

**NANYANG
TECHNOLOGICAL
UNIVERSITY**

**SETTLEMENT OF SHALLOW FOUNDATION ON
COHESIONLESS SOIL CONSIDERING MODULUS
DEGRADATION OF SOIL**

**HUANG YONGQING
SCHOOL OF CIVIL AND ENVIRONMENTAL ENGINEERING
2011**

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A thesis submitted to the Nanyang Technological University
in fulfillment of the requirement for the degree of
Doctor of Philosophy

2011

ABSTRACT

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Settlement of shallow foundation on cohesionless soil is an old topic and a number of methods have been proposed in the literature. However, accurate settlement estimation of shallow foundation on cohesionless soil is still a challenge. The main difficulty is that modulus of in situ cohesionless soil depends not only on soil properties such as relative density, but also foundation properties, such as foundation size and load on the foundation. Therefore, a rational way for estimating the foundation settlement should consider the modulus degradation of soil from small-strain stiffness G_0 . The main objective of this research is to propose a practical method for better estimation of settlement of shallow foundation of all sizes on cohesionless soil by considering the modulus degradation from small-strain stiffness.

Majority of the proposed methods for estimating settlement of shallow foundation on cohesionless soil rely on elastic solution of vertical displacement influence factor or vertical strain influence factor diagram. The effects of Poisson's ratio, foundation rigidity, foundation shape and finite soil thickness on the vertical strain influence factor diagram were investigated numerically in this research. A simplified vertical strain influence factor diagram and correction factors were proposed to account for foundation rigidity, foundation shape and finite soil thickness. Many proposed methods also adopted ultimate bearing capacity of the shallow foundation to normalize foundation load to improve settlement estimation. Therefore, a commonly recognized phenomenon of the ultimate bearing capacity of shallow foundation, i.e., the so-called "scale effect" of bearing capacity was investigated using numerical method.

Since Schmertmann's (1970, 1978) method for settlement estimation of shallow foundation on cohesionless soil is the most frequently used method, it was reviewed first and modification was proposed by using small-strain stiffness G_0 . Existing

empirical correlations to derive G_0 and effective angle of internal friction ϕ' from q_c of cone penetration test were adopted. To account for modulus degradation of soil and non-linear load-settlement behavior of foundation, a new expression of peak vertical strain influence factor considering mobilized loading level was proposed. The expression was calibrated using the load-settlement curves of plate load tests (PLT) from two sites. A modified Schmertmann's method was proposed.

Besides the indirect way of considering modulus degradation of soil adopted in the modified Schmertmann's method, direct application of average modulus degradation of soil-foundation system was introduced to the elastic solution of foundation settlement to develop the modulus degradation method to estimate settlement of shallow foundation on cohesionless soil. Normalized modulus degradation measured in laboratory was correlated to the normalized average modulus degradation of soil-foundation system using numerical analyses based on a non-linear elasto-plastic constitutive soil model. The normalized average modulus degradation of soil-foundation system was also calibrated using load-settlement curves of PLT from two sites.

The calculation procedures of modified Schmertmann's method and modulus degradation method were illustrated using an example. The two methods were evaluated using 31 case studies. It was found that the two methods were comparable and the latter was slightly better based on the 31 case studies. Significant improvement in settlement estimation was achieved for both methods compared with settlement estimation from Schmertmann's (1970, 1978) method.

ACKNOWLEDGMENTS

First and foremost, I would like to thank Prof. Leong Eng Choon for his valuable and patient guidance after he kindly took over the supervision of my research at Nanyang Technological University, Singapore. His kind supports, both mental and financial, are greatly acknowledged. I also would like to thank Prof. Chang Ming Fang for the opportunity to study at NTU and his guidance at the beginning.

My thanks are given to my family: my parents, my wife and son, and my parents in law, for their unconditional support during the progress of my PhD programme. Without their support, I would never be able to complete my dissertation.

I am also grateful to my seniors, juniors and friends for their help and encouragements. Finally, I would like to extend my thanks to all of those around me during the period of my PhD study.

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LIST of SYMBOLS

a	Foundation radius
A	Material constant
B	Foundation width or diameter
B^*	A reference foundation width or diameter
B_{eq}	Equivalent foundation width
c	Cohesion
c_f	Average factor for shearing resistance
c_t	Cutting tension off
C_C	Creep factor
C_D	Depth factor
C_g	Material constant
d_q	Depth factor for N_q
d_γ	Depth factor for N_γ
D	Foundation depth
D_{50}	50 th percentage grain size
D_r	Relative density
e	Void ratio
e_{max}	Maximum void ratio
e_{min}	Minimum void ratio
E	Young's modulus
E_I	$I=1, 2, 3 \dots$ Young's modulus of I^{th} soil layer
E_0, E_{max}	Small-strain Young's modulus
E'_0	Young's modulus of soil at ground surface
E_C	Pressuremeter modulus
E_D	Dilatometer modulus
E_{fdn}	Elastic modulus of foundation
E_i	Initial tangent modulus
E_j	Young's modulus of j^{th} soil layer
E_P	Pressuremeter modulus
E_s	Secant Young's modulus
E_{sav}	Representative average elastic modulus of soil at depth of a
E_t	Tangent Young's modulus
E_{ur}	Unload-reload Young's modulus
f	Material constant
f^s	Shear failure function
f^t	Tension failure function
f^*	Material constant
$F(e)$	Void ratio function
g	Material constant
g^s	Shear potential function
g^t	Tension potential function

g^*	Material constant
G	Shear modulus
$G_0, G_{\max}$	Small-strain shear modulus
G_s	Secant shear modulus
G_t	Tangent shear modulus
G_{ur}	Unload-reload shear modulus
h	Soil thickness
I	Displacement influence factor
I_1	First invariant
I_{10}	First invariant at initial status
I_c	Compressibility index
I_d	$1/(4.6+10K_F)$
I_h	Soil thickness factor
I_F	Foundation rigidity factor
I_j	Displacement influence factor in j^{th} soil layer
$I_{L/B}$	Foundation shape factor
I_{sj}	Displacement influence factor in j^{th} soil layer
I_{total}	Total displacement influence factor
I_z	Strain influence factor
I_{z0}	Strain influence factor at footing bottom
I_{zp}	Peak strain influence factor
j	Soil layer number
J_2	Second invariant
J_{20}	Second invariant at initial status
$J_{2\max}$	Maximum second invariant
k	Emperical constant
k_E	Rate of increase of E_s with dpth
K	Modulus number
K'	Bulk modulus
K_0	At-rest earth pressure coefficient
K_2	Material constant
$K_{2\max}$	Maximum K_2
K_D	Horizontal stress index from DMT
K_E	Rate of increase of soil Young's modulus with depth
K_F	Foundation stiffness factor
L	Foundation length
m	Material constant
m_v	Coefficient of volume change
M	Constrained modulus
n	Material constant
n_g	Material constant
n_i	Material constant
n_j	Material constant
N	Number of blow counts for 300mm penetration

N_q	Bearing capacity factor for surcharge
$\bar{N}$	Average blow counts for 300mm penetration $\alpha\beta\chi\delta\epsilon\phi\eta\theta\kappa\lambda\mu\nu\pi\rho\sigma\tau\upsilon\omega\xi\psi\zeta$
N_ϕ	$(1+\sin\phi)/(1-\sin\phi)$
N_γ	Bearing capacity factor for soil unit weight
N_γ^*	A reference value of N_γ
N_ψ	$(1+\sin\psi)/(1-\sin\psi)$
OCR	Overconsolidation ratio
P_a	Atmospheric pressure
q	Foundation pressure
q'	Deviatoric stress
q'_0	Initial deviatoric stress
q^*	Net pressure at foundation bottom
$\bar{q}$	Surcharge
q_c	Tip resistance of CPT
q_{ult}	Ultimate bearing capacity of shallow foundation
Q	Material constant
R	Material constant
R_f	Failure ratio
R_{G0}	Ratio of G_0
R_m	$1/(m_v E_D)$
s	Settlement of shallow foundation
s_i	Immediate settlement of shallow foundation
s_q	Shape factor for N_q
s_s	Secondary settlement
s_γ	Shape factor for N_γ
S_{ij}	Material constant
t	Thickness of foundation
v_y	Nodal velocity
V_s	Shear wave velocity
x	Normalized axial strain
x_L	Reference strain
x_{th}	Normalized threshold strain
z	Depth
z_i	Depth of influence
α	Empirical constant
α'	Material constant
β	Empirical constant
β'	Material constant
β^*	Material constant
ϵ	Current strain
ϵ_0	Limiting strain
ϵ_a	Axial strain
ϵ_f	Failure strain

ε_z	Vertical strain
ϕ	Angle of internal friction
ϕ_{cv}	Angle of internal friction at critical state
ϕ'	Effective angle of internal friction
ϕ'_p	Peak effective angle of internal friction
γ	Bulk density
γ'	Effective bulk density
γ_r	Reference shear strain
γ_s	Shear strain
ν	Poisson's ratio
ρ	Total density of soil
$\sigma_1, \sigma_2, \sigma_3$	Three principal stresses
σ'_i, σ'_j	Effective principal stress
σ'_m	Mean effective stress
$\sigma_x, \sigma_y, \sigma_z$	Stress components
σ'_{v0}	Effective stress due to self weight of soil at D
τ	Shear stress
τ_{max}	Maximum shear stress

Chapter 1 Introduction

1.1 Background

In foundation design, question is always raised whether to use shallow foundations or deep foundations to support a structure. Although shallow foundations such as spread footing are usually much less expensive than deep foundation systems, the latter is preferred in many cases. One of the main reasons is the lack of confidence on the performance of the foundations. Estimation of settlement of shallow foundation on cohesionless soil is a challenging topic in geotechnical engineering. A number of methods for estimating the settlement of shallow foundation have been published. However in a conference on prediction of settlement of shallow foundation on cohesionless soil held at Texas A&M University in 1994, accurate settlement estimation proved a big challenge even though extensive in situ and laboratory test results were provided, particularly when the width of the foundation varies significantly (Briaud and Gibbens, 1994).

Accurate settlement estimation of shallow foundation relies on the accurate assessment of the deformation modulus of in situ cohesionless soil, which is almost impossible due to high cost, even if it is feasible. As a result, most of these methods normally depend on one or several in situ tests, such as standard penetration test (SPT), cone penetration test (CPT), dilatometer test (DMT), field compressometer test (FCPT), self-boring pressuremeter test (SBPT), etc., to assess the deformation modulus of the in situ cohesionless soil.

On the other hand, soil behaves elastically only within a very small strain level, known as elastic threshold. Beyond the elastic threshold, the stress-strain behaviour of cohesionless soil is highly non-linear. Deformation modulus of cohesionless soil depends on many factors: stress states, strain levels, stress and strain history, relative density of soil, loading rate and creep. It is not rational to interpret a

constant deformation modulus of soil from in situ tests, and then using this constant to estimate the settlement of shallow foundation of various sizes, because the stress and strain levels beneath each foundation could vary significantly. The importance of introducing non-linear stress-strain behaviour, which can be represented by modulus degradation from small strain stiffness, into the estimation of settlement of shallow foundation on cohesionless soils has been emphasized repeatedly (Jardine et al., 1986; Fahey, 1994; Mayne, 1994; Atkinson, 2000).

Laboratory experiments demonstrated that under static loading, secant soil modulus degrades from small-strain stiffness. Small-strain stiffness was known as dynamic stiffness before and was usually measured by dynamic methods. Small-strain stiffness of cohesionless soil depends on fewer factors compared with the secant modulus. Therefore, it can be measured or estimated relatively easier and more reliably. However, it can only be applicable to the geotechnical problems at small strain levels without considering degradation. For problems at intermediate to large strains, a suitable reduction has to be applied to the small-strain stiffness. To do this, a reliable modulus degradation curve is essential.

Modulus degradation curve can be measured in the laboratory using triaxial test, torsional shear test, simple shear test, etc. Based on laboratory test results, systematic investigation can be carried out using numerical methods, such as finite element method (FEM) and finite difference method (FDM). However, this makes the calculation complicated and not convenient to be implemented in practice. In situ determination of the modulus degradation curve is still difficult. Moreover, the modulus degradation curve measured by in situ tests needs to be interpreted before application. Alternatively, only the small-strain stiffness is assessed based on in situ tests and the average modulus degradation curve of the soil-foundation is estimated based on single soil element test and systematic numerical studies. In this case, a simple closed-form expression for the modulus degradation can be generated, which is more convenient for application.

1.2 Objective and Scope

The objective of this research is to propose a practical method for estimating the settlement of shallow foundation on cohesionless soil considering the modulus degradation of soil from small-strain stiffness. In order to achieve this, the scope of this research includes:

1. Investigating the scale effect of the bearing capacity factor N_γ . Ultimate bearing capacity of the shallow foundation is usually adopted to normalize foundation loading. The normalized loading is very useful in calculating the average modulus degradation of soil-foundation system. By accounting for scale effect on N_γ , more accurate assessment of the ultimate bearing capacity of the shallow foundation can be made,
2. Exploring the correlation between the normalized modulus degradation of single soil element and the equivalent modulus degradation of the soil-foundation system,
3. Proposing a practical procedure to estimate the settlement or even the non-linear load-settlement curve of the shallow foundation under vertical loading, and
4. Calibrating and verifying the proposed procedure to justify the basis of the procedure.

1.3 Thesis Outline

This dissertation consists of eight chapters and six appendices.

Chapter 2 reviews the existing methods for estimating settlement of shallow foundation on cohesionless soil. The concept of modulus degradation of soil from small-strain stiffness is then introduced and reviewed. The knowledge gaps and difficulties in implementing the modulus degradation of soil into the estimation of settlement of shallow foundation are also discussed.

Chapter 3 focuses on the study of the vertical strain influence diagram. Compared with vertical displacement influence factor, vertical strain influence factor diagram provides a more rational method in calculating the settlement of shallow foundation on cohesionless soil particularly for soil that is inhomogeneous and layered. Finite element method software ABAQUS was utilized to investigate the effects of various factors on the vertical strain influence diagram. Simplified vertical strain influence factor diagrams are proposed, which can be applied conveniently in the calculation of foundation settlement.

Chapter 4 investigates the ultimate bearing capacity of shallow foundation and the scale effect of N_γ numerically. Finite difference method software FLAC was adopted to carry out the simulation. Modification is made to the built-in Mohr-Coulomb (MC) constitutive model provided by FLAC. Bolton's (1986) correlation between peak strength of the soil and the mean effective stress level and relative density of soil can be implemented. The simulated scale effect of N_γ is comparable with those measured in model centrifuge tests and in situ spread footing tests. Charts and closed-form solutions are provided to estimate the ultimate bearing capacity of shallow foundation considering scale effect.

Chapter 5 reviews Schmertmann's (1970, 1978) method (Schmertmann, 1970 and Schmertmann et al., 1978) and its various modifications. The limitations of the Schmertmann's (1970, 1978) method and the modifications are discussed. Modification to overcome the limitations is proposed by indirectly considering modulus degradation from small-strain stiffness.

Chapter 6 covers the investigation of the load-settlement response of rigid circular foundation on a non-linear cohesionless soil. FLAC was utilized to carry out the simulation. The built-in MC constitutive model was modified to incorporate the non-linear elasticity model proposed by Fahey and Carter (1993). The modified non-linear elastic MC constitutive model was calibrated. Typical values of the parameters f and g in the constitutive model measured in laboratory tests are reviewed and summarized. A unique relationship can be found between the load-settlement curve of foundation and the modulus degradation curve of single soil element. Based on this relationship, a non-linear load-settlement curve of foundation can be estimated based on the known modulus degradation of single soil element from small-strain stiffness and strength property. The closed-form expression proposed by Mayne (1994a) incorporating the concept of modulus degradation of soil from small-strain stiffness was calibrated using the load-settlement data measured from the plate load tests and footing load tests. The calibrated average modulus degradation of soil-foundation system is compared with the results from FLAC modelling.

Chapter 7 summarizes the proposed methods to estimate the settlement of shallow foundation on cohesionless soil. An example was used to illustrate the difference between Schmertmann's (1970, 1978) method and the proposed methods. Thirty one case studies were used to evaluate the improvement of the proposed methods over Schmertmann's (1970, 1978) method.

Conclusions are summarized in the last chapter, i.e., Chapter 8. Further researches relevant to the topic are recommended at the end of the chapter.

Appendix A lists the in situ test data from Changi East reclamation site, Singapore. Results of 15CPT tests and 15plate load tests (PLT) are given.

Appendix B lists the in situ tests data from Texas A&M University, U.S.A. Results of total five CPT tests and five footing load tests are given.

Appendix C gives the interpretation of small-strain stiffness and internal friction angle from CPT tests for the Changi East reclamation site and Texas A&M University site.

Appendix D gives the interpretation of ultimate bearing capacity from the PLT and footing load tests from the two sites. Both Chin's (1971) method and Decourt's (1999) method are adopted.

Appendix E summarizes the published data of triaxial tests and plane strain tests used to calibrate the model proposed by Fahey and Carter (1993).

Appendix F lists the modified MC constitutive model incorporating non-linear elasticity and Bolton's correlation between the mean effective stress and relative density and peak strength of cohesionless soil.

Chapter 2 Literature Review

2.1 Introduction

In a typical design of shallow foundation resting on cohesionless soil, bearing capacity and foundation settlement are two important issues. For cohesionless soil, bearing capacity is usually not a problem. As a result, allowable settlement, or bearing pressure for the allowable settlement governs the design. The settlement of shallow foundation on cohesionless soil depends on many factors, such as the stress-strain behaviour of underlying soil, the pressure distribution on the foundation, foundation size, foundation geometry, foundation rigidity, thickness of the underlying soil layer, etc. Although numerous methods have been proposed for estimating the settlement, accurate estimation remains a big challenge, particularly when the foundation size varies considerably.

In this chapter, the factors affecting the settlement of shallow foundation on cohesionless soils are examined. Existing methods for estimating the settlement of shallow foundation on cohesionless soil are reviewed briefly. Modulus degradation of soil is introduced and the factors affecting the modulus degradation are discussed. The knowledge gaps and difficulties in applying the modulus degradation of soil to the estimation of settlement of shallow foundation on cohesionless soil are also discussed.

Settlement s of a foundation resting on cohesionless soil usually consists of two components

$$s = s_i + s_s \dots\dots\dots(2.1)$$

where s_i = immediate settlement and s_s = secondary compression.

Secondary compression is time-dependent and occurs at constant effective stress. It is usually not significant in clean sand but may be noticeable in clayey or silty sand (e.g. Stuart and Graham, 1975). In this research, secondary compression is ignored unless otherwise stated.

2.2 Factors affecting Settlement of Shallow Foundation on Cohesionless Soil

The main factors affecting the settlement of shallow foundation on cohesionless soil are related to foundation, pressure or load on the foundation, and the underlying soil profile. Table 2.1 summarizes these factors.

Table 2.1: Main factors affecting settlement of shallow foundation

Relevant to	Factors affecting settlement	Remarks
Foundation	Foundation size (width/diameter B)	Small footing to large raft.
	Foundation shape (L/B: length to width ratio)	Square, rectangular and circular.
	Foundation depth (D)	Shallow foundation ($D/B < 1$)
	Foundation rigidity	
	Roughness of foundation base	
Load applied on foundation	Distribution of the load	Only vertical load is considered in this research.
	Magnitude of the load	
Underlying soil profile	Stress-strain behaviour	Linear elastic, non-linear elastic, elasto-plastic
	Bulk density (γ')	
	Depth of water table	
	Thickness of soil layer (h)	

Some factors show significant effects on the settlement, such as foundation size, foundation shape, load level and stress-strain behaviour of the underlying soil layer. Consideration of all of these factors in settlement estimation is not feasible unless numerical analysis is adopted. Hence, existing methods only consider the more important factors and simplify or neglect the less important factors.

The stress-strain behaviour of the underlying soil is particularly complex and significantly influences the settlement of shallow foundation. Unfortunately, the stress-strain behaviour of cohesionless soil depends on many properties and aspects of the in situ soil, which will be discussed in detail later.

2.3 Review of Existing Methods

Numerous methods for estimating settlement of shallow foundation on cohesionless soil have been proposed. They can be classified into three categories: empirical methods, semi-empirical methods and numerical methods. Each method has its advantages and disadvantages, which will be reviewed briefly below.

2.3.1 Empirical Methods

In empirical methods, empirical correlation is directly derived between measured foundation settlement and selected in situ test results, usually from SPT, CPT, DMT, and PLT. The empirical correlation is then used for settlement estimation according to the type of in situ test.

Typical examples of empirical methods include those methods proposed by Alpan (1964), Meyerhof (1965), Terzaghi and Peck (1967), D'Appolonia (1968) and Burland and Burbidge (1985). As an example, the empirical correlation provided by Burland and Burbidge (1985) is reviewed.

The method proposed by Burland and Burbidge (1985) is based on the analysis of over 200 case records of settlement of shallow foundations, tanks and embankments on sands and gravels. The empirical relations can be expressed simply as

$$s = q^* \cdot B^{0.7} \cdot I_c \dots\dots\dots(2.2)$$

for normally consolidated sand, and

$$s = \sigma'_{v0} \cdot B^{0.7} \cdot \frac{I_c}{3} + (q^* - \sigma'_{v0}) \cdot B^{0.7} \cdot I_c \dots\dots\dots(2.3)$$

for over consolidated (O.C.) sand. In Equations (2.2) and (2.3), s = immediate settlement of shallow foundation (mm); q^* = net pressure at the bottom of the foundation (kN/m^2); B = foundation width (m); σ'_{v0} = maximum previous effective overburden pressure (kN/m^2) and I_c = compressibility index, which can be correlated to the average SPT blow counts $\bar{N}$ over the depth of influence z_i of the foundation as follows:

$$I_c = \frac{1.71}{\bar{N}^{1.4}} \dots\dots\dots(2.4)$$

Equation (2.4) is derived based on a regression analysis of I_c versus $\bar{N}$ for more than 200 case records. Figure 2.1 shows the linear correlation between the depth of influence z_i and the foundation width B on a log-log plot adopted by Burland and Burbidge (1985). According to Burland and Burbidge, depth of influence z_i represents the depth within which 75% of the settlement of foundation takes place.

It can be seen that in Burland and Burbidge's (1985) method, settlement depends on the net pressure q' at the bottom of the foundation, the foundation width B , average SPT blow count $\bar{N}$ and maximum previous effective overburden pressure σ'_{v0} . Other factors, such as foundation shape (L/B ratio), soil thickness and time-dependent settlement are accounted for by corresponding correction factors expressed in an approximate form. The method is straightforward and estimation can be made without much calculation. Moreover, $\bar{N}$ is readily available for most projects considering the prevalence of SPT test, which contributes to the popularity of the method.

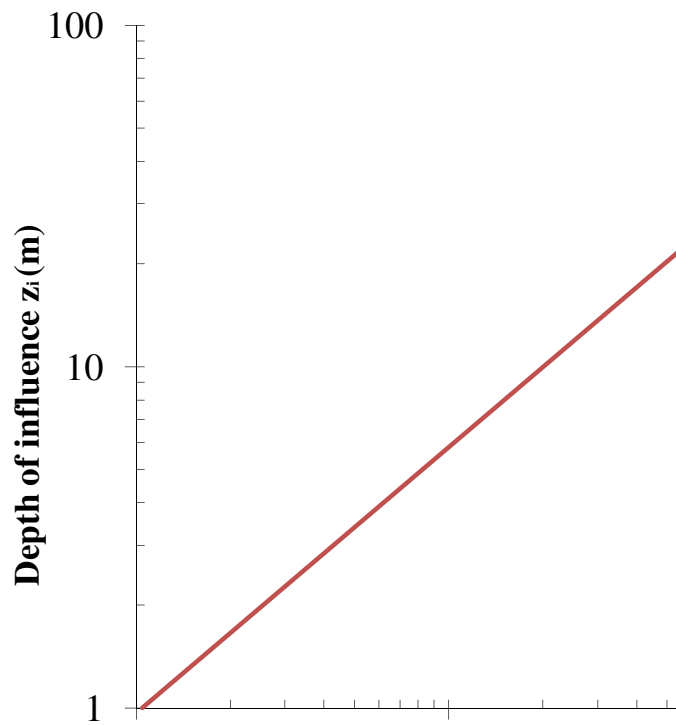


Figure 2.1: Relationship between depth of influence z_i and foundation width B by Burland and Burbidge (1985)

However, other factors listed in Table 2.1, such as foundation rigidity and particularly the stress-strain behaviour of the in situ soil are not taken into consideration in this method. The average SPT blow count $\bar{N}$ is recognized as a crude indicator of compressibility of in situ soil. Information of average SPT blow count $\bar{N}$ solely is not sufficient to generate reliable stress-strain behaviour of the in situ soil. Furthermore, SPT does not provide continuous information of blow count $\bar{N}$ with depth. Due to these limitations, Burland and Burbidge' method is not capable of accounting for the stress-strain behaviour of underlying soil accurately. The price of the convenience of application is the reduction of accuracy of the estimation. Hence, Burland and Burbidge's (1985) recommended that if more precise estimations of settlement on cohesionless soil are required, one must use direct methods of determining in situ stress-strain behaviour and not indirect methods such as SPT and CPT.

Other empirical methods follow similar principle as Burland and Burbidge's (1985) method, although different in situ tests and forms of empirical equation are adopted. In conclusion, the major advantage of the empirical methods is their simplicity in application. The disadvantage of the empirical methods is the questionable accuracy of the estimation. Therefore, it is not surprising that one rarely finds new empirical methods after Burland and Burbidge (1985).

2.3.2 Semi-empirical Methods

Semi-empirical methods are more complex compared with empirical methods, but relatively simpler in application compared with numerical methods. Most of the semi-empirical methods are developed from elastic solutions of stress and strain distribution within soil mass beneath a foundation or the simplifications of these solutions. Some of them are developed from arbitrarily assumed stress distribution which is approximate to that based on elastic theory. By incorporating the stress strain behaviour of the soil, semi-empirical methods have clearer theoretical basis and is capable of handling more factors listed in Table 2.1. Therefore, they are potentially more accurate compared with empirical methods.

The general form of the foundation settlement based on elastic theory can be expressed as:

$$s = \int \varepsilon_z dz = \frac{q}{E} \int I_z dz = \frac{qBI}{E} \dots\dots\dots(2.5)$$

where ε_z = vertical strain, I_z = vertical strain influence factor, q = applied uniform stress on foundation, B = foundation width (or diameter); E = Young's modulus of the elastic medium within the depth of influence; ν = Poisson's ratio; and I = vertical displacement influence factor. It can be seen that vertical displacement influence factor is the integration of the vertical strain influence factor within the depth of influence. Sometimes I is expressed as $I'(1-\nu^2)$ in Equation (2.5).

Semi-empirical methods mainly focus on two components of Equation (2.5) to improve the estimation of foundation settlement: one is the vertical displacement influence factor I , and the other is Young's modulus E .

(i) Modifications of Displacement Influence Factor I

The vertical displacement influence factor I depends on many factors such as foundation geometry, foundation rigidity, foundation roughness, foundation depth, soil profile and soil properties. Numerous rigorous and numerical solutions based on elastic theory have been reported. For convenience of application, they are usually presented in terms of charts or tables. Typical displacement influence factors used in practice include Steinbrenner (1934) influence factor for settlement calculation of the corner of a rectangular, flexible, uniformly loaded area, and its developments presented by Timoshenko and Goodier (1951) considering the effect of foundation depth.

Instead of using vertical displacement influence factor, semi-empirical methods sometimes adopt vertical strain influence diagram or its simplification, which is a diagram showing the vertical strain influence factor with depth, to improve settlement estimation. Schmertmann's (1970, 1978) method (Schmertmann, 1970 and Schmertmann et al., 1978) is a well-known example which adopts simplified vertical strain influence factor diagrams. A detailed review of Schmertmann's (1970, 1978) method is given in Chapter 5. Other examples include those methods proposed by Mesri and Shahien (1994), Jeanjean (1995) and Briaud (2007).

Occasionally, semi-empirical methods adopt some other assumed stress distribution profiles. For instance, Papadopoulos (1992) proposed a method of determining vertical stress distribution of an applied foundation pressure based on the assumption that the shear stress transmitted by friction is directly proportional to the in situ horizontal stress, or the in situ coefficient of earth pressure at rest, K_0 . Similar vertical stress distribution based on assumed shear stress profile was adopted by Strout (1998) in the interpretation of FCPT test.

In fact, both the strain influence diagram and the displacement influence diagram provide information of the distribution, or the percentage of the settlement with depth. Stress distribution profiles play a similar role because strain profile can be calculated based on the stress profile and Hooke's law. Modification of Equation (2.5) is essential for inhomogeneous soil profiles, where soil with stiffness increases with depth and for soil whose stiffness is related to the mean stress level. Such modification provides a more rational and accurate way to estimate settlement than using a single value of displacement factor I .

However, replacement of I in Equation (2.5) by either strain influence diagram or displacement influence diagram or assumed stress profiles make settlement estimation more complicated. Since the vertical strain influence diagrams and the vertical displacement influence factor are affected by many factors, they will be investigated in detail in Chapter 4 of this dissertation.

(ii) Investigation on Reliable Stiffness Modulus of Soil

Reliable Young's modulus is important for settlement estimation. Because of difficulty and high cost in obtaining intact samples of cohesionless soil to measure stiffness properties in the laboratory, in situ tests are preferred in developing semi-empirical methods. Early researchers focused on how to estimate a constant and representative Young's modulus based on the results of in situ tests. A number of correlations between equivalent Young's modulus E_{eq} (E in Table 2.1) or other relevant parameters such as coefficient of volume change m_v and constrained modulus M from in situ tests such as PLT, SPT, CPT, and DMT have been proposed in the literature. Table 2.2 lists some of the correlations, and the semi-empirical methods adopting these correlations.

Table 2.2: Selected correlations between soil stiffness E (m_v) and in situ test results

In situ tests	Correlation between E/m_v and test results	Method	Remarks
PLT	$E = \frac{q}{s} \cdot \frac{\pi}{4} (1 - \nu^2) I_z$	Elastic theory	I_z = influence factor; B = diameter of the plate.
SPT	$E = 53 + 1.32N$ $E = 21.2 + 1.04N$	D'Appolonia et al. (1970)	N = the number of blow counts for 300 mm penetration
	$E = 5(N + 15)$ (for submerged fine to medium sands) $E = \frac{10}{3}(N + 5)$ (for clayey sands) $E = 4(N + 12)$ (average profile)	Webb (1969)	
	$E = 4N$ (for silts or slightly cohesive silt-sand) $E = 12N$ (for sandy gravel and gravel)	Schmertmann (1970)	
CPT	$m_v = \frac{2}{3q_c}$	Meyerhof (1965)	q_c is the cone tip resistance in MPa usually
	$E = 2.5q_c$ (for square footing) $E = 3.5q_c$ (for strip footing)	Schmertmann et al. (1978)	
DMT	$E = 0.9E_D$ (NC), $E = 3.5E_D$ (OC)	Leonards and Frost (1988)	$R_m = 1/m_v E_D$, D_r is relative density and K_D is horizontal stress index, which are obtained from DMT
	$R_m = 0.14 + 2.36 \log K_D$ for $D_r \leq 0.6$ $R_m = 0.5 + 2 \log K_D$ for $D_r \geq 0.3$ $R_m = R_{m,0} + (2.5 - R_{m,0}) \log K_D$ for $0.3 \leq D_r \leq 0.6$ where $R_{m,0} = 0.14 + 0.15(D_r - 0.6)$ $R_m = 0.32 + 2.18 \log K_D$ for $K_D > 10$	Marchetti (1980)	
PMT	E_D, E_c are used directly	Menard (1965)	E_D and E_c are pressuremeter modulus within the zone of influence of the deviatoric and spherical stress tensor, respectively

With the understanding that deformation modulus of soil is highly non-linear and degrades with strain level, non-linear nature of the stress-strain behaviour of the cohesionless soil has to be considered in settlement estimation. A direct way to utilize a non-linear secant Young's modulus E_s , instead of a constant Young's modulus E in Equation (2.5) is to consider the effect of modulus degradation on the settlement of foundation. A couple of typical examples are given below.

Example 1: Oweis (1979) noted that the deformation modulus of soil depends on mean effective normal stress, strain levels and the initial compactness of the sand. It is very surprising to observe that the concept of modulus of soil at small strains ($10^{-3}\%$) measured by seismic velocity methods has been introduced by Oweis as early as 1979 for settlement estimation. According to Oweis, an empirical correlation can be established between small-strain stiffness and corrected average SPT blow count number. As a result, small-strain stiffness of the in situ soil can be estimated from SPT result. Moreover, the increase of small-strain stiffness due to the increase of mean effective normal stress during the loading of plate or foundation is also considered in his method.

On the other hand, secant modulus reduction from maximum value at small strains with the increase of average vertical strain beneath a foundation was calibrated based on the load settlement curves of plate load tests and presented in a chart. Due to the fact that resolution of measurement in plate load test is normally larger than 0.03% , a linear extrapolation of these data in a log-log plot to strains as low as 0.03% was suggested. Oweis (1979) considered this linear extrapolation to be reasonable based on cyclic shear tests on sands.

In the application of Oweis' (1979) method, the soil within the depth of influence z_i can be divided into layers to account for the change of mean effective stress level and average vertical strain level with depth. The calculation can be carried out in an explicit process with small load intervals with the soil treated as linearly elastic. Finally, a non-linear load settlement curve can be estimated.

Example 2: Ghionna et al. (1991) introduced the hyperbolic model developed by Duncan and Chang (1970) into Equation (2.5) to account for effect of the non-linear stress-strain behaviour of gravelly soil on the settlement estimation. According to Duncan and Chang (1970), the hyperbolic model can be expressed as:

$$E_t = \frac{\frac{1}{E_i}}{\left[\frac{1}{E_i} + \frac{R_f \varepsilon_a}{(\sigma_1 - \sigma_3)_f} \right]^2} \dots\dots\dots (2.6)$$

where E_t = tangent modulus; $(\sigma_1 - \sigma_3)_f$ = compressive strength, or stress difference at failure; R_f = failure ratio, which is equal to $(\sigma_1 - \sigma_3)_f / (\sigma_1 - \sigma_3)_{ult}$, where $(\sigma_1 - \sigma_3)_{ult}$ = asymptotic value of stress difference; ε_a = axial strain; E_i = initial tangent modulus, which, according to Janbu (1963) can be expressed as

$$E_i = K P_a \left(\frac{\sigma_3}{P_a} \right)^n \dots\dots\dots (2.7)$$

where K = modulus number and n = exponent describing the rate of variation of E_i with σ_3 , both are pure numbers; P_a = atmospheric pressure having the same unit as E_i and σ_3 = minor principal stress. All the parameters can be determined in conventional triaxial compression tests. Equation (2.7) was modified by Duncan et al. (1980) as

$$E_i = K (\sigma'_m)^n \dots\dots\dots (2.8)$$

where σ'_m = mean effective stress; K and n = dimensionless material numbers.

Substituting Equations (2.7) and (2.8) into Equation (2.5), one can obtain the following equation for settlement calculation:

$$s = \frac{1}{K} \frac{q B I_f (1 - \nu^2)}{(\sigma'_m)_{av}^n - \frac{q B I_f (1 - \nu^2)}{(\sigma'_m)_{av}^{1-n} c_f z}} \dots\dots\dots (2.9)$$

where c_f = average factor for shearing resistance, which mainly depends on the internal friction angle and the stress level; $(\sigma'_m)_{av}$ = average value of σ'_m ; z_i = depth of influence; other symbols are the same as defined before.

In using the method, plate load test is suggested to be conducted first to determine the parameters K and c_f in Equation (2.9). Similar to Oweis' (1979) method, the soil within the depth of influence can be divided into layers to account for the change of mean stress level with depth.

Example 3: Wahls and Gupta (1994) adopted a relationship between the shear modulus and the mean effective stress, relative density and shear strain developed experimentally by Seed and Idriss (1970). The relationship can be expressed as

$$G = 21.9K_2P_a\left(\frac{\sigma'_m}{P_a}\right)^{0.5} \dots\dots\dots(2.10)$$

where G = shear modulus; σ'_m = mean effective stress; and P_a = atmospheric pressure; K_2 = coefficient that is a function of relative density D_r , which can be estimated from SPT results, and shear strain γ . The value of K_2 can be estimated from shear strain (%), γ , as follows:

$$\frac{K_2}{K_{2max}} = 0.0133(\log \gamma_s)^4 + 0.1375(\log \gamma_s)^3 + 0.4131(\log \gamma_s)^2 + 0.0707(\log \gamma_s) + 0.068$$

..... (2.11)

Note that for $30\% < D_r < 90\%$, $K_{2max} = 0.6D_r + 16$; At shear strain $\gamma_s = 10^{-4}\%$, $K_{2max} = K_2$ can be assumed.

Other similar examples considering modulus degradation of soil in the estimation of settlement include methods proposed by Bobe and Pietsch (1981), Mayne (1994a, 1994b) and Lehane and Cosgrove (2000).

It can be seen that the three examples given above all consider modulus degradation, either Young's modulus or shear modulus, with strain levels, either the average vertical strain beneath a foundation (Oweis, 1979 and Ghionna et al., 1991) or the shear strain (Wahls and Gupta, 1994). The modulus degradation starts from either the small strain stiffness (Oweis, 1979 and Wahls and Gupta, 1994) or the initial stiffness of the hyperbolic model. The effect of the mean effective stress level on the stiffness is also considered. The incorporation of the non-linear modulus of soil depending on the mean effective stress and strain level is a remarkable improvement compared with that adopting constant deformation modulus. For the methods proposed by Oweis (1979) and Wahls and Gupta (1994), the soil within the depth of influence should be divided into layers to account for the change of mean effective stress level and shear strain level with depth. The calculation is carried out explicitly from initial loading with estimated maximum modulus.

The relationship between soil modulus and mean effective stress, relative density and shear strain are based on plate load test (Oweis, 1979), triaxial test (Ghionna et al., 1991) and torsional shear test (Wahls and Gupta, 1994). Although the modulus degradation observed from torsional shear tests is applied directly to the estimation of settlement of shallow foundation, Wahls and Gupta's (1994) reported that based on the study of 120 cases, overall the method provides better estimation both in terms of the standard deviation of the absolute difference between the estimated and measured one, and the maximum value of the difference.

However, these methods do not prevail in the estimation of settlement of shallow foundations. For example, in 1994, a conference was held at Texas A&M University discussing estimated and measured behaviour of five spread footings on sand. A total 31 papers adopting 22 methods were presented. Among them, only Oweis' (1979) method was selected twice. Schmertmann's (1970, 1978) method was the most frequently used method in the conference being adopted by 18 researchers. Several reasons can explain this observation. The first reason is that although the concept of modulus degradation is commonly recognized, there is still a gap between modulus degradation observed from laboratory tests on single soil

element and the equivalent modulus degradation of the soil mass in situ beneath a shallow foundation. The modulus degradation observed from laboratory tests on single soil element can be measured from triaxial test, torsional shear test, or simple shear test. The boundary condition and loading condition differ in each test, and differ with that of the soil mass in situ. It is not convincing to apply modulus degradation observed from laboratory tests directly to geotechnical problems.

The second reason is the accuracy and effectiveness of the description of the modulus degradation of cohesionless soil. For example, the modulus degradation proposed by Oweis (1979) is established based on PLT and presented in a chart. The normalized deformation modulus is plotted versus average vertical strains beneath a foundation, which is not known in the calculation. Ghionna et al. (1991) adopted hyperbolic equation, which is believed to be suitable for clay and sand under cyclic loading. Modification such as proposed by Fahey and Carter (1993) is necessary to make it suitable to describe the modulus degradation of normally consolidated sand under monotonic loading. Wahls and Gupta (1994) adopted Seed and Idriss' (1970) expression of shear modulus degradation of sand, which is not as convenient as the hyperbolic equation.

The third reason is the reliable determination of the modulus degradation. Plate load test suggested by Oweis (1979) and Ghionna et al. (1991) is not a good choice to determine modulus degradation or maximum modulus because only the soil within the depth of influence, which is about $2B$ (B =diameter/width of the plate), is tested. Modulus degradation and stiffness properties of the soil within this depth of influence may not be representative for foundation, which is larger in dimension than the plate used in PLT. Standard penetration test as suggested by Oweis (1979) and Wahls and Gupta (1994) is widely used in practice to estimate small-strain stiffness. However, SPT usually does not provide continuous information of blow count with depth.

The fourth reason is the convenience of carrying out the calculation by using these methods. Compared with methods using constant soil modulus in the estimation of

settlement, these methods are not convenient to be applied, because either the soil is required to be divided into layers to consider the change of mean effective stress and strain level with depth, or the calculation is proceeded in an explicit manner stepwise due to the dependence of the soil modulus on strain level. Therefore, there is more resistance in applying them because of inconvenience of application.

From the discussions above, past researchers have attempted to consider modulus degradation of soil in the estimation of settlement of shallow foundation using semi-empirical methods. However based on the current understanding of modulus degradation of soil, improvements can be made in the following areas: (i) to adopt a more accurate, effective and widely accepted modulus degradation curve; (ii) to apply modulus degradation of cohesionless soil measured in the laboratory to estimate settlement problem of foundation; (iii) to develop a more reliable method to determine small-strain stiffness in situ or in laboratory; and (iv) to achieve a better balance between accuracy and convenience in the method for estimating settlement.

2.3.3 Numerical Methods

Numerical methods are widely used in practice at present with the advancement of computer technology. Among all the numerical methods, FEM is probably the most widely and frequently used technique. Typical software includes ABAQUS, ANSYS, and PLAXIS. Besides FEM, FDM is also adopted by researchers and engineers. Typical software includes FLAC.

The advantages of numerical methods are evident. When applying numerical methods to solve geotechnical problems, both complex constitutive model of soil, and complicated boundary conditions and initial conditions can be accounted for, which are not possible with analytical methods. Examples of complicated boundary conditions include flexibility of the foundation (rigid, flexible or intermediate), complex contact conditions between foundation and the underlying soil (smooth,

rough or between), ground water table and layered strata. Complex initial condition, such as non-isotropic stress can also be investigated.

However, numerical methods also have limitations. The most important limitation is the accuracy of the analysis. Accuracy of the analysis of geotechnical problem using numerical analysis significantly depends on the constitutive model that is adopted and the determination of the input values of those parameters of the constitutive model. For instance, in order to investigate the bearing capacity of a foundation, MC model comprising of linear elasticity and perfect plasticity may be a good choice. However, it may not be as good as a MC model comprising of non-linear elasticity if the settlement problem of shallow foundation is investigated. Therefore, the understanding and choice of the constitutive model is important.

The determination of the input values of the parameters of the constitutive model is another issue that significantly affects the accuracy of numerical methods. Usually, more complicated constitutive model requires more input parameters. For example, MIT E-3 constitutive model requires 15 parameters to describe the stress strain behaviour of soil, among which, some are measured by specific tests and may not be available for many projects. Even traditional MC model comprising of linear elasticity and perfect plasticity requires five parameters. These parameters are measured in laboratory or in situ. Experiences play an important role in interpreting the laboratory test results or in situ test results to obtain the input value of these parameters.

Other factors affecting numerical accuracy include density of the mesh; tolerance error and size of time step. Although there are many commercial softwares available, the available constitutive models may not be suitable for the problem. In this case, some softwares provide option to incorporate user written subroutine.

2.4 Modulus Degradation of Cohesionless Soil

It is widely known that cohesionless soil behaves non-linear from very early loading stages (e.g. Tatsuoka and Shibuya, 1992; Lo Presti, 1994; Hicher, 1996). Accurate estimation of settlement of shallow foundation depends on correct definition of the deformation modulus of soil. Modulus degradation curve from small-strain stiffness measured in laboratory tests provides an effective way to achieve this. However, the understanding of modulus degradation from small-strain stiffness under static monotonic loading is not very long, starting from the late 1980's and early 1990's. Torsional shear tests and triaxial tests capable of determining the small-strain stiffness play important role in advancing the recent understanding of modulus degradation of soils from small-strain stiffness under static loading.

