

Performance Analysis of Adaptive MMSE Reception for DS-CDMA in Flat Nakagami Fading Channels

Wanshi Chen and Manh Anh Do

Abstract—An efficient method is derived in this paper to obviate the integral involved in the performance analysis of the minimum mean square error (MMSE) detector for direct-sequence code-division multiple-access (DS/CDMA) systems in fading environments. The new method is shown to significantly reduce the necessary computational load for the performance analysis of the MMSE detector. We then examine the behavior of the MMSE detector in flat Nakagami fading channels.

Index Terms—DS-CDMA, fading channels, MMSE receiver, multiuser detection, spread-spectrum communications.

I. INTRODUCTION

THE CONVENTIONAL detector for direct-sequence code-division multiple-access (DS/CDMA) systems is very sensitive to the multiple-access interference (MAI) and the so-called near-far problem [1]. Interference suppression schemes based on the minimum mean square error (MMSE) criterion [2]–[5] have been shown to be immune from the interference floor and the near-far problem. It is very difficult, if not impossible, to derive a closed expression for the error probability of the MMSE detector in fading environments because the bit error rate (BER) is dependent on the spreading codes, the timing delays, and the received amplitudes of all the active users. Instead, [5] takes a “semi-analytic” approach that first generates a random selection of the parameters of all the users and then numerically evaluates the integral of the error probability conditioned on the fading of the user of interest. However, the disadvantage of this method is the time-consuming evaluation of the error probability integral mentioned above. In this paper, we derive an efficient method, which removes the integral and thus lessens the necessary computational load. We then extend the analysis of the performance of the MMSE detector to Nakagami distributed fading, which can be used to characterize a wide range of fading environments.

II. SYSTEM MODEL

We consider a frequency nonselective slow fading channel. Without loss of generality, user 1 is taken as the reference user. In [5], the received signal is first converted to the complex baseband and then passed through a chip-matched filter followed by

a chip-rate sampler. The resultant received vector $\mathbf{r}(i)$ for the i th bit interval is [5]

$$\mathbf{r}(i) = d_1(i)\alpha_{1,i}e^{j\theta_{1,i}}\mathbf{c}_1 + \sum_{k=2}^K \sqrt{\frac{P_k}{P_1}} \alpha_{k,i}e^{j\theta_{k,i}}\mathbf{J}_{k,i} + \mathbf{n}(i) \quad (1)$$

where $d_1(i) = \{+1, -1\}$ is the i th data bit of user 1 and P_k , $\alpha_{k,i}$, and $\theta_{k,i}$ are the average received signal power, amplitude, and phase of the fading process, respectively. K is the number of users, \mathbf{c}_1 is the spreading code of length N for user 1, where N is the spreading gain, $\mathbf{J}_{k,i}$ is the interference vector which is a function of data bits, the spreading code, and the relative timing delay of the k th user, and $\mathbf{n}(i)$ is the noise vector [5].

A scheme was proposed in [5] to track and estimate channel phase $\theta_{1,i}$. Assuming the estimated value is $\hat{\theta}_{1,i}$, $\mathbf{r}(i)$ is modified to

$$\hat{\mathbf{r}}(i) = \text{Re} \left\{ e^{-j\hat{\theta}_{1,i}} \mathbf{r}(i) \right\}. \quad (2)$$

It was shown in [5] that the degradation of the error probability due to imperfect phase estimation with this scheme is minimal and hence reasonable results can be obtained by assuming perfect phase estimation.

