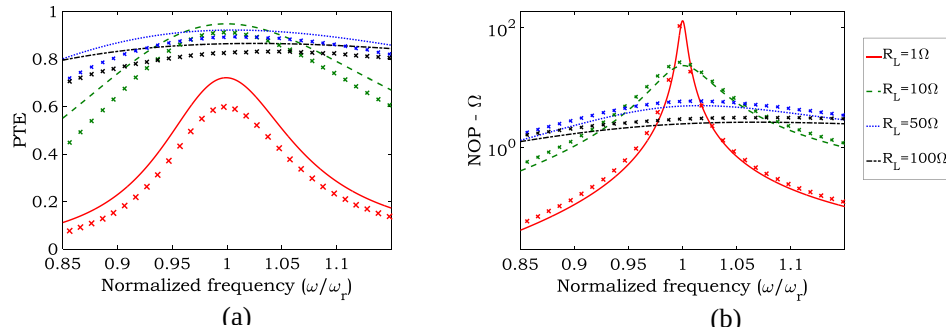
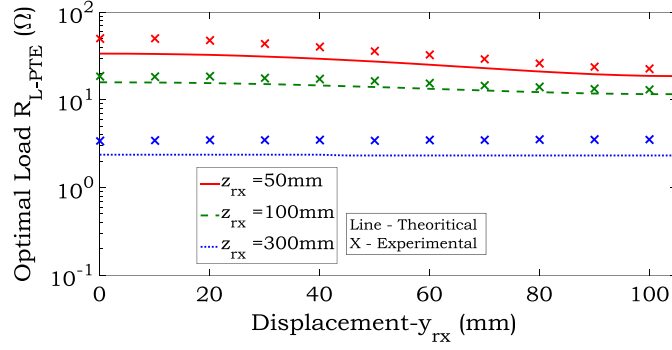
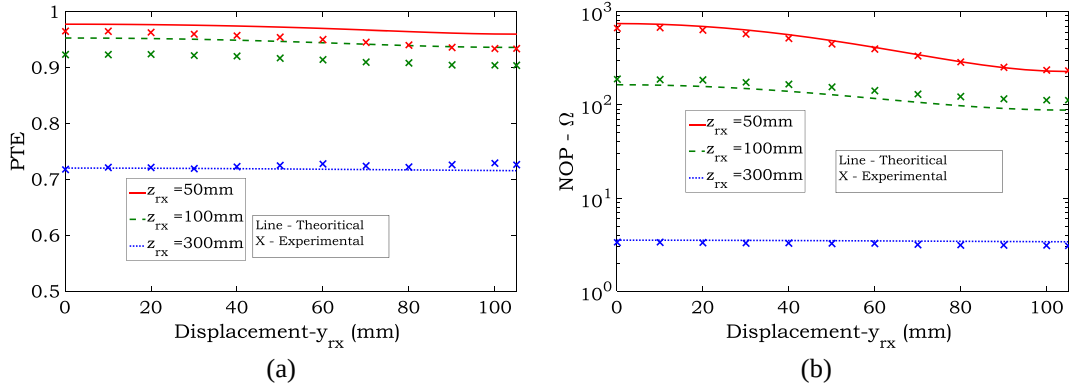
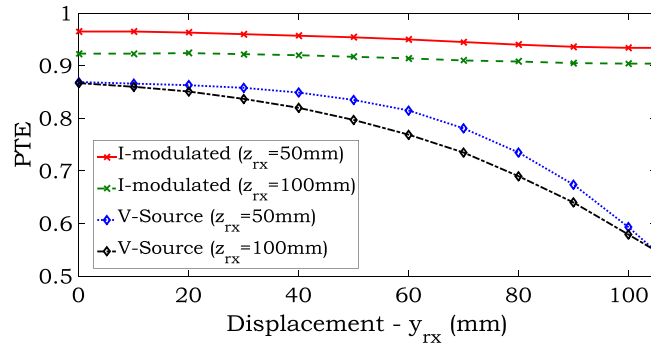


Fig. 5-18. Frequency characteristics comparison of experimental and theoretical performance ($n=3$ and $z_{rx}=100\text{mm}$). (a) – PTE and (b) – NOP.


 Fig. 5-19. Comparison between theoretical and experimental results for optimum load resistance ($n=3$).

 Fig. 5-20. Comparison between theoretical and experimental results for optimum performance. ($n=3$ and respective optimized load resistance) (a) – PTE and (b) – NOP.

Voltage source configuration is compared with the proposed modulated current source configuration in Fig. 5-21. Due to the different normalization of both sources, NOP cannot be fairly compared and hence, only PTE is used for the evaluation. While the proposed method maintains its efficiency relatively constant throughout the displacement, the PTE of the voltage source configuration degrades significantly.


 Fig. 5-21. Experimental results: Comparison of Voltage-source D-WPT discussed in Chapter 4 vs. current modulated D-WPT proposed in this chapter (load resistances are chosen to be at respective optimized values and $\omega=\omega_r$).

Overall, experimental results agree well with the theoretical analysis with slight deviations. These deviations could be due to experimental disparities such as non-identical coils, different coil self-resonance frequency and additional parasitic resistances from capacitors, cables and connectors.

5.5 Summary

This chapter investigated efficiency and output power optimization for transmitter (Tx) current modulation used in D-WPT applications. The theoretical optimum current distribution within a Tx array was derived by applying an n-dimensional coordinate analogy onto the equivalent circuit. The optimal current distribution at arbitrary receiver (Rx) position is a function of the mutual inductances between Tx coils and Rx. The optimization results were presented for an arbitrary number of Tx coils and further simplified for 2-Tx and 3-Tx setups. Supplementing this, numerical results of performance metrics provided visualization of the current distribution optimization. In addition, optimization design parameters such as operating frequency, load resistance and number of simultaneously powered Tx coils were explored. Furthermore, the theoretical upper bound of the efficiency and normalized output power was derived. Experimental results were presented to validate the theoretical analysis. Lastly, comparison between the conventional voltage source and the current source with proposed optimization method was performed. The proposed method improved the overall performance and alleviated the negative effects of misalignment in D-WPT systems. Moreover, the theoretical analysis derived in this chapter provides a platform for optimization of multi-Tx WPT systems.

Chapter 6

Figure-of-Merit for the Optimization of Wireless Power Transfer System Against Misalignment Tolerance

6.1 Introduction

Maximum power transfer efficiency (PTE), high transferred power (TP) and misalignment tolerance are key design requirements most of the WPT applications. PTE refers to the energy efficiency through WPT link whereas, TP refers to the normalized output power at the load. Major focus of chapters 3, 4 and 5 was on the improvement of the PTE, because high PTE is essential to minimize system losses. However, TP is also an important parameter to consider as it allows higher output power delivered to the load. High TP is important requirement for high power applications because the component ratings of high power applications is highly dependent on the TP characteristics. Therefore, it is important to consider both PTE and TP in designing WPT systems. The maximum TP and PTE are mainly limited by the distance between transmitter (Tx) and receiver (Rx) coils.

Among the studies reported in literature, the primary optimization objective is either PTE [14, 36] or TP [23, 25]. The focus on a single performance index does not guarantee the optimized result for the other index, therefore, the requirement of additional techniques to optimize both indices arises. For example, a typical series compensated 2-coil WPT setup operating at resonant frequency of individual coils needs an impedance matching (IM) circuitry to improve TP. Therefore, improvement of both PTE and TP simultaneously is a significant aspect for optimization. A figure-of-merit (FOM) considering PTE and TP along with prioritization scheme of both indices has been introduced in this study.

In addition to the system level performance indices such as PTE and TP, application specific criteria such as transfer distance, misalignment tolerance and nature of Rx positioning also affect the performance of the design. Typically, various optimizations

presented in literature focus on improving the performance for a desired Rx position and orientation, and subsequently, the performance is evaluated with misalignment [75, 76]. However, such an approach gives rise to inferior performance with misalignment as Rx dynamics are not taken into account. Misalignment analysis needs to be carried out in accordance with the Rx behaviour. Dynamics of Rx in each of six degrees of freedom (6DoF) misalignments modelled using Euclidean transformation at any desired Rx position and dynamics comprise of translation along three axes and rotation around three axes (Fig. 6-1). Likelihood of Rx misalignment in each of 6DoF is also an important design parameter. Probability of alignment in each 6DoF can be defined by considering Rx dynamics.

The literature lacks an analysis on incorporating such application level criteria into the design stage. A novel FOM is introduced in this study incorporating both system level performance indices and design level parameters such as PTE, TP, translational and rotational misalignment, and probability of alignment with respect to 6DoF. In order to substantiate the proposed FOM, an optimization of WPT system using repeaters is presented. The proposed FOM can be used not only for the optimization of WPT with repeaters, but also in a wide range of design approaches such as coil design, tuning, compensation circuit design and converter design for WPT.

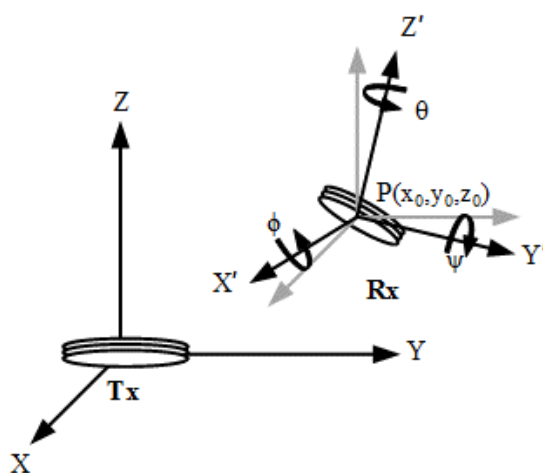


Fig. 6-1. WPT system with 6DoF misalignment.

This chapter is organized as follows. Section 6.2 introduces the proposed Tri-Spiral Repeater (TSR) with analysis and parameter calculation using equivalent circuit approach. A FOM that can be used to improve system level performance and 6DoF misalignment tolerance is introduced in Section 6.3. The proposed FOM is applied to

optimization of WPT with TSR in Section 6.4. Variable operating frequency is identified as a promising tuning approach to improve 6DoF misalignment tolerance in Section 6.5, followed by the chapter summary.

6.2 Analysis of WPT system with repeaters

6.2.1 Proposed tri-spiral repeater (TSR)

This section introduces the analysis of the proposed tri-spiral repeater (TSR). Three spiral coils are used to construct a planar TSR. Three coils are used as it is the minimum number of coils required to achieve the symmetrical structure that allows misalignment tolerance in any direction in radial plane. The proposed WPT system with two TSRs placed at Tx and Rx is illustrated in Fig. 6-2. The TSR placed at the Tx coil is named as TSR1 (coils: TxR1, TxR2 & TxR3) and the TSR placed at the Rx coil is named as TSR2 (coils: RxR1, RxR2 & RxR3) (refer to Fig. 6-2). Z_{TSR1} (or Z_{TSR2}) is the axial distance between Tx (or Rx) and TSR1 (or TSR2) while u (or v) is the radial separation of the coils in TSR1 (or TSR2). Position of the Rx is defined using coordinates x_{rx} , y_{rx} and z_{rx} with respect to the Tx. Axial distance (z_{rx}) is defined as unobstructed axial separation between TSR1 and TSR2. Angular misalignments around x, y, and z axes are defined as ϕ , ψ , and θ respectively. Misalignments in radial plane, rotation around x-axis and rotation around z-axis are illustrated in Fig. 6-2. Angular misalignments around y-axis (ψ) is similar to the angular misalignments around x-axis (ϕ) for TSR-WPT setup. Self-resonance frequency of each coil is chosen to be 1MHz.

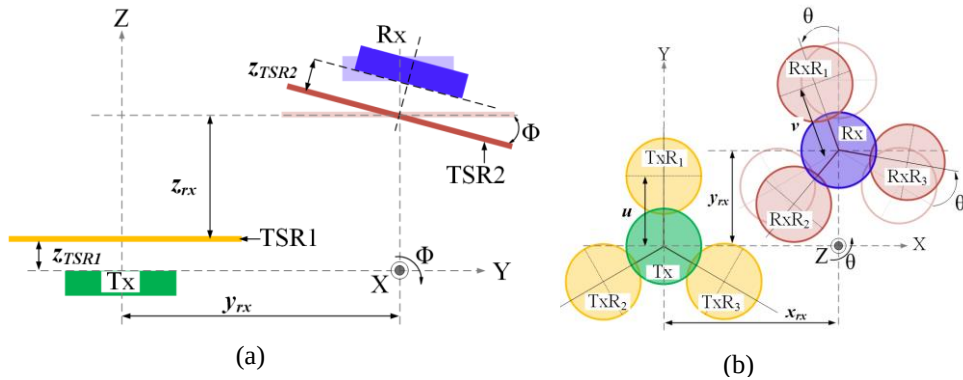


Fig. 6-2. Design parameters of the proposed WPT system with TSR (two dimensional representation)
(a) – Y-Z view and (b) – X-Y view.

The performance characteristics of WPT with TSRs are compared against the performance of equivalent single-spiral repeater (SSR) having same physical size as the

TSR. It is identified that TSR outperform SSR, and a comparison is detailed in Section IV. Nevertheless, the proposed optimization of this chapter can be applied for any kind of WPT optimization including repeaters with any number of coils. The coupling coefficient of helical shaped coils is found to be higher than that of the spiral shaped coils at higher axial and radial distances. Therefore, helical shaped coils are preferred for WPT applications. However, helical shaped coils occupy larger space compared to spiral shape due to their height, making them unsuitable for design of planar repeaters. Therefore, helical shaped coils are used as Tx and Rx coils while spiral shape is used for the design of repeaters.

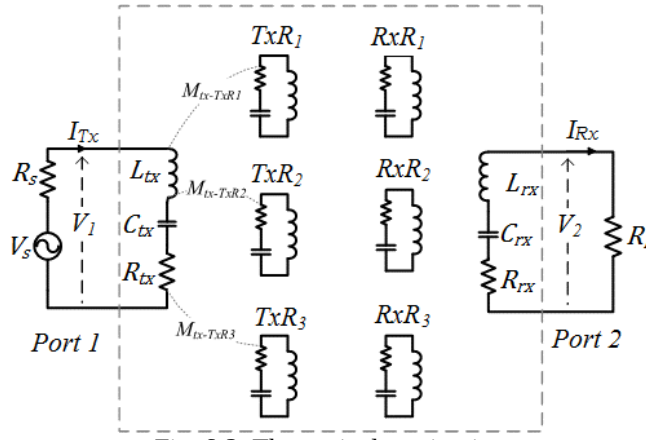


Fig. 6-3. The equivalent circuit.

6.2.2 Equivalent circuit model

For WPT system analysis, the equivalent circuit method with multiple transmitters, repeaters and receiver coils is well established in literature [30]. Fig. 6-3 illustrates the equivalent circuit of the WPT system with two TSRs placed near Tx and Rx. Equation (6-1) can be expressed to evaluate system performance using the equivalent circuit model.

$$\begin{aligned}
 [Z]_{8 \times 8} [I]_{8 \times 1} &= [V]_{8 \times 1} \\
 [Z_{kl}] &= \begin{cases} Z_k = R_k + j(\omega L_k - 1/\omega C_k), & k = l \\ X_{kl} = X_{lk} = j\omega M_{kl} = j\omega M_{lk}, & k \neq l \end{cases} \\
 [I] &= [I_{Tx} \ I_{TxR_1} \ \dots \ I_{RxR_3} \ I_{Rx}]^T \\
 [V] &= [V_1 \ 0 \ \dots \ 0]^T \\
 V_1 &= V_s - I_{Tx} R_s
 \end{aligned} \tag{6-1}$$

Diagonal elements of the impedance matrix, Z_k ($k=Tx, TxR_{1,2,3}, RxR_{1,2,3}$, and Rx) are the self-impedances of Tx, Rx and repeater coils, while L_k , C_k , and R_k are inductance, capacitance and parasitic resistance respectively. $X_{kl}=X_{lk}=j\omega M_{kl}=j\omega M_{lk}$ ($k, l= Tx, TxR_{1,2,3}, RxR_{1,2,3}$, and Rx) where $M_{kl}=M_{lk}$ is the mutual inductance between k^{th} coil and l^{th} coil. R_L is the equivalent load resistance. ω is the operating angular frequency of the source. RMS voltage across Tx coil is defined as V_1 . Once system equation is defined, RMS currents in each coil (I_k ; $k= Tx, TxR_{1,2,3}, RxR_{1,2,3}$, and Rx) can be determined. After defining the system equation, the performance indices, TP and PTE can be calculated as in (6-2), where P_{out} , P_{in} , V_s and R_s are output power, input power RMS source voltage and equivalent source resistance respectively. Input power defined in the PTE equation is the accumulation of the losses in the coils and the output power. The S-parameters based measurement technique discussed in Chapter 2 is used for the experimental validations. With the definition of system parameters and performance evaluation, the proposed FOM is introduced in the next section.

$$TP = \frac{P_{out}}{|V_s|^2/4R_s} = TP = \frac{|I_{Rx}|^2 R_L}{|V_s|^2/4R_s} \quad (6-2)$$

$$PTE = \frac{P_{out}}{P_{in}} = \frac{|I_{Rx}|^2 R_L}{\sum(|I_k|^2 R_k) + |I_{Rx}|^2 R_L}$$

6.3 New approach on the figure-of-merit

6.3.1 System performance indices

In a WPT setup, high PTE does not always guarantee a high TP, and vice versa. A figure-of-merit of system (F_{sys}), considering both PTE and TP along with prioritization number has been introduced as in (6-3). The prioritization number, ρ depends on the importance of PTE over TP in a particular application. If $\rho < 1$, the application prioritizes TP than PTE. Such a prioritization scheme is beneficial when PTE and TP show different characteristics with respect to design variables such as operating frequency, Tx/Rx/repeater position, coupling between coils and load resistance. For example, Fig. 6-4(a) shows frequency response of PTE and TP for a classical two coil setup. It can be seen that PTE becomes highest at the resonance frequency, whereas, TP shows frequency splitting phenomena (it should be noted that relatively high coupling

coefficient of 0.5 is used for clearer visualization). Therefore, the selection of operating frequency (f_s) requires a balance between PTE and TP. Fig. 6-4(b) shows PTE and TP with respect to ρ when operating frequency is chosen to maximize F_{sys} .

$$F_{sys} = (PTE)^\rho \times TP \quad (6-3)$$

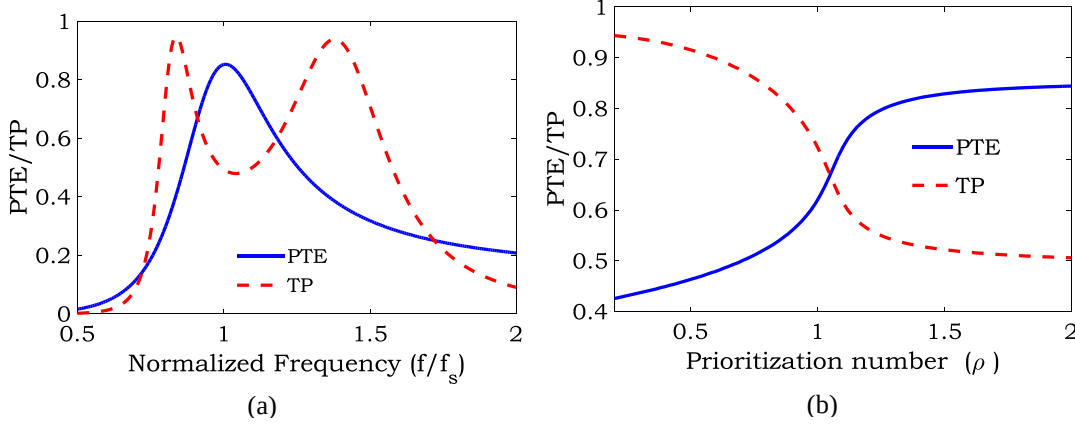


Fig. 6-4. (a) – PTE and TP variation with respect to frequency, (b) PTE and TP when operating frequency is chosen to maximize F_{sys} ($f_s=1\text{MHz}$, $k=0.5$, $R_L=30\Omega$, $R_s=50\Omega$).

In addition to the 2-coil example, the usage of F_{sys} is explained with regards to proposed TSR-WPT. Fig. 6-5(a) shows a frequency response of PTE and TP for a SSR-WPT setup at aligned position and 20 cm transfer distance (further explanation on the optimization of TSR-WPT setup is discussed in subsequent sections). The frequency splitting phenomena can be seen in TP profile for TSR-WPT scheme. Fig. 6-5(b) shows the variation of optimal PTE and TP with respect to ρ when the operating frequency is chosen to maximize F_{sys} . The respective operating frequency variation with respect to prioritization number is shown in Fig. 6-5(c). At low values of ρ , TP is close to its maximum value and PTE is substantially less than its peak since higher priority is given to TP. On the other hand at high values of ρ , PTE is close to its maximum value and TP is substantially less than its peak since PTE is prioritized. Therefore, by having F_{sys} as the optimization objective of a design process allows improvement of both PTE and TP with a prioritization between them. Subsequent analysis of this study equally prioritizes PTE and TP by defining $\rho=1$.

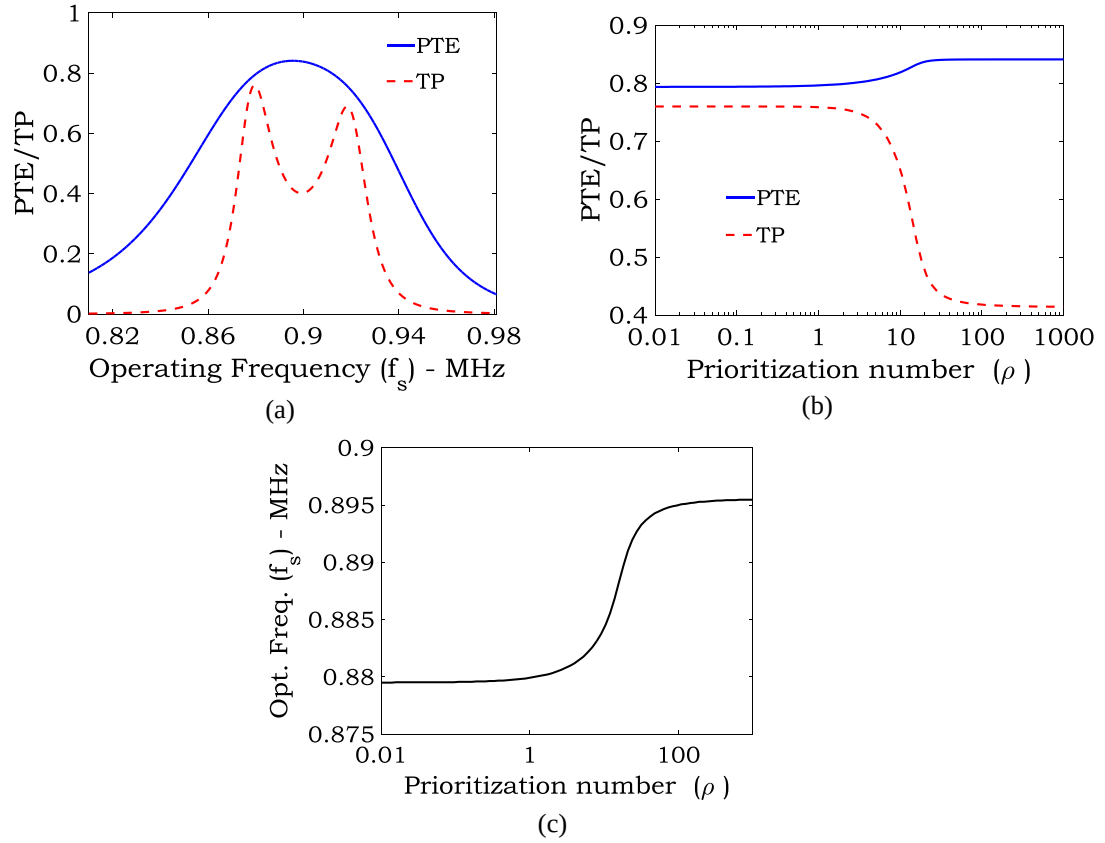


Fig. 6-5. The performance characteristics when operating frequency is chosen to maximize F_{sys} : (a) – PTE and TP variation with respect to frequency, (b) Optimum PTE and TP vs. prioritization number and (c) – respective optimum operating frequency vs. prioritization number (design parameters of the TSR-WPT setup are: $z_{rx}=20\text{cm}$, $y_{rx}=0$, $u=4.9\text{cm}$, $v=4.8\text{cm}$, $Z_{TSR1}=10\text{cm}$, and $Z_{TSR2}=12.9\text{cm}$).

Table 6-1. Design parameters of helical shaped and spiral shaped coils used for TSR and SSR (The size of SSR is chosen to be similar to the maximum footprint of SSR corresponds to $u=4.9\text{cm}$ in subsequent optimization).

	Helical (Tx, Rx)	Spiral used for TSR	Spiral used for SSR
Outer radius	8.9cm	8.9cm	12.5cm
Turns separation	4.2mm	5mm	5mm
Thickness	5cm	2mm	2mm
Number of turns	11	14	14
Inductance - Calculated	30 μH	20 μH	40 μH
Inductance - Measured	32 μH	21 μH	40 μH

6.3.2 Misalignment analysis

As discussed in previous chapters, the optimal design parameters critically dependent on the position of the receiver. Therefore, dynamic behavior of the receiver needs to be included in the optimization process. Rx misalignment can be any of Rx's 6DoF when WPT is applied to applications such as biomedical implants and wearable electronics in internet of things (IoT) applications [50]. Therefore, the FOM proposed in this chapter gives consideration to all 6DoFs.

Mainly 6DoFs of Rx with respect to the desired position can be decoupled as Euclidean geometric transformations that are comprised of translational and rotational misalignments along three axes. Each translational and rotational misalignment regions are segmented into sub regions, and each sub regions have their respective Rx positioning probabilities which is termed as the probability of alignment. Then the minimum performance in each sub region is prioritized based on the probability of alignment for the derivation of FOM as the optimization objective. A set of decoupled FOMs are defined with respect to each translational and rotational misalignment. Thereafter, a combined FOM is defined to achieve simultaneous misalignment tolerance in multiple DoFs. Mathematical formulation of FOM is explained in the following sections. It should be noted that span of the sub regions and probabilities of alignment can be determined by analyzing Rx dynamics in a particular application criteria.

6.3.3 Translational misalignment analysis

Translational misalignment along x-y axes is in the radial plane with respect to Tx, while translational misalignment along z axis is in the axial direction. Therefore, two decoupled FOMs with respect to radial (x-y plane) and axial (z axis) misalignments are defined for the translational misalignment analysis.

1) Radial Misalignment

Usually, the desired Rx position is when it is aligned with Tx in radial plane (zero misalignment position in radial plane). Therefore, sub regions for the radial direction misalignment are defined as a set of circular regions as illustrated in Fig. 6-6(a). Radius of the i^{th} region is defined as r_i . Probability that Rx can be occupied in i^{th} sub region (S_i^{xy}) is defined as δ_i^{xy} . The minimum value of F_{sys} within each sub region is prioritized

with the respective probability of alignment in deriving FOM against translational misalignment along x-y axes (F^{xy}) as defined in (6-4).

Choice of FOM as the optimization objective allows improving performance with respect to misalignment. However, if the optimization includes all the Rx positioning possibilities including those farther away positions with low probabilities, resultant may give poor performance within high probability regions. Therefore, the number of sub regions (n_{xy}) up to a certain confidence interval (CI^{xy}) are taken into account such that the summation of δ_i^{xy} is equal to CI^{xy} .

