

Cooperative Spectrum Sensing in a Medium-traffic Primary Network using Double-threshold Scheme over Imperfect Reporting Channels

Ramtin Rabiee, and Kwok Hung Li

School of Electrical and Electronic Engineering (EEE)
Nanyang Technological University (NTU), Singapore
Email: ramtin001@e.ntu.edu.sg; ekhli@ntu.edu.sg

Abstract—The status of a primary user (PU) within sensing time is one of the important parameters, which is considered to be fixed in many studies of spectrum sensing in cognitive radio networks. In this paper, we consider a medium traffic of a primary network, in which the PU changes its state (either idle or active) at most once during the sensing period of the secondary user (SU). This situation degrades the detection performance of the secondary network. We first try to find a suitable distribution for observed energy levels by SUs, and then evaluate the detection performance of the both single- and double-threshold methods in terms of the total error rate using cooperative spectrum sensing over imperfect reporting channels. The results show that the double-threshold scheme outperforms the single one.

Index Terms—Cognitive radio, Cooperative spectrum sensing, Medium traffic of primary network, Energy detector, Double-threshold detection, Imperfect reporting channels, Total error rate.

I. INTRODUCTION

Spectrum sensing is an important part of a cognitive radio network (CRN) [1] design, which has been investigated extremely in the literature to achieve a reliable and applicable detection method. Usually, the information of the primary signal and the channel condition are not accessible in a CRN, and each secondary user (SU) needs to do a blind estimation to find the status of the primary user (PU). Thus the application of the energy detection (ED) in the CRN has been considered widely [2]-[5]. After measurement, SUs must compare their observed energy of the received signal with at least one threshold to make their own local decisions on the activity of PU. The double-threshold scheme has been proposed in recent studies to have a more reliable detection [6]-[8] by decreasing the effect of uncertainty on the local decision of SUs.

Cooperative spectrum sensing (CSS) is another practical suggestion to enhance the detection performance, in which SUs send their local decisions to a fusion center (FC) through reporting channels. Then the FC fuses received information through a suitable fusion rule to make the global decision on presence or absence of the PU [9], [10].

Very often, the status of the PU has been considered invariant during the sensing time (either active or idle). This assumption can be a realistic case but we should mention that it is true whenever the traffic of the primary network is low and the mean idle/active time of the PU is high enough as

compared with the sensing time. In [11], a medium traffic of the primary network is considered in which the state of the PU changes at most once during the whole sensing period. It means that there are four situations: the licensed channel is idle all over the sensing time (H_{01}), it is first busy and then becomes idle by the end of the sensing period (H_{02}), it is busy all over the sensing time (H_{11}) or it is idle and then becomes busy by the end of the sensing period (H_{12}). They evaluated the effects of the PU's traffic rate on the performance of the spectrum sensing over additive white Gaussian noise (AWGN) sensing channels, using the CSS with perfect (error-free) reporting channels. The authors in [12] proposed an improved detector for the channel model of [11] to relieve its degraded sensing performance due to a higher traffic and analyzed it through a non-CSS scheme. In this paper, we consider the above mentioned model with a medium traffic of the primary network. We first evaluate the accuracy of both approximate gamma and Gaussian distributions for ED outputs over AWGN sensing channels. By using the double-threshold scheme, we analyze the detection performance of the CSS through imperfect reporting channels. We will show that the double-threshold method outperforms the conventional single-threshold detector in terms of the total error rate.

We define the system model and conduct the distribution evaluation in Section II, and continue by introducing single- and double-threshold methods in addition to the total error rate in Section III. Finally, we present numerical results in Section IV and conclude them in Section V.

