Annuity and insurance choice under habit formation[*]

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ABSTRACT

This paper examines the impact of habit formation on the demand for life-contingent contracts in a life-cycle model. We derive an analytical solution for the optimal consumption, portfolio choice, and life insurance/annuity purchases. We illustrate the mechanism by which the consumption habit assumption can alter the bequest motive and therefore drive the demand for life-contingent products. Based on our assumed insurance/annuity markets, we show that habit formation alone leads to low demand on either life insurance or annuity but not both. However, habit formation together with social security results in low demand in both life insurance and annuity.

Key Words: Habit formation; Life-cycle model; Life insurance; Annuity; Martingale Method
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1 Introduction

Since the seminal contribution of Yaari (1965), the utility-based life-cycle models have become a mainstream approach among academics and practitioners in quantifying the impact of mortality uncertainty, bequest motive, risk aversion, idiosyncratic labor income risk, and social security on the decision to purchase life insurance and/or annuities. See, for example, Hakansson (1969); Fischer (1973); Richard (1975); Pliska and Ye (2007); Huang et al. (2008); Koijen et al. (2010); Ekeland et al. (2012); Kwak and Lim (2014); Wei et al. (2020); Bernard et al. (2021). While the above studies attempt to address the annuity puzzle by typically assuming that individuals have precise expectations about their future survival probabilities, Heimer et al. (2019) document that young (old) people tend to overestimate (underestimate) their mortality risk and, as a result, young people save less and retirees dissave at a slower pace. See also O'Dea and Sturrock (2021) that provide additional evidence that individuals misperceive their mortality risk and this in turn has the capacity of explaining the significant low rate of annuitization.

In many of the above formulation of life-cycle models, the individual's preference is given by an additively time-separable constant relative risk aversion (CRRA) utility function. This assumption simplifies the analysis but it is unrealistic. A more plausible alternative to the standard CRRA preferences is habit formation that relaxes the time separability of preferences and allows for adjacent complementarity in consumption. The habit formation has been successfully used in economics and finance to explain the observed and puzzling behaviours. For example, Sundaresan (1989) shows that habit formation can rationalize the observed stickiness of the consumption series relative to the fluctuations in stock market wealth, and thus account for the observed consumption smoothing. Abel (1990) and Constantinides (1990) demonstrate that the habit formation can serve as a resolution of the equity premium puzzle. Kraft et al. (2017) show that an individual's optimal consumption over the life cycle can have the hump shape observed empirically if the individual's preferences exhibit habit formation. Ben-Arab et al. (1996) document that the observed phenomenon of individuals' over-purchasing insurance, such as a propensity for low deductibles, can be justified by habit formation. Despite the extensive applications of habit formation, relatively few papers examine how consumption habits can influence the household demand for mortality-contingent claims. Notably, the household decision for life-contingent claims is inextricably linked to the family's consumption over the life cycle (Hansen and İmrohoroğlu, 2008). As a result, the consumption habits that greatly affect current level of consumption and satisfaction should be taken into consideration for individuals' life insurance and annuity choices. This paper attempts to fill this gap.

We propose a life-cycle model in which households allocate their resources among saving, consumption, investment, life insurance, and annuities. Our model differs from prior work by allowing the agent's utility to depend on how present consumption compares to a standard of living defined by the weighted average of past consumption. We propose a novel way to model the bequest motive. Rather than use an exogenous function to represent the bequest we endogenize it within the model. By assuming there is only one breadwinner in the household, the survivors' consumption will be financed through the legacy upon the breadwinner's death. We show that this specification is equivalent to an endogenously updated bequest function that depends on not only the legacy but also the family's consumption habits because the breadwinner must ensure that the legacy is adequate for the survivors to maintain a certain level of consumption. The advantage of this specification is that the family's living standard is incorporated into the bequest motive to reflect the effect of habit formation.

We obtain an analytical solution for the agent's optimization problem. We derive the optimal consumption, portfolio choice, and life insurance/annuity decision. This enables us to examine the impact of habit formation on the demand for life insurance/annuities over the life cycle under different assumptions. Habit formation allows the agent to develop a "taste for the good life" by making the utility of a given current consumption level a decreasing function of the past consumption level. On the one hand, the family needs to maintain sufficient financial wealth to ensure that future consumption will not fall below the habit level before the breadwinner's death, hence reducing the bequest motive. On the other hand, the survivors also require enough legacy to maintain the minimum subsistence upon the breadwinner's death, thus increasing the bequest motive. We show that the net effect of these two conflicting effects is a reduced bequest motive. However, the presence of habit formation also strengthens the impact of wealth and labor income, thereby increasing the bequest motive. Therefore, habit formation can have various impacts on the bequest motive depending on different scenarios. We find that a young family with low risk-aversion and high living standards purchases less life insurance; while an older family with high risk-aversion and low living standards makes less annuity purchases.

In our paper, we obtain the optimal strategy under a stochastic environment and follow Purcal and Piggott (2008) to base our analysis on the expected paths of the optimal strategy which illustrate the life-cycle pattern of a "typical" family. We first consider the expected annuity choice for a 65-year-old retiree. Under our assumed insurance/annuity markets, we find that with a high initial living standard, the family has to decrease its consumption and habit levels over time to ensure there are enough resources to meet the minimum subsistence in the future, leading to low annuity demand in the subsequent years. Notably such a pattern is reversed for a family with low initial habit consumption. We then explore the scenario in which the breadwinner is 30 years old. The first observation is that the expected consumption path is hump-shaped (as observed empirically) if the preferences of the family exhibit internal habit formation while the family would prefer a decreasing consumption path over its life time in the absence of habit formation, thereby extending the result in Kraft et al. (2017) to the case with financial investment, uncertain lifetime, bequest motive, and life insurance/annuity purchases. The second observation is that as the consumption habits increase in the early ages, the bequest motive is reduced and the family purchases less life insurance. One may wonder if habit formation can address the mismatch in the annuity market. However, as the planning horizon is long enough (compared to the 65-year-old retiree) for the family to smooth consumption, the consumption habits remain at relatively high levels over the lifetime, thereby increasing the demand for annuities.

We also examine the role of retirement income (from government's social security and/or corporate pension) by allowing the family to receive a constant income equal to a fixed proportion (the "replacement ratio") of the wage at retirement. To make meaningful comparisons, we reduce the wage by the pension contributions so that the present value of future income at the beginning of the life cycle is unchanged. Bernheim (1991) find that social security annuity benefits significantly raise life insurance holdings and depress private annuity holdings among elderly individuals. Purcal and Piggott (2008) further document that in a CRRA setting, as social security becomes more important, the household will buy more life insurance and less annuities, and the motive for voluntary annuitization diminishes completely when social security is set at 50% of the pre-retirement income. Moreover, the family is likely to purchase life insurance at very old ages, which is unrealistic. In contrast, when the replacement ratio is set at 60%, the family does not annuitize at all and purchases life insurance over the life cycle, but the holdings of life insurance are reduced by habit formation. When the replacement ratio is set at 40%,[1] the family without habit formation purchases life insurance over its life time. In the presence of habit formation, the family with a 40% replacement ratio purchases life insurance before retirement and starts to buy annuities at retirement. In addition, the demand for life insurance/annuity is greatly reduced, with life insurance premia/annuity payouts less than $\$510$ per annum. These results illustrate that social security together with habit formation lead to lower demand in life insurance and annuity products for our assumed insurance/annuity markets. This phenomenon is consistent with the empirical evidence based on the actual insurance/annuity markets.

The technical analysis in our paper is of independent interest. From a technical point of view, three state-variable problems, i.e., wealth and consumption habit levels of the two family members, make the underlying Hamilton-Jaboian-Bellman (HJB) equation difficult to solve. Despite this difficulty, we manage to obtain analytical solutions via the martingale approach. One of the key steps in our solution methodology, and indeed one of our technical contributions, is to treat the differences between consumption and habit, and between legacy and a modification of the habit as new decision variables, and derive the corresponding budget constraint. The advantage of this formulation is that the optimization problem becomes a static one as the endogenously determined consumption habits are absorbed into the decision variables, and hence static optimization techniques become applicable. This is an extension of the technique in Schroder and Skiadas (2002) to incorporate life insurance/annuity purchases.

The rest of the paper is organized as follows. Section 2 introduces the economic setting, including the financial market, the insurance market, and the agent's preferences. Section 3 solves the model analytically and Section 4 calibrates our model. Section 5 and Section 6 examine the impact of habit formation on the demand for life insurance and annuity, respectively. Section 7 illustrates the impacts over the life cycle. Section 8 concludes the paper. Appendices collect all the necessary proofs as well as additional numerical comparisons.

2 Economic Setting

We consider a similar setting as in Richard (1975); Pliska and Ye (2007); Huang et al. (2008). Let W_t be a standard Brownian motion defined on a given probability space $(\Omega, \mathcal{F}, \mathbb{P})$. Let $T > 0$ be the time horizon of the family and $\mathbb{F} = \{\mathcal{F}_t, t \in [0, T]\}$ where $\mathcal{F}_t$ is the $\mathbb{P}$-augmentation of the filtration $\sigma\{W_s : 0 \leq s \leq t\}$.

2.1 Financial Market

In this paper we assume that there is a riskless asset (i.e., the bank account) earning an instantaneous, continuously compounded rate of return r, and a risky asset (e.g., stock index, market portfolio), whose price process, S_t, satisfies the following stochastic differential equation:

$$dS_t = S_t(\mu dt + \sigma dW_t),$$

where μ and σ are constants. Thus, the *market price of risk* is given by $\theta := (\mu - r)/\sigma$ and it is assumed $\theta > 0$.

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[1] Maurer et al. (2013) suggest a 40% replacement ratio as the Organisation for Economic Co-operation and Development (OECD) in 2012 reported the average replacement rates for the average U.S. earner was 39.4%.

2.2 Insurance Market

Suppose the breadwinner/wage-earner is alive at time $t = 0$. The actual life time of the breadwinner can be described as the value of a non-negative random variable τ having probability density function $f(t)$ and distribution function $F(t) := \mathbb{P}(\tau < t) = \int_0^t f(u)du$. We assume the future lifetime τ is independent of $\mathbb{F}$. The *survival* function, $\bar{F}(t)$, is defined as the probability that the survival time is greater than or equal to t, i.e., $\bar{F}(t) := \mathbb{P}(\tau \geq t) = 1 - F(t)$. The survival function can therefore be used to represent the probability that the wage-earner survives to time t. The *instantaneous force of mortality IFM curve* $\lambda_y(t)$ is defined as the probability that the breadwinner dies at time t, conditional on he/she having survived to that time, where y is the breadwinner's age at time $t = 0$. The IFM therefore represents the instantaneous death rate for the breadwinner surviving to time t, i.e.,

$$\lambda_y(t) := \lim_{\varepsilon \to 0} \frac{\mathbb{P}(t \leq \tau < t + \varepsilon | \tau \geq t)}{\varepsilon} = \frac{f(t)}{\bar{F}(t)}. \tag{1}$$

It then follows that $f(t) = \lambda_y(t)\bar{F}(t) = \lambda_y(t) \exp\left(-\int_0^t \lambda_y(u)du\right)$. In this paper, we assume $\lambda_y(t)$ is a non-negative, continuous, and deterministic function.

The family purchases short-term insurance on the life of the wage-earner, which is renegotiated and guaranteed renewable on an ongoing basis at a predetermined schedule that is driven by the IFM curve. More precisely, if the family pays the insurance premium at the rate of I_t continuously, then the family receives the lump-sum benefit $\frac{I_\tau}{\lambda_y(\tau)}$ upon death. If the premium rate I_t is negative, then the wage-earner purchases a special term pension annuity, where he/she receives the pension annuity at rate I_t. However, the amount $\frac{I_\tau}{\lambda_y(\tau)}$ should be paid by the breadwinner's beneficiary to the insurance company upon his/her death. While the above formulation of the insurance/annuity market is typical in the life-cycle model and that Dybvig and Liu (2010) argue that this resembles a term version of a life annuity since it trades wealth in the event of death for a cash inflow while living, it should be emphasized that these products are not commonly available in practice. Hence our subsequent analysis on the agent's demand for insurance/annuity applies only to the above hypothetical insurance/annuity market. A premium loading could be incorporated into our model by replacing the death benefit $\frac{I_\tau}{\lambda_y(\tau)}$ with $\frac{I_\tau}{\eta_y(\tau)}$, where η_y is referred to as the insurance *premium-payout ratio* in Pliska and Ye (2007).[2] However, as illustrated in Purcal and Piggott (2008), insurance loadings are relatively unimportant in this context so we assume that the mortality-contingent claims are actuarially fair.

2.3 Wealth Dynamics

Analogous to the literature (see, e.g., Huang and Milevsky, 2008; Huang et al., 2008; Kwak et al., 2011), we assume the wage-earner receives labor income at the rate of $w(t)$ during the period $[0, \min\{T, \tau\}]$ and the rest of the family has no labor income.[3] As the focus of

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[2] In this situation, we may also interpret $\lambda_y(\tau)$ as the decision-maker's subjective mortality belief and $\eta_y(\tau)$ as the mortality belief used by insurers for pricing, as in O'Dea and Sturrock (2021).

[3] It should be emphasized that the main conclusions of the paper remain valid even if we relax the assumption "the rest of the family has no labor income", provided that the rest of the family can survive to T and the income is deterministic. To see this, suppose the rest of the family earns income at a rate of $\tilde{w}(t)$ from time 0 to T. The time-0 value of the future income is $\tilde{x} := \int_0^T e^{-rt}\tilde{w}(t)dt$. In the absence of the borrowing constraint, the family can borrow against future income and the life-cycle problem is equivalent to the original problem in which the rest

this paper is to examine the impact of habit formation on the household's demand for life insurance/annuity, we additionally assume $w(t)$ is deterministic and uniformly bounded and thus abstract from idiosyncratic labor income risk. Further, suppose the wage-earner and the rest of the family consume at the rates of c_{1t} and c_{2t}, respectively. Denote by π_t the dollar amount of the family's wealth invested in the risky asset at time t and X_t the wealth of the family at time t. Assume the family is endowed with initial wealth x at time 0. Trading takes place continuously in a self-financing fashion and there are no transaction costs. Then X satisfies

$$dX_t = [rX_t + (\mu - r)\pi_t - c_{1t} - c_{2t} - I_t + w(t)]dt + \sigma \pi_t dW_t,\ t < \min\{T, \tau\}.$$

If the breadwinner dies at time $\tau \le T$, then the rest of the family will receive the death benefit $\frac{I_\tau}{\lambda_y(\tau)}$. Thus the breadwinner's total legacy when he/she dies at time τ with wealth $X_{\tau-}$ is $Z_\tau := X_{\tau-} + \frac{I_\tau}{\lambda_y(\tau)}$, and the surviving family will continue to live with endowment $X_\tau = Z_\tau$ so that the wealth process X_t satisfies

$$dX_t = [rX_t + (\mu - r)\pi_t - c_{2t}]dt + \sigma \pi_t dW_t,\ t > \min\{T, \tau\}.$$

2.4 Preferences

Let U_1 and U_2 denote, respectively, the utility functions for consumption of the breadwinner and the rest of the family.[4] While the rest of the family can be viewed as either the spouse or the children or all of them collectively in our model, the interpretation of the spouse is more universally acceptable as suggested by Bernheim (1991) and Inkmann et al. (2010). Following the works of Huang and Milevsky (2008), Huang et al. (2008), and Kwak et al. (2011), we assume that the rest of the family has a certain life expectancy.[5] Under the utility maximization framework, the family chooses c_1, c_2, I, and π that maximize

$$\mathbb{E}\left[\kappa_1 \int_0^{\min\{T,\tau\}} e^{-\delta t} U_1(c_{1t} - h_{1t})dt + \kappa_2 \int_0^T e^{-\delta t} U_2(c_{2t} - h_{2t})dt\right]. \tag{2}$$

The weights κ_1 and κ_2 in the above objective function, which satisfy $\kappa_1 + \kappa_2 = 1$, capture the relative weights of the utility of consumption between the breadwinner and the rest of the family. The parameter $\delta \ge 0$ is the subjective rate of time preference while h_{1t} and h_{2t} are, respectively, the time-t *habit levels* of the breadwinner and the rest of the family. The habit levels are defined by

$$dh_{it} = -(\beta_i h_{it} - \alpha_i c_{it})dt, \tag{3}$$

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of the family has no income but the family's initial wealth increased by $\tilde{x}$. Therefore, the assumption that the rest of the family also earns income has no material impact on our solution.