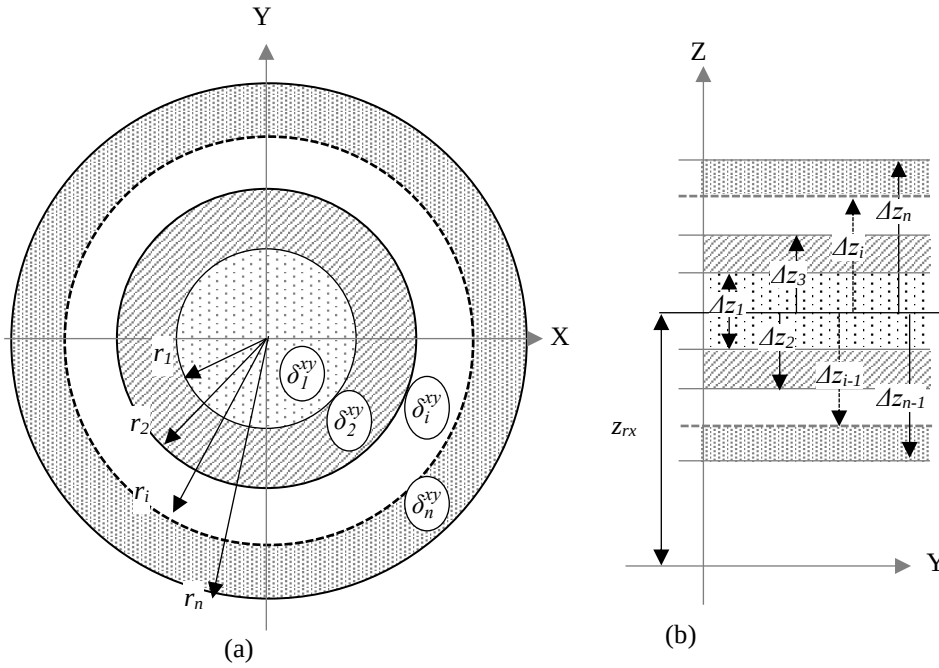


Fig. 6-6. Selection of sub regions: (a) – Radial direction misalignment, (b) – Axial direction misalignment (two dimensional representation).

$$F^{xy} = \prod_{i=1}^{n_{xy}} [\text{Min}(F_{sys} \in S_i^{xy})]^{\delta_i^{xy}} \quad (6-4)$$

$$S_i^{xy} = r_{i-1} < r \leq r_i$$

$$\sum_{i=1}^{n_{xy}} \delta_i^{xy} = CI^{xy}$$

2) Axial Misalignment

When it comes to the axial misalignment, desired axial position is the nominal transfer distance (z_{rx}) which is dependent on the application requirement. Axial misalignment can either be towards Tx (-ve z direction) or away from Tx (+ve z

direction). Therefore, sub regions with respect to axial misalignment are defined both positive and negative z-directions as shown in Fig. 6-6(b). Number of sub regions (n_z), probability of misalignment (δ_i^z) and confidence interval (CI^z) are defined in a similar manner to the radial misalignment case. Equation (6-5) defines FOM with respect to the axial misalignment.

$$F^z = \prod_{i=1}^{n_z} [\text{Min}(F_{sys} \in S_i^z)]^{\delta_i^z}$$

$$S_i^z = \{\Delta z \in z_i | \Delta z \notin z_i\}; \Delta z = z - z_{rx}$$

$$\sum_{i=1}^{n_z} \delta_i^z = CI^z$$
(6-5)

6.3.4 Angular misalignment

Angular misalignment can be modelled as rotation around three axes. Decoupled FOMs can be defined for angular misalignment with respect to each axis as in (6-6), (6-7) and (6-8) which are similar to the translational misalignment scenario. Subscripts and superscripts ϕ , ψ , and θ represent rotation around x, y and z axes respectively. Fig. 6-7 illustrates the selection of sub regions for rotation around z and x axes for the proposed WPT with TSR. With respect to θ , sub regions (S_i^θ) are defined in (6-6) with reference to the optimum angle, θ_{opt} , as illustrated in Fig. 6(a). Each sub-region comprises of clockwise and counter clockwise angular misalignment from its optimum angular position. Due to the symmetric nature of the TSR around z-axis rotation, both clockwise and counter clockwise rotations are equally divided into sub regions. However, it is possible to define asymmetric sub regions if any application requires to do so. Number of sub regions (n_θ), probability of misalignment (δ_i^θ) and confidence interval (CI^θ) with respect to angular misalignment around z-axis can be determined according to application specific requirements.

On the other hand, sub regions with respect to rotational misalignments around x-axis and y-axis (S_i^ϕ and S_i^ψ respectively) are defined with reference to zero rotational misalignment. Similarly, number of sub regions (n_ϕ and n_ψ), probabilities of misalignment (δ_i^ϕ and δ_i^ψ) and confidence interval (CI^ϕ and CI^ψ) are defined with respect to angular misalignments around x-axis and y-axis. Afterwards, decoupled FOMs with

respect to each angular misalignment are defined by prioritizing the minimum value of F_{sys} in each sub region using the respective probabilities.

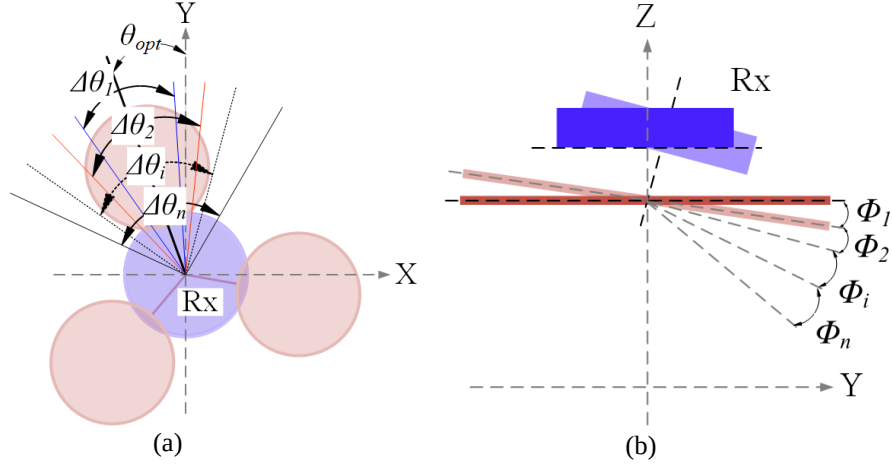


Fig. 6-7. The selection of sub regions for rotational misalignment with TSR: (a) – rotation around z-axis, (b) – rotation around x-axis (two diamentional representation).

$$F^\theta = \prod_{i=1}^{n_\theta} [\text{Min}(F_{sys} \in S_i^\theta)]^{\delta_i^\theta}$$

$$S_i^\theta = \{\Delta\theta \in \Delta\theta_i | \Delta\theta \notin \Delta\theta_{i-1}\}; \Delta\theta = \theta - \theta_{opt}$$

$$\sum_{i=1}^{n_\theta} \delta_i^\theta = CI^\theta$$
(6-6)

$$F^\phi = \prod_{i=1}^{n_\phi} [\text{Min}(F_{sys} \in S_i^\phi)]^{\delta_i^\phi}$$

$$S_i^\phi = \phi_{i-1} < \phi \leq \phi_i$$

$$\sum_{i=1}^{n_\phi} \delta_i^\phi = CI^\phi$$
(6-7)

$$F^\psi = \prod_{i=1}^{n_\psi} [\text{Min}(F_{sys} \in S_i^\psi)]^{\delta_i^\psi}$$

$$S_i^\psi = \psi_{i-1} < \psi \leq \psi_i$$

$$\sum_{i=1}^{n_\psi} \delta_i^\psi = CI^\psi$$
(6-8)

6.3.5 Figure of merit of the design

Although decoupled FOMs can be used to improve the performance in a single DoF misalignment, most WPT applications are expected to operate within misalignment tolerance in several DoFs. If there is a higher significance for a particular DoF, the

respective decoupled FOM as defined in (6-4)-(6-8) can be used. However, the proposed criteria can be extended to improve misalignment in several DoF directions in case of dynamic Rx behaviour. Combined FOM of the design (F_{des}) is defined as in (6-9) by multiplying each decoupled FOMs in (6-4) to (6-8). If the optimization does not require all 6DoFs, (6-9) can be modified by including only the selected decoupled FOMs in important misalignment directions.

$$F_{des} = (F^{xy} \times F^z \times F^\theta \times F^\phi \times F^\psi) \quad (6-9)$$

6.4 Optimization of TSR with FOM

The proposed FOMs in previous section are applied to the optimization of the design parameters of TSR. System equations, (6-1) and (6-2) are implemented as a function of optimization variables (f_s , u , v , Z_{TSR1} , Z_{TSR2} and θ as defined in section 6.2), and design variables. FOM is defined as the objective function with a set of application specific criteria as defined in (6-4) to (6-8). Matlab® optimization toolbox is used for the optimization process. Transfer distance, z_{rx} is chosen to be at nominal distance of 20cm for analysis in subsequent sections. Therefore, desired position of Rx is at aligned position with $z_{rx}=20\text{cm}$, $x_{rx}=0$, $y_{rx}=0$, $\theta=\theta_{opt}$, $\phi=0$, and $\psi=0$.

Before moving to the misalignment analysis with the proposed TSR, the performance characteristics of TSR at the aligned position is compared with the corresponding SSR-WPT setup in order to validate the use of TSR. The size of SSR is chosen to be 12.5cm which is similar to the maximum size of TSR in the following optimization (TSR corresponds to u (or v) =4.9cm), and all the other design parameters are chosen to be same as the coils in TSR (refer Table 6-1). Frequency characteristics illustrated in Fig. 6-8 compare theoretical and experimental results for TSR and SSR optimized at the aligned position. The operating frequency is normalized with corresponding optimum frequencies (f_{opt}) which give maximum F_{sys} ($F_{sys}= \text{PTE} \times \text{TP}$) (which results the peak-PTE frequency is slightly off from the optimum frequency for SSR-WPT). In addition, it should be noted that the frequency responses of WPT with repeater are dependent on a number design parameters such as coupling between Tx, Rx and repeater coils [80]. Particularly, frequency response for TP can be single peak profile or multi-peak profile depending on the setup parameters [80]. For example,

frequency response of both PTE and TP shows a single peak characteristics for TSR-WPT setup analyzed in Fig. 6-8 whereas, the frequency response of TP showed a double peak characteristics for WPT setup illustrated in Fig. 6-5.

It is evident that TSR shows better performance compared to SSR. However it should be re-emphasized that the proposed optimization technique is applicable to any kind of optimization with respect to misalignment even with SSR, and TSR is chosen as a case study to validate the proposed FOM. The following subsection presents the optimization of design parameters of setup with respect to each DoF misalignment.

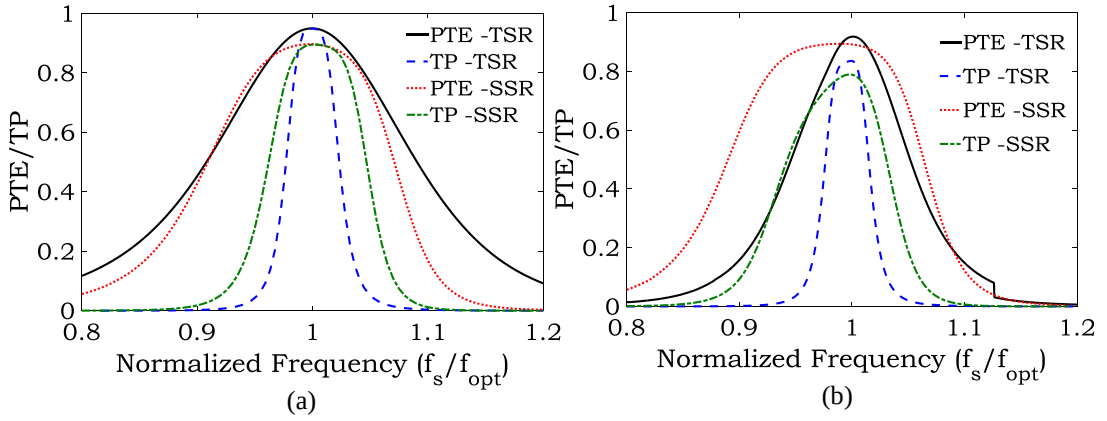


Fig. 6-8. Performance comparison with respect to normalized operating frequency. (a) – Theoretical results and (b) – Experimental results (The operating frequency is normalized with corresponding optimum frequencies (f_{opt}) which give maximum F_{sys}).

Sub regions and their respective probabilities are denoted as arrays for simpler interpretation in following sections. For example, three sub regions with respect to radial misalignment are denoted as an array of three elements as, $S^{xy}=[S_1^{xy}; S_2^{xy}; S_3^{xy}] \equiv [r_1; r_2; r_3]$, while their respective probabilities are denoted as $\delta^{xy}=[\delta_1^{xy}; \delta_2^{xy}; \delta_3^{xy}]$. With this notation, $S^{xy}=[10\text{cm}; 20\text{cm}; 25\text{cm}]$ and $\delta^{xy}=[0.85; 0.05; 0.05]$ imply that three sub regions are considered with radii 10cm, 20cm and 25cm, and probabilities of 85%, 5%, and 5% respectively. A similar notation is used for all the DoF misalignments.

6.4.1 Translational misalignment with TSR

First, radial misalignment analysis is performed by choosing three sub regions with radii 10cm, 20cm and 25cm ($S^{xy}=[10\text{cm}; 20\text{cm}; 25\text{cm}]$). Probability of alignment for each region is varied keeping CI^{xy} to be 95%. Numerical results in Fig. 6-9(a) and Fig. 6-9(b) show performance variation against radial misalignment when the design

optimized under different constraints. Resulting optimum design variables for each optimization criteria are illustrated in Table 6-2. The condition, $\delta^{xy}=1$, corresponds to the design optimized only at the aligned position. When the TSR is optimized only at the aligned position, both PTE and TP at aligned position are around 95%, and both are less than 20% with 25cm radial misalignment. Although performance can be enhanced at the aligned position, misalignment tolerance is poor when the optimization focus only to the aligned position.

Table 6-2. The optimum design variables when the design is optimized to F^{xy} at different δ^{xy} ($S^{xy} = [10\text{cm}; 20\text{cm}; 25\text{cm}]$).

$[\delta_1^{xy}; \delta_2^{xy}; \delta_3^{xy}]$	f_s (MHz)	u (cm)	v (cm)	Z_{TSR1} (cm)	Z_{TSR2} (cm)	θ_{opt} (deg.)
1	0.616	2.5	2.5	0	0	83
[0.85; 0.05; 0.05]	0.850	4.7	4.5	6.1	7.9	57
[0.50; 0.40; 0.05]	0.885	4.9	4.7	7.9	10.7	56
[0.35; 0.30; 0.30]	0.897	4.9	4.8	10	12.9	59

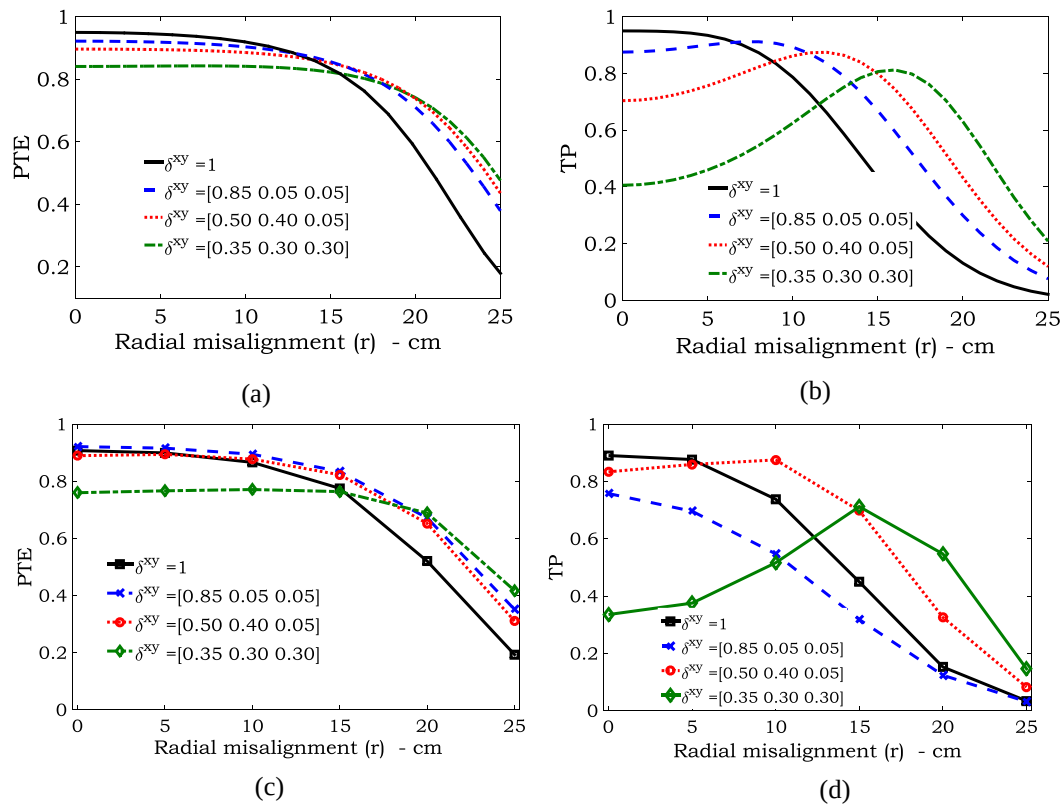


Fig. 6-9. Performance comparison with respect to radial misalignment at different design conditions: (a) – Theoretical results: PTE variation, (b) – Theoretical results: TP variation, (c) – Experimental results: PTE variation, and (d) – Experimental results: TP variation. ($S^{xy} = [10\text{cm } 20\text{cm } 25\text{cm}]$).

When F^{xy} is used as the optimization objective, higher performance can be achieved against misalignment with slight performance reduction at the aligned position. For example, using $\delta^{xy} = [0.85; 0.05; 0.05]$ as the optimization objective, TP can be improved by nearly 20% within 10cm to 20cm misalignment region. This results in TP reduction by 8% at the aligned position. On the other hand, PTE is improved from 18% to 39% at 25cm misalignment with a PTE reduction of only 3% at aligned position. Therefore, it is clear that the use of F^{xy} as the optimization objective enhances performance reduction against radial misalignment. Assigning higher probability to misaligned regions offers higher performance elevation with respect to misalignment. For example, $\delta^{xy} = [0.50; 0.40; 0.05]$ shows TP improvement of 30% within 15cm-20cm region and 25% PTE improvement at 25cm radial misalignment. These improvements result in a 5% and 15% PTE and TP reduction at aligned position respectively. However, assignment of high probabilities to misaligned regions further reduces performance at desired position. For example, choice of $\delta^{xy} = [0.35; 0.30; 0.30]$ shows 55% reduction in a TP at aligned position. This is because the optimization process gives high priority to improve low performance within misaligned region. Therefore, proper choice of optimization parameters is crucial for achieving the best performance.

The translational misalignment analysis is experimentally validated by constructing TSR as described in Section 6.2, and the experimental setup is illustrated in Fig. 6-10. S-parameters are measured by connecting Tx and Rx coils to the network analyzer. Fig. 6-9(c) and Fig. 6-9(d) show experimental results comparing four design cases illustrated in Table 6-2. The experimental results agree closely with the theoretical analysis, with an exception for TP characteristics at high misalignments. Deviations in experimental results can be attributed to the experimental disparities in coils, geometries and self-resonances. Particularly, TP characteristics is sensitive to the geometry of TSR. Overall, the experimental results follow the theoretical analysis. Theoretical results illustrated in Fig. 6-11 show the performance variation for classical SSR-WPT scheme when the similar optimization is used. Although performances of SSR are lower than that of the TSR, the proposed method can be used even with SSR for the optimization against radial misalignment. It can be clearly seen that assigning higher probabilities of alignment to misaligned sub-regions allows performance elevation against misalignment for SSR as well (design parameter corresponds to $Z_{TSR1,2}$ is used for the optimization of SSR).

Therefore, it is apparent that the proposed FOM can be used with a range of optimization approaches including the classical SSR-WPT scheme.

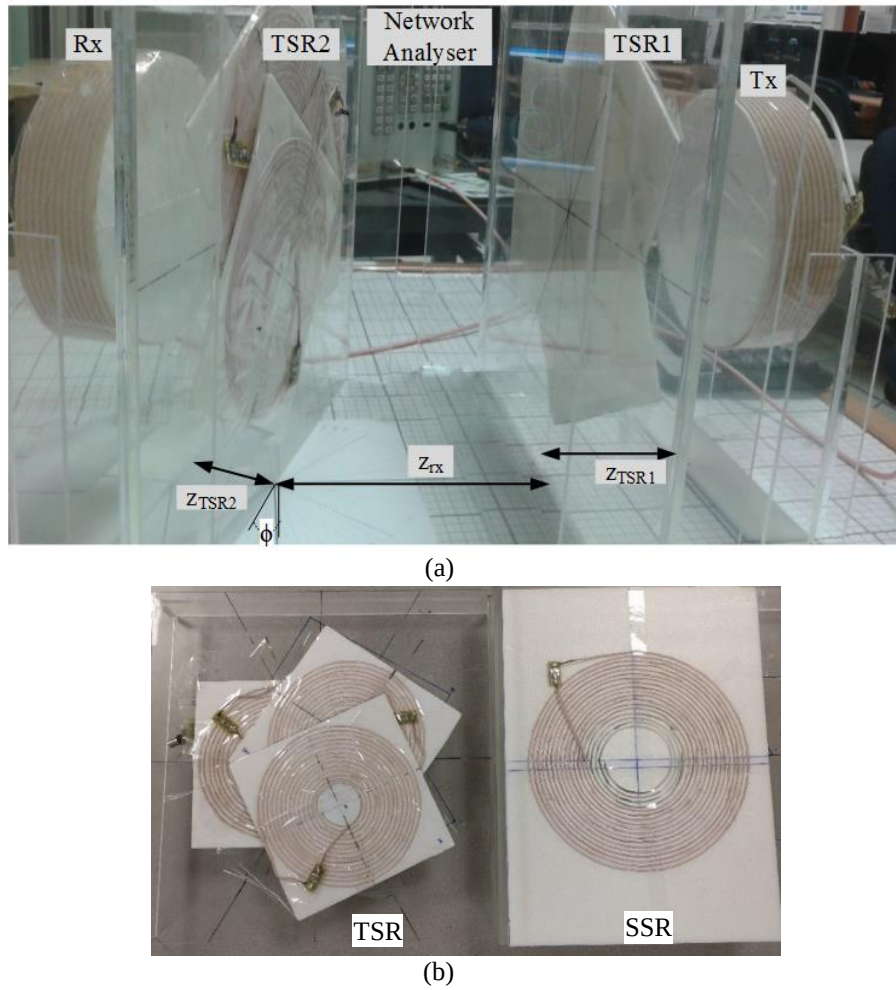


Fig. 6-10. (a) – The experimental setup and (b) – experimental realization of TSR and SSR

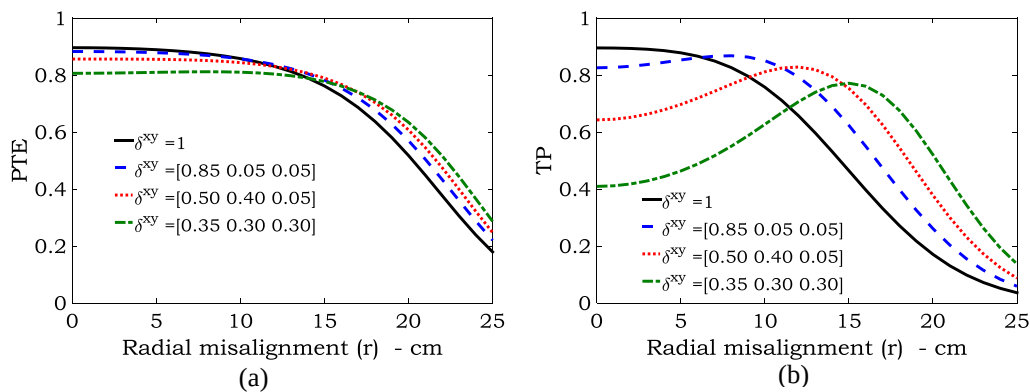


Fig. 6-11. Performance comparison (Theoretical results) with respect to radial misalignment at different design conditions for SSR-WPT: (a) –PTE variation, (b) –TP variation

Next, the misalignment analysis is extended to investigate axial misalignment. WPT system with TSR shows a different performance profile against axial distance. PTE

decreases with the increase of axial distance, whereas, TP becomes maximum only at a particular distance, and reduces with increasing or decreasing distance. TP profile is analogous to critical coupling distance (distance at which coupling coefficient equals to critical coupling) in two-coil WPT setup discussed in Chapter 3.

Performance of the WPT system is optimized to F^z using two sub regions. Two regions are chosen for prioritizing misalignment towards Tx ($S^z=[1\text{cm}; -5\text{cm}]$) and misalignment away from Tx ($S^z=[1\text{cm}; 5\text{cm}]$). Probability of alignment, δ^z is chosen as 70% and 25% distribution with 95% CI^z ($\delta^z=[0.7; 0.25]$). Two design cases are compared with the design optimized to aligned position as shown in Fig. 6-12. It can be observed that a TP increment of 10% within prioritized region incurs a 2% TP reduction in desired position. In both cases, PTE change is within 1%. However, it should be noted that prioritizing both positive and negative z-directions simultaneously is not possible with the proposed TSR due to the nature of TP profile. This limitation can be addressed with the variable frequency operation which is discussed in Section 6.5. Fig. 6-12(c) compares experimental results with the theoretical analysis against axial distance variation for the design optimized at aligned position ($\delta^z=1$).

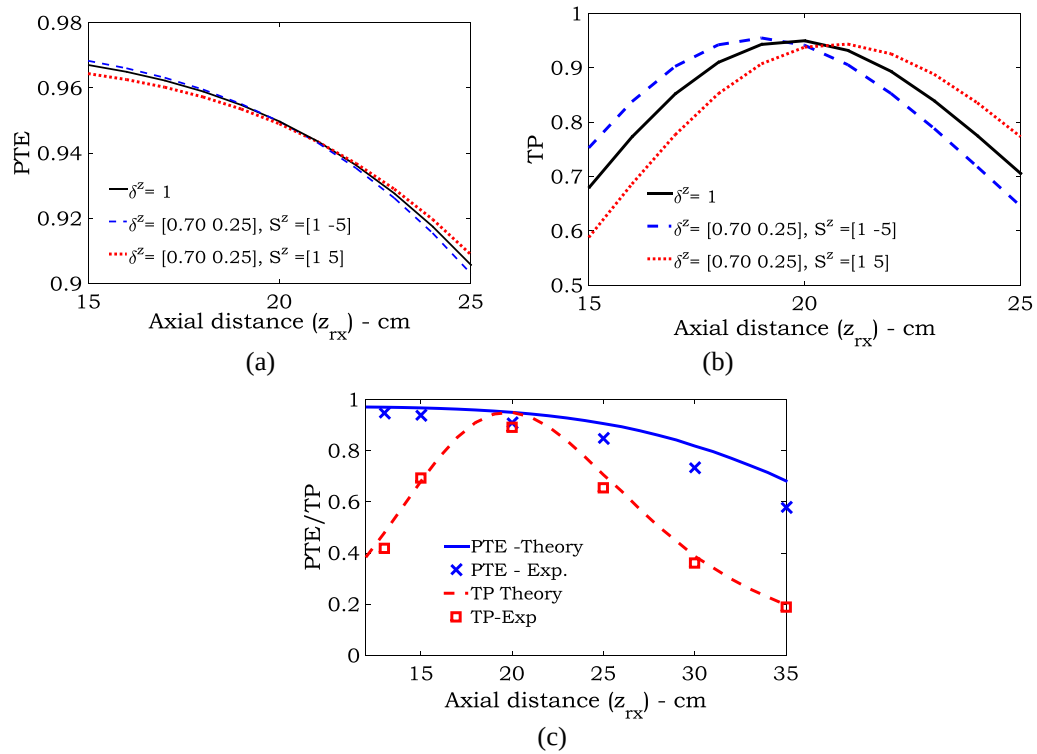


Fig. 6-12. Performance comparison with respect to axial misalignment at different design conditions: (a) – PTE variation, (b) – TP variation and (c) – Comparison of experimental results and theoretical results for the design optimized at aligned position ($\delta^z=1$).

