

Vendor Managed Inventory Contracts – Coordinating the Supply Chain while looking from the Vendor’s Perspective

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The paper studies coordination of a supply chain when the inventory is managed by the vendor (VMI). We also provide a general mathematical framework that can be used to analyze contracts under both retailer managed inventory (RMI) and VMI. Using a simple newsvendor scenario with a single vendor and single retailer, we study five popular coordinating supply chain contracts: buyback, quantity flexibility, quantity discount, sales rebate, and revenue sharing contracts. We analyze the ability of these contracts to coordinate the supply chain under VMI when the vendor freely decides the quantity. We find that even though all of them coordinate under RMI, quantity flexibility and sales rebate contracts do not generally coordinate under VMI. Furthermore, buyback and revenue sharing contracts are equivalent. Hence, we propose two new contracts which coordinate under VMI (one of which coordinates under RMI too, provided a well-known assumption holds). Finally, we extend our analysis to consider multiple independent retailers with the vendor incurring linear or convex production cost, and show that our results are qualitatively unchanged.

Keywords: supply chain management; newsvendor; retailing; buyback contract.

1. Introduction

In the last decade, many companies have changed their supply chain structure from retailer managed inventory (RMI) to vendor managed inventory (VMI) in which the vendor decides the quantity to be stocked at the retail location(s). The best known pioneer of a large-scale move towards VMI is Wal-Mart (partnering with P & G and many other suppliers), but many other companies have followed the general trend, e.g. Campbell Soup, Barilla, GE and Intel (Fry et al. 2001).

A shift from RMI to VMI can involve different changes involving the implementation of new IT systems to enable the vendor to access point-of-sale data, development of trust

between vendor and retailer, and the role of vendor's sales force (Hammond 1994). It also requires a reconsideration of the supply chain contracts. In some extreme cases, as we describe next with examples, failure to adequately reform the contractual relationship between vendor and retailer has led to failed VMI implementations. They highlight the need for an adequate contract to coordinate the supply chain under VMI.

Analyzing and understanding how supply chain contracts perform under VMI (and RMI) can be important for online retailers. Consider the case of Lazada, a recent start-up and an Amazon-style online retailer, which is gaining popularity and increasing its market share in many countries across Southeast Asia. Currently, they have a mix of three fulfillment strategies: (i) drop shipping for many of their larger "trusted suppliers" in which the merchants directly ship the products to customers, (ii) consignment (with cross-docking) for their intermediate suppliers, and (iii) fulfillment by Lazada for their smaller suppliers (Lazada 2015). The first and second strategies constitute different forms of VMI, while the third strategy pertains to RMI. In this paper, we model and analyze the performance of different supply chain contracts under VMI and RMI.

In the early 90s, Bausch and Lomb's (B & L) upper management wanted to boost sales of sunglasses as part of their strategy of aggressive organic growth. In order to meet sales targets, the company kept supplying more sunglasses to distributors and retailers. By 1994, they had almost 9 months worth of inventory in their distribution channels. When their distributors and retailers finally realized that they had too much stock, the company was forced to take back excess inventories, which deteriorated the overall supply chain performance. In addition, they had to face an SEC investigation, share price volatility and shareholder lawsuits because of reporting inflated revenues and sales (Businessweek Archives 1995). Although they did not have a formal VMI program in place at the time, since they were determining shipment quantities and timing, the supply chain was in effect practicing VMI. The presence of adequate contracts might have taken away B & L's (short term) incentives to ship arbitrarily large quantities to its distributors. In a similar situation, Chrysler in the mid 2000's had been pushing more cars than the market demanded to reluctant dealers and rental car fleets, which led to poor financial performance (Businessweek Archives 2007).

In an opposite scenario, Spartan Stores, a Michigan cooperative grocery wholesaler, had to discontinue its VMI program due to drop in inventories at the retail stores, especially during promotions (KPMG Report 1996). We hypothesize that the low inventories were partly a result of the conservative approach of vendors in response to a consignment clause that

made them responsible for inventory holding costs. The vendor’s concern about consignment inventory is also reflected in Gamble (1994) where Air Products and Chemicals is faced with a dilemma about VMI since its customers want zero inventory, thereby tying up more of its working capital.

In this article, we consider various VMI contracts in the framework of a model with a single vendor supplying a retailer/multiple independent retailers faced with a classical newsvendor problem. The newsvendor model has also been used extensively in the supply chain contracting literature (see e.g., Cachon (2003), Lariviere and Porteus (2001), and Krishnan et al. (2004)). It also closely reflects the situation in some real-life instances, e.g., newspaper distribution (Bensoussan et al. 2011), DVD sales (Infosys 2007), and book publishing (Shatzkin 1997). In these examples, the vendor typically has better information about the demand than the retailer. This aspect is another key reason for studying supply chain contracts under VMI. A contract, which (i) coordinates under both VMI and RMI, and (ii) has parameters that are independent of the demand distribution (e.g., the buyback contract, see Table 1), will likely result in truthful information sharing, less conflicts of interest, and a more harmonious relationship between vendor and retailer(s). Table 1 summarizes the performance of different types of contracts under VMI and RMI.

2. Literature Survey

We consider the extant literature in three closely related areas — VMI, supply chain contracting in a newsvendor scenario, and contracts in a more general scenario (multi-period, multiple retailers etc.). In examining the supply chain contracting literature (second and third areas mentioned above), we only consider closely related papers (for a more extensive review, see Cachon (2003)).

First, we consider the VMI literature. Some examples of research in this area include Aviv and Federgruen (1998), Wong et al. (2009), Cachon and Fisher (1997), Mishra and Raghunathan (2004), Savaşaneril and Erkip (2010), Dong and Xu (2002), and Disney and Towill (2003). This paper is closely related to Wong et al. (2009) who consider a newsvendor model under VMI with price-setting supplier and retailers that are either independent or

¹With consignment under VMI. Without consignment, there is still no coordination under VMI since $Q_v = \infty$. Also, we say that a contract *always coordinates* only if there is *arbitrary profit allocation*.

²Since vendor’s and retailer’s profit expressions are the same under VMI and RMI, we provide the vendor’s (retailer’s) profit only under VMI (RMI) for the sake of conciseness.

³Demand distribution has an increasing generalized failure rate.

Contract Type	VMI	RMI	Remarks
Wholesale Price ¹	Never coordinates.	Never coordinates.	Optimal order quantity under VMI can be less than that under RMI.
Buyback ²	Always coordinates.	Always coordinates.	Arbitrary profit allocation under both RMI and VMI.
Quantity Flexibility	Sometimes coordinates.	Always coordinates.	Arbitrary profit allocation is not possible under VMI.
Quantity Discount	General wholesale price $w(Q)$ always coordinates. Two-part tariff never does.	Again, $w(Q)$ always coordinates, but two-part tariff does so too.	Arbitrary profit allocation with $w(Q)$ under both VMI and RMI.
Incremental Sales Rebate	Never coordinates.	Always coordinates.	Optimal order quantity under VMI becomes infinite.
Revenue Sharing	Always coordinates	Always coordinates.	Arbitrary profit allocation under both RMI and VMI.
Modified Quantity Flexibility	Always coordinates.	Sometimes coordinates.	If Φ has an IGFR ³ , arbitrary profit allocation and coordination in RMI.
Modified Buyback	Always coordinates.	Sometimes coordinates.	$t = 0$ implies buyback contract. High t implies no coordination in RMI.

Table 1: Performance of different types of contracts under VMI and RMI

competing with each other. They show that sales rebate contracts can achieve coordination. However, this paper is different from Wong et al. (2009) because (i) we compare RMI vs. VMI, (ii) prices are exogenous, and (iii) we also examine other contracts.

There have been papers on shipment scheduling in the context of VMI. Cetinkaya and Lee (2000) propose a model for the supplier to coordinate her inventory and transportation decisions. Cheung and Lee (2002) analyze the benefits to a supply chain from two important attributes of VMI programs: shipment coordination and stock re-balancing.