III. PERFORMANCE ANALYSIS

The conditional error probability of the MMSE receiver is given by [5]

$$P_e^{\text{mmse}}|_{\alpha_{1,i}} = Q \left(\sqrt{\frac{\alpha_{1,i}^2 \mathbf{c}_1^T \mathbf{R}^{-1} \mathbf{c}_1}{1 - \alpha_{1,i}^2 \mathbf{c}_1^T \mathbf{R}^{-1} \mathbf{c}_1}} \right) \quad (3)$$

where¹

$$\begin{aligned} \mathbf{R} &= E \left\{ \hat{\mathbf{r}}(i) \hat{\mathbf{r}}(i)^T \right\} \\ &= \alpha_{1,i}^2 \mathbf{c}_1 \mathbf{c}_1^T + \sum_{k=2}^K \frac{P_k}{2P_1} \alpha_{k,i}^2 E \left\{ \mathbf{J}_{k,i} \mathbf{J}_{k,i}^T \right\} + \sigma^2 \mathbf{I}_{N \times N}. \end{aligned} \quad (4)$$

$E\{\}$ and T denote expectation and matrix transposition, respectively, and $\sigma^2 = N/2\gamma_1$ in which $\gamma_1 = E_b/N_{o1}$ is the bit-energy-to-noise-density of user 1.

P_e^{mmse} is obtained by averaging (3) over the probability density function of $\alpha_{1,i}$

$$P_e^{\text{mmse}} = \int_0^\infty p_{\alpha_{1,i}}(\alpha_{1,i}) P_e^{\text{mmse}}|_{\alpha_{1,i}} d\alpha_{1,i}. \quad (5)$$

$${}^1Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty \exp(-t^2/2) dt.$$

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Equations (3)–(5) show that P_e^{mmse} is dependent on the timing delays, received amplitudes, and spreading codes of K users. In [5], a “semi-analytic” approach was used to calculate P_e^{mmse} , which first randomly selects the parameters for all users and then numerically evaluates (5). It is reasonable to believe that the mean value of a relatively large number of evaluations will give a quite close approximation of the true error probability P_e^{mmse} .

Denote

$$\mathbf{R} = \alpha_{1,i}^2 \mathbf{c}_1 \mathbf{c}_1^T + \mathbf{R}_2 \quad (6)$$

$$\mathbf{R}_2 = \sum_{k=2}^K \frac{P_k}{2P_1} \alpha_{1,i}^2 E \{ \mathbf{J}_{k,i} \mathbf{J}_{k,i}^T \} + \sigma^2 \mathbf{I}_{N \times N}. \quad (7)$$

We can now rewrite $\alpha_{1,i}^2 \mathbf{c}_1^T \mathbf{R}^{-1} \mathbf{c}_1$ as (see Appendix)

$$\alpha_{1,i}^2 \mathbf{c}_1^T \mathbf{R}^{-1} \mathbf{c}_1 = \frac{2\alpha_{1,i}^2 \gamma_1^{\text{eff}}}{1 + 2\alpha_{1,i}^2 \gamma_1^{\text{eff}}} \quad (8)$$

with $\gamma_1^{\text{eff}} = \mathbf{c}_1^T \mathbf{R}_2^{-1} \mathbf{c}_1 / 2$. In the single-user case, γ_1^{eff} reduces to $\gamma_1^{\text{eff}} = \gamma_1$.

Now (3) can be simplified into

$$P_e^{\text{mmse}}|_{\alpha_{1,i}} = Q \left(\alpha_{1,i} \sqrt{2\gamma_1^{\text{eff}}} \right). \quad (9)$$

Note that γ_1^{eff} is independent of the fading of user 1, the integral in (5) thus turns out to be the usual formulation for the error probability of coherent binary PSK in the fading channels. In the case of Nakagami fading, we have [6]

$$P_e^{\text{mmse}} = \frac{\Gamma \left(m + \frac{1}{2} \right)}{2\sqrt{\pi} \Gamma(m+1)} \left(\frac{m}{m + \gamma_1^{\text{eff}}} \right)^m \left(\frac{\gamma_1^{\text{eff}}}{m + \gamma_1^{\text{eff}}} \right)^{1/2} \cdot {}_2F_1 \left(1, m + \frac{1}{2}; m + 1; \frac{m}{m + \gamma_1^{\text{eff}}} \right) \quad (10)$$