II. THE SYSTEM MODEL

Suppose a CRN with K SUs, one FC and one PU which either arrives or departs the network at most once among the sensing duration. The observed energy level of N normalized received samples at the i -th SU is obtained by

$$Y_i = \begin{cases} \sum_{m=1}^N \left(\frac{|u_i(m)|}{\sigma_n} \right)^2, & H_{01} \\ \sum_{m=1}^N \left(\frac{|s(m)+u_i(m)|}{\sigma_n} \right)^2, & H_{11} \\ \sum_{m=1}^{J_0} \left(\frac{|s(m)+u_i(m)|}{\sigma_n} \right)^2 + \sum_{m=J_0+1}^N \left(\frac{|u_i(m)|}{\sigma_n} \right)^2, & H_{02} \\ \sum_{m=1}^{J_1} \left(\frac{|u_i(m)|}{\sigma_n} \right)^2 + \sum_{m=J_1+1}^N \left(\frac{|s(m)+u_i(m)|}{\sigma_n} \right)^2, & H_{12} \end{cases} \quad (1)$$

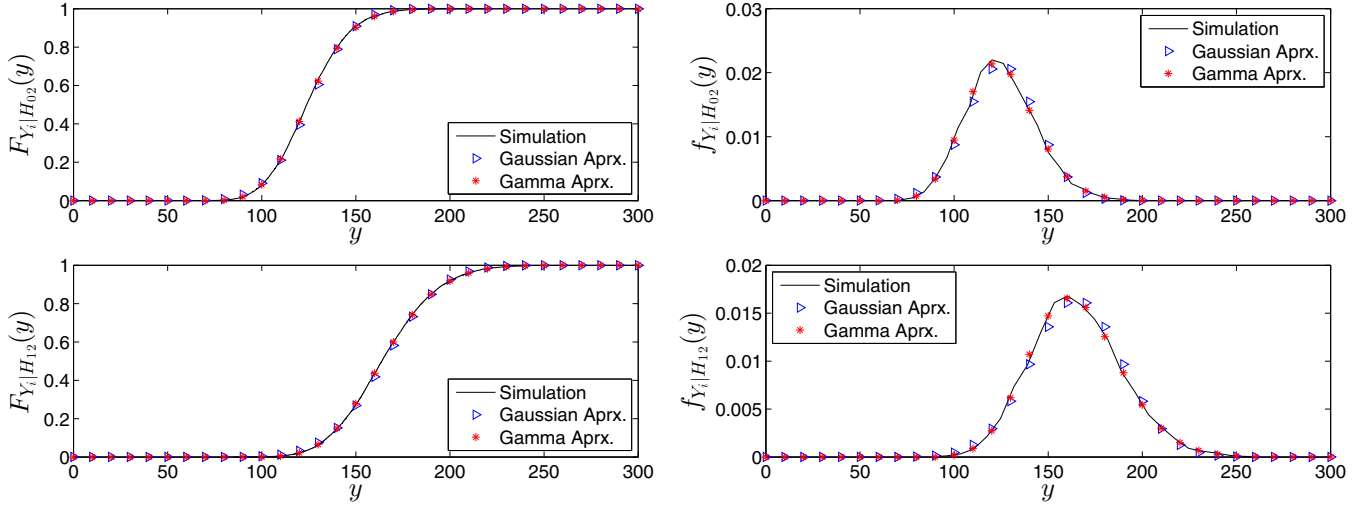


Fig. 1. CDF and PDF of ED outputs with $\gamma = 0$ dB and $N = 100$ when the PU either arrives or departs the network once during the sensing time