There is also a stream of literature which addresses the issue of information sharing in VMI. Some research works include Seidmann and Sundararajan (1998), Lee et al. (2000),

and Corbett (2001). The papers in this stream do not specifically consider the role of *supply chain* contracts.

Secondly, we consider the literature on supply chain contracts in a newsvendor scenario. We refer the reader to Pasternack (1985), Tsay (1999), Lariviere and Porteus (2001), Taylor (2002), Krishnan et al. (2004), Cachon and Lariviere (2005), and Özalp and Wei (2006) for examples of research works in this literature. Unlike our paper, the papers in this literature consider supply chain contracts only under RMI.

Thirdly, we look at more general supply chain contracts; e.g. see Cachon (2001), Darwish and Odah (2010), Fry et al. (2001), Choi et al. (2004), Gerchak and Wang (2004), Gerchak et al. (2007), Nagarajan and Rajagopalan (2008), Bernstein et al. (2006), Wang (2004), and Li and Wang (2007). While some of these papers do specifically look at VMI, the setting and analyses with the newsvendor model in this paper are very different. Next, we elaborate on the differences of this paper from research works that study about VMI.

While papers that examine VMI typically consider (i) the vendor optimizing the whole supply chain (e.g., Aviv and Federgruen (1998), Cachon and Fisher (1997), and Darwish and Odah (2010)), (ii) shipment scheduling (e.g., Cetinkaya and Lee (2000), Cheung and Lee (2002)) or (iii) information sharing (e.g., Lee et al. (2000), Corbett (2001)), we assume a “selfish vendor” who maximizes her own profits and we look at the supply chain from a contracting perspective. These aspects are considered by Bernstein et al. (2006), Nagarajan and Rajagopalan (2008) and Fry et al. (2001). However, there are key differences in our paper. While they consider some very specific contracts (wholesale prices and discounts in Bernstein et al. (2006), holding cost subsidies in Nagarajan and Rajagopalan (2008), and (z, Z) contracts in Fry et al. (2001)) in a multi-period setting, we consider contracts that are more popular and widely used in a newsvendor scenario. We also devise a mathematical mechanism for formally characterizing any supply chain contract (see Section 3), and propose two new contracts that are similar to these contracts and coordinate the supply chain under VMI (see Section 5).

In addition to the aforementioned papers, there are other recent research works that consider novel issues pertaining to supply chain contracting. Examples include Turcic et al. (2015), Altug (2016), Jadidi et al. (2016), Giri and Bardhan (2014), Feng and Lu (2013), Ai et al. (2012), Kouvelis and Zhao (2015), and Adida and Ratisoontorn (2011). None of these papers consider VMI contracts in which inventory is a key decision made by the vendor. However, some recent research works such as Guan and Zhao (2010), Chakraborty

Research Paper	VMI	RMI	News- vendor Scenario	Supply Chain Contracts
Cetinkaya and Lee (2000), Cheung and Lee (2002), Seidmann and Sundararajan (1998), Lee et al. (2000), Cachon and Fisher (1997), Savaşaneril and Erkip (2010), Corbett (2001), Mishra and Raghunathan (2004)	*			
Aviv and Federgruen (1998), Dong and Xu (2002), Disney and Towill (2003)	*	*		
Turcic et al. (2015), Hochbaum and Wagner (2015), Jadidi et al. (2016), Feng and Lu (2013), Ai et al. (2012), Kouvelis and Zhao (2015)			*	*
Pasternack (1985), Tsay (1999), Taylor (2002), Lariviere and Porteus (2001), Krishnan et al. (2004), Cachon and Lariviere (2005), Özalp and Wei (2006)		*		*
Choi et al. (2004), Gerchak et al. (2007), Chakraborty et al. (2015)	*			*
Cachon (2001), Fry et al. (2001), Nagarajan and Rajagopalan (2008), Bernstein et al. (2006)	*	*		*
Wong et al. (2009)	*		*	*
Gerchak and Wang (2004)	+	+	+	*
This paper	*	*	*	*

Table 2: Summary showing how this paper fits in with the related research literature

et al. (2015), Lee et al. (2016), and Cai et al. (2017) do consider management/ownership of inventory by vendor. analyze contracts under VMI but they assume that the demand is deterministic and use an EOQ model. Table 2 summarizes how this paper is related to other key research papers. Gerchak and Wang (2004) comes close to this paper; yet there are key differences. They do consider a newsvendor model but they have assembler and suppliers instead of retailer(s) and vendor. They consider different contracts under RMI (wholesale price) and VMI (revenue sharing). These differences fundamentally change the modeling and analysis here. In summary, to the best of our knowledge, this paper is the first to compare the performance of supply chain contracts under RMI and VMI in a newsvendor scenario.

3. General Model

We assume a price-taking newsvendor, i.e., the retail price is assumed to be exogenously given. In an RMI system the retailer decides the order quantity in order to maximize his

profit. In VMI, the quantity supplied to the retailer is decided by the vendor who maximizes her profit. We assume both players have complete information and know the true value of the parameters (production cost per unit, retail price and salvage value). We examine eight different types of contracts — the standard wholesale price contract, five popular contracts from the supply chain literature and two contracts that coordinate the supply chain under VMI and that to the best of our knowledge have not appeared in the supply chain literature before.

The following notation is common to all contracts studied:

c	production cost per unit incurred by vendor
p	retail price per unit (received by the retailer from the end customer)
D	demand (during the selling season)
Q	quantity ordered by (supplied to) retailer in RMI (VMI)
s	salvage value per unit for unsold units at the end of the selling season
$\pi_v(Q; z)$	expected vendor's profit for the season under contract z
$\pi_r(Q; z)$	expected retailer's profit for the season under contract z
$\pi_{tot}(Q)$	expected total profit of the system for the season
$\Phi(\cdot)$	cdf of demand
$\bar{\Phi}$	complementary cdf ($= 1 - \Phi(\cdot)$)
$\phi(\cdot)$	pdf of demand
$Q_v(z), Q_r(z), Q^*$	profit maximizing quantity under z in VMI, RMI, entire system
$\dot{\pi}(Q; z), \dot{\pi}_j(Q; z)$	$= d\pi(Q; z)/dQ, d\pi_j(Q; z)/dQ; j = v, r$

We assume that Φ is continuous and that there exists an interval (l, u) such that $0 \leq l \leq u \leq \infty$; $\Phi(l) = 0$, $\Phi(u) = 1$; and $\Phi(\cdot)$ is strictly increasing on (l, u) . For the problem to be non-trivial, we also assume $s < c < p$. These assumptions guarantee a non-negative demand realization and the uniqueness of the system-wide optimal quantity Q^* given by (1) below.

Let $\tilde{\pi}_r(Q, D; z)$ be the payoff under contract z to the retailer when the quantity chosen was Q and the realized demand D . Then the payoff to the vendor is $\tilde{\pi}_v(Q, D; z) = \tilde{\pi}_{tot}(Q, D) - \tilde{\pi}_r(Q, D; z)$ where $\tilde{\pi}_{tot}(Q, D) = (p - c)D - (p - c)(D - Q)^+ + (s - c)(Q - D)^+$ is the total supply chain payoff. Hence a contract z is completely characterized by its retailer's profit function $\tilde{\pi}_r(\cdot, \cdot; z)$. We assume that under RMI the retailer chooses the quantity $Q_r(z) = \arg \max\{\pi_r(Q; z) : l \leq Q \leq u\}$ while under VMI the vendor chooses the quantity $Q_v(z) = \arg \max\{\pi_v(Q; z) : l \leq Q \leq u\}$, where $\pi_r(Q; z) = E_D(\tilde{\pi}_r(Q, D; z))$ and $\pi_v(Q; z) = E_D(\tilde{\pi}_v(Q, D; z))$ are the retailer's and the vendor's expected profit functions under contract z . Since all the contracts analyzed involve only transfer payments between the

vendor and the retailer, the system optimal quantity Q^* is independent of z and satisfies

$$\dot{\pi}_{tot}(Q^*) = 0 \Leftrightarrow \Phi(Q^*) = \frac{p - c}{p - s}. \quad (1)$$

Let Z be the set of all possible contracts. We define some terms here which will be used later in the paper.