[4] In the life-cycle literature, most papers examine the optimal portfolio choices for a single individual/representative agent in the household. However, as argued in Hubener et al. (2014), these models cannot adequately describe the effects of one spouse's death on the financial and consumption choice of the surviving partner. Therefore, in this paper, we separate the utility functions to model the variation in family size. Notably, such a modelling approach has been adopted in earlier literature such as that by Brown and Poterba (2000).

[5] Hubener et al. (2014) consider a life-cycle problem with uncertain lifespan for both family members. However, their analysis relies on the assumption that the survival probabilities are independent. Our approach is also applicable to such a model setting.

or equivalently

$$h_{it} = h_{i0} e^{-\beta_i t} + \alpha_i \int_0^t e^{-\beta_i (t-s)} c_{is} ds, \ i = 1, 2, \tag{4}$$

where $h_i, \alpha_i, \beta_i, i = 1, 2$, are all non-negative constants and h_{i0} denotes the initial level of habit.[6] It follows from the above definition that the habit level is a multiple of a weighted average of past consumption rates with the weights being exponentially decreasing so that the recent consumption rates have higher weights. The parameters β and α measure, respectively, the persistence of the past and the intensity of consumption habits, i.e., the importance of the consumption history relative to the inherited initial standard of living. Persistence is high when β is low. The standard of living places more emphasis on the history of consumption when α increases. We require $\beta_i > \alpha_i, i = 1, 2$, to ensure that the habit level will decline when the investor consumes at the habit level.

In line with most of the literature, we assume U_i is a power utility exhibiting CRRA:

$$U_i(x) = \frac{x^{1-\gamma_i}}{1-\gamma_i}, \ i = 1, 2, \tag{5}$$

where γ_1, γ_2 are positive constants. Thus, the consumption rate is required to exceed the habit level so that the habit level is the minimum or subsistence consumption rate determined by past consumption rates. Note that the relative risk aversion is no longer a constant. In fact, the relative risk aversion $-\frac{c\frac{\partial^2 U_i(c-h)}{\partial c^2}}{\frac{\partial U_i(c-h)}{\partial c}} = \gamma_i \frac{c}{c-h}$ is decreasing in the consumption-to-habit ratio c/h.

A notable difference between our proposed model and other insurance models (see Yaari, 1965; Fischer, 1973; Richard, 1975; Pliska and Ye, 2007) is that we do not model the bequest motive directly. In the literature, there is no consensus on the form of the utility bequest function in the presence of habit formation. We could follow Polkovnichenko (2006) to apply a utility function (such as a CRRA function) on bequest. We argue that the bequest function should also depend on the surviving family members' consumption habit as, in our setting, the family's consumption after the breadwinner's death will be mainly financed through the bequest. Therefore, we incorporate the bequest motive into (2) through the surviving family member's consumption after the breadwinner's death. As will be clear in (7), this specification is equivalent to a utility function of bequest which depends not only on the legacy but also on the surviving family member's consumption habit.

3 Solution

If the breadwinner dies before T, that is, $\tau = t < T$, then the surviving family chooses c_2 and π to maximize

$$E\left[\int_t^T e^{-\delta(s-t)} U_2(c_{2s} - h_{2s}) ds | \mathcal{F}_t\right]. \tag{6}$$

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[6] Our approach is also valid in a model that treats the household as a single decision-making unit. However, as the impacts of the breadwinner and the surviving family's consumption habits on household decision might be different, we separate the habits of the family members in our setting. Notably, papers that do separate the household members' preferences have shown that this is a crucial consideration. Empirical evidence confirms that household members do differ in risk preference (see, e.g., Barsky et al., 1997; Kimball et al., 2008). Browning (2000) and Mazzocco (2004) further discover that the resources allocation within the family heavily affects the consumption-saving decisions when spouses have different preference.

This is a standard consumption and portfolio choice problem with habit formation, and can be easily solved via the method in Schroder and Skiadas (2002). In particular, Munk (2008) characterizes explicit solutions under various models. Let $V_2(t,z,h)$ denote the maximal value of (6) for a breadwinner that dies at $t < T$ with legacy z. In other words, $Z_t = z$ is also the wealth of the surviving family at time t, and the surviving family's habit level at time t is h, i.e., $h_{2t} = h$. From Munk (2008, Theorem 1), we obtain

Lemma 3.1.

$$V_2(t,z,h) = (H(t))^{\gamma_2} \frac{(z - hG(t))^{1-\gamma_2}}{1-\gamma_2},$$

where

$$\begin{aligned} G(t) &:= \frac{1}{r+\beta_2-\alpha_2}(1 - e^{-(r+\beta_2-\alpha_2)(T-t)}), \\ H(t) &:= \int_t^T \exp\left(-\left[\frac{\delta}{\gamma_2} + (1-\frac{1}{\gamma_2})r + \frac{1}{2\gamma_2}(1-\frac{1}{\gamma_2})\theta^2\right](s-t)\right)[1+\alpha_2 G(s)]^{1-\frac{1}{\gamma_2}} ds. \end{aligned}$$

Therefore, the family's problem (2) simplifies to

$$\mathbb{E}\left[\int_0^{\min\{T,\tau\}} e^{-\delta t}\big(\kappa_1 U_1(c_{1t}-h_{1t}) + \kappa_2 U_2(c_{2t}-h_{2t})\big)dt + \kappa_2 e^{-\delta\tau} V_2(\tau, Z_\tau, h_{2\tau})1_{\tau<T}\right]. \quad (7)$$

As discussed in the preceding section, $V_2(t,z,h)$ serves as the bequest function. By modeling the bequest function endogenously, we are able to integrate the surviving family member's consumption habit. The relative importance of the bequest is determined jointly by the exogenous weight κ_2 and the endogenous quantity $(H(t))^{\gamma_2}$. Because the family has no labor income after the breadwinner's death, the future consumption must be financed by the legacy so that $hG(t)$ reflects the minimum legacy required by the beneficiary to meet the minimum future subsistence. Our specification allows for the time-varying, endogenously-updated bequest motive, which is more realistic and can be an advantage over the fixed, exogenous bequest function. For example, a one million dollar legacy is surely more favorable to a family living in a studio apartment than to a family living in a mansion.

Because τ is independent of $\mathbb{F}$, we can treat (7) as if the breadwinner will stay alive until T.

Lemma 3.2. *We have*

$$\begin{aligned} &\mathbb{E}\left[\int_0^{\min\{T,\tau\}} e^{-\delta t}\big(\kappa_1 U_1(c_{1t}-h_{1t}) + \kappa_2 U_2(c_{2t}-h_{2t})\big)dt + \kappa_2 e^{-\delta\tau} V_2(\tau, Z_\tau, h_{2\tau})1_{\tau<T}\right] \\ &= \mathbb{E}\left[\int_0^T D(t)\big(\kappa_1 U_1(c_{1t}-h_{1t}) + \kappa_2 U_2(c_{2t}-h_{2t}) + \kappa_2 \lambda_y(t) V_2(t, Z_t, h_{2t})\big)dt\right] \end{aligned}$$

where $D(t) := e^{-\delta t - \int_0^t \lambda_y(s)ds}$.

Lemma 3.2 states that the breadwinner who facing an uncertain time of death time acts as if he/she will live until time T, but with a subjective rate of time preference adjusted by his/her "force of mortality". From a technical point of view, this lemma enables us to convert the random horizon optimization problem to a problem with a fixed terminal time. Consequently, we can consider the following optimization problem

$$\begin{aligned}
\max_{c_1,c_2,Z}\ & \mathbb{E}\left[\int_0^T D(t)\big(\kappa_1 U_1(c_{1t}-h_{1t})+\kappa_2 U_2(c_{2t}-h_{2t})+\kappa_2\lambda_y(t)V_2(t,Z_t,h_{2t})\big)dt\right]\\
\text{s.t. }\ & dX_t=[rX_t+(\mu-r)\pi_t-c_{1t}-c_{2t}-I_t+w(t)]dt+\sigma\pi_t dW_t,\ X_0=x,\\
& Z_t=X_t+\frac{I_t}{\lambda_y(t)},\\
& dh_{it}=-(\beta_i h_{it}-\alpha_i c_{it})dt,\ i=1,2.
\end{aligned}\tag{8}$$

3.1 Optimal Strategies

In the continuous-time consumption and portfolio choice literature, there are mainly two approaches: the dynamic programming approach (Merton, 1969) and the martingale approach (Pliska, 1986; Karatzas et al., 1987; Cox and Huang, 1989). We do not use the dynamic programming approach to solve (8) as the HJB equation involves three state variables: x, h_1 and h_2, which is extremely difficult to solve. By exploiting the martingale method and the transformation technique proposed by Schroder and Skiadas (2002), we are able to solve (8) in a clear and simple way.

Define $\rho_t := \exp\left(-(r+\frac{1}{2}\theta^2)t-\theta W_t\right)$, as the *state price density* in this economy. Then, in view of the standard martingale approach (see Ye, 2006; Kwak et al., 2011; Kwak and Lim, 2014), we can consider the following problem

$$\begin{aligned}
\max_{c_1,c_2,Z}\ & \mathbb{E}\left[\int_0^T D(t)\big(\kappa_1 U_1(c_{1t}-h_{1t})+\kappa_2 U_2(c_{2t}-h_{2t})+\kappa_2\lambda_y(t)V_2(t,Z_t,h_{2t})\big)dt\right]\\
\text{s.t. }\ & \mathbb{E}\left[\int_0^T (c_{1t}+c_{2t}+\lambda_y(t)Z_t)\xi_t dt\right]\le x+L(0),
\end{aligned}\tag{9}$$

where h_{it} is given by (4),

$$L(t):=\frac{1}{\xi_t}\mathbb{E}_t\left[\int_t^T w(s)\xi_s ds\right]=\int_t^T w(t)e^{-r(s-t)-\int_t^s\lambda_y(u)du}ds \tag{10}$$

is the time-t value of the wage-earner's future labor income, and

$$\xi_t:=\rho_t\exp\left(-\int_0^t\lambda_y(s)ds\right),$$

is the state price density adjusted for the mortality risk. Appendix A provides more details about the relationship between the dynamic optimization problem (8) and its martingale formulation (9).

We emphasize that (9) is not a conventional static utility maximization problem. The current consumption depends on not only its past paths through the current habit but also its future paths via the disutility attached to future habits. This is in contrast to the standard time-separable preference in which current consumption is independent of past and future paths. Therefore, (9) cannot be solved by the standard static optimization method. Inspired by Schroder and Skiadas (2002), we treat $c_{1t}-h_{1t}$, $c_{2t}-h_{2t}$, and $Z_t-h_2G(t)$ instead of c_{1t}, c_{2t}, and Z_t as decision variables and derive the corresponding budget constraint, as shown in the lemma below.

Lemma 3.3. *We have*

$$\mathbb{E}_t\left[\int_t^T c_{1s}\xi_s ds\right] = \mathbb{E}_t\left[\int_t^T (c_{1s}-h_{1s})(1+\alpha_1 G_1(s))\xi_s ds\right] + h_{1t}\xi_t G_1(t), \tag{11}$$

and

$$\begin{aligned}\mathbb{E}_t\left[\int_t^T c_{2s}\xi_s ds\right] =& \mathbb{E}_t\left[\int_t^T (c_{2s}-h_{2s})(1+\alpha_2 G_2(s))\xi_s ds\right] - \mathbb{E}_t\Big[\int_t^T \lambda_y(s)G(s)h_{2s}\xi_s ds\Big]\\ &+ h_{2t}\xi_t G_2(t),\end{aligned} \tag{12}$$

where

$$\begin{aligned}G_1(t) &:= \int_t^T G_1(s;t)ds,\\ G_2(t) &:= \int_t^T (1+\lambda_y(s)G(s))G_2(s;t)ds,\\ G_i(s;t) &:= e^{-(r+\beta_i-\alpha_i)(s-t)-\int_t^s \lambda_y(u)du},\ i=1,2.\end{aligned}$$

Therefore, we can consider the following problem

$$\begin{aligned}\max_{c_1,c_2,Z}\ & \mathbb{E}\left[\int_0^T D(t)\big(\kappa_1 U_1(c_{1t}-h_{1t}) + \kappa_2 U_2(c_{2t}-h_{2t}) + \kappa_2\lambda_y(t)V_2(t,Z_t,h_{2t})\big)dt\right]\\ \text{s.t. }\ & \mathbb{E}\left[\int_0^T (c_{1t}-h_{1t})(1+\alpha_1 G_1(t))\xi_t dt\right] + \mathbb{E}\Big[\int_0^T (c_{2t}-h_{2t})(1+\alpha_2 G_2(t))\xi_t dt\Big]\\ &+\mathbb{E}\left[\int_0^T \lambda_y(t)(Z_t - G(t)h_{2t})\xi_t dt\right] \le x + L(0) - h_{10}G_1(0) - h_{20}G_2(0).\end{aligned} \tag{13}$$

Lemma 3.3 extends Schroder and Skiadas (2002, Proposition 3) by incorporating life insurance/annuity purchases. By rewriting the budget constraint to include current and future habit levels, we are able to treat $c_{1t}-h_{1t}$, $c_{2t}-h_{2t}$, and $Z_t - G(t)h_{2t}$ as decision variables. The advantage of this formulation is that (13) is a static optimization problem, and hence standard optimization techniques become applicable.

The following proposition characterizes the optimal solution to (13).