6.4.2 Angular misalignment with TSR

Moving forward, angular misalignment analysis is performed with the use of proposed FOM. Fig. 6-13 shows performance variations against rotation around z axis for the WPT with TSR at 20cm transfer distance. It can be seen that both PTE and TP are almost constant against θ . This is because the change in relative distances between coils is negligible which makes mutual inductance change insignificant. Therefore, it is not required to use FOM with respect to rotation around z-axis for the optimization of the proposed WPT scheme with TSR.

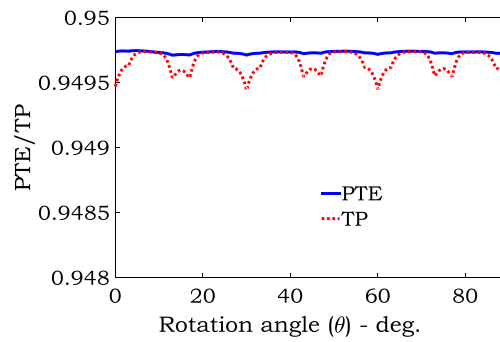


Fig. 6-13. PTE and TP variation with respect to angular rotation around z-axis.

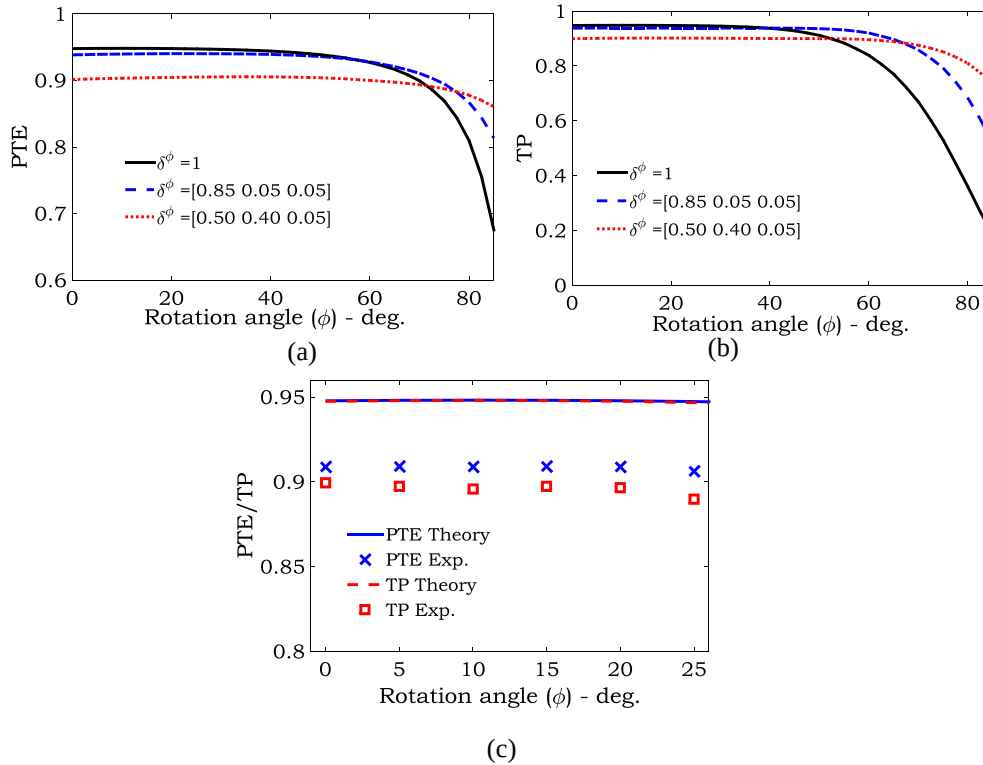


Fig. 6-14. Performance variation with respect to angular misalignment around x-axis ($S^\phi = [35^\circ; 60^\circ; 80^\circ]$), (a) – PTE, (b) – TP and (c) – Comparison of experimental results and theoretical results for the design optimized at aligned position ($\delta^\phi=1$).

Angular misalignment around x and y axes shows similar behaviour against rotation angle due to the symmetry of TSRs. Therefore, angular misalignment around x-axis is analyzed with the proposed FOM. Fig. 6-14 shows PTE and TP resulting from a set of optimization cases. It can be seen that the design optimized only at the aligned position ($\delta^\phi=1$) shows near constant PTE and TP up to 45° angular misalignment around x-axis. Both PTE and TP are higher than 90% up to 60° angular misalignment. Expected angular misalignment tolerance for most WPT applications is less than 60° . Therefore, WPT with TSR can be directly applied to many applications even without using the proposed FOM for angular misalignments around x and y axes. Theoretical results are verified using experimental results as shown in Fig. 6-14(c) for the design optimized at the aligned position. Rotation angles up to 25° are measured due to the mechanical constraint in the experimental setup. However, if it is required to operate at high misalignment angles, FOM can be applied as the optimization objective. For example, PTE and TP can be improved up to 85% and 70% respectively at 80° (using $S^\phi=[35^\circ; 60^\circ; 80^\circ]$ and $\delta^\phi=[0.85; 0.05; 0.05]$) whereas, PTE and TP are at 80% and 35% respectively for the design optimized to the aligned position. PTE and TP reduction at aligned position due to the introduction of FOM is around 1%.

With the above analysis, it is apparent that the proposed decoupled FOMs can be used to improve tolerance against each of 6DoF misalignment. The choice of sub regions and probabilities need to be carried out with proper analysis on application specific criteria.

6.5 Variable operating frequency

For all the design cases analyzed in the previous section, it is assumed that the operating frequency is fixed. Variable frequency operation can be implemented along with a location tracking mechanism [34, 35]. This section introduces the use of the proposed FOM with variable frequency operation. Frequency tuning is found to be a promising technique for improving the performance against misalignment, particularly, TP improvement against radial and axial misalignments. In addition, it is found that variable frequency operation allows performance improvement against multiple DoFs simultaneously.

6.5.1 Decoupled FOM with variable operating frequency

Fig. 6-15 shows performance variation with respect to radial misalignment when the variable frequency operation is incorporated into optimization. Sub regions and probability of alignments are chosen to be identical to those in fixed operating frequency analysis. Performance improvement against radial misalignment can be achieved with a lower performance reduction at aligned position compared with the fixed frequency operation. For example, TP reduction at aligned position for $\delta^{xy}=[0.35; 0.30; 0.30]$ is 19% whereas, it is 55% reduction in fixed frequency case. In case of $\delta^{xy}=[0.35; 0.30; 0.30]$, frequency tuning allows the system to achieve more than 80% of PTE and TP, with radial misalignments up to 18cm. Experimental results are illustrated in Fig. 6-15(c) and (d), and they agree well with the theoretical analysis.

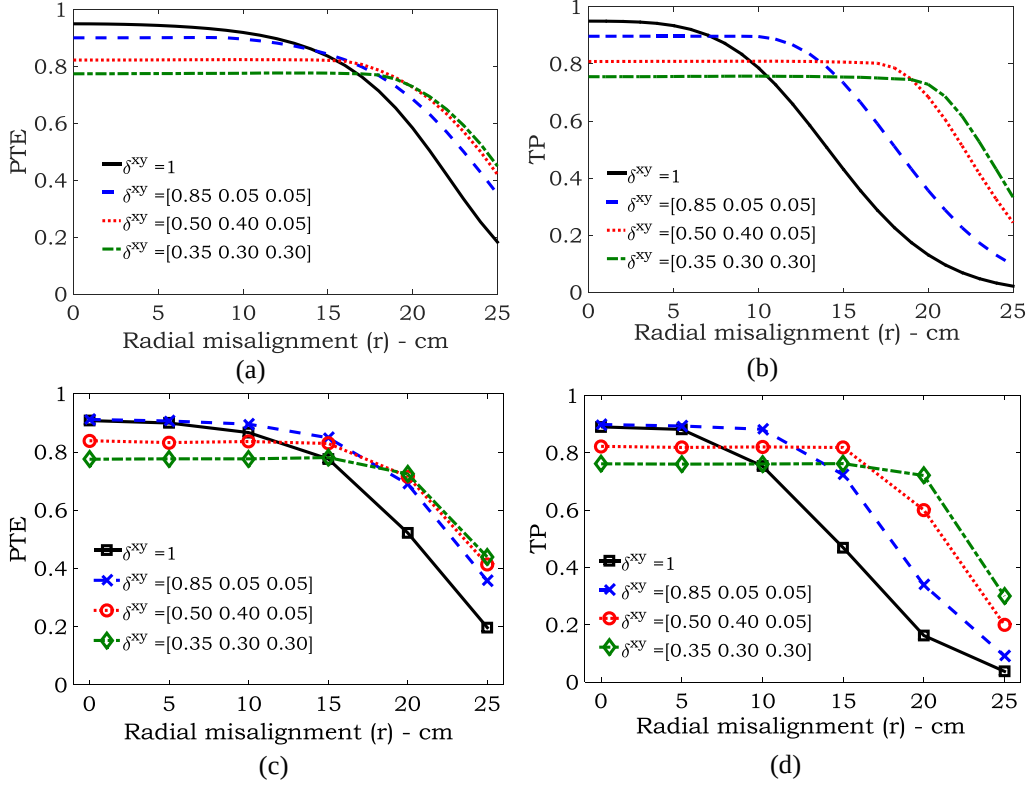


Fig. 6-15. Performance variation against radial misalignment when the design is optimized for radial misalignment tolerance with variable frequency operation (a) – PTE theoretical, (b) – TP theoretical, (c) – PTE experimental , (d) – TP experimental.

Frequency tuning approach offers a significant improvement against axial misalignment as well. Fig. 6-16 shows performance evaluation with respect to axial distance when the optimization objective, F^z is implemented along with the variable frequency operation. TP can be maintained nearly constant at its maximum value up to

critical coupling distance by employing variable frequency operation. Even if the design is optimized at 20cm aligned position, 95% PTE and TP can be achieved in axial distances up to 20cm. Therefore, misalignment towards Tx (-ve z direction) need not to be considered for the optimization process. Fig. 6-16 is generated for $S^z = [2\text{cm}; 5\text{cm}; 10\text{cm};]$ with different δ^z combinations, in which, misalignment away from Tx is taken into account. Respective optimum design variables are summarized in Table 6-3. By employing variable frequency operation, both PTE and TP can be maintained around 90% up to $z=25\text{cm}$. It is significant to note that TP can be improved to 80% from 40% at 10cm axial misalignment ($z=30\text{cm}$). Therefore, operating frequency tuning can be used in applications that require higher axial misalignment tolerance. Variable frequency operation is not analyzed with respect to angular misalignments, because the fixed frequency operation itself provided satisfactory performance needed for most of the applications.

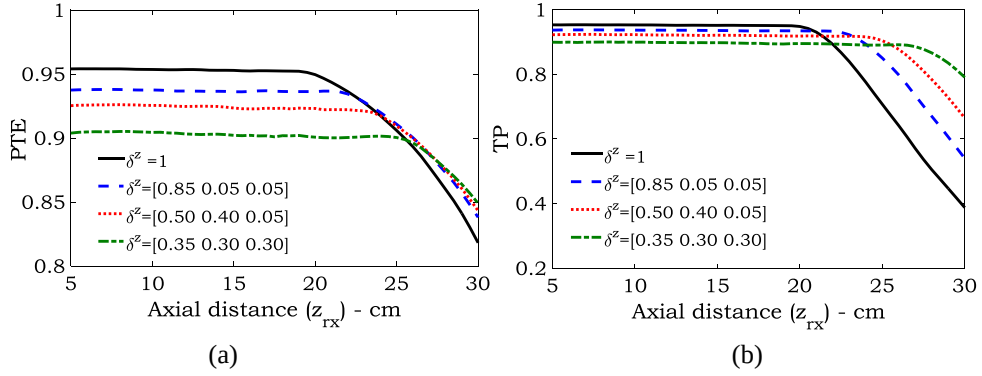


Fig. 6-16. Performance variation against axial misalignment when the design is optimized for axial misalignment tolerance with variable frequency operation.

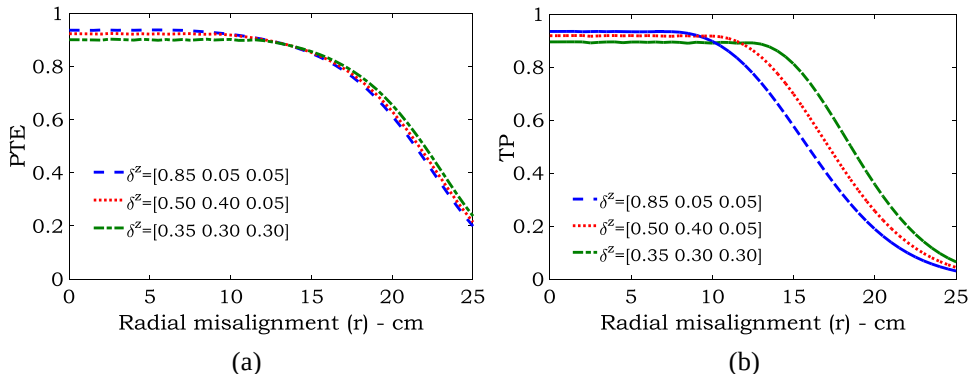


Fig. 6-17. Performance variation against radial misalignment when the design is optimized for axial misalignment tolerance with variable frequency operation (a) – PTE and (b) – TP.

6.5.2 Combined FOM of design

Thus far, misalignment tolerance for each DoF is analyzed individually using the respective decoupled FOMs. However, some WPT applications such as biomedical implants and IoT are intended to be designed with misalignment tolerance in two or more DoFs simultaneously. However, an optimization focusing on a single DoF may result poor performance against other DoF misalignments. For example, Fig. 6-17 shows performance variation against radial misalignment when the design is optimized for axial misalignment with frequency tuning. It can be seen that both PTE and TP deteriorate drastically beyond 15cm radial misalignment. On the other hand, Fig. 6-18 shows performance against axial misalignment when the design objective is on radial misalignment. Performance against axial misalignment also shows lower performance when design focus is only on radial misalignment, in particular, when higher probabilities are assigned to misaligned regions.

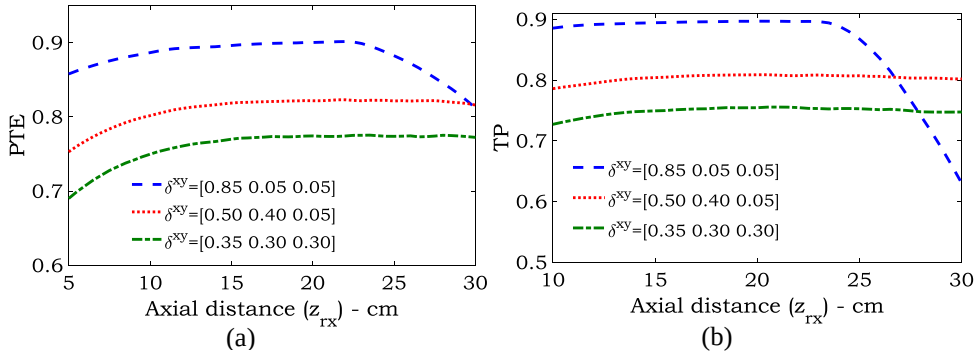


Fig. 6-18. Performance variation against axial misalignment when the design is optimized for radial misalignment tolerance with variable frequency operation (a) – PTE and (b) – TP.

Therefore, the combined FOM, F_{des} , as defined in (6-9) needs to be used as the optimization objective in order to improve the misalignment tolerance in both axial and radial DoF. Fig. 6-19 shows performance variation against both radial and axial misalignments when F_{des} is chosen as the optimization objective, and Table 6-3 shows optimum design variables for each case. Alignment probabilities for three sub regions in radial and axial directions are chosen to be identical in this analysis. It can be seen that performance can be improved against both radial and axial misalignment tolerances. The optimization condition with $\delta^{xy} = \delta^z = [0.5; 0.4; 0.05]$ provides both PTE and TP higher than 85% up to 20cm radial misalignment and 30cm axial distance. Fig. 6-20 shows performance variation against rotational misalignment when the design is

optimized to F_{des} considering radial and axial misalignment. Noteworthy, it can be seen that the proposed TSR optimized to radial and axial misalignment allows nearly constant performance against rotational misalignment. Therefore, 6DoF misalignment tolerance can be achieved with the use of TSR optimized to proposed FOM and variable frequency operation.

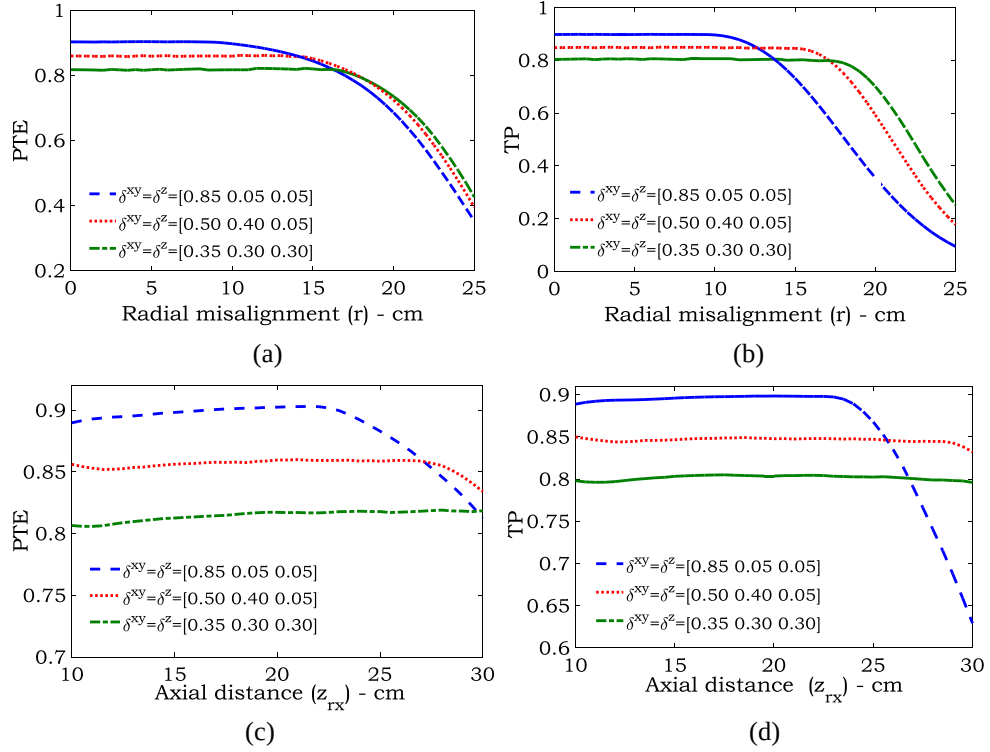


Fig. 6-19. Performance variation when the design is optimized to the combined FOM considering radial and axial misalignment ($F_{des} = F^{xy} \times F^z$) – Theoretical results: (a) – PTE vs. radial misalignment, (b) – TP vs. radial misalignment, (c) – PTE vs. axial misalignment, (d) – TP vs. axial misalignment.

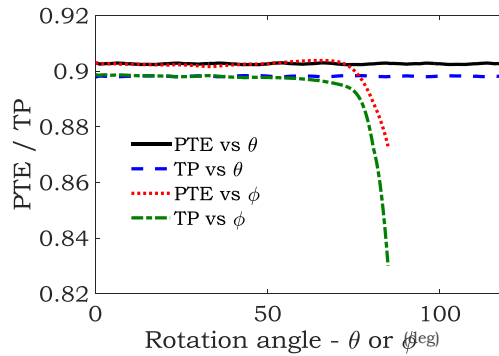


Fig. 6-20. Performance variation against rotational misalignment around z-axis (θ) and x-axis (ϕ), $\delta^{xy} = \delta^z = [0.85; 0.05; 0.05]$.

Table 6-3. Optimum design parameters for different design cases with variable frequency operation

FOM	δ	u (cm)	v (cm)	z_{TSR1} (cm)	z_{TSR2} (cm)	θ_{opt} (deg.)
F^{xy} $S^{xy}=[10; 20; 25]\text{cm}$	[0.85; 0.05; 0.05]	4.9	4.9	9.0	9.3	58
	[0.50; 0.40; 0.05]	4.9	4.9	12.2	11.9	58
	[0.35; 0.30; 0.30]	4.8	4.8	13.4	13.2	62
F^z $S^z=[2; 5; 10]\text{cm}$	[0.85; 0.05; 0.05]	1.7	1.6	0.4	0.6	84
	[0.50; 0.40; 0.05]	1.4	1.4	1.1	1.1	74
	[0.35; 0.30; 0.30]	0.9	0.9	1.9	1.8	62
$F_{des}=F^{xy} \times F^z$	[0.85; 0.05; 0.05]	4.9	4.9	8.9	9.1	69
	[0.50; 0.40; 0.05]	4.3	4.3	8.8	8.7	63
	[0.35; 0.30; 0.30]	4.3	4.3	9.9	9.9	63

Fig. 6-21 illustrates experimental results of performance against axial and radial misalignment. Experimental PTE and TP profiles match with the theoretical analysis well. However, experimental results show lower value than theoretical values at higher misalignments. This could be due to the experimental disparities during the construction of TSRs. In particular, the performance is sensitive to radial separation which is imperfect due to manual construction of TSR.

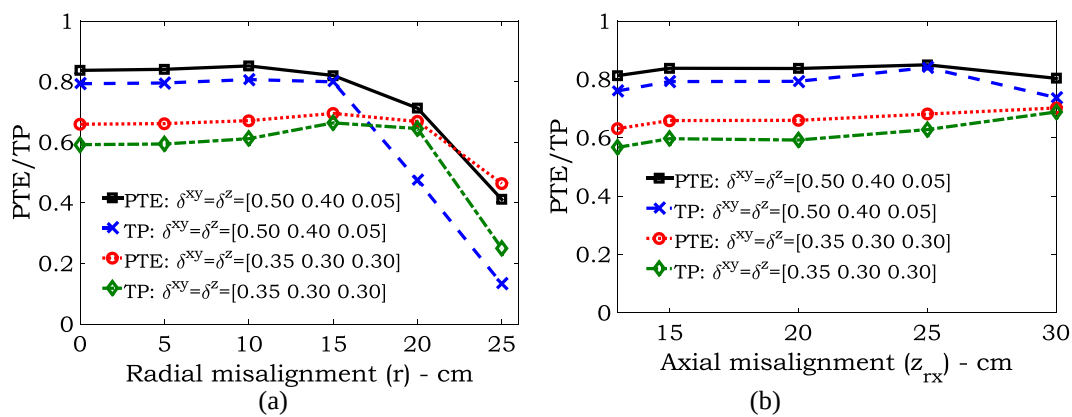


Fig. 6-21. Experimental results: Performance variation against (a) – axial and (b) – radial misalignment when the design is optimized to the combined FOM considering radial and axial misalignment

$$(F_{des}=F^{xy} \times F^z).$$

6.6 Summary

This chapter presented an optimization of WPT system to improve the performance against misalignment. A novel figure-of-merit (FOM) was introduced as an optimization objective function in order to improve system performance and misalignment tolerance simultaneously. FOM is defined by incorporating system level performance indices such as power transfer efficiency (PTE) and transferred power (TP), as well as the application specific design constraints such as misalignment tolerance in any of Rx's DoF and probability of alignment. It was investigated that FOM allows improved performance against misalignment with a slight performance reduction at the aligned position. Theoretical analysis was verified using the experiments. The proposed FOM provides design freedom to select its parameters such as sub-regions in each of Rx's misalignment DoF and probability of alignment in each sub region based on the application. Therefore, the proposed approach can be implemented with diverse design scenarios and applications. Furthermore, the proposed FOM can be applied not only to the WPT with repeaters, but also in a variety of optimization approaches such as compensation circuit, coil design and power electronics control.

Chapter 7

Repeater Tuning for Load Variation and Misalignment Tolerance

7.1 Introduction

The key WPT performance indices include power transfer efficiency (PTE), transferred power (TP), misalignment tolerance and load variation characteristics. The investigations in previous chapters allowed the improvement of PTE, TP and misalignment tolerance for a given nominal load condition. For instance, the performance improvement against misalignment was investigated for a nominal load of 50Ω in Chapter 6. Although the proposed investigations in Chapter 6 can be applied to an arbitrary nominal load, the performance decreases when the load resistance deviates from the nominal value, which is inevitable in most of the applications. On the other hand, in order to achieve best possible performance, the optimal load resistance has been derived for different operating conditions. For instance, with the assumption of optimum load condition, both quality factor of the coils and coupling coefficient between coils are used as the main optimization objectives in Chapter 3 for the optimization of WPT coils. Although, the investigation on the optimal load resistance is useful for the initial design stage, it is not always possible to operate at optimal load condition during operational environments. The value of the load is dependent on the application specific criteria. For example, equivalent load resistance of a battery charging application is dependent on the battery state-of-charge (SOC). In addition in case of WPT for ventricular assist device, the equivalent load resistance varies according to heart pressure and pump speed. Therefore, it is important to improve the system performance for non-optimal load conditions.

Therefore, this chapter investigates an adaptive optimization scheme for WPT with repeaters for varying load conditions. The theoretical analysis is performed for two types of repeaters, namely, single spiral repeater (SSR) and tri-spiral repeater (TSR), which were also used for the study in Chapter 6. Although the TSR-WPT setup has not been studied in the literature, WPT schemes involving SSR-WPT setup can be found in

literature. Usually, SSR-WPT setups are termed as 3-coil WPT [33, 60, 79, 80]. For example, it is proved in [79] that the 3-coil WPT outperforms its 2-coil counterpart for respective optimum load conditions. On the other hand in [80], frequency characteristics and the effects of geometric position of repeater is investigated. Their study focused on PTE and TP profiles for a nominal load value, and enhancement of load characteristic was not investigated. In addition, a coupling independent region has been investigated for strongly coupled region in [109]. However, a generalized optimization approach with the use of repeaters has not been investigated in literature. In particular, simultaneous improvement of both PTE and TP against load variations has not been investigated. Therefore, the theoretical contribution of this chapter is useful for the improvement of load characteristics. In addition to the improvement of load characteristics, misalignment tolerance can also be improved by employing the same resonance frequency tuning approach. Such tuning can be useful for applications when the optimization method proposed in Chapter 6 does not give satisfactory performance. For example, assignment of high probabilities to larger misaligned regions in multiple directions results in lower performance at aligned position. The investigation in this chapter shows that the TSR-WPT scheme significantly improved performance against misalignment.