Definition A contract $z \in Z$, is said to *coordinate* under RMI (VMI) if and only if Q^* is the unique global maximizer of $\pi_r(Q; z)$ ($\pi_v(Q; z)$) and $\pi_j(Q^*; z) > 0$ for $j = r, v$.

Definition A contract, z , is said to *potentially coordinate* under RMI (VMI) if and only if $\dot{\pi}_r(Q^*; z) = 0$ ($\dot{\pi}_v(Q^*; z) = 0$) and $\pi_j(Q^*; z) > 0$ for $j = r, v$.

Since $\pi_j(Q; z)$ is differentiable in Q for $j = r, v$ for all the contracts that we consider (we show this result in Appendix C), we can always restrict ourselves to potentially coordinating contracts when trying to find coordinating contracts (any coordinating contract is a potentially coordinating contract, although the reverse is not necessarily true). We denote by Z_{PV} (Z_{PR}) the set of all potentially coordinating contracts under VMI (RMI), and Z_V (Z_R) is the set of all coordinating contracts. Next, we show how Z_{PV} and Z_{PR} are related. All proofs are in the Appendix.

Proposition 1. $Z_{PV} = Z_{PR}$.

Definition We say that contract type $Z' \subset Z$ is *structured to coordinate* under RMI (VMI) if for every $0 < y < \pi_{tot}(Q^*)$ there exists a unique contract $z \in Z' \cap Z_R$ ($Z' \cap Z_V$) such that $\pi_r(Q^*; z) = y$.

The types of contracts that are structured to coordinate are easier to implement because their parameters can be determined by knowing only the profit split between the vendor and the retailer. Note that, by our definition, any type of contract which is structured to coordinate under RMI (VMI) also achieves arbitrary allocation of profits between the vendor and the retailer.

Proposition 2. *Suppose $Z' \subset Z$ is such that $Z' \cap Z_R = Z' \cap Z_{PR}$. Then*

(i) $Z' \cap Z_V \subset Z' \cap Z_R$.

(ii) *Further, suppose Z' is structured to coordinate under RMI. It is structured to coordinate under VMI too if and only if $Z' \cap Z_V = Z' \cap Z_R$.*

Proposition 2 establishes the relationship between how contracts perform under VMI and RMI under certain conditions. The condition $Z' \cap Z_R = Z' \cap Z_{PR}$ states that every potentially coordinating contract in Z' is coordinating, or, equivalently, that the first order condition is sufficient to ensure optimality in the retailer's optimization. Then *any contract in Z' that coordinates under VMI also coordinates under RMI*. Furthermore, suppose Z' is also structured to coordinate under RMI. Then it is structured to coordinate under VMI too if and only if *the same set of contracts in Z' coordinate both under VMI and RMI*.

We discuss related work in Cachon (2003) and the contribution of this paper before we analyze the extant contracts in Section 4. The proofs for the coordination under RMI in the case of buyback, quantity flexibility, quantity discount and sales rebate contracts can be found in the literature (Cachon 2003). They are provided here for the sake of completeness. However, to the best of our knowledge, our research work is the first to analyze how the contracts perform under VMI. Cachon (2003) does analyze whether it coordinates under voluntary compliance. However, that analysis is different because the vendor can only supply the retailer with any quantity less than or equal to retailer's order quantity, while the vendor can freely choose the order quantity under VMI in this paper. Hence contracts which coordinate under VMI always coordinate under voluntary compliance but the reverse statement is not necessarily true. Some examples of this are given by two part tariffs when $A > c$ (see Section 4.4) and the example given in Figure 1 in the analysis of QF contracts (see Section 4.3). We also extend the analysis in Cachon (2003) in cases of QF and sales rebate contracts, and propose two new contracts in section 5 that coordinate under VMI. We provide a general mathematical framework for analyzing contracts under both RMI and VMI, and extend our analysis to consider multiple retailers in section 6. These are our primary contributions.

4. Analysis of Extant Contracts

We analyze the performance of all these contracts under VMI in this section. Analyses of these contracts under RMI exist in the literature but we provide them in the Appendix for completeness.

4.1 Wholesale Price Contract

We assume that unsold inventory at the end of the season is the vendor's responsibility and that the retailer pays only for the goods sold. In other words, the vendor's goods are consigned to the retailer. We look at the case with consignment for three different reasons:

- Without consignment, the vendor has no responsibility for the inventory and hence her incentive is to supply the retailer with infinite quantity.
- Many VMI agreements have a consignment clause in real life.
- We are interested in knowing whether consignment alone can achieve channel coordination.

In the consignment case, $\tilde{\pi}_r(Q, D; z) = (p - w)D - (p - w)(D - Q)^+$ and $\tilde{\pi}_v(Q, D; z) = (w - c)D - (w - c)(D - Q)^+ + (s - c)(Q - D)^+$ for some $w \in S^{WPC} = \{w : c < w < p\}$. Like before, let Z^{WPC} be the set of corresponding wholesale price with consignment contracts. Since all the costs are shifted from the retailer to the vendor, the vendor is “penalized” more for stocking than is warranted and hence there is no coordination as Proposition 3 shows. So $Z^{WPC} \cap Z_V = \emptyset$.

Proposition 3. *Let $Q_r(z)$ and $Q_v(z)$ be the corresponding profit maximizing quantities under wholesale price contract without and with consignment respectively. Under VMI, the wholesale price contract with consignment does not coordinate the supply chain, and $Q_v(z) < Q^*$. Also $Q_v(z) < (=, >)Q_r(z)$ if and only if $w < (=, >)w^* = s + \sqrt{(c - s)(p - s)}$.*

An important corollary from the above proposition is that consignment alone does not coordinate the supply chain though it might take us nearer to the optimal solution if the wholesale price is high enough i.e. $w > w^*$ (this is due to the fact that the profit of the total system is concave in Q so the closer we are to Q^* the higher is the system profit).

4.2 Buyback Contract

This contract was first introduced by Pasternack (1985). It is implemented in many industries, notably publishing and farm products (especially perishable goods). In this contract, the retailer pays the vendor a price of w per unit purchased while the vendor agrees to provide the retailer an amount of b in addition to the salvage value s for each unsold unit. For a non trivial problem, we require $(b, w) \in S^B = \{(b, w) : c < w < p \text{ and } b + s < w\}$ with the corresponding set of contracts Z^B . The payoff functions for the retailer and vendor are given by $\tilde{\pi}_r(Q, D; z) = p(\min(D, Q)) - wQ + (b + s)(Q - D)^+$ and $\tilde{\pi}_v(Q, D; z) = (w - c)Q - b(Q - D)^+$.

Proposition 4. *Any buyback contract which coordinates under RMI also coordinates the supply chain under VMI.*

An important corollary of Proposition 4 is that if the existing contract is a coordinating buyback contract, a change from RMI to VMI does not necessitate a new contract. The quantity decisions will not change and the two players earn exactly the same profit in VMI as in RMI. Hence $Z^B \cap Z_V = Z^B \cap Z_R$, and Proposition 2 implies that the buyback contract is also structured to coordinate under VMI.

4.3 Quantity Flexibility Contract

Notable firms which use quantity flexibility (QF) contracts are Toyota Corporation, HP and IBM (Tsay and Lovejoy 1999). In this contract for a given parameter Δ ($0 < \Delta < 1$) the vendor charges w per unit and agrees to take back up to ΔQ unsold units from the retailer for w per unit where $(\Delta, w) \in S^{QF} = \{(\Delta, w) : c < w < p \text{ and } 0 < \Delta < 1\}$ with the corresponding set of contracts Z^{QF} . The payoff for the retailer and vendor are given by $\tilde{\pi}_r(Q, D; z) = p(\min(D, Q)) - wQ + w(\min(\Delta Q, (Q - D)^+)) + s((Q - D)^+ - \Delta Q)^+$ and $\tilde{\pi}_v(Q, D; z) = (w - c)Q - (w - s)(\min(\Delta Q, (Q - D)^+))$. In the retailer's payoff, the first term represents the revenue from sales during the selling season, the second term the amount paid to the vendor for the quantity Q ordered, the third term accounts for the "partial return credit" obtained by the retailer from the vendor and the last term accounts for the salvage value obtained by the retailer for units neither sold nor taken back by the vendor.