Proposition 3.1. *Suppose that* $x + L(0) - h_{10}G_1(0) - h_{20}G_2(0) > 0$. *We have*

(a) the optimal consumption strategies are

$$\begin{aligned}c_{1t}^* &= h_{1t}^* + \Big(\frac{v^*}{\kappa_1}\Big)^{-\frac{1}{\gamma_1}}\Big(\frac{1+\alpha_1 G_1(t)}{D(t)}\Big)^{-\frac{1}{\gamma_1}}\xi_t^{-\frac{1}{\gamma_1}},\\ c_{2t}^* &= h_{2t}^* + \Big(\frac{v^*}{\kappa_2}\Big)^{-\frac{1}{\gamma_2}}\Big(\frac{1+\alpha_2 G_2(t)}{D(t)}\Big)^{-\frac{1}{\gamma_2}}\xi_t^{-\frac{1}{\gamma_2}},\end{aligned}$$

where $h_{it}^*, i=1,2$ *are the habit levels induced by the optimal consumption strategies* $c_{it}^*, i=1,2,$ *respectively;*

(b) the optimal legacy is

$$Z_t^* = h_{2t}^* G(t) + \left(\frac{v^*}{\kappa_2}\right)^{-\frac{1}{\gamma_2}} (D(t))^{\frac{1}{\gamma_2}} H(t) \xi_t^{-\frac{1}{\gamma_2}},$$

and the optimal face value of the life insurance is $\frac{I_t^*}{\lambda_y(t)} = Z_t^* - X_t^*$,*where* X_t^* *is the wealth process induced by the optimal strategies and is given by*

$$\begin{aligned}
X_t^* = &- L(t) + h_{1t}^* G_1(t) + h_{2t}^* G_2(t) \\
&+ \xi_t^{-\frac{1}{\gamma_1}} \left(\frac{v^*}{\kappa_1}\right)^{-\frac{1}{\gamma_1}} \int_t^T (1+\alpha_1 G_1(s))^{1-\frac{1}{\gamma_1}} (D(s))^{\frac{1}{\gamma_1}} F_1(s;t) ds \\
&+ \xi_t^{-\frac{1}{\gamma_2}} \left(\frac{v^*}{\kappa_2}\right)^{-\frac{1}{\gamma_2}} \int_t^T (1+\alpha_2 G_2(s))^{1-\frac{1}{\gamma_2}} (D(s))^{\frac{1}{\gamma_2}} F_2(s;t) ds \\
&+ \xi_t^{-\frac{1}{\gamma_2}} \left(\frac{v^*}{\kappa_2}\right)^{-\frac{1}{\gamma_2}} \int_t^T \lambda_y(s) (D(s))^{\frac{1}{\gamma_2}} H(s) F_2(s;t) ds,
\end{aligned}$$

with $F_i(s;t) := \mathbb{E}_t\left[\left(\frac{\xi_s}{\xi_t}\right)^{1-\frac{1}{\gamma_i}}\right] = e^{-(1-\frac{1}{\gamma_i})\int_t^s \lambda_y(u)du-(1-\frac{1}{\gamma_i})(r+\frac{\theta^2}{2\gamma_i})(s-t)}$, $i = 1, 2;$

(c) the optimal investment strategy is

$$\begin{aligned}
\pi_t^* = \Bigg\{ &\frac{1}{\gamma_1} \xi_t^{-\frac{1}{\gamma_1}} \left(\frac{v^*}{\kappa_1}\right)^{-\frac{1}{\gamma_1}} \int_t^T (1+\alpha_1 G_1(s))^{1-\frac{1}{\gamma_1}} (D(s))^{\frac{1}{\gamma_1}} F_1(s;t) ds \\
&+ \frac{1}{\gamma_2} \xi_t^{-\frac{1}{\gamma_2}} \left(\frac{v^*}{\kappa_2}\right)^{-\frac{1}{\gamma_2}} \int_t^T (1+\alpha_2 G_2(s))^{1-\frac{1}{\gamma_2}} (D(s))^{\frac{1}{\gamma_2}} F_2(s;t) ds \\
&+ \frac{1}{\gamma_2} \xi_t^{-\frac{1}{\gamma_2}} \left(\frac{v^*}{\kappa_2}\right)^{-\frac{1}{\gamma_2}} \int_t^T \lambda_y(s) (D(s))^{\frac{1}{\gamma_2}} H(s) F_2(s;t) ds \Bigg\} \frac{\theta}{\sigma};
\end{aligned}$$

(d) v^* *is the (positive) Lagrangian multiplier that solves the budget constraint*

$$\begin{aligned}
&\mathbb{E}\left[\int_0^T (c_{1t}^* - h_{1t}^*)(1+\alpha_1 G_1(t)) \xi_t dt\right] + \mathbb{E}\left[\int_0^T (c_{2t}^* - h_{2t}^*)(1+\alpha_2 G_2(t)) \xi_t dt\right] \\
&\quad + \mathbb{E}\left[\int_0^T \lambda_y(t)(Z_t^* - G(t) h_{2t}^*) \xi_t dt\right] \\
&= \left(\frac{v^*}{\kappa_1}\right)^{-\frac{1}{\gamma_1}} \int_0^T (1+\alpha_1 G_1(t))^{1-\frac{1}{\gamma_1}} (D(t))^{\frac{1}{\gamma_1}} F_1(t;0) dt \\
&\quad + \left(\frac{v^*}{\kappa_2}\right)^{-\frac{1}{\gamma_2}} \int_0^T (1+\alpha_2 G_2(t))^{1-\frac{1}{\gamma_2}} (D(t))^{\frac{1}{\gamma_2}} F_2(t;0) dt \\
&\quad + \left(\frac{v^*}{\kappa_2}\right)^{-\frac{1}{\gamma_2}} \int_0^T \lambda_y(t) (D(t))^{\frac{1}{\gamma_2}} H(t) F_2(t;0) dt, \\
&= x + L(0) - h_{10} G_1(0) - h_{20} G_2(0).
\end{aligned} \tag{14}$$

Proposition 3.1 highlights the technical contribution of the paper that solves the consumption, portfolio choice, and life insurance/annuity purchase problem in the presence of habit formation analytically. We complement the literature on portfolio choice with habit formation by including the decision for life insurance/annuity, and the literature of life insurance/annuity demand over the life-cycle by incorporating habit formation.

With the help of Proposition 3.1, we can express the optimal strategies in feedback forms when $\gamma_1 = \gamma_2$.

Corollary 3.1. *If* $x + L(0) - h_{10}G_1(0) - h_{20}G_2(0) > 0$ *and* $\gamma_1 = \gamma_2 = \gamma$*, we have*

(a) the optimal consumption strategies are

$$c_{1t}^* = h_{1t}^* + \Big(\frac{1+\alpha_1 G_1(t)}{\kappa_1 D(t)}\Big)^{-\frac{1}{\gamma}} \frac{X_t^* + L(t) - G_1(t)h_{1t}^* - G_2(t)h_{2t}^*}{H_\gamma(t)},$$
$$c_{2t}^* = h_{2t}^* + \Big(\frac{1+\alpha_2 G_2(t)}{\kappa_2 D(t)}\Big)^{-\frac{1}{\gamma}} \frac{X_t^* + L(t) - G_1(t)h_{1t}^* - G_2(t)h_{2t}^*}{H_\gamma(t)},$$

where $h_{it}^*, i = 1,2$ *are the habit levels induced by the optimal consumption strategies* $c_{it}^*, i = 1,2$*, respectively;*

(b) the optimal legacy and the optimal face value of life insurance are given, respectively, as

$$Z_t^* = h_{2t}^* G(t) + (\kappa_2 D(t))^{\frac{1}{\gamma}} H(t) \frac{X_t^* + L(t) - G_1(t)h_{1t}^* - G_2(t)h_{2t}^*}{H_\gamma(t)}, \tag{15}$$
$$\frac{I_t^*}{\lambda_y(t)} = Z_t^* - X_t^*, \tag{16}$$

where X_t^* *is the wealth process induced by the optimal strategies and is given by*

$$X_t^* = -L(t) + h_{1t}^* G_1(t) + h_{2t}^* G_2(t) + \xi_t^{-\frac{1}{\gamma}} (v^*)^{-\frac{1}{\gamma}} H_\gamma(t),$$

and

$$\begin{aligned} H_\gamma(t) :=& \int_t^T \Big[\kappa_1^{\frac{1}{\gamma}} (1+\alpha_1 G_1(s))^{1-\frac{1}{\gamma}} + \kappa_2^{\frac{1}{\gamma}} (1+\alpha_2 G_2(s))^{1-\frac{1}{\gamma}} + \kappa_2^{\frac{1}{\gamma}} \lambda_y(s) H(s)\Big] \\ & \times (D(s))^{\frac{1}{\gamma}} e^{-(1-\frac{1}{\gamma})\int_t^s \lambda_y(u)du - (1-\frac{1}{\gamma})(r+\frac{\theta^2}{2\gamma})(s-t)} ds, \\ v^* =& \Big(\frac{x + L(0) - h_{10}G_1(0) - h_{20}G_2(0)}{H_\gamma(0)}\Big)^{-\gamma}; \end{aligned}$$

(c) the optimal investment strategy is $\pi_t^* = \frac{\theta}{\gamma\sigma}[X_t^* + L(t) - h_{1t}^* G_1(t) - h_{2t}^* G_2(t)]$.

3.2 Expected Life-Cycle Path

We obtain the optimal behavior of a family in a stochastic environment. To characterize the life-cycle pattern, we follow Purcal and Piggott (2008) and base our analysis on the expected paths of the optimal strategy to give us some sense of the life-cycle pattern of a "typical" family. The following corollary characterizes the expected paths of the optimal strategy.

Corollary 3.2. *Suppose that* $x + L(0) - h_{10}G_1(0) - h_{20}G_2(0) > 0$*. For* $t \in [0, T]$*, we have* *(a) The expected time-t habit levels are*

$$\mathbb{E}[h_{it}^*] = h_{i0}e^{-(\beta_i-\alpha_i)t}+\alpha_i\Big(\frac{v^*}{\kappa_i}\Big)^{-\frac{1}{\gamma_i}}\int_0^t e^{-(\beta_i-\alpha_i)(t-s)}\Big(\frac{1+\alpha_i G_i(s)}{\kappa_i D(s)}\Big)^{-\frac{1}{\gamma_i}}\mathbb{E}\Big[\xi_s^{-\frac{1}{\gamma_i}}\Big]ds,\ i=1,2,$$

where $\mathbb{E}[\xi_t^{-\frac{1}{\gamma_i}}] = \exp\Big(\frac{1}{\gamma_i}\int_0^t \lambda_y(s)ds + \frac{1}{\gamma_i}(r+(1+\frac{1}{\gamma_i})\frac{\theta^2}{2})t\Big)$;

(b) the expected time-t consumption levels are

$$\mathbb{E}[c_{it}^*] = \mathbb{E}[h_{it}^*] + \Big(\frac{v^*}{\kappa_i}\Big)^{-\frac{1}{\gamma_i}}\Big(\frac{1+\alpha_i G_i(t)}{D(t)}\Big)^{-\frac{1}{\gamma_i}}\mathbb{E}[\xi_t^{-\frac{1}{\gamma_i}}],\ i=1,2;$$

(c) the expected time-t legacy is

$$\mathbb{E}[Z_t^*] = \mathbb{E}[h_{2t}^*]G(t) + \Big(\frac{v^*}{\kappa_2}\Big)^{-\frac{1}{\gamma_2}}(D(t))^{\frac{1}{\gamma_2}}H(t)\mathbb{E}[\xi_t^{-\frac{1}{\gamma_2}}],$$

and the expected time-t face value of the life insurance is

$$\mathbb{E}\Big[\frac{I_t^*}{\lambda_y(t)}\Big] = \mathbb{E}[Z_t^*] - \mathbb{E}[X_t^*],$$

where $\mathbb{E}[X_t^*]$ is the expected time-t wealth process induced by the optimal strategies and is given by

$$\begin{aligned}\mathbb{E}[X_t^*] =& -L(t) + \mathbb{E}[h_{1t}^*]G_1(t) + \mathbb{E}[h_{2t}^*]G_2(t)\\ &+\mathbb{E}[\xi_t^{-\frac{1}{\gamma_1}}]\Big(\frac{v^*}{\kappa_1}\Big)^{-\frac{1}{\gamma_1}}\int_t^T(1+\alpha_1G_1(s))^{1-\frac{1}{\gamma_1}}(D(s))^{\frac{1}{\gamma_1}}F_1(s;t)ds\\ &+\mathbb{E}[\xi_t^{-\frac{1}{\gamma_2}}]\Big(\frac{v^*}{\kappa_2}\Big)^{-\frac{1}{\gamma_2}}\int_t^T(1+\alpha_2G_2(s))^{1-\frac{1}{\gamma_2}}(D(s))^{\frac{1}{\gamma_2}}F_2(s;t)ds\\ &+\mathbb{E}[\xi_t^{-\frac{1}{\gamma_2}}]\Big(\frac{v^*}{\kappa_2}\Big)^{-\frac{1}{\gamma_2}}\int_t^T\lambda_y(s)(D(s))^{\frac{1}{\gamma_2}}H(s)F_2(s;t)ds;\end{aligned}$$

(d) the expected time-t investment strategy is

$$\begin{aligned}\mathbb{E}[\pi_t^*] =&\Bigg\{\frac{1}{\gamma_1}\mathbb{E}[\xi_t^{-\frac{1}{\gamma_1}}]\Big(\frac{v^*}{\kappa_1}\Big)^{-\frac{1}{\gamma_1}}\int_t^T(1+\alpha_1G_1(s))^{1-\frac{1}{\gamma_1}}(D(s))^{\frac{1}{\gamma_1}}F_1(s;t)ds\\ &+\frac{1}{\gamma_2}\mathbb{E}[\xi_t^{-\frac{1}{\gamma_2}}]\Big(\frac{v^*}{\kappa_2}\Big)^{-\frac{1}{\gamma_2}}\int_t^T(1+\alpha_2G_2(s))^{1-\frac{1}{\gamma_2}}(D(s))^{\frac{1}{\gamma_2}}F_2(s;t)ds\\ &+\frac{1}{\gamma_2}\mathbb{E}[\xi_t^{-\frac{1}{\gamma_2}}]\Big(\frac{v^*}{\kappa_2}\Big)^{-\frac{1}{\gamma_2}}\int_t^T\lambda_y(s)(D(s))^{\frac{1}{\gamma_2}}H(s)F_2(s;t)ds\Bigg\}\frac{\theta}{\sigma}.\end{aligned}$$

Moreover, if $\gamma_1 = \gamma_2 = \gamma$, we have

$$\mathbb{E}[c_{1t}^*] = \mathbb{E}[h_{1t}^*] + \Big(\frac{1+\alpha_1 G_1(t)}{\kappa_1 D(t)}\Big)^{-\frac{1}{\gamma}} \frac{\mathbb{E}[X_t^*] + L(t) - G_1(t)\mathbb{E}[h_{1t}^*] - G_2(t)\mathbb{E}[h_{2t}^*]}{H_\gamma(t)},$$
$$\mathbb{E}[c_{2t}^*] = \mathbb{E}[h_{2t}^*] + \Big(\frac{1+\alpha_2 G_2(t)}{\kappa_2 D(t)}\Big)^{-\frac{1}{\gamma}} \frac{\mathbb{E}[X_t^*] + L(t) - G_1(t)\mathbb{E}[h_{1t}^*] - G_2(t)\mathbb{E}[h_{2t}^*]}{H_\gamma(t)},$$
$$\mathbb{E}[Z_t^*] = \mathbb{E}[h_{2t}^*]G(t) + (\kappa_2 D(t))^{\frac{1}{\gamma}} H(t) \frac{\mathbb{E}[X_t^*] + L(t) - G_1(t)\mathbb{E}[h_{1t}^*] - G_2(t)\mathbb{E}[h_{2t}^*]}{H_\gamma(t)},$$
$$\mathbb{E}\Big[\frac{I_t^*}{\lambda_y(t)}\Big] = \mathbb{E}[Z_t^*] - \mathbb{E}[X_t^*],$$
$$\mathbb{E}[\pi_t^*] = \frac{\theta}{\gamma\sigma}(\mathbb{E}[X_t^*] + L(t) - G_1(t)\mathbb{E}[h_{1t}^*] - G_2(t)\mathbb{E}[h_{2t}^*]).$$

4 Calibration

This section discusses the calibration issues as well as the parameter values that will be assumed in our subsequent analysis. We assume the stock index's inflation-adjusted return $\mu = 7\%$ per year with an annualized volatility $\sigma = 20\%$, and the real risk-free rate $r = 2\%$ per year. These values are consistent with the portfolio choice literature. The dynamics of the breadwinner's real labor income are given by

$$\frac{dw(t)}{w(t)} = g_y(t)dt, \; y + t < 65,$$

where y is the breadwinner's age at time $t = 0$, $w(0)$ is the breadwinner's initial real wage, and $g_y(t)$ is the deterministic growth rate of the labor income to capture the hump-shaped pattern in labor income over the life cycle. Following Koijen et al. (2010), we set

$$g_y(t) = 0.1682 - 0.00646(y+t) + 0.00006(y+t)^2, \; y + t < 65,$$

which corresponds to an individual with a high school education in the estimates of Cocco et al. (2005); Munk and Sørensen (2010). We assume that the breadwinner retires at age 65 and the family has no labor income during the breadwinner's retirement, that is, $w(t) = 0, \; y + t \geq 65$. Following Huang et al. (2008), the insurance prices and mortality rates are driven by the Gompertz law of mortality, i.e.,

$$\lambda_y(t) = \frac{1}{9.5} e^{\frac{y+t-86.3}{9.5}}.$$

These numbers are consistent with survival rates implicit in pension-based mortality tables.