where m is the Nakagami fading figure, $\Gamma(\cdot)$ is the gamma function, and ${}_2F_1(a, b; c; z)$ is the hypergeometric function [7]

$${}_2F_1(a, b; c; z) = \sum_{k=0}^{\infty} \frac{(a)_k (b)_k z^k}{(c)_k k!},$$

$$(a)_k = a(a+1) \cdots (a+k-1), (a)_0 = 1.$$

When m is an integer, (10) can be further simplified to

$$P_e^{\text{mmse}} = \frac{1}{2} \left[1 - \mu \sum_{k=0}^{m-1} \binom{2k}{k} \left(\frac{1-\mu^2}{4} \right)^k \right],$$

$$\text{with } \mu = \sqrt{\frac{\gamma_1^{\text{eff}}}{m + \gamma_1^{\text{eff}}}}. \quad (11)$$

For the particular case when $m = 1$

$$P_e^{\text{mmse}} = \frac{1}{2} (1 - \mu), \quad \text{with } \mu = \sqrt{\frac{\gamma_1^{\text{eff}}}{1 + \gamma_1^{\text{eff}}}} \quad (12)$$

which corresponds to the well-known expression of the error probability for Rayleigh fading channels.

When using (10)–(12), we must be aware that γ_1^{eff} is dependent on the received amplitudes, spreading codes, and relative timings of the $K - 1$ interfering users, as well as the spreading code of the reference user. Therefore, to evaluate the average error probability of the MMSE detector, we still have to take the “semi-analytic” approach as in [5].

However, in the “semi-analytic” approach for each independent run by using (5), although it is possible to avoid the calculation of \mathbf{R}^{-1} by specific implementations, the necessary computational load for all numerical integral intervals is still very high. On the contrary, (10) obviates the need to calculate the integral, and its main computational load comes from the evaluation of the hypergeometric function ${}_2F_1(a, b; c; z)$, which is easier [7]. Therefore, the computational load involved in the calculation of P_e^{mmse} is significantly lightened.

IV. NUMERICAL RESULTS

The performance of the MMSE detector is numerically evaluated in this section. The random codes of length $N = 31$ are employed. The improved Gaussian approximation (IGA) method [8] is adopted to evaluate the error probability of the conventional detector. We assume that all the users are subject to statistically identical Nakagami fading. All the curves shown below are averaged over 500 independent runs. Also, as in [5], the error probability P_e shown in all the following figures is multiplied by a factor of $2(1 - P_e)$ to account for the differential encoding and decoding.

Fig. 1 shows the error probability of user 1 as a function of γ_1 with $K = 15$ simultaneously active users for the conventional detector (CD) and the MMSE detector in perfect power control systems. The values of the fading figure of Nakagami fading are taken from $m = 0.5, 1, 2, 5$, and 100 , which cover a wide range of fading environments, from the most severe fading ($m = 0.5$), to the Rayleigh fading ($m = 1$), and finally to an approximately nonfading case ($m = 100$). It can be seen that the MMSE detector offers substantial performance gain over the conventional detector in varying degrees of fading channels. It should be noted that the error probability of the MMSE detector decreases almost inversely with SNR in severe fading channels ($m = 0.5, 1$, and 2) and becomes exponential degradation in a nonfading case ($m = 100$). This is consistent with the property of the error probability of a single user in fading and nonfading channels [9]. An intuitive explanation of this phenomenon is the Gaussian assumption of the output of the MMSE detector [10], which results in an approximate equivalence of the MMSE detector in a multiuser environment to the single-user case.