Tsay (1999) shows that in the QF contract, vendor's profit is strictly increasing in Δ . This immediately implies that the QF contract is structured to coordinate under RMI. For any $z \in Z^{QF}$, the retailer's profit is concave in Q and has a unique optimum, in the interior, as shown in the proof. Therefore, Proposition 2 applies. So, while analyzing coordination under VMI, we only consider those contracts $z \in Z^{QF} \cap Z_R$. Further, note that $Z^{QF} \cap Z_R = Z^{QF} \cap Z_{PR} = Z^{QF} \cap Z_{PV}$ in which the first equality is because $\pi_r(Q; z)$ is concave in Q , while the second equality follows from Proposition 1.

We now look at the profit maximizing quantity which will be determined by the vendor under the contract. The derivative of vendor's expected profit function is given by

$$\begin{aligned} \dot{\pi}_v(Q; z) &= w_\Delta - c - (w_\Delta - s)\Delta + (w_\Delta - s)\bar{\Phi}(Q) - \\ &\quad (w_\Delta - s)(1 - \Delta)(\Phi(Q) - \Phi((1 - \Delta)Q)) \\ &= w_\Delta - c - (w_\Delta - s)\Phi(Q) + (w_\Delta - s)(1 - \Delta)\Phi((1 - \Delta)Q). \end{aligned}$$

The vendor's profit function, in general, is not concave in Q (the last term in the derivative above is increasing in Q) so the first order condition may have multiple solutions and in

fact $\pi_v(Q; z)$ may not have a finite maximum at all. So the question becomes, “Under what conditions is there a unique maximum for the vendor’s profit function at Q^* ”? Note that since $Z^{QF} \cap Z_R = Z^{QF} \cap Z_{PV}$, Q^* always satisfies the first order condition $\dot{\pi}_v(Q; z) = 0$. The derivative of $\pi_v(Q; z)$ can be rewritten as

$$\begin{aligned}\dot{\pi}_v(Q; z) &= w_\Delta - c - (w_\Delta - s)\Phi(Q) + (w_\Delta - s)(1 - \Delta)\Phi((1 - \Delta)Q) \\ &= (w_\Delta - s) \left(\frac{w_\Delta - c}{w_\Delta - s} - g(Q, \Delta) \right) \\ &= (w_\Delta - s) (g(Q^*, \Delta) - g(Q, \Delta)),\end{aligned}$$

where $g(Q, \Delta) = \Phi(Q) - (1 - \Delta)\Phi((1 - \Delta)Q)$.

By studying the normalized derivative $g(Q^*, \Delta) - g(Q, \Delta)$ (note that since $w_\Delta > s$, the derivative is just a positive constant times the normalized derivative), we only need to consider two parameters (Δ and Q^*) to analyze the QF contract. The following scenarios are possible: 1. $Q_v(z) = \infty$, the contract does not coordinate under VMI; 2. $Q_v(z) \neq Q^*$ and is finite, the contract does not coordinate under VMI; and 3. $Q_v(z) = Q^*$, the contract coordinates under VMI.

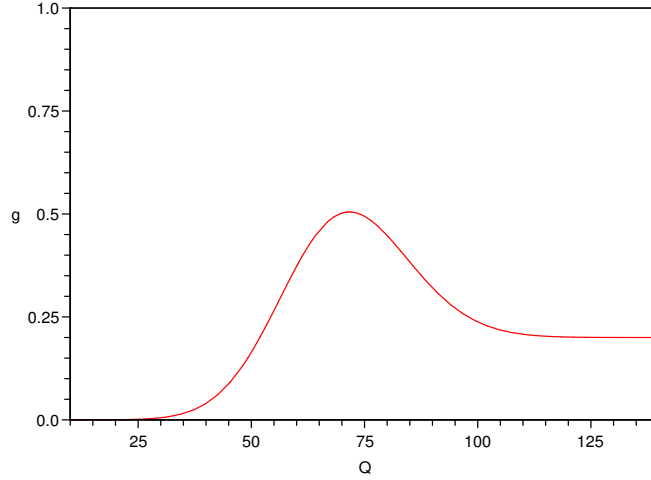


Figure 1: $g(Q, 0.2)$ for normal demand distribution with $\mu = 60$ and $\sigma = 12$

We next provide instances where the first two scenarios occur and later propose fairly general sufficient conditions that will result in the third scenario.

Scenario 1: Assume demand follows a normal distribution with mean 60 and standard deviation 12, and that $\Delta = 0.2$ (with wholesale price equal to $w_{0.2}$). From Figure 1 it can be

clearly seen that if $Q^* = 60$ then $Q_v(z) = \infty$ — the derivative of vendor's profit is positive for arbitrarily large values of Q and hence $\pi_v(Q; z) \rightarrow \infty$ as $Q \rightarrow \infty$.

Scenario 2: Let the pdf and cdf of demand be as plotted in Figures 2 and 3. Figure 4 shows a plot of $g(Q, 0.375)$ as a function of Q . Figure 5 shows the normalized derivative when $Q^* = 1$. Hence, for the contract z parametrized by $(0.375, w_{0.375})$, $Q_v(z) = 5/3$ (though derivative is 0 at $Q = 7/8$ and $Q = 1$, they are a local maximum and a local minimum respectively). Since $Q^* = 1$, $Q_v(z) \neq Q^*$.

Proposition 5 gives the sufficient conditions, that depend on Δ and the demand distribution, under which the QF contract does coordinate under VMI (Scenario 3). Before stating the proposition, we first define a function $\mathcal{F}(x)$ to be *weakly unimodal* if $\exists \mathcal{M}$ s.t. $\mathcal{F}(x)$ is strictly increasing for all $x \leq \mathcal{M}$ and $\mathcal{F}(x)$ is (weakly) decreasing for all $x \geq \mathcal{M}$.

Proposition 5. *If $g(Q, \Delta)$ is weakly unimodal and $g(Q^*, \Delta) < \Delta$ for a given contract z , then it coordinates under VMI. Further, if the former condition holds but the latter does not, then $Q_v(z) = \infty$ and the contract does not coordinate under VMI.*

The condition $g(Q^*, \Delta) < \Delta$ is actually equivalent to a lower bound on Δ i.e. $\Delta > \Delta_{min}$ where Δ_{min} satisfies the equation $g(Q^*, \Delta_{min}) = \Delta_{min}$ (this is due to the fact that $g(Q^*, \Delta)$ is a monotonically decreasing function in Δ). In fact, the example given by Figure 1 violated this condition (though weak unimodality was satisfied) and hence $Q_v(z) = \infty$. Proposition 6 shows that the condition on weak unimodality in Proposition 5 is actually satisfied by popular demand distributions.