In terms of preference parameters, we assume that $\gamma_1 = \gamma_2 = \gamma$ and consider low ($\gamma = 2$) and high ($\gamma = 6$) risk aversion. Moreover, we assume $\kappa_1 = \kappa_2 = 0.5$ so that the breadwinner values himself/herself and his/her family equally. We also follow Kraft et al. (2017) by setting $\delta = 0.1$. This is in accordance with the experimental studies of Andersen et al. (2008), Love (2009), and others. Moreover, Kraft et al. (2017) show that an impatient individual would prefer a decreasing consumption path over the life cycle in the absence of habit formation, while the individual's consumption path can have the hump shape observed empirically if the individual's preference exhibits internal habit formation. For cases with habit formation, we assume that $\alpha_1 = \alpha_2 = \alpha$, $\beta_1 = \beta_2 = \beta$ and $h_{10} = h_{20} = h$. This is not a restrictive assumption as members within a family typically exhibit similar consumption patterns. To the best of our knowledge, there is limited empirical study that provides direct estimates of the habit parameters and initial habits except Kraft et al. (2017). Thus, we consider various combinations of the habit parameters which are similar to values considered in Constantinides (1990); Munk (2008); Kraft et al. (2017). In the special case with $\alpha_1, \alpha_2, \beta_1, \beta_2, h_{10}$, and h_{20} equal to zero, we recover the optimal strategies under the standard CRRA preferences, i.e., there is no habit formation.

For different initial ages y, we choose the initial wealth x and the initial wage $w(0)$ to be comparable to the median net household wealth and income, respectively, of the education group where the breadwinner of the family has a high school education which is constructed by Lusardi et al. (2017) from the Panel Study of Income Dynamics (PSID).

5 Demand for Life Insurance

This section investigates the impact of habit formation on the demand for life insurance. Because life insurance is primarily used to protect family's income, we focus on the breadwinner's pre-retirement phase. We consider two realistic scenarios, labelled as Scenario A and Scenario B. For Scenario A, we assume the primary breadwinner is $y = 30$ years old, earns $w(0) = \$25,000$ per year in real terms, and currently has $x = \$35,000$ saved. The breadwinner has 35 years to retirement and the family horizon is $T = 70$ years. For Scenario B, we set $y = 55$, $w(0) = \$40,000$, $x = \$125,000$, and $T = 45$. To be consistent with how insurance is discussed in practice, we focus on the optimal face value of life insurance (i.e. death benefit) instead of the amount spent on life insurance. Table 1 displays the optimal face amount of life insurance $(\frac{I_t}{\lambda_y(t)})$ for a family with low ($\gamma = 2$) and high risk aversion ($\gamma = 6$), with different combinations of the intensity level α, the persistence level β, the habit level h, and for both Scenario A and Scenario B.

Table 1 indicates that the 30-year-old breadwinner (i.e. Scenario A) will optimally purchase around $\$500,000$ of life insurance, which is more than 20 times the breadwinner's annual wage. The impacts of α, β, and h on the optimal amount of life insurance purchased are marginal. Habit formation can increase the face value of life insurance purchased by at most 1.6% (compare 512.23 from $\gamma = 6, \alpha = 0.4$, $\beta = 0.5$, and $h = 0$, to 503.95, the case without habit formation). We also find that the optimal amount of life insurance purchased is less sensitive to the family's risk aversion, a 4% increase when γ increases from 2 to 6 (compare 512.23 from $\gamma = 6, \alpha = 0.4$, $\beta = 0.5$, and $h = 0$, to 491.66 from $\gamma = 2, \alpha = 0.4$, $\beta = 0.5$, and $h = 0$).

By increasing the breadwinner's age from 30 to 55 (i.e. from Scenario A to Scenario B), the optimal face value of life insurance decreases to around $\$140,000$, representing more than 3.5 times the breadwinner's annual wage. With habit formation, it can increase the face value of life insurance purchased by at most 7.29% (compare 154.59 from $\gamma = 6, \alpha = 0.4, \beta = 0.5, h = 0$, to 144.09, the case without habit formation). However, a very high habit level can decrease the life insurance demand by 2.86% (compare 139.97 from $\gamma = 6, \alpha = 0.1, \beta = 0.2, h = 10$, to 144.09, the case without habit formation). The effect of risk aversion is also higher, a 11.86% increase when γ increases from 2 to 6 (compare 138.20 from $\gamma = 2, \alpha = 0.4, \beta = 0.5, h = 0$ to 154.59 $\gamma = 6, \alpha = 0.4, \beta = 0.5, h = 0$).

Intuitively, the breadwinner purchases less life insurance as he/she approaches retirement, which is consistent with the human capital perspective that life insurance protects the family against the loss of the labor income. In contrast to the result in Huang et al. (2008), we find

Table 1: Optimal Face Value of Life Insurance $\frac{I_t}{\lambda_y(t)}$ (in Thousands)

γ	α	β	$h = 0$	$h = 2$	$h = 4$	$h = 6$	$h = 8$	$h = 10$
	Scenario A with $y = 30, T = 70, w(0) = 25,000, x = 35,000$							
2	No habit formation		487.62					
2	0.1	0.2	489.97	489.64	489.31	488.98	488.65	488.32
2	0.1	0.3	488.30	488.11	487.93	487.75	487.56	487.38
2	0.2	0.3	490.88	490.52	490.16	489.80	489.44	489.08
2	0.1	0.4	487.94	487.81	487.69	487.56	487.44	487.31
2	0.3	0.4	491.36	490.99	490.61	490.24	489.86	489.49
2	0.1	0.5	487.80	487.71	487.61	487.52	487.42	487.33
2	0.4	0.5	**491.66**	491.28	490.89	490.51	490.12	489.74
6	No habit formation		**503.95**					
6	0.1	0.2	508.74	507.79	506.85	505.90	504.95	504.00
6	0.1	0.3	505.52	505.02	504.53	504.03	503.54	503.04
6	0.2	0.3	510.61	509.60	508.59	507.58	506.57	505.56
6	0.1	0.4	504.68	506.46	504.05	503.66	504.75	503.13
6	0.3	0.4	511.61	510.57	509.52	508.48	507.44	506.39
6	0.1	0.5	504.40	504.03	503.90	503.65	503.38	503.11
6	0.4	0.5	**512.23**	511.16	510.10	509.03	507.97	506.91
	Scenario B with $y = 55, T = 45, w(0) = 40,000, x = 125,000$							
2	No habit formation		131.50					
2	0.1	0.2	135.30	134.29	133.28	132.27	131.26	130.25
2	0.1	0.3	132.89	132.17	131.46	130.75	130.03	129.32
2	0.2	0.3	136.84	135.73	134.61	133.50	132.39	131.27
2	0.1	0.4	132.20	131.68	131.16	130.64	130.12	129.60
2	0.3	0.4	137.68	136.51	135.34	134.16	132.99	131.82
2	0.1	0.5	131.92	131.51	131.11	130.71	130.30	129.90
2	0.4	0.5	**138.20**	136.99	135.79	134.58	133.38	132.17
6	No habit formation		**144.09**					
6	0.1	0.2	149.98	147.98	145.98	143.97	141.97	**139.97**
6	0.1	0.3	146.36	145.13	143.91	142.68	141.45	140.23
6	0.2	0.3	152.42	150.25	148.08	145.92	143.75	141.58
6	0.1	0.4	145.26	144.40	143.53	142.67	141.81	140.95
6	0.3	0.4	153.75	151.49	149.23	146.98	144.72	142.46
6	0.1	0.5	144.80	144.14	143.47	142.81	142.15	141.49
6	0.4	0.5	**154.59**	152.27	149.96	147.65	145.33	143.02

that risk aversion has an impact on the life insurance decision near retirement. A more risk averse family purchases more life insurance to protect its financial vulnerability. In general, the presence of habit formation increases the demand for life insurance, yet the impact is marginal at earlier stages. However, a high consumption habit level can also decrease the demand for life insurance compared to the case without habit formation.

Notably, in our model, the family allocates resources among consumption and legacy across periods (as reflected in the budget constraint). In the absence of habit formation, there are two types of trade-offs. The first type is that each member within the family needs to trade immediate consumption against future consumption. The second is the bequest motive that the breadwinner needs to trade off his/her lifetime consumption against legacy, which is linked to the surviving member's consumption upon his/her death. Habit formation introduces the third type: each member trades off immediate consumption against the effects of early consumption on the standard of living in the later life. In the presence of (linear) habit formation, the breadwinner must ensure that the family's future consumption does not fall below the habit level. On the one hand, habit formation increases the bequest motive as the breadwinner needs to ensure the legacy is enough to finance the family's consumption above the habit level upon his/her death, thus increasing the demand for life insurance. On the other hand, the breadwinner must maintain enough financial wealth (plus future labor income) to finance the family's subsistence before his/her death, reducing the bequest motive and thus the demand for life insurance. The overall effect of habit formation on the bequest motive determines its effect on the demand for life insurance.

The above arguments can be made concrete by revisiting (15) and (16). Let

$$\phi(t) = \frac{(\kappa_2 D(t))^{\frac{1}{\gamma}} H(t)}{H_\gamma(t)}, \qquad \phi_i(t) = \phi(t) G_i(t), \; i = 1, 2.$$

Then the optimal legacy (15) with the habit formation can be rewritten as

$$Z_t^* = G(t)h_{2t}^* + \phi(t)X_t^* + \phi(t)L(t) - \phi_1(t)h_{1t}^* - \phi_2(t)h_{2t}^*. \tag{17}$$

The quantity $G(t)h_{2t}^*$ corresponds to the least capital required by the family to maintain minimum subsistence upon the breadwinner's death and $G(t)$ measures the (marginal) bequest motive increased by the family's consumption habit. $G_1(t)h_{1t}^*$ and $G_2(t)h_{2t}^*$ are the least capital required by the breadwinner and the rest of the family to maintain minimum subsistence before the breadwinner's death. Thus, $\phi_1(t)$ and $\phi_2(t)$ measure, respectively, the (marginal) bequest motive decreased by the family's consumption habits. The net effect of h_{2t}^* is measured by $G(t) - \phi_2(t)$. Recall that the optimal face value of the life insurance is given by the difference between the legacy and wealth, i.e., $\frac{I_t^*}{\lambda_y(t)} = Z_t^* - X_t^*$. Hence the life insurance demand is driven by wealth and future labor income with their impacts being measured by $\phi(t) - 1$ and $\phi(t)$, respectively.

Similarly, by setting $\alpha = \beta = h_{10} = h_{20} = 0$ in (15) or equivalently (17), the optimal legacy in the absence of habit formation becomes

$$Z_t^* = \psi(t)X_t^* + \psi(t)L(t),$$

where $\psi(t)$ is defined similarly as $\phi(t)$ but with the addition conditions that $\alpha = \beta = h_{10} = h_{20} = 0$. Analogously, $\psi(t) - 1$ and $\psi(t)$ measure, respectively, the impacts of wealth and future labor income on the demand for life insurance. Hence comparing $\phi(t)$ with $\psi(t)$ provides additional insights on the impact of habit formation.

Figure 1 plots $\phi(t)$ and $\psi(t)$ over the life cycle, assuming that $y = 30, \alpha = 0.1, \beta = 0.2$, and $\gamma = 4$ (the medium risk aversion). Both $\phi(t)$ and $\psi(t)$ exhibit hump-shaped, starting from around 0.52 at age 30, reaching maximum at around 0.68, decreasing to 0.55 at later ages. The values of $\phi(t)$ and $\psi(t)$ imply that the family will purchase more life insurance as the labor income increase and the increased face value is a proportion of the increase in the present value of the future labor income, which is consistent with the intuition that life insurance protects the family against the loss of the labor income. In contrast, because both values are less than one, a wealthier family will purchases less life insurance, that is in line with the previous studies such as Inkmann et al. (2010). The presence of habit formation increases the bequest motive resulting from labor income and wealth. Moreover, habit formation has a relatively small impact in the early ages.

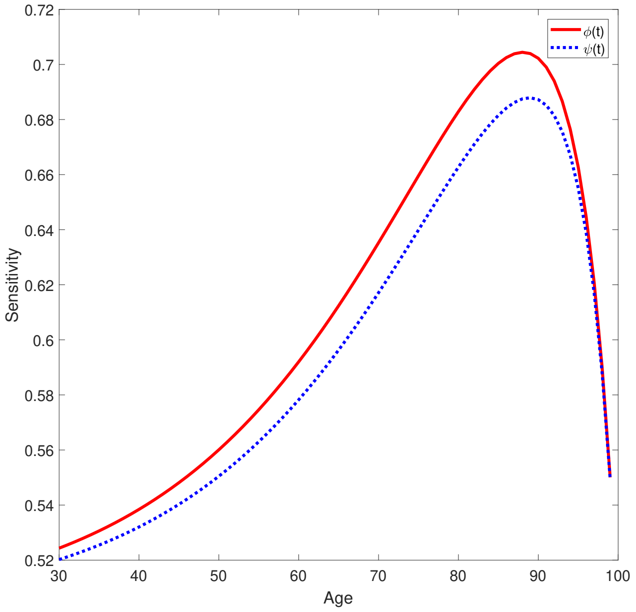

Figure 1: Sensitivities of Wealth and Labor Income. Red solid line: $\phi(t)$; blue dotted line: $\psi(t)$. Parameter values: $y = 30, \alpha = 0.1, \beta = 0.2, \gamma = 4$. All other parameters are as of Section 4.

Figure 2 plots $G(t), \phi_1(t), \phi_2(t)$, and $G(t) - \phi_2(t)$ over the life cycle. Because $G(t) - \phi_2(t)$ is always smaller than $\phi_1(t)$, the demand for life insurance decreases as the levels of the family's consumption habits rise. Moreover, this effect is stronger as the breadwinner approaches retirement. When the consumption habits are low, the effect of wealth and labor income dominates and we see the increase in the life insurance demand. When the consumption habits are high and/or the breadwinner is near retirement, the effect of consumption habits dominate and we see the decrease in the life insurance demand. This explains the ambiguous findings in the preceding examples.