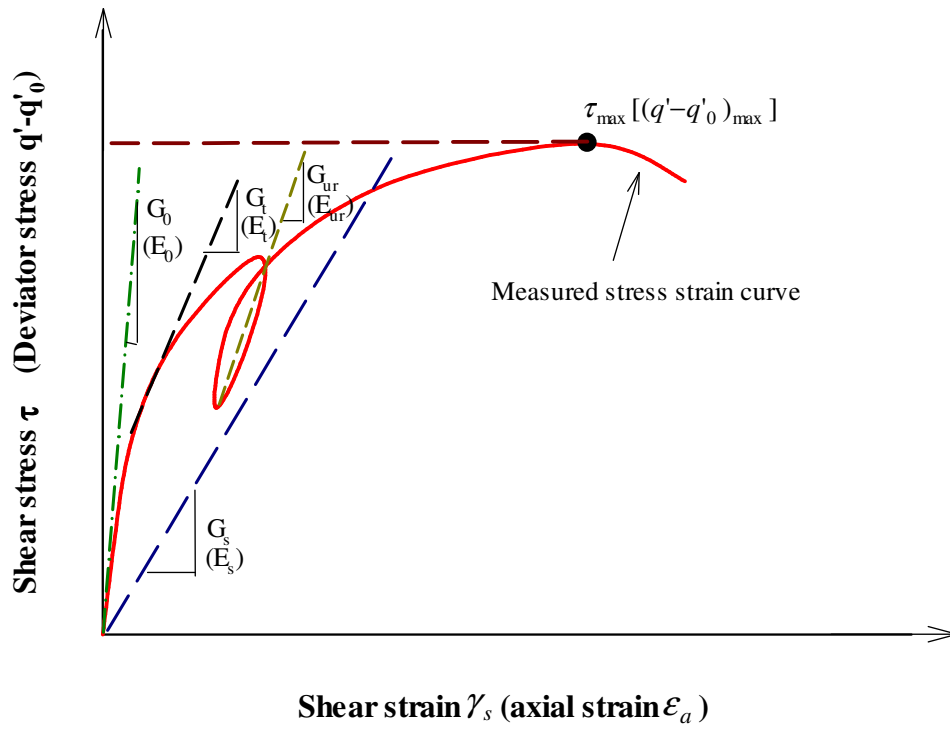


Figure 2.2: Definition of soil stiffness

Figure 2.2 shows the definitions of various shear moduli from torsional shear test: secant shear modulus, $G_s = \tau/\gamma_s$; tangent shear modulus, $G_t = d\tau/d\gamma_s$; unload-reload

shear modulus, G_{ur} , and small-strain shear modulus, G_{max} or G_0 . Similarly, based on triaxial test, a series of Young's moduli E_s , E_t , E_{ur} and E_{max} can be defined, as shown in brackets in Figure 2.2.

Based on laboratory test results, it is understood that soils behave purely elastically only in a very small range of strain, usually within a threshold shear strain of about 0.001% (e.g. Tatsuoka and Shibuya, 1992, Shibuya et al., 1992, Lo Presti et al., 1993, Tatsuoka et al., 1994, Lo Presti, 1995, Hicher, 1996 and Kohata et al, 1997). The stiffness in this range is defined as small-strain stiffness G_{max} or G_0 , and is formerly known as dynamic shear modulus modulus G_{dyn} , because it is measured using dynamic methods in earlier times. Within the elastic range, secant modulus, tangent modulus and unload-reload modulus are all equal to the small-strain modulus.

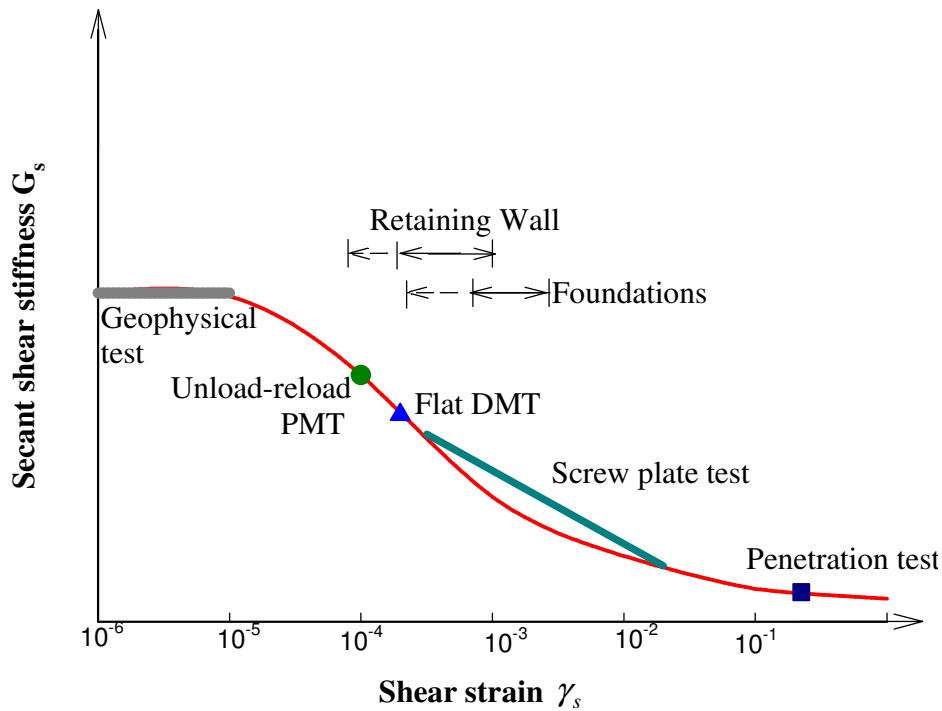


Figure 2.3: Modulus degradation of soil with typical strain ranges for in situ tests and structures (Modified from Mayne and Schneider, 2001)

The soil behaviour becomes non-linear as shear strain exceeds the threshold shear strain up till failure. The secant shear stiffness G_s reduces significantly with increasing shear strain, as shown in Figure 2.3. Figure 2.3 also shows the typical strain ranges of soil involved for retaining wall and foundation problems. As a comparison, the typical strain ranges of the soil under various in situ tests are also given in Figure 2.3.

2.4.1 Soil Stiffness within Elastic Range

Stress-strain behaviour of cohesionless soil within the elastic range can be described by Hooke's law and small-strain modulus. Small-strain modulus in this range is usually recognized as stress-dependent, anisotropic, and strain-rate-independent. Many factors affect the small-strain modulus. Table 2.3 lists the factors reported in the literature.

Table 2.3: Factors affecting small-strain stiffness

Factor	Reference
Void ratio (e)	Hardin and Black (1966), Iwasaki et al (1978), Lo Presti (1995)
Effective stress components	Hardin and Black (1966), Roseler (1979), Lo Presti (1995), Hoque and Tatsuoka (1998)
Mean effective stress (σ'_m)	Hardin (1978), Yamashita et al. (2003)
Stress history (OCR)	Hardin (1978), Hardin and Blandford (1989)
Soil structure (aging, cementation, etc.)	Belletti et al (1997), Sharma and Fahey (2003). Christopher and James, (2004)
Silt content	Salgado et al. (2000)

Void ratio and stress state have significant effect on small-strain stiffness. Many researches showed that the mechanical loading history does not have a significant effect on small-strain stiffness (Shibuya et al., 1994, Kohata et al., 1994) except for crushable carbonate sands (Fioravante et al., 1994a).

Hardin and Blandford (1989) suggested that the Young's modulus for elastic compressive strain increments in a certain direction is a unique function of normal

stress in that direction. Lo Presti et al. (1995) and Hoque and Tatsuoka (1998) reported similar results for triaxial tests performed on clay and sand.

Soil structure such as cementation and aging has significant effects on small-strain stiffness (e.g. Schmertmann, 1991, Belletti et al., 1997 and Sharma and Fahey, 2003). Salgado et al. (2000) reported that small-strain stiffness of Ottawa sand decrease dramatically at a given relative density and confining stress level with the addition of a small percentage of silt.

Empirical equation to estimate G_0 accounting for these factors are proposed in case measurement of G_0 is not available. According to Hardin and Blandford (1989), G_0 can be related to the current state of a soil by means of the following relationship:

$$G_0 = S_{ij} F(e) (OCR)^k P_a^{1-n_i-n_j} \sigma_i'^{n_i} \sigma_j'^{n_j} \dots\dots\dots (2.12)$$

where S_{ij} = nondimensional material constant of given soil that also reflects its fabric; k = empirical exponent; σ_i and σ_j = effective principal stresses acting on plane in which G_0 is measured; n_i and n_j = empirical exponents; and p_a = reference stress; $F(e)$ = void ratio function, which = $(2.17-e)^2/(1+e)$ according to Iwasaki et al. (1978); $F(e) = e^{-1.3}$ is proposed by Lo Presti (1989) and for Quiou sand $F(e) = (3.82-e)^2/(1+e)$ is proposed by Fioravante et al. (1994a).

However, the following empirical expression proposed by Hardin and Black (1976) is more widely adopted in estimating small-strain shear modulus:

$$G_0 = C_g F(e) P_a^{1-n_g} \sigma_m'^{n_g} \dots\dots\dots (2.13)$$

where C_g , n_g = material constants and σ'_m = mean effective stress.

Reliable value of G_0 can be measured in the laboratory and in situ. Laboratory tests consist of both dynamic and static tests. Dynamic tests include resonant column (RC) tests (e.g. Hardin and Drnevich, 1972) and seismic tests, such as bender element (BE) tests (e.g. Dyvik and Madhus, 1985). Compared with static tests, such as triaxial test (e.g. Jardine et al., 1984) and torsional shear test (e.g. Teachavorasinskun et al., 1991), dynamic tests is probably more accurate. However, the main advantage of static tests is that not only small-strain modulus, but the moduli outside the elastic range can also be measured, such that a continuous modulus degradation curve can be obtained.

In situ geophysical tests can also provide reliable measurement of small-strain modulus of in situ soil. These include down-hole method (e.g. Woods, 1978; Campanella et al., 1994), cross-hole method (e.g. Hoar and Stokoe, 1978), seismic cone penetration test (e.g. Robertson et al, 1986b), and seismic flat dilatometer test (e.g. Hepton, 1988). As shown in Figure 2.3, in situ static tests are not sufficiently accurate to measure G_0 . However, based on seismic test results, some empirical correlations have been proposed to correlate G_0 with in situ test results, such as CPT (e.g. Mayne and Rix, 1993) and DMT (e.g. Hryciw, 1991). Considering the fact that laboratory test results based on undisturbed samples are usually not available for cohesionless soil, empirical correlations provide important compensation, although accuracy is not comparable to measurement by seismic tests.

2.4.2 Modulus Degradation of Soil outside Elastic Range

Stress-strain behaviour of cohesionless soil outside the elastic range is more complex than that within the elastic range. Plasticity develops once shear strain exceeds the threshold shear strain (i.e., shear strain of about 0.001%). The secant modulus degrades dramatically outside the elastic range with increasing shear strain level as shown in Figure 2.3. Some difference is observed between the modulus degradation curve measured from strain-rate controlled triaxial test and torsional shear test. But generally, the modulus degradation curve is affected by stress state, shear strain level (or mobilized shear stress ratio), relative density of soil, stress and

strain history, shearing rate and creep. As far as immediate settlement of shallow foundation is concerned, the latter two factors are not evident.

Shear modulus degradation with increasing shear strain has been investigated since 1970's using resonant column apparatus or torsional shear apparatus (Seed and Idriss, 1970; Iwasaki et al., 1978). The results of cyclic shear tests are normally presented in terms of normalized modulus $G/G_{\max}$ versus shear strains (e.g., Seed and Idriss), or normalized modulus $G/G_{\max}$ versus normalized shear strains γ_s/γ_r (where $\gamma_r = \tau/\tau_{\max}$) (e.g., Hardin and Drnevich, 1972a and 1972b). The normalized shear strain (γ_s/γ_r) can also be converted to normalized shear stress $\tau/\tau_{\max}$ (or mobilized shear stress ratio) (Fahey, 1991). As pointed out by Fahey (1991), the normalized shear stress approach is better in the sense that the relationship is linear for cyclic shear test results and the physical meaning is clearer. In addition, the relationship is more straightforward in application.

Figure 2.4 shows two examples of the normalized shear modulus degradation curves of Toyoura sand and Hamaoka sand measured using cyclic torsional shear tests. It can be seen that degradation of the measured normalized modulus approximately follows a hyperbolic relationship. This means that under cyclic torsional shear, the effect of confining stress and mobilized stress ratio on the normalized shear modulus degradation can be eliminated by normalization. As a result, a hyperbolic equation can be used to describe the normalized shear modulus degradation with mobilized shear stress ratio.

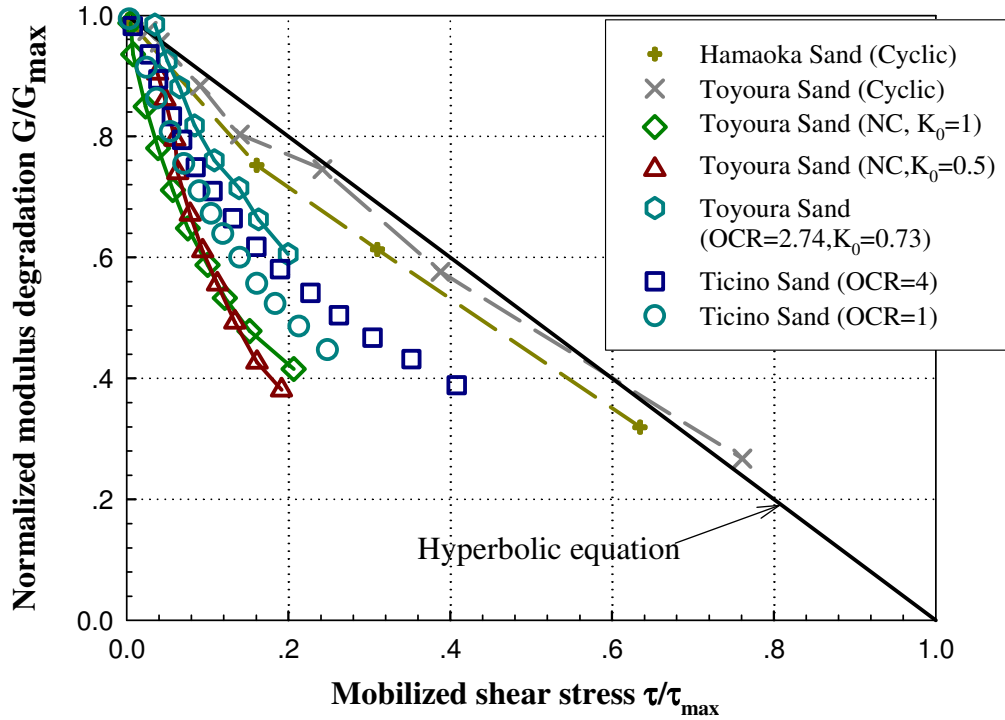


**Figure 2.4: Normalized shear modulus degradation from torsional shear tests
(Modified from Lo Presti, 1993 and Teachavorasinskun et al., 1991B)**

Figure 2.4 also shows that for Ticino and Toyoura sands under monotonic torsional shear test, the normalized modulus degradation is not linear in the normalized plot. For both overconsolidated (OC) Ticino sand and Toyoura sand, slower modulus degradation can be observed compared with that measured on the normally consolidated (NC) samples at similar relative density D_r and mean effective stress σ'_m . But for Toyoura sand under isotropic consolidation ($K_0=1.0$) and K_0 consolidation ($K_0=0.5$), no significant difference of the normalized shear modulus degradation can be found between the two, as shown in Figure 2.4. In conclusion, overconsolidation has more significant effect than K_0 condition on the normalized modulus degradation curve measured in monotonic torsional shear test. No existing literature suggests that the normalized modulus degradation shown in Figure 2.4 depends on the relative density of the sample.

More complicated normalized Young's modulus degradation can be observed from triaxial tests. Figure 2.5 shows several examples of normalized Young's modulus degradation observed from triaxial compression tests. Figure 2.5 clearly shows the significant effect of overconsolidation on normalized Young's modulus degradation, for both Toyoura sand and silty sand consisting of Ottawa sand and fines. Similar to that observed from torsional shear test, the OC sand shows slower degradation of normalized Young's modulus. However, the effect of overconsolidation is much more significant for Young's modulus than shear modulus.

Figure 2.5 also shows that confining stress has significant effect on the normalized Young's modulus degradation compared with Figure 2.5. For the same OC Toyoura sand, slower normalized Young's modulus degradation can be observed for $K_0 = 0.69$ compared with that of $K_0 = 0.46$. Similar results can be found for NC sand as well, although not presented here. It has also been found that the relative density D_r (or void ratio e) has significant effect on the normalized Young's Modulus degradation as well (Lee and Salgado, 1999). Figure 2.6 shows the effect based on the observation of Ottawa sand in triaxial tests. It can be seen that denser sample shows slower degradation of normalized Young's modulus.

The difference between the effects of initial confining stress and relative density on the normalized Young's modulus degradation and normalized shear modulus degradation is partially due to the difference in the loading condition between triaxial test and torsional shear test. For torsional shear test, the mean effective stress does not change during the test. However for triaxial test, the mean effective stress increases as axial stress increases. As a result, small-strain stiffness $E_{\max}$ increases according to Equation (2.12) or (2.13). This increase of $E_{\max}$ also depends on the void ratio function, and therefore, the initial void ratio or relative density of sand.

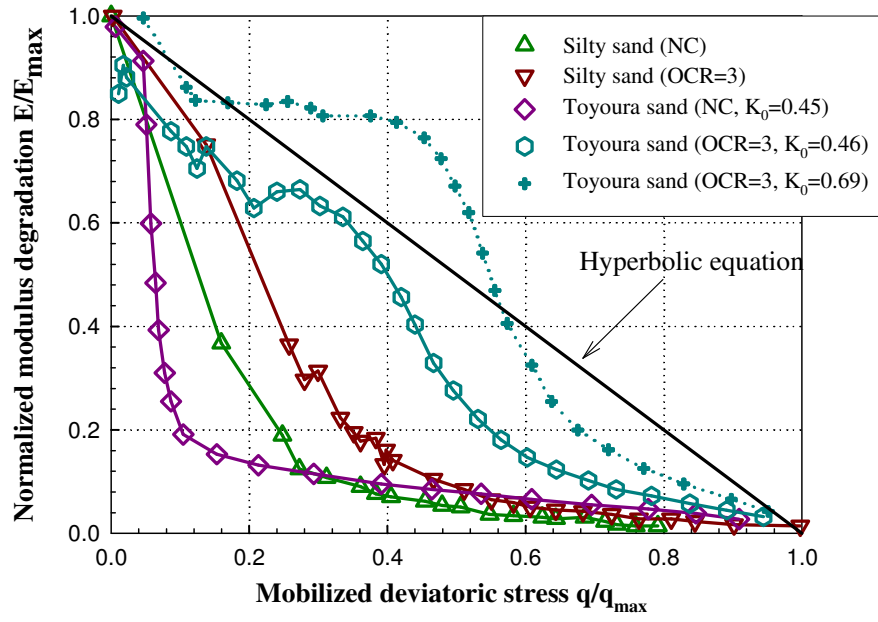


Figure 2.5: Normalized Young's modulus degradation observed from triaxial tests (Modified from Lo Presti, 1993 and Lee et al., 2004)

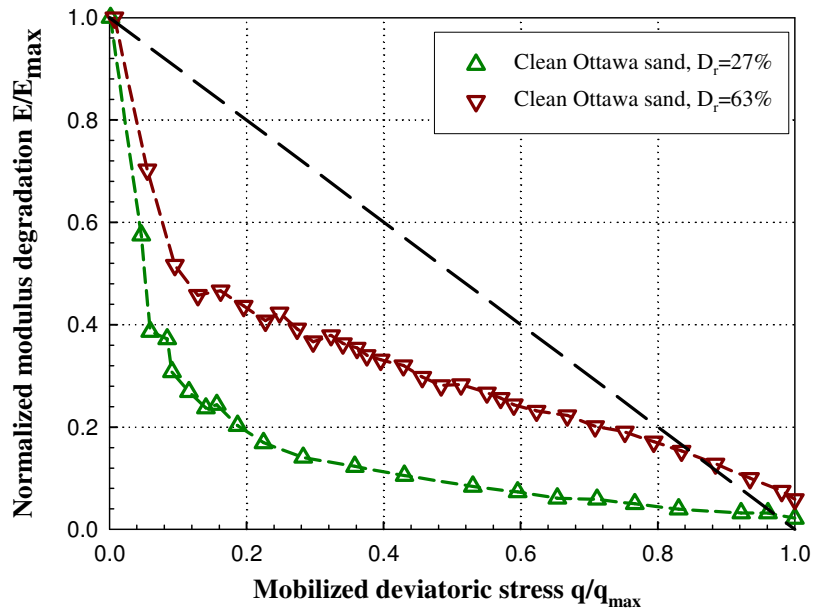


Figure 2.6: Effect of relative density on the normalized Young's modulus degradation observed from triaxial tests (After Lee and Salgado, 1999)

However, the dependence of small-strain stiffness on the confining stress and void ratio is insufficient to explain the huge difference the effect of OCR has on the normalized modulus degradation. The mean effective stress and void ratio during unloading and reloading do not vary significantly compared with first time loading in triaxial test. However, the normalized Young's modulus $E/E_{\max}$ degrades much slower compared with the original loading.

In conclusion, shear modulus degradation of cohesionless soil outside the elastic range observed in torsional shear test mainly depends confining stress, shear strain (or shear stress level) and stress and strain history. For sand, normalization of G_s with $G_{\max}$ and τ with $\tau_{\max}$ can eliminate the effects of confining stress and shear stress. No experimental evidence shows that the relatively density of cohesionless soil has significant effect on the normalized modulus degradation curve. Hyperbolic equation and its modification can be used to describe the normalized shear modulus degradation.

Compared with shear modulus, Young's modulus degradation of cohesionless soil outside the elastic range observed from triaxial test is more complicated and depends on confining stress, shear strain (or shear stress level), relatively density and stress strain history. Aging and cementation also has significant effect, but are not well investigated compared with other factors. For sand, normalization of E_s with $E_{\max}$ and q with $q_{\max}$ cannot entirely eliminate the effects of confining stress and deviatoric stress, due partially to the variation of effective mean stress during loading. Although modified hyperbolic equation can be used to describe the normalized Young's modulus degradation, the values of the parameters may not be unique in contrast to normalized shear modulus degradation, and depend on additional factors such as relative density and mean effective stress level.

2.5 Application of Concept of Modulus Degradation and Knowledge Gaps between the Concept and its Application

The concept of normalized modulus degradation in terms of $G/G_{\max}$ or $E/E_{\max}$ has been widely implemented in various geotechnical problems relevant to static loading since early 1990's (e.g. Wahls and Gupta 1994; Mayne, 1994; Fahey et al., 1994; Zhu and Chang, 2002, etc.). Table 2.4 lists several examples of these applications in literature. The advantage of adopting $G_{\max}$ as a starting point is evident: one is that $G_{\max}$ is a fundamental property dependent on fewer variables compared with secant shear modulus G_s . This means that $G_{\max}$ can be estimated relatively conveniently even if no measured data are available; the other is that $G_{\max}$ can be reliably measured by dynamic method both in laboratory and in situ; and finally $G_{\max}$ is the maximum value of G_s , which can be applied to all types of the geotechnical problems by reducing it to an appropriate value of shear modulus based on mean effective stress level and strain level.

The detailed process of the applications listed in Table 2.4 can be either complicated or relatively simple. For instance, Fahey and Carter (1993) used FEM and MC model incorporating modified hyperbolic model describing modulus degradation to interpret pressuremeter test. Similar approach was proposed by Fahey et al. (1994) to analyze the settlement of shallow foundation on sand. Lee and Salgado (1999) also adopted FEM to conduct their analysis of model plate load test. The modified hyperbolic equation was extended to 3-D case. Zhu and Chang (2000) established a closed-form expression to estimate the pile-soil response. Mayne (1994) also proposed a simple closed-form expression to estimate the settlement of shallow foundation on sands.

Although a lot of applications are in the literature. There are some knowledge gaps between the concept of modulus degradation and its application in practice. Figure 2.7 shows the possible directions to bridge the gaps. First of all, it should be mentioned that although in situ reliable measurement of $G_{\max}$ is not difficult, the measurement of a reliable modulus degradation curve is another issue, which is at

least difficult, if not infeasible. In situ tests, such as SBPT, DMT and SCPT, have been reported to be used to measure the modulus degradation of cohesionless soil from small-strain stiffness. However, they are still not widely accepted mainly due to the accuracy of the tests. In addition, the deformation modulus measured by these tests need to be interpreted before its application to estimation of settlement of shallow foundation, such as the direction 1 shown in Figure 2.7. For example, Briaud (2007) established a correlation between load-displacement curve measured by SBPT and load-settlement curve of foundation, such that a full scale load settlement curve of foundation can be estimated based on SBPT.

Table 2.4: Application of concept of modulus degradation

Geotechnical problem	Reference
Settlement of shallow foundation on cohesionless soil	Oweis, 1979; Wahls and Gupta 1994; Mayne, 1994; Fahey et al., 1994; Lehane and Cosgrave, 2000
Load transfer curve of bored pile	Zhu and Chang, 2002
Interpretation of pressuremeter test	Fahey and Carter, 1993
Interpretation of chamber plate load test	Lee and Salgado, 1999
General routine design purpose	Atkinson, 2000

Alternatively, a more practical way, i.e., direction 2 in Figure 2.7 is to measure or estimate the small-strain stiffness of in situ soil only. The modulus degradation curve measured in laboratory test can be adopted to assess the modulus degradation of soil mass in situ. The main drawback is that some important aspects, such as stress and strain history of the in situ soil, cannot be duplicated in the laboratory test. However, modulus degradation curve measured in laboratory test provides valuable reference for the assessment.

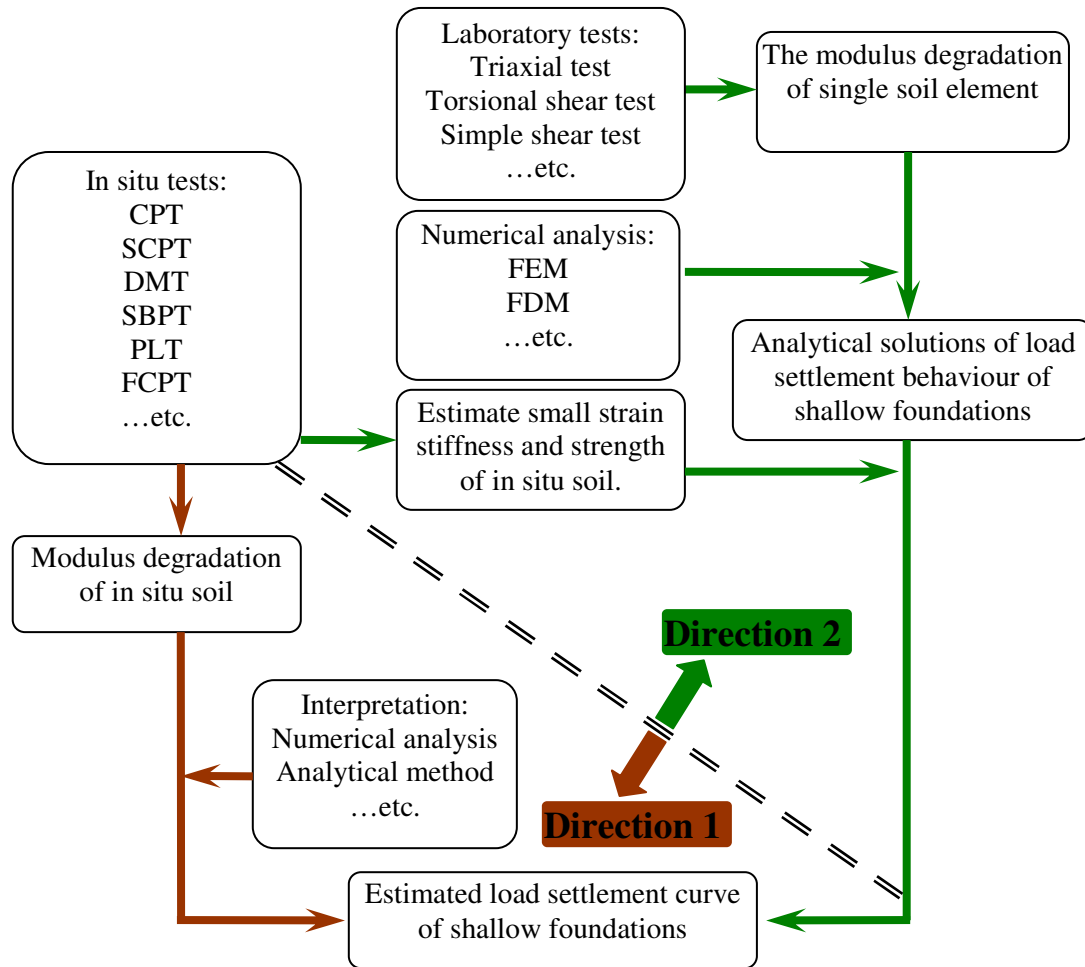


Figure 2.7: Schematic diagram illustrating the possible measures to estimate the settlement of shallow foundation on cohesionless soil

Secondly, as discussed in Section 2.4.2, the normalized modulus degradation shows different characteristics because of the initial condition and loading condition. Hence, particular attention should be given in the application of the concept of modulus degradation. For example, it may seem reasonable to apply the concept of normalized shear modulus degradation in estimating pile-soil response, particularly, for the case that reaction from pile base is not significant. For friction pile, the pile-soil response is close to the pure shear situation tested in torsional shear test. During loading of the pile, it is expected that the mean effective stress does not change considerably.

However, straightforward application of normalized shear modulus degradation to estimate settlement of shallow foundation on sand may not be a good idea, because at least the increase of the small-strain stiffness due to the increase of the mean effective stress is not accounted for by normalized shear modulus degradation. During loading of the foundation, mean effective stress is expected to increase. As compensation, one may consider to adopt either Equation (2.12) or Equation (2.13) in the calculation instead of using small-strain stiffness to consider the effect of changing mean effective stress (e.g. Fahey, 1993; Lee and Salgado, 1999). However, this makes the computation inconvenient and is usually solved by using FEM or explicit computation proposed by Wahls and Gupta (1994). Moreover, the effect of the overconsolidation cannot be accounted for accurately unless the unload-reload modulus is also considered. For example, Fahey (1993) analyzed unload-reload response of pressuremeter test using modified hyperbolic model. However, an assumed unload-reload modulus was used in his analysis.

Alternatively, one may consider applying the normalized Young's modulus degradation measured in triaxial test to estimate the settlement of shallow foundation on sand (e.g. Mayne, 1994 and Lehane and Cosgrove 2000). Although the loading condition is not exactly the same as experienced by the soil beneath the shallow foundation, triaxial test provides the closer loading condition compared with torsional shear test or simple shear test. In both triaxial test and foundation loading, the mean effective stress increases. In this case, the hyperbolic model and its modification can still be used by assuming that Poisson's ratio does not change significantly during loading (e.g. Lee and Salgado, 1999). The effects of overconsolidation and initial confining stress can be catered for in the triaxial tests and represented by hyperbolic model implicitly by adopting appropriate material constants.

When a shallow foundation is loaded or a PLT is conducted, the applied pressure q' increases from zero to possibly the ultimate bearing capability q_{ult} . Accordingly, the average strain ϵ_{av} ($\epsilon_{av}=s/z_i$, where s =settlement and z_i =depth of influence) induced inside the soil mass increases from zero to its maximum value ϵ_{max} . The equivalent

soil stiffness modulus E_{eq} ($E_{eq} = q'/\epsilon_{av}$), decreases from its maximum value of E'_0 (E_{max}) to a minimum at failure. This process is quite similar as that shown in Figure 2.2. Therefore, there is a need to investigate if there is some correlation between the modulus degradation measured on a single element in laboratory tests and the equivalent modulus degradation measured in footing tests.

2.6 Summary

In this chapter, the main factors affecting the settlement of shallow foundation on cohesionless soil were summarized. Existing methods for estimating the settlement of shallow foundation on cohesionless soil were reviewed. The advantages and disadvantages of these methods were discussed. The modulus degradation of soils observed from laboratory tests and the factors affecting the modulus degradation of cohesionless soil were reviewed. Some examples of the application of the concept of the modulus degradation of soil from small strain stiffness were given. The knowledge gaps between the concept established from laboratory test results and the applications in practical geotechnical problems were identified. Possible measures to cover the knowledge gaps were discussed.

Chapter 3 Vertical Strain Influence Factor Diagram

3.1 Introduction

For those semi-empirical methods based on elastic theory to estimate settlement of shallow foundation on cohesionless soil, displacement influence factor needs to be known. However, it is more rational and accurate to adopt vertical strain influence factor diagrams or vertical displacement influence diagrams, rather than using a displacement influence factor, considering the fact that soil profile is rarely homogeneous and always layered in situ. In this chapter, FEM was used to study the vertical strain influence diagrams of uniformly loaded, circular and rectangular foundations resting on homogeneous elastic layer underlain by a rigid base. The effects of the Poisson's ratio, foundation rigidity, foundation geometry and finite depth of the soil layer on the strain influence factor diagrams were investigated. Compared with the vertical displacement influence factor diagrams, the vertical strain influence factor diagrams were found to be more straightforward and effective to handle those aspects affecting it. Based on the analyses of vertical strain influence factor diagrams, simplified vertical strain influence factor diagrams for calculating settlement of shallow foundation on layered soil was proposed.

3.2 Background

Most methods for estimating settlement of shallow foundation on cohesionless soil are semi-empirical and developed from elastic method. Compared with purely empirical methods, such as Burland and Burbidge's method (1985), semi-empirical methods prevailed in researches of the settlement estimation of shallow foundation, because they have clearer theoretical basis and are potentially more accurate.

The general form of foundation settlement equation based on elastic theory can be expressed as either Equation (2.1) or:

$$s = \frac{qBI(1-\nu^2)}{E_s} \dots\dots\dots (3.1)$$

where q = applied uniform stress, B = foundation width (or diameter); E_s = equivalent elastic Young's modulus of the medium within the depth of influence; ν = Poisson's ratio; and I = vertical displacement influence factor, which depends on many factors such as foundation geometry, foundation rigidity, foundation roughness, foundation depth, soil profile and soil properties.

In using Equation (3.1), approximation has to be made to determine two parameters. One is vertical displacement influence factor I and the other is E_s . It can be seen from Equation (3.1) that the accuracy of vertical displacement influence factor I affects the accuracy of settlement estimation proportionally. Theoretically, one may be able to find a suitable value of I based on foundation shape, foundation rigidity and thickness of underlying soil layer. However, considering the fact that a great number of situations can exist, it is not convenient to do so (Mayne and Poulos, 1999). Typical displacement influence factors used in practice include the Steinbrenner's (1934) influence factor for the settlement calculation at the corner of a rectangular, flexible, uniformly loaded area. Enhancement of the Steinbrenner's influence factor was presented by Timoshenko and Goodie (1951) considering the effect of foundation depth. One may also calculate approximate value of I by following Mayne and Poulos' (1999) method using spreadsheet based on the solution of elastic theory.

The determination of the equivalent Young's modulus E_s of cohesionless soils is more difficult. One method is to calculate the average value of the Young's modulus of soils by thicknesses of soil layers within the depth of influence z_i , as suggested by Bowles (1986 and 1987). In this case, the depth of influence z_i must be determined beforehand. According to Burland and Burbidge (1985), for practical

purpose, the depth of the influence can be assumed to be the depth at which the settlement is 25% of the surface settlement. For a uniformly distributed circular load on isotropic homogeneous elastic half space, this depth is usually taken as $2B$. However, this may not be true for rigid foundations, or foundations of other shapes, although sometimes $2B$ is also taken as depth of influence for these foundations without being questioned. Bowles (1986 and 1987) suggested that depth of influence can be taken as $5B$ in settlement estimation. In fact, the depth of influence z_i depends on those factors affecting the vertical displacement influence factor or vertical strain influence diagram. For accurate settlement estimation, vertical displacement influence factor diagram or vertical strain influence factor diagram should be relied upon instead of a single valued displacement influence factor.

Furthermore, even if the depth of influence z_i is reasonably estimated, it is unsatisfactory to average Young's modulus by soil thickness. Figure 3.1 illustrates an example of two different layered soil profiles. Averaging Young's modulus by thickness of soil layers within the normalized depth of influence (assuming $2B$) of the two cases leads to the same value of E_s , which implies no difference between the calculated settlements according to Equation (1) for the two cases. This is in conflict with experiences.

A more accurate value of E_s can be obtained by averaging Young's modulus of soils by the influence on settlement of each layer. However, because the displacement influence factor diagrams are affected by so many factors, it is not feasible to take into account of all these factors. Fraser and Wardle (1976) used a similar idea to average Young's modulus. However, for convenience of implementation, only the displacement influence factor diagrams beneath centre of a perfectly smooth, uniformly loaded, square foundation were considered at three values of Poisson's ratios. Others factors, such as foundation rigidity, foundation geometry and finite thickness of soil layers were accounted for using a correction factor provided in a series of charts. This definitely decreases the accuracy of the estimation for case of layered cohesionless soil.

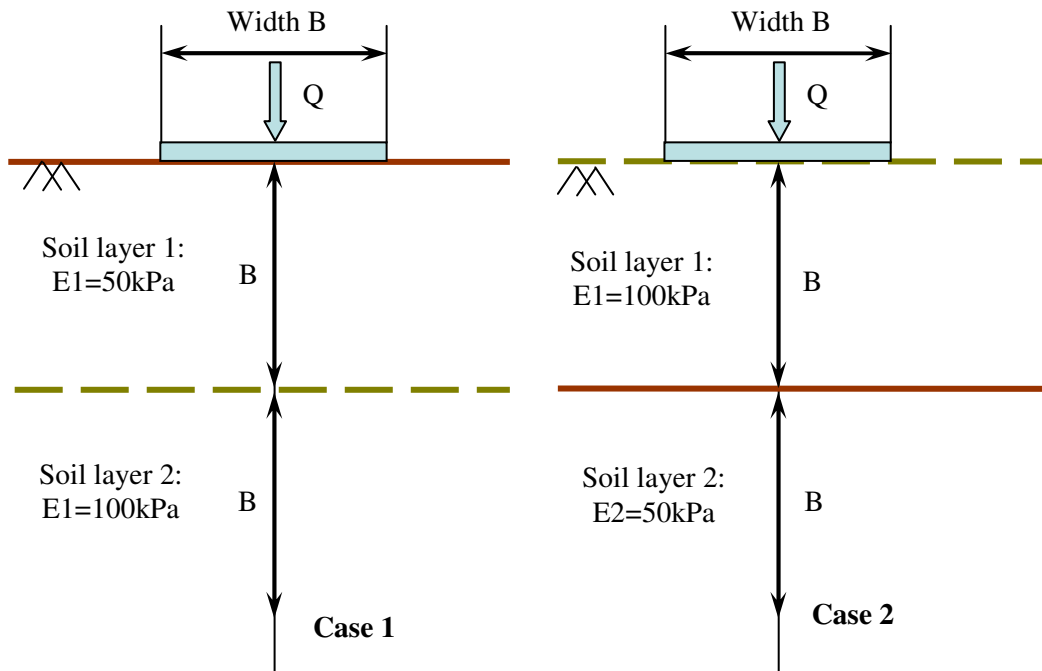


Figure 3.1: Two cases of the layered soil profiles

The vertical strain influence factor diagrams can be used to improve the accuracy of averaging E_s . Typical example is the well-known Schmertmann's (1970, 1978) method. Compared with vertical displacement influence factor diagrams proposed by Fraser and Wardle (1976), vertical strain influence factor diagrams are more straightforward in handling those factors affecting them. As a result, several simplified vertical strain influence factor diagrams can be found in the literature (e.g. Meri and Shahien, 1994; Jeanjean, 1995 and Briaud, 2007). Figure 3.2 shows three examples of the simplified vertical strain influence factor diagrams. These diagrams simplified the strain influence diagrams using three points and two lines: point A1 is the strain influence factor at ground surface; point B2 is the maximum strain influence factor at a specified depth and point C3 is the strain influence factor of zero at maximum depth of influence. The area of enclosed by the two lines between three points and the two axes should be equal or close to the displacement influence factor.

It can be seen from Figure 3.2 that Schmertmann's (1970, 1978) method adopts two different strain influence diagrams for square and strip footing, respectively. Jeanjean (1995) and Briaud (2008) preferred to use one simple diagram with larger maximum strain influence factor at ground surface compared with Schmertmann's method. Mesri and Shahien (1994) used the following expression to determine the depth of influence z_i :

$$\frac{z_i}{B} = 2(1 + \log \frac{L}{B}) \dots\dots\dots (3.2)$$

where $1 \leq L/B \leq 10$, B = footing width; L = footing length. Chang et al. (2005) found that the following expression produces a closer displacement influence factor compared to elastic theory:

$$\frac{z_i}{B} = 2.5(1 + \log \frac{L}{B}) \dots\dots\dots (3.3)$$

However, for convenience, all these vertical strain influence factor diagrams were simplified significantly at the cost of accuracy. In this chapter, FEM was used to investigate the effects of Poisson's ratio, foundation rigidity, foundation geometry, finite thickness of soil layer and Young's modulus of soil on the vertical strain influence diagrams based on elastic theory. It was found that the effects of these factors on the vertical strain influence factor diagrams can be captured without much effort. Based on the investigation, simplified vertical strain influence factor diagrams considering these factors are proposed together with correction factors of displacement influence factor. Therefore, a better balance between convenience of application and accuracy of settlement estimation was achieved based on the proposed simplified vertical strain influence factor diagrams and correction factors.

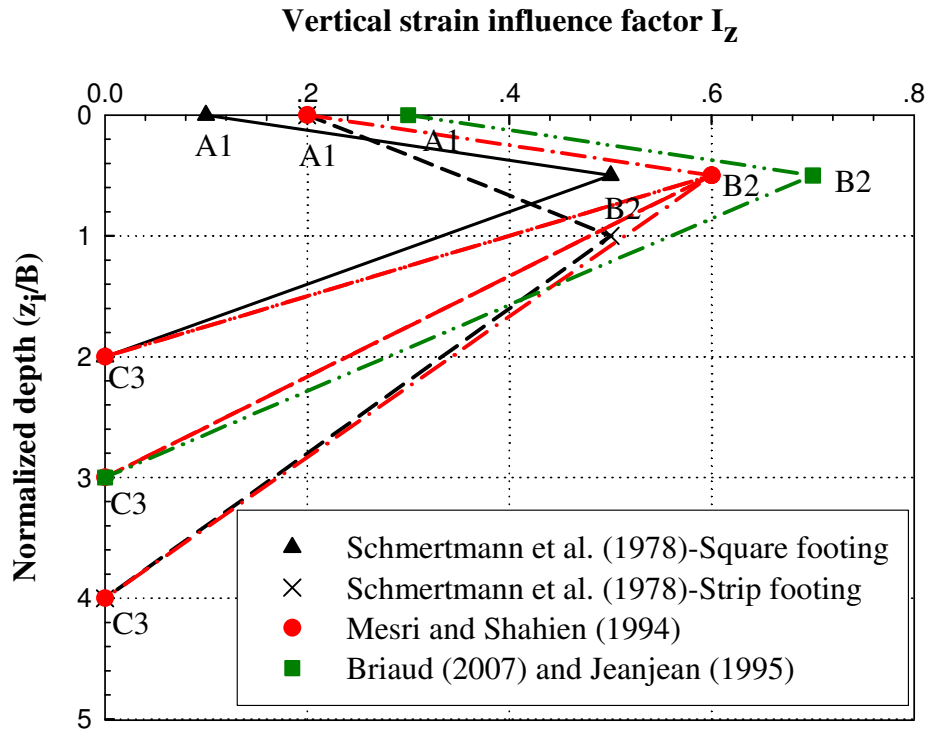


Figure 3.2: Examples of simplified vertical strain influence factor diagrams

3.3 FEM Simulation and Setup

In this chapter, FEM software ABAQUS was used to investigate the effects of various factors on the vertical strain influence factor diagrams. In the subsequent sections, “foundation” is used to refer to the structural component, i.e., footing or raft of a foundation system. Circular foundation was simulated using axisymmetric models. Square and rectangular foundations were simulated in three dimensions. Due to the symmetry, only a quarter model was used in the simulation. Foundations were modeled in three dimensions to study the effect of foundation rigidity on the vertical strain influence factor diagrams. The width of the foundation was 1m; the length of the foundation was varied from 1, 2, and 4 to 10m. The thickness of the foundation was 0.25m. The lateral boundary was ten times of the length of the footing size. The bottom boundary was varied according to the thickness of the soil assumed. Table 3.1 gives the summary of the dimensions of foundation and soil in the FEM model.

Table 3.1: Dimensions of foundation and finite element soil model in the simulation

Model		Dimension
Foundation	Width B	1m
	Length L	1m, 2m, 4m and 10m
	Thickness t	0.25m
Soil model	Lateral boundary	10 times of the foundation size
	Vertical boundary	$10\sqrt{BL}$, $4\sqrt{BL}$, $2\sqrt{BL}$, $\sqrt{BL}$, $0.5\sqrt{BL}$, $0.25\sqrt{BL}$

In all simulations, the interface between the base of the foundation and the soil was assumed to be rough. Normal contact provided by ABAQUS was applied between foundation and the soil. The foundation bottom is rough. To simulate a perfectly rigid foundation, a very large Young's modulus can be assigned to the foundation, or alternatively a uniform displacement condition on the area of the foundation can be applied. The two methods gave similar results in term of both displacement influence factor and strain influence diagram. For fully flexible foundations, a uniform pressure was applied on the ground surface.