Proposition 6. *When the demand is uniform, normal (truncated at zero), exponential or gamma distributed then $g(Q, \Delta)$ is weakly unimodal for $0 < \Delta < 1$.*

Propositions 5 and 6 provide us some insights about the performance of QF contracts under VMI. For the set of popular demand distributions, the coordination does not depend on the type of distribution. However, it depends on Δ , the degree of flexibility. There is a lower bound on Δ which has to be satisfied for the contract to coordinate. In fact, $Z^{QF} \cap Z_V = \{(z \in Z^{QF} : \Delta_{min} < \Delta < 1)\}$. Hence the contract cannot achieve arbitrary allocation of profits between vendor and retailer. So it is not structured to coordinate under VMI (this can also be inferred from Proposition 2 since $Z^{QF} \cap Z_V \neq Z^{QF} \cap Z_R$). Further, a high value of Δ discourages the vendor from pushing more goods towards the

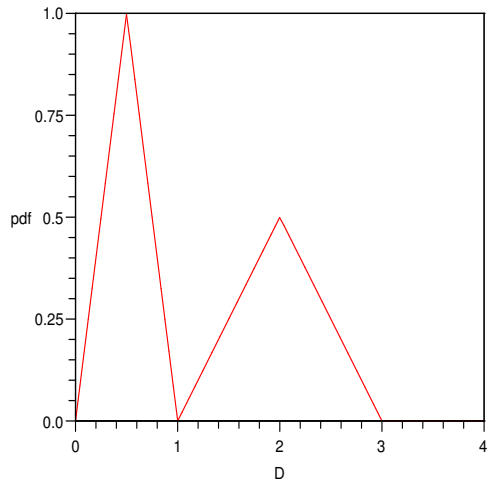


Figure 2: pdf of demand in Scenario 2 example

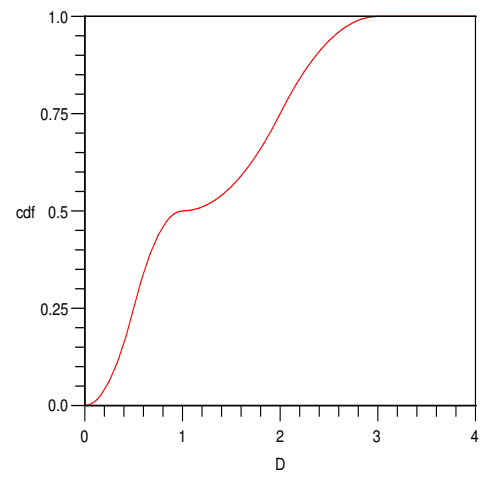


Figure 3: cdf of demand in Scenario 2 example

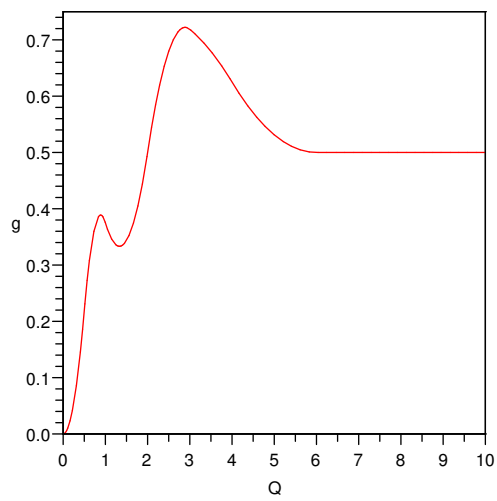


Figure 4: $g(Q, 0.375)$ for Scenario 2 example

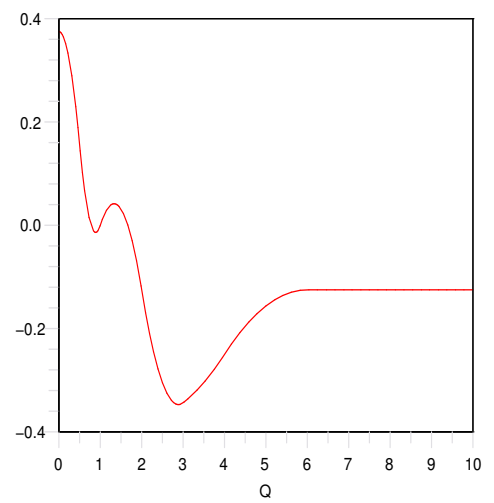


Figure 5: Normalized vendor's derivative i.e. $\hat{\pi}_v / (w_\Delta - s)$ for Scenario 2 example

retailer because he has a responsibility of taking back most of the unsold goods. However, as mentioned before, a high value of Δ also implies a higher share for the vendor's profits. Hence, if the retailer's profit had been sufficiently high under RMI (low Δ), the retailer has to give up some of those profits to the vendor (by increasing Δ) in order for coordination to be achieved. Otherwise, supply chain inefficiency will result since the vendor will try to push too much inventory to the retailer. Of course, the vendor could compensate the retailer with a lump sum payment for this loss without affecting $Q_v(z)$.

4.4 Quantity Discount Contract

In this contract, the wholesale price is a function $w(Q)$ of quantity. The payoffs for the retailer and vendor are given by $\tilde{\pi}_r(Q, D; z) = p \min(D, Q) - w(Q)Q + s(Q - D)^+$ and $\tilde{\pi}_v(Q, D; z) = (w(Q) - c)Q$ respectively.

It can be seen that for any contract z by setting

$$w(Q) = c + \frac{\lambda \pi_{tot}(Q)}{Q},$$

$$\pi_v(Q; z) = \lambda \pi_{tot}(Q), \pi_r(Q; z) = (1 - \lambda) \pi_{tot}(Q) \quad \forall Q.$$

Hence this type of contract coordinates under both VMI and RMI for any (vendor's) *profit share* λ satisfying $0 < \lambda < 1$. Further, the term "quantity discount" is a misnomer here because $\pi_{tot}(Q)/Q$ might not be monotonically decreasing in Q . We now also look at the two-part tariff contract which is a special case of the quantity discount contract. In this contract, $w(Q) = A + B/Q$. The derivative of retailer's profit function with respect to Q is given by

$$\dot{\pi}_r(Q; z) = p - A - (p - s)\Phi(Q).$$

The retailer's expected profit is concave in Q and the contract coordinates under RMI when $A = c$. Arbitrary allocation of total profits between vendor and retailer can be achieved by varying B from 0 to $\pi_{tot}(Q^*)$.

The vendor's expected profit is given by $\pi_v(Q; z) = (w(Q) - c)Q = B + (A - c)Q$. We can have the following three scenarios: 1. $A < c$, in this case $Q_v(z) = 0$; 2. $A > c$, in this case $Q_v(z) = \infty$; and 3. $A = c$, in this case $\pi_v(Q; z) = B$ and is independent of Q so there is no coordination. Hence the two-part tariff never coordinates under VMI.

4.5 Incremental Sales Rebate (ISR) Contract

This type of contract is particularly useful when we not only need to coordinate the supply chain but the vendor also has to consider the change in demand due to retailer's sales efforts. In that case, it can be shown that an incremental sales rebate contract along with a returns contract can achieve coordination (Taylor 2002). However we do not look at the sales force effects here rather we look at how the incremental sales rebate contract performs under VMI and RMI. In this contract, the wholesale price is w and the retailer gets a rebate of x for every unit of sales made above a threshold t where $(x, t, w) \in S^{ISR} = \{(x, t, w) : x > 0, t > 0 \text{ and } c < w < p\}$ with the corresponding set of contracts Z^{ISR} . The retailer's and the vendor's payoff functions are given by $\tilde{\pi}_r(Q, D; z) = p(\min(D, Q)) - wQ + s(Q - D)^+ + x(\min(D, Q) - t)^+$ and $\tilde{\pi}_v(Q, D; z) = (w - c)Q - x(\min(D, Q) - t)^+$ respectively. Note that when $Q \leq t$, the last term in both the retailer's and vendor's profit function equals zero.

The derivative of vendor's expected profit is given by

$$\dot{\pi}_v(Q; z) = \begin{cases} w - c & \text{if } Q < t \\ w - c - x + x\Phi(Q) & \text{if } Q > t \end{cases}$$

Hence, it can be seen that $Q_v(z) = \infty$ because the vendor can achieve infinite profits by shipping arbitrarily large quantities since

$$\lim_{Q \rightarrow \infty} \dot{\pi}_v^{ISR}(Q; z) = w - c > 0.$$

Note that for all values of $z \in Z^{ISR}$, $Q_v(z) = \infty$ and there is no coordination. Interestingly, it is true even for large values of x ($x \gg w$). The reasoning is as follows — the vendor only needs to pay an incentive of x to the retailer on the units sold above the threshold t ; since the expected demand is finite, this liability is finite while the vendor's profit margin per unit remains at $w - c > 0$. Hence, $Z^{ISR} \cap Z_V = \emptyset$.