7.1.1 WPT system with repeaters

The WPT scheme analyzed in this chapter comprises of a planar repeater placed near the Tx, as illustrated in Fig. 7-1 (a). The planar repeater can be constructed with spiral coils as discussed in previous chapters. Two types of repeaters, single-spiral-repeater (SSR) and tri-spiral repeater (TSR) are employed for the analysis in this chapter. As the names imply, SSR consists of a single spiral coil while TSR consists of three symmetrically placed spiral coils. Schematics in Fig. 7-1 (b) and (c) show SSR and TSR respectively. The overall physical sizes of SSR and TSR are chosen to be of the same footprint for a fair comparison. All coils in TSR (x_1 , x_2 and x_3) are symmetrically placed with a distance, u , from the center of the repeater. The axial distance between Tx and repeater is denoted as z_x . Transfer distance (z_{rx}) is considered as the unobstructed axial separation between the repeater and the Rx, while the misalignments, x_{rx} and y_{rx} refer to the displacements in x-direction and y-direction respectively.

This chapter is organized as follows. Section 7.2 introduces the theoretical analysis of WPT scheme using two-port theory. In addition, an optimization approach is proposed to improve both TP and PTE for SSR-WPT and TSR-WPT setups. Next, a numerical case study is presented in Section 7.3 where its load variation characteristics are improved by applying the proposed optimization. The numerical analysis is validated using experimental results in Section 7.4. The same optimization principle is applied to improve the performance against misalignment in Section 7.5 which is followed by the chapter summary.

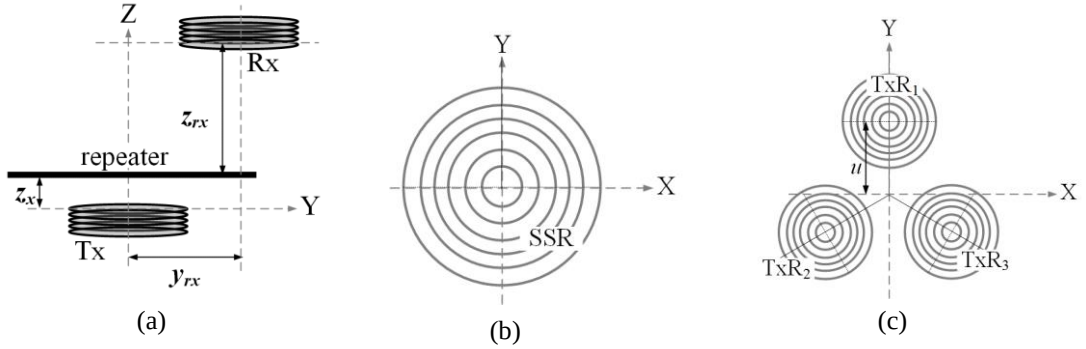


Fig. 7-1. (a) - WPT setup with a planar repeater placed at the Tx. (b) Single spiral repeater and (b) – tri-spiral repeater.

7.2 Theoretical analysis of WPT with a repeater

This section introduces the theoretical analysis of the WPT with a repeater. First, two-port model is applied for the analysis of the WPT system. The performances are evaluated in terms of generalized two port impedance parameters, and the conditions for maximum PTE and TP are derived. Next, the generalized theory is applied to SSR-WPT and TSR WPT along with the numerical analysis and experimental validations.

7.2.1 General two-port analysis

The two-port representation of a WPT setup with a repeater is shown in Fig. 7-2. Input port (port 1) is considered as the terminal of Tx coil (including compensation) while the output port (port 2) is considered as the terminals of load resistance (R_L). Therefore, compensated Tx, Rx and the repeater are encapsulated within the two-port network. As discussed in Chapter 2, the two-port impedance parameter matrix can be defined as in (7-1), where V_1 and V_2 are RMS voltages across port 1 and port 2 respectively, and I_1 and I_2 are RMS currents through port 1 and port 2 respectively. Impedance parameters of two port network Z_{11} , Z_{22} and Z_{12} can be calculated using

equivalent circuit method by knowing the coil arrangements within the two-port network which are comprised of coil parameters (resistances, self-inductances and compensation capacitances) and mutual inductances between coils. Similar to the all the other chapters in this thesis, the source impedance (R_s) and load impedance (R_L) are assumed to be purely resistive for the analysis.

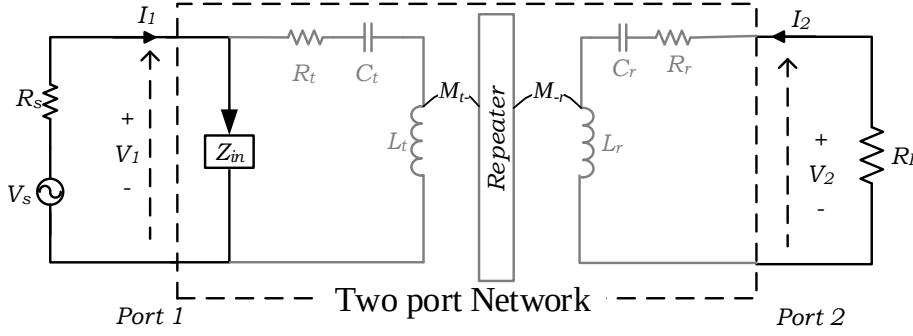


Fig. 7-2. Two-port network model of a WPT scheme with a repeater

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \quad (7-1)$$

Input impedance, Z_{in} , seen at port 1 can be determined as in (7-2). Afterwards, input power (P_{in}) and output power (P_{out}) are derived in (7-3) using impedance parameters as discussed in Chapter 2. Accordingly, the performance indices PTE and TP can be computed as in (7-4).

$$Z_{in} = \frac{V_1}{I_1} = Z_{11} - \frac{Z_{21}Z_{12}}{Z_{22} + R_L} \quad (7-2)$$

$$P_{in} = |I_1|^2 R_{in} = \frac{|V_s|^2}{|R_s + Z_{in}|^2} R_{in} \quad (7-3)$$

$$P_{out} = |I_2|^2 R_L = \frac{|Z_{21}|^2 |V_s|^2}{|(Z_{in} + R_s)(Z_{22} + R_L)|^2} R_L$$

where $R_{in} = \text{Re}(Z_{in})$

$$PTE = \frac{P_{out}}{P_{in}} = \frac{|Z_{12}|^2}{|R_L + Z_{22}|^2} \frac{R_L}{R_{in}} \quad (7-4)$$

$$TP = \frac{P_{out}}{|V_s|^2/4|R_s|} = \frac{4|Z_{21}|^2|R_s|R_L}{|(Z_{in} + R_s)(Z_{22} + R_L)|^2}$$

In order to further analyze the performance characteristics, the real and imaginary components of the equivalent impedance parameters can be defined as in (7-5) where r_{ij} and x_{ij} are real and imaginary components respectively. A notable difference between classical 2-coil WPT and WPT with a repeater is that the off-diagonal term ($Z_{12}=Z_{21}$) contains a non-zero real part for the WPT with repeaters (i.e., $r_{12} \neq 0$), whereas, Z_{12} is purely imaginary ($r_{12}=0$) for classical 2-coil WPT. Therefore, optimum conditions derived for the classical 2-coil WPT is not valid for a WPT with repeaters.

$$Z_{11} = r_{11} + jx_{11} \quad Z_{12} = Z_{21} = r_{12} + jx_{12} \quad Z_{22} = r_{22} + jx_{22} \quad (7-5)$$

With the definition of (7-5), performance indices, PTE and TP can be characterized with respect to impedance parameters. The expression for PTE is obtained as in (7-6), by substituting (7-5) into (7-4).

$$PTE = \frac{(x_{12}^2 + r_{12}^2)R_L}{r_{11}(R_L + r_{22})^2 + (R_L + r_{22})(x_{12}^2 - r_{12}^2) + x_{22}(r_{11}x_{22} - 2r_{12}x_{12})} \quad (7-6)$$

For a given two-port network the optimum value of x_{22} which gives maximum PTE, x_{22,PTE_max} is computed by using differential technique as in (7-7), and respective maximum PTE is derived in (7-8). For classical 2-coil WPT, x_{22} represents of the reactive impedance of Rx coil. According to (7-7), for the classical 2-coil WPT, x_{22,PTE_max} is zero since the value of r_{12} is zero which coincides with the well-known relation of maximum PTE at resonance frequency of Rx.

$$x_{22,PTE_max} = \frac{r_{12}x_{12}}{r_{11}} \quad (7-7)$$

$$PTE = \frac{r_{11}(x_{12}^2 + r_{12}^2)R_L}{[r_{11}(R_L + r_{22}) + x_{12}^2][r_{11}(R_L + r_{22}) - r_{12}^2]} \quad (7-8)$$

Next with the assumption of optimum x_{22} , the optimum load resistance that gives maximum PTE (R_{L,PTE_max}) can be derived as (7-9) using the differential technique.

$$R_{L,PTE_max} = \frac{\sqrt{(r_{11}r_{22} + x_{12}^2)(r_{11}r_{22} - r_{12}^2)}}{r_{11}} \quad (7-9)$$

On the other hand, the expression for TP is obtained as in (7-10), by substituting (7-5) into (7-4).

$$TP = \frac{4(x_{12}^2 + r_{12}^2)R_L R_s}{[\sigma_2(R_L + r_{22}) + \sigma_1 x_{22}]^2 + [\sigma_1(R_L + r_{22}) - \sigma_2 x_{22}]^2} \quad (7-10)$$

$$\text{where } \sigma_1 = R_s + r_{11} - \frac{(R_L + r_{22})(r_{12}^2 - x_{12}^2) + 2r_{12}x_{12}x_{22}}{(R_L + r_{22})^2 + x_{22}^2}, \text{ and } \sigma_2 = x_{11} + \frac{r_{22}(r_{12}^2 - x_{12}^2) - 2r_{12}x_{12}(R_L + r_{22})}{(R_L + r_{22})^2 + x_{22}^2}$$

For a given two-port network the optimum value of x_{22} which gives maximum TP, x_{22,TP_max} is computed by using differential technique as in (7-11).

$$x_{22,TP_max} = \frac{x_{11}(x_{12}^2 - r_{12}^2) + 2r_{12}x_{12}(R_s + r_{11})}{(R_s + r_{11})^2 + x_{11}^2} \quad (7-11)$$

TP characteristics is dependent on the input impedance. Real and imaginary part of the input impedance (R_{in} and X_{in} respectively) is derived in (7-12).

$$\begin{aligned} \text{Re}(Z_{in}) = R_{in} &= r_{11} - \frac{(R_L + r_{22})(x_{12}^2 - r_{12}^2) - 2r_{12}x_{12}x_{22}}{(R_L + r_{22})^2 + x_{22}^2} \\ \text{Im}(Z_{in}) = X_{in} &= x_{11} + \frac{x_{22}(x_{12}^2 - r_{12}^2) - 2r_{12}x_{12}(R_L + r_{22})}{(R_L + r_{22})^2 + x_{22}^2} \end{aligned} \quad (7-12)$$

According to the theory of impedance matching, TP is maximized when the conditions in (7-13) and (7-14) are satisfied [110].

$$X_{in} = 0 \quad (7-13)$$

$$R_{in} = R_s \quad (7-14)$$

As discussed in previous chapters, both PTE and TP cannot be optimized simultaneously for a classical 2-coil WPT setup. However, it was numerically investigated in Chapter 6 that both PTE and TP can be improved simultaneously for WPT with repeaters. This is because the two port network parameters can be optimized by manipulating the arrangement of repeater (thereby changing mutual inductances) and

the resonance frequencies of the coils (thereby changing reactive impedances). However, physical arrangement cannot be used as an optimization candidate for a dynamic tuning approach. Instead, self-resonance frequency can be varied with a use of electronics tuning approaches [101]. However, all the design condition derived for maximum PTE and maximum TP in (7-6) to (7-14) cannot be achieved simultaneously. Therefore, an optimization scheme need to be derived to obtain maximum possible PTE and TP by selecting a proper design conditions which do not contradict each other. For example, impedance matching condition responds to real input impedance in (7-14) cannot be fully satisfied when coupling between Tx and Rx is very poor. Therefore, the following sub-sections propose an approach to obtain an optimal solution, analyzing the theoretical basis of the optimization with regards to SSR and TSR for arbitrary load conditions.

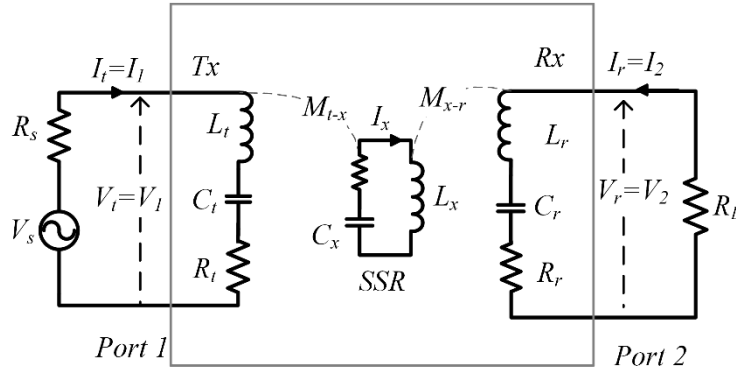


Fig. 7-3. The equivalent circuit and two-port model illustration for SSR-WPT

7.2.2 Two-port analysis of SSR-WPT setup

The general two-port analysis is applied to the SSR-WPT scheme. The equivalent circuit of the SSR-WPT setup is illustrated in Fig. 7-3. The system equation is defined in (7-15) using the equivalent circuit theory.

$$\begin{bmatrix} V_t \\ 0 \\ V_r \end{bmatrix} = \begin{bmatrix} Z_t & j\omega M_{t,x} & j\omega M_{t,r} \\ j\omega M_{t,x} & Z_x & j\omega M_{x,r} \\ j\omega M_{t,r} & j\omega M_{x,r} & Z_r \end{bmatrix} \begin{bmatrix} I_t \\ I_x \\ I_r \end{bmatrix} \quad (7-15)$$

where $Z_k = R_k + jX_k$, $X_k = \omega L_k - \frac{1}{\omega C_k}$, $V_t = V_s - I_t R_s$, and $V_r = I_r R_L$

Diagonal elements of the impedance matrix, Z_k ($k=t, x$, and r) are the self-impedances of Tx, Rx and repeater coils, while L_k , C_k , and R_k are inductance, capacitance and

parasitic resistance respectively. X_k ($k=t, x$, and r) represents the reactive impedances of coils at operating frequency (f_s). $j\omega M_{k,l}=j\omega M_{l,k}$ ($k,l=t, x$, and r) where $M_{k,l}=M_{l,k}$ is the mutual inductance between k^{th} coil and l^{th} coil, and ω is the operating angular frequency of the source. R_L is the equivalent load resistance. Once the system equation is defined, currents in each coil (I_k ; $k=t, x$, and r) can be determined. Subscripts t, x , and r , represent Tx, Rx and repeater respectively. It should be emphasized that the format of the system equation in (7-15) is slightly modified from the system equation defined in previous chapters, by including the voltage across load (V_r) to the left hand side of the equation. This is useful in deriving the two port equivalent impedance parameters in the following analysis. The two port equivalent of the SSR-WPT scheme can be defined as,

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{12} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \quad (7-16)$$

where $V_1 = V_t$, $V_2 = V_r$, $I_1 = I_t$ and $I_2 = I_r$

Afterwards, the elements of the impedance matrix in (7-16) can be evaluated by manipulating the impedance matrix in (7-15) as in (7-17). It can be seen that the real and imaginary components of the two port impedance parameters are modified from its 2-coil counterpart due to the addition of SSR. For example, an additional term is added to the real part of the diagonal impedance terms (r_{11} and r_{22}) compared to its 2-coil counterpart. This additional term includes resistance of SSR and mutual impedance from SSR to Tx (or, to Rx) which can be considered as the added loss component incurred due to the addition of SSR. On the other hand, real component of the off-diagonal impedance term (r_{12}) becomes a non-zero value which is equal to the square root of the multiplication of two additional terms introduced for diagonal real components. This non-zero term, r_{12} has positive impact on the PTE expression as identified in previous section. Similarly, imaginary components of impedance terms are also modified accordingly. Therefore, unlike classical 2-coil WPT setup, an optimization cannot be carried out by considering individual terms in the impedance matrix for SSR-WPT setup.

The impedance matrix in (7-16) can be optimized by changing the self-resonances of the coils, thereby changing the respective reactive impedances, X_t , X_x , and X_r . Therefore, resonance frequencies of Tx (f_{Tx}), Rx (f_{Rx}), SSR (f_x) and the operating frequency (f_s) are used as the optimization variables. An optimized design for PTE and

TP is introduced by considering performance characteristics and resonance theory as defined in following steps.

$$\bar{Z}_{11} = r_{11} + jx_{11}, \quad \bar{Z}_{12} = r_{12} + jx_{12}, \quad \bar{Z}_{22} = r_{22} + jx_{22} \quad (7-17)$$

where

$$r_{11} = R_t + \frac{R_x(\omega M_{t.x})^2}{R_x^2 + X_x^2}$$

$$r_{22} = R_r + \frac{R_x(\omega M_{x.r})^2}{R_x^2 + X_x^2}$$

$$r_{12} = \frac{R_x \omega^2 M_{t.x} M_{x.r}}{R_x^2 + X_x^2}$$

$$x_{11} = X_t - \frac{X_x(\omega M_{t.x})^2}{R_x^2 + X_x^2}$$

$$x_{22} = X_r - \frac{X_x(\omega M_{x.r})^2}{R_x^2 + X_x^2}$$

$$x_{12} = \omega M_{t.r} - \frac{X_x \omega^2 M_{t.x} M_{x.r}}{R_x^2 + X_x^2}$$

Step 1 – First, according to the resonance theory, the maximum energy is absorbed to Rx when it resonates at the same frequency as the exciting frequency of the magnetic field. Therefore, the reactive impedance of the Rx coil should be zero at the optimum operating frequency as shown in (7-18).

$$X_r = 0 \quad (7-18)$$

Step 2– Next, the optimum reactive impedance of Tx (X_t) is derived as (7-19) and conforms to reactive impedance matching condition in (7-13).

$$X_t = \frac{\omega R_t M_{t.r} M_{t.x} M_{x.r} \{2R_t(\omega M_{x.r})^4 + R_x[(\omega M_{x.r})^2[(\omega M_{t.r})^2 + 2R_t \Lambda_L] - \Lambda_L^2(\omega M_{t.x})^2]\}}{\Lambda_L^2 R_x^2 [(\omega M_{t.r})^2 M_{t.x}^2 + R_t^2 M_{x.r}^2] + 2\omega^2 R_x R_t^2 \Lambda_L M_{x.r}^4 + \omega^4 R_t^2 M_{x.r}^6} \quad (7-19)$$

where $\Lambda_L = (R_L + R_r)$

Step 3– Next, the optimum reactive impedance of SSR (X_x) can be determined using differential method where TP becomes maximum. However, the closed form solution for the optimum value of X_x results in a very complicated expression, therefore, fourth order polynomial form of a solution is expressed as in (7-20). In order to obtain the optimal value of X_x , (7-20) need to be solved for real roots.

$$a_4 X_{x-opt}^4 + a_3 X_{x-opt}^3 + a_2 X_{x-opt}^2 + a_1 X_{x-opt} + a_0 = 0 \quad (7-20)$$

$$a_0 = -M_{t.r} M_{t.x} M_{x.r} [R_x \Lambda_L + (\omega M_{x.r})^2]^2 \{R_x \Lambda_L^2 (R_x \Lambda_S + (\omega M_{t.x})^2) + \Lambda_L [2R_x (\omega M_{x.r})^2 \Lambda_S - (\omega M_{t.x})^2 R_x^2 - (\omega M_{t.x})^2 (\omega M_{x.r})^2] + \Lambda_S (\omega M_{x.r})^4 + R_x (\omega M_{t.r})^2 (\omega M_{x.r})^2\},$$

$$a_1 = \omega [(\omega M_{x.r})^2 + R_x \Lambda_L] \{\Lambda_L^3 M_{t.x}^2 [2R_x^2 M_{t.r}^2 + \omega^2 M_{t.x}^2 M_{x.r}^2 - R_x M_{x.r}^2 \Lambda_S] - \Lambda_L^2 M_{x.r}^2 [(\omega M_{t.x})^2 M_{x.r}^2 \Lambda_S + R_x M_{t.r}^2 - 2M_{t.r}^2 R_x^2 \Lambda_S] + 3\Lambda_L (\omega M_{t.r})^2 M_{x.r}^4 [R_x \Lambda_L - (\omega M_{t.x})^2] + (\omega M_{t.r})^2 M_{x.r}^4 [\Lambda_S (\omega M_{x.r})^2 + R_x (\omega M_{t.r})^2]\},$$

$$a_2 = -3\omega^2 M_{t.r} M_{t.x} M_{x.r} \Lambda_L [(\omega M_{x.r})^2 + R_x \Lambda_L] [\Lambda_L^2 M_{t.x}^2 - \omega^2 M_{t.r}^2 M_{x.r}^2],$$

$$a_3 = \omega \Lambda_L \{\Lambda_L^3 M_{t.x}^2 [2R_x^2 M_{t.r}^2 - M_{x.r}^2 \Lambda_S] + \Lambda_L^2 M_{t.r}^2 M_{x.r}^2 [3(\omega M_{t.x})^2 + 2\Lambda_S R_x] + \Lambda_L \Lambda_S (\omega M_{t.r})^2 M_{x.r}^4 - (\omega M_{t.r})^4 M_{x.r}^4\},$$

$$a_4 = M_{t.r} M_{t.x} M_{x.r} \Lambda_L^2 [\Lambda_L \Lambda_S - (\omega M_{t.r})^2], \Lambda_S = (R_s + R_t) \text{ and } \Lambda_L = (R_L + R_r)$$

Steps 1 to 3 define the optimum values of the reactive impedances of Tx, Rx, and SSR. Therefore, the optimum self-resonance frequencies of Tx (f_t), Rx (f_r) and SSR (f_x) can be determined by knowing the optimal reactive impedances using the relationship in (7-21).

$$f_k = \frac{1}{2\pi} \sqrt{\omega^2 - \frac{\omega X_k}{L_k}} \quad (7-21)$$

Step 4–After the above three steps, both PTE and TP can be expressed as a function of single unknown variable of operating frequency. Finally, the optimum operating frequency can be determined by a simple variable sweep with respect to operating frequency.

7.2.3 Two-port analysis of TSR-WPT setup

Next, the general two-port analysis is applied to the TSR-WPT scheme. The system equation (7-22), can be expressed based on the equivalent circuit shown in Fig. 7-4. The introduction of SSR makes the rank of the system equation to be five. Three coils in TSR are termed as x_1 , x_2 and x_3 .

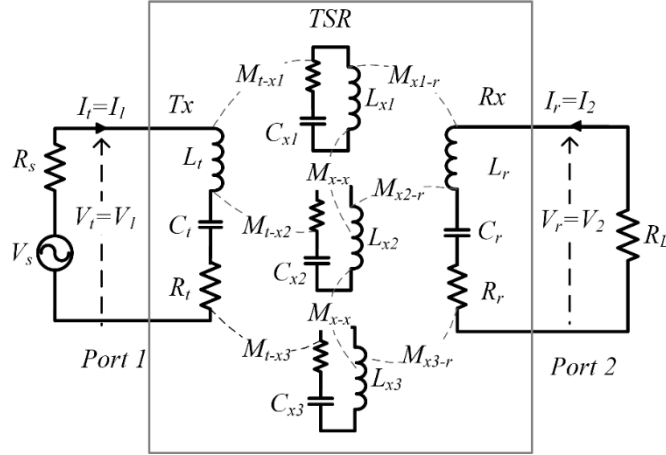


Fig. 7-4. The equivalent circuit and two-port model illustration for TSR-WPT

$$\begin{bmatrix} V_t \\ 0 \\ 0 \\ 0 \\ V_r \end{bmatrix} = \begin{bmatrix} Z_t & j\omega M_{t,x1} & j\omega M_{t,x2} & j\omega M_{t,x3} & j\omega M_{t,r} \\ j\omega M_{t,x1} & Z_{x1} & j\omega M_{x1,x2} & j\omega M_{x1,x3} & j\omega M_{x1,r} \\ j\omega M_{t,x2} & j\omega M_{x1,x2} & Z_{x2} & j\omega M_{x2,x3} & j\omega M_{x2,r} \\ j\omega M_{t,x3} & j\omega M_{x1,x3} & j\omega M_{x2,x3} & Z_{x3} & j\omega M_{x3,r} \\ j\omega M_{t,r} & j\omega M_{x1,r} & j\omega M_{x2,r} & j\omega M_{x3,r} & Z_r \end{bmatrix} \begin{bmatrix} I_t \\ I_{x1} \\ I_{x2} \\ I_{x3} \\ I_r \end{bmatrix} \quad (7-22)$$

where $Z_k = R_k + jX_k$, $X_k = \omega L_k - \frac{1}{\omega C_k}$, $V_t = V_s - I_t R_s$ and $V_r = I_r R_L$

Diagonal elements of the impedance matrix, Z_k ($k=t, x1, x2, x3$, and r) are the self-impedances of Tx, Rx and repeater coils, while L_k , C_k , and R_k are inductance, capacitance and parasitic resistance respectively. $j\omega M_{k,l} = j\omega M_{l,k}$ ($k, l=t, x1, x2, x3$, and r) where $M_{k,l} = M_{l,k}$ is the mutual inductance between k^{th} coil and l^{th} coil. R_L is the equivalent load resistance. ω is the operating angular frequency of the source. Once system equation is defined, currents in each coil (I_k ; $k=t, x1, x2, x3$, and r) can be determined. Similar to the SSR-WPT, the system equation for TSR-WPT is rearranged by including the voltage across load (V_r) to the left hand side of the system equation for the derivation of two port equivalent impedance parameters. Because of the symmetric structure of TSR, following mutual inductance relationship can be obtained.

$$M_{t,x1} = M_{t,x2} = M_{t,x3} = M_{t,x} \text{ and } M_{x1,x2} = M_{x2,x3} = M_{x1,x3} = M_x \quad (7-23)$$

It is assumed that the Rx is positioned at the aligned position for the subsequent analysis with TSR-WPT scheme. Therefore the following mutual inductance relationships can be obtained for the aligned position ($x_{rx}=y_{rx}=0$).

$$M_{x1.r} = M_{x2.r} = M_{x3.r} = M_{x.r} \quad (7-24)$$

The assumption of identical repeater coils makes the following resistance relationship.

$$R_{x1} = R_{x2} = R_{x3} = R_x \quad (7-25)$$

With the above simplifications for mutual inductances and coil resistances, the two port equivalent of the TSR-WPT scheme can be defined as,

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{12} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \quad (7-26)$$

where $V_1 = V_t$, $V_2 = V_r$, $I_1 = I_t$ and $I_2 = I_r$

Afterwards, the elements of the impedance matrix in (7-26) can be evaluated by manipulating the impedance matrix in the system equation, (7-22), according to the following relationships,

$$\begin{aligned} Z_{11} &= Z_t + \frac{3(\omega M_{t.x1})^2}{Z_{x1} + 2j\omega M_{x1.x2}} \\ Z_{12} &= j\omega M_{t-r} + \frac{3\omega^2 M_{t.x1} M_{x1.r}}{Z_{x1} + 2j\omega M_{x1.x2}} \\ Z_{22} &= Z_r + \frac{3(\omega M_{x1.r})^2}{Z_2 + 2j\omega M_{x1.x2}} \end{aligned} \quad (7-27)$$

Real and imaginary components of the impedance parameters, which are evaluated in (7-28) can be further analyzed for the optimum conditions. Similar to the SSR-WPT, it can be clearly seen that the addition of SSR has modified its impedance parameters compared to classical 2-coil WPT.