Figure 3.3 plots a typical discrete model of a square footing and the influenced soil in the simulation. It can be seen that biased mesh is used. A solid element named C3D8R by ABAQUS was adopted. It provides reduced integration scheme to avoid so-called shear locking in the simulation.

In the discussion below, the vertical strain diagrams plot the vertical strain influence factors at various depths calculated beneath the centre of the foundation. Following the definition of Ahlvin and Ulery (1962), I_z , the vertical strain influence factors at depth z , can be calculated based on Hooke's law and the three normal stress components at this depth from FEM simulation. It can be expressed as:

$$\varepsilon_z = \frac{1}{E} [\sigma_z - \nu(\sigma_x + \sigma_y)] = \frac{q}{E} \left[\frac{\sigma_z - \nu(\sigma_x + \sigma_y)}{q} \right] = \frac{q}{E} I_z \dots\dots\dots(3.4)$$

where ε_z = vertical strain; σ_x , σ_y , σ_z = three normal stress components; E and ν = Young's modulus and Poisson's ratio of soil; q = applied pressure on the foundation. For axisymmetric condition, $\sigma_x = \sigma_y$ = radial stress. σ_z = vertical stress. The integration of I_z within normalized depth of influence z_i/B gives the displacement influence factor I defined by Davis and Poulos (1968). Accordingly, the integration of the vertical strain ε_z within the depth of influence gives the surface vertical displacement. In fact, integrating the results obtained from FEM and Equation (3.4) within the normalized depth z_i/B gives almost the same displacement influence factor I in Equation (3.1).

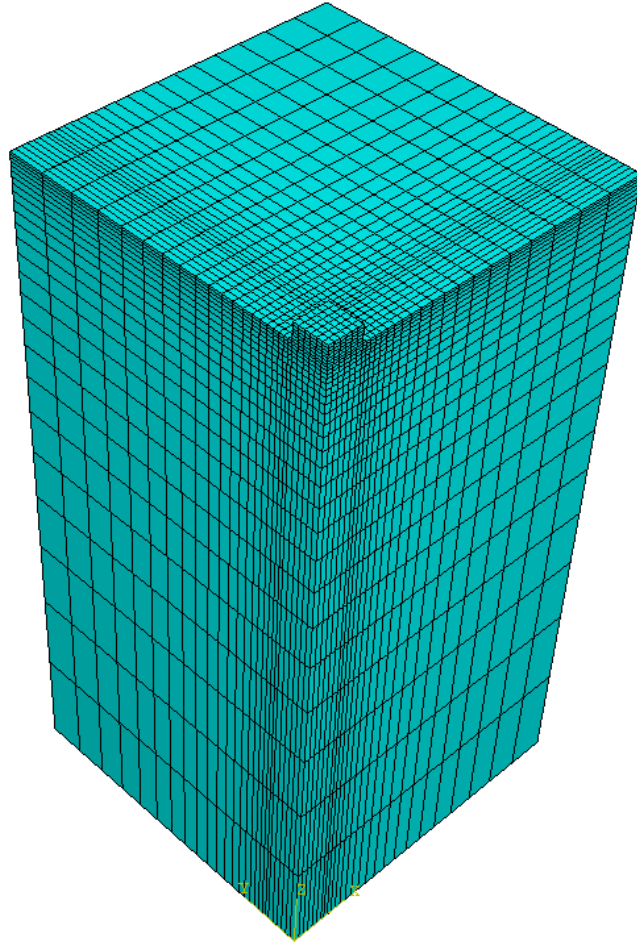


Figure 3.3: Discrete model of a square foundation

3.3.1 Effect of Poisson's Ratio

Poisson's ratio ν has considerable effect on the displacement influence factor as shown in Equation (3.1), so does the vertical strain influence factor diagram. The magnitude of I reduces by 25% as ν varied from 0 to 0.5. Figure 3.4 shows the vertical strain influence factor diagrams beneath the centre of a circular flexible and a circular rigid footing foundation, for various Poisson's ratios (0, 0.2 and 0.5). The thickness h of the soil layer was $10B$, although in Figure 3.4 the y-coordinates is up to $5B$ only.

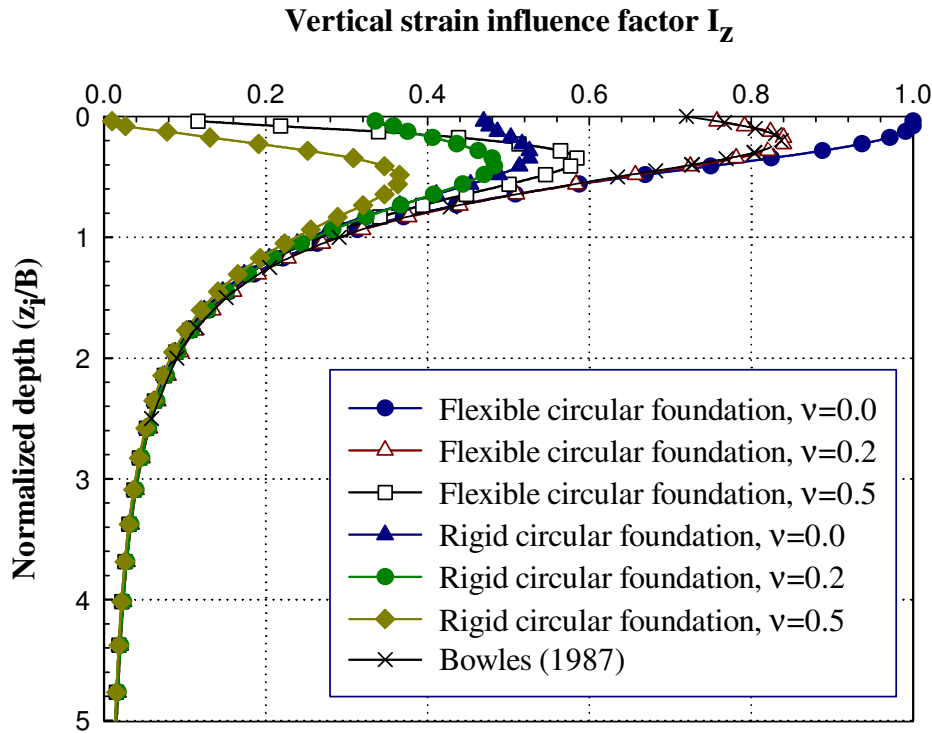


Figure 3.4: Poisson's ratio effect on vertical strain influence factor diagrams of circular foundations

It can be seen that only for flexible foundation and $\nu = 0$, the maximum strain influence factor occurs in the soil immediately beneath the bottom of the foundation. For flexible foundations and other values of Poisson's ratio, the maximum strain occurs at some depths no more than $0.35B$ below the foundation. While for rigid foundation, the maximum strain influence factor occurs at depth from about $0.5B$ to

0.4B as Poisson's ratio decreases from 0.5 to 0. Figure 3.4 also shows that regardless of the rigidity of the foundation, the variation of Poisson's ratio only causes significant changes of strain influence factor diagram within depth around B. For the strain influence diagram of depth greater than B, there is negligible change induced by Poisson's ratio.

It is possible that the strain influence factor within depth of B is affected by singularity point in stress distribution field near the foundation edge. To investigate the effect, a similar analysis of rigid foundation assuming $\nu = 0.5$ was performed by using element C3D20 with 20 nodes, instead of C3D8R, which only have 8 nodes each element. Figure 3.5 compares the vertical strain influence factor diagram beneath the centre and edge of foundation based on the two elements. It can be seen that by using element C3D20, the stress singularity point at the edge of the foundation can be better simulated, compared with element C3D8R. However, almost no difference can be observed between the strain influence factor diagrams beneath the foundation centre. Therefore, it can be concluded that the effect of stress singularity near the foundation edge on the vertical strain influence factor diagram beneath the foundation centre is not significant and can be neglected.

The area enclosed by the curves and the two axes was integrated. Theoretically, the integration of the strain influence diagram is equal to the displacement influence factor. Base on the strain influence diagrams shown in Figure 3.4, the integrations were between 95.3% ($\nu = 0.5$) and 97.7% ($\nu = 0$) of that based on rigorous solutions (Brown, 1969a and b) for semi-infinite half space. For rigid footings, the ratios of the areas between of numerical results and rigorous solutions were between 92.9% and 93.3%. This error is small and acceptable compared with that in the measurement of the in situ soil stiffness. Figure 3.4 also shows strain influence diagram for flexible smooth circular uniformly loaded foundation from Bowles (1987) based on Timoshenko and Goodler (1951) equation. They fit perfectly well for all Poisson's ratio, although only $\nu = 0.2$ is plotted in Figure 3.4. For rectangular footings, Poisson's ratio shows similar effect on the vertical strain influence diagrams, i.e., only causing the change within the soil at depth within about 1B.

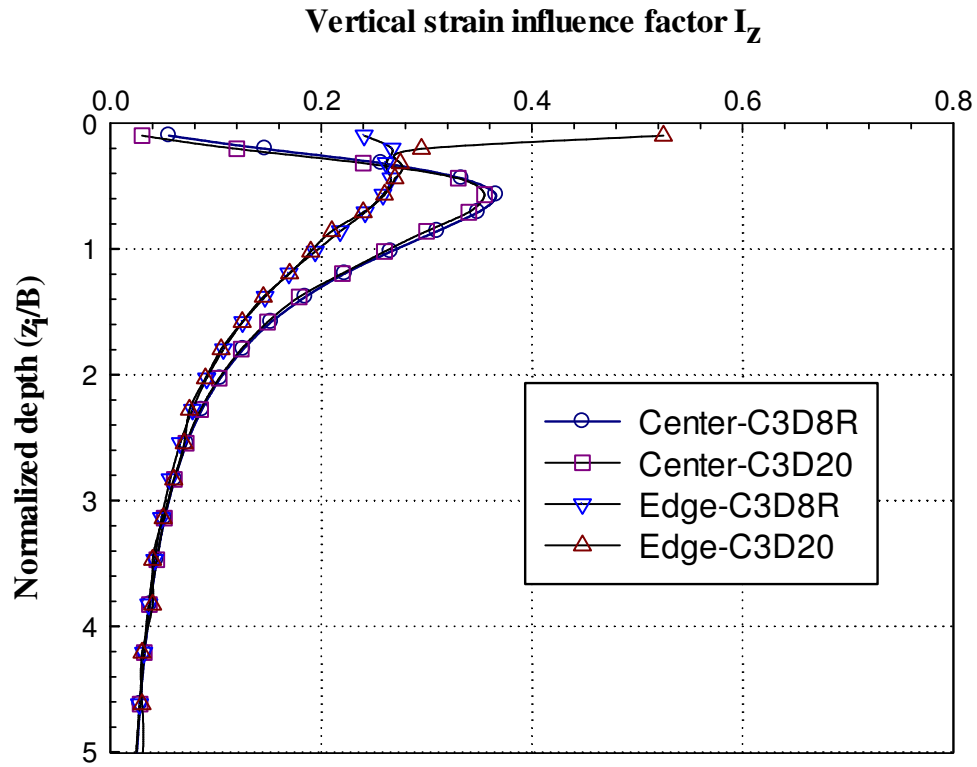


Figure 3.5: Comparison of vertical strain influence factor beneath the centre and edge of foundation based on element C3D8R and C2D20

Fortunately, typical magnitudes of Poisson's ratio for cohesionless soil under strains of working load on foundation do not vary in such a big range from 0 to 0.5. In early studies, Poisson's ratio is usually taken as 0.3 for dry cohesionless soil and 0.4 to 0.45 for wet cohesionless soil (Bowles, 1987). Probably that is the reason why in Schmertmann's method, I_z at ground surface is 0.1 and 0.2 for square and strip foundations. Recently, more accurate laboratory measurements by researchers such as Tatsuoka et al. (1994) and Lo Presti et al. (1995) indicate that, Poisson's ratio of cohesionless soil at small to intermediate strains is between 0.1 and 0.2. Given that ν varies between 0.1 and 0.2, Equation (3.1) does not vary significantly. Therefore, in the subsequent investigation, Poisson's ratio has been assumed to be 0.2 unless otherwise stated.

3.3.2 Effect of Foundation Rigidity

Foundation rigidity affects both the stress and strain distributions beneath the shallow foundation. Many methods estimating settlement of shallow foundation only account for foundation rigidity implicitly for convenience. Usually, small size footings can be assumed to be rigid but it seems more reasonable to treat rafts and mats as flexible.

Mayne and Poulos (1999) presented the following approximate equation to assess the effect of foundation rigidity on displacement influence factor:

$$I_F \approx \frac{\pi}{4} + I_d \dots\dots\dots (3.5)$$

where I_F = correction factor to account for foundation rigidity; $I_d = \frac{1}{4.6 + 10 \cdot K_F}$
(for fully rigid foundation $I_d \approx 0$ and for fully flexible foundation, $I_d = 1/4.6 \approx 0.22$.)
and K_F = foundation stiffness factor, which following Brown (1969b) can be approximated as:

$$K_F \approx (E_{fdn} / E_{sAV})(t / a)^3 \dots\dots\dots (3.6)$$

where E_{fdn} = elastic modulus of foundation; E_{sAV} = representative average elastic modulus of soil at depth of a ; t = foundation thickness; a = foundation radius.

From Equation (3.5), the value of I_F for a perfectly rigid circular foundation is $\frac{\pi}{4}$, and around 1.0 for a fully flexible foundation. As suggested by Mayne and Poulos (1999), $K_F > 10$ indicates a rigid foundation; $K_F < 0.01$ indicates a flexible foundation; and while $0.01 < K_F < 10$ indicates a foundation of intermediate rigidity.

Equation (3.6) may also be applied to rectangular foundations by converting L and B to equivalent radius by $a = \sqrt{4LB/\pi}$.

Figure 3.6 shows the vertical strain influence factor diagrams beneath the centre of a square foundation for cases of rigid, flexible and $E_{fdn}/E_{sAV} = 4$. It can be seen from Figure 3.6 that based on the elastic theory, variation of relative foundation rigidity only cause significant change of strain influence diagram within the depth of $0.5B$. The strain influence factor at ground surface increases from 0.35 to 0.76 as the foundation rigidity change from rigid to flexible ($\nu = 0.2$). The maximum strain influence factor increases from 0.48 at a depth of $0.41B$ to 0.84 at a depth of $0.17B$.

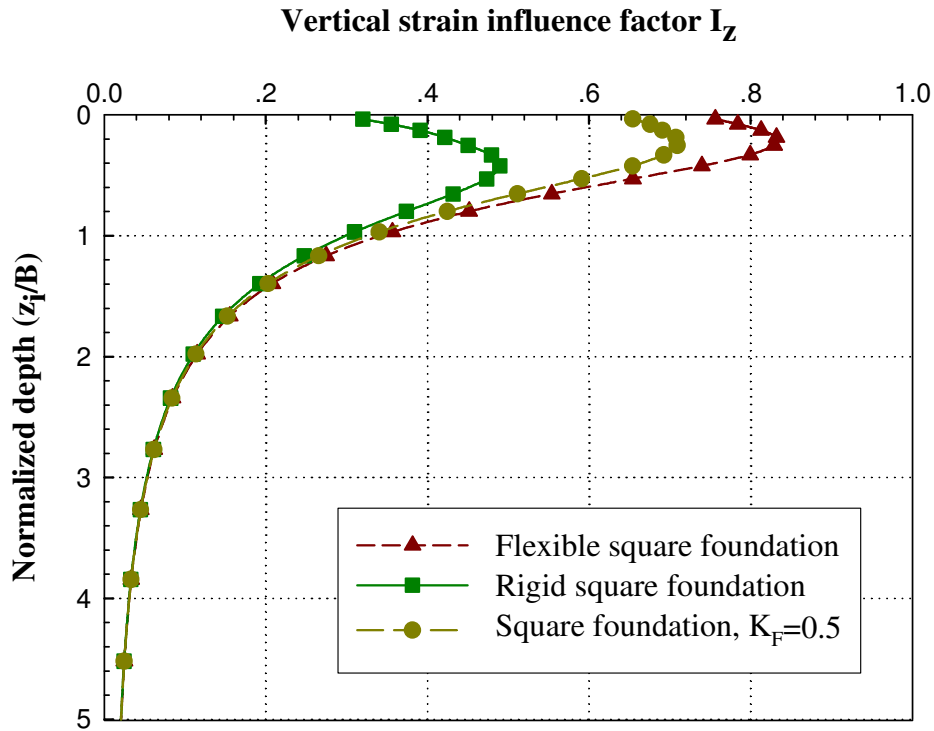


Figure 3.6: Effect of foundation rigidity on the vertical strain influence factor diagrams of square foundations

According to Equation (3.6) for a circular foundation with $t = 0.25\text{m}$ and $a = 0.5\text{m}$, K_F is about 0.5. The results show similar characteristic to those of circular foundations in Figure 3.4. It can be seen that, as the foundation becomes more flexible, the soil within $0.5B$ depth plays a crucial role in settlement. In settlement calculation, the effect of foundation rigidity can be accounted for more reasonably by adjusting the strain influence diagram, instead of factoring IF in Equation (3.5) especially if non-homogeneous soil occurs within this depth.

3.3.3 Effect of Foundation Geometry

Foundation geometry is another important factor influencing both the displacement influence factor and the strain influence diagram. Among several modifications of Schmertmann's method, one way to improve is to account for effect of L/B ratios on strain influence diagrams, for example, by introducing either Equation (3.2) or (3.3).

Figure 3.7 shows the strain influence diagrams of rigid and flexible foundations, respectively, for $L/B = 1, 2, 4$ and 10 and soil thickness $h = 10B_{eq}$, where $B_{eq} = \sqrt{LB}$. It can be seen from Figure 3.7a that for rigid foundations with various L/B ratios, the strain influence factors show consistent trend varying from around 0.37 at ground surface to 0.5 at depth of around $0.5B$. Below $0.5B$, with the increase of the L/B ratio, the strain influence factor increases. This implies that more vertical settlement occurs within the depth deeper than $0.5B$ as L/B increases. From another point of view, one may check the normalized depth where a certain I_z occurs, let's say 0.1. For $L/B = 1, 2, 4$, and 10 , the corresponding depths for $I_z = 0.1$ are about $2B, 3B, 4B$ and $5.5B$, i.e., the depth of influence increases as L/B increases.

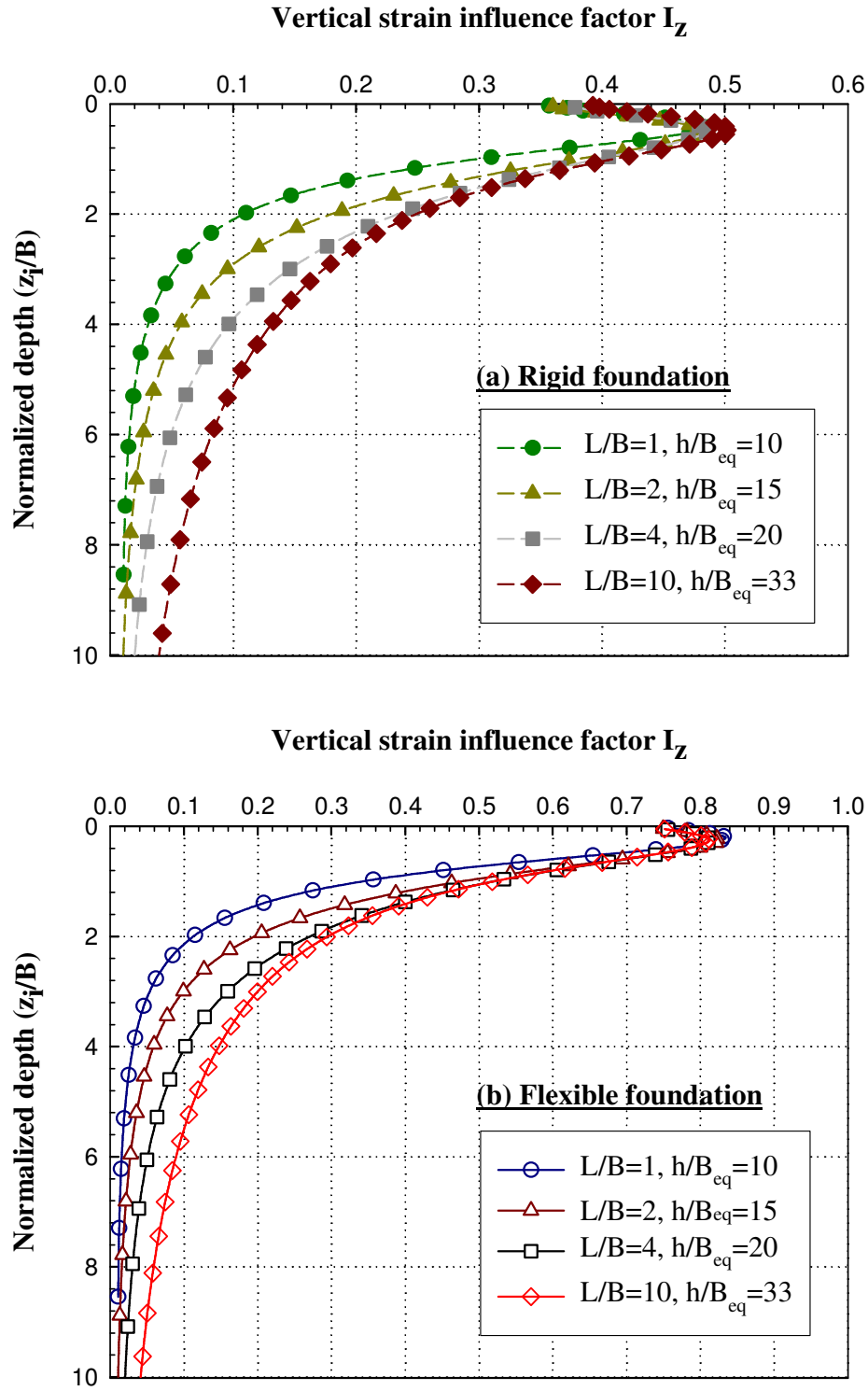


Figure 3.7: Effect of foundation geometry (L/B) on the vertical strain influence factor diagrams of (a) rigid (b) flexible rectangular foundations

Figure 3.7b also shows that for flexible foundations, the strain influence factors near ground surface are slightly more than two times of those of rigid foundations. The other difference is that the maximum strain occurs at shallower depth, i.e., around half of depth as that of the rigid foundations. With the increase of the normalized depth, the difference between strain influence factors of rigid and flexible foundations becomes smaller. It seems that when normalized depth is larger than $2\sqrt{LB}$, rigid foundations and flexible foundations have almost the same strain influence factors. This is consistent with the observation on the effect of foundation rigidity on the strain influence diagrams.

3.3.4 Effect of Finite Thickness of Soil Layer

The effect of finite thickness of the soil layer on both the displacement influence factor and the strain influence diagram are investigated in this section. Rigid and flexible foundations with different L/B ratios on finite thickness of soil layer h , which was varied from $h/\sqrt{LB} = 10$ to $h/\sqrt{LB} = 0.25$, with $L/B = 1, 2, 4$ and 10 were simulated. The resulting strain influence diagrams were integrated to produce the displacement influence factors, which were compared with published results.

Firstly, circular foundations were investigated. Figure 3.8 shows the strain influence diagrams within finite soil layer of rigid and flexible circular foundations. It can be seen that as the soil layer thickness h decreases from $10B$ to $4B$, the strain influence diagram only shifts slightly to the right near the bottom of the soil layer. Within the depth of $2B$, no apparent difference can be detected. As the thickness h decreases to $2B$, there is slight increase of the strain influence factor, particularly near the bottom of the soil layer. That means the soil near the bottom contribute more in terms of the settlement compared with thicker soil layer situations. However, the increase is not significant and there is no significant change of the profile of the strain influence diagram.

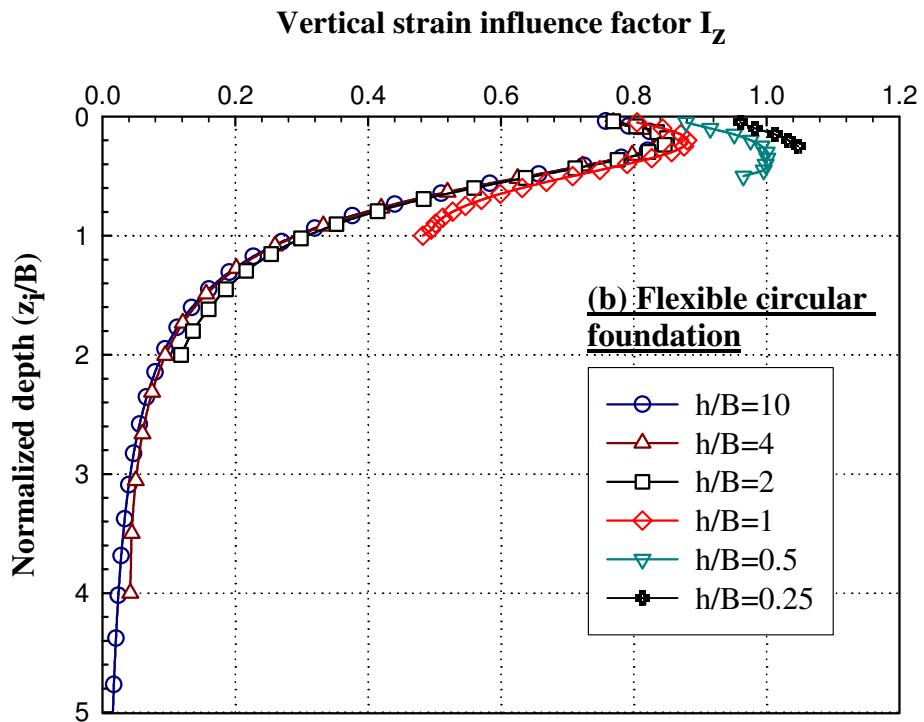
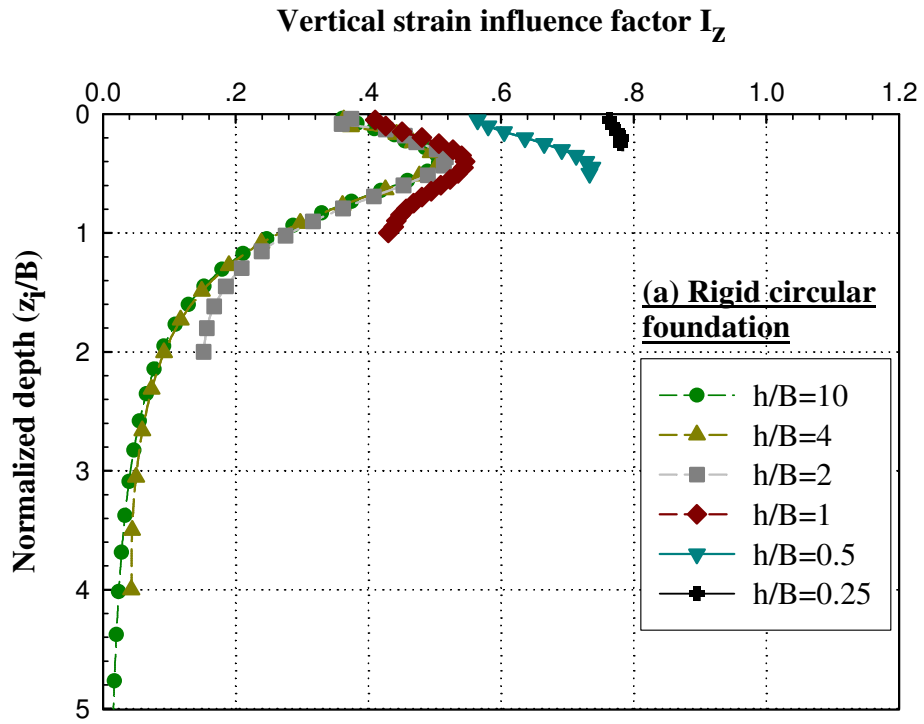


Figure 3.8: Effect of soil layer thickness on the vertical strain influence factor diagrams of (a) rigid circular foundation; (b) flexible circular foundation

For the case of $h = 1B$, the whole vertical strain influence factor diagram shifts to the right, with a big shift in the bottom part. As h decreases to less than $0.5B$, the increase of the value of strain influence factor is significant. However, the strain influence diagram still maintains a similar profile under these circumstances. This indicates that although the strain influence factor increases due to decrease of thickness of soil layer, the percentage of the settlement occurring within the soil at various depths may not change greatly compared to the situations of large soil thickness.

The areas enclosed by the vertical strain influence factor diagrams and the two axes are integrated. The results are compared with the displacement influence factors reported by others. Ueshita and Meyerhof (1968) obtained the rigorous solutions of the displacement influence factors for centre of circular area on finite thickness under distributed loading for flexible footings. Poulos (1968) reported the vertical displacement influence factor of rigid circular plate on finite elastic layer. Figure 3.9 compares the results based on FEM of this study and the two rigorous solutions with $\nu = 0.2$. It can be seen that the results are in very good agreement.

Secondly, rectangular foundations are investigated. Figure 3.10 to Figure 3.13 plot the vertical strain influence diagrams of the flexible and rigid rectangular foundations with $L/B = 1, 2, 4$ and 10 for cases of $h/B_{eq} = 1, 2, 4$ and 10 . It can be seen that similar effect of the finite soil thickness on the strain influence diagram as that of circular foundations can be observed. Generally, when soil layer becomes thinner, the profiles of the vertical strain influence factors are similar except near the bottom of the soil layer where I_z increase slightly, when h is larger than $2B$. When h is less than B , the vertical strain influence factor diagrams shift right. For rigid foundations (Figure 3.10a), when the thickness is small ($h < B$), the variation of the vertical strain influence factors with depth is not considerable. This is also applicable to flexible foundations for $h < 0.5B$. Similar observations are found for rectangular foundations with L/B varying from 2 to 10 , as shown in Figure 3.11 to Figure 3.13. Comparing rigid and flexible foundations, the strain influence factors for flexible foundations show bigger changes with depth. In this respect, it is more

crucial to use average Young's modulus weighted with displacement influence factor in settlement calculation for flexible foundations.

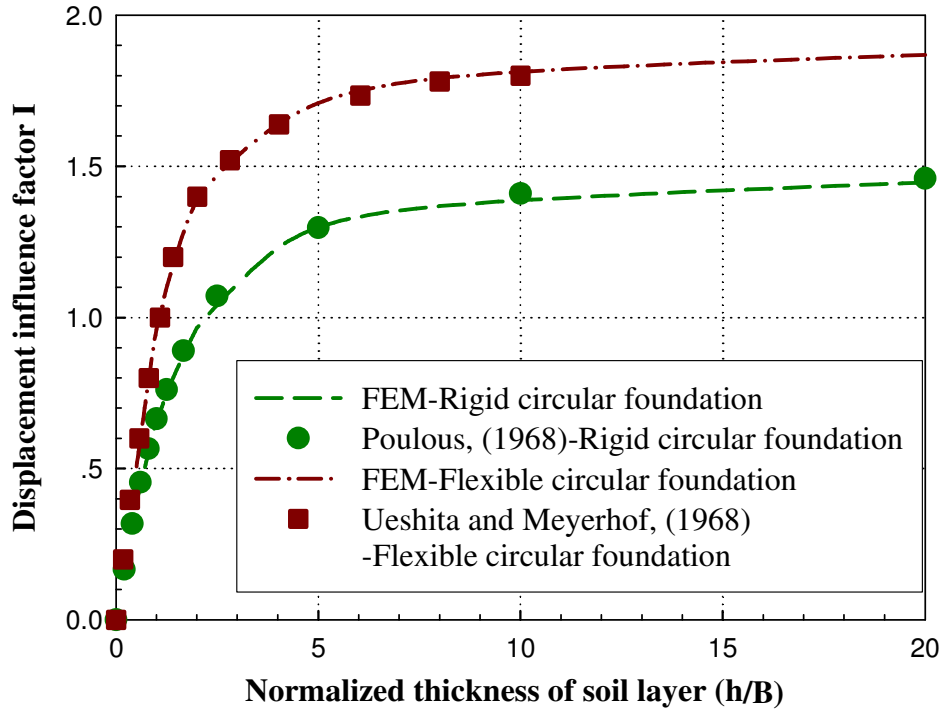


Figure 3.9: Displacement influence factors for circular foundations on finite soil layer

The area enclosed by the vertical strain influence factor diagrams and the two axes are integrated and the strain influence factors are compared with those reported by others. Harr (1966) presented the rigorous solutions of displacement influence factors for flexible foundation on finite to infinite elastic layers of smooth interface. Figure 3.14 shows that the integrated displacement influence factors based on FEM simulation of this study compare very well with those reported by Harr (1966) for rectangular foundations with $L/B = 1$ and 2 . Due to the limited layer depth (maximum $h/B_{eq} = 10$) simulated in the FEM, the displacement influence factors corresponding to the maximum thickness of soil layer is slightly smaller than those of infinite layer reported by Harr (1966).

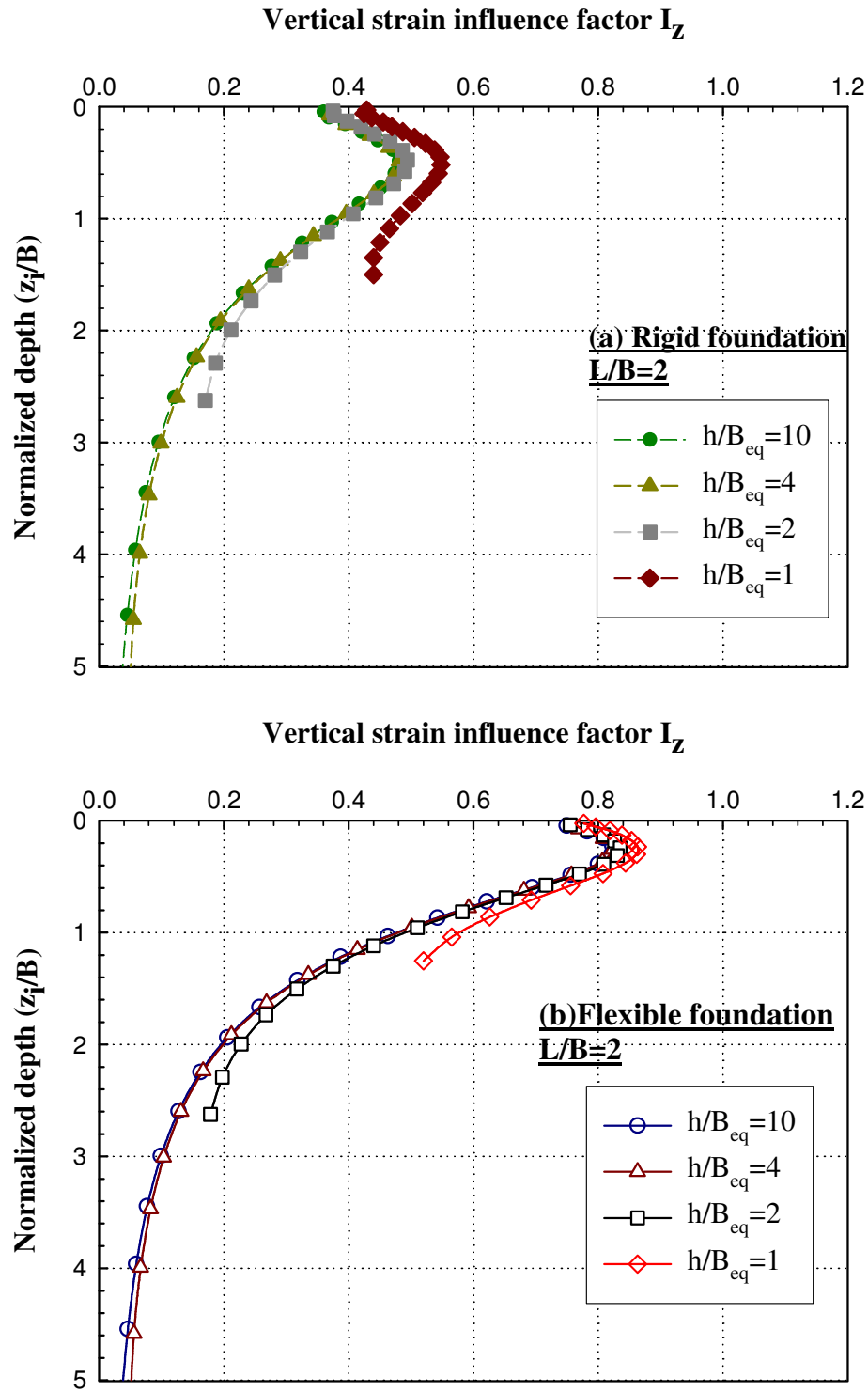


Figure 3.11: Effect of soil layer thickness on the vertical strain influence factor diagrams of (a) rigid; (b) flexible rectangular foundations ($L/B = 2$)

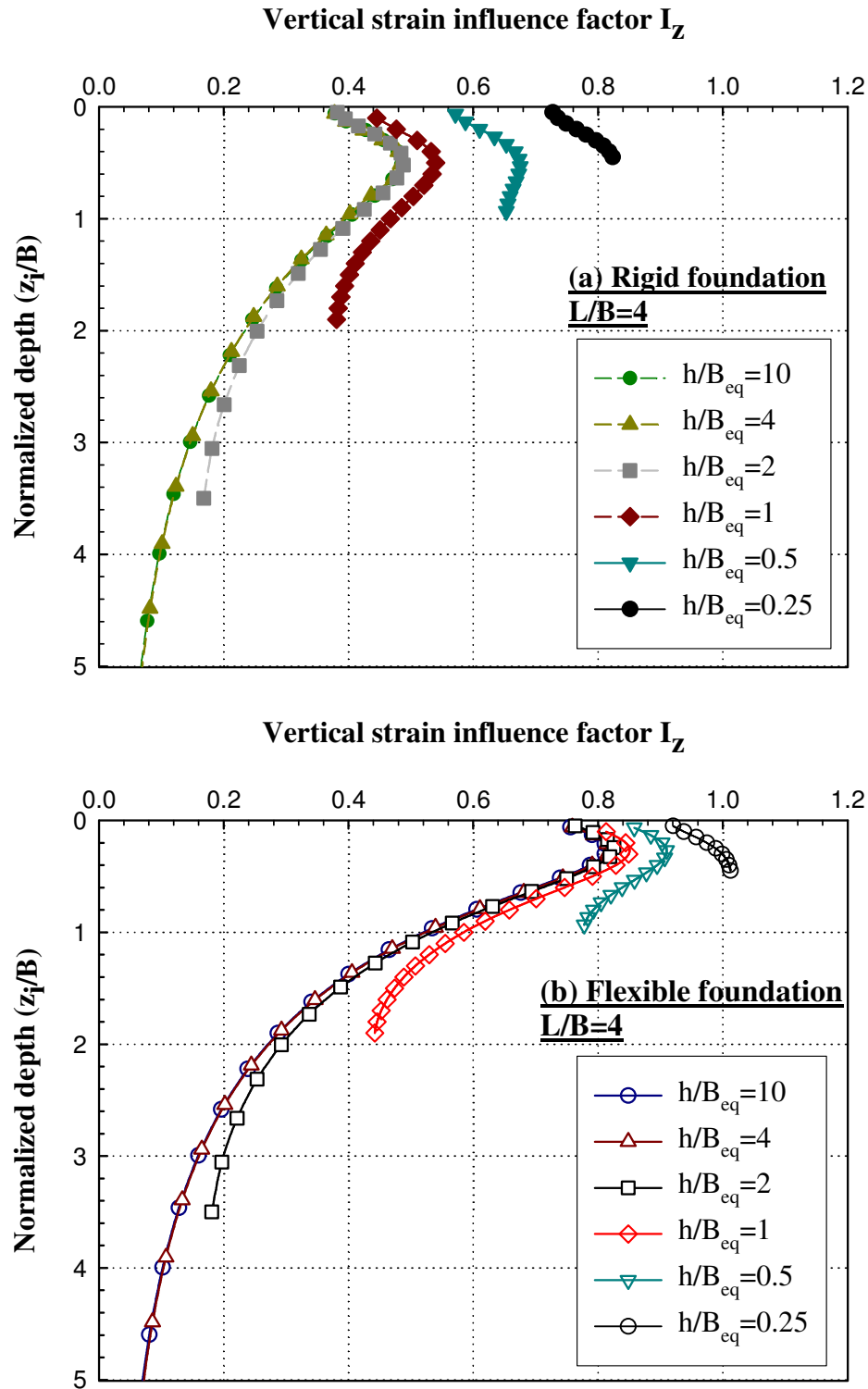


Figure 3.12: Effect of soil layer thickness on the vertical strain influence factor diagrams of (a) rigid; (b) flexible rectangular foundations ($L/B = 4$)

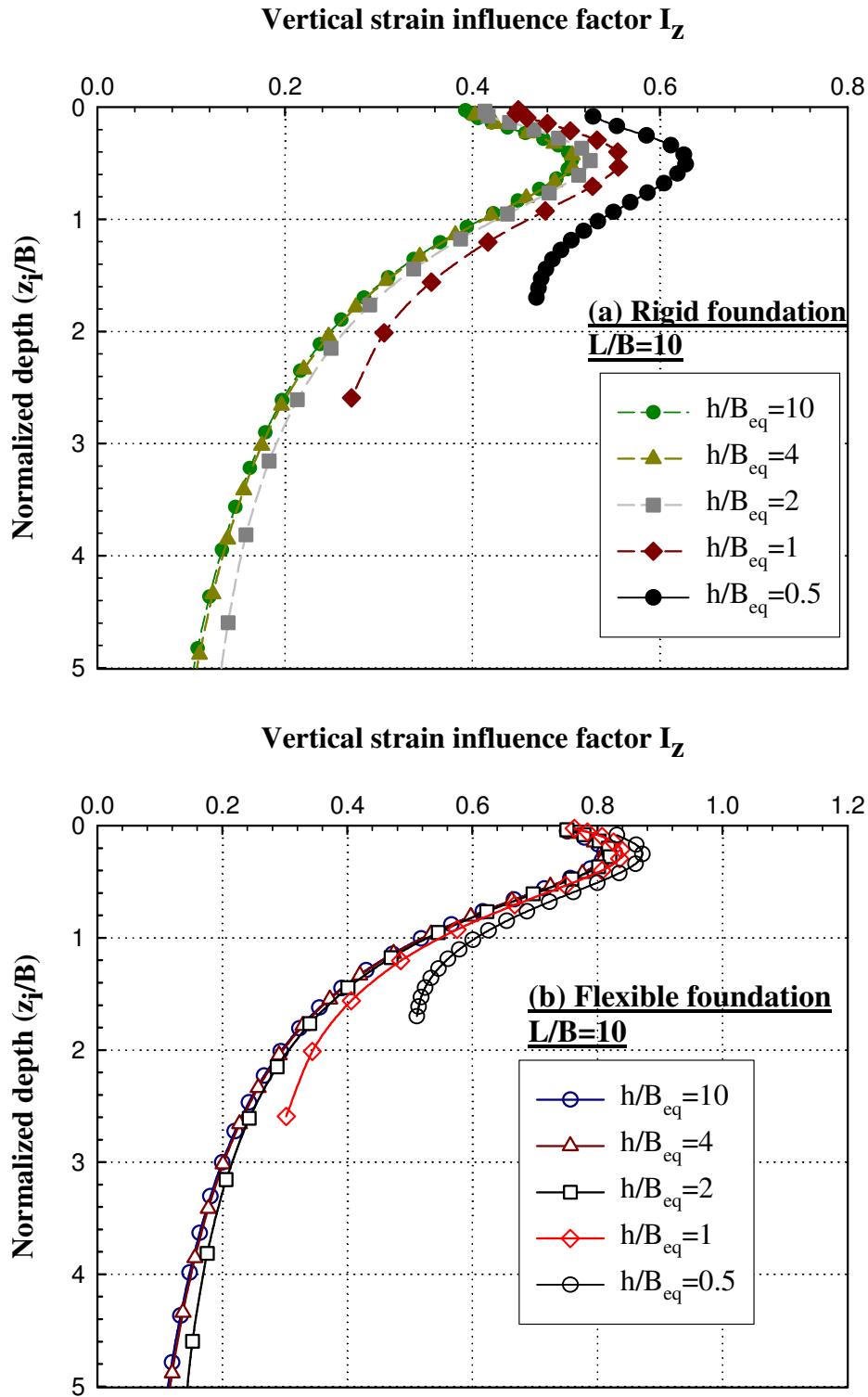


Figure 3.13: Effect of soil layer thickness on the vertical strain influence factor diagrams of (a) rigid; (b) flexible rectangular foundations ($L/B = 10$)

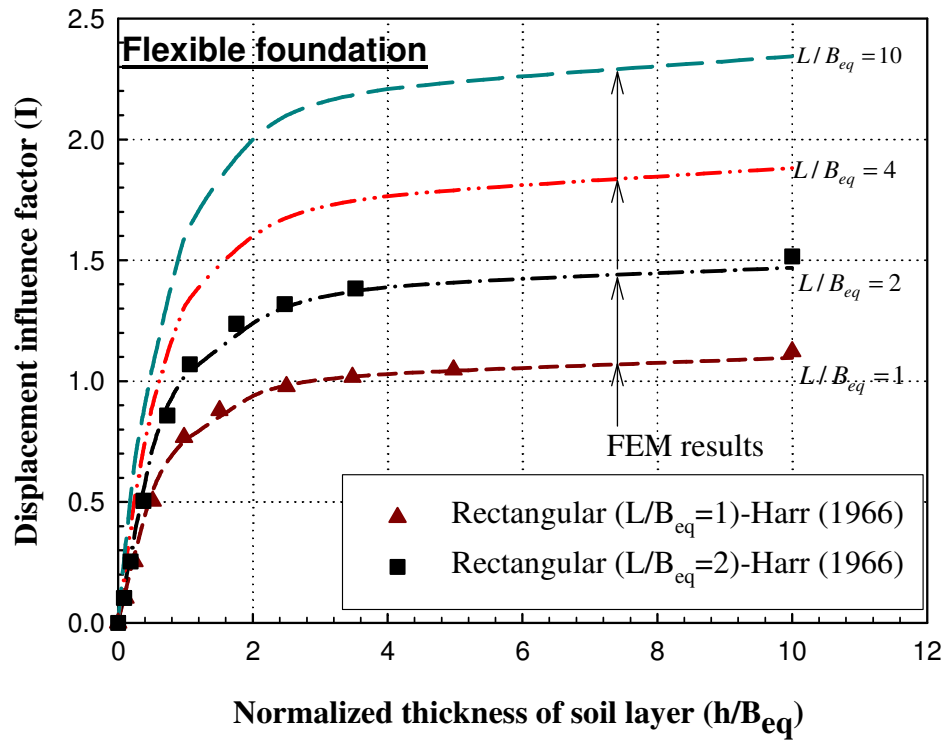


Figure 3.14: Displacement influence factors for flexible rectangular foundations on finite soil layer

Figure 3.15 shows the displacement influence factors for rigid rectangular foundations based on the FEM simulation in this study. Unfortunately, no rigorous solutions under exactly the same assumptions are available for comparison. However, the results compare well with FEM results. Whitman and Richart (1967) quoted the approximate solutions of displacement influence factors of rigid rectangular foundations on semi-infinite elastic medium. Figure 3.15 shows the comparison of their results with those from FEM simulation of this study. It can be seen that Whitman and Richart's (1967) approximate results are about 10% larger than FEM results.