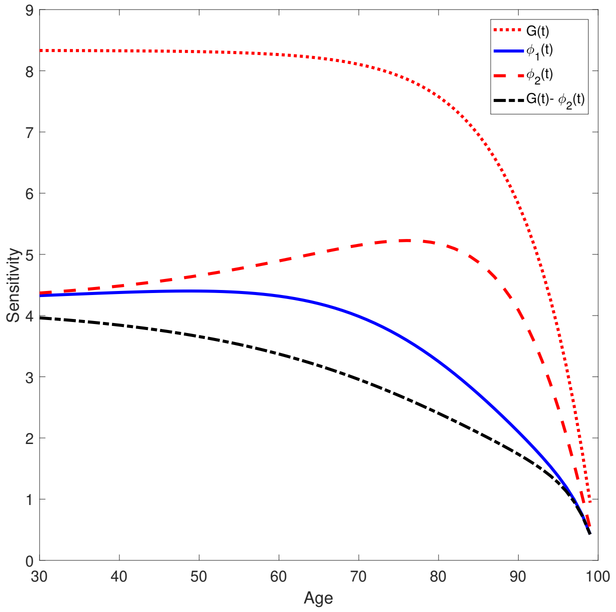

Figure 2: Sensitivities of Consumption Habits. Red dotted line: $G(t)$; blue solid line: $\phi_1(t)$; red dashed line: $\phi_2(t)$; black dash-dot line: $G(t) - \phi_2(t)$. Parameter values: $y = 30, \alpha = 0.1, \beta = 0.2, \gamma = 4$. All other parameters are as of Section 4.

6 Demand for Annuities

This section assesses the impact of habit formation on the demand for annuities. Because annuities help the family to alleviate the risk of outliving their financial wealth, we focus on the breadwinner's post-retirement phase. As in the preceding section, we consider two realistic scenarios, labelled as Scenario C and Scenario D. For Scenario C, we assume the breadwinner is $y = 65$ years old, i.e., just at retirement, and currently has $x = \$200,000$ saved. The family horizon is $T = 35$ years. For Scenario D, we set $y = 75$ (i.e. the breadwinner has already been retired for 10 years), $x = \$250,000$, and $T = 25$. Table 2 displays the optimal capital used for annuity purchase (i.e. $-\frac{I_t}{\lambda_y(t)}$) for a family with different combinations of γ, α, β, and h and for Scenario C and Scenario D.

Table 2 indicates that for Scenario C the family will optimally purchase around $\$80,000$ of annuity, which is about 40% of the savings. Habit formation can decrease the amount of annuity purchased by at most 7.95% (compare 72.74 from $\gamma = 6, \alpha = 0.4, \beta = 0.5, h = 0$, to 79.02, the case without habit formation). However, a very high habit level can increase the annuity demand by 10.36% (compare 87.17 from $\gamma = 6, \alpha = 0.1, \beta = 0.2, h = 10$, to 79.02, the case without habit formation). Moreover, when the family's risk aversion γ increases from 2 to 6, the capital used for annuity purchase decreases by 9.66% (compare 72.74 from $\gamma = 6, \alpha = 0.4, \beta = 0.5, h = 0$ to 80.51 from $\gamma = 2, \alpha = 0.4, \beta = 0.5, h = 0$). Appendix C provides some additional discussions on how risk-aversion affects the optimal life insurance/annuity purchase.

For Scenario D with older breadwinner, the family will optimally purchase around $\$90,000$ of annuity, which is about 36% of the savings. Habit formation can decrease the amount of annuity purchased by at most 11.44% (compare 78.08 from $\gamma = 6, \alpha = 0.4, \beta = 0.5, h = 0$, to 88.17, the case without habit formation). However, a very high habit level can increase the annuity demand by 7.11% (compare 94.44 from $\gamma = 6, \alpha = 0.1, \beta = 0.3, h = 10$, to 88.17, the case without habit formation). Moreover, when the family's risk aversion γ increases from 2 to 6, the capital used for annuity purchase decreases by 11.06% (compare 78.08 from $\gamma = 6, \alpha = 0.4, \beta = 0.5, h = 0$ to 87.80 from $\gamma = 2, \alpha = 0.4, \beta = 0.5, h = 0$). The lowest annuitization rate is 31.23% (78.08 from $\gamma = 6, \alpha = 0.4, \beta = 0.5, h = 0$).

To conclude, we note that the annuitization rate is low (less than 50%) when there is no habit formation. However, the annuitization is even lower when there is a habit formation. The endogenously-updated bequest function gives rise to great bequest motive, thus reducing the demand for annuity. In general, high consumption habits decrease the demand for annuity while low consumption habits increase the demand. Moreover, a more risk averse family purchases less annuity.

Notably, because the purchase of a term annuity can be viewed as selling a term insurance, the arguments in Section 5 carry over to the case of annuities. The values of $\phi(t)$ and $\psi(t)$ imply that a wealthier family will purchases more annuities, which is consistent with the observation in Inkmann et al. (2010). The presence of habit formation increases the bequest motive resulted from labor income and wealth and thus reduces the demand for annuity. As $G(t) - \phi_2(t)$ is always smaller than $\phi_1(t)$, the demand for annuity increases as the levels of the family's consumption habits rise. Moreover, this effect is stronger near the breadwinner retirement. When the consumption habits are low, the effect of wealth dominates and we observe the decrease in the annuity demand. When the consumption habits are high, the effect of consumption habits dominate and we see the increase in the life annuity demand. This explains the ambiguous findings in the preceding examples.

7 Life-Cycle Pattern

In this section, we present and interpret the (expected) optimal strategies over the life cycle based on the analytic formulas derived in Section 3.2. Unless otherwise stated, all parameters are as in Section 4.

7.1 Annuity Demand for Retiree

We first consider a 65-year-old retiree (i.e. Scenario C) and examine his expected annuity choice. The situation is similar to Davidoff et al. (2005) who study the annuity choice for a retiree with internal habit formation albeit the utility is of the habit-ratio specification and the model is in discrete time. They conclude that in a complete annuity market in which the retiree can choose any annuity payout trajectory, full annuitization is always optimal; while in an incomplete market in which the retiree is constrained to purchase a constant real annuity, full annuitization is optimal if the level of initial habit is low and a very high initial habit can reduce the annuitization rate to two-third. Our model also differs from them in the sense that we include the bequest motive which is endogenously updated by the family's consumption habits. Nevertheless, some of the intuition and results carry over to our model.

We assume that the family currently has $x = \$200,000$ saved and $\alpha = 0.1, \beta = 0.2$, and $\gamma = 4$. Figure 3 plots the expected consumption and habit paths for the retiree with different initial habits. We do not plot the expected consumption path for the other member because it is very similar to that of the retiree. Figures 4 and 5 plot the expected capital for annuity purchase

Table 2: Optimal Capital Used for Annuity Purchase $-\frac{I_t}{\lambda_y(t)}$ (in Thousands)

γ	α	β	$h = 0$	$h = 2$	$h = 4$	$h = 6$	$h = 8$	$h = 10$
		Scenario C with $y = 65, T = 35, x = 200,000$						
2	No habit formation		84.71					
2	0.1	0.2	82.39	83.68	84.97	86.27	87.56	88.85
2	0.1	0.3	83.73	84.79	85.85	86.92	87.98	89.04
2	0.2	0.3	81.40	82.84	84.28	85.73	87.17	88.61
2	0.1	0.4	84.19	85.01	85.82	86.64	87.45	88.27
2	0.3	0.4	80.86	82.38	83.90	85.43	86.95	88.48
2	0.1	0.5	84.39	85.04	85.69	86.34	86.99	87.64
2	0.4	0.5	**80.51**	82.09	83.66	85.24	86.81	88.39
6	No habit formation		**79.02**					
6	0.1	0.2	75.59	77.91	80.22	82.54	84.86	**87.17**
6	0.1	0.3	77.53	79.13	80.74	82.34	83.94	85.55
6	0.2	0.3	74.10	76.64	79.18	81.72	84.27	86.81
6	0.1	0.4	78.22	79.40	80.58	81.76	82.93	84.11
6	0.3	0.4	73.27	75.94	78.60	81.27	83.93	86.60
6	0.1	0.5	78.52	79.45	80.37	81.30	82.22	83.15
6	0.4	0.5	**72.74**	75.48	78.23	80.98	83.72	86.47
		Scenario D with $y = 75, T = 25, x = 250,000$						
2	No habit formation		94.62					
2	0.1	0.2	91.01	92.34	93.66	94.98	96.30	97.62
2	0.1	0.3	92.80	94.11	95.42	96.73	98.04	99.35
2	0.2	0.3	89.36	90.86	92.36	93.86	95.36	96.86
2	0.1	0.4	93.58	94.67	95.77	96.87	97.97	99.07
2	0.3	0.4	88.41	90.01	91.62	93.22	94.82	96.42
2	0.1	0.5	93.95	94.87	95.79	96.71	97.62	98.54
2	0.4	0.5	**87.80**	89.47	91.13	92.80	94.47	96.14
6	No habit formation		**88.17**					
6	0.1	0.2	82.92	85.11	87.31	89.50	91.69	93.89
6	0.1	0.3	85.48	87.27	89.06	90.86	92.65	**94.44**
6	0.2	0.3	80.45	82.91	85.37	87.83	90.29	92.75
6	0.1	0.4	86.60	88.03	89.45	90.88	92.31	93.73
6	0.3	0.4	79.02	81.64	84.25	86.87	89.48	92.10
6	0.1	0.5	87.16	88.32	89.49	90.65	91.82	92.98
6	0.4	0.5	**78.08**	80.80	83.52	86.23	88.95	91.66

and expected annuity flow, respectively. Without habit formation, the expected consumption rate decreases over time. In the presence of habit formation, current consumption increases the subsistence level and thus the marginal utility in the future. When the initial habit is low ($h = 0$), the habit level gradually increases and the family defers consumption, making annuity more desirable. In contrast, if the initial habit is high ($h = 10$) such that the habit level must decrease over time to ensure that the family is able to finance the minimum subsistence level in the future, habit formation pushes the consumption earlier, thereby reducing the demand for annuities in the future. We complement Davidoff et al. (2005)'s result by showing that even in a complete annuity market, habit formation can still reduce the demand for annuities in the presence of a bequest motive.

We emphasize that the above result is not contrary to the claim in Section 6 that a high consumption habit level increases the immediate annuity demand. In fact, as can be seen from Figures 4 and 5, the high initial habits increase the initial annuity demand, but the subsequent low habits (and thus low consumption rates) reduce the annuity demand.

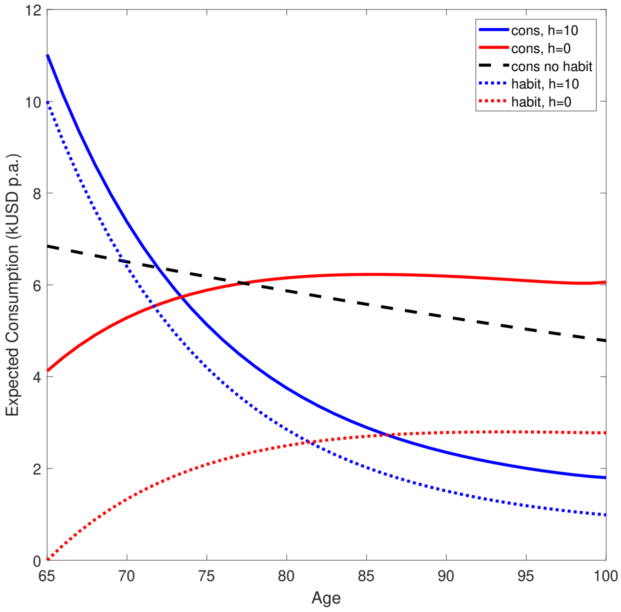

Figure 3: Expected Consumption of the Breadwinner (US$ p.a.). The blue solid line shows the expected consumption path of the breadwinner with habit formation and initial habit level $h = 10$. The blue dotted line shows the corresponding expected path of the habit level. The red solid line shows the expected consumption path of the breadwinner with habit formation and initial habit level $h = 0$. The red dotted line shows the corresponding expected path of the habit level. The black dashed line depicts the expected consumption path of the breadwinner without habit formation. All other parameters are equal.

7.2 Life Insurance/Annuity over the Life Cycle

We now explore to what extent our model can address the mismatches in the life insurance and annuity market by considering Scenario A and with $\alpha = 0.1$, $\beta = 0.2$, and $\gamma = 4$. These

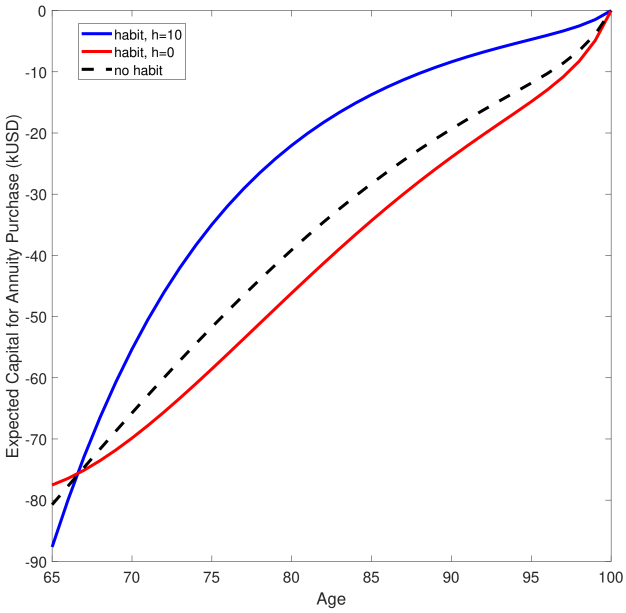

Figure 4: Expected Capital for Annuity Purchase (- Value) (US$ in thousand). The blue solid line shows the expected capital for annuity purchase (- value) with habit formation and initial habit level $h = 10$. The red solid line shows the value with habit formation and initial habit level $h = 0$. The black dotted line shows the value without habit formation. All other parameters are equal.

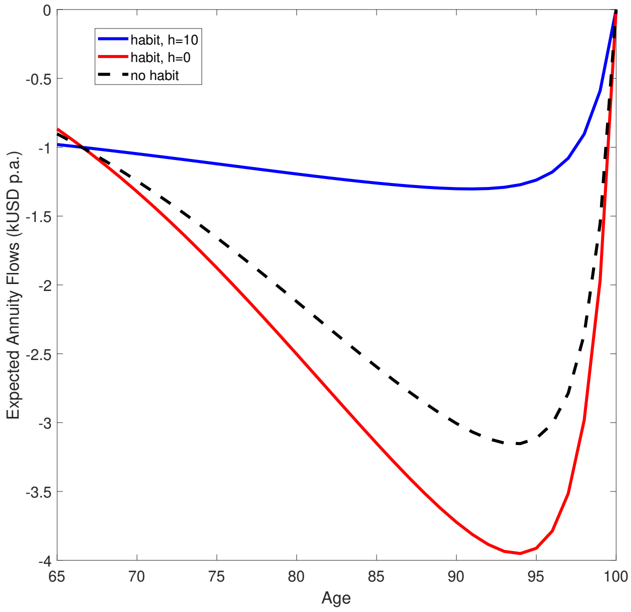

Figure 5: Expected Annuity Flows (- Value) (US$ in thousand p.a.). The blue solid line shows the expected annuity payout (- value) with habit formation and initial habit level $h = 10$. The red solid line shows the value with habit formation and initial habit level $h = 0$. The black dotted line shows the value without habit formation. All other parameters are equal.