Similar to SSR-WPT setup, the impedance matrix in (7-26) can be optimized by changing the self-resonances of the coils, thereby changing the respective reactive

impedances, X_t , X_x , and X_r . Therefore similar to the SSR-WPT scheme, resonance frequencies of Tx (f_{Tx}), Rx (f_{Rx}), SSR (f_x) and the operating frequency (f_s) are used as the optimization variables for the performance optimization of TSR-WPT scheme. Therefore, the optimum frequencies can be derived using the same design steps as SSR-WPT case. However, the optimum design variables for TSR-WPT can be directly deduced by comparing (7-28) with (7-17). It can be seen that the two-port impedance matrix for TSR-WPT can be derived by substituting parameters of SSR-WPT as described in (7-29). Therefore, four steps introduced to obtain the optimal design condition for SSR-WPT can be directly applied to deduce the optimal conditions for TSR-WPT.

$$Z_{11} = r_{11} + jx_{11}, Z_{12} = r_{12} + jx_{12}, Z_{22} = r_{22} + jx_{22} \quad (7-28)$$

$$\text{where } r_{11} = R_t + \frac{3R_x(\omega M_{t,x})^2}{\sigma}, r_{12} = \frac{3R_x\omega^2 M_{t,x}M_{x,r}}{\sigma}, r_{22} = R_r + \frac{3R_x(\omega M_{x,r})^2}{\sigma}$$

$$x_{11} = X_t - \frac{3(X_x + 2\omega M_x)(\omega M_{t,x})^2}{\sigma}, x_{12} = \omega M_{t,r} - \frac{3(X_x + 2\omega M_x)\omega^2 M_{t,x}M_{x,r}}{\sigma},$$

$$x_{22} = X_r - \frac{3(X_x + 2\omega M_x)(\omega M_{x,r})^2}{\sigma} \text{ and } \sigma = (X_x + 2\omega M_x)^2 + R_x^2$$

$$M_{t,x} \rightarrow \sqrt{3}M_{t,x} \quad (7-29)$$

$$M_{x,r} \rightarrow \sqrt{3}M_{x,r}$$

$$X_x \rightarrow X_x + 2\omega M_x$$

With the above described method, the optimal design conditions for SSR-WPT and TSR-WPT schemes can be derived with respect to self-resonance frequencies of coils and the operating frequency. The subsequent sections of this chapter apply the proposed theory for the optimizations of SSR-WPT and TSR-WPT schemes against load variations and Rx misalignment.

7.3 Optimization against load variation

In order to substantiate the above theoretical analysis, a case study is presented in this section. It is investigated that the self-resonance frequencies of Tx, Rx and repeater along with the operating frequency can be tuned to improve the performance against

load variations. The same SSR-WPT and TSR-WPT schemes introduced in Chapter 6 is used for the analysis in this chapter. The physical properties of Tx, Rx, SSR and TSR are illustrated in Table 7-1. It should be noted that the arrangement of TSR is chosen to provide increased misalignment tolerance according to the investigations in Chapter 6. The following subsections present numerical analysis and experimental verifications for the optimization of SSR-WPT and TSR-WPT schemes. For this analysis, Rx is assumed to be at the aligned position.

Table 7-1. Design parameters of coils used for TSR and SSR

Coil Parameters				Setup Properties	
	Tx, Rx	TSR	SSR		
Outer radius	8.9 cm	8.9 cm	12.5 cm	TxR radial location (u)	4 cm
Turns separation	4.2 mm	5 mm	5 mm	Transfer distance (z_{rx})	15 cm
Number of turns	11	14	14	Z_{X-TSR}	0
Inductance	30 μ H	20 μ H	40 μ H	Z_{X-SSR}	0

7.3.1 Optimization of SSR-WPT setup against load variation

The proposed optimization technique is applied to SSR-WPT scheme. Equations (7-18) to (7-21) are employed to obtain the optimal operating conditions, and respective optimal performance is evaluated using (7-4). The variation of optimal frequencies with respect to load resistance are illustrated in Fig. 7-5(a), while the respective optimal PTE and TP are shown in Fig. 7-5(b). It can be seen that optimal frequencies increase with the increase of load resistance. In addition, the optimal PTE and TP show almost identical profiles, which curves overlapped in Fig. 7-5(a). Therefore, it is evident that the proposed optimization procedure allows simultaneous improvement of both PTE and TP. In order to implement the dynamic tuning strategy, a set of switched capacitor bank connected to coils can be used to tune the self-resonance similar to the approach used for adaptive impedance matching [101].

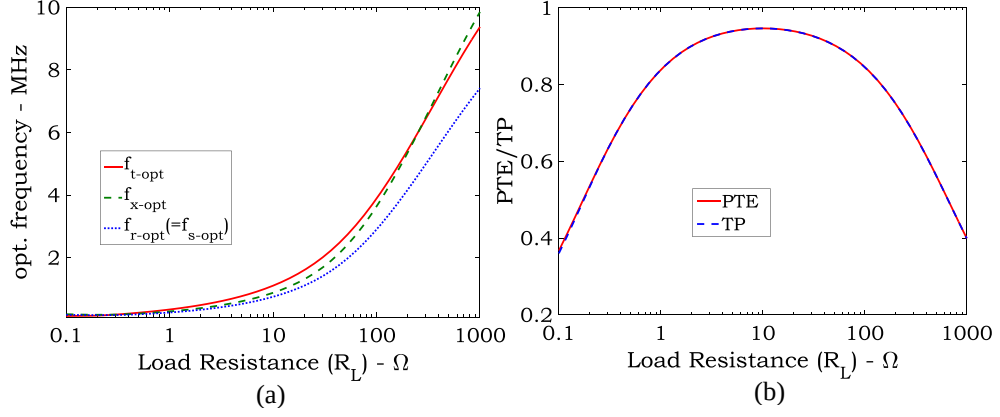


Fig. 7-5. Theoretical results: (a) – The variation of optimal frequencies (f_{t-opt} –Tx self-resonance, f_{x-opt} –SSR self-resonance, f_{r-opt} – Rx self-resonance, f_{s-opt} – operating frequency) with respect to load resistance and (b) – The respective optimal PTE and TP variations.

In order to evaluate the performance improvement that can be obtained with the dynamic tuning of self-resonance frequencies, Fig. 7-6 illustrates PTE and TP variation with respect to load resistance when SSR-WPT scheme is optimized to different nominal load resistances ($R_{L-nominal}$). It can be seen that a significant performance improvement can be obtained by frequency tuning approach. For example, 90% PTE and TP can be achieved at 50Ω load when the design is optimized for 50Ω load. However, such a design optimized to 50Ω load shows PTE and TP pairs as low as (40%, 7%) and (70%, 45%) at 1Ω and 250Ω loads respectively. On the other hand, both PTE and TP can be improved to 84% and 71% at 1Ω and 250Ω load values respectively when the design variables are optimized to the respective load resistances. Therefore, it is apparent that the tuning of self-resonances and operating frequency can be used to improve the performance for a wide load range.

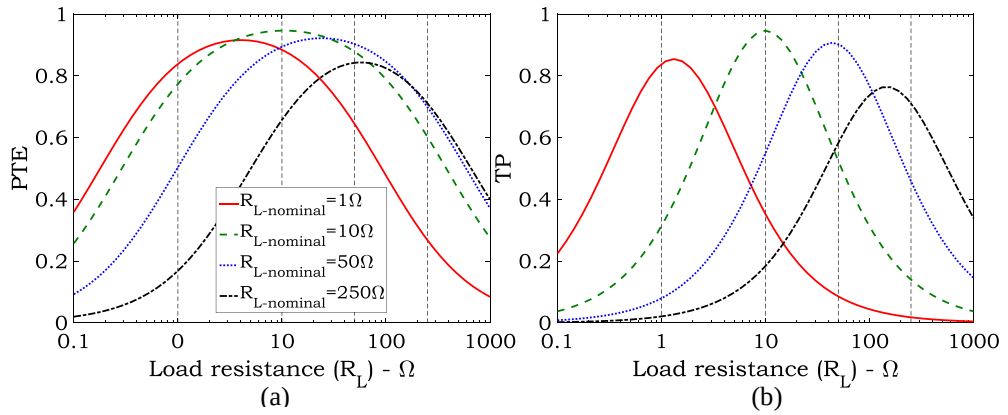


Fig. 7-6. Theoretical results: (a) – PTE variation and (b) – TP variation with respect to load resistance when SSR-WPT scheme is optimized to different nominal load resistance values ($R_{L-nominal}$)

It is interesting to note that, for a design optimized at a particular nominal load, the maximum PTE (or TP) occurs at a load value different from nominal load resistance. This is because the objective of the optimization is to achieve simultaneous improvement of PTE and TP. In order to show the impedance matching at optimum condition, Fig. 7-7 illustrates the variations of real and imaginary components of input impedance with respect to load resistance when SSR-WPT scheme is optimized to different nominal load resistance values. It can be clearly seen that the reactive impedance matching condition is fulfilled at the nominal load resistances. In addition, the real part of the input impedance matches closely with the source impedance (50Ω). However, the exact value of the real input impedance does not match exactly to 50Ω .

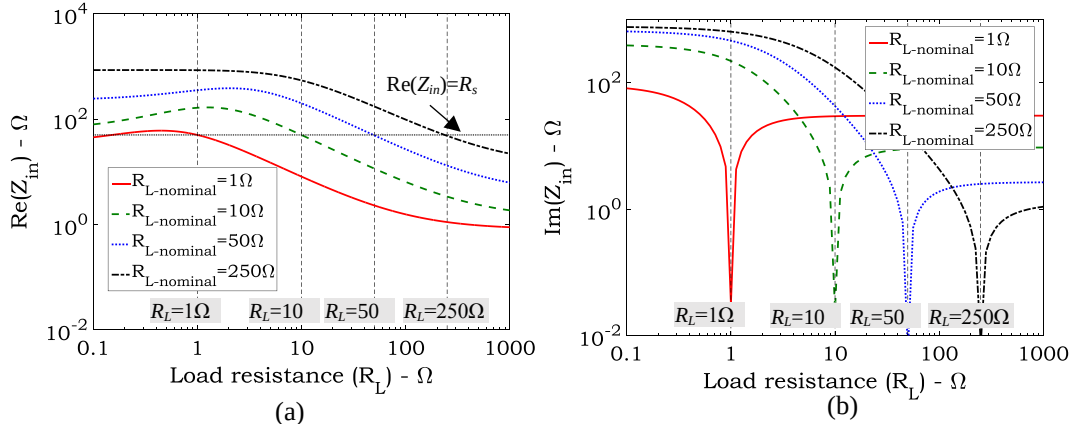


Fig. 7-7. Theoretical results: (a) – real and (b) – imaginary components of input impedance variations with respect to load resistance when SSR-WPT scheme is optimized to different nominal load resistance values ($R_{L-nominal}$).

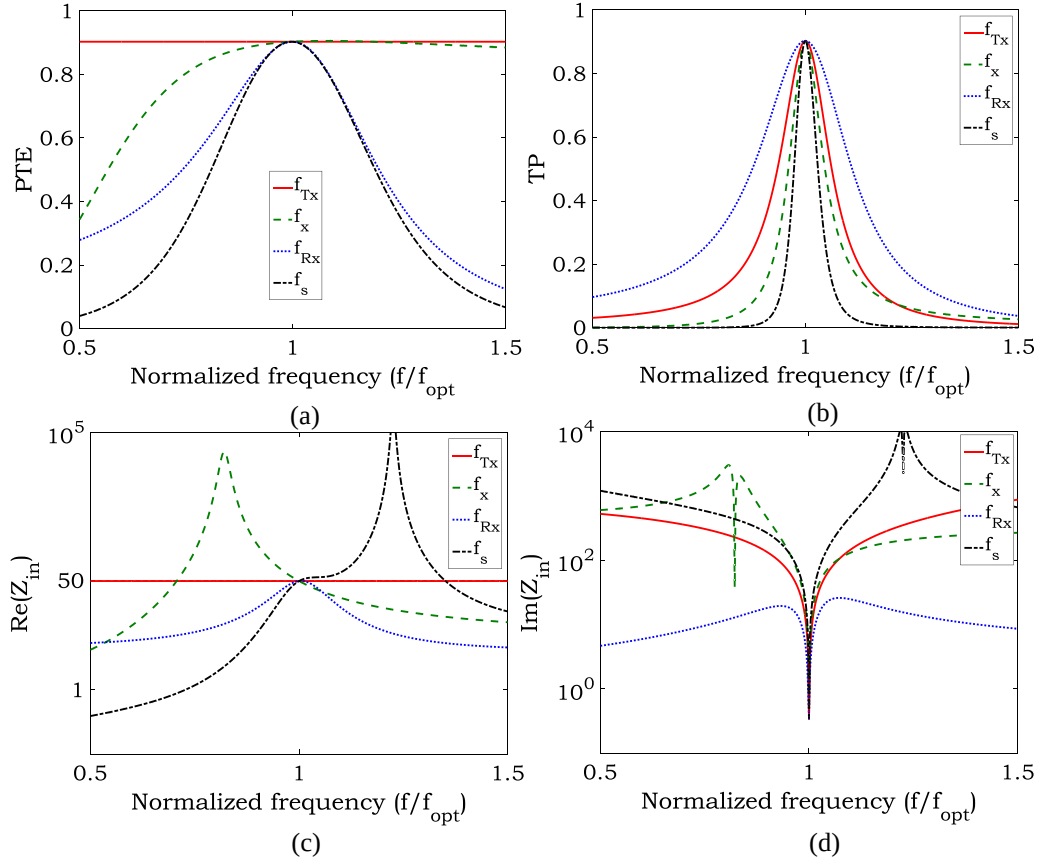


Fig. 7-8. Theoretical results: (a) – PTE, (b) – TP, (c) – real input impedance and (d) – imaginary input impedance variation against each frequency (each frequency is normalized to its optimal value (f_{opt})) for SSR-WPT scheme optimized to 50Ω load resistance.

Next, the performance variations are analyzed with respect to the optimization variables (i.e., f_t , f_x , f_t and f_s). Fig. 7-8 illustrates the performance variations with respect to each frequency variable (i.e., f_t , f_x , f_t and f_s) for the design optimized at a load resistance of 50Ω. The performance characteristics is generated by individually varying each frequency variable while all the other conditions are kept at their respective optimal values. For instance, when generating the curve corresponding to f_t , all the other parameters are kept at their respective optimum values. At the optimum point (normalized frequency = 1), it can be seen that both PTE and TP become maximum which forms a local maxima. In fact, the values of PTE and TP obtained with the above optimization procedure are almost identical to the global maxima, which will be shown in the subsequent section. In addition, it is apparent from the impedance characteristics shown in Fig. 7-8(c) and (d) that the optimum frequencies follow impedance matching conditions.

7.3.2 Optimization of TSR-WPT setup against load variation

Next, the proposed optimization technique is applied to TSR-WPT scheme. The setup parameters are depicted in Table 7-1. The variation of optimal frequencies with respect to load resistance is illustrated in Fig. 7-9(a), while the respective optimal PTE and TP are shown in Fig. 7-9(b). It can be seen that optimal frequencies increase with the increase of load resistance. In addition, the optimal PTE and TP show almost identical profiles, which curves overlapped in Fig. 7-9(b).

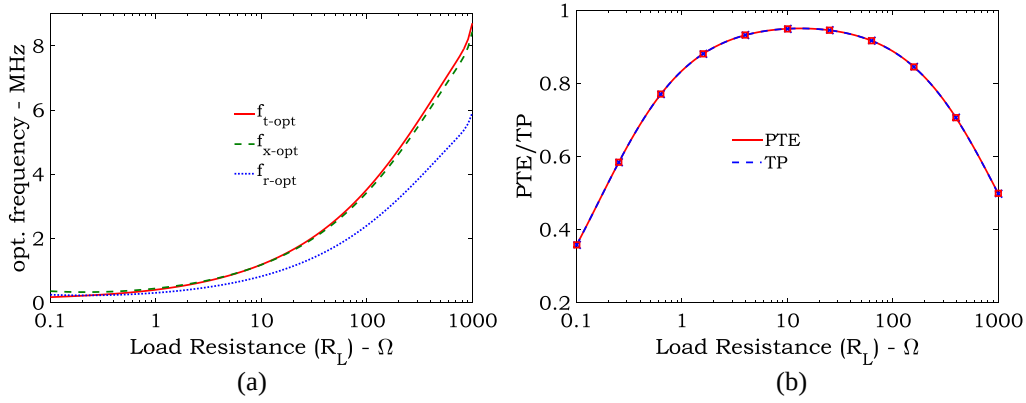


Fig. 7-9. Theoretical results: (a) – The variation of optimal frequencies (f_{t-opt} – Tx self-resonance, f_{x-opt} – TSR self-resonance, f_{r-opt} – Rx self-resonance, f_{s-opt} – operating frequency) with respect to load resistance and (b) – The respective optimal PTE and TP variations.

Therefore, it is apparent that the proposed optimization procedure allows simultaneous improvement of both PTE and TP. A dynamic tuning strategy can be implemented to track the optimal frequencies. A set of switched capacitor bank connected to coils can be used to tune the self-resonance similar to the approach used for adaptive impedance matching [101].

In order to evaluate the performance improvement that can be obtained with the dynamic tuning of self-resonance frequencies, Fig. 7-10 illustrates PTE and TP variation with respect to load resistance when TSR-WPT scheme is optimized to different nominal load resistance values ($R_{L-nominal}$). It can be seen that a significant performance improvement can be obtained by frequency tuning approach. For example, 90% PTE and TP can be achieved at 50Ω load when the design is optimized for 50Ω load. However, such a design optimized to 50Ω load shows PTE and TP pairs as low as (40%, 7%) and (70%, 45%) at 1Ω and 250Ω loads respectively. On the other hand, both PTE and TP can be improved to 84% and 71% at 1Ω and 250Ω load values respectively when

the design variables are optimized to the respective load resistances. Therefore, it is apparent that the tuning of self-resonances and operating frequency can be used to improve the performance in a wide load range.

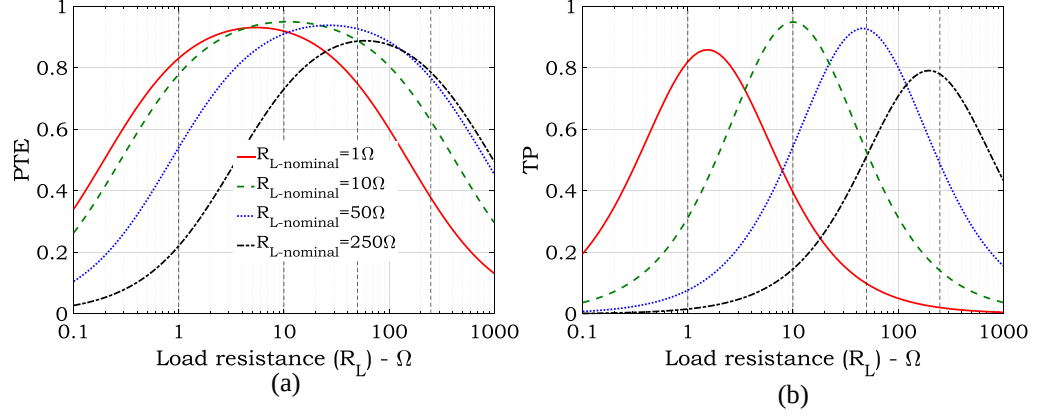


Fig. 7-10. Theoretical results: (a) – PTE variation and (b) – TP variation with respect to load resistance when TSR-WPT scheme is optimized to different nominal load resistance values ($R_{L-nominal}$)

It is interesting to note that, for a design optimized at a particular nominal load, the maximum PTE (or TP) occurs at a load value different from nominal load resistance. This is because the objective of the optimization is to achieve simultaneous improvement of PTE and TP. In order to show the impedance matching at optimum condition, Fig. 7-11 illustrates the variations of real and imaginary components of input impedance with respect to load resistance when SSR-WPT scheme is optimized to different nominal load resistance values. It can be clearly seen that the impedance matching conditions are fulfilled at the nominal load resistances. In addition, the real part of the input impedance matches closely with the source impedance (50 Ω). However, the exact value of the real input impedance does not match exactly to 50 Ω .

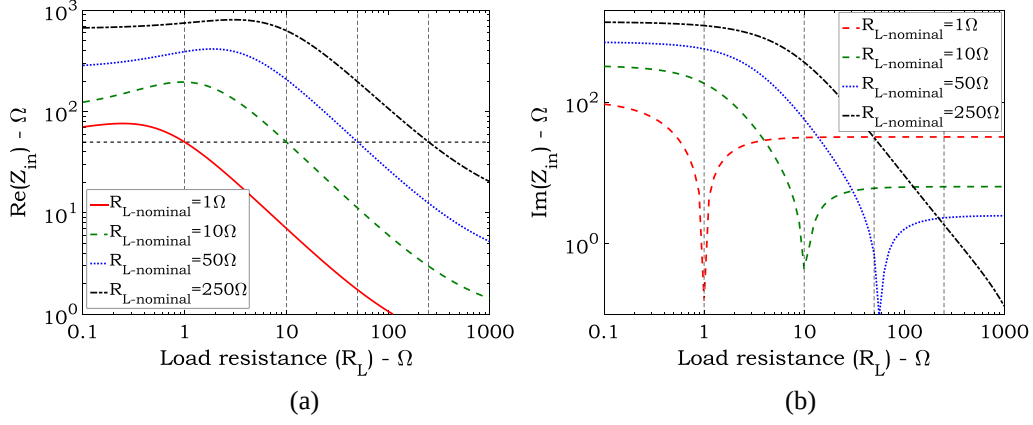


Fig. 7-11. Theoretical results: (a) – real and (b) – imaginary components of input impedance variations with respect to load resistance when TSR-WPT scheme is optimized to different nominal load resistance values ($R_{L-nominal}$).

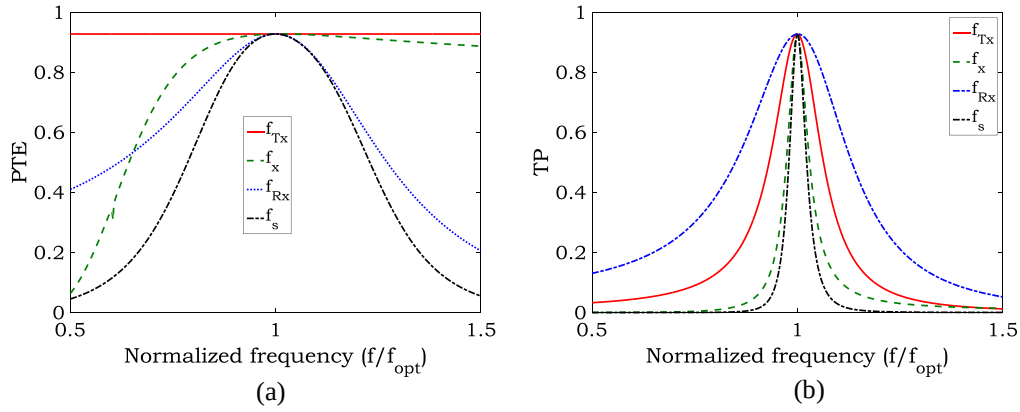


Fig. 7-12. Theoretical results: (a) – PTE and (b) – TP variation against each frequency (each frequency is normalized to its optimal value (f_{opt})) for TSR-WPT scheme optimized to 50Ω load resistance.

Next, the performance variations are analyzed with respect to the optimization variables (i.e., f_t , f_x , f_t and f_s). Fig. 7-12 illustrates the performance variations with respect to each of the frequency for the design optimized at a load resistance of 50Ω . The performance characteristics is generated by individually varying each frequency while all the other conditions are kept at their respective optimal values. For instance, when generating the curve corresponding to f_t , all the other parameters are kept at their respective optimum values. It can be seen that both PTE and TP become maximum at the optimum point (normalized frequency =1) proving that the derived optimal frequencies provide a local maxima for the performance variation. In fact, the values of PTE and TP obtained with the above optimization procedure are almost identical to the global maxima, which will be shown in subsequent section. In addition, it is apparent from the impedance characteristics shown in Fig. 7-13 that the optimum frequencies follow impedance matching conditions.

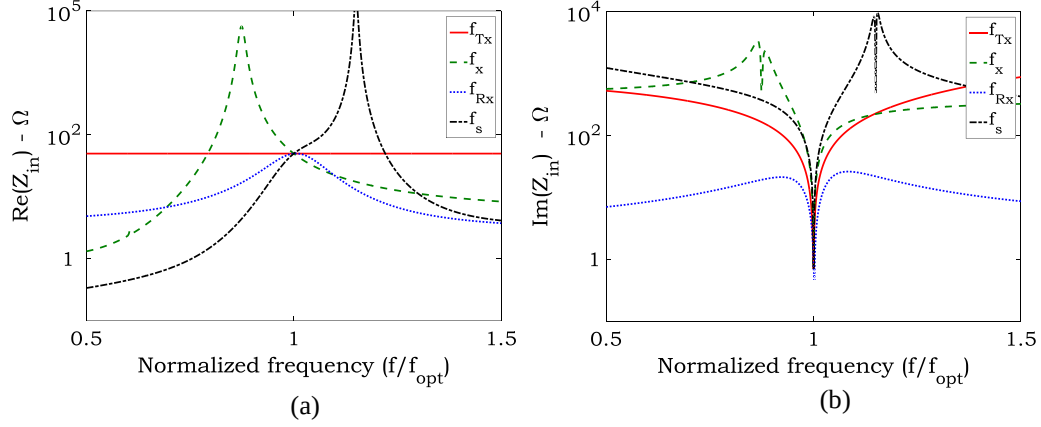


Fig. 7-13. Theoretical results: The variation of (a) – real and (b) – imaginary components of input impedance against each frequency (each frequency is normalized to its optimal value (f_{opt})) for TSR-WPT scheme optimized to 50Ω load resistance.

7.3.3 Comparison of analytical solution with numerical optimization

It is evident from the above numerical case study that the proposed optimization procedure improves the performance with respect to load variations, and at the same time the optimum design variables locate a maxima for both PTE and TP. However, it is not investigated whether the derived optimal solution for self-resonance frequencies provide a local maxima or global maxima. The optimization of multiple objectives simultaneously (in this case, PTE and TP) in a non-linear multi-variable system (four frequencies) may contain multiple optimal solutions. Therefore, numerical optimization is carried out to compare the validity of the proposed analytical procedure. The optimization problem for numerical optimization is defined in (7-30), and it is implemented using Matlab[®] optimization toolbox. The multiplication of PTE and TP is used as the maximization objective function.

$$\begin{aligned}
 &\text{Maximize}[f = F]: f(f_t, f_x, f_r, f_s) \\
 &s. t. 100\text{kHz} < f_t, f_x, f_r, f_s < 10\text{MHz} \\
 &F = PTE \times TP
 \end{aligned} \tag{7-30}$$

Fig. 7-14 compares the analytical solution obtained from the proposed approach and the optimum solution obtained from numerical optimization for SSR-WPT scheme. A similar comparison for TSR-WPT is illustrated in Fig. 7-15. It can be seen that the performances obtained from the proposed approach for both SSR-WPT and TSR-WPT

setups are almost identical to the global optimum derived from numerical optimization. Therefore it is evident that the proposed optimization approach provides an analytical approach to find the optimal solution for WPT with repeaters. However due to the nature of the non-linear optimization problem with multiple solutions, the optimum frequencies obtained from analytical approach are slightly different from the solution of numerical optimization.