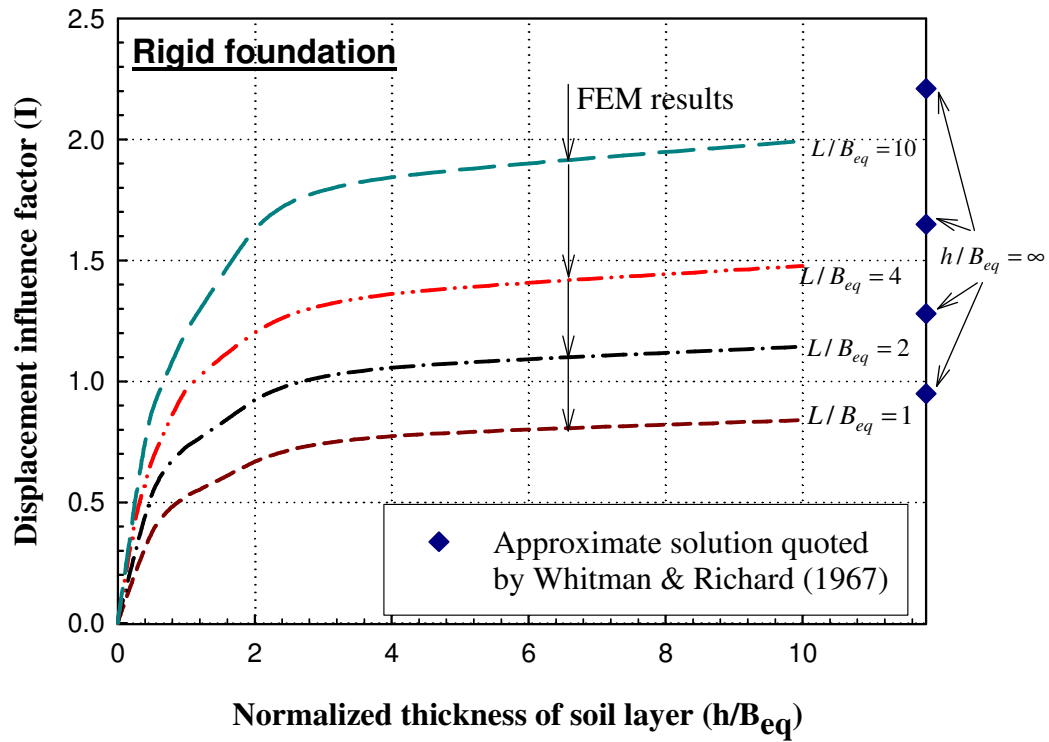


Figure 3.15: Displacement influence factors for rigid rectangular foundations on finite soil layer

For the convenience of application, two factors, i.e., soil thickness factor I_h and foundation shape factor $I_{L/B}$ which were based on the assumption of a linear elastic soil, will be used to reflect the effect of finite thickness of soil layer and foundation shape on displacement influence factor I , respectively. Figure 3.16 shows correlation between soil thickness factor I_h , which is displacement influence factor I normalized with the displacement influence factor I corresponding to $h/B_{eq} = 10$ and the normalized soil thickness h/B_{eq} . Both flexible and rigid foundations are included. It is noted that for rigid foundation, the magnitude of I_h is slightly less than that for flexible foundation except when $h/B = 10$. For convenience, this small difference can be neglected conservatively in practice. Therefore, only best match based on the flexible foundation is obtained, which can be expressed as:

$$I_h = 0.3 \ln(h/B) - 0.04(h/B) + 0.72 \quad (0.5 < h/B < 10) \dots\dots\dots (3.7)$$

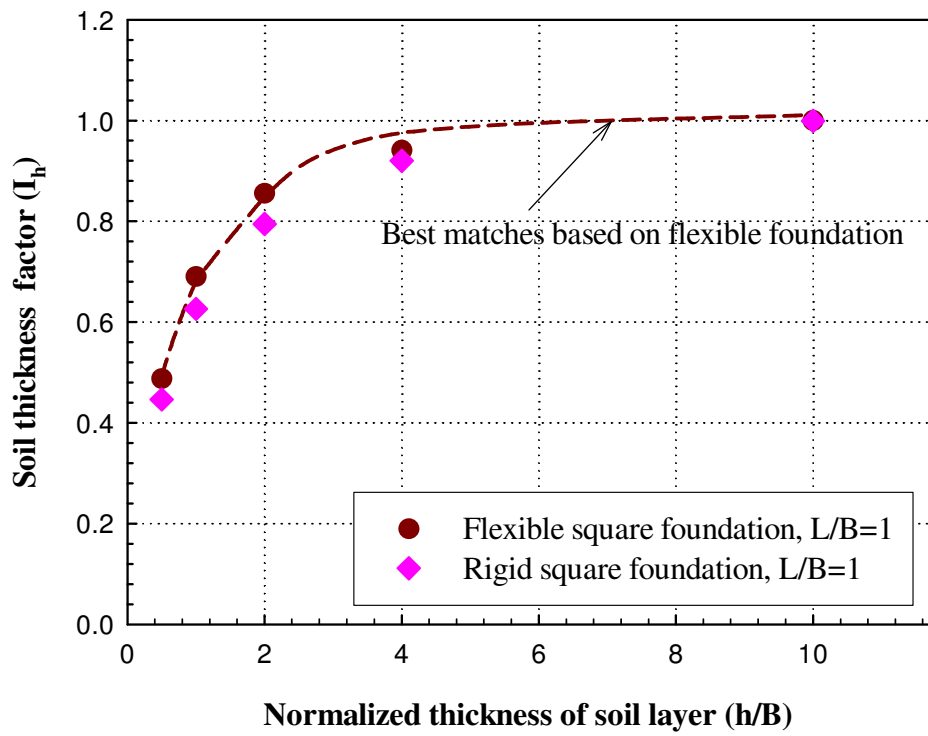


Figure 3.16: Soil thickness factor

Figure 3.17 shows the correlation between the foundation shape factor $I_{L/B}$ and the ratio of L/B for rectangular foundations. The foundation shape factor $I_{L/B}$ is calculated by normalizing the displacement influence factor with that of $L/B = 1$, when $h/B_{eq} = 0.5, 1, 2, 4$ and 10 . Best matches are established for both flexible and rigid foundation. They can be expressed as:

$$I_{L/B} = 0.6 \ln(L/B) + 1 \quad \text{for flexible foundation} \dots \dots \dots (3.8)$$

$$I_{L/B} = 0.5 \ln(L/B) + 1 \quad \text{for rigid foundation} \dots \dots \dots (3.9)$$

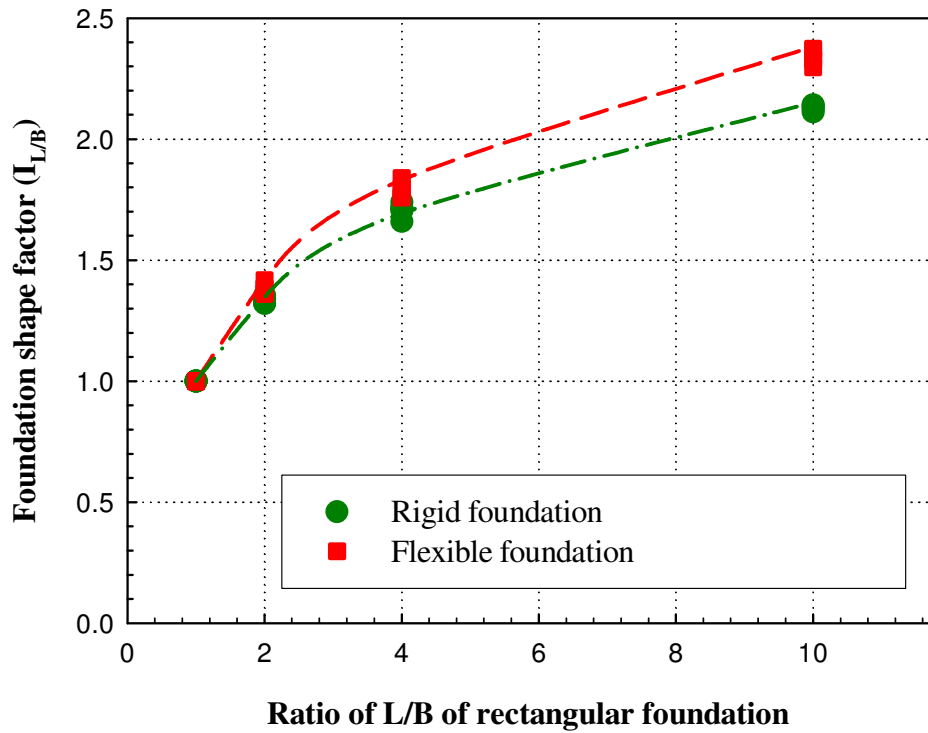


Figure 3.17: Foundation shape factor

3.3.5 Effect of Two-Layered Soil Profiles

Displacement influence factors relevant to layered soil profiles can be found in the literature. Compared with the factors mentioned above, there are fewer publications, possibly because it is too tedious in terms of the numerous combinations of number of soil layers, layer thickness, and elastic parameters of each soil layer. Approximate solutions of vertical displacement influence factors for multi-layer systems were summarized by Poulos and Davis (1974). One solution assumes that the stress profile of the layered system can be approximated by some rigorous solution, such as Bossinesq's (1885) solution; another solution use the equivalent Young's modulus.

Here, the vertical strain influence factor diagrams based on a two-layer soil system was investigated using FEM for a rigid circular foundation. The main purpose is to

examine the difference between the vertical strain influence diagrams of homogeneous and layered soil systems. If the difference is not significant, the vertical strain influence diagrams in homogeneous soils can be approximately applied to layered soils, such as Schmertmann's (1970, 1978) method. The interface between the two layers was assumed at depth of B . Poisson's ratios of the two layers were both 0.2. The ratios of the Young's modulus of the upper layer over lower layer $E_1/E_2 = 0.2, 0.5$ to 2 and 5. The total thickness of the both soil layers was $10B$.

Figure 3.18 shows the vertical strain influence factor diagrams for the two-layer soil system. It can be seen that for the cases that the upper soil stiffness E_1 is larger than the lower soil stiffness E_2 , the vertical strain influence factor diagrams are on the left side of that of the homogenous soils. It is interesting to observe that there is a sudden decrease of the vertical strain influence factor at the interface. For the cases that the upper soil stiffness E_1 is smaller than the lower soil stiffness E_2 , the vertical strain influence diagrams are on the right side of that of the homogenous soils. In this case, no sudden change of the vertical strain influence factor is observed at the interface.

One may be interested in the area enclosed by the vertical strain influence factor diagrams and the two axes, i.e., the magnitude of displacement influence factors of each case. Compared with homogeneous situation, the area was about 24% and 10% smaller for the cases of $E_1/E_2 = 5$ and 2, respectively. On the other hand, there is about 8% and 17% increase for the cases of the $E_1/E_2 = 0.2$ and 0.5, respectively. It is important to see that the profiles of the strain influence diagrams do not differ much. This difference of displacement influence factor indicates the possible error in the settlement calculation, given that the vertical strain influence diagram of the homogeneous soil, instead of the accurate one similar to that shown in Figure 3.18 is adopted. For instance, Schmertmann et al.'s method (Schmertmann et al., 1978) adopts simplified vertical strain influence diagrams based on homogeneous soil profile. In application, the layered soil is accounted for by taking different Young's modulus of each soil layer. More accurate estimation can be achieved if vertical

strain influence factor diagram based on layered soil is adopted. However, this is not feasible in practice.

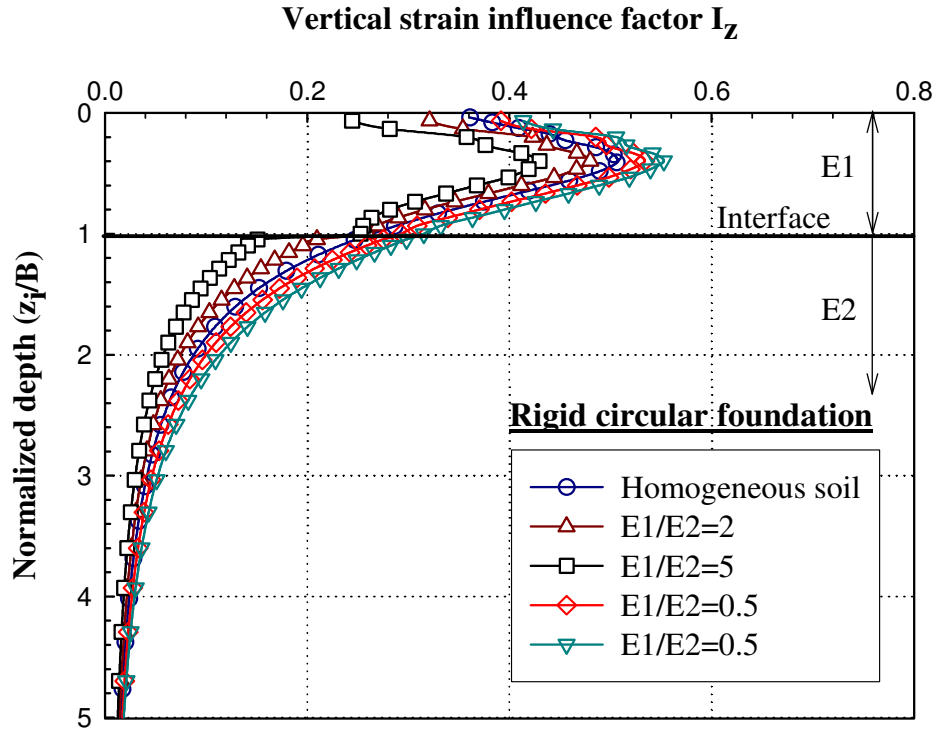


Figure 3.18: Vertical strain influence factor diagrams for rigid round foundations on two-layered soils

The error in not considering the vertical strain influence factor diagrams of layered soil in existing semi-empirical methods may be acceptable as actual soil profiles in situ are possibly much more complex than the two-layer soil system. However the effect of the layered soil on the vertical displacement influence factor or vertical strain influence factor diagram should be taken into account for accurate estimation of settlement of shallow foundation on cohesionless soil.

3.3.6 Effect of Gibson Soil

A special case of non-homogeneous soil was first studied by Gibson (1967) and later called Gibson soil. In Gibson soil the elastic Young's modulus E_s increases linearly with depth as:

$$E_s = E'_0 + k_E \cdot z \dots\dots\dots (3.10)$$

where E'_0 = Young's modulus of soil at ground surface ($z = 0$); k_E = rate of the increase of the E_s with depth z . For convenience of presenting the results, a normalized Gibson modulus ratio is defined as $\beta = E'_0 / (k_E B)$. According to Gibson (1967), the displacement influence factor depends on β value only, despite various combinations of the E'_0 and $(k_E B)$.

The vertical displacement influence factors for Gibson soils with various β values are available, (Gibson, 1967, and Mayne and Poulos, 1999). However, in practice, the concept of Gibson soil is not frequently implemented, probably because it is difficult to determine the normalized Gibson modulus ratio β . For small footing size, the increase of Young's modulus of soil within depth of influence is not obvious. In addition, the value of B is small compare to the value of E'_0 . As a result, the value of β can be very large, and the soil can be treated as homogeneous. In this chapter, a series of simulations on Gibson soil was carried out for completeness.

A User-Defined Material (UMAT) was incorporated into ABAQUS to simulate Gibson soil. Using UMAT, Young's modulus of soil was correlated to the vertical stress in the geostatic stage. In the loading stage, the Young's modulus was kept unchanged. This geostatic stage has no effect on the calculated vertical strain influence factor diagrams. In the simulation, $E'_0 = 5$ MPa, $B = 1$ m, $k_E = 1, 5, 10$ and 50 MPa/m. Accordingly, $\beta = 5, 1, 0.5$ and 0.1 .

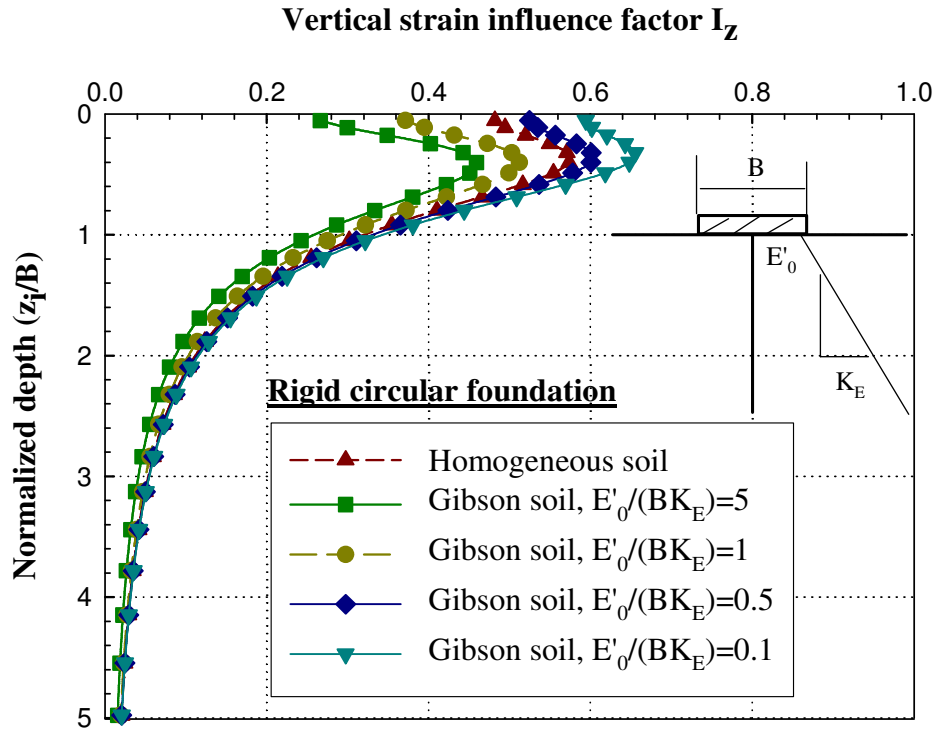


Figure 3.19: Vertical strain influence factor diagrams for rigid square foundations on Gibson soils

Figure 3.19 shows the simulated vertical strain influence factor diagrams of rigid square foundations on Gibson soils with various β values. It should be noted that the strain influence diagram also depends on β only, despite the different combinations of E'_0 and $(k_E B)$. From Figure 3.19 it can be seen that as β increases, the strain influence diagrams shift to the right. If the strain influence diagram of homogeneous soil is used for Gibson soil, it may underestimate the settlement. For example in this simulation for the case of $\beta = 0.1$, the integrated displacement influence factor is around 1.5 times of that of homogeneous soil.

3.4 Discussion of Simplified Vertical Strain Influence Factor Diagrams

At present, the rational way to account for the non-homogeneous soil profiles in settlement estimation of shallow foundation is to adopt vertical strain influence factor diagrams, or vertical displacement influence factor diagrams. Among the

existing methods, Wardle and Fraser (1976) adopted the vertical displacement influence factor diagram beneath a flexible square foundation. Briand (2007) and Jeanjean (1995) adopted vertical strain influence factor diagram with area of 1.125, which is equal to the displacement influence factor of flexible square foundations. The non-linearity of the load-settlement curve is accounted for by using a variable equivalent Young's modulus of soil. For Schmertmann's (1970, 1978) method, modified vertical strain influence factor diagrams based on rigid square and strip foundations are used. A variable maximum I_{zmax} dependent on loading q and self-weight of soil γ' is defined to account for the non-linear behaviour of soil. Thus, the displacement influence factor varies with q and γ' .

It can be seen that these simplifications did not accurately consider the effect of finite soil layer thickness, foundation rigidity and foundation geometry, except for Schmertmann's (1970, 1978) method, which separates square footings and strip footings. For homogeneous soil, using a correction factors to the displacement influence factor to account for these effects may produce reasonable settlement estimations. However, for inhomogeneous soil, simply using a correction factor for I_z may reduce the accuracy of the settlement estimation. A more rational way is to calculate the average Young's modulus E_s based on the influence of the displacement on each soil layer.

3.5 Proposed Simplified Vertical Strain Influence Factor Diagram

Wardle and Fraser (1976) proposed the following equation to calculate the average Young's modulus E_s of multi-layer soils consisting of n layers, by considering the influence of displacement in each soil layer:

$$\frac{1}{E_s} = \sum_{j=1}^n \frac{1}{E_j} \frac{I_j}{I_{total}} \dots\dots\dots (3.11)$$

Where E_j = Young's modulus of the j^{th} soil layer; I_j = vertical displacement influence factor in j^{th} soil layer and I_{total} = vertical displacement influence factor within depth of influence z_i .

For Wardle and Fraser (1976), the determination of the I_j and I_{total} is from the vertical displacement influence factor diagram of a fully flexible square footing, as shown in Figure 3.20. The finite thickness of the soil layer was also accounted for by using Figure 3.20.

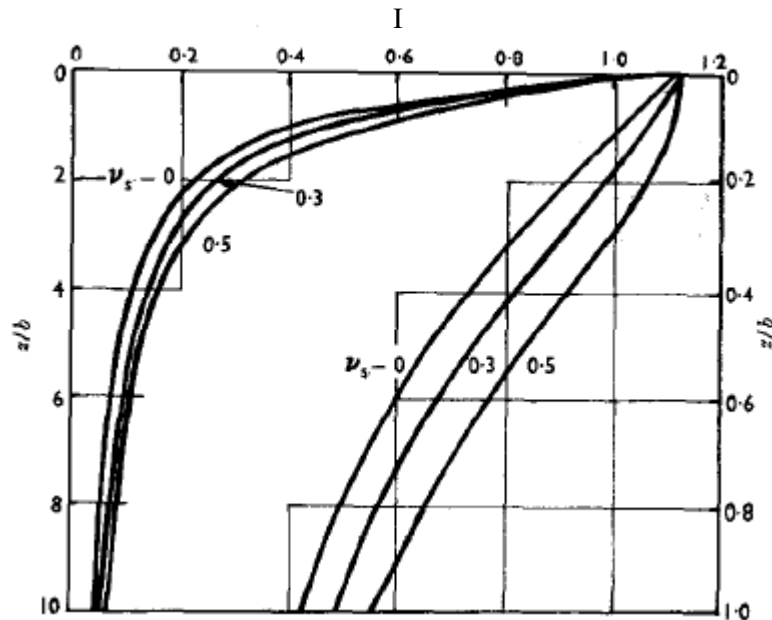


Figure 3.20: Vertical displacement influence factor diagrams for calculating average Young's modulus of soil in Wardle and Fraser (1976)

Improvement can be made by calculating the I_j and I_{total} using the proposed simplified vertical strain influence factor diagram shown in Figure 3.21 where foundation shape and foundation stiffness can also be considered.

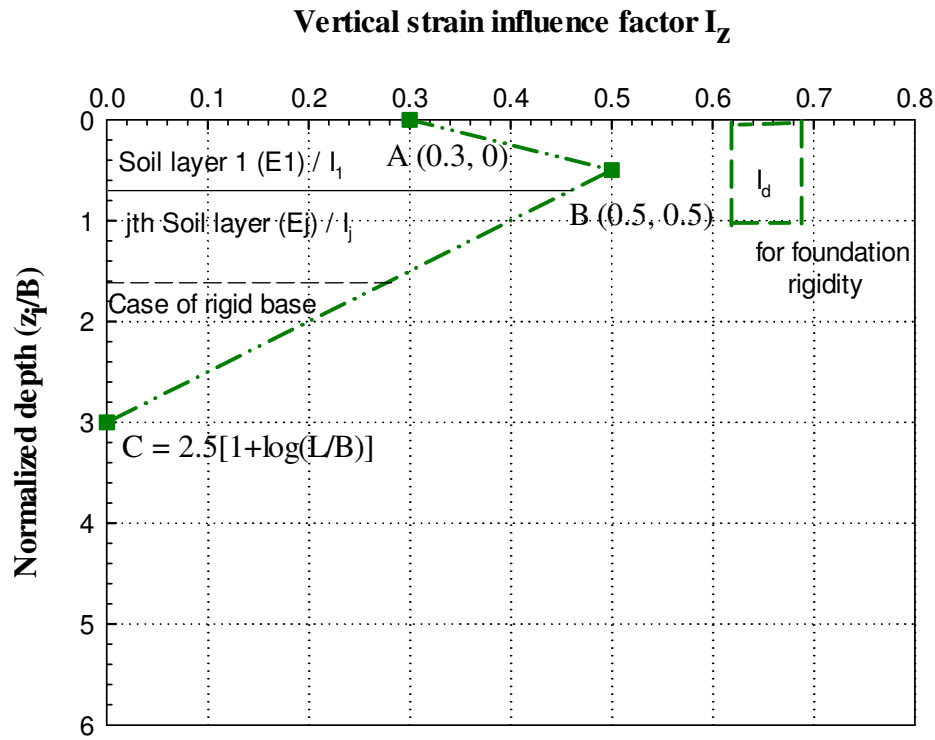


Figure 3.21: Proposed simplified vertical strain influence factor diagram

In Figure 3.21, the vertical strain influence factor at ground surface is 0.3 (Point A); the maximum strain influence factor is 0.5 at depth of $0.5B$ (Point B); the zero strain is at depth of influence z_i , where $z_i/B = 2.5[1+\log(L/B)]$, $1 \leq L/B \leq 10$ (Point C). The introduction of Point C is to account for the effect of L/B , or foundation shape, based on the study in Section 3.3.3. As L/B increase from 1 to 10, the normalized depth of influence increases from $2.5B$ to $5.0B$.

To take into consideration of effect of foundation rigidity, I_d given in Equation (3.5) is added to I_j within depth of B as shown in Figure 3.21. The effect of finite thickness of soil layer can be considered directly by terminating the vertical strain influence factor diagram at the depth of the rigid base, as illustrated in Figure 3.21. The values of I_j and I_{total} are the displacement influence factor in j^{th} soil layer, i.e., the area enclosed by the vertical strain influence factor diagram, the interface between soil layers and the two axes shown in Figure 3.21. The average Young's modulus can be calculated based on Equation (3.11).

Based on the above, settlement of shallow foundation on cohesionless soil can be estimated using average Young's modulus E_s , displacement influence factor I and correction factors I_F , I_h and $I_{L/B}$ as follows:

$$s = \frac{qBI}{E_s} I_F I_h I_{L/B} \dots\dots\dots (3.12)$$

where $I = 1$; I_F , I_h and $I_{L/B}$ are correction factors given in Equation (3.5), Equation (3.7), Equation (3.8) and Equation (3.9).

3.6 Summary

For inhomogeneous soil, the vertical strain influence factor diagrams or the displacement influence factor diagrams are necessary in order to estimate the settlement of shallow foundation on cohesionless soil more accurately. Existing semi-empirical methods for estimating the settlement usually adopt simplified diagrams, which omit many important features, such as foundation shape, foundation rigidity and finite thickness of soil layers. In this chapter, the vertical strain influence factor diagrams beneath the centre of the foundation were investigated based on FEM and elastic theory. The effects of Poisson's ratio, foundation rigidity, foundation geometry, and finite thickness of soil layer on the vertical strain influence diagrams were studied. The possible error by applying diagram from homogeneous soil to two-layered soil and Gibson soil was examined. The enclosed area of the obtained diagrams and two axes, i.e., the vertical displacement influence factors agree well with published results. Based on the understanding of the effect of these factors on the vertical strain influence factor diagrams, simplified vertical strain influence factor diagram and correction factors are proposed to account for finite soil thickness, foundation shape and foundation rigidity.

Chapter 4 Numerical Studies of Scale Effect of Bearing

Capacity Factor N'_γ

4.1 Introduction

In settlement estimation of shallow foundation, ultimate bearing capacity of shallow foundation is an important property and frequently used to normalize the foundation load. For shallow foundation on cohesionless soil, the ultimate bearing capacity does not increase linearly with increase of foundation width. This scale effect of bearing capacity factor N'_γ has been observed in laboratory tests. Several factors are believed to contribute to the scale effect. Among these factors, is stress- and density-dependent peak effective angle of internal friction of cohesionless soil ϕ'_p . Bolton's (1986) equation describing correlation between ϕ'_p and critical state friction angle ϕ_{cv} at constant volume shearing, relative density D_r and mean effective stress σ'_m is frequently used to estimate ϕ'_p of cohesionless soil. FLAC is used to evaluate the scale effect of bearing capacity of rigid, rough circular footing resting on the surface of cohesionless soils in this chapter. A user-defined modified MC constitutive model, incorporating Bolton's expression of stress- and density-dependent ϕ'_p , was implemented into FLAC so that peak strength of cohesionless soils can be computed based on mean effective stress σ'_m and relative density D_r during loading. Both associated and non-associated flow rules were assumed in the analyses. The effect of footing width on N'_γ (which is N_γ accounting for foundation shape) is investigated. The computed results are compared with published data based on both centrifuge tests and spread footing tests. Based on numerical results, charts were developed for determining ultimate bearing capacity considering the scale effect of N'_γ .

4.2 Background

Bearing capacity of a rough, rigid foundation in cohesionless soils under vertical loading can be described by classical Vesic's (1973, 1975) equation:

$$q_{ult} = \bar{q}N_q s_q d_q + 0.5B\gamma N_\gamma s_\gamma d_\gamma \dots\dots\dots (4.1)$$

where q_{ult} = ultimate bearing capacity; $\bar{q} = \gamma'D$, is the surcharge due to the embedment D ; N_q and N_γ = bearing capacity factor for surcharge and soil unit weight, respectively, which depends on the effective peak angle of internal friction ϕ'_p ; s_q, d_q, s_γ and d_γ = shape factors and depth factors.

For foundation resting on cohesionless soils, only the self weight component is considered. Sometimes, $N'_\gamma = N_\gamma s_\gamma$ was used by some researchers to describe the bearing capacity factor, which accounts for the shape factor already. It has been observed from model tests and commonly acknowledged that N_γ decreases with increasing footing width B . In other words, the ultimate bearing capacity does not increase linearly with B . De Beer (1963) first described this phenomenon as “scale effect”. In the following 40 years, extensive experimental results have demonstrated the scale effect of N_γ (e.g. Graham and Stuart, 1971; Hettler and Gudehus, 1988; Ueno et al., 2001; Zhu et al., 2001). Prototype spread footing tests with different footing widths at the same test site also showed strong evidence of scale effect of N_γ (Briaud and Gibbson, 1994).

At least two factors are believed to contribute to the scale effect of N_γ . The first is the stress- and density-dependent peak strength of cohesionless soil. In other words, for cohesionless soil of the same density, the higher the mean effective stress level beneath a larger footing, the smaller is the effective angle of friction; the second is “grain size effect” or “particle size effect” due to progressive failure. These two factors are studied to explain scale effect observed from model tests and centrifuge experiments. However, for prototype-scale footings, more factors could be involved.

For example, Hettler and Gudehus (1988) listed the following factors: non-uniform distributions of density and effective angle of internal friction of in situ soil beneath the footing, particularly when the ground surface was compacted; and cohesion due to cementation or suction, if the soil is above the ground water table.

Current researches focused on the former two factors, especially the first one, i.e., the stress- and density-dependent peak strength of cohesionless soil. For the latter, i.e., grain size effect, the studies primarily relied on centrifuge tests and model tests (Habib, 1974; Kimura et al., 1985; Tatsuoka et al., 1991; Cerato and Lutenegger, 2007). These researches indicate that grain size effect becomes insignificant once B/d_{50} is larger than a threshold value, where d_{50} is the 50th percentile grain size. Kusakabe (1995) suggested a value of 50 to 100 for B/d_{50} and Habib (1974) suggested a value of 200. The results imply that grain size effect may not be significant for prototype-scale footing since B/d_{50} in most cases will be larger than the threshold value reported by Kusakabe (1995) and Habib (1974). On the other hand, these studies indicated that grain size effect probably leads to overestimation of bearing capacity factor N_γ in model tests or centrifuge tests. Therefore, small-scale footing test results are not applicable to footing design without correction.

To investigate the effect of stress- and density-dependent peak strength of cohesionless soil on the bearing capacity factor N_γ , both experimental methods (triaxial tests, model tests and centrifuge tests) and analytical or numerical methods have been utilized. Analytical methods include limit analysis method and method of stress characteristics. Numerical methods include FEM and FDM.

Stress- and density-dependent peak strength of cohesionless soil is usually measured from triaxial tests or plane strain tests. The correlation between ϕ'_p and D_r and σ'_m is then incorporated into the analytical or numerical analyses to study scale effect of N_γ . Hettler and Gudehus (1988) proposed an empirical correlation between ϕ'_p and confining pressure based on triaxial tests. An equivalent value of angle of internal friction ϕ_{eq} was assumed to represent the ϕ'_p of soil mass beneath a footing under load. The ultimate bearing capacity of footing, which depends on stress level,

is calculated iteratively. Graham and Hovan (1986) investigated the effect using critical state model and stress characteristics method. Zhu et al. (2001) incorporated a stress-dependent ϕ'_p of silica sand based on triaxial test into method of characteristics to investigate the scale effect. Centrifuge test results on sand at the same relative density were used to verify the analyses. Kumar and Khatri (2008) used a limit analysis approach and a relationship between ϕ'_p and the effective confining pressure in their analyses. Veiskarami et al. (2010) conducted their analyses using so-called ZEL method. Bolton's (1986) equation (Equation 4.2) for determining ϕ'_p of sand was applied in their investigation. The analyses were calibrated using centrifuge test results from the literature.

The success of all these investigations depends firstly on how accurate these analytical or numerical methods are capable of estimating the bearing capacity factor N_γ of shallow foundation. For instance, although method of characteristics was widely applied to study N_γ , the accuracy of the results is affected by unrealistic assumptions, such as soil is weightless and the associated plastic flow rule at the slip surface. For rough footings, the precise boundary condition that should be applied at the interface between the base of the footing and the soil is not clear (Frydman and Burd, 1997). Compared with method of characteristics, limit analysis partially overcome some of the drawbacks. It is able to deal with more complex boundary conditions. However, associated plastic flow rule must be assumed in the analyses (Lyamin et al., 2007, Loukidis and Salgado, 2009). Apparently, this assumption conflicts with the fact that ψ is significantly lower than ϕ for soils. In addition, according to critical state concept and those theories based on this concept, peak strength can be estimated by critical-state internal friction angle ϕ_{cv} and dilation angle ψ , which depends mainly on the relative density D_r and mean effective stress σ'_m . Empirical correlations between peak strength and stress level have already taken into consideration the effect of ψ on the strength of sand. Therefore, ψ was double accounted for when these correlations were incorporated into the analyses with associated flow rule.

Unfortunately, existing researches have demonstrated that ψ has significant effect on bearing capacity factor N_γ . Generally, the higher the value of ψ , the larger is the value of N_γ . Researches based on FDM (Frydman and Burd, 1997, Yin et al. 2001, Erickson and Drescher, 2002) and FEM (Loukidis and Salgado, 2009) also showed that the larger the angle of internal friction ϕ' , the more significant is the effect of ψ on N_γ . Therefore, results based on assumption of associated flow rule may lead to misleading conclusions.

Normally, numerical researches on scale effect of N_γ are based on MC model. However, other advanced constitutive model, such as MIT-S1 model was adopted by Yamamoto et al (2007) and a multi-surface kinematic constitutive model was adopted by Banimahd and Woodward (2006) in their numerical investigation on the scale effect of N_γ . In these numerical studies, non-associated flow rule can be simulated. However, accurate limit load was not easy to define for some cases with high non-associativity and large internal angle of friction. Therefore, footing load corresponding to a certain settlement (e.g. 10% of footing width B) was usually adopted as the ultimate bearing capacity.

This chapter investigates the scale effect of bearing capacity factor N'_γ using FDM (FLAC). The as-built MC model was modified and incorporated into FLAC. The purpose of the modification is to incorporate Bolton's (1986) equation of ϕ'_p . Based on the modified MC model, parameters such as bulk density γ , lateral earth pressure coefficient at rest K_0 , Poisson's ratio ν and Young's modulus E were examined, since they are supposed to influence stress distribution and hence ϕ'_p , which depends on the stress level. Both associated and non-associated plastic flow rules were considered in the analyses. The different behaviors of soil due to assumption of flow rule were compared. The computed N'_γ were compared with those measured from centrifuge tests and prototype-scale footing tests.

4.3 Numerical Analysis of N'_γ and Scale Effect

Not many publications relevant to the numerical study of scale effect of N'_γ can be found in the literature, simply because the numerical study of bearing capacity factor N'_γ itself is still not perfect. Numerical analyses, using FEM and FDM, are not considered as accurate as method of characteristics and limit analysis. This is firstly due to the fact that accuracy of numerical analysis depends on the density of discretization. The finer the discretization, the more accurate will be the result. However, the finer discretization implies longer computation time, which makes some analyses infeasible. Secondly, numerical instability occurs when non-associated flow rule is assumed, particularly when there is a big difference between ϕ and ψ . In this case the footing load oscillates, which makes it difficult to determine the accurate value of ultimate footing load and hence N'_γ . It is even worse to observe that a finer discretization causes more severe oscillation of load (Loukidis and Salgado, 2009). Therefore, it seems difficult to ensure accuracy of numerical results.

In spite of these limitations, numerical analysis is becoming prevalent in investigating bearing capacity-related problems. The main reason is that numerical analysis is capable of dealing with complex boundary conditions. Moreover, non-associated flow rule can be considered in numerical analysis.

Loukidis and Salgado (2009) recently demonstrated that FEM is able to estimate ultimate bearing capacity and N'_γ accurately. Finite difference method using FLAC has also been demonstrated to be capable of estimating N'_γ with acceptable accuracy (Erickson and Drescher, 2002). Both associated and non-associated plastic flow rule can be considered in these analyses. Loukidis and Salgado (2009) assumed that ψ is less than ϕ and used different values of ψ . Values of ψ ranging from zero to ϕ were assumed in Erickson and Drescher, (2002).

The difference between ultimate bearing capacity values from numerical limit analysis (Hjiaj et al. 2004) and from FEM with associated flow rule (Loukidis and Salgado, 2009) is negligible, for both strip and circular footings. The difference between ultimate bearing capacity values from FEM (Loukidis and Salgado, 2009) and FDM (Erickson and Drescher, 2002) is about 10% for associated flow rule assumption.

For cases of non-associated flow rule, researchers are still facing difficulty to obtain the ultimate bearing capacity accurately due to numerical stability problem and oscillation of footing load. Loukidis and Salgado (2009) took the maximum footing load observed in their simulation as the ultimate bearing capacity. Yin et al. (2001) and Erickson and Drescher (2002) calculated the mean value of oscillations as the ultimate bearing capacity.

4.3.1 Modified MC Constitutive Model

Program FLAC was used in this research to investigate scale effect on N'_γ . Program FLAC provides a built-in MC model comprising linear elasticity before yielding and perfect plasticity after yielding. The shear failure criterion of the model is a straight line in the meridional plane, which means the shear failure envelope is linear with stress level. Modification was made to the built-in MC model by correlating effective internal angle of friction ϕ' to the mean effective stress level σ'_m and relative density D_r of cohesionless soil. Bolton's (1986) equation for ϕ'_p shown below was adopted:

$$\phi'_p = \phi_{cv} + 0.8\psi \dots\dots\dots(4.2)$$

where ϕ'_p = peak angle of internal friction; ϕ_{cv} = angle of internal friction at critical state; ψ = dilation angle, which depends σ'_m and D_r .

Based on the study of extensive data measured in triaxial and plane strain tests on 17 sands, Bolton (1986) proposed that ϕ'_p can be expressed as follows:

$$\phi'_p = \phi_{cv} + A[D_r(Q - \ln \sigma'_m) - R] \dots\dots\dots (4.3)$$

where D_r = relative density of sand; σ'_m = mean effective stress; A, Q and R = material constants; A is 5 for plane strain condition and 3 for triaxial condition; Q depend on the mineral type, which is 10 for quartz and feldspar); R = fitting parameter, which is equal to 1.0.

Equation (4.3) is implemented into the built-in MC model subroutine using FISH language (FLAC manual). Basically, the modified subroutine does the following:

- 1) At the beginning of each step, the stresses from the previous step are used to calculate the value of ϕ'_p , which is then assumed to be a constant to calculate the stress component later in the current step;
- 2) If the calculated ϕ'_p is larger than 45° , then $\phi'_p = 45^\circ$; and
- 3) If the calculated ϕ'_p is lesser than ϕ_{cv} , then $\phi'_p = \phi_c$.

Step (1) implies that time step must be controlled carefully. As FLAC is an explicit finite difference method, the dynamic equation of motion in conjunction with incremental constitutive laws must be solved over small time steps to obtain meaningful results. Therefore, field variables propagate step by step as a real physical distribution and no iteration is required at each step. In this study, each simulation involves about million steps. Verification will be provided subsequently that Step (1) does not cause detectable error in the simulation.

Steps (2) and (3) were based on the understanding that theoretically, ϕ'_p cannot be larger than 45° and less than ϕ_{cv} . Although cases of ϕ'_p greater than 45° have been reported in some experiments (e.g. Bolton, 1986 and Zhu et al., 2002), no numerical simulation considered ϕ'_p to be more than 45° was found in the literature. Due to Step (2), the maximum value of N'_γ for circular rough footing is 45° based on associated flow rule and 198 based on non-associated flow rule according to Erickson and Drescher (2002).