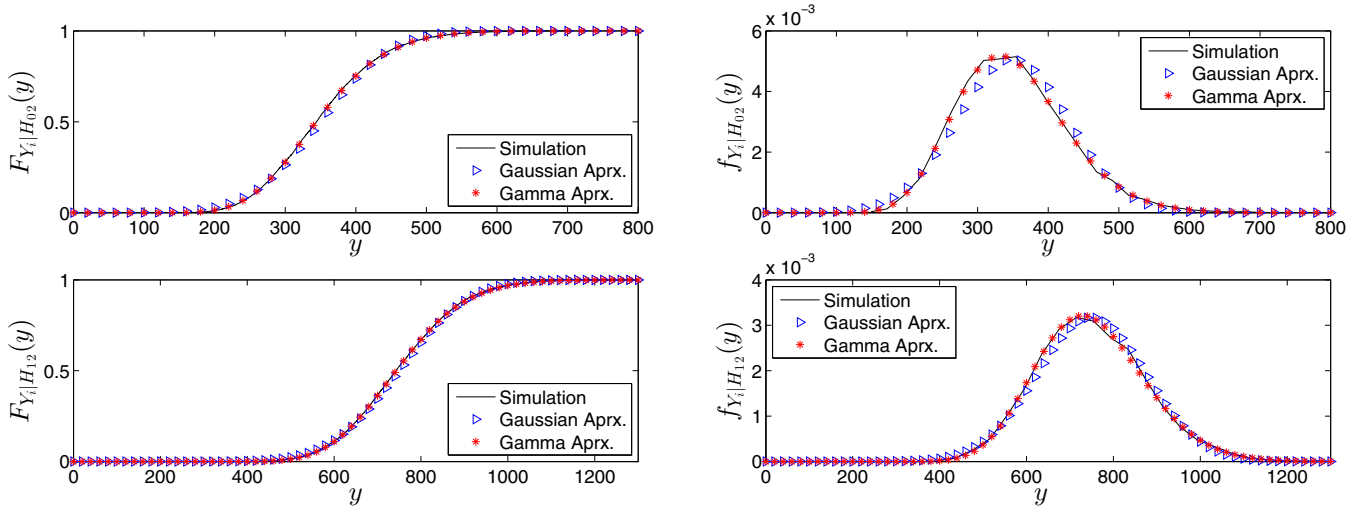


Fig. 2. CDF and PDF of ED outputs with $\gamma = 10$ dB and $N = 100$ when the PU either arrives or departs the network once during the sensing time

where $s(m)$ is the zero-mean primary signal with average power σ_s^2 and independent of the Gaussian noise $u_i(m)$ with variance σ_n^2 . J_0 and J_1 are the number of samples in which the status of the PU remains unchanged and after that it becomes either idle or active (busy), respectively. Note that both idle and busy times are exponentially distributed with means μ_i and μ_b [11], [13]. Therefore, the probabilities of being idle and active (busy) for the PU at each time instant are $P_i = \mu_i/(\mu_i + \mu_b)$ and $P_b = \mu_b/(\mu_i + \mu_b)$, respectively. P_b is also known as the duty cycle.

The observed energy level Y_i is gamma distributed where the state of the PU is fixed within the sensing period (H_{01} and H_{11}). There is no closed form distribution for other two cases (H_{02} and H_{12}) and we need to approximate them. From (1), one can see that there is a summation of two gamma distributed parts whose parameters have different values in both H_{02} and H_{12} cases. Thus one approach is to approximate them as gamma distribution with the probability density function

(PDF) of

$$f_{Y_i|H_{uv}}(y) = \frac{1}{\theta_{uv}^{k_{uv}} \Gamma(k_{uv})} y^{k_{uv}-1} \exp\left(-\frac{y}{\theta_{uv}}\right), \quad y \geq 0$$

$$\begin{aligned} & , \quad u = \{0, 1\} \\ & , \quad v = \{1, 2\} \end{aligned} \quad (2)$$

where its shape and scale parameters are

$$k_{uv} = \frac{\mathbb{E}^2 \{Y_i|H_{uv}\}}{\text{Var} \{Y_i|H_{uv}\}}$$

$$\theta_{uv} = \frac{\text{Var} \{Y_i|H_{uv}\}}{\mathbb{E} \{Y_i|H_{uv}\}}, \quad u = \{0, 1\}, \quad v = \{1, 2\} \quad (3)$$

The mean and variance of different cases are given in Table I, where $\gamma = \sigma_s^2/\sigma_n^2$ is the average signal-to-noise ratio (SNR) of sensing channels. Another approach is to approximate them as Gaussian distribution with the means and variances of

Table I. Note that N must be high enough to consider Gaussian approximation based on the central limit theorem (CLT).