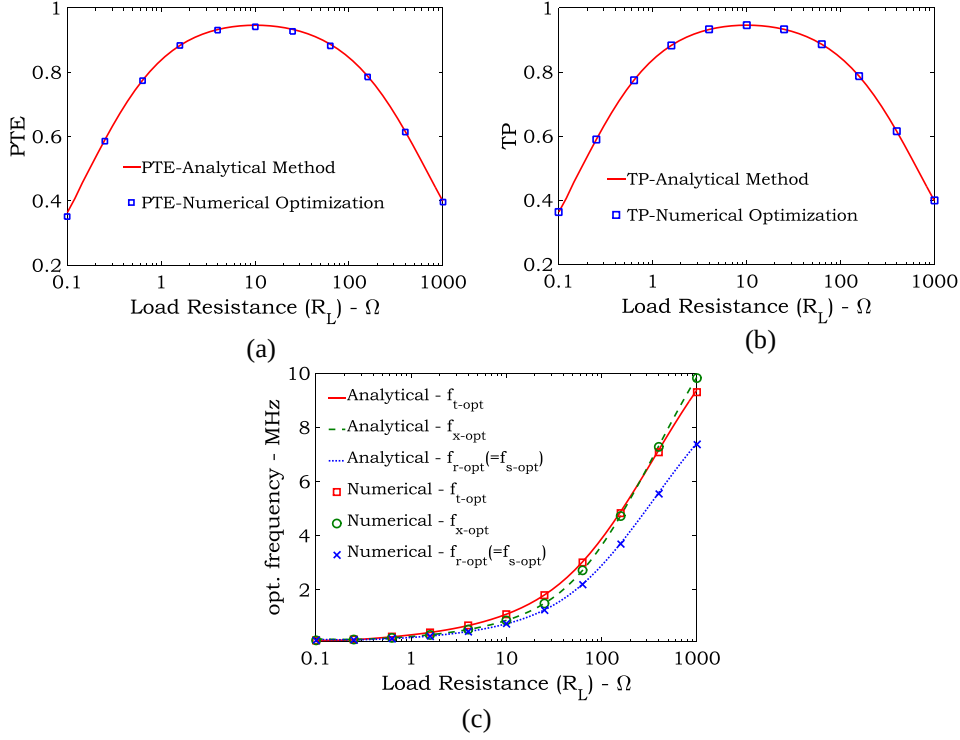


Fig. 7-14. Comparison of analytical solution obtained from the proposed approach with the optimum solution obtained from numerical optimization for SSR-WPT scheme (a) – PTE, (b) – TP and (c) – optimum frequencies.

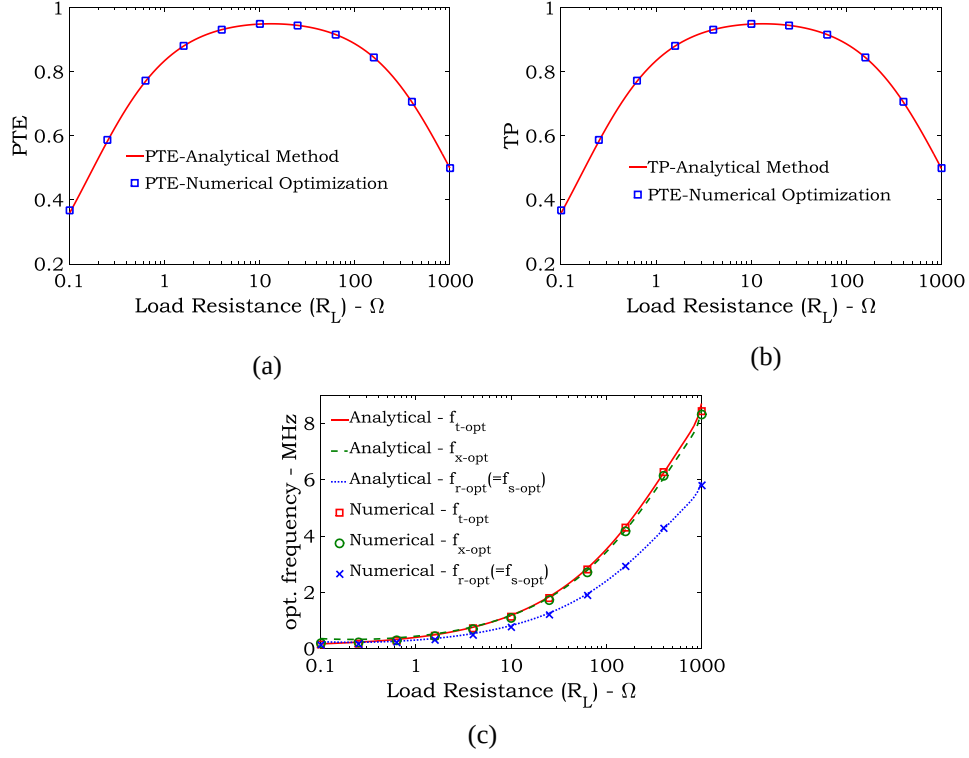


Fig. 7-15. Comparison of analytical solution obtained from the proposed approach with the optimum solution obtained from numerical optimization for TSR-WPT scheme (a) – PTE, (b) – TP and (c) – optimum frequencies.

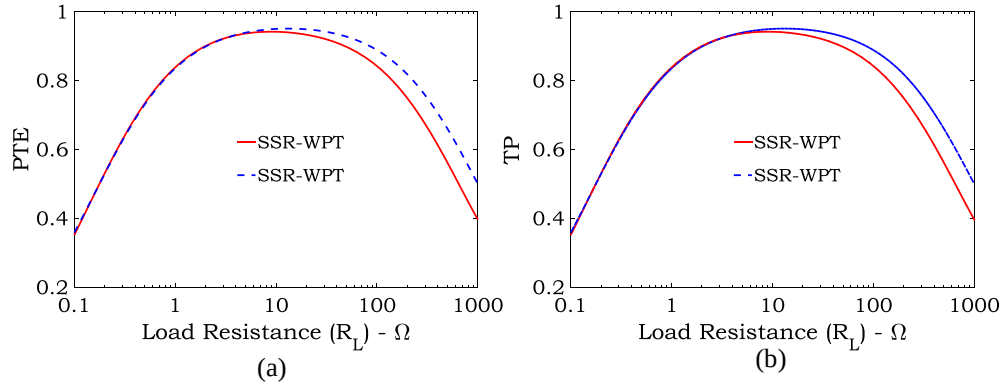


Fig. 7-16. Performance comparison of TSR-WPT and SSR-WPT schemes against load variation: (a) – PTE and (b) – TP.

7.3.4 Comparison of SSR-WPT vs. TSR-WPT

Finally, the performance of SSR-WPT and TSR-WPT are compared against load variations. Fig. 7-16 compares the optimal performances of SSR-WPT setup and TSR-WPT setup against load resistance. It can be seen that the TSR-WPT scheme outperforms SSR-WPT in high load resistance region whereas both setups show similar performance within the low load region. For example, TSR-WPT setup offers both PTE

and TP improvement around 5% to 10% compared to SSR-WPT setup when the load resistance is higher than 60Ω . Therefore, the repeater configuration can be chosen according to the load resistance variation in an application.

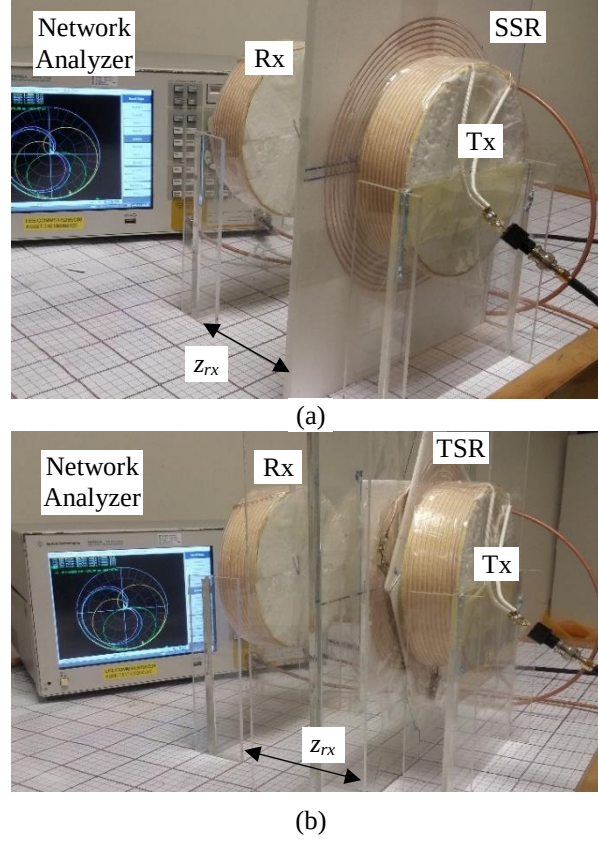


Fig. 7-17. The experimental setup (a) – SSR-WPT setup and (b) – TSR-WPT setup

7.4 Experimental verification

Theoretical analysis is validated with an experimental setup using S-parameter measurement technique. Fig. 7-17 shows the experimental setup for SSR-WPT and TSR- WPT schemes. WPT coils are constructed to match the design parameters in Table 7-1 (which are the same as the coils constructed previous chapters). Similar to the experimental procedures in all the previous chapters, network analyzer measurements are used for the experimental verification of this chapter. Measured S-parameters are processed using Agilent Advanced Design System (ADS) tool to obtain performance in different operating conditions. For both setups, frequencies are tuned manually for selected nominal load resistances because power electronics control and dynamic tuning are beyond the scope of this thesis. Each coil is manually tuned with the use of high Q lumped capacitors to obtain desired resonance frequencies for each design case.

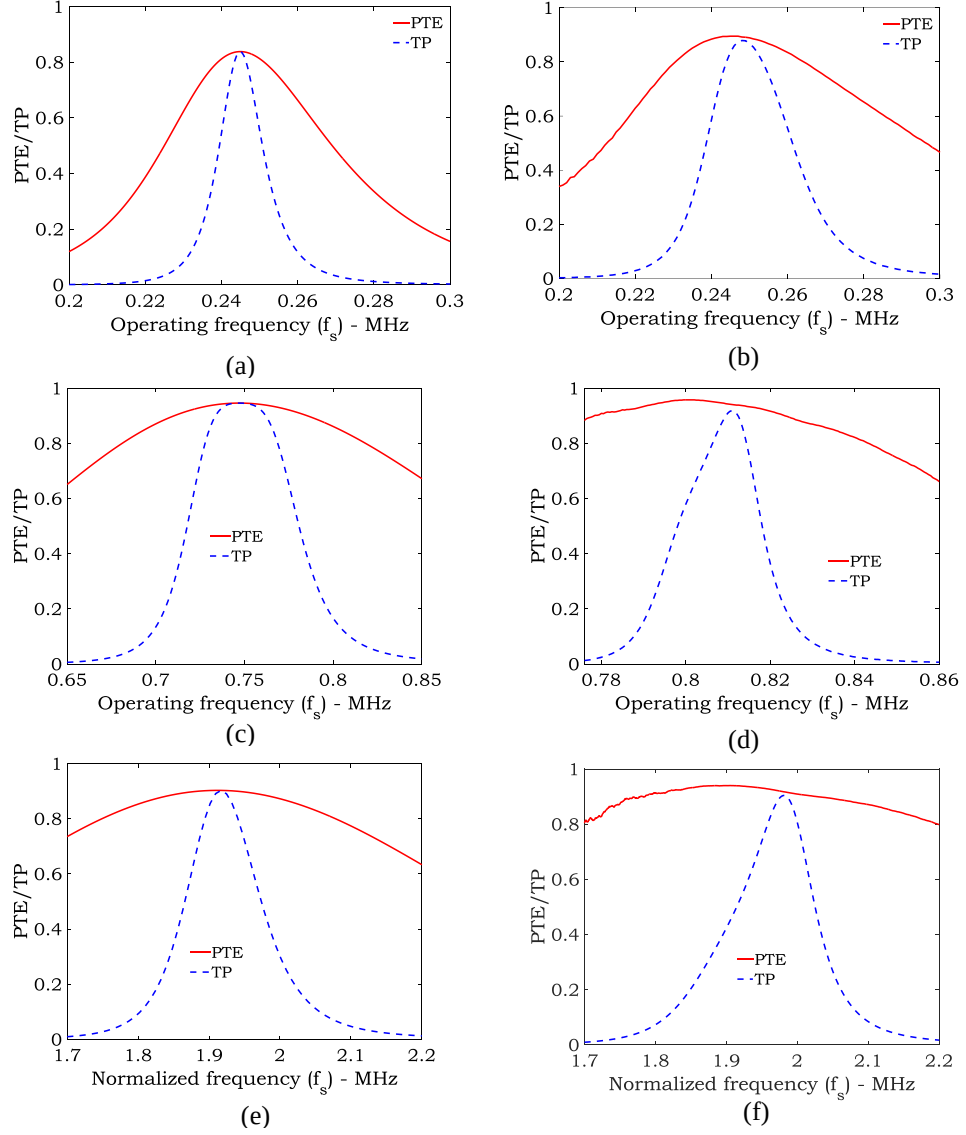


Fig. 7-18. Comparison of the performance variation against operating frequency when SSR-WPT setup is optimized to different operating conditions: (a) – theoretical for nominal load of 1Ω , (b) – experimental results nominal load of 1Ω , (c) – theoretical for nominal load of 10Ω , (d) – experimental results nominal load of 10Ω , (e) – theoretical for nominal load of 50Ω and (f) – experimental results nominal load of 50Ω .

7.4.1 Experimental validation of SSR-WPT setup

The experimental study for SSR-WPT scheme is carried out considering three nominal load conditions. Theoretical optimal frequencies and experimentally tuned frequencies are illustrated in Table 7-2 for different nominal load conditions. The frequency responses of experimental setups for each nominal load conditions are compared with respective theoretical counterparts in Fig. 7-18. The experimental results agree well with theoretical results for all design cases. It should be noted that the

frequency responses of experimental TP profile are slightly shifted compared to their theoretical TP profiles. This is due to the disparities in experimentally tuned frequencies as seen in Table 7-2.

Next, experimental results are analyzed with respect to load resistance when operating at optimum frequencies corresponding to each nominal loads, as illustrated in Fig. 7-19. It can be clearly seen that the proposed optimization scheme of tuning coil resonance frequencies allows significant improvement against load variations for the SSR-WPT setup. For example, the design optimized to nominal load of 50Ω shows PTE and TP improvements around 10% to 30% for load resistance higher than 50Ω compared to the design optimized to nominal load of 10Ω . On the other hand, the design optimized to nominal load of 10Ω shows PTE and TP improvements around 20% to 30% for load resistance lower than 50Ω , compared to the design optimized to nominal load of 50Ω .

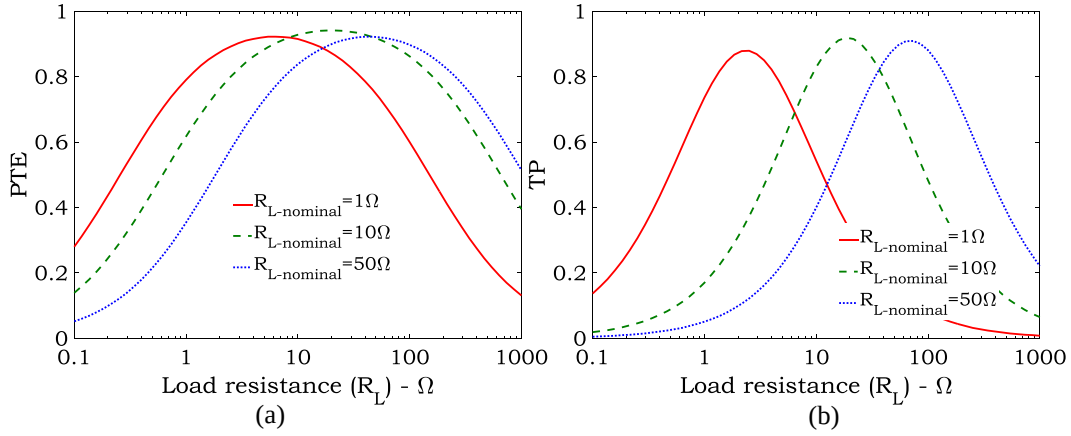


Fig. 7-19. Experimental results: Performance variation with respect to load resistance when SSR-WPT setup is optimized for different nominal load resistances. (a) – PTE variation and (b) – TP variation.

Table 7-2. Theoretical and experimental resonance frequencies for SSR-WPT (Δ - percentage difference between theoretical and experimental frequencies)

	$R_{L-nominal} = 1\ \Omega$			$R_{L-nominal} = 10\ \Omega$			$R_{L-nominal} = 50\ \Omega$		
	Theory	Exp.	Δ	Theory	Exp.	Δ	Theory	Exp.	Δ
f_t (MHz)	0.341	0.338	0.88%	1.102	1.095	0.64%	2.666	2.670	0.15%
f_r (MHz)	0.280	0.281	0.36%	0.875	0.879	0.46%	2.337	2.325	0.51%
f_{x1} (MHz)	0.245	0.247	0.82%	0.743	0.742	0.14%	1.917	1.896	1.09%

7.4.2 Experimental validation of TSR-WPT setup

The performance variation against operating frequency for the WPT scheme optimized to load resistance of 50Ω is compared with the theoretical counterpart in Fig.

7-20. Near 90% PTE and TP can be observed in experimental results. Experimental results agree well with the theoretical analysis with a slight shift in experimental optimum frequency compared to that of the theoretical value. This could be due to the experimental disparities in coil resonances (shown in Table 7-3) and geometries. The frequency and load characteristics shown in Fig. 7-21 also show that the experimental results agree well with the theoretical analysis.

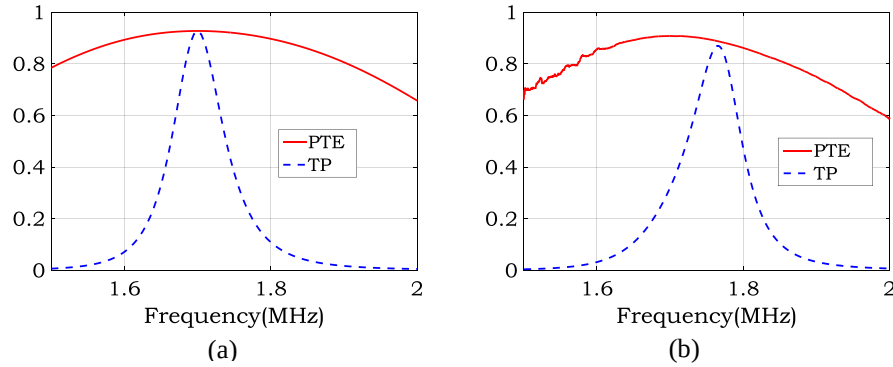


Fig. 7-20. Performance variation against operating frequency: Comparison of (a) – theoretical and (b) – experimental results.

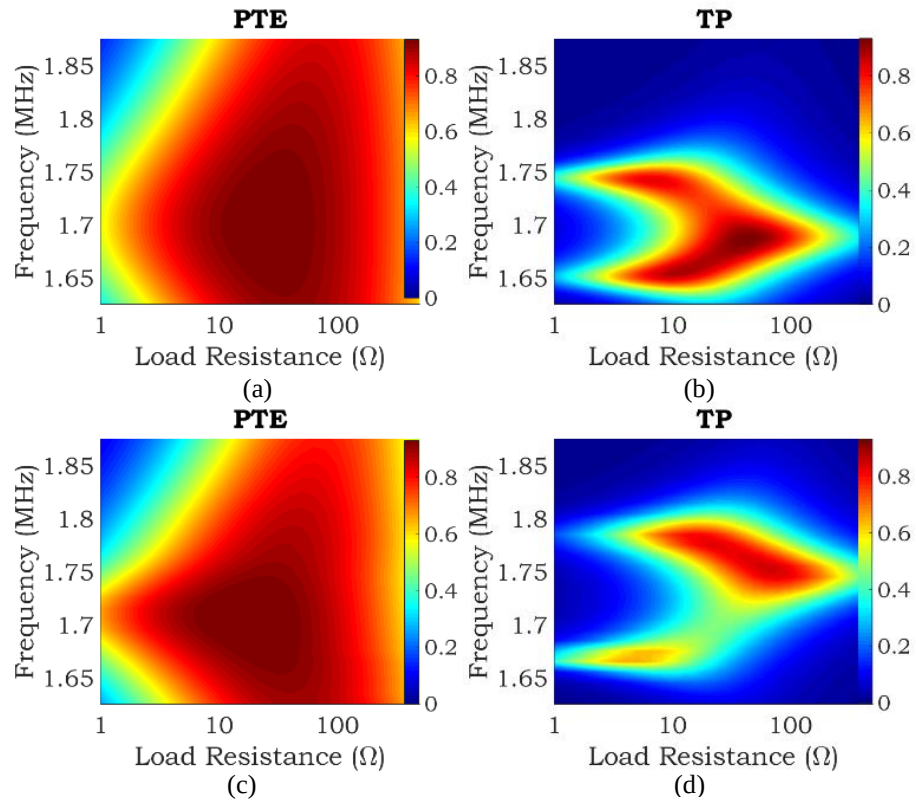


Fig. 7-21. Performance variation against operating frequency and the load resistance. (a) – PTE (Theory), (b) – TP (Theory), (c) – PTE (Experiment), and (d) – TP (Experiment).

Table 7-3. Comparison of theoretical and experimental resonance frequencies for TSR-WPT (nominal load resistance of 50 Ω)

	Theoretical	Experimental	Percentage deviation
f_t (MHz)	2.512	2.475	1.473%
f_r (MHz)	1.700	1.702	0.118%
f_{x1} (MHz)	2.418	2.410	0.331%
f_{x2} (MHz)	2.418	2.420	0.083%
f_{x3} (MHz)	2.418	2.401	0.703%

7.5 Optimization against misalignment

In the previous section, load characteristics optimization has been carried out when Rx is at the aligned position. However in real operational conditions, Rx may experience misalignment. Therefore the proposed performance optimization and tuning approach are extended to misalignment tolerance improvement in this section. Both SSR-WPT and TSR-WPT schemes are studied.

7.5.1 Optimization of SSR-WPT setup against misalignment

The optimization approach proposed in the previous section is applied to obtain the optimum self-resonance of each coil and optimum operating frequency with respect to Rx misalignment. The variation of the optimal frequencies with respect to misalignment is illustrated in Fig. 7-22(a). The respective PTE and TP variations with respect to misalignment are compared with the design optimized only at the aligned position in Fig. 7-22(b). It can be seen that a significant TP improvement is resulted from the frequency tuning, whereas, PTE variation resulted from frequency tuning is almost identical to the design optimized to the aligned position. For example, at 15cm misalignment, both PTE and TP can be improved up to 56% when the design is optimized to misaligned position whereas the design optimized to aligned position shows 59% and 32% of PTE and TP respectively.

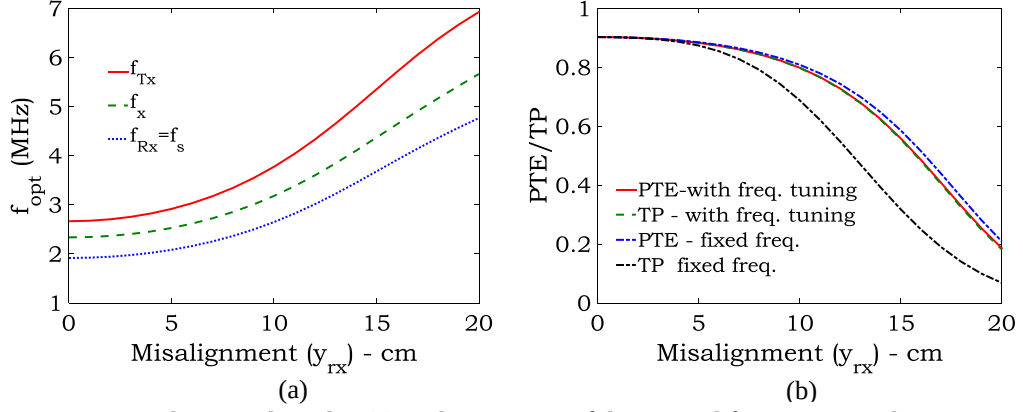


Fig. 7-22. Theoretical results: (a) – The variation of the optimal frequencies with respect to misalignment for SSR-WPT and (b) – Comparison of performance with respect to misalignment ($R_L=50\Omega$).

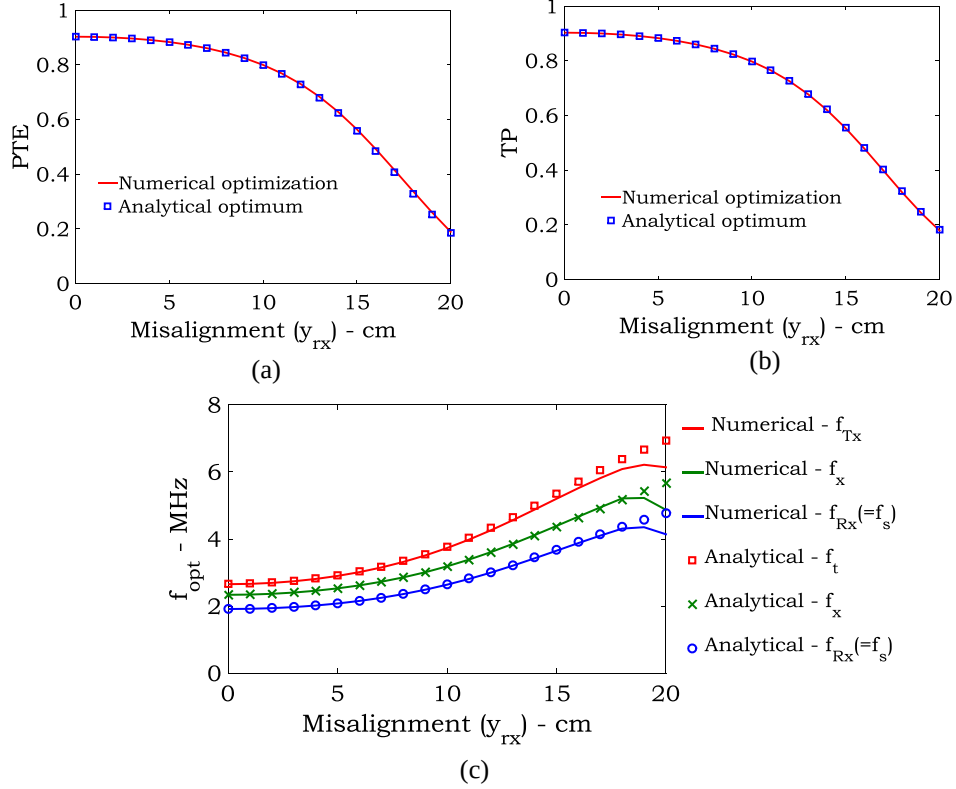


Fig. 7-23. Comparison of analytical solution obtained from the proposed approach with the optimum solution obtained from numerical optimization for SSR-WPT scheme (a) – PTE vs. misalignment, (b) – TP vs. misalignment and (c) – optimum frequencies vs. misalignment.