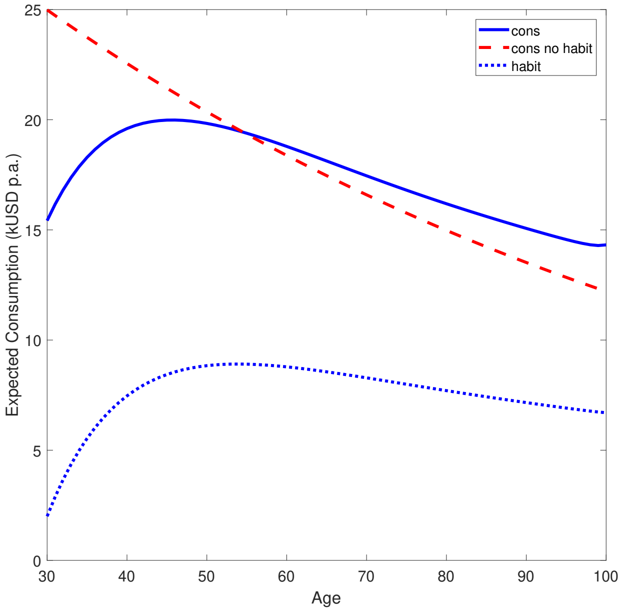

Figure 6: Expected Consumption of the Breadwinner (US$ in thousand p.a.). The blue solid line shows the expected consumption path of the breadwinner with habit formation. The blue dotted line shows the corresponding expected path of the habit level. The red dashed line depicts the expected consumption path of the breadwinner without habit formation. All other parameters are equal.

numbers are similar to those considered in Kraft et al. (2017). Figure 6 plots the expected consumption and habit paths for the 30-year-old breadwinner. In the absence of habit formation, the consumption path is decreasing over life, attributing to the high discount rate. Habit formation has a dramatic effect on the expected consumption path including the middle-life consumption hump often observed empirically. Therefore, we extend the result in Kraft et al. (2017) to the case with bequest motive and life insurance/annuity purchases.

Figure 7 reports the expected face value of life insurance (positive value)/capital for annuity purchase (negative value). The breadwinner will purchase life insurance of $\$500,000$ in the early age, which decreases as the human capital decreases. He/She begins to purchase annuity in his/her late 50s and the annuity demand is the highest at the age of retirement. This demand then decreases as the family approaches the planning horizon. Figure 8 depicts the expected insurance premium (positive value)/annuity payout (negative value). The life insurance premium starts at a low level as the mortality rate is very small at young ages which more than offsets the required huge face value of the life insurance. Despite the fact that the required face value of the life insurance decreases, the increasing mortality rate pushes up the premium. The face value and the premium decrease as the breadwinner approaches retirement and eventually he/she starts to receive annuity payout. Similar to Purcal and Piggott (2008), we find a kink in the annuity payout at retirement. The annuity payout increases dramatically until the age of 90, as attribute to the increasing mortality rate.

The presence of habit formation reduces the demand for life insurance throughout the pre-retirement period. However, in this case, habit formation also increases the annuity purchase over the life cycle. The difference between this observation and the result in Section 7.1 lies in

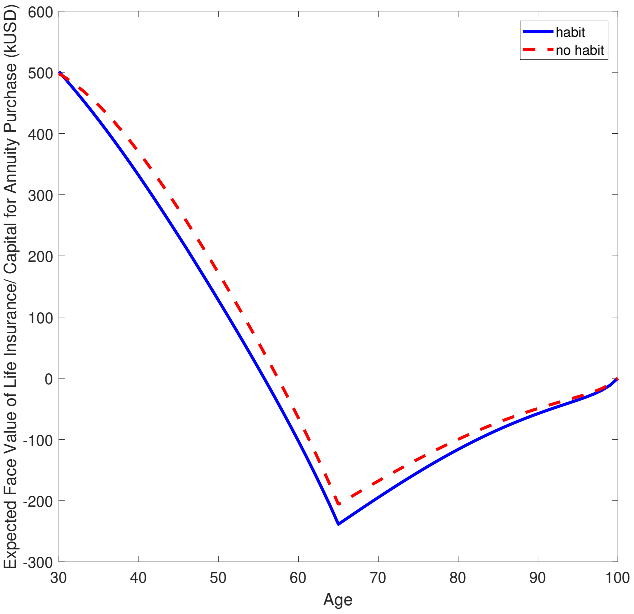

Figure 7: Expected Face Value of Life Insurance (+ Value)/Capital for Annuity Purchase (- Value) (US$ in thousand). The blue solid line shows the expected face value of life insurance (+ value)/capital for annuity purchase (- value) with habit formation. The blue dotted line shows the value without habit formation.

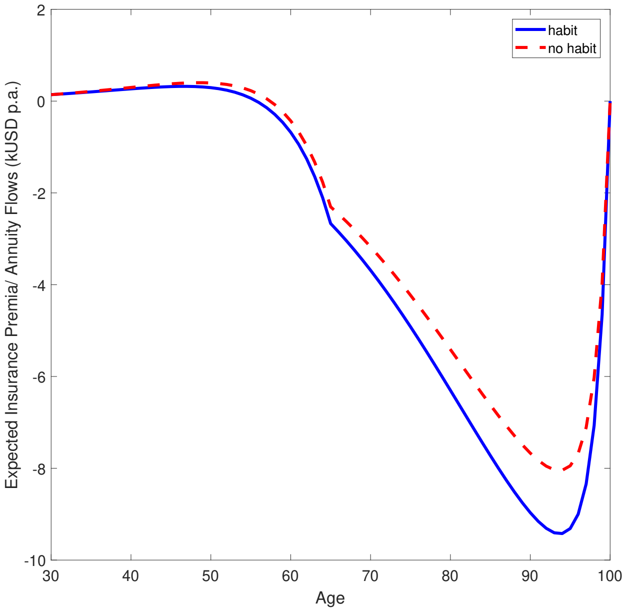

Figure 8: Expected Insurance Premia (+ Value)/Annuity Flows (- Value) (US$ in thousand p.a.). The blue solid line shows the expected life insurance premium (+ value)/annuity payout (- value) with habit formation. The blue dotted line shows the value without habit formation. All other parameters are equal.

the fact that when the planning horizon is long, the family tries to smooth the future consumption and the consumption rate does not drop dramatically in the old ages. In fact, the expected consumption rate is even higher in the presence of habit formation during retirement, thereby requiring more annuity. In contrast, a retiree with a very high living standard has to lower the consumption rate and habit and the consumption rate is lower than that without habit formation, reducing the annuity demand.

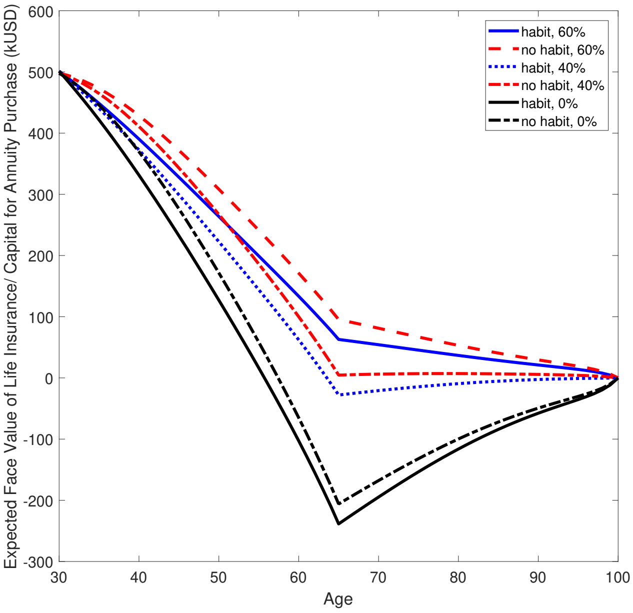

Figure 9: Expected Face Value of Life Insurance (+ Value)/Capital for Annuity Purchase (- Value) (US$ in thousand). The blue solid line shows the expected face value of life insurance (+ value)/capital for annuity purchase (- value) with habit formation and 60% replacement ratio. The red dashed line shows the value with 60% replacement ratio but without habit formation. The blue dotted line shows the value with habit formation and 40% replacement ratio. The red dash-dot line shows the value with 40% replacement ratio but without habit formation. The black solid line shows the value with habit formation but without social security. The black dash-dot shows the value without habit formation or social security. The initial wage is modified such that the present value of future income at the beginning of the life cycle is unchanged. All other parameters are equal.

7.3 Impact of Social Security

In the previous analysis, the family loses all labor income upon the breadwinner's retirement. We relax this assumption in this section by examining the role of social security. Purcal and Piggott (2008) document that as social security becomes more important, the household will buy more life insurance and less annuities, and will postpone the annuity purchase, because social security provides income after retirement but does not carry any survivor's benefit. Moreover, the motive for voluntary annuitization diminishes completely when social security is set at 50% of the pre-retirement income. However, there are at least two limitations. First, the increase in the life insurance demand is contrary to the mismatch in the life insurance market. Second, the family is likely to purchase life insurance at very old ages, which is not realistic at all.

Our treatment of social security is similar to that of Purcal and Piggott (2008). We allow the family to receive a constant income equal to a fixed proportion (the "replacement ratio") of the wage at retirement. The initial wage is reduced such that the present value of future income at the beginning of the life cycle is unchanged. Figure 9 reports the expected face value of life insurance (positive value)/capital for annuity purchase (negative value) with different replacement ratio. Figure 10 depicts the corresponding expected insurance premium (positive value)/annuity payout (negative value). Social security increases the demand for life insurance and decrease the demand for annuity, which is in line with the result in Purcal and Piggott (2008). With a 60% replacement ratio, there is no annuitization outside of social security but the family also purchases significantly more life insurance even at very old ages when the premium rate is formidable to afford. Habit formation addresses these problem. In the presence of habit formation, the family with a 60% replacement ratio purchases less life insurance over the life cycle. The breadwinner purchases even less life insurance in early ages and starts to annuitize slightly at retirement when the replacement ratio is reduced to 40%. In particular, Figure 10 shows that the the expected insurance premium (positive value)/ annuity payout (negative value) is highly limited throughout the life when there is habit formation and the replacement ratio is 40%.[7]

Table 3: Expected Insurance Premia (+ Value)/Annuity Flows (- Value) (US$ in thousand p.a.) Columns 2, 4, and 6 report the expected life insurance premium (+ value)/annuity payout (-value) at different ages with habit formation and $0\%, 40\%$, and 60% replacement ratio, respectively. Columns 3, 5, and 7 report corresponding values without habit formation

	0%		40%		60%	
Age	habit	no habit	habit	no habit	habit	no habit
40	0.27	0.30	0.30	0.33	0.31	0.34
50	0.29	0.40	0.51	0.62	0.61	0.71
60	-0.68	-0.43	0.41	0.66	0.88	1.13
70	-3.69	-3.18	-0.39	0.12	1.03	1.54
80	-6.30	-5.41	-0.51	0.38	1.99	2.88
90	-8.97	-7.67	-0.41	0.89	3.28	4.58

8 Conclusion

This paper explored the role of habit formation in the household's demand for life-contingent claims, namely life insurance and annuities. We proposed a continuous time life-cycle model featuring consumption, investment and life-contingent claims. The bequest motive was endogenously modeled as the utility of the survivor's future consumption which depends on not only the bequest but also the living standard. We solved the life-cycle model analytically and concluded habit formation has a large impact on the demand for life-contingent claims. We

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[7] As mentioned in Maurer et al. (2013), 40% replacement ratio is in accordance with OECD (2012) report for the average U.S. earner of 39.4% percent.

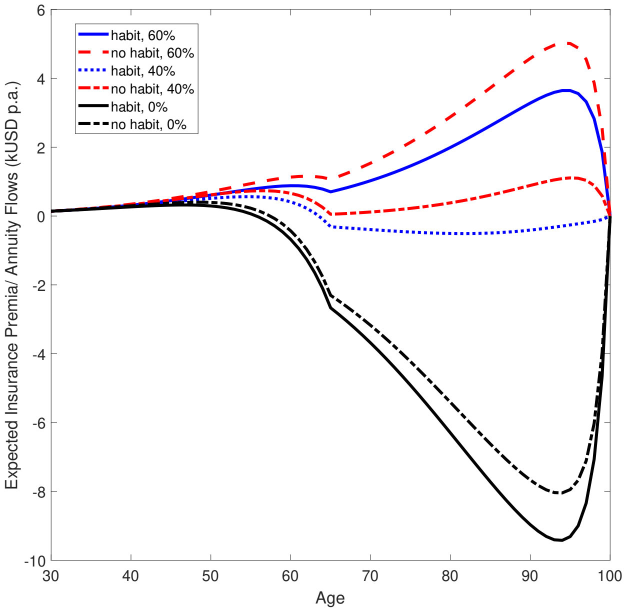

Figure 10: Expected Insurance Premia (+ Value)/Annuity Flows (- Value) (US$ in thousand p.a.). The blue solid line shows the expected life insurance premium (+ value)/annuity payout (- value) with habit formation and 60% replacement ratio. The red dashed line shows the value with 60% replacement ratio but without habit formation. The blue dotted line shows the value with habit formation and 40% replacement ratio. The red dash-dot line shows the value with 40% replacement ratio but without habit formation. The black solid line shows the value with habit formation but without social security. The black dash-dot shows the value without habit formation or social security. The initial wage is modified such that the present value of future income at the beginning of the life cycle is unchanged. All other parameters are equal.

demonstrated that the social security together with habit formation further reduces the demand for life insurance market and the voluntary annuity market outside of social security. This is consistent with what is observed empirically though our analysis is based on our hypothetical insurance/annuity market.

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A Martingale Approach

In this appendix, we illustrate that the dynamic optimization problem (8) can be solved by considering its martingale formulation (9) under some technical conditions. The proof essentially follows from Chapter 5 of Ye (2006).

We say a consumption, portfolio, and insurance strategy (c_1, c_2, π, I) is admissible if

1. c_{it} is non-negative, progressively measurable, and satisfies $\int_0^T c_{it} dt < \infty$, a.s., $i = 1, 2$;
2. π_t is progressively measurable and satisfies $\int_0^T \pi_t^2 dt < \infty$, a.s.;
3. I_t is progressively measurable and satisfies $\int_0^T |I_t| dt < \infty$, a.s.;
4. the corresponding wealth X satisfies $X_t + L(t) \geq 0$, a.s., a.e. $t \in [0, T]$;
5. the corresponding legacy Z satisfies $Z_t \geq 0$, a.s., a.e. $t \in [0, T]$.

Denote by $\mathcal{A}$ the set of all admissible strategies. We first show that the static budget constraint in (9) is satisfied for any admissible strategy $(c_1, c_2, \pi, I) \in \mathcal{A}$. From Ito's lemma, we have

$$d(\xi_t X_t) = -\xi_t \left(c_{1t} + c_{2t} + \lambda_y(t) Z_t - w(t)\right) dt + \xi_t(\sigma \pi_t - \theta X_t) dW_t,$$

and

$$\xi_t X_t = x - \int_0^t \xi_s \left(c_{1s} + c_{2s} + \lambda_y(s) Z_s - w(s)\right) ds + \int_0^t \xi_s(\sigma \pi_s - \theta X_s) dW_s.$$

Noting that

$$L(t) = \frac{1}{\xi_t} \mathbb{E}_t \left[\int_t^T w(s) \xi_s ds\right],$$

we have

$$\xi_t L(t) + \int_0^t w(s) \xi_s ds = \mathbb{E}_t \left[\int_0^T w(s) \xi_s ds\right],$$

and thus

$$\xi_t \left(X_t + L(t)\right) + \int_0^t \xi_s \left(c_{1s} + c_{2s} + \lambda_y(s) Z_s\right) ds = x + \mathbb{E}_t \left[\int_0^T w(s) \xi_s ds\right] + \int_0^t \xi_s(\sigma \pi_s - \theta X_s) dW_s. \tag{18}$$

From the definition of $\mathcal{A}$, the LHS of (18) is non-negative. Moreover, the RHS of (18) is a local martingale so it is a non-negative local martingale and hence a super-martingale. Therefore,

$$\mathbb{E}\left[\int_0^T \xi_s \left(c_{1s} + c_{2s} + \lambda_y(s) Z_s\right) ds\right] \leq x + \mathbb{E}\left[\int_0^T w(s) \xi_s ds\right].$$

Next, we show that if the consumption and legacy processes (c_1, c_2, Z) satisfy

$$\mathbb{E}\left[\int_0^T \xi_s \left(c_{1s} + c_{2s} + \lambda_y(s) Z_s\right) ds\right] = x + \mathbb{E}\left[\int_0^T w(s) \xi_s ds\right], \tag{19}$$

then there exist portfolio and insurance processes (π, I) that satisfy the wealth dynamics

$$dX_t = [rX_t + (\mu - r)\pi_t - c_{1t} - c_{2t} - I_t + w(t)]dt + \sigma\pi_t dW_t,\ X(0) = x, \tag{20}$$

provided that $x + L(0) \geq 0$.