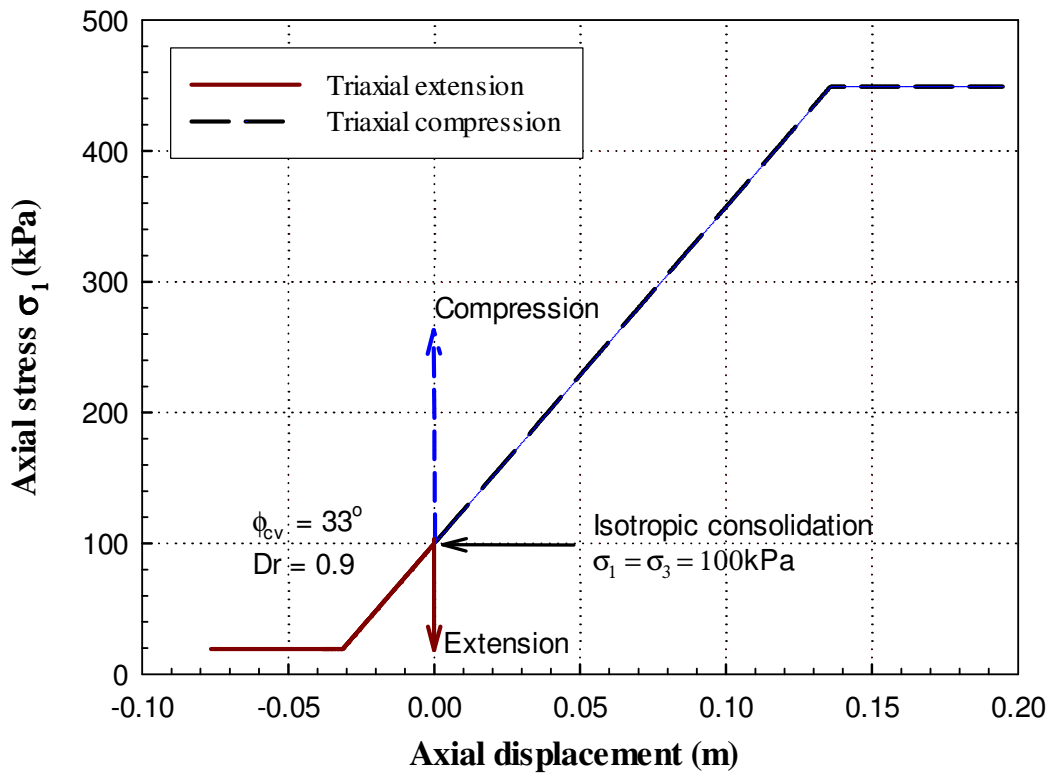


Figure 4.1: Load-displacement curves of simulated triaxial test on single soil element

Verification of modified MC constitutive model is carried out by using an element test in both triaxial compression and extension. The dimension of the simulated soil element is 1m in radius and 1m in height. Figure 4.1 shows the load displacement curve for the soil element test. The confining stress σ'_3 is 100 kPa. It is assumed that ϕ_{cv} of the sand is 33° and D_r is 0.9. It can be seen that under compression

condition, the major principal stress, i.e., σ'_1 is 449.2 kPa. Therefore, $\sigma'_m = 216.4$ kPa and according to Equation (4.3), the value of ϕ'_p is 39.48° . Based on MC criterion, $\sin\phi'_p = (\sigma'_1 - \sigma'_3) / (\sigma'_1 + \sigma'_3)$, ϕ'_p can be calculated and is exactly equal to 39.48° . For the same soil element under triaxial extension condition, the minor principal stress is 19.44 kPa, which is σ'_3 . The confining stress of 100 kPa is σ'_1 . Thus ϕ'_p can be calculated as 42.4° according to both Bolton's equation (Equation 4.3) and MC criterion.

4.3.2 FLAC Simulations of Load Tests on Footings

Circular, rough and rigid footing on the surface of cohesionless soil was simulated using FLAC. A similar set-up and procedure reported by Erickson and Drescher (2002) was adopted in the simulation of the footing load test. Briefly, due to axisymmetry, only half of the soil mass and the footing were modeled. The soil was discretized into 40 square elements along the r (horizontal) axis and 25 elements along the z (vertical) axis, as shown in Figure 4.2. Square elements were adopted because they have been proven to be able to provide more stable and accurate results than rectangular elements (Erickson and Drescher, 2002). The bottom and right boundaries of the soil mass were fixed. The boundary along the axis of symmetry was constrained in the lateral direction. The dimensions of elements vary from 0.083m to 0.83m as the footing size increases from 1m to 10m. Before loading, initial geostatic stresses increasing from zero at the ground surface were applied. The gradient of the horizontal stress can be varied to simulate different K_0 conditions.

Applied load on the footing is simulated by displacement control method on the nodal points corresponding to the contact between footing and soil. These points are constrained laterally to simulate rough interface between footing and soil except for the outermost nodal point. The free lateral movement of the outermost nodal point has been proven to be able to improve the accuracy of the simulation (Erickson and Drescher, 2002).

Erickson and Drescher (2002) simulated a footing with radius of 6m. The footings simulated in this study have radii varying from 1m to 10m. Simulations of load on footings with built-in MC model and associated flow rule produced identical value of N'_γ . This observation implies there is no scale effect due to size of element based on built-in MC model and associated flow rule. This is crucial because otherwise it is difficult to separate scale effect due to stress- and density-dependent peak strength of sand from that due to size of elements.

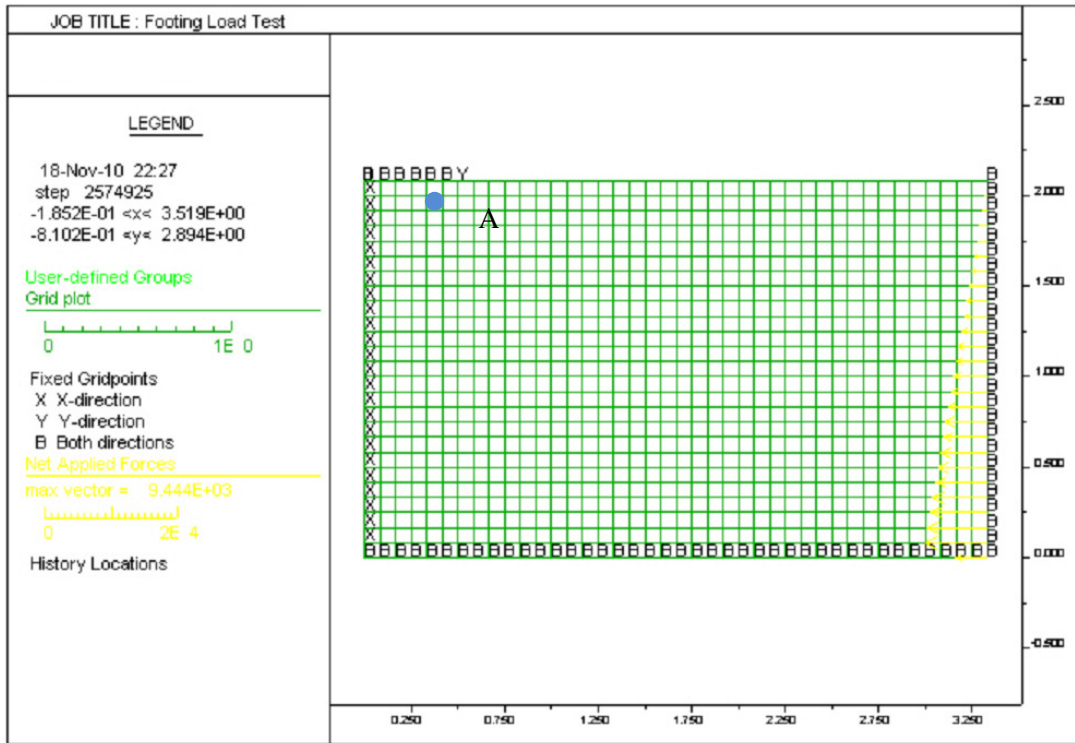


Figure 4.2: Detailed set-up of simulation of footing load test

In the simulation, footing load was calculated by summing the vertical reactions on the nodal points where displacement were applied. For associated flow rule condition, it was found that more accurate results can be obtained using smaller velocity v_y , as shown in Figure 4.3. The reduction is due to the nodal velocity being zero at this stage. Ultimate bearing capacity is the residual value shown in Figure 4.3. The y-axis of Figure 4.3 is in terms of bearing capacity factor N'_γ instead of footing load. Figure 4.3 also shows that N'_γ increases with decrease in footing nodal velocity. This is understandable because of the dependence of ϕ'_p on the mean

stress level. The overestimated footing load in the process of computation leads to smaller ϕ'_p , which leads to smaller N'_γ . This phenomenon is not observed for built-in MC model. However, it can be seen from Figure 4.4 that the difference is less than 1% between the two cases with smaller nodal velocities.

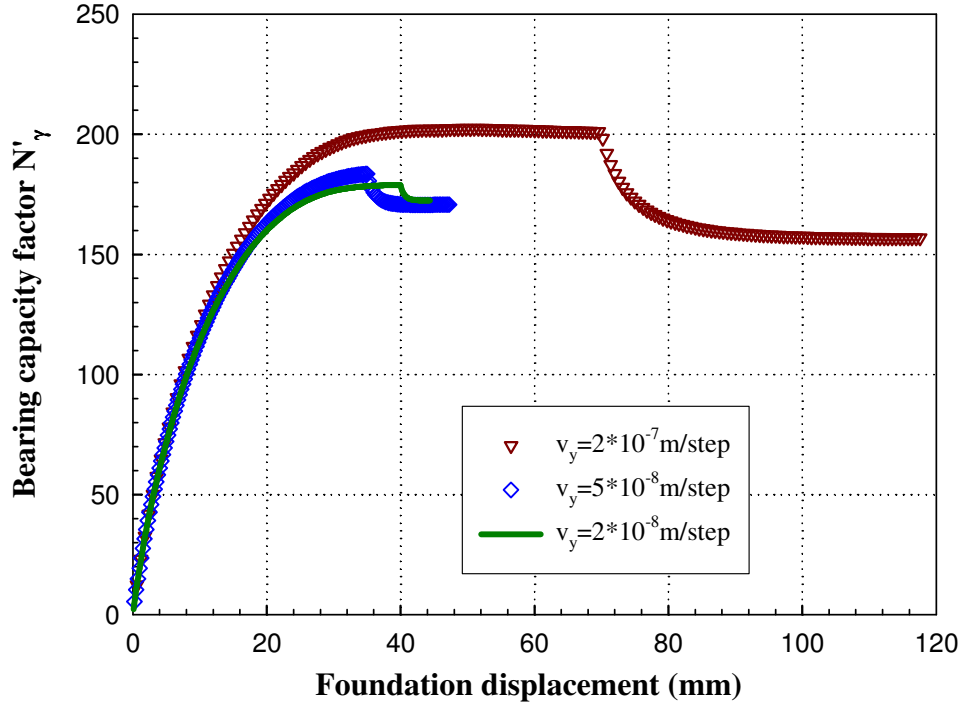


Figure 4.3: Effect of nodal velocity on N'_γ (Associated flow rule)

For non-associated flow rule, it was found that the results were not significantly affected by the applied nodal velocity based on built-in MC model. This is also observed for the modified MC model. Figure 4.5 shows the load-settlement curves of three simulations with applied nodal velocities from 1.8×10^{-7} m/step to 5.0×10^{-9} m/step. It can be seen that the curves are almost identical in the elastic range and the beginning portion of the plastic range. The magnitudes of oscillation are slightly bigger for the two cases with larger nodal velocities. It should be noted that for the case of nodal velocity of 5.0×10^{-9} m/step, more than ten million steps were needed to obtain the results shown in Figure 4.5. The maximum unbalance force was controlled to within 1N. Typically, one million steps were used to obtain the bearing capacity factor in the subsequent analyses.

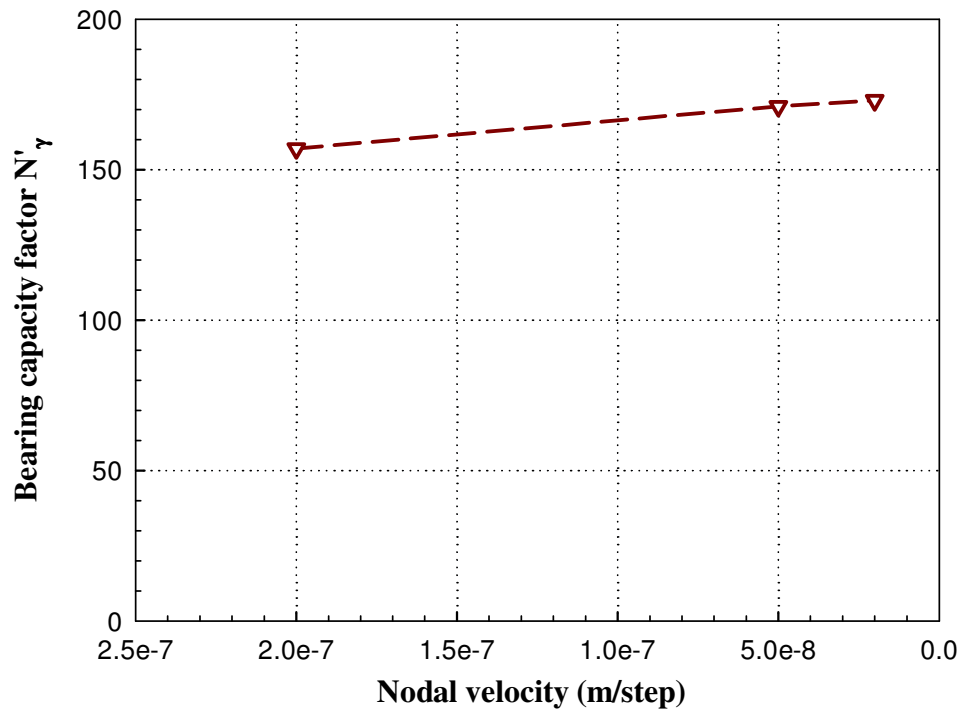


Figure 4.4: Increase of N'_γ with decrease of nodal velocity (Associated flow rule)

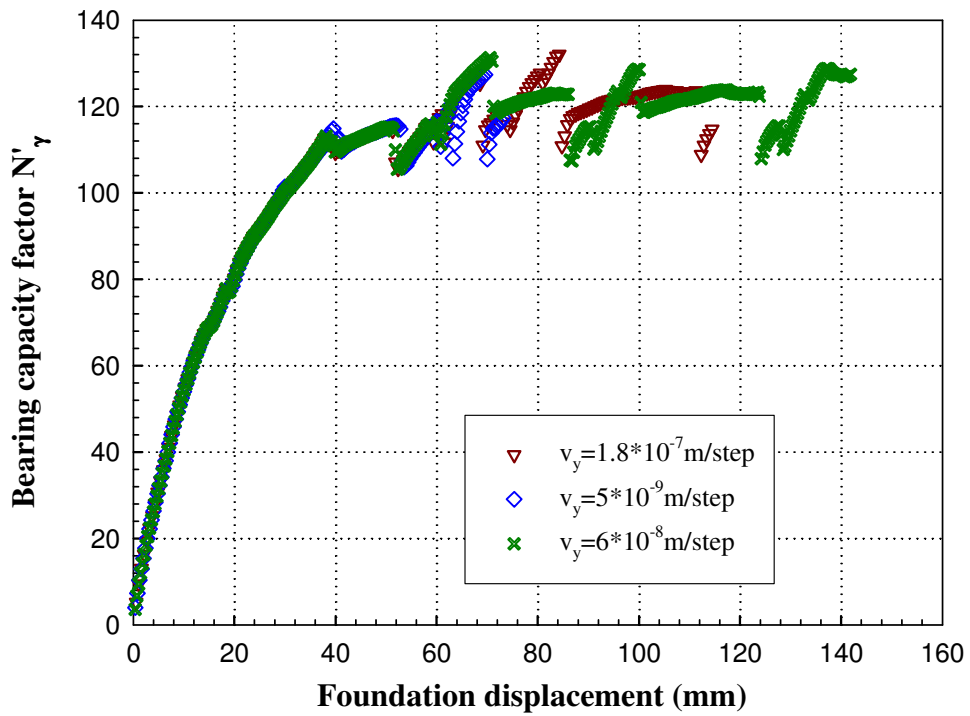


Figure 4.5: Effect of nodal velocity on N'_γ (Non-associated flow rule)

4.3.3 Parametric Studies

A series of parametric studies were carried out to investigate the effects of Young's modulus, Poisson's ratio, coefficient of lateral earth pressure at rest K_0 and bulk density of soil on the ultimate bearing capacity based on modified MC model. For built-in MC model, these parameters have been proven to have no effect on ultimate bearing capacity of footing (Erickson and Drescher, 2002). However, these factors were seldom investigated for sand with stress- and density-dependent strength. Since the stress distributions in the analyses depend on these factors and the strength of sand depends on the stress distributions based on the modified MC model, any possible effect of these factors on the bearing capacity of footing should be examined. Table 4.1 summarizes the input parameters used in the parametric studies.

Table 4.1: Input parameters for parameter studies

Parametric studies	Input parameters						
	E (MPa)	ν	K_0	γ' (kN/m ³)	D_r	B(m)	$\phi_{cv} (^{\circ})$
Young's Modulus	25	0.3	1.0	17	0.6	1.0	33
	50						
	100						
Poisson's ratio	50	0.0	1.0	17	0.6	1.0	33
		0.2					
		0.4					
At-rest earth pressure coefficient	50	0.3	0.5	17	0.6	1.0	33
			1.0				
			2.0				
Bulk density	100	0.3	1.0	13	0.9	10	30
				15			
				17			

In the first parametric study, Young's modulus of sand was examined. Figure 4.6 shows the load settlement curves of a 1m diameter footing on sand with Young's moduli of 25 MPa, 50 MPa and 100 MPa. Other properties of sand were exactly identical for the three simulations. It can be seen that almost no difference can be found between the bearing capacity factors N'_γ among the three cases, although the

curves show different stiffness. This means that effect of Young's modulus of sand on N'_γ can be neglected.

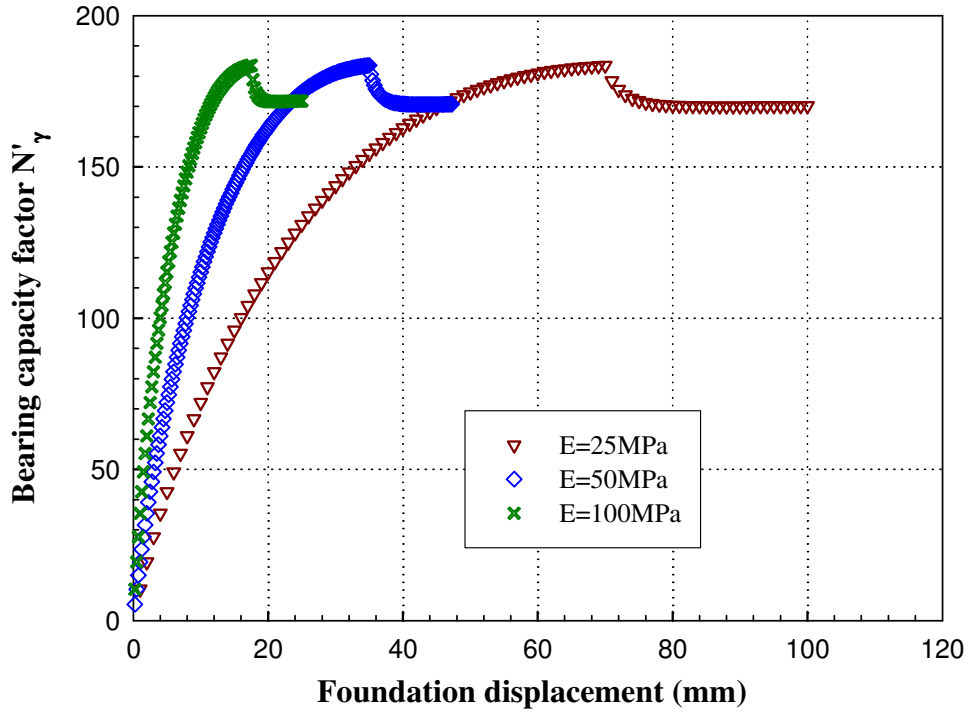


Figure 4.6: Effect of Young's modulus on N'_γ

In the second parametric study, Poisson's ratio was studied. Computations were carried out by using Poisson's ratios of 0, 0.2, 0.3 and 0.4. Other properties of sand were identical for the four simulations. Figure 4.7 shows the computed load settlement curves using various values of Poisson's ratio. Almost no difference of the bearing capacity factors N'_γ among these computations can be observed. Therefore the effect of Poisson's ratio of sand on N'_γ can be neglected.

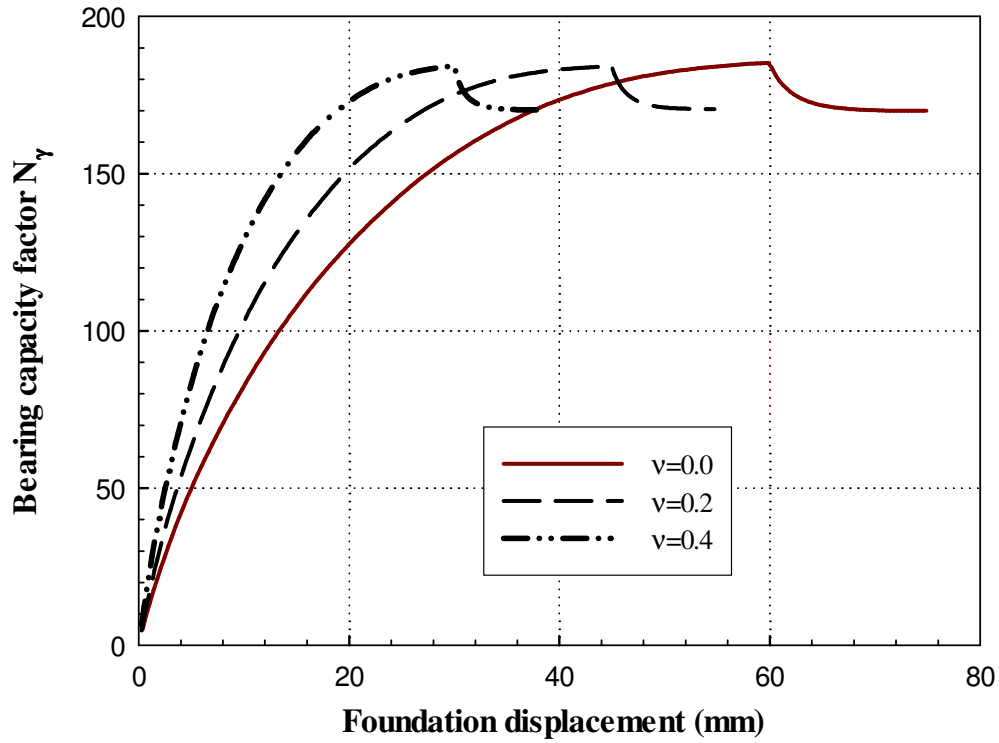


Figure 4.7: Effect of Poisson's ratio on N'_γ

In the third parametric study, the at-rest earth pressure coefficient K_0 was studied. The magnitudes of K_0 used were 0.5, 1.0 and 2.0. The other properties of sand were identical for the simulations. Figure 4.8 shows the simulated results. No apparent difference of the ultimate bearing capacity among these simulations can be observed. This is in conflict with some results reported in the literature. For example, Lee and Salgado (2005) reported that as the K_0 increases, the ultimate bearing capacity increases. However, in their analyses, the ultimate bearing capacity was determined by the footing load at a certain large footing displacement ($s = 0.1B$), instead of ultimate footing load as defined in this study. Moreover, the constitutive model used in their analysis correlated the Young's modulus of sand to K_0 , which led to stiffer load-settlement response for the case of larger K_0 . This probably leads to overestimation of the ultimate bearing capacity as K_0 increases.

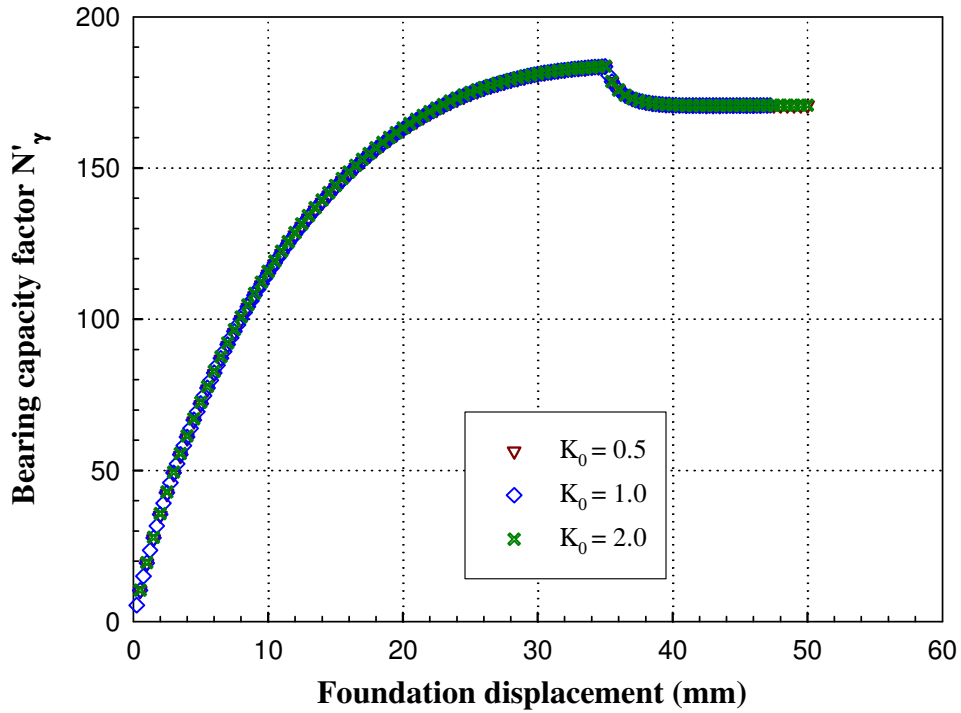


Figure 4.8: Effect of K_0 on N'_γ

In the last parametric study, the effect of bulk density γ of sand on the bearing capacity of footing was examined. Figure 4.9 shows the simulated results. It can be seen that unlike for built-in MC constitutive model, bulk density of sand do affect the ultimate bearing capacity for modified MC constitutive model. Basically, N'_γ increases as bulk density of the sand decreases. A series of more comprehensive parametric studies show that the maximum difference between N'_γ when γ is 13 kN/m³ and 17 kN/m³ is about 10%.

In conclusion, except for the bulk density, no apparent effect of other properties on N'_γ can be observed as shown by the results above. The results indicate that the Young's modulus, Poisson's ratio and K_0 of sand do not affect the final stress distribution in the simulation of footing load test. They may affect the stress distribution in the beginning or middle of the computation. For example, the load-settlement curves show different slopes. The curve based on larger Young's modulus shows stiffer response. However, final stress status does not differ much

between each other, which explain why the bearing capacity factors observed from these simulations are almost same.

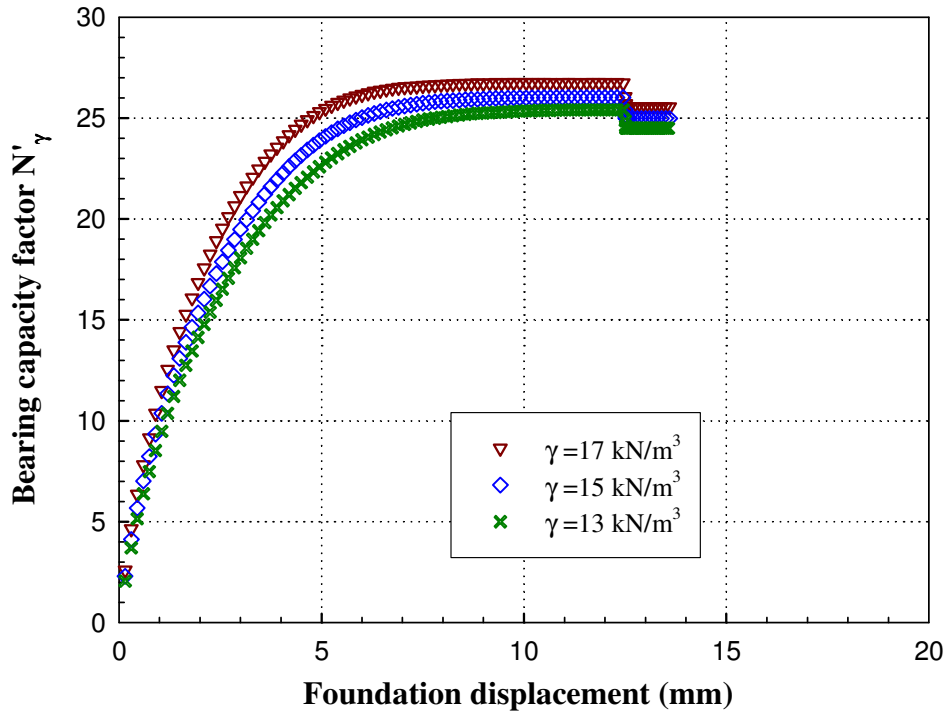


Figure 4.9: Effect of bulk density of soil γ on N'_γ

4.4 Scale effect on N'_γ

Scale effect of N'_γ was studied using the modified MC model. Circular, rough and rigid footings were considered. Four footing diameters were simulated, i.e., 1m, 2m, 4m and 10m. Three relative densities of soil were considered, i.e., 0.3, 0.6 and 0.9. Three critical state angles of friction were investigated, i.e., 30° , 33° and 36° . Other properties of sand were assigned typical values. Table 1 summarizes the input parameters adopted in the simulations. It should be noted that for cases with non-associated flow rule, dilation angle ψ equal to zero is assumed.

Figure 4.10 to Figure 4.12 show the computed bearing capacity factors of the cases listed in Table 4.2 for ϕ_{cv} of 30° , 33° and 36° , respectively. It can be seen that as the

footing size increases, N'_γ decreases. Figure 4.10 to Figure 4.12 show that this scale effect is more significant for dense sand than loose sand and scale effect is more significant for associated flow rule than for non-associated flow rule. As expected, the bearing capacity factors for cases with non-associated flow rule are smaller than those corresponding cases with associated flow rule. Higher bearing capacity factors were obtained for footings resting on denser sands.

Table 4.2: Input parameters to study scale effect on N'_γ

Cases	D _r	Density (kN/m ³)	E (MPa)	ν	ϕ _{cv}	Ψ	c (kPa)	B (m)						
30-0.3-N	0.3	13	12.5	0.3	30°	0 (for non-associated flow rule) and ϕ' _p (for associated flow rule)	0	1,2,4,10						
30-0.3-A	0.3	13	12.5	0.3										
30-0.6-N	0.6	15	25	0.3										
30-0.6-A	0.6	15	25	0.3										
30-0.9-N	0.9	17	50	0.3										
30-0.9-A	0.9	17	50	0.3										
33-0.3-N	0.3	13	12.5	0.3	33°				0 (for non-associated flow rule) and ϕ' _p (for associated flow rule)	0	1,2,4,10			
33-0.3-A	0.3	13	12.5	0.3										
33-0.6-N	0.6	15	25	0.3										
33-0.6-A	0.6	15	25	0.3										
33-0.9-N	0.9	17	50	0.3										
33-0.9-A	0.9	17	50	0.3										
36-0.3-N	0.3	13	12.5	0.3	36°							0 (for non-associated flow rule) and ϕ' _p (for associated flow rule)	0	1,2,4,10
36-0.3-A	0.3	13	12.5	0.3										
36-0.6-N	0.6	15	25	0.3										
36-0.6-A	0.6	15	25	0.3										
36-0.9-N	0.9	17	50	0.3										
36-0.9-A	0.9	17	50	0.3										

Note: “N”- non-associated flow rule; “A” – associated flow rule.

Figure 4.10 to Figure 4.12 also compare N'_γ with back calculated N'_γ values from centrifuge tests and spread footing load tests conducted at Texas A&M University. Table 4.3 lists the detailed information of the soil properties and test conditions of the centrifuge tests and footing load tests. From Table 4.3, it can be seen that the footings and footing models are rigid, either circular or rectangular, and bottom is rough. Except for the in situ footing load tests, where the footing embedment is

about 0.75m, the centrifuge tests are all with model footing resting on the surface of sand.

A series of PLT were conducted at Texas A&M University (Briaud and Gibbens, 1994). The subsoil layer was silty sand, with estimated in situ relative density of about 55%. The embedment of these footings was about 0.75m. Results based on triaxial compression test on intact sample from depths of 0.6m and 3.0m show that ϕ_{cv} was about 30° and 32° , respectively. The ground water table was at a depth of about 4.9m.

Results from footing load tests show significant scale effect of N'_γ compared with simulated results and those measured from centrifuge tests. Very large values of N'_γ were back calculated for footings of width 1.0m and 1.5m. Several factors can contribute to this unrealistic large value of N'_γ . First and most likely is cohesion of shallow soil layer near ground surface. The cohesion can be due to cementation or suction. The component of bearing capacity due to cohesion was included in the back calculated N'_γ . Secondly, a dense shallow soil layer near the ground surface could lead to higher N'_γ . As footing width increases to 2.5m and 3.0m, the measured N'_γ approaches the simulated values. This means that the contribution of any possible cohesion or densification becomes insignificant as footing width increases.

CT 1 was reported by Okamura et al. (1997) based on centrifuge tests. Toyoura sand at relatively density of about 88% was tested. Several researchers (e.g. Verdugo and Ishihara 1996, Wang et al. 2002) have measured and reported ϕ_{cv} of Toyoura sand, ranging from 31.1° to 34.4° , with an average value of 32.8° under triaxial condition (Fukushima and Tatsuoka 1984; Tatsuoka 1987). It should be noted that the degree of saturation of the samples in CT conducted by Okamura et al. (1997) ranges from 95% to 100%. CT2 was reported by Ueno et al. (1994). Toyoura sand at relative density of 70% was used in the centrifuge tests.

Table 4.3: Description of Centrifuge Tests and PLT

Series	Sand				Footing Details	Reference
	Type	ϕ_{cv}	Relative Density (%)	Water condition		
PLT	Silt Sand	30° 32°	~55	Above Ground Water Table	Square /Rough/Embedment (0.76m)	Briaud and Gibbens, (1994)
CT 1	Toyoura sand	32.8°	88	Degree of Saturation 95%~100%	Circular/Rough /Surface	Okamura et al. (1997)
CT 2	Toyoura sand	32.8°	70		Circular/Rough /Surface	Ueno et al. (1994)
CT 3	Inagi Sand	32°	81.8	Dry	/Rough/Surface	Kusakabe et al. (1991)
CT 4	Silica Sand	~35.6°	90	Dry	Circular/Rough /Surface	Zhu et al. (2001)
CT 5	Monterey 0/30 Sand	36.5°	93~95	Dry	Circular/Rough /Surface	Kutter et al. (1988)

Figure 4.11 shows that N'_γ results of CT1 are in good agreement with the numerical N'_γ values based on associated flow rule when B is no more than 2m and the numerical N'_γ values were overestimated when B is 3m. For those cases with B equal to 3m, the sand used in the tests was not fully saturated, which could contribute to higher N'_γ because of suction. The measured values are about 1.25 to 1.50 times of the numerical values using non-associated flow rule.

CT 3 was reported by Kusakabe et al. (1991). Dry Inagi sand at average relative density of 81.8% was used. Typical value of ϕ_{cv} of Inagi sand is 32° (Simonini, 1993). Figure 4.11 shows that the measured data scatter between the numerical results of relative density of 60% and 90%. The trend N'_γ decreases with increases of B is in good agreement with the test results.

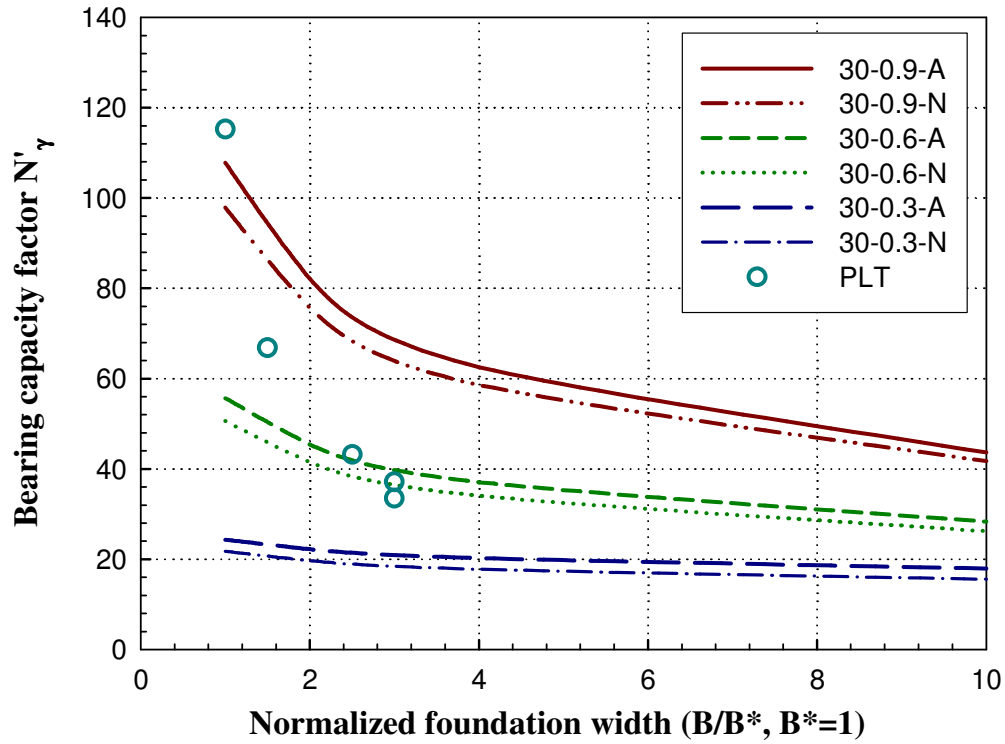


Figure 4.10: Numerical and measured N'_γ vs (B/B^*) ($\phi_{cv} = 30^\circ$)

CT 4 was reported by Zhu et al. (2001). Silica sand at relative density of 90% was used in their tests. Matching the results from triaxial compression test at various confining stresses using Equation (4.3) show that ϕ_{cv} was about 35.6° . Figure 4.12 shows the comparison between the measured bearing capacity factors and the numerical values. It can be seen that the back calculated data agree better with the numerical values based on non-associated flow rule.

CT 5 was reported by Kutter et al. (1988). Dry Monterey 0/30 sand at relative density of 93%~95% was used in their tests. The value of ϕ_{cv} was about 36.5° , according to Lade and Duncan (1973) and Steven and Craig (2000). Figure 4.12 shows that the magnitude and trend of the numerical N'_γ value agree reasonably well with the back calculated values.

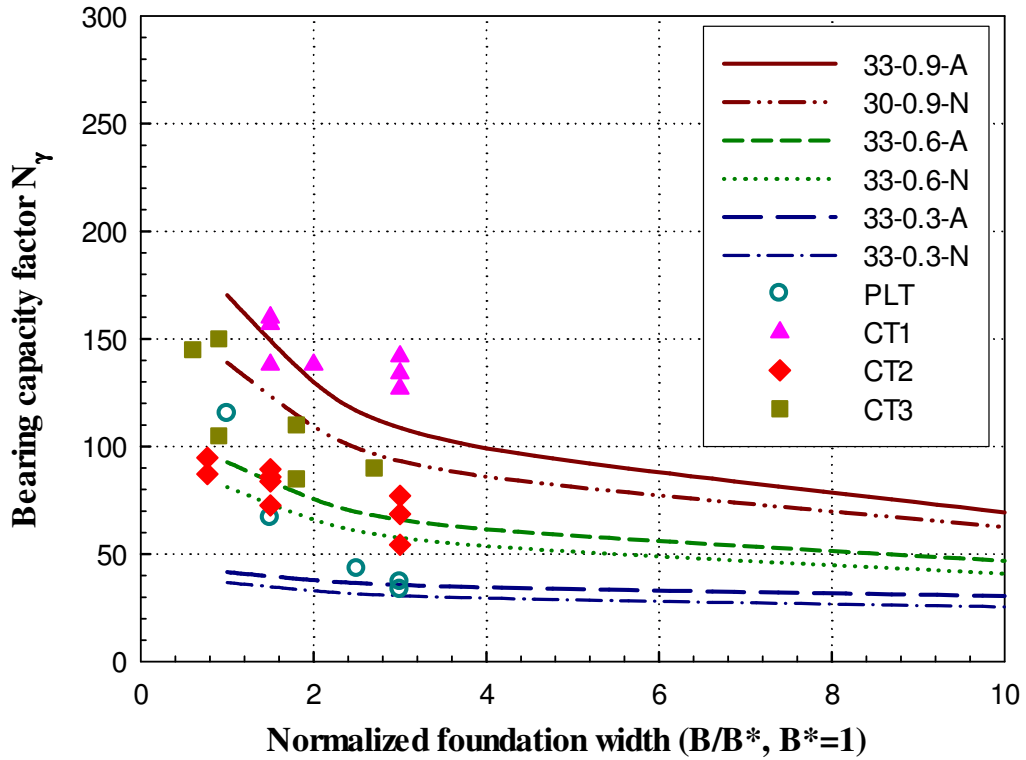


Figure 4.11: Numerical and measured N'_γ vs $(B/B^*, B^*=1)$ ($\phi_{cv} = 33^\circ$)

In conclusion, Figures 4.9 to 4.12 show that the numerical N'_γ values were comparable with those measured in centrifuge tests. However, higher N'_γ values were observed for small footings with B less than 2.0m in the PLT tests. As the footing width increases to 2.5m and 3.0m, the N'_γ values become comparable with the numerical N'_γ values. Therefore, the observed scale effect from in situ tests was more significant compared with those observed in centrifuge test. The numerical N'_γ values underestimated the bearing capacity of footings of small width.

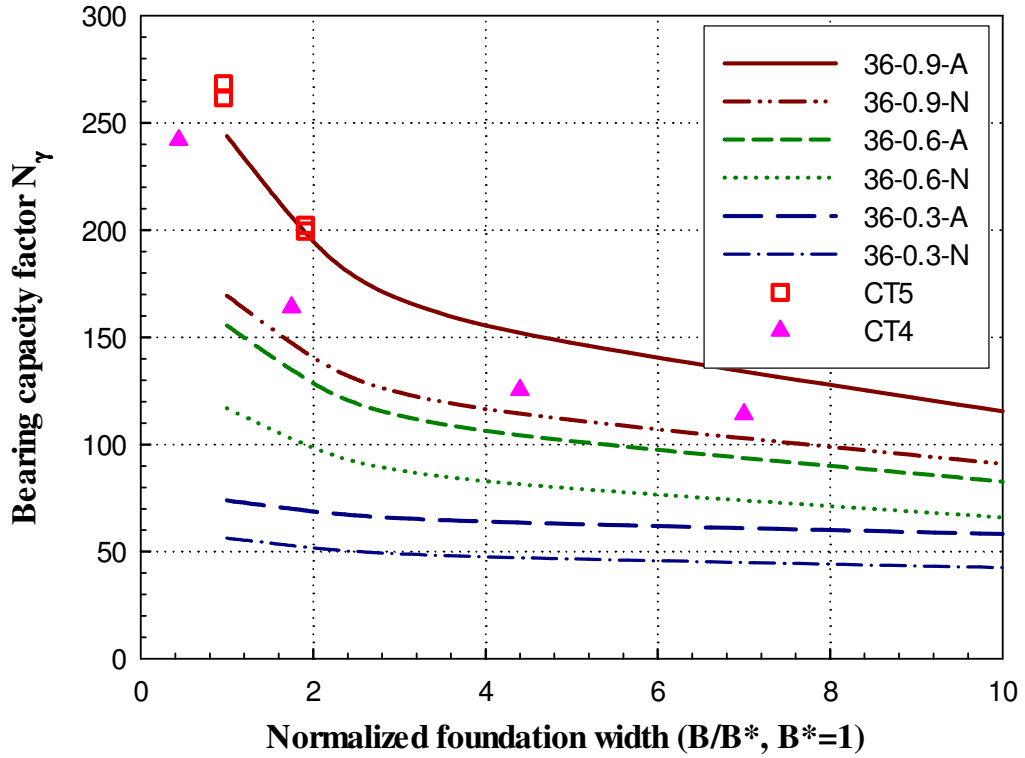


Figure 4.12: Numerical and measured N'_γ vs (B/B^*) ($\phi_{cv} = 36^\circ$)

The numerical results in Figures 4.9 to 4.12 were curve fitted using power function recommended by Shiraishi (1990) as follows:

$$N'_\gamma = N_\gamma^* \left(\frac{B}{B^*} \right)^{-\beta^*} \quad \dots\dots\dots (4.4)$$

where N_γ^* = a reference value of N'_γ ; B^* = a reference footing width or diameter; β^* = a fitting parameter reflecting the dependency of N'_γ on the stress level. It should be noted that B^* is not clearly defined by Shiraishi (1990), although Shiraishi (1990) adopted B^* as 1.4m. The magnitude of β is independent of B^* , which means that β controls the decreasing speed of N'_γ with B/B^* only. The absolute value of N'_γ was dominated by N_γ^* given a constant β value. Others, such as Zhu et al. (2001)

and Ueno et al. (2001) prefer to adopt the following expression to keep the units consistent:

$$N'_\gamma = N_\gamma^* \left(\frac{\gamma' B}{P_a} \right)^{-\beta^*} \dots\dots\dots (4.5)$$

Where γ' = effective bulk density of sand and P_a = reference pressure, taken as atmospheric pressure (100 kPa).