Next, the results from analytical optimization approach are compared with the numerical optimization results which finds the global optimum of the optimization variables. The numerical results in Fig. 7-23 compare the performances of the design optimized with the proposed analytical approach with the performances of the numerical optimization. It can be clearly seen that the proposed analytical approach achieves a performance almost identical to the numerical optimization method. In addition, results

in Fig. 7-23(c) show that the optimum frequencies of analytical approach are closely following the results of numerical optimization. Therefore, it is evident that the optimization approach proposed in this chapter allows to optimize the SSR-WPT scheme against misalignment.

7.5.2 Optimization of TSR-WPT setup against load variation

The analytical optimization approach proposed in this chapter cannot be applied directly for the optimization of TSR-WPT scheme against misalignment. This is because the mutual inductance relationship in (7-24) becomes invalid with the Rx misalignment. However, the concept of optimization of coil resonance frequency and operating frequency is still viable approach. Therefore, this subsection introduces the performance optimization of TSR-WPT against misalignment using a numerical optimization approach. The variation of the optimal frequencies with respect to misalignment is illustrated in Fig. 7-24(a). The respective PTE and TP variations with respect to misalignment are compared with the design optimized only at the aligned position in Fig. 7-24(b). For example at 15cm misalignment, both PTE and TP can be improved up to 65% when the design is optimized to misaligned position whereas the design optimized to aligned position shows 66% and 33% of PTE and TP respectively.

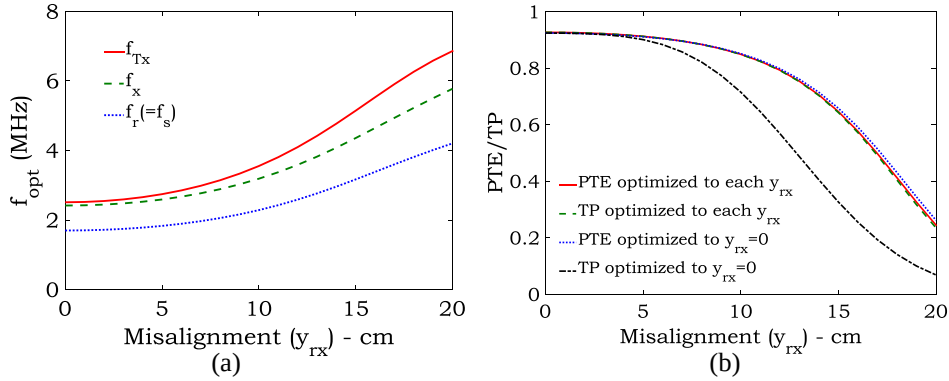


Fig. 7-24. Theoretical results: (a) – The variation of the optimal frequencies with respect to misalignment for TSR-WPT setup (b) – Comparison of performance with respect to misalignment ($R_L=50\Omega$).

The performance of TSR-WPT scheme against misalignment can be further improved by selectively tuning the self-resonance of TS coils where three coils in TSR (x_1 , x_2 and x_3) are tuned to different resonant frequencies (f_{x1} , f_{x2} and f_{x3}). This approach improves the performance misalignment because different mutual coupling between TSR coils can be compensated individually. Fig. 7-25(a) shows the variation of optimal

frequencies with respect to misalignment when each coil in TSR is selectively tuned. It can be seen that the optimal value of f_{x1} is less than the optimal values of f_{x2} and f_{x3} . In addition, the optimal values of f_{x2} and f_{x3} are identical due to the symmetry of x_2 and x_3 in y-direction. The respective performance variation is illustrated in Fig. 7-25(a).

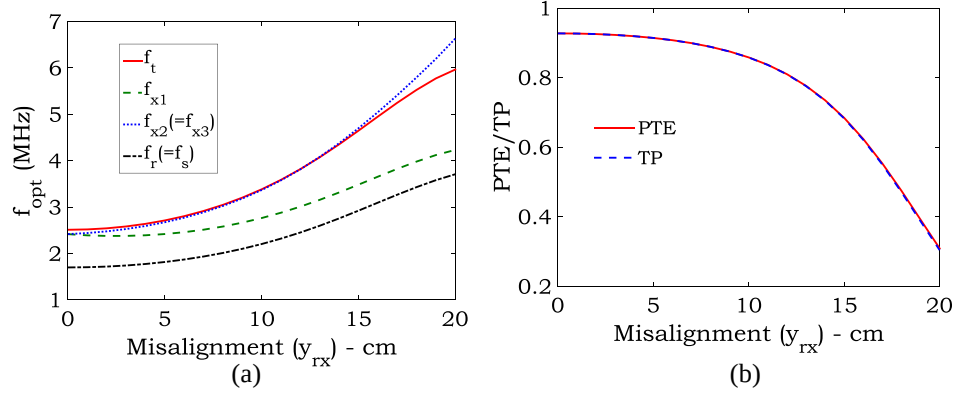


Fig. 7-25. Theoretical results: (a) – The variation of the optimal frequencies with respect to misalignment when TSR coils are selectively tuned to different frequencies (b) – Comparison of performance with respect to misalignment when the selective tuning approach is employed ($R_L=50 \Omega$).

7.5.3 Comparison of SSR-WPT vs. TSR-WPT

Performance comparison between SSR-WPT, symmetric TSR-WPT (all coils in TSR having same resonance frequency) and selective tuning of TSR-WPT is presented in this subsection. Numerical results in Fig. 7-26 compare the performances of SSR-WPT, symmetric TSR-WPT and selectively tuned TSR-WPT schemes with respect to misalignment. It can be seen that the selective tuning of TSR coils outperforms other two setups, particularly, with higher misalignment positions. This is because the selective tuning of TSR coils optimizes reactive impedances of each coil in TSR which compensates the effect of mutual inductance deficiencies due to misalignment. However, implementation of such a selective tuning approach may increase the control complexity of tuning circuitry. Therefore, application specific characteristics such as expected Rx misalignment need to be considered for the choice of tuning approach. For example, when misalignment is higher than 15cm, selective tuning of TSR yielded performance improvements up to 7% and 18% when compared to symmetric-TSR-WPT and SSR-WPT respectively. On the other hand, when misalignment is less than 10cm, both symmetric TSR-WPT and selectively tuned TSR-WPT schemes offer similar performances which are slightly higher than SSR-WPT. Therefore, if the expected

misalignment of an application is within 10 cm limit, selective tuning of TSR would not be a suitable choice for above analyzed WPT scheme.

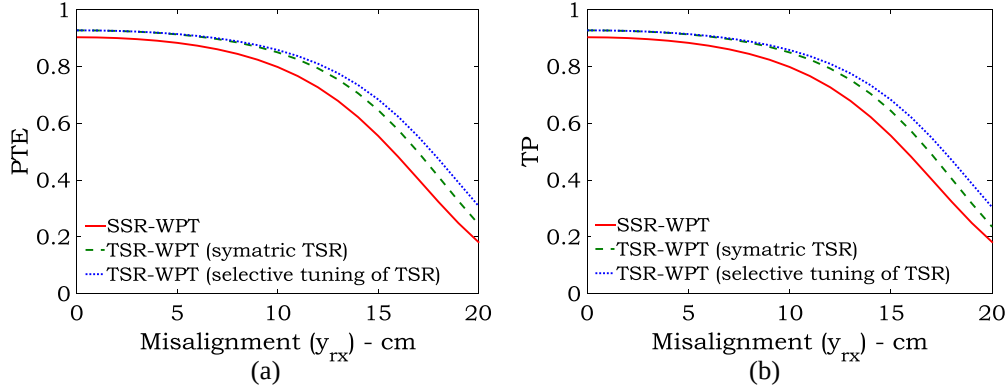


Fig. 7-26. Performance comparison between SSR-WPT, identical TSR-WPT and selective tuning of TSR-WPT with respect to misalignment (all the designs are optimized to nominal load resistance of 50Ω) (a) – PTE variation and (b) – TP variation.

7.6 Summary

This chapter presented an optimization approach for WPT systems with the use of a repeater. It was investigated that the self-resonance frequencies of transmitter (Tx), receiver (Rx) and repeater coils and the operating frequency can be optimized to improve the load characteristics and misalignment tolerance. At the beginning of the chapter, an analysis of WPT system with the use of a repeater was presented by employing the two-port theory. The generalized theory was applied to two types of repeaters, single-spiral-repeater (SSR) and tri-spiral-repeater (TSR), and optimization steps are derived with regards to both types of repeaters. It was investigated that the proposed optimization of this chapter allows the improvement of both PTE and TP against load variations and misalignment tolerance. It was also found that the proposed optimization approach was capable of reaching the global optimum value of the performance with respect to design variables at given operating conditions. Therefore the theoretical contribution of this chapter can be implemented along with a tuning approach for a wide range of applications. Theoretical analysis was verified with the experimental results.

Chapter 8

Conclusion and Recommendations

8.1 Conclusion

Wireless power transfer technology (WPT) has been identified as a potential technology that can be employed in a wide range of applications to eliminate drawbacks associated with power cables. However, it still needs to overcome numerous technical challenges to become one of the next epoch-making technologies. Performance improvement of wireless link is one such major technical challenge that needs extensive investigations. The study in this thesis has proposed several techniques that significantly improves the wireless link performance.

Key performance indices of WPT are power transfer efficiency (PTE) and transferred power (TP) through the wireless link. The performances are severely restricted by a number of design constraints such as transfer distance, Rx misalignment and variation in the load. In addition, PTE and TP characteristics show dissimilar profiles with respect to design parameters such as operating frequency, coupling coefficient and load resistance. For example when operating at the resonance frequency, PTE declines drastically with respect to transfer distance whereas, TP becomes maximum at a particular distance corresponds to critical coupling. Conversely, PTE is maximized when the operating frequency is same as the resonance frequency whereas, TP shows a frequency splitting phenomena when coupling coefficient is higher than critical coupling. Therefore, each of the characteristics has to be considered for the optimization of WPT systems. Considering all these characteristics for optimization of WPT systems in initial design stage is extremely difficult. Hence, the study in this thesis has been carried out in a top-down manner. PTE optimization was taken as the primary optimization objective in the initial chapters (chapters 3 and 4) of this study, while the simultaneous optimization of both PTE and TP was attempted in the later part (chapters 5, 6 and 7) of this study.

The optimization of the WPT coils plays an imperative role in the WPT performance improvements. For the coil optimization, maximum PTE (when load conditions are

assumed to be optimal), which is a function of quality factor and coupling coefficient, is identified as the key optimization index for WPT coil design. A detailed study on the improvement of WPT coils has been provided in Chapter 3 along with a set of generalized guidelines for coil design. For example, as a rule of thumb, optimum pitch should be chosen as four times of the wire radius if the coil radius is higher than 150 times of the wire radius to obtain maximum quality factor. In addition, empirical formulas are derived to obtain generalized coil design parameters for a WPT system when size restrictions are imposed on Rx footprint area.

Dynamic WPT (D-WPT) has been identified as a revolutionary extension of classical stationary WPT, because it allows Rx to be moved freely while receiving power wirelessly. Efficiency characteristics against design parameters such as, operating frequency, load resistance, number of simultaneously powered Tx coils and Tx coil spacing have been presented in Chapter 4. Misalignment problem dominates the technical challenges in designing D-WPT systems with segmented Tx arrays. Tx coils farther away from the Rx coil experience higher currents as compared to Tx coils nearer to Rx coil due to lower reflected impedance induced in the Tx coils that are farther away from Rx. Frequency tuning has been identified as a possible approach to address the misalignment problem. In addition, a double spiral repeater for D-WPT has been introduced to improve the performance. The investigations in Chapter 4 has been extended to derive the theoretical optimum current distribution within a Tx array in Chapter 5. An n-dimensional coordinate analogy has been applied onto the equivalent circuit to derive the optimum current distribution. The theoretical upper bound of the performances in D-WPT has been derived for arbitrary Rx position. The maximum efficiency for D-WPT system has been derived as a similar form to the maximum PTE formula for classical two-coil WPT. The theoretical analysis in Chapter 5 provides a fundamental platform for the optimization of D-WPT systems.

Moving forward in Chapter 6, a novel figure-of-merit (FOM) is proposed to improve system performance and misalignment tolerance simultaneously. The FOM is defined by incorporating system performance indices such as PTE and TP, as well as the application specific design constraints such as misalignment in Rx's degrees of freedom (DoF). Dynamics of Rx in each of six DoF (6DoF) misalignments has been integrated into the FOM by defining a set of sub-regions along with the probability of alignment

with respect to each 6DoF (probability of alignment refers to the likelihood of Rx misalignment in each of sub-region). The performance in each sub region is prioritized based on their respective probabilities of alignment in defining the FOM. When the proposed FOM is used as the optimization objective, the performance can be improved significantly with respect to misalignment with a slight performance reduction at the aligned position. In addition, the design parameters such as sub-regions in each of Rx's misalignment DoF and probability of alignment in each sub region can be chosen according to the application requirements. Therefore, the theoretical contribution of Chapter 6 can be applied not only to the WPT with repeaters, but also in a variety of optimization approaches such as compensation circuit, coil design and power electronics control.

For the investigations of this study up to Chapter 6, load resistance is assumed to be at a constant value. However, in a real implementation, load resistance variation is inevitable due to the dynamic nature of most applications. Although the theoretical contributions investigated up to Chapter 6 can be applied to different values of nominal load, the performance may decline when the load resistance differs from the nominal load. Therefore, the performance improvement against load variations is also tackled during this study. Chapter 7 presented an optimization approach for WPT systems against load variations with the use of a repeater. It is investigated that the self-resonance frequencies of Tx, Rx and repeater coils and the operating frequency can be optimized to improve the load variation characteristics. Two-port method is adopted to analyze the WPT scheme with a repeater, and it is applied to two types of repeaters, single-spiral-repeater (SSR) and tri-spiral-repeater (TSR). Optimization steps are derived to obtain the optimal condition analytically with regards to both types of repeaters. It is investigated that the proposed optimization procedure discussed in Chapter 7 allows the improvement of both PTE and TP against load variations. The proposed optimization approach is capable of reaching the global optimum value of the performance with respect to its design variables at given operating conditions. Furthermore, the proposed optimization approach can also be applied to improve the performance against misalignment. Selective tuning of TSR coils has been identified as a promising approach to improve the misalignment tolerance.

8.2 Recommendations for future research

As with any research, there is room for more extensions and development of the proposed work. Some of these are identified and outlined below.

- The coil design equations proposed in Chapter 3 has been evaluated for circular shaped air-core coils. A similar method can be applied with a variety design conditions such as rectangular shaped coils and coil design with ferrite materials. The basic considerations proposed in this study will be still valid for the different coil shapes. For example, proximity effect resistance will be significant for tightly wound coils. However, coupling mutual inductance characteristics will be changed when the coil shape is altered. Therefore it would be interesting to further explore the proposed approach with regards to different coil designs conditions.
- The theoretical analysis provided in Chapter 5 can be applied as a guideline for power converter design for D-WPT systems. The dynamic control of the converter current to track the optimal conditions derived in this work would be an interesting area to explore in future research. Moreover, the proposed theory in Chapter 5 can be applied to different kinds of multi-Tx, multi-Rx WPT schemes such as WPT pad (where multiple Tx coils are used as a charging pad for portable devices).
- The investigation on the simultaneous optimization of PTE, TP and misalignment tolerance can be easily applied along with a range of optimization approaches such as coil design, compensation circuit design and different repeater schemes. Moreover, Rx dynamics and its misalignment probabilities are arbitrarily chosen to validate the proposed concept. However, actual studies can be conducted to analyze Rx behaviour in different applications which would substantiate realistic probability distribution. Undoubtedly, this work would push forward the concept of free-positioning WPT where Rx can be placed in any position or orientation.
- The concept of tuning the coil resonance frequencies to improve the load variation characteristics has been validated using a manual tuning technique in

Chapter 7. A dynamic tuning strategy can be implemented with use of a set of switched capacitor networks and a proper control method in a load varying application. Exploration of such a dynamic tuning approach would be an interesting research topic.

- Although technical feasibility of the proposed investigations are verified with the laboratory experiments, the health and safety concerns have not been addressed within the scope of this thesis. Hence, human safety in terms of electromagnetic exposure levels needs to be carefully examined before the deployment of WPT technology to any kind of application. Therefore, the study of health and safety issues of WPT technology is an interesting topic to explore.

Appendix

Derivation of the simplified transfer impedance ratio (Kn)

This appendix illustrates the trigonometric simplification procedure for the derivation of (5.6). First the transfer impedance ratio, K_n is rewritten as,

$$K_n = \frac{\omega^2 I^2 R_L}{(R_r + R_L)^2 + X_r^2} \Gamma_n \quad (\text{A.1})$$

where,

$$\Gamma_n = \{M_{1,r} \cos(\theta_1) + \dots + M_{n-1,r} \sin(\theta_1) \dots \sin(\theta_{n-2}) \cos(\theta_{n-1}) + M_{n,r} \sin(\theta_1) \dots \sin(\theta_{n-2}) \sin(\theta_{n-1})\}^2$$

The last two terms of Γ_n can be rearranged as,

$$\Gamma_n = \{M_{1,r} \cos(\theta_1) + \dots + \sin(\theta_1) \dots \sin(\theta_{n-2}) [M_{n-1,r} \cos(\theta_{n-1}) + M_{n,r} \sin(\theta_{n-1})]\}^2$$

$$\Gamma_n = \{M_{1,r} \cos(\theta_1) + \dots + \sin(\theta_1) \dots \sin(\theta_{n-2}) [M_{n-1,r} \cos(\theta_{n-1}) + M_{n,r} \sin(\theta_{n-1})]\}^2$$

with the definition of $\alpha_{n-1} = \tan^{-1} \left\{ \frac{M_{n-1,r}}{M_{n,r}} \right\}$, Γ_n becomes

$$\Gamma_n = \left\{ M_{1,r} \cos(\theta_1) + \dots + \sin(\theta_1) \dots \sin(\theta_{n-2}) [\sin(\alpha_{n-1}) \cos(\theta_{n-1}) + \cos(\alpha_{n-1}) \sin(\theta_{n-1})] \times \sqrt{M_{n,r}^2 + M_{n-1,r}^2} \right\}^2$$

Γ can be further simplified to,

$$\Gamma_n = \left\{ M_{1,r} \cos(\theta_1) + \dots + \sin(\theta_1) \dots \sin(\theta_{n-2}) \sin(\alpha_{n-1} + \theta_{n-1}) \times \sqrt{M_{n,r}^2 + M_{n-1,r}^2} \right\}^2$$

Next, considering the last two terms of the expression,

$$\Gamma_n = \left\{ M_{1r} \cos(\theta_1) + \dots + \sin(\theta_1) \dots \sin(\theta_{n-3}) \left[M_{n-2,r} \cos(\theta_{n-2}) + \sqrt{M_{n,r}^2 + M_{n-1,r}^2} \sin(\alpha_{n-1} + \theta_{n-1}) \sin(\theta_{n-2}) \right] \right\}^2$$

with the definition of $\alpha_{n-2} = \tan^{-1} \left\{ \frac{M_{n-2,r}}{\sqrt{M_{n-1,r}^2 + M_{n,r}^2} \sin(\theta_{n-1} + \alpha_{n-1})} \right\}$, Γ_n becomes

$$\Gamma_n = \left\{ M_{1r} \cos(\theta_1) + \dots + \sqrt{M_{n-2,r}^2 + (M_{n-1,r}^2 + M_{n,r}^2) \sin^2(\theta_{n-1} + \alpha_{n-1})} \sin(\theta_1) \dots \sin(\theta_{n-3}) [\sin(\alpha_{n-2}) \cos(\theta_{n-2}) + \cos(\alpha_{n-2}) \sin(\theta_{n-2})] \right\}^2 \text{ and}$$

$$\Gamma_n = \left\{ M_{1r} \cos(\theta_1) + \dots + \sqrt{M_{n-2,r}^2 + (M_{n-1,r}^2 + M_{n,r}^2) \sin^2(\theta_{n-1} + \alpha_{n-1})} \sin(\alpha_{n-2} + \theta_{n-2}) \sin(\theta_1) \dots \sin(\theta_{n-3}) \right\}^2$$

In a similar manner, angles $\alpha_{n-3}, \dots, \alpha_1$, can be defined as,

$$\alpha_{n-3} = \tan^{-1} \left\{ \frac{M_{n-3,r}}{\sqrt{[(M_{n-1,r}^2 + M_{n,r}^2) \sin^2(\theta_{n-1} + \alpha_{n-1}) + M_{n-2,r}^2] \sin(\theta_{n-2} + \alpha_{n-2})}} \right\},$$

⋮

$\alpha_1 =$

$$\tan^{-1} \left\{ \frac{M_{1r}}{\sqrt{[(M_{n-1,r}^2 + M_{n,r}^2) \sin^2(\theta_{n-1} + \alpha_{n-1}) + M_{n-2,r}^2] \sin^2(\theta_{n-2} + \alpha_{n-2}) + M_{n-3,r}^2 \dots + M_{2,r}^2} \sin(\theta_2 + \alpha_2)} \right\}$$

and Γ_n becomes, $\Gamma_n = \left\{ \left[\left((M_{n,r}^2 + M_{n-1,r}^2) \sin^2(\theta_{n-1} + \alpha_{n-1}) + M_{n-2,r}^2 \right) \sin^2(\theta_{n-2} + \alpha_{n-2}) + M_{n-3,r}^2 \dots + M_{2,r}^2 \right] \sin^2(\theta_2 + \alpha_2) + M_{1,r}^2 \right\} \sin^2(\theta_1 + \alpha_1)$

Then the simplified Γ_n can be substitute to (A.1) to obtained (5.6).

Authors Publications

Book Chapters

- [1] D.M. Vilathgamuwa and J.P.K. Sampath, "Wireless Power Transfer (WPT) for Electric Vehicles (EVs)—Present and Future Trends," in *Plug In Electric Vehicles in Smart Grids*, ed: Springer, 2015, pp. 33-60.

Journal Papers

- [2] J. P. K. Sampath, D. M. Vilathgamuwa, and A. Alphones, "Efficiency Enhancement for Dynamic Wireless Power Transfer System With Segmented Transmitter Array," *IEEE Transactions on Transportation Electrification*, vol. 2, pp. 76-85, 2016.
- [3] J. P. K. Sampath, A. Alphones, and D. M. Vilathgamuwa, "Figure-of-Merit for the Optimization of Wireless Power Transfer System against Misalignment Tolerance," *IEEE Transactions on Power Electronics*, vol. 32, no. 6, pp. 4359-4369, June 2017.
- [4] J. P. K. Sampath, A. Alphones, and D. M. Vilathgamuwa, "Optimum Transmitter Current Distribution for Dynamic Wireless Power Transfer (D-WPT) with segmented Array," *IEEE Transactions on Microwave Theory and Techniques*, vol.PP, no.99, pp.1-1 (Early access)
- [5] J. P. K. Sampath, A. Alphones, and D. M. Vilathgamuwa, "Optimization of Repeater for Wireless Power Transfer Against Load Variation," *IEEE Transactions on Industrial Electronics*, vol.PP, no.99, pp.1-1 (Early access).

Conference Papers

- [6] J. P. K. Sampath, A. Alphones, and Hitoshi Shimasaki D. M. Vilathgamuwa, "Coil design Guidelines for High Efficiency Wireless Power Transfer," 2016, in *TENCON 2015 - 2015 IEEE Region 10 Conference*, Singapore, 2016, pp. 726-729.
- [7] J. P. K. Sampath, A. Alphones and D. M. Vilathgamuwa, "Coil optimization against misalignment for wireless power transfer," *2016 IEEE 2nd Annual Southern Power Electronics Conference (SPEC)*, Auckland, 2016, pp. 1-5

- [8] J. P. K. Sampath, A. Alphones, and D. M. Vilathgamuwa, "Repeater tuning against Load variation for Wireless Power Transfer," in *11th International Conference on Power Electronics and ECCE Asia (ICPE-ECCE Asia)*, Hefei, 2016, pp. 3173-3177.
- [9] J. P. K. Sampath, A. Alphones, D. M. Vilathgamuwa, M.-H. Hsieh, and D. G. Dorrell, "Design of High efficiency Wireless Power Transfer System with Miniaturized Receiver Coil," in *IEEE Magnetics Conference (INTERMAG)*, 2016, pp. 1-1.
- [10] J. P. K. Sampath, A. Alphones, and D. M. Vilathgamuwa, "Optimization of double spiral metamaterial for wireless power transfer," in *9th International Conference on Power Electronics and ECCE Asia (ICPE-ECCE Asia)* 2015, pp. 1937-1943.
- [11] J. P. K. Sampath, A. Alphones, L. Y. Y. Kenneth, and D. M. Vilathgamuwa, "Analysis on normalized distance and scalability in designing wireless power transfer," in *Emerging Technologies: Wireless Power (WoW), 2015 IEEE PELS Workshop on*, 2015, pp. 1-6.
- [12] J. P. K. Sampath, A. Alphones, and D. M. Vilathgamuwa, "Tunable metamaterials for optimization of wireless power transfer systems," in *Antennas and Propagation & USNC/URSI National Radio Science Meeting, 2015 IEEE International Symposium on*, 2015, pp. 107-108.
- [13] A. Alphones and J. P. K. Sampath, "Metamaterial assisted wireless power transfer system," in *2015 Asia-Pacific Microwave Conference (APMC)*, 2015, pp. 1-3.
- [14] J. P. K. Sampath, A. Alphones, D. M. Vilathgamuwa, A. Ong, and X. B. Nguyen, "Coil enhancements for high efficiency wireless power transfer applications," in *40th Annual Conference of the IEEE Industrial Electronics Society, IECON*, 2014, pp. 2978-2983.