Now let us consider the MMSE detector in a near–far environment. As in [5], we assume that the average received power levels of interfering signals conform to the lognormal distribution with a standard deviation of σ_P dB. The mean power level of user 1 is assumed to be one standard deviation lower than the mean values of all other interfering users. Fig. 2 shows that in a wide range of imperfect power control, the performance of the MMSE detector degrades slightly, while the conventional detector is very vulnerable to the MAI. This verifies the near–far

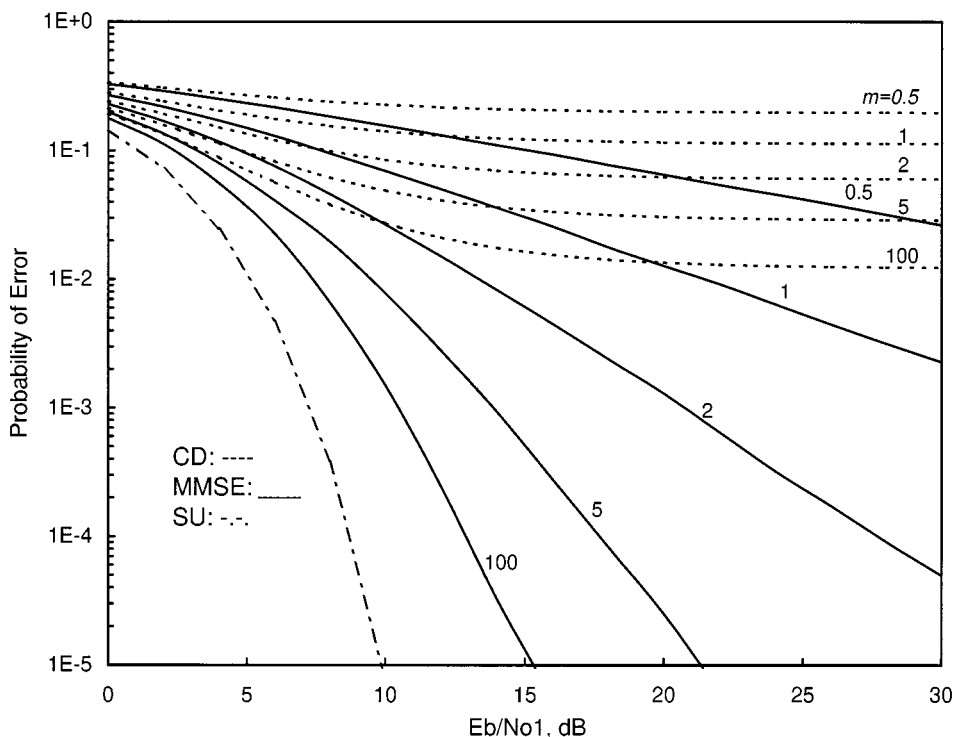


Fig. 1. Error probability of the conventional detector (CD) and the MMSE detector as a function of E_b/N_{o1} , $N = 31$, $K = 15$, $m = 0.5, 1, 2, 5$, and 100 . Single-user (SU) case is also shown for comparison.