Cumulative distribution function (CDF) and PDF of the estimated energy of the received signal have been shown in Figures 1 and 2, where the PU changes its status once during the sensing time (H_{02} and H_{12}). They represent both approximation approaches in comparison with the simulation results over different SNRs. One can see that the gamma approximation is more accurate. By comparing two figures, It is also revealed that the estimation error of the Gaussian approximation increases in the higher SNR. The same results are valid for other conditions of H_{01} and H_{11} . Hence, the gamma approximation works better and we consider it for our subsequent detection performance analysis.

TABLE I
MEAN AND VARIANCE OF DIFFERENT CASES

State	\mathbb{E}	$\mathbb{V}\text{ar}$
H_{01}	N	$2N$
H_{11}	$N(1 + \gamma)$	$2N(1 + \gamma)^2$
H_{02}	$J_0(1 + \gamma) + (N - J_0)$	$2J_0(1 + \gamma)^2 + 2(N - J_0)$
H_{12}	$J_1 + (N - J_1)(1 + \gamma)$	$2J_1 + 2(N - J_1)(1 + \gamma)^2$

III. DETECTION PERFORMANCE

The status changing process of the PU is a two-state process in which the holding times are independent exponential random variables with parameters $\lambda_i = 1/\mu_i$ and $\lambda_b = 1/\mu_b$. Thus for their transition probabilities we have [14, p. 782]

$$P(t) = \begin{pmatrix} P_{00}(t) & P_{01}(t) \\ P_{10}(t) & P_{11}(t) \end{pmatrix} = \frac{1}{\lambda_i + \lambda_b} \begin{pmatrix} \lambda_b + \lambda_i e^{-(\lambda_i + \lambda_b)t} & \lambda_i - \lambda_i e^{-(\lambda_i + \lambda_b)t} \\ \lambda_b - \lambda_b e^{-(\lambda_i + \lambda_b)t} & \lambda_i + \lambda_b e^{-(\lambda_i + \lambda_b)t} \end{pmatrix} \quad (4)$$

and the probabilities of different hypotheses are defined as

$$P(H_{01}) = P_i \cdot (P_{00}(T_s))^N \quad (5)$$

$$P(H_{11}) = P_b \cdot (P_{11}(T_s))^N \quad (6)$$

$$P(H_{02}, J_0) = P_b \cdot (P_{11}(T_s))^{J_0} \cdot P_{10}(T_s) \cdot (P_{00}(T_s))^{(N - J_0 - 1)} \quad (7)$$

$$P(H_{12}, J_1) = P_i \cdot (P_{00}(T_s))^{J_1} \cdot P_{01}(T_s) \cdot (P_{11}(T_s))^{(N - J_1 - 1)} \quad (8)$$

where T_s is sampling time.

A. Single-threshold scheme

Each SU needs to compare the observed energy of the received signal with a suitable threshold to make its own local decision on activity or idleness of the PU as

$$D_i = \begin{cases} H_0 & \lambda \geq Y_i \geq 0 \\ H_1 & Y_i \geq \lambda \end{cases}, \quad \lambda > 0 \quad (9)$$

The conditional probabilities of false alarm and detection under different hypotheses can be obtained by

$$\begin{aligned} P(H_1|H_{01}) &= \Pr\{Y_i > \lambda|H_{01}\} \\ P(H_1|H_{11}) &= \Pr\{Y_i > \lambda|H_{11}\} \\ P(H_1|H_{02}, J_0) &= \Pr\{Y_i > \lambda|H_{02}, J_0\} \\ P(H_1|H_{12}, J_1) &= \Pr\{Y_i > \lambda|H_{12}, J_1\} \end{aligned} \quad (10)$$