4.6 Revenue Sharing (RS) Contract

Cachon and Lariviere (2005) analyze the performance of revenue sharing contracts under different scenarios; however, note that they do not consider VMI. In this contract, the wholesale price is w and the retailer gets a share, β , of the total supply chain revenue where $(w, \beta) \in S^{RS} = \{(w, \beta) : c < w < p \text{ and } 0 < \beta < 1\}$ with the corresponding set of contracts Z^{RS} . Hence, the retailer's and vendor's payoff functions are given by $\tilde{\pi}_r(Q, D; z) = \beta p \min(D, Q) - wQ + s(Q - D)^+$ and $\tilde{\pi}_v(Q, D; z) = (w - c)Q + (1 - \beta)p \min(D, Q)$. Next, we analyze when the RS contract coordinates under VMI.

Proposition 7. *The revenue sharing contract coordinates under VMI if and only if $w = \frac{(p-c)s + \beta p(c-s)}{p-s}$. Furthermore, the contract is structured to coordinate, and the same set of contracts coordinate under both VMI and RMI.*

Based on the analyses in §4.1-§4.6, we find that buyback, quantity discount, and revenue sharing contracts can coordinate under both VMI and RMI. Next, we compare these three contracts, and examine how they are related to each other.

Proposition 8. *The buyback and RS contracts are “equivalent”, i.e., there exists a one-to-one mapping between their parameters, which results in the same payoffs for the retailer and vendor. In the QD contract, the vendor’s payoff is independent of demand D .*

5. Analysis of New Contracts

All the contracts in Section 4 coordinate the supply chain in the case of RMI. But the QF contract and the sales rebate contract do not coordinate under VMI in general. Further, the buyback and RS contracts which coordinate always under VMI are actually equivalent (see Proposition 8), while the retailer bears all the risk associated with uncertain demand under QD contract. Hence, we propose two new types of contracts which always coordinate the supply chain in VMI. Additional coordinating contract types can be useful for a number of reasons: contracts with a single price parameter may be preferred over those with two price parameters such as buyback contracts (Tsay 1999); vendor and/or retailer might consider risk or variation in the profits in addition to the expected profits; implementation issues may favor one type of coordinating contract over another. Since these two contracts are proposed for coordination under VMI, we will first look at VMI followed by RMI. Before we examine them, we first provide a proposition which is similar to Proposition 2 but considers contracts from the VMI perspective.

Proposition 9. *Suppose $Z' \subset Z$ is such that $Z' \cap Z_V = Z' \cap Z_{PV}$. Then*

(i) $Z' \cap Z_R \subset Z' \cap Z_V$.

(ii) *Further, suppose Z' is structured to coordinate under VMI. It is structured to coordinate under RMI too if and only if $Z' \cap Z_V = Z' \cap Z_R$.*

5.1 Modified Quantity Flexibility Contract

In this contract, which is similar to the original QF contract, the vendor charges w as the wholesale price and agrees to take back every unsold unit above ΓQ units ($0 < \Gamma < 1$) for w (instead of upto ΓQ units under the original quantity flexibility contract) where $(\Gamma, w) \in S^{MQF} = \{(\Gamma, w) : 0 < \Gamma < 1 \text{ and } c < w < p\}$ with the corresponding set of contracts Z^{MQF} . The payoff functions in this contract for the vendor and retailer are given by $\tilde{\pi}_v(Q, D; z) = (w - c)Q - (w - s)((Q - D)^+ - \Gamma Q)^+$ and $\tilde{\pi}_r(Q, D; z) = (p - w)Q - (p - s)(Q - D)^+ + (w - s)((Q - D)^+ - \Gamma Q)^+$ respectively.

VMI

The derivative of the vendor's expected profit function can be written as $\dot{\pi}_v(Q; z) = w - c - (w - s)(1 - \Gamma)\Phi((1 - \Gamma)Q)$. Since the derivative $\dot{\pi}_v(Q; z)$ is decreasing in Q , vendor's profit is concave in Q . For a given Γ , equating the derivative at Q^* to zero, the supply chain is coordinated when the wholesale price is given by

$$w_\Gamma = \left(\frac{c - s(1 - \Gamma)\Phi((1 - \Gamma)Q^*)}{1 - (1 - \Gamma)\Phi((1 - \Gamma)Q^*)} \right). \quad (2)$$

Hence the set of parameters which enable modified QF contract to coordinate under VMI is given by $Z^{MQF} \cap Z_V = \{(\Gamma, w_\Gamma) : 0 < \Gamma < 1\}$.

Proposition 10. *The wholesale price w_Γ is strictly decreasing in Γ . Further, $c < w_\Gamma < p \forall 0 < \Gamma < 1$.*

Intuitively, as Γ increases the vendor accepts less leftover inventory and so the wholesale price necessary for coordination decreases. When the vendor agrees to take back all the leftover inventory ($\Gamma = 0$), the wholesale price should be equal to p , the retail price and when she does not take back any leftover inventory ($\Gamma = 1$), the wholesale price should be equal to c , the production cost. When $\Gamma = 0$, vendor's profit is equal to the total system profit and when $\Gamma = 1$ vendor's profit is equal to zero; vendor's profit continuous in Γ and hence the contract achieves arbitrary allocation of total profits.

RMI

Note that the vendor's profit is concave in Q and has a unique optimum, in the interior, $\forall z \in Z^{MQF}$. Hence it can be inferred from Proposition 9 (part (i)) that we only need to consider $z \in Z^{MQF} \cap Z_V$ for coordination under RMI. The derivative of the profit function

of the retailer in this case is given by

$$\begin{aligned}\dot{\pi}_r(Q; z) &= p - w_\Gamma - (p - s)\Phi(Q) + (w_\Gamma - s)(1 - \Gamma)\Phi((1 - \Gamma)Q) \\ &= (p - w_\Gamma)(1 - \Phi(Q)) - (w_\Gamma - s)(\Phi(Q) - (1 - \Gamma)\Phi((1 - \Gamma)Q)).\end{aligned}$$

In general, the derivative is not decreasing in Q and hence the profit function is not concave in Q . But Q^* might still be the optimum and hence we might be able to achieve coordination. We will propose a general sufficient condition under which this is true. Proposition 11 will help us in proving the sufficiency of the condition given by Proposition 12.

Proposition 11. *Under the modified QF contract, the retailer's optimal order quantity, $Q_r(z)$, is always finite and positive.*

Proposition 11 implies that the contract does not incentivize the retailer to order arbitrarily large quantities. The reasoning is as follows — as the retailer orders more inventory (Q increases), the leftover inventory that the retailer is responsible for, ΓQ , also increases proportionately and it deters the retailer from ordering too much.

Proposition 12. *If Φ is an IGFR (increasing generalized failure rate) distribution then $\forall 0 < \Gamma < 1$, the modified QF contract $z \in Z^{MQF} \cap Z_V$ coordinates under RMI as well.*

It is to be noted that Φ being IGFR is a sufficient but not a necessary condition (see Appendix for details). However, the reason we consider IGFR is because this criterion is already popular in the supply chain contracting literature (Lariviere and Porteus 2001). Most of the well-known probability distributions have an IGFR. Further, since the contract remains the same and coordinates for all Γ , arbitrary allocation of total profits can be achieved. Hence in this case, $Z^{MQF} \cap Z_R = Z^{MQF} \cap Z_V$.

If we compare the original QF contract with the modified QF contract, the modified one is preferable for coordination because in the former, a constraint is needed on Δ for the contract to coordinate while there is no such constraint on Γ in the latter. In the former contract, there is an incentive for the vendor to ship arbitrarily large quantities but there is no incentive here for the retailer to order arbitrarily large quantities. And finally, the IGFR assumption is well known in the literature while $g(Q, \Delta)$ being unimodal (which is a sufficient condition for the original QF contract to coordinate in VMI) is less often employed.

5.2 Modified Buyback Contract

The vendor charges wholesale price w and offers rebate x in addition to the salvage value s to the retailer for every unit of leftover inventory beyond a threshold t where $(x, t, w) \in S^{MB} = \{(x, t, w) : x > 0, 0 < t < Q^* \text{ and } c < w < p\}$ with the corresponding set of contracts Z^{MB} . The payoff functions are given by $\pi_v(Q, D; z) = (w - c)Q - x(Q - t - D)^+$ and $\pi_r(Q, D; z) = (p - w)Q + x(Q - t - D)^+ - (p - s)(Q - D)^+$ respectively.