References

- [1] N. Tesla, "High frequency oscillators for electro-therapeutic and other purposes," *The Electrical Engineer*, vol. XXVI, no. 550, November 17, 1898.
- [2] N. TESLA, "Apparatus for transmitting electrical energy," Patent 1,119,732, December 1914.
- [3] M. Fu, T. Zhang, C. Ma, and X. Zhu, "Efficiency and Optimal Loads Analysis for Multiple-Receiver Wireless Power Transfer Systems," *IEEE Transactions on Microwave Theory and Techniques*, vol. 63, no. 3, pp. 801-812, 2015.
- [4] J. J. Casanova, Z. N. Low, and J. Lin, "A Loosely Coupled Planar Wireless Power System for Multiple Receivers," *IEEE Transactions on Industrial Electronics*, vol. 56, no. 8, pp. 3060-3068, 2009.
- [5] J. Yin, D. Lin, C. K. Lee, T. Parisini, and Y. S., "Front-End Monitoring of Multiple Loads in Wireless Power Transfer Systems Without Wireless Communication Systems," *IEEE Transactions on Power Electronics*, vol. 31, no. 3, pp. 2510-2517, 2016.
- [6] A. Dukju and H. Songcheol, "Effect of Coupling Between Multiple Transmitters or Multiple Receivers on Wireless Power Transfer," *IEEE Transactions on Industrial Electronics*, vol. 60, no. 7, pp. 2602-2613, 2013.
- [7] Y. You, B. H. Soong, S. Ramachandran, and W. Liu, "Palm size charging platform with uniform wireless power transfer," in *Control Automation Robotics & Vision (ICARCV), 2010 11th International Conference on*, 2010, pp. 85-89.
- [8] A. K. RamRakhyani, S. Mirabbasi, and M. Chiao, "Design and Optimization of Resonance-Based Efficient Wireless Power Delivery Systems for Biomedical Implants," *IEEE Transactions on Biomedical Circuits and Systems*, vol. 5, no. 1, pp. 48-63, 2011.
- [9] M. Kiani and M. Ghovanloo, "A Figure-of-Merit for Designing High-Performance Inductive Power Transmission Links," *IEEE Transactions on Industrial Electronics*, vol. 60, no. 11, pp. 5292-5305, 2013.
- [10] S. C. Tang and N. J. McDannold, "Power Loss Analysis and Comparison of Segmented and Unsegmented Energy Coupling Coils for Wireless Energy Transfer," *IEEE Journal of Emerging and Selected Topics in Power Electronics*, vol. 3, no. 1, pp. 215-225, 2015.
- [11] S. C. Tang, "A Low-Operating-Voltage Wireless Intermediate-Range Scheme for Energy and Signal Transmission by Magnetic Coupling for Implantable Devices," *IEEE Journal of Emerging and Selected Topics in Power Electronics*, vol. 3, no. 1, pp. 242-251, 2015.
- [12] S. Li and C. C. Mi, "Wireless Power Transfer for Electric Vehicle Applications," *IEEE Journal of Emerging and Selected Topics in Power Electronics*, vol. 3, no. 1, pp. 4-17, 2014.
- [13] C.-G. Kim, D.-H. Seo, J.-S. You, J.-H. Park, and B. H. Cho, "Design of a contactless battery charger for cellular phone," *IEEE Transactions on Industrial Electronics*, vol. 48, no. 6, pp. 1238-1247, 2001.
- [14] J. Kim, H. C. Son, D. H. Kim, and Y. J. Park, "Optimal design of a wireless power transfer system with multiple self-resonators for an LED TV," *IEEE Transactions on Consumer Electronics*, vol. 58, no. 3, pp. 775-780, 2012.
- [15] S. Y. Hui, "Planar Wireless Charging Technology for Portable Electronic Products and Qi," *Proceedings of the IEEE*, vol. 101, no. 6, pp. 1290-1301, 2013.
- [16] K. Kandasamy, D. M. Vilathgamuwa, U. K. Madawala, and K. J. Tseng, "Inductively coupled modular battery system for electric vehicles," *IET Power Electronics*, vol. 9, no. 3, pp. 600-609, 2016.
- [17] S. Choi, J. Huh, W. Y. Lee, S. W. Lee, and C. T. Rim, "New Cross-Segmented Power Supply Rails for Roadway-Powered Electric Vehicles," *IEEE Transactions on Power Electronics*, vol. 28, no. 12, pp. 5832-5841, 2013.
- [18] N. P. Suh, D. H. Cho, and C. T. Rim, "Design of On-Line Electric Vehicle (OLEV)," in *Global Product Development*, A. Bernard, Ed.: Springer Berlin Heidelberg, 2011, pp. 3-8.

- [19] J. M. Miller, P. T. Jones, L. Jan-Mou, and O. C. Onar, "ORNL Experience and Challenges Facing Dynamic Wireless Power Charging of EV's," *IEEE Circuits and Systems Magazine*, vol. 15, no. 2, pp. 40-53, 2015.
- [20] N. Kyungmin, J. Heedon, M. Hyunggun, and F. Bien, "Tracking Optimal Efficiency of Magnetic Resonance Wireless Power Transfer System for Biomedical Capsule Endoscopy," *IEEE Transactions on Microwave Theory and Techniques*, vol. 63, no. 1, pp. 295-304, 2015.
- [21] J. P. C. Smeets, T. T. Overboom, J. W. Jansen, and E. A. Lomonova, "Comparison of Position-Independent Contactless Energy Transfer Systems," *IEEE Transactions on Power Electronics*, vol. 28, no. 4, pp. 2059-2067, 2013.
- [22] J. P. C. Smeets, T. T. Overboom, J. W. Jansen, and E. A. Lomonova, "Modeling Framework for Contactless Energy Transfer Systems for Linear Actuators," *IEEE Transactions on Industrial Electronics*, vol. 60, no. 1, pp. 391-399, 2013.
- [23] S. Cheon, Y. H. Kim, S. Y. Kang, M. L. Lee, J. M. Lee, and T. Zyung, "Circuit-Model-Based Analysis of a Wireless Energy-Transfer System via Coupled Magnetic Resonances," *IEEE Transactions on Industrial Electronics*, vol. 58, no. 7, pp. 2906-2914, 2011.
- [24] W. Chwei-Sen, G. A. Covic, and O. H. Stielau, "Power transfer capability and bifurcation phenomena of loosely coupled inductive power transfer systems," *IEEE Transactions on Industrial Electronics*, vol. 51, no. 1, pp. 148-157, 2004.
- [25] L. Chen, S. Liu, Y. C. Zhou, and T. J. Cui, "An Optimizable Circuit Structure for High-Efficiency Wireless Power Transfer," *IEEE Transactions on Industrial Electronics*, vol. 60, no. 1, pp. 339-349, 2013.
- [26] S. Li, W. Li, J. Deng, T. D. Nguyen, and C. C. Mi, "A Double-Sided LCC Compensation Network and Its Tuning Method for Wireless Power Transfer," *IEEE Transactions on Vehicular Technology*, vol. 64, no. 6, pp. 2261-2273, 2015.
- [27] Q. Zhu, Y. Guo, L. Wang, C. Liao, and F. Li, "Improving the Misalignment Tolerance of Wireless Charging System by Optimizing the Compensate Capacitor," *IEEE Transactions on Industrial Electronics*, vol. 62, no. 8, pp. 4832-4836, 2015.
- [28] R. Jegadeesan and Y. X. Guo, "Topology Selection and Efficiency Improvement of Inductive Power Links," *IEEE Transactions on Antennas and Propagation*, vol. 60, no. 10, pp. 4846-4854, 2012.
- [29] T. D. Nguyen, S. Li, W. Li, and C. C. Mi, "Feasibility study on bipolar pads for efficient wireless power chargers," in *Twenty-Ninth Annual IEEE Applied Power Electronics Conference and Exposition (APEC) 2014*, pp. 1676-1682.
- [30] S. Y. R. Hui, Z. Wenxing, and C. K. Lee, "A Critical Review of Recent Progress in Mid-Range Wireless Power Transfer," *IEEE Transactions on Power Electronics*, vol. 29, no. 9, pp. 4500-4511, 2014.
- [31] W. Zhong, C. K. Lee, and S. Y. R. Hui, "General Analysis on the Use of Tesla's Resonators in Domino Forms for Wireless Power Transfer," *IEEE Transactions on Industrial Electronics*, vol. 60, no. 1, pp. 261-270, 2013.
- [32] B. L. Cannon, J. F. Hoburg, D. D. Stancil, and S. C. Goldstein, "Magnetic Resonant Coupling As a Potential Means for Wireless Power Transfer to Multiple Small Receivers," *IEEE Transactions on Power Electronics*, vol. 24, no. 7, pp. 1819-1825, 2009.
- [33] B. Lee, M. Kiani, and M. Ghovanloo, "A Triple-Loop Inductive Power Transmission System for Biomedical Applications," *IEEE Transactions on Biomedical Circuits and Systems*, vol. 10, no. 1, pp. 138-148, 2016.
- [34] A. P. Sample, D. T. Meyer, and J. R. Smith, "Analysis, Experimental Results, and Range Adaptation of Magnetically Coupled Resonators for Wireless Power Transfer," *IEEE Transactions on Industrial Electronics*, vol. 58, no. 2, pp. 544-554, 2011.
- [35] N. Y. Kim, K. Y. Kim, J. Choi, and C. W. Kim, "Adaptive frequency with power-level tracking system for efficient magnetic resonance wireless power transfer," *Electronics Letters*, vol. 48, no. 8, pp. 452-454, 2012.

- [36] H. H. Wu, A. Gilchrist, K. D. Sealy, and D. Bronson, "A High Efficiency 5 kW Inductive Charger for EVs Using Dual Side Control," *IEEE Transactions on Industrial Informatics*, vol. 8, no. 3, pp. 585-595, 2012.
- [37] U. K. Madawala, M. Neath, and D. J. Thrimawithana, "A Power-Frequency Controller for Bidirectional Inductive Power Transfer Systems," *IEEE Transactions on Industrial Electronics*, vol. 60, no. 1, pp. 310-317, 2013.
- [38] M. Eghtesadi, "Inductive power transfer to an electric vehicle-analytical model," in *IEEE 40th Vehicular Technology Conference*, 1990, pp. 100-104.
- [39] U. K. Madawala, J. Stichbury, and S. Walker, "Contactless power transfer with two-way communication," in *30th Annual Conference of IEEE Industrial Electronics Society, 2004 (IECON) 2004*, pp. 3071-3075.
- [40] A. Kurs, A. Karalis, R. Moffatt, J. D. Joannopoulos, P. Fisher, and M. Soljačić, "Wireless power transfer via strongly coupled magnetic resonances," *science*, vol. 317, no. 5834, pp. 83-86, 2007.
- [41] L. Chi Kwan, W. X. Zhong, and S. Y. R. Hui, "Effects of Magnetic Coupling of Nonadjacent Resonators on Wireless Power Domino-Resonator Systems," *IEEE Transactions on Power Electronics*, vol. 27, no. 4, pp. 1905-1916, 2012.
- [42] C. Chih-Jung, C. Tah-Hsiung, L. Chih-Lung, and J. Zeui-Chown, "A Study of Loosely Coupled Coils for Wireless Power Transfer," *IEEE Transactions on Circuits and Systems II: Express Briefs*, vol. 57, no. 7, pp. 536-540, 2010.
- [43] M. Dionigi, M. Mongiardo, and R. Perfetti, "Rigorous Network and Full-Wave Electromagnetic Modeling of Wireless Power Transfer Links," *IEEE Transactions on Microwave Theory and Techniques*, vol. 63, no. 1, pp. 65-75, 2015.
- [44] V. T. Nguyen, S. H. Kang, J. H. Choi, and C. W. Jung, "Magnetic resonance wireless power transfer using three-coil system with single planar receiver for laptop applications," *IEEE Transactions on Consumer Electronics*, vol. 61, no. 2, pp. 160-166, 2015.
- [45] R. F. Xue, K. W. Cheng, and M. Je, "High-Efficiency Wireless Power Transfer for Biomedical Implants by Optimal Resonant Load Transformation," *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. PP, no. 99, pp. 1-1, 2012.
- [46] R. Jegadeesan and Y. X. Guo, "Modeling of wireless power transfer link for retinal implant," in *2016 IEEE/ACES International Conference on Wireless Information Technology and Systems (ICWITS) and Applied Computational Electromagnetics (ACES)*, 2016, pp. 1-2.
- [47] R. Jegadeesan, S. Nag, K. Agarwal, N. V. Thakor, and Y. X. Guo, "Enabling Wireless Powering and Telemetry for Peripheral Nerve Implants," *IEEE Journal of Biomedical and Health Informatics*, vol. 19, no. 3, pp. 958-970, 2015.
- [48] L. Xie, Y. Shi, Y. T. Hou, and H. D. Sherali, "Making sensor networks immortal: An energy-renewal approach with wireless power transfer," *IEEE/ACM Transactions on Networking (TON)*, vol. 20, no. 6, pp. 1748-1761, 2012.
- [49] R. Zhang and C. K. Ho, "MIMO Broadcasting for Simultaneous Wireless Information and Power Transfer," *IEEE Transactions on Wireless Communications*, vol. 12, no. 5, pp. 1989-2001, 2013.
- [50] B. H. Choi, E. S. Lee, Y. H. Sohn, G. C. Jang, and C. T. Rim, "Six Degrees of Freedom Mobile Inductive Power Transfer by Crossed Dipole Tx and Rx Coils," *IEEE Transactions on Power Electronics*, vol. 31, no. 4, pp. 3252-3272, 2015.
- [51] U. K. Madawala and D. J. Thrimawithana, "A Bidirectional Inductive Power Interface for Electric Vehicles in V2G Systems," *IEEE Transactions on Industrial Electronics*, vol. 58, no. 10, pp. 4789-4796, 2011.
- [52] B. X. Nguyen *et al.*, "An Efficiency Optimization Scheme for Bidirectional Inductive Power Transfer Systems," *IEEE Transactions on Power Electronics*, vol. 30, no. 11, pp. 6310-6319, 2015.
- [53] L. Seung-Hwan and R. D. Lorenz, "A design methodology for multi-kW, large air-gap, MHz frequency, wireless power transfer systems," in *IEEE Energy Conversion Congress and Exposition (ECCE)*, 2011, pp. 3503-3510.

- [54] WiTricity. (Accessed: 01-05-2016). *WiT-3300 Deployment Kit*. Available: http://www.witricity.com/assets/WiT-3300_data_sheet.pdf
- [55] M. Budhia, J. T. Boys, G. A. Covic, and H. Chang-Yu, "Development of a Single-Sided Flux Magnetic Coupler for Electric Vehicle IPT Charging Systems," *IEEE Transactions on Industrial Electronics*, vol. 60, no. 1, pp. 318-328, 2013.
- [56] K. Jiseong *et al.*, "Coil Design and Shielding Methods for a Magnetic Resonant Wireless Power Transfer System," *Proceedings of the IEEE*, vol. 101, no. 6, pp. 1332-1342, 2013.
- [57] O. C. Onar, J. M. Miller, S. L. Campbell, C. Coomer, C. P. White, and L. E. Seiber, "A novel wireless power transfer for in-motion EV/PHEV charging," in *Twenty-Eighth Annual IEEE Applied Power Electronics Conference and Exposition (APEC)*, 2013 2013, pp. 3073-3080.
- [58] L. Pengfei and R. Bashirullah, "A Wireless Power Interface for Rechargeable Battery Operated Medical Implants," *IEEE Transactions on Circuits and Systems II: Express Briefs*, vol. 54, no. 10, pp. 912-916, 2007.
- [59] M. Kiani and M. Ghovanloo, "An RFID-Based Closed-Loop Wireless Power Transmission System for Biomedical Applications," *IEEE Transactions on Circuits and Systems II: Express Briefs*, vol. 57, no. 4, pp. 260-264, 2010.
- [60] A. K. RamRakhyani and G. Lazzi, "On the Design of Efficient Multi-Coil Telemetry System for Biomedical Implants," *IEEE Transactions on Biomedical Circuits and Systems*, vol. 7, no. 1, pp. 11-23, 2013.
- [61] S. Kim, J. S. Ho, and A. S. Y. Poon, "Midfield Wireless Powering of Subwavelength Autonomous Devices," *Physical Review Letters*, vol. 110, no. 20, p. 203905, 05/17/ 2013.
- [62] A. Leung, W. J. Luksemburg, A. S. Wong, and M. H. Wong, "Spatial distribution of polybrominated diphenyl ethers and polychlorinated dibenzo-p-dioxins and dibenzofurans in soil and combusted residue at Guiyu, an electronic waste recycling site in southeast China," *Environmental science & technology*, vol. 41, no. 8, pp. 2730-2737, 2007.
- [63] AirFuel-Alliance. (Accessed: 20-05-2016). "Wireless charging headlines from CES 2016 ". Available: <http://airfuel.org/news/airfuel-news-articles/item/429-ifs-wireless-charging-headlines-from-ces-2016>
- [64] Sony-Corp. (Accessed: 01-05-2016). "Sony develops highly efficient wireless power transfer system based on magnetic resonance. ". Available: <http://www.sony.net/SonyInfo/News/Press/200910/09-119E/>
- [65] H. Chang-Yu, J. T. Boys, G. A. Covic, and M. Budhia, "Practical considerations for designing IPT system for EV battery charging," in *IEEE Vehicle Power and Propulsion Conference, 2009 (VPPC)*, 2009, pp. 402-407.
- [66] H. Haus and W. P. Huang, "Coupled-mode theory," *Proceedings of the IEEE*, vol. 79, no. 10, pp. 1505-1518, 1991.
- [67] A. Karalis, J. D. Joannopoulos, and M. Soljačić, "Efficient wireless non-radiative mid-range energy transfer," *Annals of Physics*, vol. 323, no. 1, pp. 34-48, 2008.
- [68] K. Kusaka and J. I. Itoh, "Proposal of Switched-mode Matching Circuit in power supply for wireless power transfer using magnetic resonance coupling," in *Applied Power Electronics Conference and Exposition (APEC)*, 2012 Twenty-Seventh Annual IEEE, 2012, pp. 653-660.
- [69] T. Beh, M. Kato, T. Imura, S. Oh, and Y. Hori, "Automated Impedance Matching System for Robust Wireless Power Transfer via Magnetic Resonance Coupling," *IEEE Transactions on Industrial Electronics*, vol. 60, no. 9, pp. 3689-3698, 2012.
- [70] B. Wang, K. H. Teo, T. Nishino, W. Yezazunis, J. Barnwell, and J. Zhang, "Experiments on wireless power transfer with metamaterials," *Applied Physics Letters*, vol. 98, no. 25, pp. 254101-254101-3, 2011.
- [71] L. Wang-Sang, S. Wang-Ik, O. Kyoung-Sub, and Y. Jong-Won, "Contactless Energy Transfer Systems Using Antiparallel Resonant Loops," *IEEE Transactions on Industrial Electronics*, vol. 60, no. 1, pp. 350-359, 2013.

- [72] M. Budhia, G. A. Covic, and J. T. Boys, "Design and Optimization of Circular Magnetic Structures for Lumped Inductive Power Transfer Systems," *IEEE Transactions on Power Electronics*, vol. 26, no. 11, pp. 3096-3108, 2011.
- [73] S.-H. Lee and R. D. Lorenz, "Surface spiral coil design methodologies for high efficiency, high power, low flux density, large air-gap wireless power transfer systems," in *Applied Power Electronics Conference and Exposition (APEC), 2013 Twenty-Eighth Annual IEEE*, 2013, pp. 1783-1790.
- [74] C. Kainan and Z. Zhengming, "Analysis of the Double-Layer Printed Spiral Coil for Wireless Power Transfer," *IEEE Journal of Emerging and Selected Topics in Power Electronics*, vol. 1, no. 2, pp. 114-121, 2013.
- [75] K. Fotopoulou and B. W. Flynn, "Wireless Power Transfer in Loosely Coupled Links: Coil Misalignment Model," *IEEE Transactions on Magnetics*, vol. 47, no. 2, pp. 416-430, 2011.
- [76] B. W. Flynn and K. Fotopoulou, "Rectifying loose coils: Wireless power transfer in loosely coupled inductive links with lateral and angular misalignment," *IEEE Microwave Magazine*, vol. 14, no. 2, pp. 48-54, 2013.
- [77] Z. Fei, S. A. Hackworth, F. Weinong, L. Chengliu, M. Zhihong, and S. Mingui, "Relay Effect of Wireless Power Transfer Using Strongly Coupled Magnetic Resonances," *IEEE Transactions on Magnetics*, vol. 47, no. 5, pp. 1478-1481, 2011.
- [78] B. Luo, S. Wu, and N. Zhou, "Flexible Design Method for Multi-Repeater Wireless Power Transfer System Based on Coupled Resonator Bandpass Filter Model," *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 61, no. 11, pp. 3288-3297, 2014.
- [79] W. X. Zhong, C. Zhang, X. Liu, and S. Y. R. Hui, "A Methodology for Making a Three-Coil Wireless Power Transfer System More Energy Efficient Than a Two-Coil Counterpart for Extended Transfer Distance," *IEEE Transactions on Power Electronics*, vol. 30, no. 2, pp. 933-942, 2015.
- [80] D. Ahn and S. Hong, "A Study on Magnetic Field Repeater in Wireless Power Transfer," *IEEE Transactions on Industrial Electronics*, vol. 60, no. 1, pp. 360-371, 2013.
- [81] Y. Urzhumov and D. R. Smith, "Metamaterial-enhanced coupling between magnetic dipoles for efficient wireless power transfer," *Physical Review B*, vol. 83, no. 20, p. 205114, 2011.
- [82] Y. Fan, L. Li, S. Yu, C. Zhu, and C.-H. Liang, "Experimental study of efficient wireless power transfer system integrating with highly sub-wavelength metamaterials," *Progress In Electromagnetics Research*, vol. 141, pp. 769-784, 2013.
- [83] B. H. Waters, A. P. Sample, and J. R. Smith, "Adaptive Impedance Matching for Magnetically Coupled Resonators," in *PIERS Proceedings*, 2012, pp. 694-701.
- [84] D. R. Nayanisiri, D. M. Vilathgamuwa, and D. L. Maskell, "Current-controlled resonant circuit based photovoltaic micro-inverter with half-wave cycloconverter," in *IEEE Industry Applications Society Annual Meeting*, 2013, pp. 1-6.
- [85] S. Byeong-Mun, R. Kratz, and S. Gurol, "Contactless inductive power pickup system for Maglev applications," in *37th IAS Annual Meeting Industry Applications Conference, 2002*, 2002, vol. 3, pp. 1586-1591 vol.3.
- [86] R. Mecke and C. Rathge, "High frequency resonant inverter for contactless energy transmission over large air gap," in *IEEE 35th Annual Power Electronics Specialists Conference, 2004. PESC 04*, 2004, vol. 3, pp. 1737-1743 Vol.3.
- [87] J. Millan, P. Godignon, X. Perpina, A. Perez-Tomas, and J. Rebollo, "A Survey of Wide Bandgap Power Semiconductor Devices," *IEEE Transactions on Power Electronics*, vol. 29, no. 5, pp. 2155-2163, 2014.
- [88] A. Bindra, "Wide-Bandgap-Based Power Devices: Reshaping the power electronics landscape," *IEEE Power Electronics Magazine*, vol. 2, no. 1, pp. 42-47, 2015.
- [89] S. Yin, K. J. Tseng, C. F. Tong, R. Simanjorang, C. J. Gajanayake, and A. K. Gupta, "A 99% efficiency SiC three-phase inverter using synchronous rectification," in *2016 IEEE Applied Power Electronics Conference and Exposition (APEC)*, 2016, pp. 2942-2949.

- [90] A. Cai, A. Pereira, R. Tanzania, Y. K. Tan, and L. Siek, "A high frequency, high efficiency GaN HFET based inductive power transfer system," in *2015 IEEE Applied Power Electronics Conference and Exposition (APEC)*, 2015, pp. 3094-3100.
- [91] E. Waffenschmidt, "Homogeneous Magnetic Coupling for Free Positioning in an Inductive Wireless Power System," *IEEE Journal of Emerging and Selected Topics in Power Electronics*, vol. 3, no. 1, pp. 226-233, 2015.
- [92] N. Minh Quoc, Z. Hughes, P. Woods, S. Young-Sik, S. Rao, and J. C. Chiao, "Field Distribution Models of Spiral Coil for Misalignment Analysis in Wireless Power Transfer Systems," *IEEE Transactions on Microwave Theory and Techniques*, vol. 62, no. 4, pp. 920-930, 2014.
- [93] C. M. Zierhofer and E. S. Hochmair, "Geometric approach for coupling enhancement of magnetically coupled coils," *IEEE Transactions on Biomedical Engineering*, vol. 43, no. 7, pp. 708-714, 1996.
- [94] L. Xun and S. Y. Hui, "Optimal Design of a Hybrid Winding Structure for Planar Contactless Battery Charging Platform," *IEEE Transactions on Power Electronics*, vol. 23, no. 1, pp. 455-463, 2008.
- [95] J. Kim and Y. Park, "Approximate Closed-Form Formula for Calculating Ohmic Resistance in Coils of Parallel Round Wires with Unequal Pitches," *IEEE Transactions on Industrial Electronics*, vol. PP, no. 99, pp. 1-1, 2014.
- [96] D. Thuc Phi and L. Jong-Wook, "Experimental Results of High-Efficiency Resonant Coupling Wireless Power Transfer Using a Variable Coupling Method," *IEEE Microwave and Wireless Components Letters*, vol. 21, no. 8, pp. 442-444, 2011.
- [97] Z. Yang, W. Liu, and E. Basham, "Inductor Modeling in Wireless Links for Implantable Electronics," *IEEE Transactions on Magnetics*, vol. 43, no. 10, pp. 3851-3860, 2007.
- [98] C. Akyel, S. I. Babic, and M.-M. Mahmoudi, "Mutual inductance calculation for non-coaxial circular air coils with parallel axes," *Progress In Electromagnetics Research*, vol. PIER 91, pp. 287-301, 2009.
- [99] J. Kim, D. H. Kim, and Y. J. Park, "Analysis of Capacitive Impedance Matching Networks for Simultaneous Wireless Power Transfer to Multiple Devices," *IEEE Transactions on Industrial Electronics*, vol. 62, no. 5, pp. 2807-2813, 2015.
- [100] L. Kibok, Z. Pantic, and S. M. Lukic, "Reflexive Field Containment in Dynamic Inductive Power Transfer Systems," *IEEE Transactions on Power Electronics*, vol. 29, no. 9, pp. 4592-4602, 2014.
- [101] B. Teck Chuan, M. Kato, T. Imura, O. Sehoon, and Y. Hori, "Automated Impedance Matching System for Robust Wireless Power Transfer via Magnetic Resonance Coupling," *IEEE Transactions on Industrial Electronics*, vol. 60, no. 9, pp. 3689-3698, 2013.
- [102] I. Awai, "Basic characteristics of "Magnetic resonance" wireless power transfer system excited by a 0 ohm power source," *IEICE Electronics Express*, vol. 10, no. 21, pp. 20132008-20132008, 2013.
- [103] M. Kiani and M. Ghovanloo, "The Circuit Theory Behind Coupled-Mode Magnetic Resonance-Based Wireless Power Transmission," *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 59, no. 9, pp. 2065-2074, 2012.
- [104] S. Zhou and C. Mi, "Multi-Paralleled LCC Reactive Power Compensation Networks and Its Tuning Method for Electric Vehicle Dynamic Wireless Charging," *IEEE Transactions on Industrial Electronics*, vol. PP, no. 99, pp. 1-1, 2015.
- [105] D. Lin, C. Zhang, and S. Y. R. Hui, "Mathematical Analysis of Omnidirectional Wireless Power Transfer: Part-II 3-Dimensional Systems," *IEEE Transactions on Power Electronics*, vol. PP, no. 99, pp. 1-1, 2016.
- [106] D. Ahn, S. Kim, J. Moon, and I.-K. Cho, "Wireless Power Transfer with Automatic Feedback Control of Load Resistance Transformation," *IEEE Transactions on Power Electronics*, vol. PP, no. 99, pp. 1-1, 2016.

- [107] S. Kai, Z. Chunbo, K. Kim Ean, T. Imura, and Y. Hori, "Wireless power transfer for running EV powering using multi-parallel segmented rails," in *IEEE PELS Workshop on Emerging Technologies: Wireless Power (WoW)*, 2015, 2015, pp. 1-6.
- [108] C. Zhang, D. Lin, and S. Y. Hui, "Basic Control Principles of Omnidirectional Wireless Power Transfer," *IEEE Transactions on Power Electronics*, vol. 31, no. 7, pp. 5215-5227, 2016.
- [109] F. Mastri, A. Costanzo, and M. Mongiardo, "Coupling-Independent Wireless Power Transfer," *IEEE Microwave and Wireless Components Letters*, vol. 26, no. 3, pp. 222-224, 2016.
- [110] N. Inagaki, "Theory of Image Impedance Matching for Inductively Coupled Power Transfer Systems," *IEEE Transactions on Microwave Theory and Techniques*, vol. 62, no. 4, pp. 901-908, 2014.