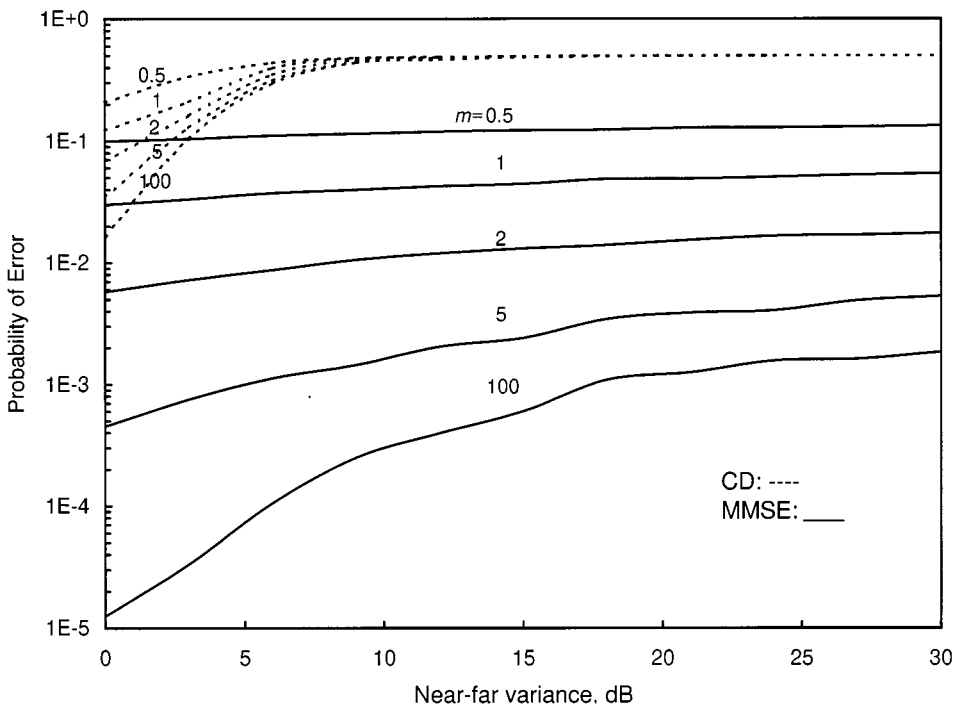


Fig. 2. Error probability of the conventional detector (CD) and the MMSE detector as a function of near-far variance, $E_b/N_{o1} = 15$ dB, $N = 31$, $K = 15$, $m = 0.5, 1, 2, 5$, and 100 .

resistance capability of the MMSE detector under various fading conditions. For the particular case of $m = 1$, i.e., in the Rayleigh fading environments, it is found that the numerical results shown in Fig. 2 are consistent with the results shown in [5, Fig. 6].

Our simulations were implemented by MATLAB and carried out on SUN SPARC workstations. Compared with the method

in [5], it was found that the new method substantially shortened the computer running time in obtaining numerical results. For instance, when $K = 15$, $N = 31$, and $m = 5$, it took the method in [5] more than 20 h to obtain P_e^{mmse} averaged over 500 independent runs, while the new method requires only about 30 min. The time efficiency of the new method was justified.

V. CONCLUSIONS

We extend Barbosa and Miller's work [5] in this paper. To eliminate the time-consuming evaluation of the integral involved in the "semi-analytic" performance analysis of the MMSE detector, we exploit the matrix inverse theorem to remove the integral over the fading of the reference user and derive a more efficient solution. We then extend the analysis of the behavior of the MMSE detector for the Nakagami fading environments.

APPENDIX

We derive (8) and (10) in this Appendix. The matrix inversion theorem [11, p. 267, Theorem 12.4] tells us that if $\mathbf{B} = \mathbf{A} - \mathbf{U}\mathbf{Q}\mathbf{V}$, where \mathbf{A} , \mathbf{B} are nonsingular matrices, we will have

$$\mathbf{B}^{-1} = \mathbf{A}^{-1} + \mathbf{A}^{-1}\mathbf{U}\mathbf{Q}(\mathbf{I} - \mathbf{V}\mathbf{A}^{-1}\mathbf{U}\mathbf{Q})^{-1}\mathbf{V}\mathbf{A}^{-1}. \quad (\text{A1})$$

Since both \mathbf{R} and \mathbf{R}_2 , defined in (6) and (7), respectively, are nonsingular matrices and $\mathbf{R} = \mathbf{R}_2 + \alpha_{1,i}^2 \mathbf{c}_1 \mathbf{c}_1^T$, we can thus use (A1) to calculate \mathbf{R}^{-1} . Define $\mathbf{B} = \mathbf{R}$, $\mathbf{A} = \mathbf{R}_2$, $\mathbf{U} = \mathbf{V}^T = \mathbf{a}_1$, $\mathbf{Q} = -\alpha_{1,i}^2$, we will have

$$\mathbf{R}^{-1} = \mathbf{R}_2^{-1} \left(\mathbf{I}_{N \times N} - \alpha_{1,i}^2 \frac{\mathbf{c}_1 \mathbf{c}_1^T \mathbf{R}_2^{-1}}{1 + 2\alpha_{1,i}^2 \gamma_1^{\text{eff}}} \right) \quad (\text{A2})$$

where

$$\gamma_1^{\text{eff}} = \frac{1}{2} \mathbf{c}_1^T \mathbf{R}_2^{-1} \mathbf{c}_1. \quad (\text{A3})$$