VMI

The derivative of vendor's profit function is given by $\dot{\pi}_v(Q; z) = w - c - x\Phi(Q - t)$. It is equal to a constant for all $Q < t$ and decreasing when $Q > t$. Hence $\pi_v(Q; z)$ is concave in Q if the vendor decides to cross the threshold. She will always cross the threshold because

$$\dot{\pi}_v(t; z) = w - c > 0.$$

For any threshold $t < Q^*$, we can make the derivative of $\pi_v(Q; z)$ with respect to Q at Q^* equal to zero by choosing

$$x = \frac{w - c}{\Phi(Q^* - t)}. \quad (3)$$

Hence $Z^{MB} \cap Z_V = \{z \in Z^{MB} : c < w < p, 0 < t < Q^* \text{ and } x = (w - c)/\Phi(Q^* - t)\}$. It can also be seen that when threshold $t = 0$, the contract is equivalent to a buyback contract so the modified contract (more general than buyback contract) does achieve arbitrary allocation of total profits but it is not structured to coordinate under VMI (e.g., setting $w = c$ makes vendor's profit zero, regardless of the value of t). However, what happens to profit allocation when $t > 0$? The proposition below provides the answer.

Proposition 13. *For any $t > 0$, if Φ is log-concave then any arbitrary profit allocation can be achieved by changing w from c to a threshold less than p .*

Proposition 13 shows that, even when $t > 0$, arbitrary profit allocation is achieved for well-known demand distributions such as uniform, normal, and exponential distributions (Bagnoli and Bergstrom 2005).

RMI

Again, $\pi_v(Q; z)$ is concave in Q and $Q_v(z)$ is unique and interior $\forall z \in Z^{MB}$. Hence, as a consequence of Proposition 9 (part (i)) we will only consider the set $Z^{MB} \cap Z_V$ for coordination under RMI.

When $t = 0$, the contract clearly coordinates under RMI too. However, it does not always coordinate under RMI as shown by this counter example. Consider a contract z parametrized by $t = Q^* - \Phi^{-1}((w - c)/(p - s))$. In this case, $t > 0$ and from (3), $x = p - s$. Then $\pi_r(Q; z) = (p - w)Q + (p - s)E(Q - t - D)^+ - (p - s)E(Q - D)^+$, and hence $Q_r(z) = \infty$ because $\lim_{Q \rightarrow \infty} \dot{\pi}_r(Q; z) = \lim_{Q \rightarrow \infty} (p - w + (p - s)(\Phi(Q - t) - \Phi(Q))) = p - w > 0$. Proposition 14 states a sufficient condition for the threshold t under which the contract will not coordinate under RMI. Intuitively, it says that for high values of the threshold t (which entail high values of x), the retailer has the incentive to order arbitrarily large quantities. Note also that the condition is sufficient irrespective of the type of demand distribution.

Proposition 14. *If $Q^* - \Phi^{-1}((w - c)/(w - s)) < t < Q^*$, then $Q_r(z) = \infty$ and hence the modified buyback contract does not coordinate under RMI.*

The insight behind Proposition 14 is as follows — when the threshold is high, a high value of x is needed to deter the vendor from pushing arbitrarily large quantities; however under RMI this value of x is too high: the retailer does not lose anything on the leftover inventory and prefers to order arbitrarily large quantities. Hence, $Z^{MB} \cap Z_R \neq Z^{MB} \cap Z_V$ in general.

6. Multiple Retailers

We consider the performance of different supply chain contracts under VMI in the case of n independently operating retailers and a single vendor. Retailer-dependent parameters are indicated with a subscript i , so Φ_i is the cdf of demand for retailer i , $\tilde{\pi}_i$ is the profit for retailer i , etc. Since all retailers sell the same product, we fix the retail price per unit to p and the salvage value per unit to s for all retailers. The total production cost is $m(\sum_i Q_i)$, so the overall supply chain profit can now be written as

$$\pi_{tot}(\mathcal{Q}) = p \sum_i Q_i - (p - s) \sum_i E_{D_i}(Q_i - D_i)^+ - m(\sum_i Q_i), \quad (4)$$

where $\mathcal{Q} = (Q_1, \dots, Q_n)$. When the function m is concave, π_{tot} may not be concave (this is true even when $n = 1$), and could have multiple local maxima. We will not consider this case further here. Instead, we consider two possibilities.

6.1 Linear Production Cost

Assume $m(x) = cx$. In this case, $\mathcal{Q}^* = (Q_1^*, \dots, Q_n^*) = \operatorname{argmax}\{\pi_{tot}(Q_1, \dots, Q_n) : Q_i \geq 0; i = 1, \dots, n\}$, where Q_i^* maximizes $\pi_{tot,i}(Q) = (p - c)Q - (p - s)E_{D_i}(Q - D_i)^+$ for $i = 1, \dots, n$.

Hence, we can use any contract from §4 or 5, and all our results will continue to hold for both RMI and VMI.

When multiple retailers are involved, there is another consideration, in addition to coordination: are the contractual terms among the retailers identical? If the vendor's profit shares from all retailers are the same, the buyback and revenue sharing contracts (which are equivalent from Proposition 8) yield identical contracts for all retailers, since the parameters for those contracts are *independent of the demand distribution*. In the quantity discount contract, the combined expected profit of the vendor and retailer ($\pi_{tot,i}$) can be different from one retailer to the next, so if the profit shares are the same for all retailers, the effective price for retailer i would be larger than that for retailer j if retailer i is *larger* than retailer j , i.e., $D_i >_{st} D_j$.⁴ This is an unlikely outcome in practice because a larger retailer with more bargaining power is likely to be able to negotiate a lower price, and therefore, different profit shares (λ 's) are required. Hence, in this situation, different contract parameters need to be used for different retailers. The practical and legal issues (Federal Trade Commission 2017) associated with setting different wholesale price functions $w_i(Q_i)$ for different retailers therefore is likely to be a barrier to implementing the quantity discount contract in the multiple retailer setting. However, having identical wholesale price requires the term $\lambda_i \pi_{tot,i}(Q_i^*)/Q_i^*$ to be the same for any retailer i , and thereby highly limits the set of parameters that can be used.

For the modified QF and the modified buyback contract with threshold $t > 0$, the contract parameters depend on the demand distribution in a non-trivial way, and will need to be individually adjusted for each retailer.

6.2 Convex Production Cost

We assume $m : \mathbb{R} \rightarrow \mathbb{R}$ is a twice differentiable, increasing, strictly convex function with $s \leq m'(\sum_i l_i) < p$ and $m(0) = 0$. Convex production costs can result from a variety of causes: when a raw material has limited supply, increased use can drive up its market price; capacity may be limited and hence the use of overtime or less efficient resources causes marginal costs to increase; higher running rates of equipment tends to increase wear and tear, etc. Since $m(x)$ is convex, $m(\sum_i Q_i)$ is jointly convex, and hence $\pi_{tot}(\mathcal{Q})$ is jointly concave in Q_1, \dots, Q_n . The following proposition characterizes the system optimal solution for the supply chain.

⁴First-order stochastic dominance of demand so that $\Phi_i(x) \leq \Phi_j(x) \forall x$.

Proposition 15. For every $\sum_i l_i < \nu < \sum_i u_i$, let w_ν be the unique solution to the equation $\sum_i \Phi_i^{-1}(w) = \nu$, define $Q_i(\nu) = \Phi_i^{-1}(w_\nu)$, $i = 1, \dots, n$, and let $f(\nu) = \pi_{\text{tot}}(\mathcal{Q}(\nu))$. Then f is a strictly concave function with a unique maximizer $\nu^* \in (\sum_i l_i, \sum_i u_i)$, and $\mathcal{Q}^* = \mathcal{Q}(\nu^*)$ is the unique maximizer of $\pi_{\text{tot}}(\mathcal{Q})$. Furthermore, $m'(\sum_i Q_i^*) = m'(\nu^*) = p - (p - s)\Phi(Q_i^*)$ for all $i = 1, \dots, n$.