In fact, from (19), we know that $\mathbb{E}_t\left[\int_0^T \xi_s \left(c_{1s} + c_{2s} + \lambda_y(s)Z_s - w(s)\right) ds\right]$ is a martingale. From the martingale representation theorem, there exists a progressively measurable process ζ such that

$$\mathbb{E}_t\left[\int_0^T \xi_s \left(c_{1s} + c_{2s} + \lambda_y(s)Z_s - w(s)\right) ds\right] = x + \int_0^t \zeta_s dW_s.$$

Define

$$X_t = \frac{1}{\xi_t}\mathbb{E}_t\left[\int_t^T \xi_s \left(c_{1s} + c_{2s} + \lambda_y(s)Z_s - w(s)\right) ds\right].$$

We have

$$\begin{aligned}\xi_t X_t =& \mathbb{E}_t\left[\int_t^T \xi_s \left(c_{1s} + c_{2s} + \lambda_y(s)Z_s - w(s)\right) ds\right]\\ =& x + \int_0^t \zeta_s dW_s - \int_0^t \xi_s \left(c_{1s} + c_{2s} + \lambda_y(s)Z_s - w(s)\right) ds,\end{aligned}$$

and

$$d(\xi_t X_t) = -\xi_t \left(c_{1t} + c_{2t} + \lambda_y(t)Z_t - w(t)\right) dt + \zeta_t dW_t.$$

From Ito's lemma, we have

$$dX_t = -\left(c_{1t} + c_{2t} + \lambda_y(t)Z_t - w(t)\right) dt + X_t\left(r + \lambda_y(t) + \theta^2\right) dt + \frac{\zeta_t}{\xi_t}\theta dt + \frac{\zeta_t}{\xi_t}dW_t + X_t\theta dW_t.$$

It is straightforward to verify that (20) is satisfied with $I_t = (Z_t - X_t)\lambda_y(t)$ and

$$\pi_t = \frac{\frac{\zeta_t}{\xi_t} + \theta X_t}{\sigma}.$$

The above analysis implies that we can solve the dynamic optimization problem (8) by solving the static optimization problem (9). Once we obtain the optimal solution to (9), we can derive the optimal solution to (8) via the martingale representation theorem.

B Proofs

B.1 Proof of Lemma 3.2

Proof of Lemma 3.2.

$$\begin{aligned}
&\mathbb{E}[\int_0^{\min\{T,\tau\}} e^{-\delta t}\big(\kappa_1 U_1(c_{1t}-h_{1t})+\kappa_2 U_2(c_{2t}-h_{2t})\big)dt+\kappa_2 e^{-\delta\tau}V_2(\tau,Z_\tau,h_{2\tau})1_{\tau<T}]\\
=&\mathbb{E}[\int_0^T 1_{t\le\tau}e^{-\delta t}\big(\kappa_1 U_1(c_{1t}-h_{1t})+\kappa_2 U_2(c_{2t}-h_{2t})\big)dt+\kappa_2 e^{-\delta\tau}V_2(\tau,Z_\tau,h_{2\tau})1_{\tau<T}]\\
=&\mathbb{E}[\mathbb{E}[\int_0^T 1_{t\le\tau}e^{-\delta t}\big(\kappa_1 U_1(c_{1t}-h_{1t})+\kappa_2 U_2(c_{2t}-h_{2t})\big)dt+\kappa_2 e^{-\delta\tau}V_2(\tau,Z_\tau,h_{2\tau})1_{\tau<T}|\mathcal{F}_0]]\\
=&\mathbb{E}[\int_0^T \bar{F}(t)e^{-\delta t}\big(\kappa_1 U_1(c_{1t}-h_{1t})+\kappa_2 U_2(c_{2t}-h_{2t})+\kappa_2\lambda_y(t)V_2(t,Z_t,h_{2t})\big)dt]\\
=&\mathbb{E}[\int_0^T e^{-\delta t-\int_0^t\lambda_y(u)du}\big(\kappa_1 U_1(c_{1t}-h_{1t})+\kappa_2 U_2(c_{2t}-h_{2t})+\kappa_2\lambda_y(t)V_2(t,Z_t,h_{2t})\big)dt]\\
=&\mathbb{E}[\int_0^T D(t)\big(\kappa_1 U_1(c_{1t}-h_{1t})+\kappa_2 U_2(c_{2t}-h_{2t})+\kappa_2\lambda_y(t)V_2(t,Z_t,h_{2t})\big)dt].
\end{aligned}$$

□

B.2 Proof of Lemma 3.3

Proof of Lemma 3.3. We first show (11). For a given consumption process c_{1t}, the habit level h_{1t} satisfies

$$dh_{1t}=-(\beta_1 h_{1t}-\alpha_1 c_{1t})dt.$$

Because

$$dh_{1t}=-\big((\beta_1-\alpha_1)h_{1t}-\alpha_1(c_{1t}-h_{1t})\big)dt,$$

we have

$$h_{1s}=e^{-(\beta_1-\alpha_1)(s-t)}h_{1t}+\alpha_1\int_t^s e^{-(\beta_1-\alpha_1)(s-u)}(c_{1u}-h_{1u})du.$$

Consequently,

$$\begin{aligned}
\mathbb{E}_t[\int_t^T c_{1s}\xi_s ds]=&\mathbb{E}_t[\int_t^T (c_{1s}-h_{1s})\xi_s ds]+\mathbb{E}_t[\int_t^T h_{1s}\xi_s ds]\\
=&\mathbb{E}_t[\int_t^T (c_{1s}-h_{1s})\xi_s ds]+\mathbb{E}_t[\int_t^T e^{-(\beta_1-\alpha_1)(s-t)}\xi_s ds]h_{1t}\\
&+\alpha_1\mathbb{E}_t[\int_t^T \xi_s\int_t^s e^{-(\beta_1-\alpha_1)(s-u)}(c_{1u}-h_{1u})duds].
\end{aligned}$$

By Fubini's Theorem,

$$\begin{aligned}
\mathbb{E}_t[\int_t^T \xi_s\int_t^s e^{-(\beta_1-\alpha_1)(s-u)}(c_{1u}-h_{1u})duds]&=\mathbb{E}_t[\int_t^T (c_{1u}-h_{1u})\int_u^T \xi_s e^{-(\beta_1-\alpha_1)(s-u)}dsdu]\\
&=\mathbb{E}_t[\int_t^T (c_{1s}-h_{1s})\int_s^T \xi_u e^{-(\beta_1-\alpha_1)(u-s)}duds]
\end{aligned}$$

$$
\begin{aligned}
&= \mathbb{E}_t\big[\int_t^T (c_{1s} - h_{1s}) \int_s^T e^{-(\beta_1-\alpha_1)(u-s)} \mathbb{E}_s[\xi_u] du ds\big] \\
&= \mathbb{E}_t\big[\int_t^T (c_{1s} - h_{1s}) \xi_s \int_s^T G_1(u; s) du ds\big].
\end{aligned}
$$

We then have

$$
\begin{aligned}
\mathbb{E}_t[\int_t^T c_{1s}\xi_s ds] =& \mathbb{E}_t[\int_t^T (c_{1s} - h_{1s})\xi_s ds] + \mathbb{E}_t[\int_t^T e^{-(\beta_1-\alpha_1)(s-t)}\xi_s ds] h_{1t} \\
&+ \alpha_1 \mathbb{E}_t\big[\int_t^T (c_{1s} - h_{1s})\xi_s \int_s^T G_1(u; s) du ds\big] \\
=& \mathbb{E}_t[\int_t^T (c_{1s} - h_{1s})\big(1 + \alpha_1 \int_s^T G_1(u; s) du\big)\xi_s ds] + h_{1t}\xi_t \int_t^T G_1(s; t) ds.
\end{aligned}
$$

Next, we prove (12). For a given consumption process c_{2t}, the habit level h_{2t} satisfies

$$
h_{2s} = e^{-(\beta_2-\alpha_2)(s-t)} h_{2t} + \alpha_2 \int_t^s e^{-(\beta_2-\alpha_2)(s-u)} (c_{2u} - h_{2u}) du.
$$

Consequently,

$$
\begin{aligned}
\mathbb{E}_t[\int_t^T c_{2s}\xi_s ds] =& \mathbb{E}_t[\int_t^T (c_{2s} - (1 + \lambda_y(s)G(s))h_{2s})\xi_s ds] + \mathbb{E}_t[\int_t^T (1 + \lambda_y(s)G(s))h_{2s}\xi_s ds] \\
=& \mathbb{E}_t[\int_t^T (c_{2s} - (1 + \lambda_y(s)G(s))h_{2s})\xi_s ds] \\
&+ \mathbb{E}_t[\int_t^T (1 + \lambda_y(s)G(s)) e^{-(\beta_2-\alpha_2)(s-t)}\xi_s ds] h_{2t} \\
&+ \alpha_2 \mathbb{E}_t[\int_t^T (1 + \lambda_y(s)G(s))\xi_s \int_t^s e^{-(\beta_2-\alpha_2)(s-u)} (c_{2u} - h_{2u}) du ds].
\end{aligned}
$$

By Fubini's Theorem,

$$
\begin{aligned}
&\mathbb{E}_t[\int_t^T (1 + \lambda_y(s)G(s))\xi_s \int_t^s e^{-(\beta_2-\alpha_2)(s-u)} (c_{2u} - h_{2u}) du ds] \\
=& \mathbb{E}_t[\int_t^T (c_{2u} - h_{2u}) \int_u^T (1 + \lambda_y(s)G(s))\xi_s e^{-(\beta_2-\alpha_2)(s-u)} ds du] \\
=& \mathbb{E}_t[\int_t^T (c_{2s} - h_{2s}) \int_s^T (1 + \lambda_y(u)G(u))\xi_u e^{-(\beta_2-\alpha_2)(u-s)} du ds] \\
=& \mathbb{E}_t\big[\int_t^T (c_{2s} - h_{2s}) \int_s^T (1 + \lambda_y(u)G(u)) e^{-(\beta_2-\alpha_2)(u-s)} \mathbb{E}_s[\xi_u] du ds\big] \\
=& \mathbb{E}_t\big[\int_t^T (c_{2s} - h_{2s})\xi_s \int_s^T (1 + \lambda_y(u)G(u)) G_2(u; s) du ds\big].
\end{aligned}
$$

We then have

$$\begin{aligned}
\mathbb{E}_t[\int_t^T c_{2s}\xi_s ds] =&\mathbb{E}_t[\int_t^T (c_{2s}-(1+\lambda_y(s)G(s))h_{2s})\xi_s ds]\\
&+\mathbb{E}_t[\int_t^T (1+\lambda_y(s)G(s))e^{-(\beta_2-\alpha_2)(s-t)}\xi_s ds]h_{2t}\\
&+\alpha_2\mathbb{E}_t\big[\int_t^T (c_{2s}-h_{2s})\xi_s\int_s^T (1+\lambda_y(u)G(u))G_2(u;s)duds\big]\\
=&\mathbb{E}_t[\int_t^T (c_{2s}-h_{2s})\big(1+\alpha_2\int_s^T (1+\lambda_y(u)G(u))G_2(u;s)du\big)\xi_s ds]\\
&+h_{2t}\xi_t\int_t^T (1+\lambda_y(s)G(s))G_2(s;t)ds-\mathbb{E}_t[\int_t^T \lambda_y(s)G(s)h_{2s}\xi_s ds].
\end{aligned}$$

□

B.3 Proof of Proposition 3.1

Proof of Proposition 3.1. Because $x+L(0)-h_{10}G_1(0)-h_{20}G_2(0)>0$, it is easy to see that there exists a unique $v^*>0$ that solves (14). Next, for any strategy $\{c_{1t},c_{2t},Z_t\}_{t\in[0,T]}$ and the associated habit levels $\{h_{1t},h_{2t}\}_{t\in[0,T]}$ that satisfy the budget constraint, we have

$$\begin{aligned}
&\mathbb{E}[\int_0^T D(t)\big(\kappa_1U_1(c_{1t}-h_{1t})+\kappa_2U_2(c_{2t}-h_{2t})+\kappa_2\lambda_y(t)V_2(t,Z_t,h_{2t})\big)dt]\\
\le&\mathbb{E}[\int_0^T D(t)\big(\kappa_1U_1(c_{1t}-h_{1t})+\kappa_2U_2(c_{2t}-h_{2t})+\kappa_2\lambda_y(t)V_2(t,Z_t,h_{2t})\big)dt]\\
&-v^*\big\{\mathbb{E}[\int_0^T (c_{1t}-h_{1t})(1+\alpha_1G_1(t))\xi_t dt]+\mathbb{E}[\int_0^T (c_{2t}-h_{2t})(1+\alpha_2G_2(t))\xi_t dt]\\
&+\mathbb{E}[\int_0^T \lambda_y(t)(Z_t-G(t)h_{2t})\xi_t dt]-x-L(0)+h_{10}G_1(0)+h_{20}G_2(0)\big\}\\
\le&\mathbb{E}[\int_0^T D(t)\big(\kappa_1U_1(c^*_{1t}-h^*_{1t})+\kappa_2U_2(c^*_{2t}-h^*_{2t})+\kappa_2\lambda_y(t)V_2(t,Z^*_t,h^*_{2t})\big)dt]\\
&-v^*\big\{\mathbb{E}[\int_0^T (c^*_{1t}-h^*_{1t})(1+\alpha_1G_1(t))\xi_t dt]+\mathbb{E}[\int_0^T (c^*_{2t}-h^*_{2t})(1+\alpha_2G_2(t))\xi_t dt]\\
&+\mathbb{E}[\int_0^T \lambda_y(t)(Z^*_t-G(t)h^*_{2t})\xi_t dt]-x-L(0)+h_{10}G_1(0)+h_{20}G_2(0)\big\}\\
=&\mathbb{E}[\int_0^T D(t)\big(\kappa_1U_1(c^*_{1t}-h^*_{1t})+\kappa_2U_2(c^*_{2t}-h^*_{2t})+\kappa_2\lambda_y(t)V_2(t,Z^*_t,h^*_{2t})\big)dt].
\end{aligned}$$

The expression for X^*_t can be obtained from the well-known relationship

$$X^*_t\xi_t=\mathbb{E}[\int_t^T (c^*_{1s}+c^*_{2s}+\lambda_y(s)Z^*_s)\xi_s ds|\mathcal{F}_t]-\xi_tL(t).$$

Applying Ito's lemma to X^*_t, we obtain the expression for π^*_t. □

B.4 Proof of Corollary 3.1

Proof of Corollary 3.1. The proof is straightforward and we omit it. □

B.5 Proof of Corollary 3.2

Proof of Corollary 3.2. Because

$$\begin{aligned} d\mathbb{E}[h_{it}^*] =& -(\beta_i \mathbb{E}[h_{it}^*] - \alpha_i \mathbb{E}[c_{it}^*])dt \\ =& -(\beta_i \mathbb{E}[h_{it}^*] - \alpha_i \mathbb{E}[h_{it}^*] - \alpha_i (\frac{v^*}{\kappa_i})^{-\frac{1}{\gamma_i}} (\frac{1+\alpha_i G_i(t)}{D(t)})^{-\frac{1}{\gamma_i}} \mathbb{E}[\xi_t^{-\frac{1}{\gamma_i}}])dt, \end{aligned}$$

we have

$$\mathbb{E}[h_{it}^*] = h_{i0} e^{-(\beta_i - \alpha_i)t} + \alpha_i \int_0^t (\frac{v^*}{\kappa_i})^{-\frac{1}{\gamma_i}} e^{-(\beta_i-\alpha_i)(t-s)} (\frac{1+\alpha_i G_i(s)}{\kappa_i D(s)})^{-\frac{1}{\gamma_i}} \mathbb{E}[\xi_s^{-\frac{1}{\gamma_i}}])ds.$$

The rest of the proof is straightforward and we omit it. □

C Effects of Risk-Aversion on Optimal Legacy

Figure 11 shows the effect of risk-aversion on $\phi(t)$, the sensitivity of optimal legacy to wealth or labor income. Because $\phi(t)$ is positive, a higher level of wealth or labor income will lead to a higher level of legacy. Moreover, $\phi(t)$ increases as γ increases, indicating that the effect of wealth/labor income is more pronounced for a more risk-averse family. We have assumed in Sections 5 and 6 that the family's consumption habits are equal. If the family increases consumption habits h_{1t}^* and h_{2t}^* simultaneously by 1, then the optimal legacy will decrease by $\phi_1(t) - (G(t) - \phi_2(t))$. Figure 12 implies that $\phi_1(t) - (G(t) - \phi_2(t))$ increases as γ increases from 2 to 6, so the effect of consumption habits is also more pronounced under a higher level of risk-aversion.