Subsequently, the local probabilities of false alarm, detection and missed detection are, respectively,

$$P_f = \frac{P(H_1|H_{01})P(H_{01}) + \sum_{J_0=1}^{N-1} P(H_1|H_{02}, J_0)P(H_{02}, J_0)}{P(H_{01}) + \sum_{J_0=1}^{N-1} P(H_{02}, J_0)} \quad (11)$$

$$P_d = \frac{P(H_1|H_{11})P(H_{11}) + \sum_{J_1=1}^{N-1} P(H_1|H_{12}, J_1)P(H_{12}, J_1)}{P(H_{11}) + \sum_{J_1=1}^{N-1} P(H_{12}, J_1)} \quad (12)$$

and

$$P_m = 1 - P_d \quad (13)$$

To apply the CSS, we use the OR fusion rule which protects the primary signal from interference strictly. Hence, for total probabilities of false alarm, detection and missed detection over imperfect reporting channels, we have

$$\begin{cases} Q_f = 1 - [(1 - P_f)(1 - P_e) + P_f P_e]^K \\ Q_d = 1 - [P_m(1 - P_e) + P_d P_e]^K \\ Q_m = 1 - Q_d \end{cases} \quad (14)$$

in which the reporting channels are considered to be binary symmetric channel (BSC) with error probability P_e .

B. Double-threshold scheme

Each SU uses two thresholds to decrease the effect of uncertainty on its own local decision as follows:

$$D_i = \begin{cases} H_0 & \lambda_1 \geq Y_i \geq 0 \\ \text{No Decision} & \lambda_2 > Y_i > \lambda_1, \quad \lambda_2 > \lambda_1 > 0 \\ H_1 & Y_i \geq \lambda_2 \end{cases} \quad (15)$$

They send no information to the FC whenever their estimated energy levels fall between two thresholds. The conditional probabilities of the double-threshold scheme will be defined as

$$\begin{aligned} P(H_1|H_{01}) &= \Pr\{Y_i > \lambda_2|H_{01}\} \\ P(H_1|H_{11}) &= \Pr\{Y_i > \lambda_2|H_{11}\} \\ P(H_1|H_{02}, J_0) &= \Pr\{Y_i > \lambda_2|H_{02}, J_0\} \\ P(H_1|H_{12}, J_1) &= \Pr\{Y_i > \lambda_2|H_{12}, J_1\} \\ P(H_0|H_{01}) &= \Pr\{Y_i < \lambda_1|H_{01}\} \\ P(H_0|H_{11}) &= \Pr\{Y_i < \lambda_1|H_{11}\} \\ P(H_0|H_{02}, J_0) &= \Pr\{Y_i < \lambda_1|H_{02}, J_0\} \\ P(H_0|H_{12}, J_1) &= \Pr\{Y_i < \lambda_1|H_{12}, J_1\} \end{aligned} \quad (16)$$

Here, we define three more probabilities as compared with the single-threshold case: local probability of detecting H_0 when the PU is idle (P_{d_0}), local probability of no decision when the PU is idle (Δ_0) and local probability of no decision when the PU is active (Δ_1). Local probabilities of false alarm and detection can be obtained from (11) and (12) and the rest of probabilities are

$$P_{d_0} = \frac{P(H_0|H_{01})P(H_{01}) + \sum_{J_0=1}^{N-1} P(H_0|H_{02}, J_0)P(H_{02}, J_0)}{P(H_{01}) + \sum_{J_0=1}^{N-1} P(H_{02}, J_0)} \quad (17)$$

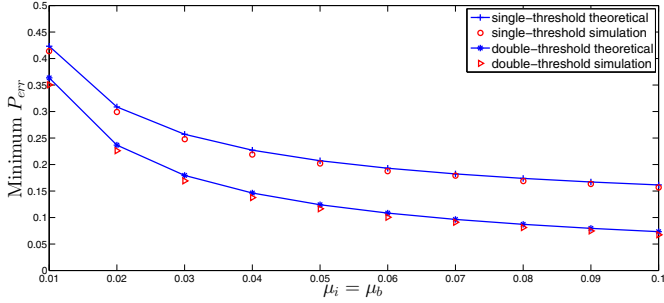


Fig. 3. Minimum total error rate vs. mean holding times with $\gamma = 0$ dB, $N = 100$, $K = 10$, $P_e = 0.01$ and $P_b = 0.5$ (i.e., $\mu_i = \mu_b$) by the OR fusion rule