However, Equation (4.5) can be transformed into Equation (4.4) by adopting different value of N_γ^* . In this study, Equation (4.4) was used to determine β and N_γ^* , and B^* was set to 1.0m.

Figure 4.13 compares the β values from curve fitting and those from measurements based mainly on dense sand samples. Numerical values are on the safe side as larger β values means faster reduction of the bearing capacity with the increase in footing width. Figure 4.14 compares the N_γ^* value obtained using curve fitting with those from measurements.

Figure 4.10 to Figure 4.12 can be used to obtain the bearing capacity factor N'_γ which accounts for scale effect given the relative density D_r and the critical-state friction angle ϕ_{cv} of the soil. Bearing capacity factor N_γ can also be estimated using Equation (4.4), Figure 4.13 and Figure 4.14 given D_r and ϕ_{cv} . Given values of D_r and ϕ_{cv} , values of β and N_γ^* can be determined using Figure 4.13 and Figure 4.14, respectively. Then based on Equation (4.4), N'_γ can be estimated.

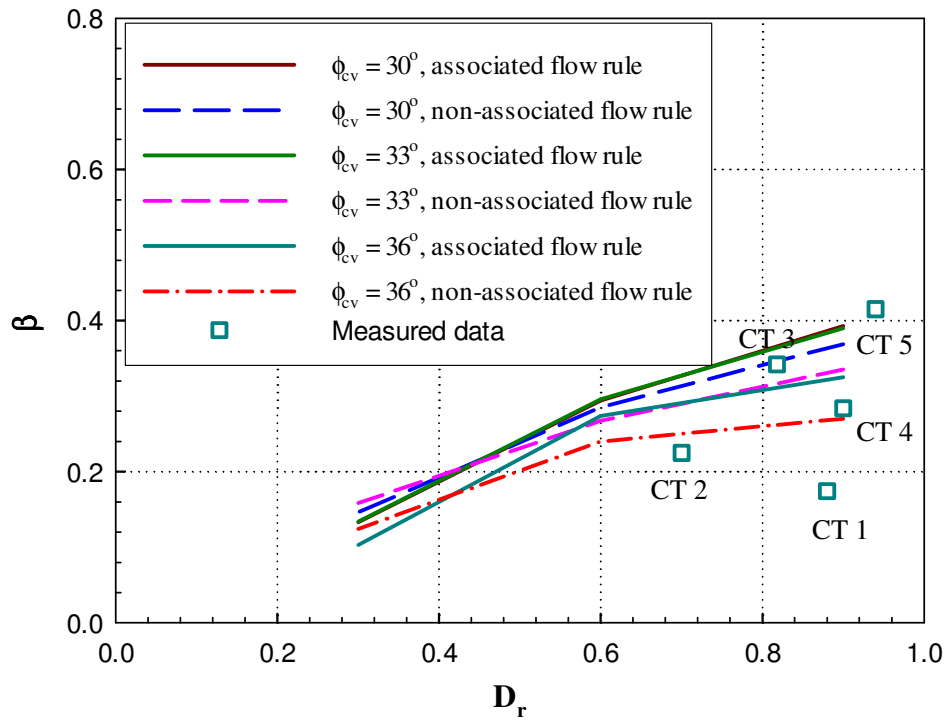


Figure 4.13: Comparison of β^* values between simulations and measurements

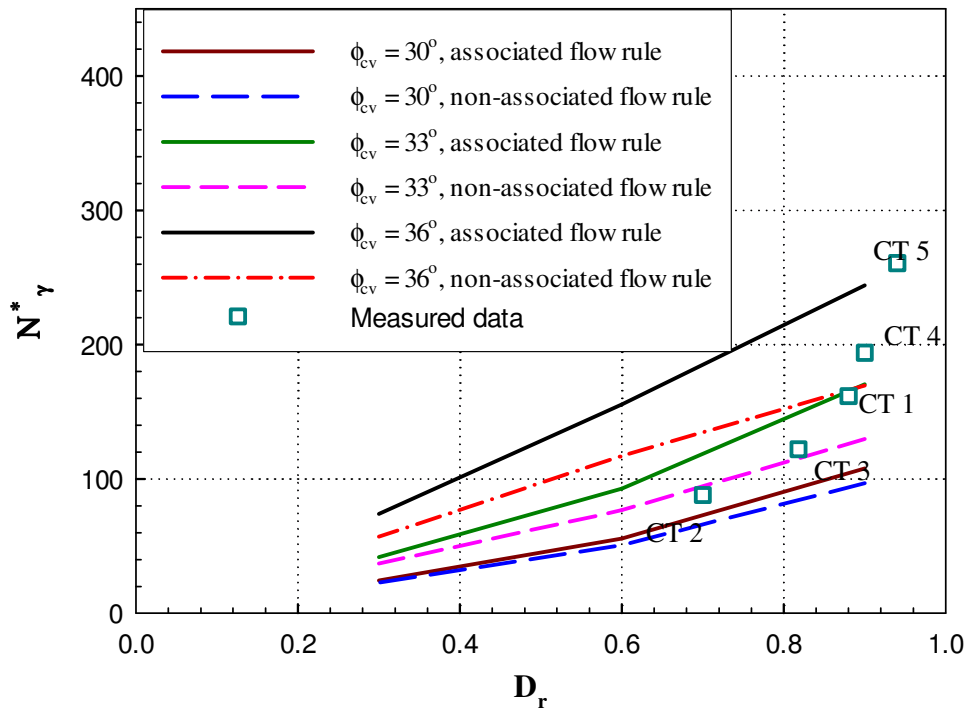


Figure 4.14 Comparison of N_γ^* values between simulations and measurements

4.5 Observations in the Simulation

Figures 4.15 and Figure 4.16 show the distributions of ϕ'_p beneath a 1m diameter footing resting on sand with relative density of 0.9 and ϕ_{cv} of 33° , assuming non-associated and associated flow rule, respectively. It can be seen that for non-associated flow rule, the magnitude of ϕ'_p is larger at the same position beneath the footing than that of associated-flow rule. However, the footing load is smaller due to the non-associated flow rule. Figure 4.17 shows the developments of effective mean stresses at Point A beneath the footing in Figure 4.2 for both associated and non-associated flow rule. Figure 4.18 shows the decrease of ϕ'_p with increase of mean stress level at the same point. It can be used to double check whether Bolton's correlation between ϕ'_p mean stresses are followed or not in the computation.

Figure 4.19 and Figure 4.20 show the mean stress distributions. It can be seen from Figure 4.20 that for associated flow rule, very high stresses were observed at the edge of the footing. This probably contributes to the higher ultimate footing load than that assuming non-associated flow rule.

Figure 4.21 and 4.22 shows the displacement fields affected by the assumption of associated and non-associated flow rule. Both the total and horizontal displacements were plotted. It can be seen that for associated flow rule, significant heave can be observed at the edge of the footing. The large displacement value at the footing edge is due to the large dilation angle used in the analyses and may not be realistic. In general, when the dilation angle is large, significant displacement is restrict to the exterior region close to the footing edge. The horizontal displacement is also larger for non-associated flow rule.

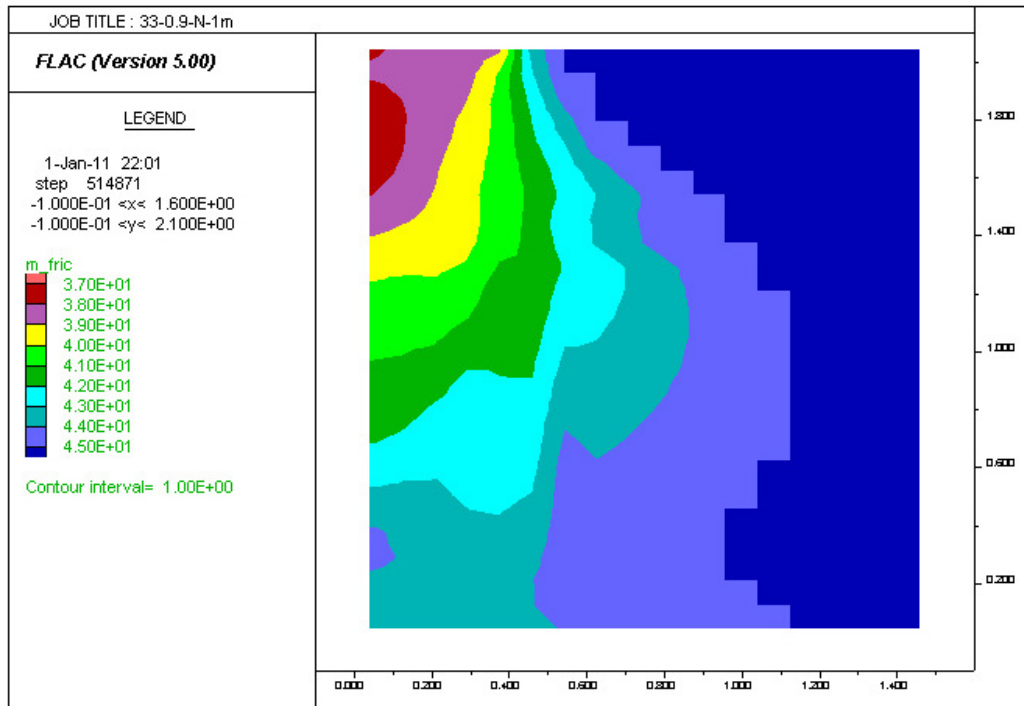


Figure 4.15: Distributions of ϕ'_p (Case 33-0.9-N-1m)

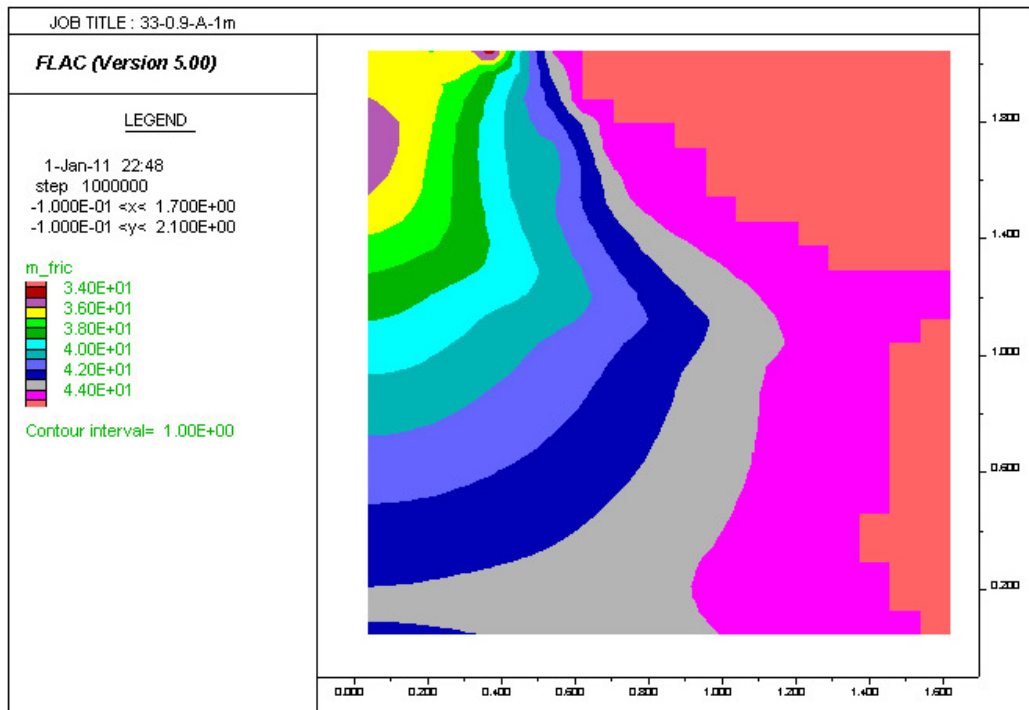


Figure 4.16: Distributions of ϕ'_p (Case 33-0.9-A-1m)

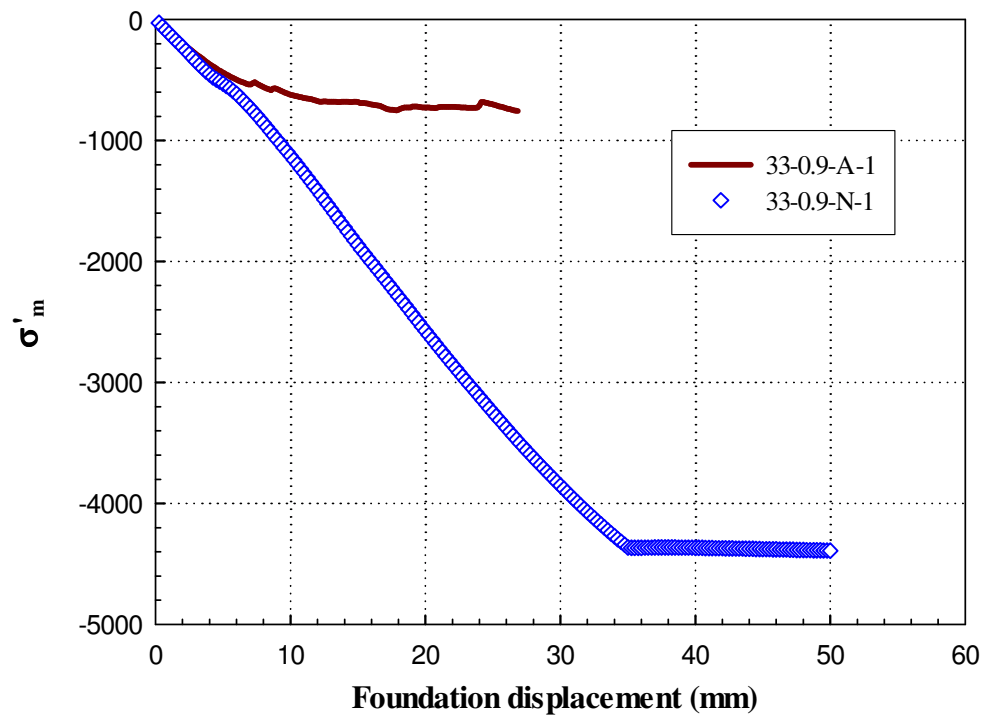


Figure 4.17: Development of mean stress beneath the footing

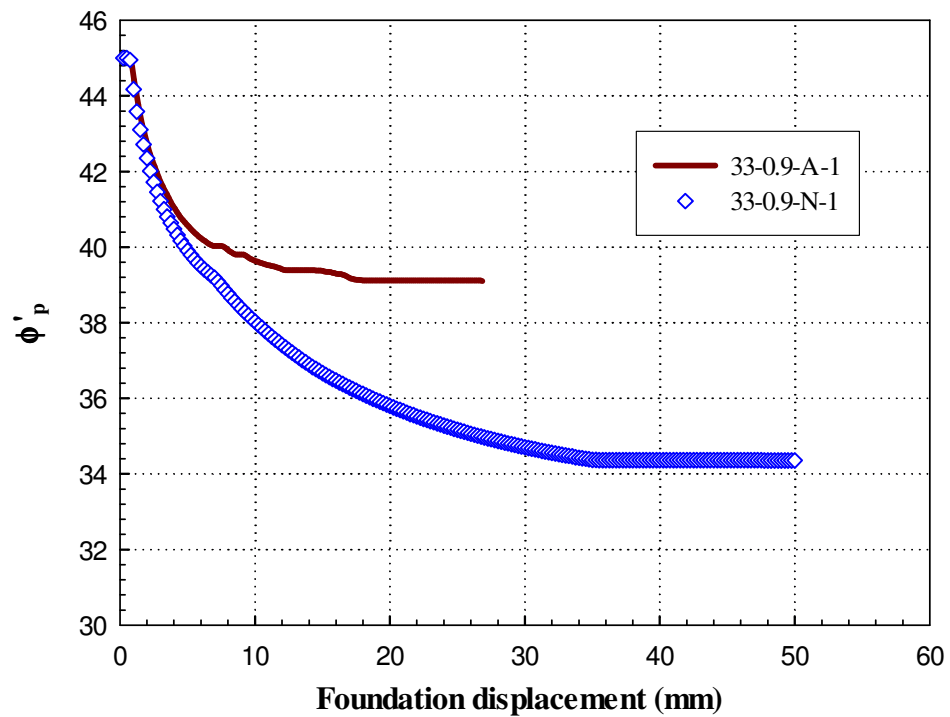


Figure 4.18: Decrease of ϕ'_p beneath the footing

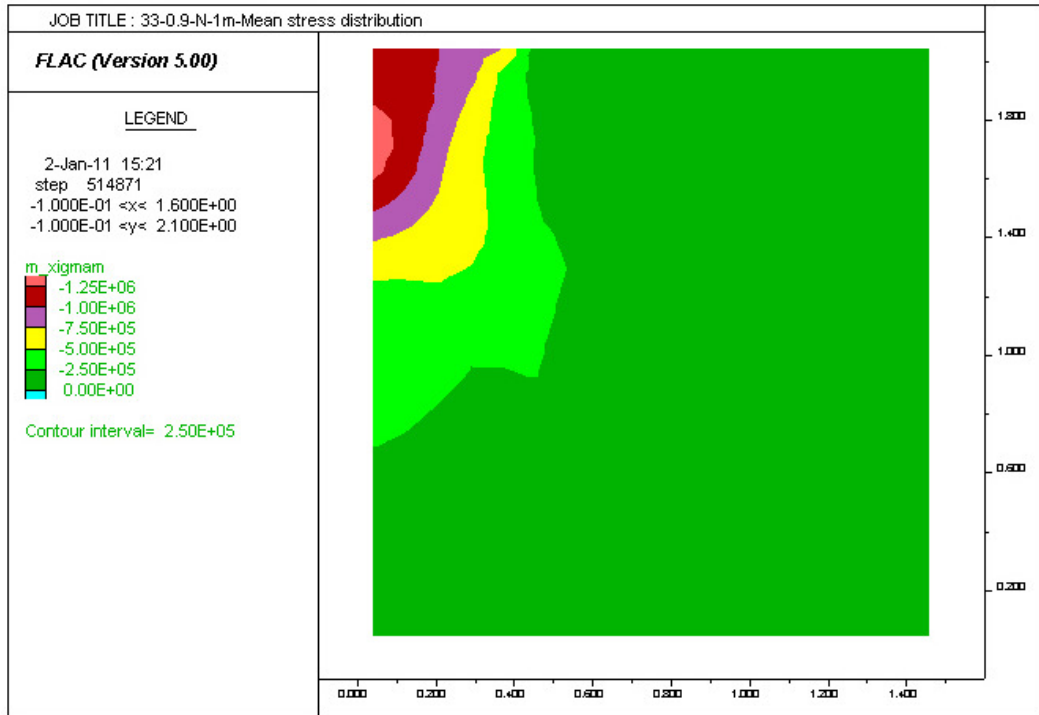


Figure 4.19: Mean stress distributions (Non-associated flow rule)

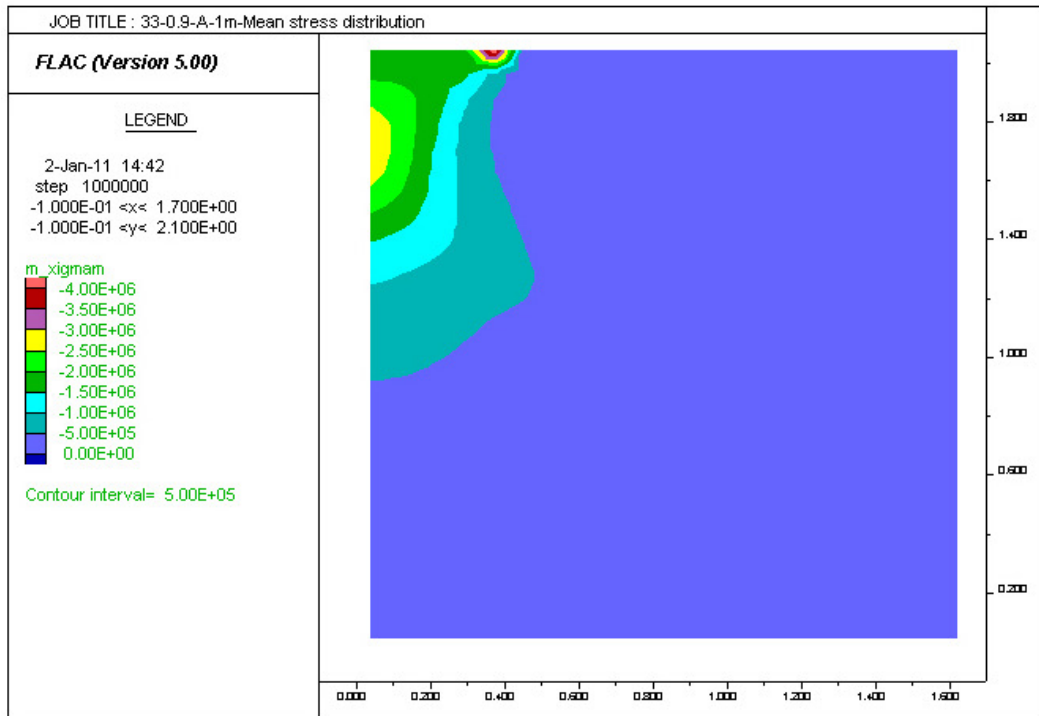


Figure 4.20: Mean stress distributions (Associated flow rule)

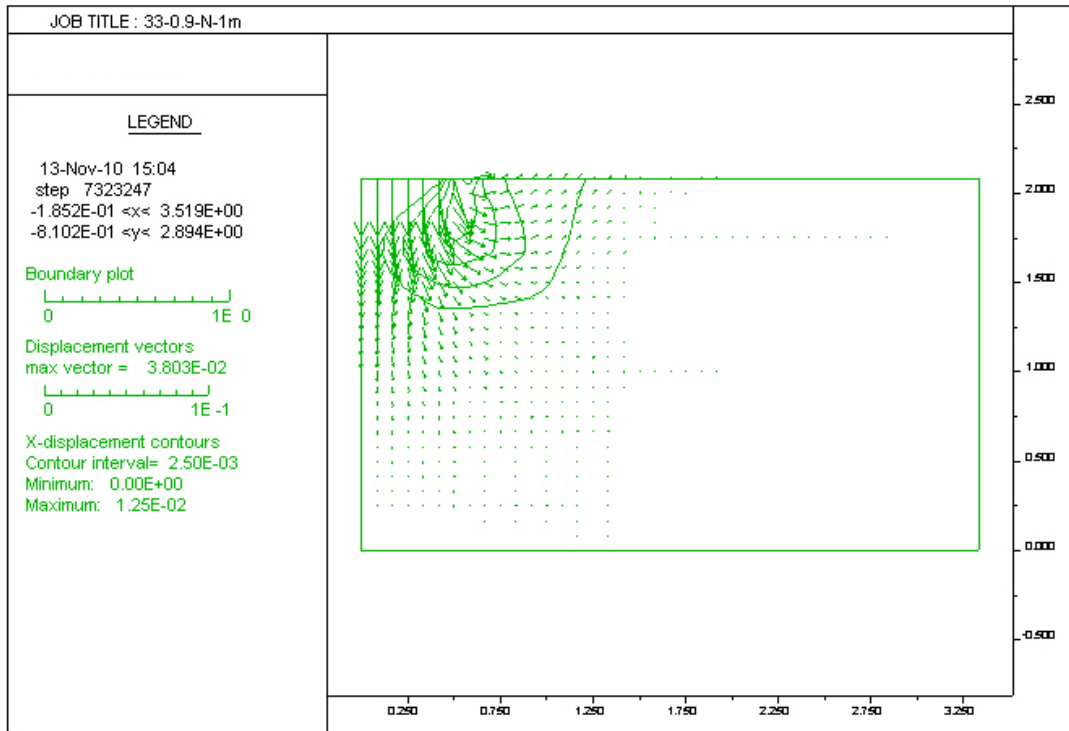


Figure 4.21: Displacement field (Non-associated flow rule)

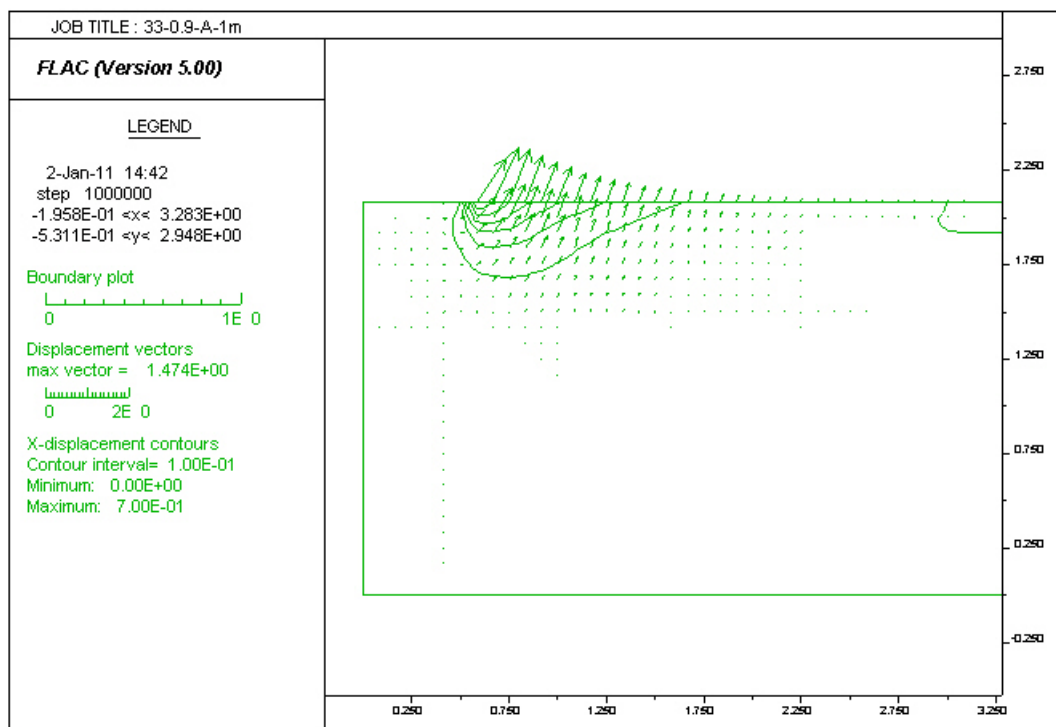


Figure 4.22: Displacement field (Associated flow rule)

4.6 Summary

This chapter presented the numerical study of the scale effect of the bearing capacity of rigid, rough and circular footings resting on surface of cohesionless soil. The study focused on the effect of stress- and density-dependent peak strength ϕ'_p of cohesionless soil on the bearing capacity. FLAC was used to carry out the numerical simulations. Bolton's (1986) equation (Equation 4.3) describing the relationship between peak strength ϕ' , relative density D_r , and mean effective stress σ'_m was adopted in this study. The built-in MC constitutive model in FLAC was modified to incorporate the Bolton (1986) equation. The modified MC constitutive model is capable of correlating the peak strength of cohesionless soil ϕ'_p to the relative density and effective confining stress of the soil in the simulation. Therefore, scale effect due to the stress- and density-dependent peak strength of sand can be investigated.

The modified MC constitutive model was verified using an element test under both triaxial compression and extension. Parametric studies were carried out to investigate the effect of Young's modulus, Poisson's ratio, bulk density and at-rest earth pressure coefficient on the ultimate bearing capacity of the footing. Based on the parametric studies, Young's Modulus, Poisson's ratio, bulk density and at-rest earth pressure coefficient do not have any detectable effect on the ultimate bearing capacity of footing.

Another series of parametric study was carried out to determine N'_γ that accounts for scale effect. Relative densities of 0.3, 0.6 and 0.9 were considered. Footing widths of 1m, 2m, 4m to 10m were examined. Three critical state angles of internal friction were investigated, i.e., 30° , 33° and 36° . Both associated and non-associated flow rule were used. The computed bearing capacity factors were curve fitted with a power function. The exponent β^* in the power function ranges from 0.10 to 0.39. The fitting parameter N^*_γ ranges from 56.8 to 244. The numerical values compared well with measured values from centrifuge tests. The effect of associated and non-

associated flow rule on PLT was also examined in terms of the distributions of peak angle of internal friction and mean stress level.

Chapter 5 Schmertmann's (1970, 1978) Method and its

Modification Considering Small-Strain Stiffness

5.1 Introduction

Schmertmann's (1970, 1978) method (Schmertmann, 1970; Schmertmann et al., 1978) is probably one of the most frequently used methods for estimating settlement of shallow foundations on cohesionless soil. A number of modifications aiming at improving the accuracy of settlement estimation have been proposed. This chapter reviews Schmertmann's method and these modifications. The limitations of the method and the modifications were discussed. A new modification considering small-strain stiffness based on Schmertmann's (1970, 1978) method was developed. The new method considered existing empirical correlation between small-strain stiffness G_0 and tip resistance q_c from CPT; empirical correlation between effective angle of internal friction ϕ' and q_c ; methods of estimating ultimate bearing capacity of shallow foundation from ϕ' ; and a new expression of peak strain influence factor I_{zp} as a function of mobilized load ratio to indirectly account for the modulus degradation of soil and non-linear nature of load-settlement curves of vertically loaded foundations. The expression of peak strain influence factor I_{zp} was calibrated using 14 load-settlement curves from PLT at two sites, i.e., Changi East reclamation site, Singapore and Texas A&M University, USA. The proposed modified Schmertmann's method was summarized.

5.2 Schmertmann's (1970, 1978) Method

Schmertmann (1970) developed a method to estimate settlement of shallow foundation on cohesionless soil based on Finite Element Method (FEM) study of strain influence diagrams, model tests, and correlation between tip resistance q_c from CPT and nominal E_s used in settlement calculation. Subsequently,

Schmertmann et al. (1978) proposed several improvements: using improved strain influence diagrams for square and strip footings; using different expression of the maximum strain influence factor I_{zp} ; and finally, using different correlation between E_s and q_c for square and strip footings. According to Schmertmann et al. (1978), the Schmertmann et al.'s (1978) method is similar or better than the original Schmertmann's (1970) method.

According to Schmertmann's (1970, 1978) method, settlement of shallow foundation on sands can be calculated using the following expression:

$$s = C_D C_C \Delta q \sum_0^{z_i} \frac{I_z \Delta z}{E_s} \dots\dots\dots (5.1)$$

where s = footing settlement; Δq = net pressure on footing; B = footing width or diameter; Δz = thickness of the stratum; z_i = depth of influence; C_D , C_C = depth correction factor and creep factor respectively, which are defined as:

$$C_D = 1 - 0.5 \left(\frac{\sigma_{v0}}{\Delta q} \right) \dots\dots\dots (5.2)$$

where σ_{v0} = the overburden pressure at the foundation level.

$$C_C = 1 + 0.2 \log \left(\frac{t(\text{year})}{0.1} \right) \dots\dots\dots (5.3)$$

I_z = vertical strain distribution factor, which can be calculated based on the improved strain influence factor diagrams as shown in Figure 5.1, where the peak strain influence factor I_{zp} is expressed as

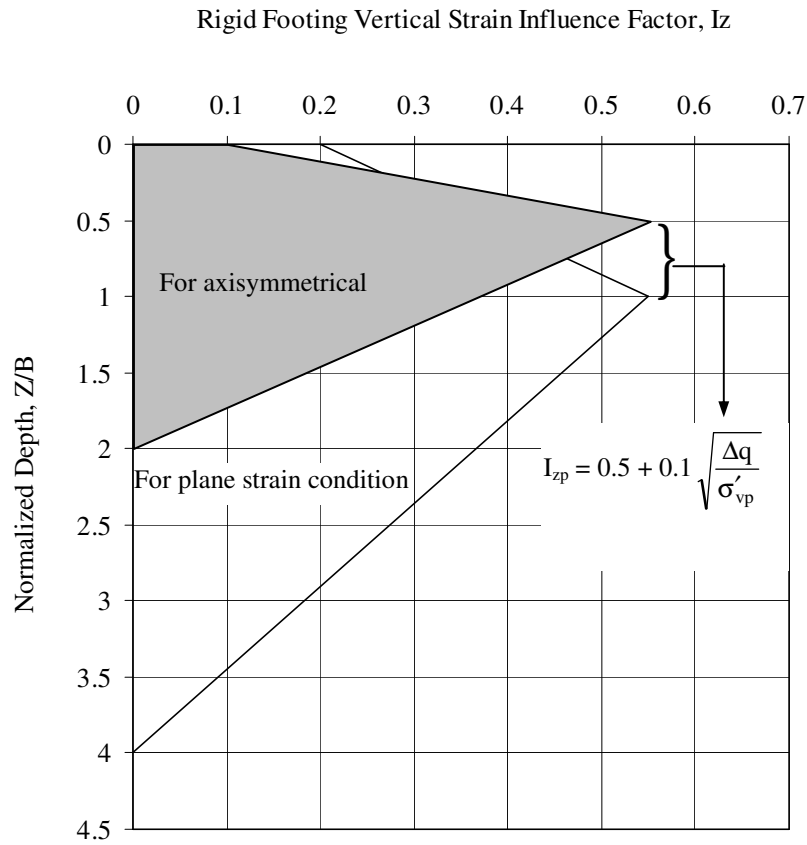


Figure 5.1 Vertical strain influence factor distributions (after Schmertmann et al. 1978)

$$I_{zp} = 0.5 + 0.1 \sqrt{\frac{\Delta q}{\sigma'_{vp}}} \dots\dots\dots (5.4)$$

E_s = Young's modulus of the soil stratum, which can be obtained from q_c of CPT by

$$\begin{aligned} E_{s(axisym)} &= 2.5q_c \\ E_{s(planestrain)} &= 3.5q_c \end{aligned} \dots\dots\dots (5.5)$$

Although Schmertmann's (1970, 1978) method is frequently used and reported to give reasonable estimations of settlement of footings, there are shortcomings. First of all, Schmertmann's (1970, 1978) method tends to overestimate the settlement of

large foundations in practice (Briaud and Gibbens, 1994, Shad et al., 2003); secondly, for very loose sand, Schmertmann's (1970, 1978) method is reported to underestimate the settlement (Marangos, 1995); thirdly, the estimated load-settlement curves using Schmertmann's (1970, 1978) method are not flexible enough to match the whole measured load-settlement curves from the very beginning of loading to failure (Chang et al. 2005). As a result, a number of modifications have been proposed to overcome these shortcomings.

5.3 Existing Modifications to Schmertmann's (1970, 1978) Method

A number of modifications to Schmertmann's (1970, 1978) method can be found in the literature. These modifications focus on the following two aspects: E_s/q_c ratio shown in Equation (5.5) and strain influence diagram shown in Figure 5.1.

5.3.1 E_s/q_c Ratio

The correlation between Young's modulus E_s used in Equation (5.5) and q_c from CPT is one of the most important factors affecting accuracy of estimation of settlement using Schmertmann's (1970, 1978) method. This correlation has been discussed even before the publication of Schmertmann's (1970, 1978) method. The correlation could be expressed as follows (David, 1987):

$$E_s = \alpha'(q_c) + \beta' \dots\dots\dots (5.6)$$

where β' can be approximated to be zero according to many investigations (David, 1987); Values of α' suggested by researchers generally vary from about 1.5 to 3, depending on the sand type, relative density and the stress history of sand. However, according to Robertson and Campanella (1983) α' may be 3 to 6 times greater for overconsolidated sands compared to normally consolidated sands. According to David (1987), a value of 2.5 for E_s/q_c ratio was considered typical by most researchers.

The advantage of using constant E_s/q_c ratio is apparent, i.e., convenience in calculation. However, the price is the accuracy of estimation, particularly for large footings, where, Schmertmann's (1970, 1978) method generally overestimates the settlement considerably. In fact, the back calculated E_s/q_c ratio for large foundation based on best-match of measured load-settlement curve is found to be much higher than 2.5 or even 3.5. For example, Shad et al. (2003) found that using $E_s/q_c = 6$ leads to less overestimation of settlement in their analyses.

Some researchers attributed the higher E_s/q_c ratio to overconsolidation of in situ soils, (Robertson and Campanella, 1983). Apparently, overconsolidation is one of the possible reasons. However, it has been discussed in Chapter 2 that E_s is not a constant, but dependent on factors such as strain level, relative density and stress history, so does E_s/q_c ratio (Bellotti et al., 1986). Robertson (1991) related E_s/q_c ratio to the degree of loading as a measure of strain level, as shown in Figure 5.2. They suggested that values from Figure 5.2 be reduced by a factor of 2 for aged NC sand and by a factor of 3 for young NC sands. It can be seen that E_s/q_c ratio can be as large as 26 at small degree of loading, which is comparable to the back analysis of the E_s/q_c ratio of the 31 cases shown later.

The E_s/q_c ratio is believed to vary with load level because the mobilized averaged strain beneath the footing changes with load level. It is commonly understood that the typical range of strain levels in the soil under a foundation is between 0.01% and 0.2% (Jardine et al. 1985 and Burland 1989). At these strain levels, the bearing pressure is usually much smaller than the ultimate bearing capacity. In the early studies, E_s/q_c ratio was mostly assumed to be 2.5 (David, 1987), which generally corresponds to strain level larger than the average strain levels encountered in foundation problems. Schmertmann (1970) correlated E_s from FCPT to q_c from the CPT, with the prevailing average strain possibly comparable with the average strains beneath small footings. The observed E_s/q_c ratio of 2.5 is therefore suitable for estimating settlement of small footings. Thus, E_s/q_c ratio of 2.5 would unavoidably result in a significant overestimation of settlement for large foundations.

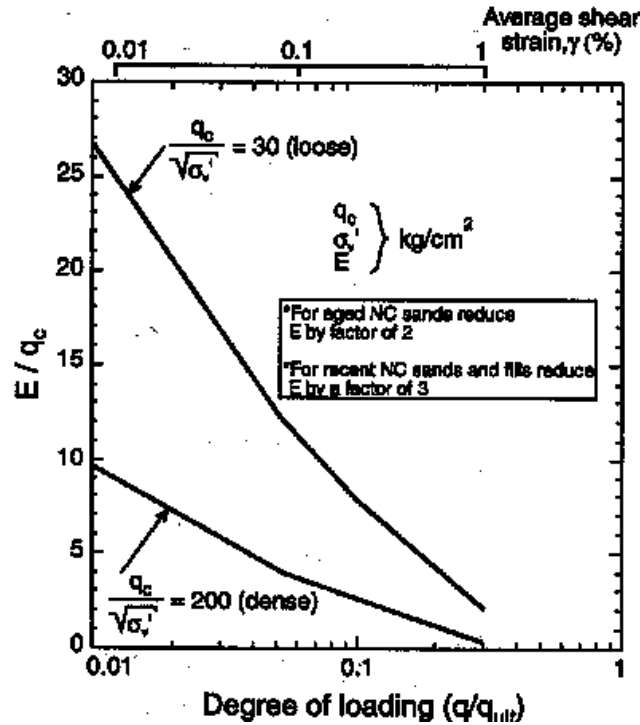


Figure 5.2: Estimation of equivalent Young's modulus for sand based on degree of loading (after Robertson, 1991)

Figure 5.2 clearly shows the dependence of E_s/q_c ratio on stress history, in addition to relative density and strain levels. Several attempts have been made to measure the overconsolidation ratio of in situ sand, however, there are no reliable methods to estimate stress history by CPT at present (Robertson and Powell, 1997).

From Equation (5.5), E_s/q_c ratio may depend on the shape of the footing. There are modifications that can be found in the literature relevant to this issue, for example, Sargand et al. (2003) used a continuous function to replace Equation (5.5). There are suggestions of obtaining E_s from other in situ tests, such as using DMT (Leonards and Frost, 1988).

5.3.2 Strain Influence Factor Diagram

The strain influence diagram has been discussed in detail in Chapter 3. Besides considering the non-linear nature of stress-strain behaviour, modifications of I_{zp} can also be found in the literature (Mesri and Shahien, 1994 and Chang et al., 2005). The modifications are mainly driven by the need to better account for the non-linear stress-strain behaviour of sand. For instance, Mesri and Shahien (1994) found that Schmertmann's (1970, 1978) method tends to underestimate the settlement of foundation on loose sand. To overcome this, they modified I_{zp} as:

$$I_{zp} = 0.5 + 0.25 \frac{\sqrt{F.S.}}{\sqrt{F.S.} - 1} \log \frac{1}{1 - D_r} \quad (\text{for dense sand}) \dots\dots\dots (5.7)$$

$$I_{zp} = 0.5 + 4 \frac{\sqrt{F.S.}}{\sqrt{F.S.} - 1} \log \frac{1}{1 - D_r} \quad (\text{for loose sand})$$

where F.S. = factor of safety against foundation failure, and D_r = relative density of sand.

Following Schmertmann's (1970, 1978) approach, Chang et al. (2005) found that better matches of measured and calculated load-settlement curves are possible by introducing the following expression:

$$I_{zp} = 0.5 + n \left(\frac{\Delta q}{\sigma'_{vp}} \right)^m \dots\dots\dots (5.8)$$

where m and n = constants that can be determined by fitting the load-settlement curves of foundations. Based on the analysis of 15 PLT tests from a reclamation site and consideration of convenience in application, Chang et al. (2005) found that m = 0.5 is acceptable; while typical n values of 0.04 for medium to very dense sand and 0.3 for loose sand were suggested. It can be seen that compared with Equation (5.4),

the suggested n value in Equation (5.8) leads to smaller estimations of I_{zp} for dense sand and larger estimation for loose sand.

5.3.3 Discussion on the Modifications of Schmertmann's (1970, 1978) Method

Schmertmann's (1970, 1978) method is widely adopted in practice because it effectively balances accuracy of estimation and convenience of application. Existing modifications generally aimed at improving the accuracy of the settlement estimation without causing too much additional inconvenience in the application. Among the various proposed modifications, some are conceptually correct but does not improve the accuracy of settlement estimation significantly. For instance, the modification of strain influence factor diagram by assuming $I_{z0} = 0.35$ rather than $I_{z0} = 0.1$ for axisymmetric condition is probably more reasonable compared with the results based on elastic theory, yet this modification does not change the results considerably. Other modifications that significantly changed the settlement estimation were justified based on results of PLT, such as Chang et al. (2005), or model tests on sand deposits, such as Mesri and Shahien (1994). The validity for their applications elsewhere has not been fully evaluated. Most of these tests were conducted using plates of 0.5m in diameter or 0.4m in width. The results based on these tests may not be applicable to footings of larger sizes.

The proposed modification of E_s/q_c ratio by Robertson (1991) shown in Figure 5.2, considering the dependence of E_s on strain level, relative density and stress history of sand, has the most significant effect on settlement estimations using the Schmertmann's (1970, 1978) method. For large foundations, due to the lower average strains beneath the foundations, the E_s/q_c ratio can be much higher than 2.5 or 3.5 as reported by some researchers (e.g. Sargand et al. 2003). The existing modifications usually lead to considerable overestimation of settlement probably because of inadequate consideration of strain-dependent stiffness of cohesionless soil.

However, Figure 5.2 proposed by Robertson (1991) is inconvenient to apply in practice. Besides, non-linearity of the load-settlement curve has already been taken into account in Schmertmann's (1970, 1978) method by Equation (5.4), therefore Figure 5.2 should be used with caution, in terms of the appropriate E_s/q_c ratio to be used at which strain level.

Small-strain stiffness E_0 can be used to replace E_s in Equation (5.1). Compared to complicated correlation between E_s and q_c which depends on many factors, the correlation between E_0 and q_c is simpler. Moreover, small-strain stiffness E_0 provides a good benchmark for non-linearity of load-settlement curves regardless of the size and shape of foundation. One problem in using E_0 in Equation (5.1) is that E_0 is too stiff and therefore an appropriate measure to account for modulus degradation should also be adopted. In this study, Equation (5.4) was modified to account for using E_0 in Equation (5.1) to indirectly cater for the modulus degradation of soil and the non-linear nature of the load-settlement curve. In the subsequent sections, modifications of Schmertmann's (1978) method based on CPT data are proposed. Fourteen data sets of measured CPT and PLT curves are used to calibrate the proposed modifications. The proposed modifications were evaluated using 31 case studies from Jeyapalan and Boehm (1984).

5.4 Proposed Modifications to Schmertmann's (1970, 1978) Method

The proposed modifications of Schmertmann's (1970, 1978) method are:

- 1) Using small-strain stiffness E_0 instead of E_s in Equation (5.1). Small-strain stiffness E_0 within the depth of influence can be estimated from q_c based on existing empirical correlation between the two.
- 2) Using strain influence factor diagrams for square foundation as shown in Figure 5.1.