Hence we will say that a contract z coordinates under VMI if (Q_1^*, \dots, Q_n^*) is the unique maximizer of $\pi_v(Q_1, \dots, Q_n; z)$ and the participation constraints $\pi_i(Q_i^*; z) \geq 0$ for $i = 1, \dots, n$ and $\pi_v(Q_1^*, \dots, Q_n^*; z) \geq 0$ are satisfied.

Consider any of the contracts in §4 or 5. Let $b_i(Q_i)$ be the expected net benefit that the vendor receives from its contract with retailer i , excluding manufacturing cost [e.g., for the buyback contract, we have $b_i(Q_i) = wQ_i - bE_{D_i}(Q_i - D_i)^+$]. The vendor's profit is then $\pi_v(Q_1, \dots, Q_n) = \sum_i b_i(Q_i) - m(\sum_i Q_i)$. The following result establishes a key property pertaining to the vendor's profit.

Proposition 16. Let $c = m'(\nu^*)$, and suppose Q_i^* is the unique maximum of $b_i(Q) - cQ$, $i = 1, \dots, n$. Then $\mathcal{Q}^* = (Q_1^*, \dots, Q_n^*)$ is the unique maximum of $\pi_v(\mathcal{Q})$.

Proposition 16 implies that if a contract coordinates under VMI in the single retailer case, it also does so in the multi-retailer case with convex production costs. A related question involves how the performances of different contracts under single retailer and multi-retailer cases compare *under RMI*. Next, we address this question. The expected profit of retailer i ($i = 1, 2, \dots, n$) under any contract becomes $\pi_i(Q_i) = p \cdot E(\min(D, Q)) - b_i(Q_i)$. Therefore, the expected profit function does not depend on the production cost function $m(\cdot)$. Hence, if the contract coordinates under RMI in the single retailer case (with production cost $c = m'(\nu^*)$), it will also coordinate under RMI in the multi-retailer case as retailer i is maximizing the same expected profit.

7. Conclusions and Future Research

To summarize, we find that of five contracts analyzed in the literature, only the buyback, quantity discount, and revenue sharing contracts can coordinate the supply chain both in VMI and RMI to achieve any arbitrary division of the total system profit. Also, the buyback and revenue sharing contracts are equivalent; further, since the parameters in these contracts are independent of the demand distribution, both vendor and retailer benefit from sharing

truthful information. The QF contracts coordinate under RMI for all values of Δ . But in VMI, their performance in general depends on the type of demand distribution though we show that for most of the typical distributions the QF contract coordinates in VMI for high Δ 's. We also develop a lower bound for Δ which, if satisfied, ensures coordination by the QF contracts for these typical distributions. Hence the contract does not achieve arbitrary allocation of total profits under VMI.

Contract	VMI	RMI	Remarks
Wholesale Price ⁵	Never coordinates. $\tilde{\pi}_v = (w - c) \min(D, Q)$ $+ (s - c)(Q - D)^+$ $\tilde{\pi}_r = (p - w) \min(D, Q)$	Never coordinates. $\tilde{\pi}_v = wQ$ $\tilde{\pi}_r = (p - w) \min(D, Q)$ $+ (s - w)(Q - D)^+$	$Q_v < Q_r$ if $w < s$ $+ \sqrt{(c - s)(p - s)}$
Buyback ⁶	Always coordinates. $\tilde{\pi}_v = (w - c)Q$ $- b(Q - D)^+$	Always coordinates. $\tilde{\pi}_r = p \min(D, Q) - wQ$ $+ (b + s)(Q - D)^+$	Arbitrary profit allocation under both RMI and VMI.
Quantity Flexibility	Sometimes coordinates. $\tilde{\pi}_v = (w - c)Q - (w - s) \cdot \min(\Delta Q, (Q - D)^+)$	Always coordinates. $\tilde{\pi}_r = p(\min(D, Q)) - wQ$ $+ s((Q - D)^+ - \Delta Q)^+$ $+ w \min(\Delta Q, (Q - D)^+)$	Arbitrary profit allocation is not possible under VMI.
Quantity Discount	General wholesale price $w(Q)$ always coordinates. Two-part tariff never does. $\tilde{\pi}_v = (w(Q) - c)Q$	Again, $w(Q)$ always coordinates, but two-part does so too. $\tilde{\pi}_r = p \min(D, Q)$ $- w(Q)Q + s(Q - D)^+$	Arbitrary profit allocation with $w(Q)$ under both VMI and RMI.
Incremental Sales Rebate	Never coordinates. $\tilde{\pi}_v = (w - c)Q$ $- x \min(D, Q) - t)^+$	Always coordinates. $\tilde{\pi}_r = p \min(D, Q)$ $- wQ + s(Q - D)^+$ $+ x(\min(D, Q) - t)^+$	$Q_v = \infty$
Revenue Sharing	Always coordinates. $\tilde{\pi}_v = (w - c)Q$ $+ (1 - \beta)p \min(D, Q)$	Always coordinates. $\tilde{\pi}_r = \beta p \min(D, Q)$ $- wQ + s(Q - D)^+$	Arbitrary profit allocation under both RMI and VMI.
Modified Quantity Flexibility	Always coordinates. $\tilde{\pi}_v = (w - c)Q - (w - s) \cdot ((Q - D)^+ - \Gamma Q)^+$	Sometimes coordinates. $\tilde{\pi}_r = (p - w)Q - (p - s) \cdot (Q - D)^+ + (w - s) \cdot ((Q - D)^+ - \Gamma Q)^+$	If Φ has an IGFR ⁷ , arbitrary profit allocation and coordination in RMI.
Modified Buyback	Always coordinates. $\tilde{\pi}_v = (w - c)Q$ $- x(Q - t - D)^+$	Sometimes coordinates. $\tilde{\pi}_r = (p - w)Q$ $+ x(Q - t - D)^+$ $- (p - s)(Q - D)^+$	$t = 0 \Rightarrow$ buyback. If $t > 0$, arbitrary profit allocation in VMI with log-concave Φ .

Table 3: Performance of different types of contracts under VMI and RMI (with their payoff functions)

Although quantity discount contracts in general can coordinate under both VMI and RMI, in the special case of the two part tariff contract, the vendor's profit becomes independent of the quantity ordered/supplied, and hence that contract does not coordinate in VMI. In an incremental sales rebate contract we find that the problem of the vendor pushing goods to the retailer is not adequately alleviated and hence the contract does not coordinate in VMI.

The two contracts which we introduce as modifications for the QF and sales rebate contract both coordinate the supply chain under VMI. However, under RMI, whether the modified buyback contract coordinates depends on the value of the threshold t . The other contract, modified QF contract, coordinates under RMI (independent of the value of Γ) if the demand distribution satisfies the IGFR property.

Based on our analyses in §'s 4-6, we can classify the contract types we looked at into 4 categories — those that never coordinate under RMI or VMI (wholesale price), those that always coordinate under one of them but never coordinate under other (ISR), those that always coordinate under one of them and do coordinate sometimes under the other (QF, modified buyback) and those that always coordinate under both of them (buyback, revenue sharing, QD and modified quantity flexibility). Table 3, which is similar to Table 1 but also includes the payoff functions, summarizes the performance of different contracts.

We first consider contracts with a single vendor single retailer newsvendor model. We then extend the analysis to examine multiple independent retailers with the vendor incurring linear or convex production cost, and show that our results are qualitatively unchanged. A potential area for future research is to model competition between retailers and/or vendors to see if this influences the change of inventory control from RMI to VMI. Also, we have assumed a single shot approach to contracting but one could also look at contracts that evolve over time, i.e., relational contracts (Plambeck and Taylor 2006), and analyze their performance under VMI and RMI.

⁵With consignment under VMI. Without consignment, there is still no coordination under VMI since $Q_v = \infty$. Also, we say that a contract *always coordinates* if and only if there is *arbitrary profit allocation*.

⁶Since vendor's and retailer's profit expressions are the same under VMI and RMI, we provide the vendor's (retailer's) profit only under VMI (RMI) for the sake of conciseness.

⁷Demand distribution has an increasing generalized failure rate.

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