$$P_m = \frac{P(H_0|H_{11})P(H_{11}) + \sum_{J_1=1}^{N-1} P(H_0|H_{12}, J_1)P(H_{12}, J_1)}{P(H_{11}) + \sum_{J_1=1}^{N-1} P(H_{12}, J_1)} \quad (18)$$

$$\Delta_0 = 1 - P_f - P_{d_0} \quad (19)$$

and

$$\Delta_1 = 1 - P_d - P_m \quad (20)$$

Similarly to the single-threshold method, we apply the CSS with the OR fusion rule and imperfect reporting channels. Therefore, the total probabilities over the double-threshold scheme are

$$\begin{cases} Q_f = \left[1 - \sum_{K_d=1}^K \binom{K}{K_d} [P_{d_0}(1 - P_e) + P_f P_e]^{K_d} \Delta_0^{K-K_d} \right] \\ \quad \times (1 - \Delta_0^K) \\ Q_d = \left[1 - \sum_{K_d=1}^K \binom{K}{K_d} [P_m(1 - P_e) + P_d P_e]^{K_d} \Delta_1^{K-K_d} \right] \\ \quad \times (1 - \Delta_1^K) \\ Q_m = 1 - Q_d \end{cases} \quad (21)$$

where K_d is the number of SUs who make distinct decisions on the status of the PU for sending to the FC. Note that Δ_0^K and Δ_1^K are probabilities of the sensing failure under hypotheses H_0 and H_1 , respectively, in which no SU sends decision to the FC.

C. Total error rate

For the detection performance analysis of both single- and double-threshold methods, we consider total error rate as follows:

$$P_{err} = Q_m + Q_f \quad (22)$$

and try to minimize it by finding optimum thresholds in each method. We will do it numerically in the next section and justify it by simulation.

IV. NUMERICAL RESULTS

In this section, the detection performance of the two comparison methods will be shown for the case of medium traffic in PU's network. We use the gamma approximation of the energy distribution observed by SUs, as described in

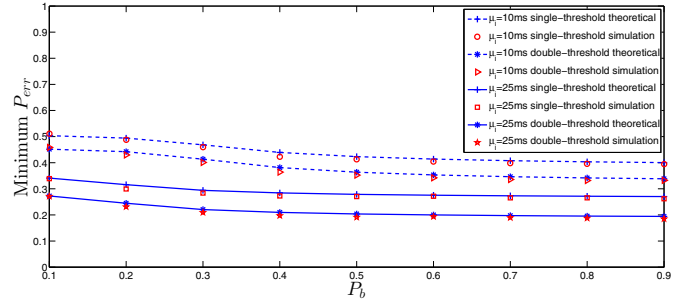


Fig. 4. Minimum total error rate vs. duty cycle with $\gamma = 0$ dB, $N = 100$, $K = 10$ and $P_e = 0.01$ by the OR fusion rule

Section II, to analyze the total error rate related to different parameters.

Suppose that the system is working with $N = 100$ and $T_s = 100 \mu\text{s}$, which implies a sensing time of $\tau = NT_s = 10$ ms. By considering $K = 10$ and $P_e = 0.01$, we apply the CSS over imperfect reporting channels. We first conduct our analysis over $\gamma = 0$ dB, and then present the effect of the SNR on the detection performance. We also justify our theoretical results by simulation.