3) Using the following expression of I_{zp} to account for the non-linearity of load-settlement curve of PLT or foundations,

$$I_{zp} = 0.5 + m \left(\frac{\sqrt{q/q_{ult}}}{1 - \sqrt{q/q_{ult}}} \right)^n \dots\dots\dots (5.9)$$

where m and n = constants needed to be determined by fitting load-settlement curves of PLT; q_{ult} = ultimate bearing capacity of foundation, which can be estimated using Vesic's (1970) Equation and effective angle of internal friction ϕ' estimated based on q_c from CPT test.

Similar to Equation (5.4), Equation (5.9) is empirical and the main purpose of introduction of Equation (5.9) is to account for the modulus degradation of soil and non-linearity of load-settlement curves. The main difference between Equation (5.9) and Equation (5.4) is that a mobilized stress ratio is adopted in the former. It can be seen from Equation (5.9) that when $q = 0$, $I_{zp} = 0.5$ and when $q = q_{ult}$, $I_{zp} = \infty$. The efficacy of Equation (5.9) was examined and m and n were calibrated using 14 load-settlement curves from two sites in subsequent sections. But before that, existing correlations between small-strain stiffness E_0 and q_c , and internal angle of friction ϕ' and q_c were reviewed based on the two sites described.

5.4.1 Description of the Test Sites and In Situ Tests

In the development of the modified procedure, CPT and PLT results measured on two sites were analyzed: the first is Changi East reclamation site, Singapore and the second is Texas A&M University, USA (Briaud and Gibbens, 1994). For the former site, all the PLT were conducted on plate with diameter of 0.5m. But the sand varies significantly in relative density and stress history. For the latter, the sands have similar relative density, but the spread footing tested varies in width from 1m to 3m. On both sites, several other in-situ tests have been conducted, so that the interpreted

soil properties from various tests can be compared and evaluated. Brief descriptions of the two sites and results of CPT and PLT were presented below.

At Changi East reclamation site, Singapore, a total of 15 PLT and 15 CPT were conducted at three lots, Lot-1, Lot-2 and Lot-3. At each lot, five PLT and five CPT were conducted in five stages. The details are shown in Appendix A. The PLT and CPT were carried out at Stage-1 first. Then an overburden of 3m was applied and maintained for about 9 months. After that the sand was carefully removed layer by layer to a certain elevation, and PLT and CPT were carried out at this elevation. At the same elevation level, CPT was located at the centre point where PLT would be conducted. More detailed description of the site and in-situ tests can be found in Na et al. (2002).

At Changi East reclamation site, the sand used in the reclamation work was of marine origin. The fill material consisted primarily of coarse sand, classified as hard and "sub-angular", according to ASTM D2488 (1993). The specific gravity of the sand was 2.66. The sand was relatively clean with fines content less than 2.1%. The grain size distribution of the sand varied across the test area. Figure 5.3 shows the ranges of grain size distributions for samples recovered at various levels between elevations of 3.0m and 12.5m in Lot-1. The lines in Figure 5.3 indicate the envelope of the grain size distributions. The grain size distributions were found to be similar to the sand in Lot-2. For the sand in Lot 3, the sand particles were more uniform, compared with those sands from Lot-1 and Lot-2. The characteristic particle size D_{60} was about 0.5mm at all three lots and the coefficient of uniformity C_u was generally between 2 and 6 for Lot-1 and Lot-2 and around 2.9 for Lot-3. The sand at Changi East reclamation site was classified as SP (poorly graded sand) based on the Unified Soil Classification System.

Two filling methods were used in the reclamation, i.e. hydraulic pumping, adopted at Lot-1 and Lot-2, and direct dumping, adopted at Lot-3. Due to the different filling methods, Lot-1 and Lot-2 showed quite different soil properties from Lot-3. The sands in Lot-1 and Lot-2 were medium dense to very dense with relative

density ranging typically from 53% to 100%. The typical relative density of the sand in Lot-3 was between 30% and 40%, except the two layers, which had been compacted by vehicular traffics during the period of the reclamation work.

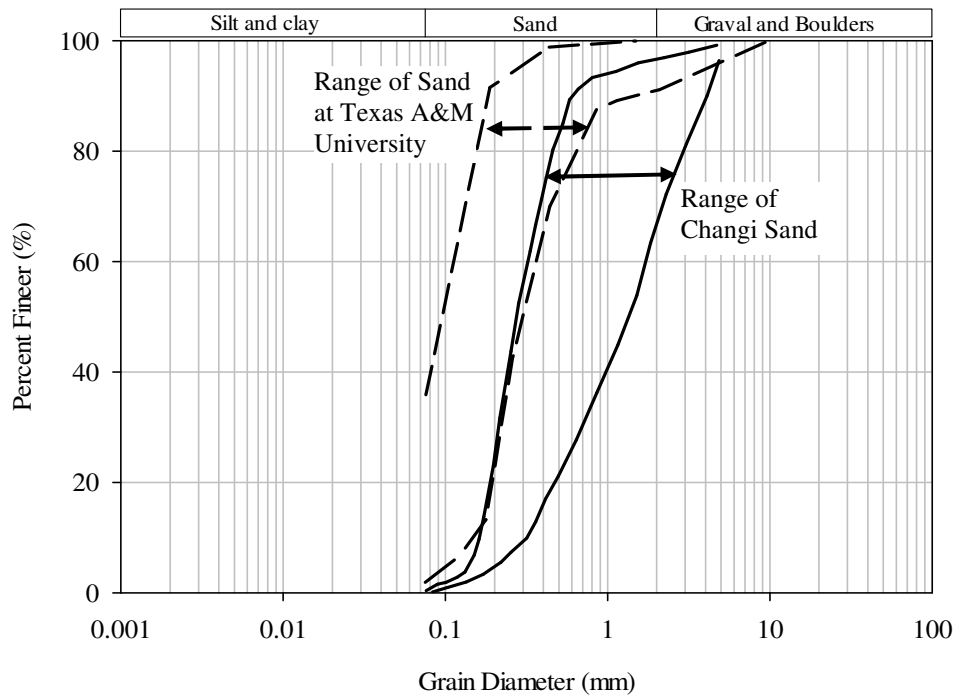


Figure 5.3: Range of grain size distributions at Changi East reclamation site and Texas A&M University (after Na. 2002 and Briaud and Gibbens, 1994)

Appendix A also shows 15 q_c profiles from CPT conducted at the three lots. It can be seen that generally at Lot-1 and Lot-2, q_c is larger than that at Lot-3. However, at Lot-3, there are two particularly dense layers with very high q_c value. The particularly stiff layer corresponds to the two layers compacted by vehicular traffic. Since q_c depends not only on the relative density, but also on stress components, normalized q_c profiles are plotted in Figure 5.4 in order to have a better understanding of the CPT results. Furthermore, considering that G_0 will be interpreted from q_c , q_c is normalized by $(\sigma_{v0})^{0.358}$, which is the stress component in the empirical correlation between q_c and small-strain stiffness G_0 presented by Hegazy and Mayne (1995). From Figure 5.4, it can be seen that at Lot-1, normalized q_c profiles from different stages are comparable if the overburden when

the CPT were conducted was sufficiently large. For example, comparing Stage-1 and Stage-2, one can find that the q_c profiles above elevation of about 8.8m differ slightly. However, below elevation of 8.8m, the two profiles are quite close to each other. Comparing Stage-3 with Stage-1 and Stage-2, the q_c profile from Stage-3 differs a little from surface of Stage-3 (elevation 9.5m) to elevation of 9.0m. Below evaluation of 9.0m, the q_c profiles from the three stages are comparable. So is the q_c profile from Stage-4. The inconsistency of q_c profiles near ground surface may be due to disturbance when removing the overburden sand. However, it may also imply that CPT results near the ground surface were not reliably repeated, possibly because of the extremely low confining pressure.

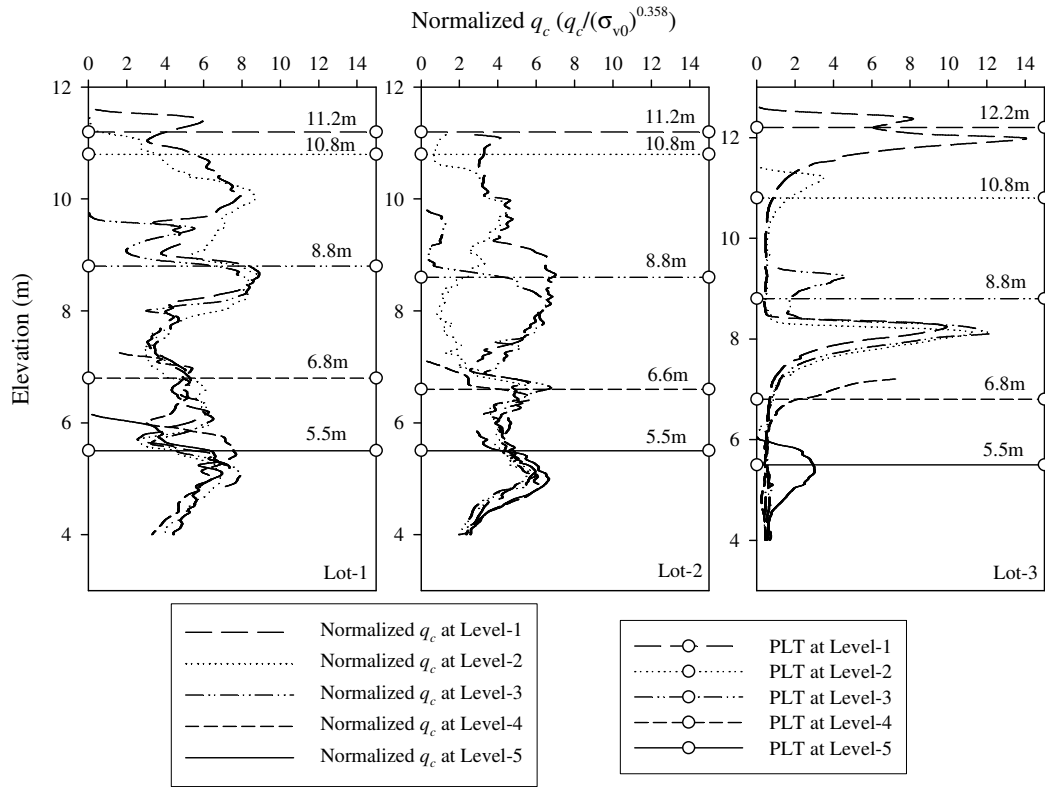


Figure 5.4: Normalized q_c profiles at Changi East reclamation site

At Lot-2, q_c profiles from various stages show similar trend described above, except for Stage-2. The q_c profile for Stage-2 differs a lot even at very large depths (from elevation about 9.5m to 7.0m). At Lot-3, similar trend can be observed. However,

for Lot-1 and Lot-2, because the sand was relatively dense, the q_c values near ground surface are smaller than what are expected due to surface disturbance. At Lot-3, the sand was very loose generally. The q_c value near the ground surface at each stage tends to be larger than expected due to surface disturbance.

Appendix A also gives the 15 load-settlement curves of PLT at Changi East reclamation site. It can be seen from Figure A.3 that at Lot-1 and Lot-2, the PLT curves are much stiffer compared to those of Lot-3, except for Stage-1. A careful check of the PLT curves of Lot-3 shows that for Stage-2, the PLT curve was stiffer compared to the other three stages at the beginning of the loading, till around 0.1MPa. This is probably due to the sand being compacted by vehicular traffic as described earlier, which makes the modulus degradation matching for the PLT curve quite different from the others which were not so stiff at the beginning. The modulus degradation matching for the PLT curve will be elaborated later.

In Texas A&M University, USA, five PLT and five CPT were carried out on an 11m thick sand layer. The layout of the CPT and PLT are shown in Appendix B. It can be seen from Figure B.2 that CPT-2, 5, 6 and 7 were located within the area where PLT-2.5m, 3mN, 1.5m, 3mS were conducted, respectively. CPT-1 was located close to where PLT-1m was conducted. As a result, each corresponding CPT data was used to analyze load-settlement curve for each PLT.

The 11m sand layer at Texas A&M University consisted of a 3.5m-thick medium dense silty sand layer, a 3.5m-thick medium dense silty sand layer with clay and gravel, followed by a layer of medium dense silty sand and sandy clay mixed with gravel. Properties of the sands at depth of 0.6m and 3.0m were measured: Specific gravities of the sands were 2.64 and 2.66, respectively and the maximum dry unit weights were 15.5kN/m^3 and 16.1kN/m^3 , respectively. There were strong evidences showing that the area experienced pre-consolidation stress. The analyses of CPT data showed that the average OCR was about 6 (Mayne, 1994). Other detail information of the site and tests can be found in Briaud and Gibbens (1994).

Five PLT were conducted using square plates of different sizes: two using concrete spread footing with width of 3m, one using footing with width of 2.5m, one using footing with width of 1.5m and the last one using footing with width of 1.0m. All footings were founded at a depth of 0.76m in the sand. The results of the CPT and PLT are given in Appendix B.

5.4.2 *Small-strain Stiffness G_0 from CPT*

Jamiolkowski et al. (1988) showed that soil density and in situ effective confining stress are two main factors affecting both q_c and G_0 . Hence, a correlation between q_c and G_0 can logically be found for uncemented and unaged cohesionless soils. Based on calibration chamber test results and field measurements, Rix and Stoke (1992) suggested the following correlation for uncemented quartz sands:

$$G_0 = 1634(q_c)^{0.25}(\sigma'_{v0})^{0.375} = 29057(q_c / P_a)^{0.25}(\sigma'_{v0} / P_a)^{0.375} \dots\dots\dots (5.10)$$

where G_0 = small-strain stiffness, in kPa; q_c = tip resistance from CPT, in kPa; and σ'_{v0} = vertical effective stress, in kPa, P_a = reference pressure, equal to 100 kPa.

Another empirical correlation between q_c and G_0 can be derived from the well-known q_c - V_s relationship reported by Hegazy and Mayne (1995):

$$V_s = 13.18(q_c)^{0.192}(\sigma'_{v0})^{0.179} = 72.76\left(\frac{q_c}{P_a}\right)^{0.192}\left(\frac{\sigma'_{v0}}{P_a}\right)^{0.179} \dots\dots\dots (5.11)$$

Where V_s = shear wave velocity, in m/s. q_c and σ'_{v0} are in kPa, P_a = reference pressure, equal to 100 kPa. Small-strain shear modulus G_0 and shear wave velocity are related as follows

$$G_0 = \rho V_s^2 = 52.94 \rho \left(\frac{q_c}{P_a} \right)^{0.384} \left(\frac{\sigma'_{v0}}{P_a} \right)^{0.358} P_a \dots\dots\dots (5.12)$$

where ρ = total mass density of in situ soil.

According to Mayne (1994), Equation (5.10) produced comparable estimations with G_0 measured using cross-hole test (CHT) on Texas A&M University site. According to Na et al. (2005), Equation (5.11) and (5.12) produced comparable estimations with G_0 measured from SCPT at Changi East Reclamation site. However, Equation (5.10) generally produces higher estimation than Equation (5.11) and (5.12). The ratio, R_{G_0} , between the two estimations can be calculated as:

$$R_{G_0} = \frac{(G_0)_1}{(G_0)_2} = \frac{5.49}{\rho} \left(\frac{q_c}{P_a} \right)^{-0.134} \left(\frac{\sigma'_{v0}}{P_a} \right)^{0.017} \dots\dots\dots (5.13)$$

where $(G_0)_1$ and $(G_0)_2 = G_0$ estimated from Equation (5.10) and Equations (5.11), (5.12), respectively.

Equation (5.13) shows that R_{G_0} increases with increase of σ'_{v0} , or depth, and decreases with increase of q_c ; A simple numerical test can be carried out to check the effect of variation of σ'_{v0} and q_c on R_{G_0} , assuming q_c is constant with depth. Figure 5.5 plots the variation of R_{G_0} with depth at different q_c values by assuming $\rho = 1.7 \text{ Mg/m}^3$. It can be seen that R_{G_0} increases a little near ground surface. However, at deeper depths ($z > 1\text{m}$), the increase of R_{G_0} with depth is very slow and can be neglected in the depth of influence for shallow foundations. Figure 5.5 also shows that with increase of q_c , R_{G_0} increases. With q_c increasing from 3MPa to 14MPa, R_{G_0} increases from around 1.5 to 2.1. However, one should remember that a constant ρ was assumed here. For the case that q_c is small, it is reasonable to assume

a ρ smaller than 1.7 Mg/m^3 , so that from Equation (5.13), R_{G_0} should increase slightly more and vice versa for cases when q_c is large.

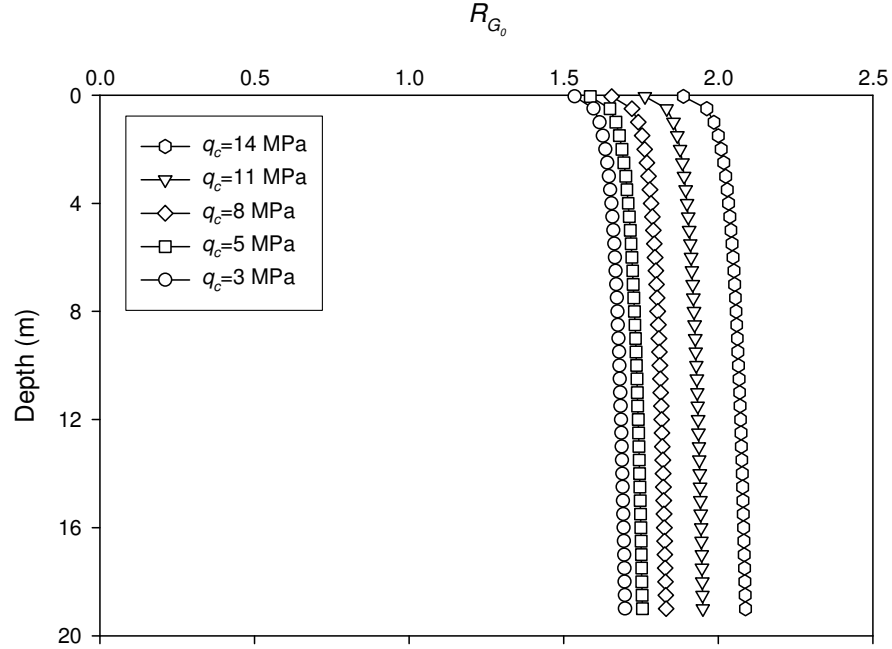


Figure 5.5: R_{G_0} versus depth at different q_c values

In the subsequent sections, both empirical correlations between q_c and G_0 are adopted since evidences show that Equation (5.10) is suitable for Texas A&M University site and Equations (5.11) and (5.12) are suitable for Changi East reclamation site. However, in the proposed procedure in estimating settlement of shallow foundation on sands, Equations (5.11) and (5.12) are recommended if there is no evidence to show that Equation (5.10) is better, because Equations (5.11) and (5.12) are based on statistical analysis of 24 different field sand sites world-wide, while Equation (5.10) is only for uncemented quartz sands. Furthermore, Equations (5.11) and (5.12) tend to produce conservative estimates in terms of smaller G_0 compared to Equation (5.10). However, it is important to note that compared with the ultimate bearing capacity estimated from CPT that will be discussed subsequently, G_0 or E_0 estimated from CPT has a greater effect on the estimation of settlement, which will be elaborated later.

In the interpretation using Equations (5.10), (5.11) and (5.12), information of in-situ vertical effective stress σ'_{v0} is required for both methods. In this chapter, γ' is assumed to be 17 kN/m^3 for medium to dense sand when q_c is larger than 5.5 MPa and 15 kN/m^3 for loose sand, when q_c is less than 5.5 MPa . Poisson's ratio is assumed to be 0.2 at small and intermediate strains, so that E_0 can be calculated from G_0 .

In the interpretation, G_0 corresponding to each q_c measured at different depths, or different σ'_{v0} , are first estimated using two methods. Then, the interpreted G_0 values within the depth of influence, i.e., two times of width or diameter B of the plate according to elastic theory, are averaged, based on the area equilibrium theory. The weighted-averaged G_0 values within $2B$ were used to fit the load-settlement curves from PLT in subsequent analysis. Alternatively, one can average q_c value within the depth of influence based on the area equilibrium first. Then calculate the corresponding G_0 according to Equation (5.10) or Equations (5.11) and (5.12) using σ'_{v0} at depth of $1B$. The two methods gave close results. This implies that for those cases where detailed profiles of q_c are not available, one may use σ'_{v0} at depth of $1B$ and the average q_c to calculate small strain stiffness G_0 .

Figure 5.6 shows an example of interpretation of G_0 from q_c measured at Texas A&M University. It can be seen that estimations from the two methods are comparable near the ground surface, but differ as depth increases. Within a certain depth, the ratio of R_{G_0} ranges from 1.4 near the ground surface to slightly more than 2.0 at deeper depths, if q_c is small. Table 1 lists the interpreted values of G_0 of the 14 cases that will be used later. In this study, for the cases at Texas A&M University, there was a 0.76 m overburden when CPT was performed. For those CPT conducted at Changi East reclamation site, overburden ranges from 0.5 m to 0.9 m because CPT results at different stages were used.

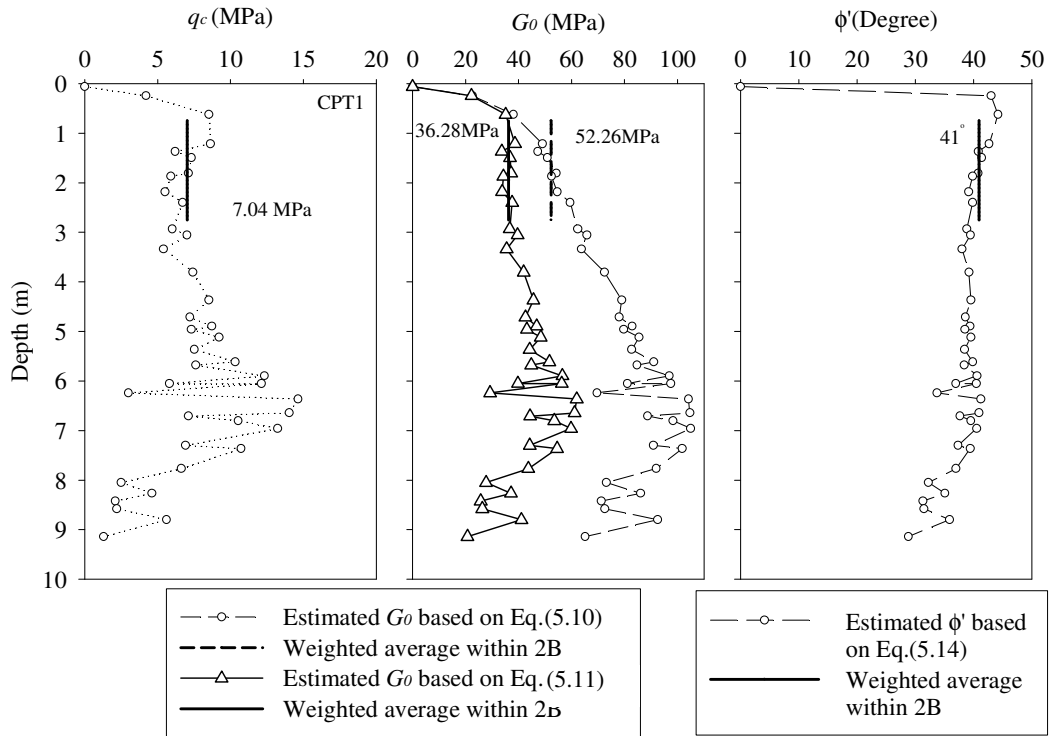


Figure 5.6: Example of interpretation of G_0 and ϕ' from CPT1 at Texas A&M University

5.4.3 Effective Angle of Internal Friction ϕ' from CPT Test

To normalize the foundation pressure, ultimate bearing capacity of the foundation must be known. For cohesionless soil, bearing capacity is normally estimated from effective angle of internal friction ϕ' . Principally, for a given cohesionless soil, two main factors controlling strength property are soil density or relative density and effective confining stress, which are similar to those affecting q_c of CPT. Thus, q_c from CPT is frequently correlated to ϕ' . A number of correlations have been published. Many of these methods are based on the assumption of known values for properties such as relative density, grain size distribution, coefficient of in situ lateral stress, etc, which are difficult to obtain reliably in-situ. For convenience, two commonly referred methods provided by Robertson and Campanella (1983) and Kulhawy and Mayne (1990) are examined. They are:

$$\phi' = 17.6 + 11.0 \log \frac{q_c / P_a}{\sqrt{\sigma'_{v0} / P_a}} \quad (\text{Kulhawy and Mayne, 1990}) \dots\dots\dots (5.14)$$

$$\phi' = \tan^{-1}[0.1 + 0.38 \log(q_c / \sigma'_{v0})] \quad (\text{Robertson and Campanella, 1983}) \dots\dots\dots (5.15)$$

where P_a = reference pressure, equal to 100 kPa.

Both methods are empirical. According to Mayne (1994) and Na (2005), Equation (5.15) overestimates ϕ' near the ground surface. For the Texas A&M University site, ϕ' near ground surface is close to 50° using Equation (5.15). At Changi East reclamation site, ϕ' is larger than 50° using Equation (5.15). This is not logical for uncemented cohesionless soil. Hence, Equation (5.14) will be adopted in this paper. For greater depths, however, Na et al. (2005) showed that both Equations (5.14) and (5.15) tend to produce conservative estimates of ϕ' when compared with those obtained from self-boring pressuremeter test. In this study, Equation (5.14) for estimating ϕ' was adopted to interpret weighted-average ϕ' . Figure 5.6 also shows an example of interpreting ϕ' from CPT.

5.4.4 Ultimate Bearing Capacity

A number of bearing capacity equations are available for shallow foundation on cohesionless soils using ϕ' based on limit equilibrium theory (e.g. Brinch Hansen, 1963; de Beer, 1970; Vesic, 1973). Regardless of the equation adopted, the calculated bearing capacity is usually less than that observed in PLT for small plates on surface of sand, but comparable with those observed in large foundations. As discussed in Chapter 4, this is attributed to the so-called scale effect of bearing capacity of shallow foundation (de Beer, 1963). Several possible factors contribute to this phenomenon, among which the most studied is the stress- and density-dependent effective angle of internal friction. However, as demonstrated in Chapter 4, the stress- and density- dependent angle of internal friction may be sufficient to explain the observed scale effect for centrifuge tests. For actual footing load test in

situ, particularly when the footing size is small (for example, $B < 1.5\text{m}$), other factors, such as cohesion due to any origin, may contribute considerably. These possible reasons are less studied and evaluated.

In order to reconcile the difference between results calculated from bearing capacity equation and measured from test, a correction factor is applied. The correction factor is determined by comparing the ultimate bearing capacity observed in PLT tests with those calculated according to Vesic's equation (1973, 1975) based on estimated ϕ' from CPT according to Equation (5.14). In order to do this, q_{ult} must be assessed from PLT first.

Various criteria for accessing bearing capacity of shallow foundation are available (e.g. Brinch Hansen, 1963; De Beer, 1970; Vesic, 1973, Decourt, 1999 and Chin, 1971). Some of them estimate the ultimate load at the start of the yielding of the soil. Decourt's (1999) zero stiffness method estimated the ultimate load at very large strains (or secant Young's modulus is equal to zero), corresponding to the asymptotic load of the load-settlement curve. Chin's (1971) method interpreted the ultimate load from the slope of the load versus settlement curve. In this chapter, both methods are examined.

Figure 5.7 shows the examples of the application of Decourt's (1999) zero stiffness method on tests at Lot-2 on Changi East reclamation site, Singapore. According to the method, secant stiffness K_s is plotted against the plate load q . The plate load corresponding to zero stiffness gives the ultimate bearing capacity, as shown in Figure 5.7 for stage-1, 2 and 4. However, in order to apply Decourt's method, the plate must be loaded to a sufficiently large displacement so that a stable plastic deformation develops. Otherwise, it is not possible to obtain a reliable $(q_{ult})_m$, such as Stage-3 and Stage-5 shown in Figure 5.7 where no stable plastic stages were observed. As a result, Decourt's (1999) zero stiffness method cannot be applied successfully for these tests.

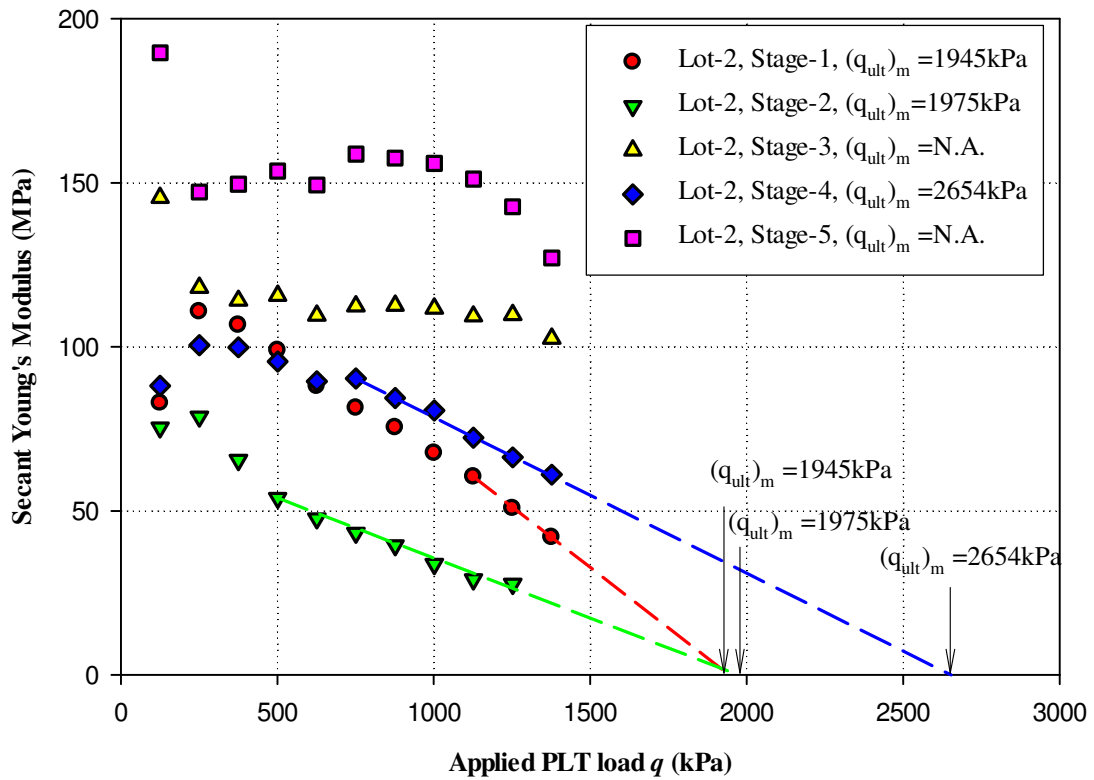


Figure 5.7: Application of Decourt's (1999) zero stiffness method to determine $(q_{ult})_m$ from PLT

Figure 5.8 shows the examples of the application of Chin's (1971) method on tests at Lot-2 on Changi East reclamation site, Singapore. According to the method, pseudo-strain $\epsilon_s = s/(2B)$ is plotted against ϵ_s/q . The gradient of the best fit lines at large pseudo strains equals the inverse of the ultimate bearing capacity q_{ult} . Similarly, the test must be loaded to a sufficiently large pressure so that there is stable plastic deformation. Figure 5.8 shows the successful application of Chin's (1971) method on tests for Stage-1, 2 and 4 and unsuccessful applications on tests of Stage-3 and Stage-5 due to insufficient plate load.

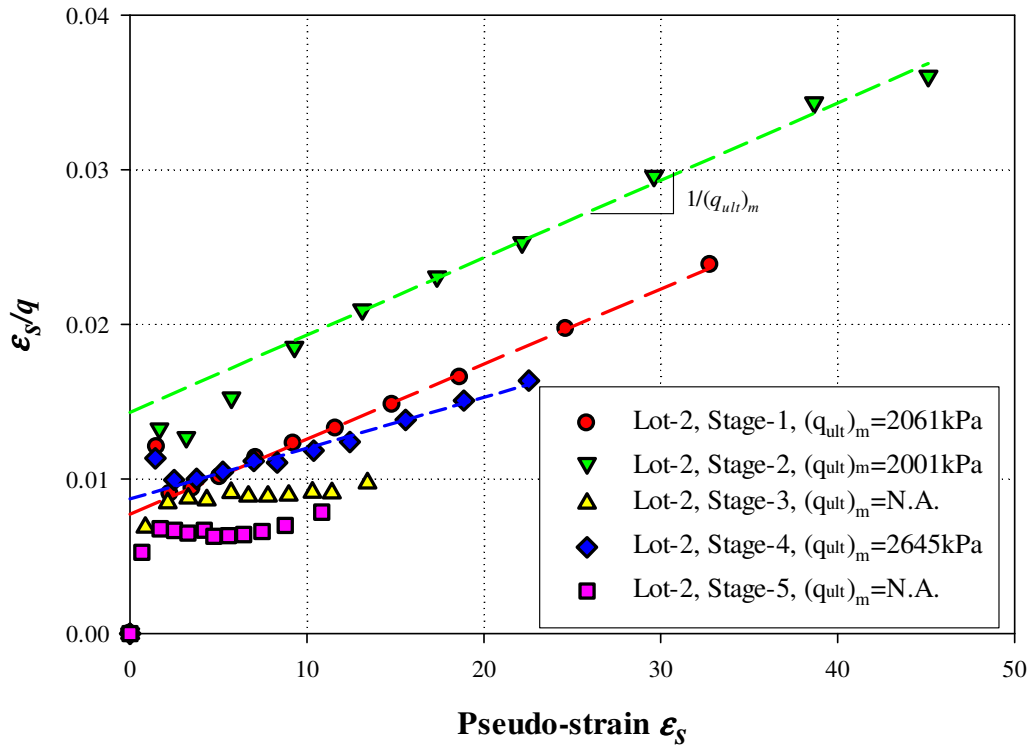


Figure 5.8: Application of Chin's (1971) method to determine $(q_{ult})_m$

Comparing the $(q_{ult})_m$ interpreted from the two methods, one may find that in fact if the same data points from load-settlement curve are used, the two methods produce very close $(q_{ult})_m$. Figure 5.9 compares the two interpretations of $(q_{ult})_m$ for 14 PLT load-settlement curves. It can be seen that the two methods provide almost similar results.

Table 5.1 summarizes the interpreted ultimate bearing capacity $(q_{ult})_m$ of 14 PLT from the three sites using Decourt's method. In Table 5.1, the average effective angle of internal friction ϕ' interpreted from CPT using Equation (5.14) and bearing capacity $(q_{ult})_v$ calculated using Vesic's equations and ignoring effect of embedment, i.e., Equation (4.1), are also listed. The ratios of the measured and calculated ultimate bearing capacity are also given.

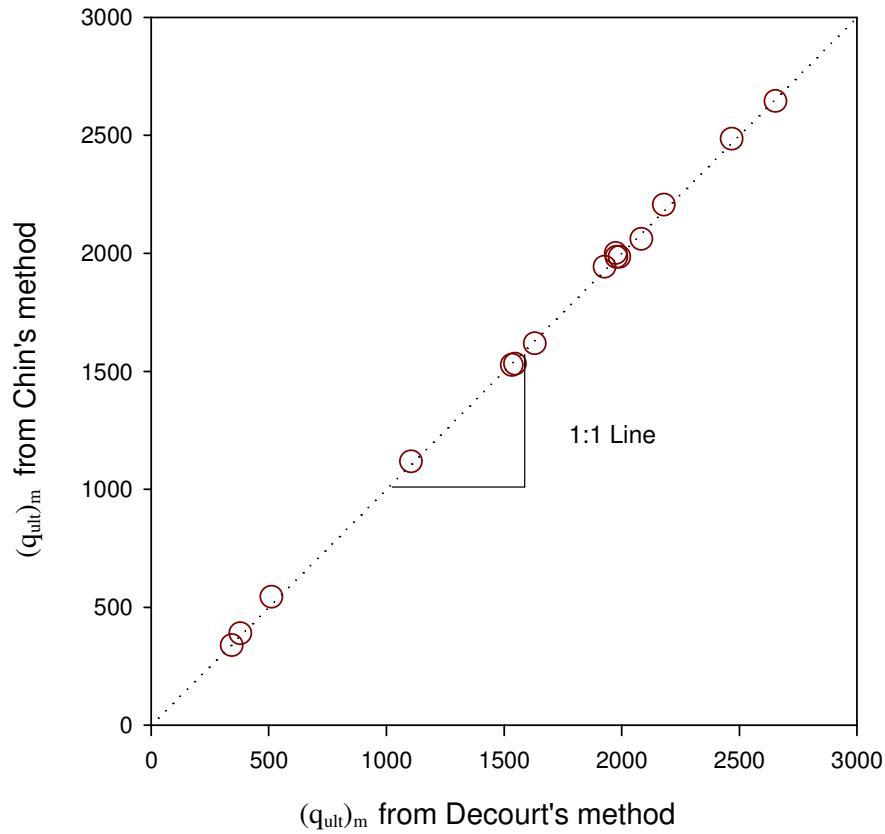


Figure 5.9: Comparison of $(q_{ult})_m$ between Decourt's and Chin's method

Figure 5.10 shows the ratios of measured and calculated bearing capacity versus dimension of foundation. From Figure 5.10, the ratio $m_q = (q_{ult})_m / (q_{ult})_v$ decreases with foundation dimension B . Generally, for round plate of diameter 0.5 m on Changi sand, the average ratio is about 3; for square footings at Texas A&M University, m_q fluctuates from 1.5 for 1m footing to around unity for 3m footing. It should be noted that although interpretation of individual CPT data shows that ϕ' ranges from about 39° to 41° , an average value of 40° was adopted in calculation for all five tests. Due to the extremely limited data of $(q_{ult})_m$, it is difficult to establish any definite trend between m_q and foundation dimension B . Before more case studies are investigated, a simplified bilinear relationship shown in Figure 5.10 is recommended temporarily.

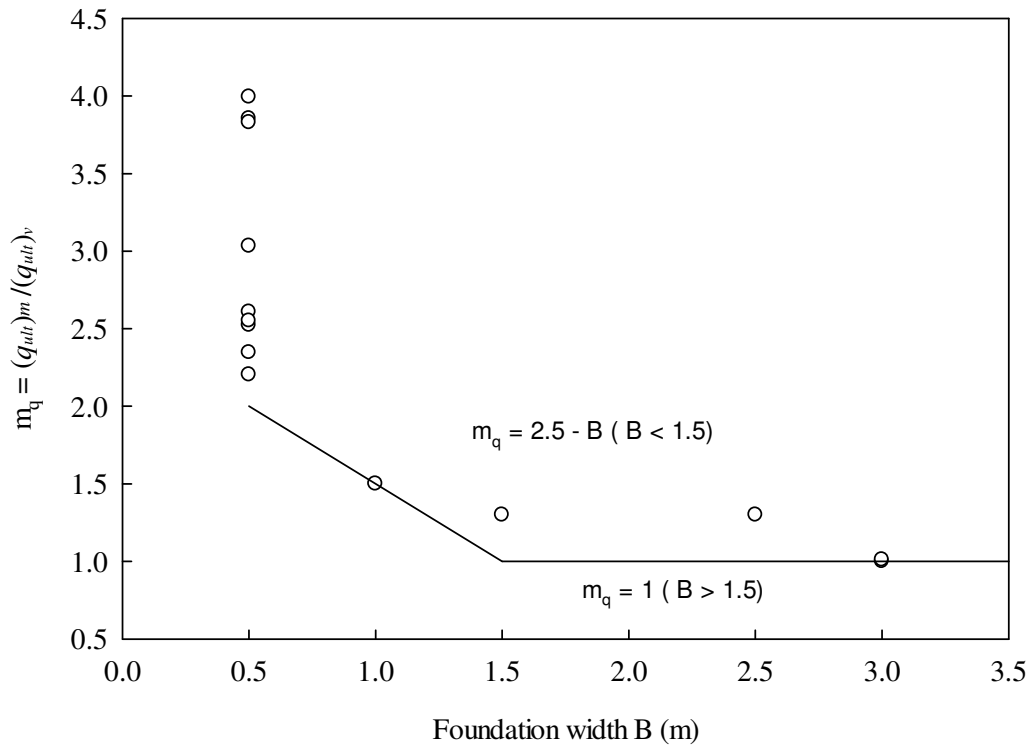


Figure 5.10: Relationship between m_q and B

Table 5.1: Interpretation of G_0 and q_{ult} from CPT

Changi East ($B = 0.5\text{m}$)	q_c (MPa)	$G_0(1)$ (MPa)	$G_0(2)$ (MPa)	$\frac{G_0(1)}{G_0(2)}$	$\phi', (^\circ)$	$(q_{ult})_v$ (kPa)	$(q_{ult})_m$ (kPa)	$\frac{(q_{ult})_m}{(q_{ult})_v}$
Lot-1, Stage-1	14.9	49.4	31.0	1.6	46.0	842.4	1976.9	2.3
Lot-1, Stage-4	14.9	53.9	33.6	1.6	45.5	763.5	1926.6	2.5
Lot-2, Stage-1	9.0	45.5	26.6	1.7	43.5	521.7	2084.4	4.0
Lot-2, Stage-2	10.9	54.3	26.5	2.0	43.5	521.7	2011.3	3.9
Lot-2, Stage-4	12.4	50.8	30.9	1.6	45.0	693.0	2653.6	3.8
Lot-3, Stage-1	16.1	49.1	28.9	1.7	45.5	718.6	1991.0	2.8
Lot-3, Stage-2	2.3	33.2	15.1	2.2	36.5	146.5	342.6	2.3
Lot-3, Stage-4	2.1	30.1	13.5	2.2	36.5	146.5	379.2	2.6
Lot-3, Stage-5	2.5	32.9	15.2	2.2	37	158.9	511.6	3.2
Texas A&M University								
$B=1\text{m}$	7.0	52.3	36.3	1.4	40.0	1314	1986	1.5
$B=2.5\text{m}$	8.7	68.3	43.0	1.6	40.0	1272	1631	1.3
$B=3\text{m, North}$	9.4	72.6	45.2	1.6	40.0	1526	1533	1.0
$B=1.5\text{m}$	5.1	52.1	32.0	1.6	40.0	1569	2052	1.3
$B=3\text{m, South}$	5.7	62.4	33.9	1.8	40.0	1526	1547	1.0

Note: $G_0(1)$ and $G_0(2)$ are small-strain stiffness based on Equation (5.10) and Equation (5.11) and (5.12); $(q_{ult})_v$ and $(q_{ult})_m$ are ultimate bearing capacity calculated based on Vesic's equation (the embedment effect is not accounted for for $D/B < 0.5$) and measured based on PLT, respectively.

5.5 Calibration of Parameters m and n

The constants m and n in Equation (5.9) were determined by fitting the load-settlement curves from PLT measured on the two sites. Since all the tests were conducted using circular plate or square plate, the depth of influence was assumed to be $2B$. The settlement s can be expressed as

$$s = \frac{qB}{G_0(1-2\nu)} \left\{ \left[(0.3 + 0.5 + m \left(\frac{\sqrt{q/(q_{ult})_m}}{1 - \sqrt{q/(q_{ult})_m}} \right)^n) \right] \frac{0.5}{2} + \left[0.5 + m \left(\frac{\sqrt{q/(q_{ult})_m}}{1 - \sqrt{q/(q_{ult})_m}} \right)^n \right] \frac{2.0}{2} \right\} \quad (5.16)$$

where q = plate load; $(q_{ult})_m$ = ultimate bearing capacity from PLT; G_0 = small-strain stiffness; ν = Poisson's ration, which is assumed to be constant equal to 0.2.

In the curve fitting, interpreted values of q_{ult} from load-settlement curves were adopted instead of q_{ult} estimated based on q_c . As there were no in situ measurements of small-strain stiffness for each PLT test, G_0 was determined using Equations (5.9), (5.10) and (5.11). The fitted m and n values for the 14 tests are listed in Table 5.2. It can be seen that values of $n(2)$ for Changi East and $n(1)$ for Texas A&M University ranges from 0.52 to 1.04, with an average value of about 0.8.

Similar to Chang et al. (2005), almost a perfect match of the load-settlement curve can be obtained for any test by varying both m and n . Figure 5.11 shows a typical example of the best fitted curve of load-settlement curve from PLT by varying both m and n . As a comparison, a match by varying only m value while setting $n = 0.8$ is also plotted in Figure 5.11.