Holding X_t^*, $L(t)$, h_{1t}^*, and h_{2t}^* fixed, when γ increases, $\phi(t)X_t^* + \phi(t)L(t)$ increases but $G(t)h_{2t}^* - \phi_1(t)h_{1t}^* - \phi_2(t)h_{2t}^*$ decreases. To evaluate the net effect, we need to determine the magnitude of each term. For instance, Table 4 presents $\phi(t)$ and $\phi_1(t) - (G(t) - \phi_2(t))$ at ages 30 and 65. As can be seen from the table, the change in $\phi_1(t) - (G(t) - \phi_2(t))$ is around 15 times higher than the change in $\phi(t)$, as γ increases from 2 to 6. In our case studies, at age 30, the breadwinner's present value of labor income is more than \$700,000 when the initial wage is \$25,000 per year. At age 65, the family's wealth is at least \$200,000. In both cases, the habit level is at most \$10,000 per year. The effect of wealth/labor income outweighs the effect of habit and the required legacy increases as risk-aversion increase. Therefore, the demand for life annuity (insurance) decreases (increases) as γ increases.

D Additional Results with $\alpha \approx \beta$

This section provides additional numerical results for the case when α is approximately equal to β by setting $\alpha = 0.1$ and $\beta = 0.11$, and repeats some of the numerical examples. As β decreases from 0.2 to 0.11, the consumption habits are more persistent. As a counterpart of Figure 1, Figure 13 plots $\phi(t)$ and $\psi(t)$ but with $\alpha = 0.1, \beta = 0.11$. A comparison between

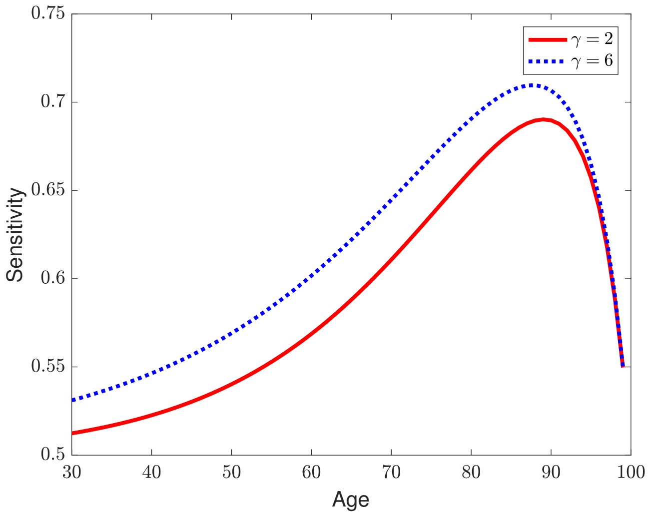

Figure 11: Sensitivities of Wealth and Labor Income, $\phi(t)$. Red solid line: $\gamma = 2$; blue dotted line: $\gamma = 6$.

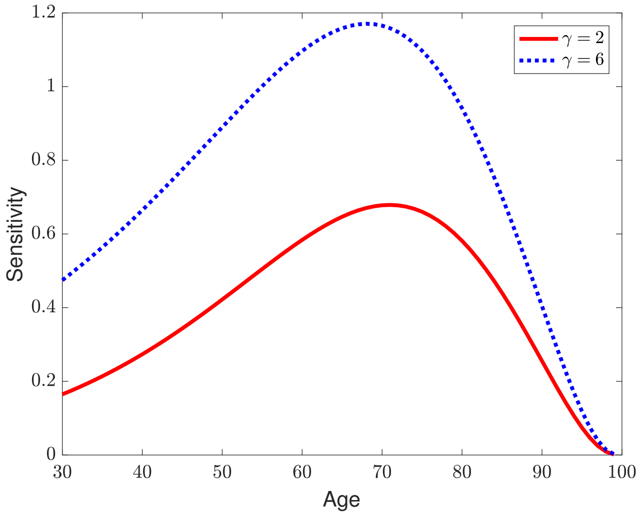

Figure 12: Sensitivities of Consumption Habits, $\phi_1(t) - (G(t) - \phi_2(t))$. Red solid line: $\gamma = 2$; blue dotted line: $\gamma = 6$.

Table 4: Sensitivities of Wealth/Labor Income and Consumption Habits

Age		30	65
$\phi(t)$	$\gamma = 2$	0.512	0.588
	$\gamma = 6$	0.531	0.622
	difference	0.019	0.034
$\phi_1(t) - (G(t) - \phi_2(t))$	$\gamma = 2$	0.165	0.647
	$\gamma = 6$	0.474	1.158
	difference	0.310	0.512

the two figures shows that a decrease in β slightly increases $\phi(t)$. Recall that $\phi(t)$ and $\phi(t)-1$ ($\psi(t)$ and $\psi(t)-1$) measure, respectively, the impacts of wealth and future labor income on the demand for life insurance with (without) habit formation. As β decreases, the bequest motive is more sensitive to future labor income $L(t)$ ($\phi(t)$ is larger) but less sensitive to wealth X_t ($|\phi(t)-1|$ is smaller). Furthermore, as β decreases, the difference between $\phi(t)$ and $\psi(t)$, indicating a more pronounced effect of habit formation.

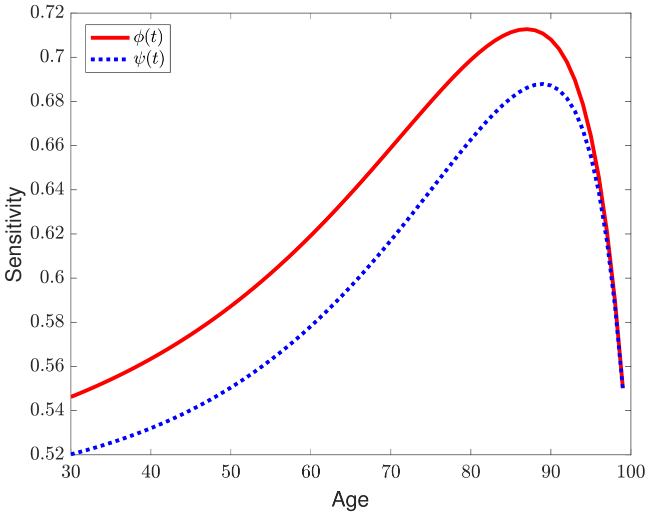

Figure 13: Sensitivities of Wealth and Labor Income. Red solid line: $\phi(t)$; blue dotted line: $\psi(t)$. Parameter values: $y=30, \alpha=0.1, \beta=0.11, \gamma=4$.

As a counterpart of Figure 2, Figure 14 plots $G(t), \phi_1(t), \phi_2(t)$, and $G(t)-\phi_2(t)$ but with $\alpha=0.1, \beta=0.11$. A comparison between the two figures shows that a decrease in β significantly increases the sensitivities of bequest motive with respect to consumption habits. In other words, when consumption habits are more persistent, the legacy decreases (increases) even more as the breadwinner's habit h_{1t} (family's habit h_{2t}). Moreover, $G(t)-\phi_2(t)$ is still smaller than $\phi_1(t)$ but the difference is still small. Thus, if the breadwinner and family's habits increase by the same amount, then the bequest motive slightly decreases.

Figure 15 plots $\phi_1(t)-(G(t)-\phi_2(t))$, for $\beta=0.2$ and $\beta=0.11$. As can be seen from the plot, the bequest motive decreases as the whole family's habits increase simultaneously, and this effect is stronger in earlier (later) years when consumption habits are more (less) persistent.

Figure 16 depicts the expected insurance premium (positive value)/annuity payout (negative value) over the life-cycle with a 40% replacement ratio. As β decreases from 0.2 to 0.11, the demand for life insurance decreases in early years and the demand for annuity significantly increases in mid-to-late years. Moreover, when $\beta=0.11$, the breadwinner starts to annuitize even before retirement. The figure shows the corresponding expected consumption over the life-cycle with a 40% replacement ratio. As β decrease from 0.2 to 0.11, the consumption habit is more persistent and the consumption in early years has a more pronounced effect on consumption in late years. Thus, the family consumes less at early ages and more at old ages. Moreover, the family purchases more annuity to meet the consumption demand in late years. As can be seen from Figures 16 and 17, the life/insurance and consumption patterns are no more consistent with empirical observations when β is close to α.

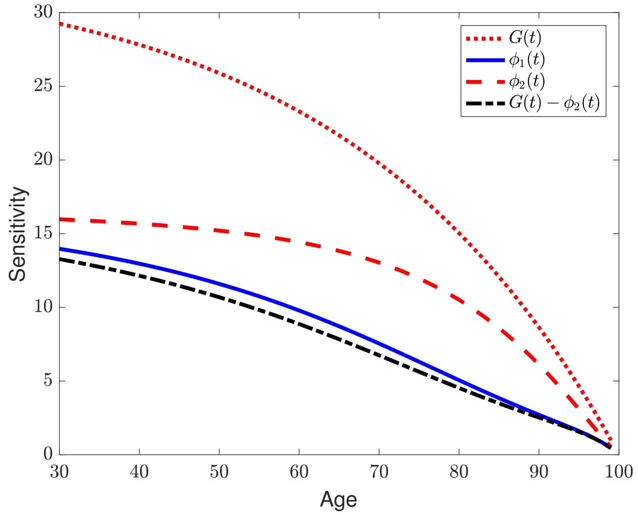

Figure 14: Sensitivities of Consumption Habits. Red dotted line: $G(t)$; blue solid line: $\phi_1(t)$; red dashed line: $\phi_2(t)$; black dash-dot line: $G(t) - \phi_2(t)$. Parameter values: $y = 30, \alpha = 0.1, \beta = 0.11, \gamma = 4$.

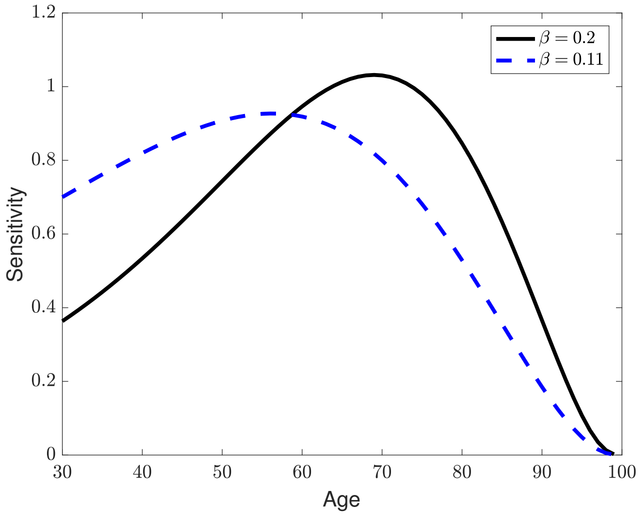

Figure 15: $\phi_1(t) - (G(t) - \phi_2(t))$. Black solid line: $\beta = 0.2$; blue dashed line: $\beta = 0.11$.

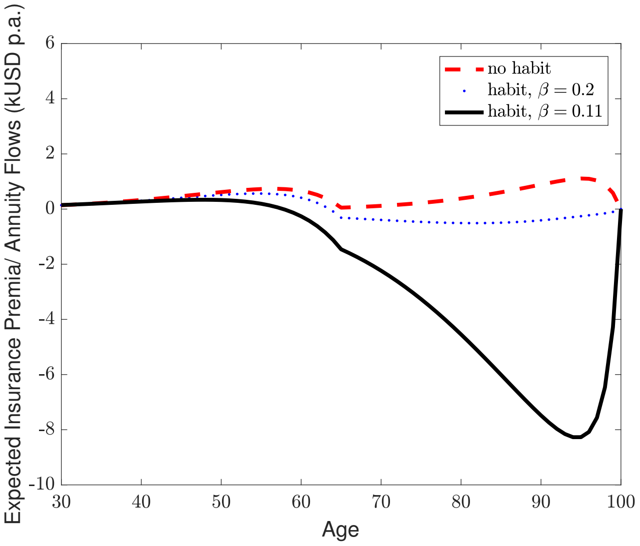

Figure 16: Expected Insurance Premia (+ Value)/Annuity Flows (- Value) (US$ in thousand p.a.). The replacement ratio is 40%. The blue solid line shows the expected life insurance premium (+ value)/annuity payout (- value) with habit formation and 60% replacement ratio. The red dashed line shows the value without habit formation. The blue dotted line shows the value with habit formation and $\beta = 0.2$. The black solid line shows the value with habit formation and $\beta = 0.11$. All other parameters are equal.

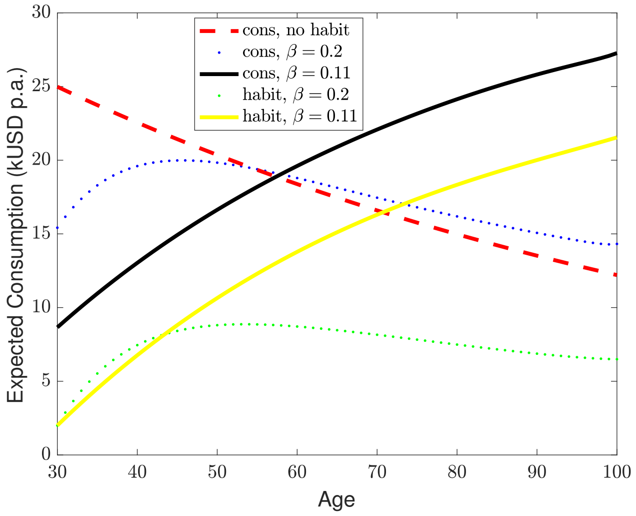

Figure 17: Expected Consumption of the Breadwinner (US$ in thousand p.a.). The blue (green) dotted dashed line shows the expected consumption (habit) path of the breadwinner with habit formation and $\beta = 0.2$. The black (yellow) solid line shows the expected consumption (habit) path of the breadwinner with habit formation and $\beta = 0.11$. The red dashed line depicts the expected consumption path of the breadwinner without habit formation. All other parameters are equal.