Figure 3 represents the minimum achievable total error rate versus mean holding times when they are assumed to be equal (i.e., $\mu_i = \mu_b$). Since the status of the PU will be more stable for higher mean holding times, SUs can detect it with lower uncertainty. Thus the detection performance improves by the increase in mean holding times. It is clear that the double-threshold method has lower detection error as compared with the single one, and their difference becomes greater for a higher value of mean holding times. The minimum total error rate versus the duty cycle is shown in Figure 4 for different values of the mean idle time. For a fixed mean idle time of the PU, a lower value of P_b means that the mean active time of the PU is smaller as compared with the whole sensing time ($\mu_b \ll \tau$) and $P(H_{11})$ is subsequently low (while $P(H_{12}, J_1)$ is high), which leads to a worse detection due to the instability of the PU. On the other hand, when P_b is greater, the probability of having a stable active PU during the sensing time ($P(H_{11})$) is higher. Therefore, the detection performance improves. Again, the performance of the double-threshold method is better than that of the conventional single-threshold scheme.

To analyze the effect of the CSS, we evaluate the minimum achievable total error rate with respect to P_e in Figure 5. One can see that detection error rates are the same when $P_e = 0$. Subsequently, it is revealed that the performance gain of the double-threshold scheme dominates the single-threshold one whenever the reporting channel becomes erroneous. It is also obtained that the single-threshold method is very sensitive to the quality of the reporting channel while the variation of the double-threshold scheme is very slight and the minimum P_{err} is almost the same for different values of P_e . In other words, the detection performance of the single-threshold method degrades with reducing the quality of the

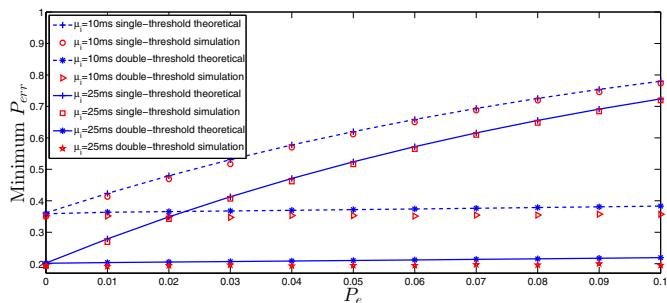


Fig. 5. Minimum total error rate vs. P_e with $\gamma = 0$ dB, $N = 100$, $K = 10$ and $P_b = 0.5$ by the OR fusion rule

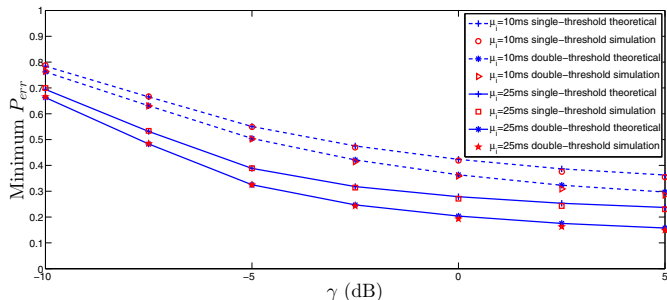


Fig. 6. Minimum total error rate vs. SNR with $N = 100$, $K = 10$, $P_e = 0.01$ and $P_b = 0.5$ by the OR fusion rule

reporting channels. The double-threshold scheme can relieve this degradation, which implies the performance gain of the double-threshold method in the case of the CSS with imperfect reporting channels. As a consequence, when the reporting channels are erroneous, the CSS can be considered as a useful method to improve the detection performance of the secondary network when utilized besides the double-threshold method for this particular primary network model.

Furthermore, the effect of the SNR on the minimum achievable total error rate is presented in Figure 6. It is reasonable to have better detection performance in a higher SNR. However, our purpose is to show the SNR gain of the double-threshold method. Its detection performance is still higher than the single-threshold method, and it becomes better by increasing the SNR.