In the single-user case, $\mathbf{R}_2 = \sigma^2 \mathbf{I}_{N \times N}$, γ_1^{eff} then reduces to $\gamma_1^{\text{eff}} = N/2\sigma^2 = \gamma_1$. Using (A2), $\alpha_{1,i}^2 \mathbf{c}_1^T \mathbf{R}^{-1} \mathbf{c}_1$ becomes

$$\alpha_{1,i}^2 \mathbf{c}_1^T \mathbf{R}^{-1} \mathbf{c}_1 = \frac{2\alpha_{1,i}^2 \gamma_1^{\text{eff}}}{1 + 2\alpha_{1,i}^2 \gamma_1^{\text{eff}}}. \quad (\text{A4})$$

Thus, the conditional error probability of the MMSE detector in (3) becomes

$$P_e^{\text{mmse}}|_{\alpha_{1,i}} = Q \left(\alpha_{1,i}^2 \sqrt{2\gamma_1^{\text{eff}}} \right). \quad (\text{A5})$$

Note that γ_1^{eff} is dependent on the spreading code of the reference user and the matrix \mathbf{R}_2 , which in turn is dependent on the received amplitudes, relative timings, and spreading codes of all the interfering users.

REFERENCES

- [1] S. M. Moshavi, "Multiuser detection for DS-CDMA communications," *IEEE Commun. Mag.*, pp. 124–136, Oct. 1996.
- [2] Z. Xie, R. T. Short, and C. K. Rushforth, "A family of sub-optimum detectors for coherent multiuser communications," *IEEE J. Select. Areas Commun.*, vol. 8, pp. 683–690, May 1990.
- [3] U. Madhow and M. L. Honig, "MMSE interference suppression for direct-sequence spread-spectrum CDMA," *IEEE Trans. Commun.*, vol. 42, pp. 3178–3188, Dec. 1994.
- [4] S. L. Miller, "An adaptive direct-sequence code-division multiple-access receiver for multiuser interference rejection," *IEEE Trans. Commun.*, vol. 43, pp. 1746–1755, Feb./Mar./Apr. 1995.
- [5] A. N. Barbosa and S. L. Miller, "Adaptive detection of DS/CDMA signals in fading channels," *IEEE Trans. Commun.*, vol. 46, pp. 115–124, Jan. 1998.
- [6] T. Eng and B. M. Milstein, "Coherent DS-CDMA performance in Nakagami multipath fading," *IEEE Trans. Commun.*, vol. 43, pp. 1134–1143, Feb./Mar./Apr. 1995.
- [7] S. Zhang and J. Jin, *Computation of Special Functions*. New York: Wiley, 1996.
- [8] R. M. Buehrer and B. D. Woerner, "Analysis of adaptive multistage interference cancellation for CDMA using an improved Gaussian approximation," *IEEE Trans. Commun.*, vol. 44, pp. 1308–1321, Oct. 1996.
- [9] J. G. Proakis, *Digital Communications*, 3rd ed. New York: McGraw-Hill, 1995.
- [10] H. V. Poor and S. Verdú, "Probability of error in MMSE multi-user detection," *IEEE Trans. Inform. Theory*, vol. 43, pp. 858–871, May 1997.
- [11] M. Fielder, *Special Matrices and Their Applications in Numerical Mathematics*. Hingham, MA: Martinus Nijhoff, 1986.

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