V. CONCLUSION

In this paper, we considered a cognitive radio network in which the primary user's network has a medium traffic rate. It means that the PU arrives or departs at most once during the sensing time. This situation degrades the detection performance of secondary users due to the increase in the uncertainty of the PU's activity. Since there is no closed form distribution for observed energy, we have first tried to evaluate gamma and Gaussian approximations to find a more accurate method. We showed that the gamma approximation works better, especially with higher SNR values, which allows us to analyze the effect of the SNR on the detection performance with more reliability. We then presented the performance of

both single- and double-threshold schemes in terms of the minimum achievable total error rate. The results showed that the double-threshold method outperforms the single one by decreasing the effect of additional uncertainty in different mean holding times, duty cycles and SNRs when the reporting channels are imperfect. It was also revealed that the single-threshold method is very sensitive to the quality of reporting channels while the double-threshold method is not. Hence, over imperfect reporting channels, the double-threshold based CSS is more useful to increase the detection performance in this particular cognitive radio network model. Finally, a good agreement was obtained between theoretical and simulated results.

REFERENCES

- [1] Y.-C. Liang, K.-C. Chen, G. Li, and P. Mahonen, "Cognitive radio networking and communications: An overview," *IEEE Transactions on Vehicular Technology*, vol. 60, no. 7, pp. 3386–3407, Sep. 2011.
- [2] H. Urkowitz, "Energy detection of unknown deterministic signals," *Proceedings of the IEEE*, vol. 55, no. 4, pp. 523–531, Apr. 1967.
- [3] V. Kostylev, "Energy detection of a signal with random amplitude," in *IEEE International Conference on Communications (ICC)*, vol. 3, 2002, pp. 1606–1610.
- [4] F. Digham, M.-S. Alouini, and M. K. Simon, "On the energy detection of unknown signals over fading channels," in *IEEE International Conference on Communications (ICC)*, vol. 5, 2003, pp. 3575–3579.
- [5] D. Cabric, S. Mishra, and R. Brodersen, "Implementation issues in spectrum sensing for cognitive radios," in *Conference Record of the Thirty-Eighth Asilomar Conference on Signals, Systems and Computers*, vol. 1, Nov. 2004, pp. 772–776.
- [6] C. Sun, W. Zhang, and K. Letaief, "Cooperative spectrum sensing for cognitive radios under bandwidth constraints," in *IEEE Wireless Communications and Networking Conference (WCNC)*, 2007, pp. 1–5.
- [7] D. Duan and L. Yang, "Cooperative Spectrum Sensing with Ternary Local Decisions," *IEEE Communications Letters*, vol. 16, no. 9, pp. 1512–1515, 2012.
- [8] S.-Q. Liu, B.-J. Hu, and X.-Y. Wang, "Hierarchical cooperative spectrum sensing based on double thresholds energy detection," *IEEE Communications Letters*, vol. 16, no. 7, pp. 1096–1099, July 2012.
- [9] W. Zhang, R. Mallik, and K. Letaief, "Optimization of cooperative spectrum sensing with energy detection in cognitive radio networks," *IEEE Transactions on Wireless Communications*, vol. 8, no. 12, pp. 5761–5766, Dec. 2009.
- [10] D. Duan, L. Yang, and J. Principe, "Cooperative diversity of spectrum sensing for cognitive radio systems," *IEEE Transactions on Signal Processing*, vol. 58, no. 6, pp. 3218–3227, 2010.
- [11] T. Wang, Y. Chen, E. Hines, and B. Zhao, "Analysis of effect of primary user traffic on spectrum sensing performance," in *Fourth International Conference on Communications and Networking in China (ChinaCOM)*, Aug. 2009, pp. 1–5.
- [12] N. Beaulieu and Y. Chen, "Improved energy detectors for cognitive radios with randomly arriving or departing primary users," *IEEE Signal Processing Letters*, vol. 17, no. 10, pp. 867–870, Oct. 2010.
- [13] A. Ghasemi and E. Sousa, "Optimization of spectrum sensing for opportunistic spectrum access in cognitive radio networks," in *4th IEEE Consumer Communications and Networking Conference (CCNC)*, Jan. 2007, pp. 1022–1026.
- [14] A. Papoulis and S. U. Pillai, *Probability, Random Variables and Stochastic Processes*, 4th ed. McGraw-Hill, 2002.