Table 5.2: Results of m and n from best matching PLT curves

Changi East (B = 0.5m)	q_c (MPa)	$G_0(1)$ (MPa)	q_{ult} (kPa)	m(1)	n(1)	$G_0(2)$ (MPa)	m(2)	n(2)
Lot-1, Stage-1	14.9	49.4	1976.9	2.38	0.80	31.0	1.29	0.88
Lot-1, Stage-4	14.9	53.9	1926.6	1.51	0.69	33.6	0.75	0.78
Lot-2, Stage-1	9.0	45.5	2084.4	1.46	0.77	26.6	0.64	0.91
Lot-2, Stage-2	10.9	54.3	2011.3	4.27	0.57	26.5	1.80	0.63
Lot-2, Stage-4	12.4	50.8	2653.6	2.05	0.52	30.9	1.01	0.62
Lot-3, Stage-1	16.1	49.1	1991.0	1.52	0.79	28.9	0.69	0.90
Lot-3, Stage-2	2.3	33.2	342.6	3.45	0.89	15.1	1.36	0.94
Lot-3, Stage-4	2.1	30.1	379.2	7.48	0.65	13.5	3.09	0.68
Lot-3, Stage-5	2.5	32.9	511.6	8.90	0.53	15.2	3.79	0.56
Texas A&M University								
B=1m	7.0	52.3	1986	2.75	0.80	36.3	1.77	0.83
B=2.5m	8.7	68.3	1631	1.90	0.90	43.0	1.04	0.96
B=3m,North	9.4	72.6	1533	1.41	0.85	45.2	0.71	0.94
B=1.5m	5.1	52.1	2180	2.71	0.72	32.0	1.48	0.47
B=3m, South	5.7	62.4	1520	1.94	0.93	33.9	0.85	1.04

Note: m(1) and n(1) are best matched based on $G_0(1)$; and m(2) and n(2) are best matched based on $G_0(2)$.

Figure 5.12 plots the m values versus q_c from CPT, together with a best-fit dashed straight line. For a conservative estimate, an upper bound of the data in Figure 5.12 is obtained by translating the best-fit dashed line. As shown in Figure 5.12, the m in Equation (5.9) can be estimated by the following expression:

$$m = 3.7 - 0.2q_c \dots\dots\dots (5.17)$$

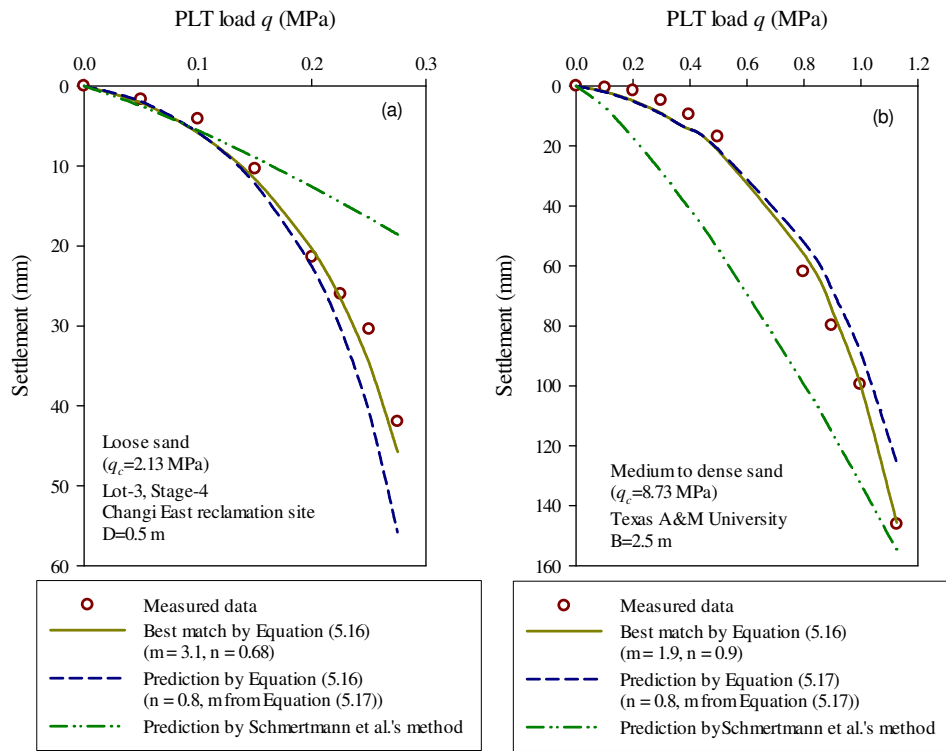


Figure 5.11: Comparison between measured and matched load-settlement curve

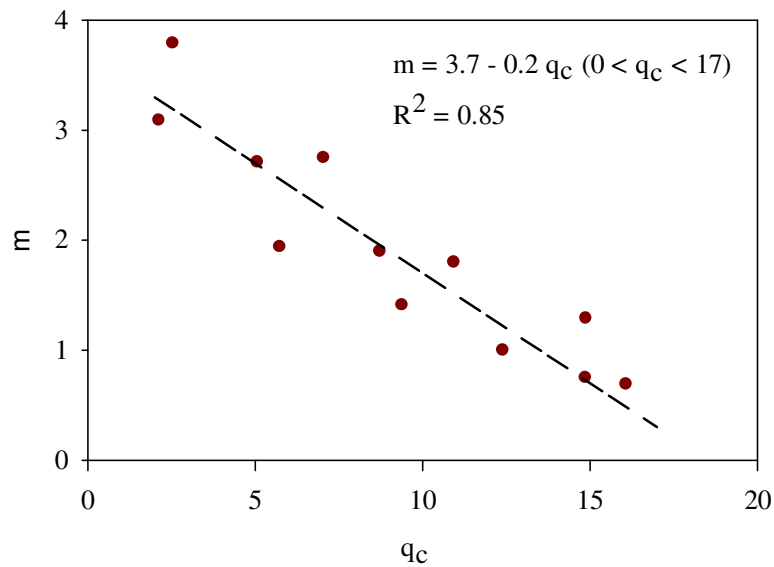


Figure 5.12: Correlation between q_c and m

5.6 Proposed Modified Schmertmann's Method for Estimating Settlement of Shallow Foundation

In summary, the proposed modification can be described as follows:

- 1) Estimate small-strain stiffness G_0 within the depth of influence z_i from q_c of CPT based on Equations (5.11) and (5.12), unless there is evidence showing that Equation (5.10) is more suitable. Estimate small-strain stiffness E_0 from G_0 with an assumed ν , which can be taken as 0.2.
- 2) Estimate effective angle of internal friction ϕ' within depth of B from q_c of CPT based on Equation (5.14). Then estimate the ultimate bearing capacity of the foundation $(q_{ult})_v$ using Vesic's equation (Equation 4.1). Conservatively, the effect of embedment on $(q_{ult})_v$ is neglected. For footing width $B < 1.5\text{m}$, a corrected $(q_{ult})_{corr}$ based on Figure 5.10 is obtained.
- 3) Calculate the peak strain influence factor I_{zp} using the following equation:

$$I_{zp} = 0.5 + (3.7 - 0.2q_c) \left(\frac{\sqrt{q/(q_{ult})_v}}{1 - \sqrt{q/(q_{ult})_v}} \right)^{0.8} \quad (0 \leq q_c \leq 17) \dots\dots\dots (5.18)$$

- 4) Plot the vertical strain influence factor diagram shown in Figure 5.13. In Figure 5.13, I_{zp} is calculated in Step 3 at depth of $0.5B$; $I_{z0} = 0.3$ at $z = 0$; $I_{zi} = 0$ at $z = 2.5(1 + \log(L/B))$.
- 5) For a flexible foundation, where $K_F \leq 10$, apply a correction for I_z at depth less than B , such that $I_{zf} = I_z + I_d$, where $I_d = 1 / (4.6 + 10K_F)$.
- 6) Calculate the settlement using the following equation:

$$s = C_D C_C \sum_0^{z_i} \left(\frac{I_z \Delta z}{E_0} \right) \dots \dots \dots (5.19)$$

where depth correction factor C_D is given in Equation (5.2) and creep factor C_C is given in Equation (5.3).

- 7) Given that the thickness of soil layer h is less than $2.5[1+\log(L/B)]$, similar procedure as described above can be adopted, except that depth of influence $z_i = h$. After immediate settlement s is obtained using Equation (5.19), it should be corrected by multiplying s with soil thickness factor I_h given in Equation (3.20).

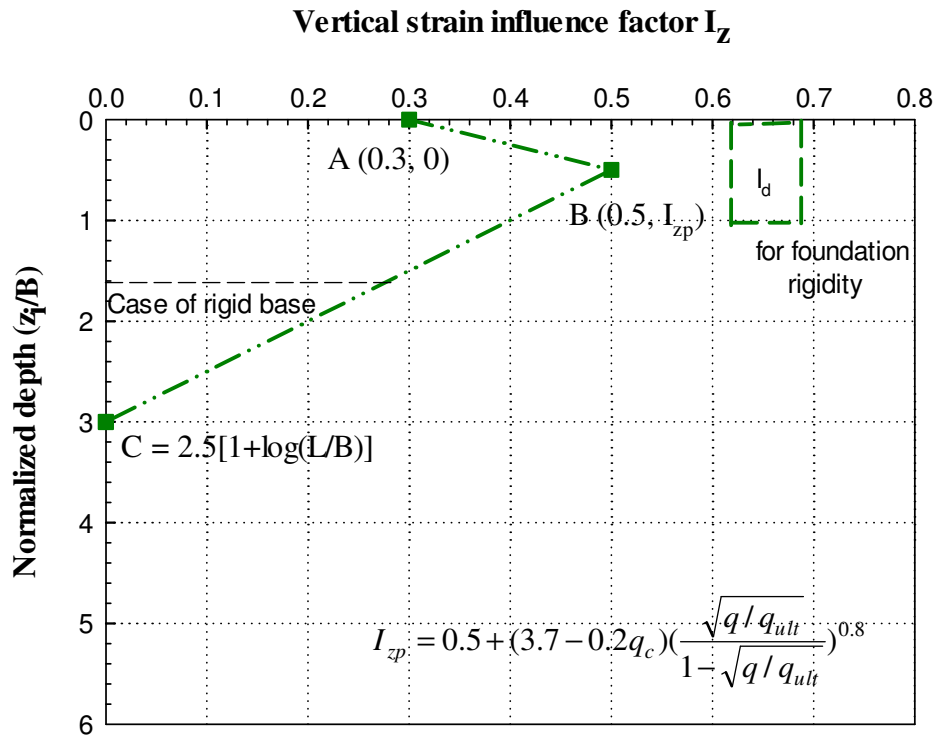


Figure 5.13: Simplified strain influence factor diagram for modified Schmertmann's method

5.7 Summary

Schmertmann's (1970, 1978) method and its modifications were reviewed. The shortcomings of Schmertmann's (1970, 1978) method and the existing modifications were discussed. A new modification of Schmertmann's (1970, 1978) method was proposed to overcome the shortcomings by adopting small-strain stiffness. Existing empirical correlations between small-strain stiffness G_0 , effective angle of internal friction ϕ' and tip resistance q_c from CPT were compared and selected. A correction factor for $(q_{ult})_v$ was suggested based on comparison of calculated $(q_{ult})_v$ using Vesic's equation and $(q_{ult})_m$ estimated from measured data from PLT. A new expression of peak strain influence factor I_{zp} dependent on mobilized load ratio was proposed to indirectly account for the modulus degradation of soil and the non-linear load-settlement behavior of the foundation. The expression was calibrated using 14 load-settlement curves from the two sites. Illustration and evaluation of the modified Schmertmann's method are given in Chapter 7.

Chapter 6 Load-Settlement Behaviour of Circular Footing on Non-linear Cohesionless Soil

6.1 Introduction

This chapter reviews the f-g non-linear elastic model proposed by Fahey and Carter's (1993). The f-g model was calibrated using the data measured in torsional shear tests, triaxial tests and plane strain tests. Typical values of the two constants f and g were summarized. Built-in MC constitutive model provided by FLAC was modified by incorporating the f-g model to represent the non-linear elastic behaviour. The modified f-g elasto-plastic model with MC failure criterion (f-g-MC) was written in FISH language (FLAC, 2005). The f-g-MC model was verified based on simulation of triaxial compression and extension using single element.

This chapter also investigates the load-settlement behaviour of circular, rigid and rough footing resting on the surface of cohesionless soil by using f-g-MC model and FLAC. Parametric studies were carried out to investigate the effect of input soil parameters on the load-settlement behaviour of the footing. Based on the simulated load-settlement behaviour of the footing, average modulus degradation curves of the soil-foundation system were computed and normalized. A unique correlation between the input modulus degradation of soil element and the simulated equivalent modulus degradation of the soil mass beneath a footing can be established. An approximate equivalent closed-form solution for estimating the non-linear load-settlement behaviour of a footing was established for a known modulus degradation of single element of soil. Besides, the normalized modulus degradation of soil-foundation system was calibrated using the 14 PLT load-settlement curves described in Chapter 5. The calibrated modulus degradation was found comparable with the numerical simulation. A modulus degradation method was proposed for estimating foundation settlement.

6.2 f-g-MC Model

The stress-strain behaviour of soil is highly non-linear from the very beginning, once the shear strain exceeds about 0.001%. The non-linear stress strain behaviour of soil can be described using either non-linear elastic model (e.g. Jardine et al., 1986; Fahey and Carter's, 1993; Kohata et al., 1994; Puzrin and Burland, 1996; Shibuya et al., 1997; Lee and Salgado, 1999; Lehane and Cosgrave, 2000), or elastic-plastic model, which usually involves at least a yield surface and a bounding surface, or even more surfaces (e.g. Mroz et al., 1979; Hashiguchi, 1985; Stallebrass and Taylor, 1997; Gajo and Wood, 1999). These models vary in degree of complexity and capability of describing the stress-strain behaviour of soil. Basically, models requiring more input parameters can better represent the soil behaviour. However, more input parameters indicate more tests and higher costs. In addition, users need more time and knowledge to understand the model, for instance, the effect of each parameter on the simulated problems.

Non-linear elastic models generally are simpler and require less input parameters, which can be determined easily. They are capable of representing the soil behaviour before failure reasonably accurate. However, post-failure soil response cannot be modeled effectively by non-linear elastic model. A good balance could be the combination of a non-linear elastic model with a plastic model, such as the "HS Small" model available in the commercial software PLAXIS. In this research, f-g model proposed by Fahey and Carter (1993) was adopted to describe the non-linear elastic stress-strain behaviour of the soil. The MC model was adopted to describe the post-failure soil behaviour, since MC constitutive model is probably the most widely accepted model for cohesionless soil.

6.2.1 f-g Model

Non-linear elastic model is usually developed based on curve-fitting, i.e., seeking an equation to fit the stress-strain curve measured from tests. A number of equations are available in the literature. Table 6.1 lists some examples of the equations that involve small-strain stiffness.

Table 6.1: Examples of modulus degradation from small-strain modulus

Expression	Reference and Remarks
Equation (2.11)	Seed and Idriss (1970). Based on resonant column tests.
$\frac{G}{G_{\max}} = \frac{1}{1 + \gamma_s / \gamma_h}$ $\gamma_h = \frac{\gamma_s}{\gamma_r} [1 + ae^{-b(\gamma_s / \gamma_r)}]$ where G and $G_{\max}$ = secant shear modulus and maximum shear modulus, respectively; reference shear strain $\gamma_r = \tau / \tau_{\max}$; τ and $\tau_{\max}$ = shear stress and maximum shear stress, respectively; γ_s = current shear strain; a and b = material constants.	Harden and Drenevich, (1972a, 1972b). Based on resonant column tests.
$Y = Y_{el} + \frac{X - X_{el}}{1 + \frac{C1(X) + \frac{X - X_{el}}{C2(X)}}{1}}$ where normalized deviatoric stress $Y = \Delta q / \Delta q_{\max}$; Δq and $\Delta q_{\max}$ = deviatoric stress and maximum deviatoric stress, respectively; normalized axial strain $X = \epsilon_a / (\epsilon_a)_r$; ϵ_a = axial strain; reference strain $(\epsilon_a)_r = \Delta q_{\max} / E_{\max}$; $C1(X)$ and $C2(X)$ = material constants	Tasuoka and Shibaya (1992) Based on cyclic triaxial tests.
$\frac{G}{G_{\max}} = 1 - f \left(\frac{\tau}{\tau_{\max}} \right)^g$ where G and $G_{\max}$ = secant shear modulus and maximum shear modulus, respectively; τ and $\tau_{\max}$ = shear stress and maximum shear stress, respectively; f and g = material constants	Fahey and Carter (1993) Based on torsional shear tests.
$\frac{G}{G_{\max}} = \left(1 - \frac{\tau}{\tau_{\max}} \right)^m$ where G and $G_{\max}$ = secant shear modulus and maximum shear modulus, respectively; τ and $\tau_{\max}$ = shear stress and maximum shear stress, respectively; m = material constant.	Mayne (1994) Based on triaxial tests.
$\frac{E}{E_{\max}} = 1 - \alpha \cdot \frac{x - x_{th}}{x} \cdot [\ln(1 + x - x_{th})]^R$ where E and $E_{\max}$ = secant Young's modulus and maximum Young's modulus, respectively; x = normalized axial strain; x_{th} = normalized threshold strain. $\alpha = \frac{x_L - 1}{x_L [\ln(1 + x_L)]^R}; \quad R = \frac{c(1 + x_L) \ln(1 + x_L)}{x_L (x_L - 1)};$	Puzrin and Burland (1996, 1998) Based on triaxial tests.

$x_L = \frac{\varepsilon_f q_{ult}}{E_{max}} ;$	
$\frac{E_t}{E_{max}} = [1 - (\frac{\Delta q}{\Delta q_{max}})^m]^n$ where E_t and E_{max} = tangent Young's modulus and maximum Young's modulus, respectively; Δq and Δq_{max} = deviatoric stress and maximum deviatoric stress, respectively; m and n = material constants.	Shibuya et al. (1997)
$\frac{G}{G_{max}} = [1 - f(\frac{\sqrt{J_2} - \sqrt{J_{20}}}{\sqrt{J_{2max}} - \sqrt{J_{20}}})^g](\frac{I_1}{I_{10}})^{n_g}$ where G and G_{max} = secant shear modulus and maximum shear modulus, respectively; $(J_2)^{1/2}$ = second invariant, 3D equivalent of τ; $(J_{20})^{1/2}$ = second invariant at initial status; $(J_{2max})^{1/2}$ = maximum second invariant. I_1 and I_{10} = first invariant and first invariant at initial status; f, g and n_g = material constants.	Lee and Salgado (1999) Extend Fahey and Carter's (1993) expression to 3D condition. Effect on mean effect stress on the G_{max} is also considered. By assuming Poisson's ratio is fixed, the expression is used to conduct the analysis based on triaxial test data. Three material constants.
$\frac{E_t}{E_{max}} = \frac{1 - (\varepsilon_f / \varepsilon)^r}{1 - (\varepsilon_f / \varepsilon_o)^r}$ where E_t and E_{max} = tangent Young's modulus and maximum Young's modulus, respectively; ε_f, ε and ε_o = failure strain, current strain and limiting strain, respectively.	Atkinson (2000) Based on triaxial tests.
$\frac{E}{E_{max}} = \frac{1}{1 + (\frac{\varepsilon - \varepsilon_{th}}{\varepsilon_r - \varepsilon_{th}})^n}$ where E and E_{max} = secant Young's modulus and maximum Young's modulus, respectively; ε_{th}, ε and ε_r = elastic threshold strain, current strain and reference strain at which $E = E_{max}/2$, respectively; n = material constant	Lehane and Cosgrove (2000) Based on triaxial tests.

The expressions listed in Table 6.1 share similar characteristics, i.e., the modulus of soil is normalized with small-strain modulus; stress is normalized with strength; and strain is normalized with the reference strain. Then the normalized modulus is correlated with either normalized stress or normalized strain. In addition, these expressions are all based on single element tests, i.e., triaxial test, torsional shear test or resonant column test. As mentioned by Fahey (1991), correlation between normalized modulus and the normalized stress is more convenient to implement compared with that based on normalized strain.

One family of the most frequently used non-linear elastic models is the hyperbolic model. Duncan and Chang (1970) developed their classical hyperbolic model based on Kondner's (1963) discovery that hyperbolic function can fit the measured stress strain curve. The original hyperbolic model is frequently used to model stress-strain behaviour of clay and sand under cyclic loading. For sand under monotonic loading, Fahey proposed the following expression:

$$\frac{G}{G_0} = 1 - f \left(\frac{\tau}{\tau_{\max}} \right)^g \dots\dots\dots(6.1)$$

where G and G_0 = secant shear modulus and small-strain shear modulus, respectively; τ and $\tau_{\max}$ = current and peak shear stress; respectively; f and g = empirical constants determined by curve fitting.

Figure 6.1 shows the comparison between f-g model by Fahey and Carter (1993) and original hyperbolic model by Duncan and Chang (1970). It can be seen that by introducing two additional material constants, f and g , more flexible curves can be obtained to match the measured data. From Figure 6.1, it can also be seen that parameter f controls the magnitude or the extent of degradation, whereas parameter g dictates the rate of the degradation and the curvature of the curve.

For both linear and non-linear elasticity, Hooke's law can be used to describe the stress-strain behaviour in numerical methods. For linear elasticity, the elastic properties are constant in calculation once they are defined. For non-linear elasticity, the properties depend on the stress or strain status in the current calculation step. In numerical modeling, tangent modulus instead of secant modulus in Equation (6.1) is preferred, because the analysis is performed incrementally. Therefore, by differentiating Equation (6.1), the relationship between normalized tangent shear modulus and mobilized shear stress is given as (Fahey and Carter, 1993):

$$\frac{G_t}{G_0} = \frac{\left(\frac{G}{G_0}\right)^2}{1 - f(1-g)\left(\frac{\tau}{\tau_{\max}}\right)^g} \dots\dots\dots (6.2)$$

where G_t = tangent shear modulus and the other parameters are defined in Equation (6.1).

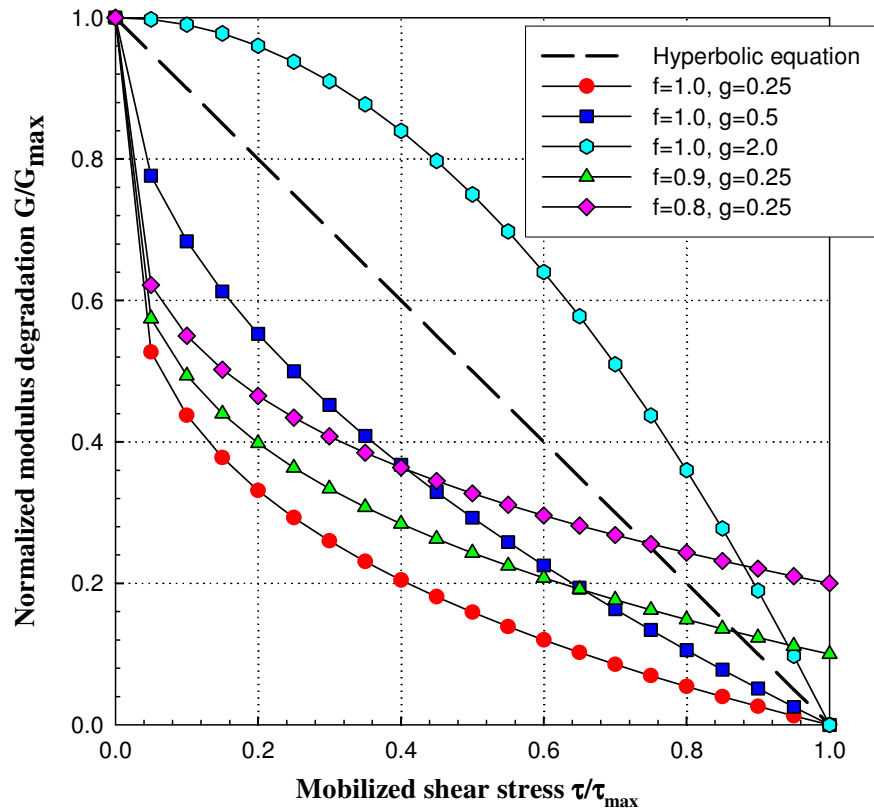


Figure 6.1: Theoretical modulus degradation curves

To extend the application of Equation (6.1) and (6.2) to three dimensional (3D) conditions, axisymmetrical and plane strain condition, second invariants of the deviatoric stress tensor corresponding to 3D equivalents of the current and maximum shear stresses, $J_2^{1/2}$ and $J_{2\max}^{1/2}$ can be used to replace the τ and $\tau_{\max}$ in Equation (6.1) and (6.2). The second invariant $J_2^{1/2}$ can be expressed as

$$\sqrt{J_2} = \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2} \dots\dots\dots (6.3)$$

where σ_1 , σ_2 and σ_3 = three principal stresses.

According to MC model, the maximum shear stress $J_{2\max}^{1/2}$ can be expressed as:

$$\sqrt{J_{2\max}} = I_1 \times \tan \phi' + c \dots\dots\dots(6.4)$$

where $I_1 = (\sigma_1 + \sigma_2 + \sigma_3)/3$, the first invariant of stress tensor; ϕ' = effective angle of internal friction and c = cohesion.

Besides the tangent shear modulus G_t , another property needs to be defined and that is either the bulk modulus K' or the Poisson's ratio ν . Some softwares prefer to express Hooke's law using Young's modulus E and Poisson's ratio ν , such as PLAXIS. Others prefer to use shear modulus G and bulk modulus K' , such as FLAC. For the former, because G_t reduces with the mobilized shear stress following Equation 6.2, unrealistic low value of bulk modulus K' can be encountered in the analysis if a constant Poisson's ratio ν is defined. To overcome this, one may consider assuming bulk modulus K' is constant. Therefore, the Poisson's ratio ν can be calculated as done by Fahey and Carter (1993). For the latter, one may define the bulk modulus K' as constant, as suggested by Duncan and Chang (1980). The two expressions are inter-related.

6.2.2 Typical Values of f and g

Table 2.1 lists the typical values of f and g based on the measurements of torsional shear tests, triaxial tests and plane strain compression tests in the laboratory. The detailed calibration of the measurements is given in Appendix E. It was observed that the normalized modulus degradation curve measured based on triaxial test on

overconsolidated sand sample does not match as well as that based on normally consolidated sample by f-g model, as shown in Figure E.1 and Figure. E.2

Table 6.2: Typical values of f and g

Test type	Sand type	f	g	Reference	Remarks
Torsional shear test	Toyoura sand (NC)	1.07	0.35	Teachavorasinskun et al. (1991)	$e=0.79, K_0=0.41$
	Toyoura sand (OC)	1.11	0.58		$e=0.78, K_0=0.98$
	Hamaoka sand	1.07	0.38		$e=0.628, D_{50}=0.237\text{mm}$
	Kentucky sand	1.00	0.47	Tatsuoka and Shibuya (1991)	
	Ticino sand (NC)	1.00	0.48	LoPresti et al. (1993)	$e=0.71, D_{50}=0.54\text{mm}$
	Ticino sand (OC)	1.00	0.56		
	Quiou sand	1.00	0.47	LoPresti et al. (1993)	$D_r = 46\% \sim 89\%$
Triaxial test	Toyoura sand (NC)	1.1	0.28	LoPresti et al. (1995)	$e = 0.84 \sim 1.07$
	Toyoura sand (OC)	1.17	1.44		
	Quiou sand (NC)	1.11	0.26	Floravante et al. (1995)	$e = 0.84 \sim 1.07$
	Quiou sand (OC)	1.22	0.58		
	Silty sand	0.97	0.1~0.6	Lee and Salgado (1999)	D_r varies
	Hime gravel	1.07	1.02	Teachavorasinskun et al. (1991)	$e_0 = 0.548, K_0 = 1$
Plane strain compression test	Toyoura sand ($e=0.67$)	1	0.32	Tatsuoka and Shibuya (1991)	$e_{\max} = 0.985, e_{\min}=0.602, D_{50} = 0.22 \text{ mm}$
	Toyoura sand ($e=0.83$)	1.06	0.23		
	S.L.B sand (NC)	1.1	0.4	Tatsuoka and Kohata (1995)	$e=0.557, D_{50}=0.62\text{mm}$
	S.L.B sand (OC)	1.11	0.49		$e=0.563$

From Table 6.2, the observed values of parameter f are all close to 1.0 and parameter g ranges from 0.35 to 0.56 for torsional shear tests. The values of g observed based on plane strain compression tests range from 0.23 to 0.49. For triaxial tests, normally consolidated samples show similar value of g to the torsional shear tests. However, for overconsolidated samples, extremely high values of g as much (as high as 1.44) can be observed.

6.2.3 MC Plastic Model

The built-in MC Model provided by FLAC is adopted to model the post-failure behaviour of soil. Generally, even the simplest plastic model consists of three components: elastic behaviour before failure; failure criterion and plastic flow rule, which depends on plastic potential function. For the MC plastic model discussed here, the elastic behaviour will be modified by a non-linear elastic model, i.e., f-g model as discussed above. No modification is made to the failure criterion and plastic potential function. Therefore, only a brief description of the two components is given in the subsequent paragraphs.

Figure 6.2 illustrates the MC failure criterion in (σ_1 and σ_3) plane in FLAC (2005). Shear failure shown from Point A to B is defined by function f^s :

$$f^s = \sigma_1 - \sigma_3 N_\phi - 2c\sqrt{N_\phi} \dots\dots\dots (6.5)$$

Tension failure from Point B to C is defined as:

$$f^t = \sigma^t - \sigma_3 \dots\dots\dots (6.6)$$

where σ_1 and σ_3 = minimum and maximum principal stresses, (in FLAC, compressive stresses are negative, therefore $\sigma_1 < \sigma_2 < \sigma_3$); $N_\phi = (1+\sin\phi)/(1-\sin\phi)$; c = cohesion and σ^t = tension strength.

The shear potential function corresponding to a non-associated flow rule is defined as:

$$g^s = \sigma_1 - \sigma_3 N_\psi \dots\dots\dots (6.7)$$

where $N_\psi = (1+\sin\psi)/(1-\sin\psi)$ and ψ = dilation angle.

The tension potential function corresponding to an associated flow rule is defined as

$$g^t = -\sigma_3 \dots\dots\dots (6.8)$$

Figure 6.2 illustrates the domain where the two plastic potential functions applied. It should be noted that by defining a function $h(\sigma_1, \sigma_3)$ in FLAC, which represents the diagonal of the two failure surface, a unique definition of the flow rule at the shear-tension edge can be determined. The pre-requirement of this technique is the small strain increments. The details of the definition of function $h(\sigma_1, \sigma_3)$ are described in the FLAC manual and will not be given here.

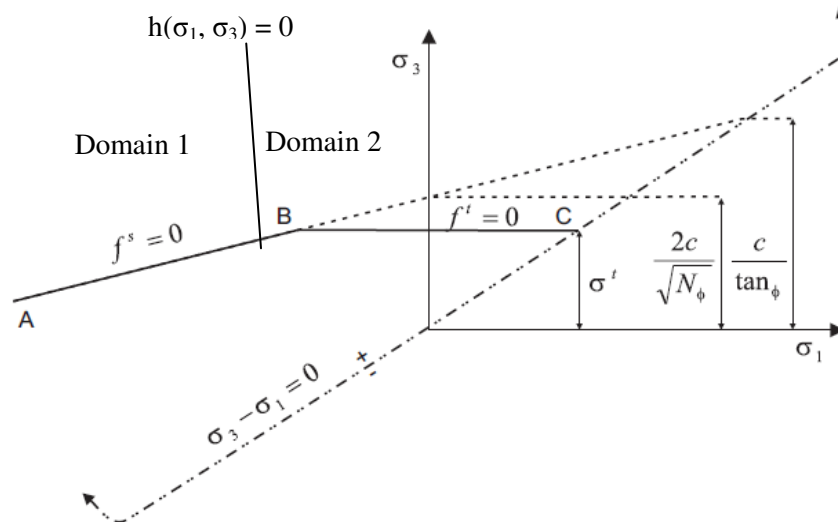


Figure 6.2: MC failure criterion in FLAC (modified after FLAC, 2005)

Once the elastic stresses estimated violate the failure criterion, the location of the stress point is determined based on the function $h(\sigma_1, \sigma_3)$, which separates the domain into domain 1 and domain 2 as shown in Figure 6.2. Then a plastic correction is applied to the elastic guessed stresses to obtain the new stresses.

6.3 Verification of f-g-MC Model

The f-g-MC model written in FISH language (FLAC, 2005) was verified based on simulation of triaxial test using single element, as shown in Figure 6.3. In the verification, axisymmetrical model is used to simulate the cylindrical soil sample in triaxial test. The element was 1m by 1m in dimension. The consolidation stress was 100 kPa. Small-strain shear modulus was 100 MPa.

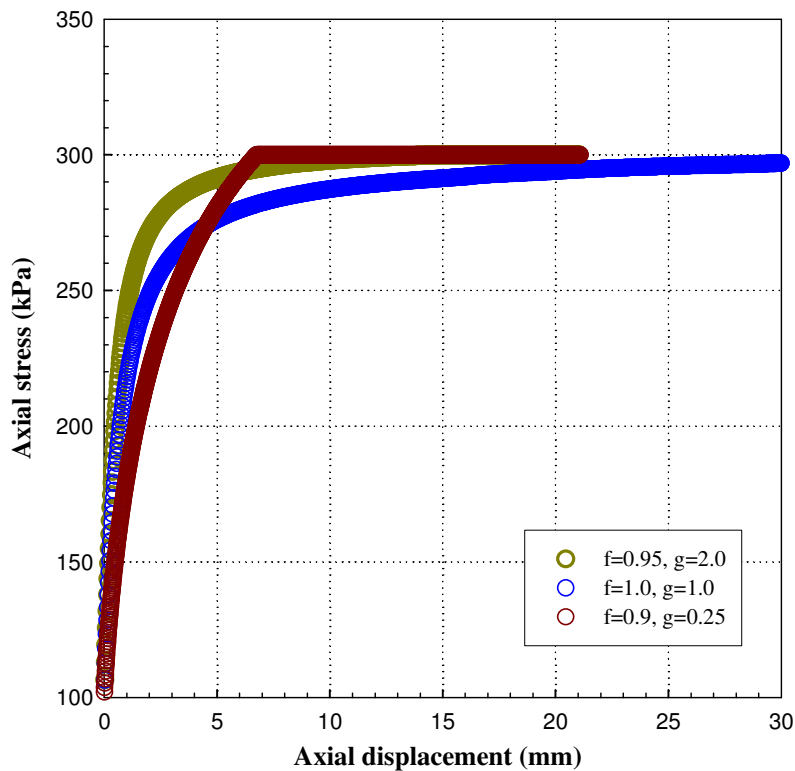


Figure 6.3 Simulated load-displacement curves of triaxial test

Three combinations of parameters f and g were tested ($f = 0.95$, $g = 2.0$; $f = 1.0$, $g = 1.0$ and $f = 0.9$, $g = 0.25$), as shown in Figure 6.3. The ϕ' of the soil is 30° . Dilation angle and cohesion c were both equal to zero. Figure 6.3 shows the simulated load-displacement curves of triaxial test. Figure 6.4 shows that the numerical simulation results based on f-g-MC model are exactly the same as the analytical results for the various combinations of f and g values.

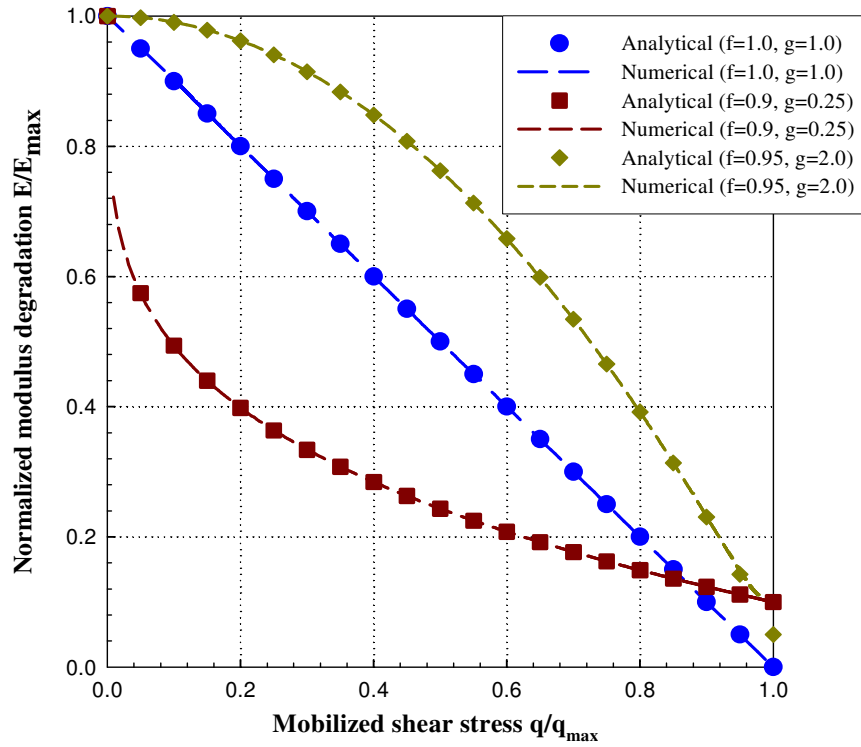


Figure 6.4: Comparison of results between theoretical and numerical normalized modulus degradation based on f-g-MC model

Figure 6.3 also shows that under triaxial compression, maximum axial stress was exactly 300kPa, which satisfied the MC criterion perfectly no matter what f and g values were used. In addition, a more straightforward comparison between the results from modified subroutine and built-in MC model are given in Figure 6.5. Figure 6.5 compares the numerical results based on built-in MC model and the modified f-g-MC model by setting f to be zero, which means linear elastic behavior before failure. The parameters for MC model are identical for the built-in and modified models. As expected, the load-settlement curves were exactly the same, as shown in Figure 6.5.

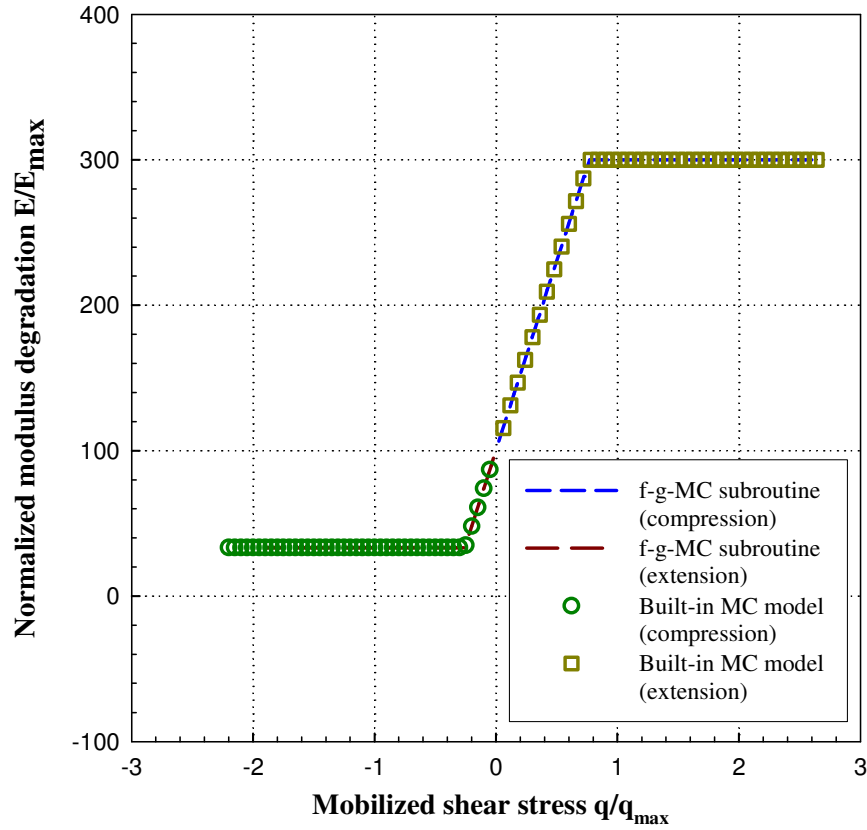


Figure 6.5: Comparison of results of load-settlement curves based on built-in MC model and f-g-MC model

6.4 Load-settlement Behaviour of Shallow Foundation on Non-linear Cohesionless Soil

The f-g-MC model was used to carry out the simulation of loading of shallow foundation. Figure 6.6 shows the mesh for the simulation. Due to symmetry, only the right half of the problem was modeled. The setup of the model was similar as that adopted in the studies of the scale effect on the bearing capacity factor, except that the boundary in the present simulation was further away, i.e., 10 times of the footing radius (10B) both in horizontal and vertical directions. Unbiased mesh with square elements were used to ensure better stability than biased mesh as suggested by Yin et al. (2001) and Erickson and Drescher (2002). A total of 2500 elements were used.

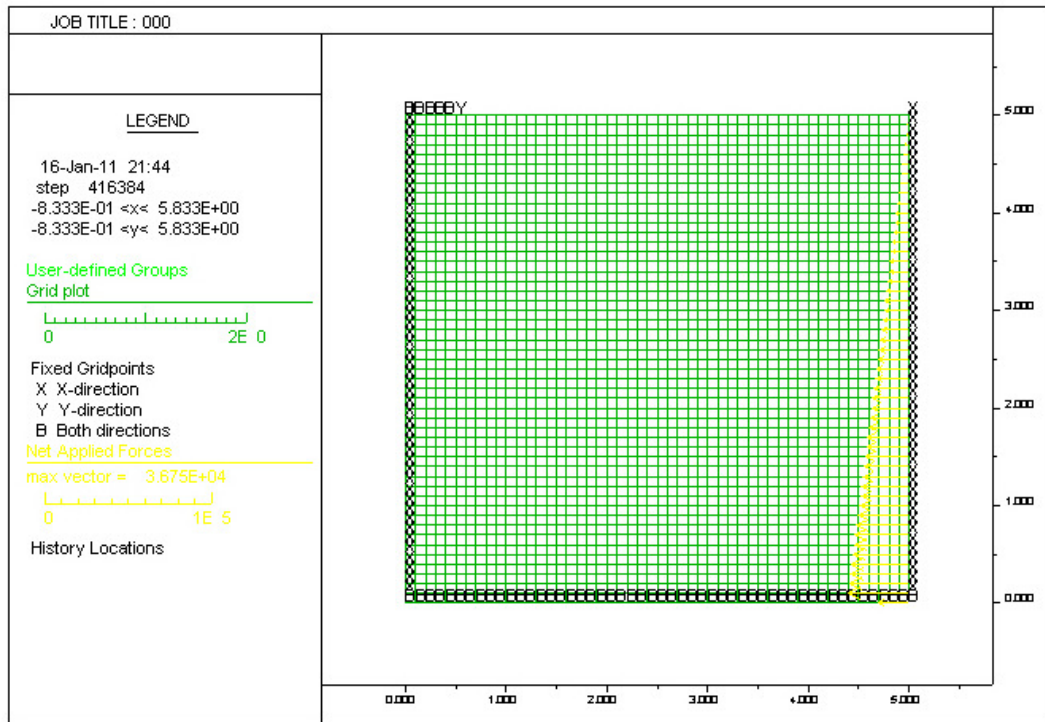


Figure 6.6: Mesh for simulation of foundation loading test

Rigid footing was modeled by six nodes on the left top of the model, as shown in Figure 6.6. The nodes were given a vertical displacement at a constant rate. All the six nodes were constrained laterally except for the node at the edge of the foundation. The free lateral movement of the node at the edge of the foundation has been found to be beneficial to the stability of the analysis. The footing load was calculated by summing the node forces of the six nodes.

6.5 Normalized Average Modulus Degradation of Soil-foundation System

A parametric study has been performed to investigate the effect of a few parameters on the load-settlement behaviour of a foundation. Table 6.3 lists these parameters together with their notation and values. In the subsequent figures, the numbers in the legend denote the input values of B , ϕ' , G_0 , and g , respectively. For example, “1-30-100-0.5” indicates that the simulation was based on a 1m diameter footing, ϕ'

was 30° , small-strain stiffness of soil was 100 MPa, and the modulus degradation parameter g was 1.0.

Table 6.3: Notation and input values of the parameters in the simulations

Parameters	Notation	Values
Footing diameter/width, B	1	1m
	2	2m
	3	3m
Internal friction angle, ϕ'	30	30°
	35	35°
	40	40°
Small-strain stiffness, G_0	50	50 MPa
	100	10 MPa
	200	200 MPa
Modulus degradation parameter, g	0.125	0.125
	0.25	0.25
	0.5	0.5
	1.0	1.0

For all these simulations, the bulk density of the dry cohesionless soil was assumed to be 15kN/m^3 . The at-rest earth pressure coefficient K_0 was assumed to be equal to unity. The dilation angle was equal to zero for all, which means non-associated flow rule was adopted in the simulations. The cohesion c and tension cut-off c_t were both 0 kPa. Based on laboratory measurements (Lee and Salgado, 1999), the constant parameter f , which describes the magnitude of modulus degradation of the soil, was assumed to be 0.97.

Figure 6.7 shows the simulated load-settlement responses of three cases. For these three cases, ϕ' were all 30° . Input values of g were all 0.5. The footing diameters were 1m, 2m and 4m. Input values of small-strain shear modulus were 50 MPa, 100 MPa and 200 MPa.

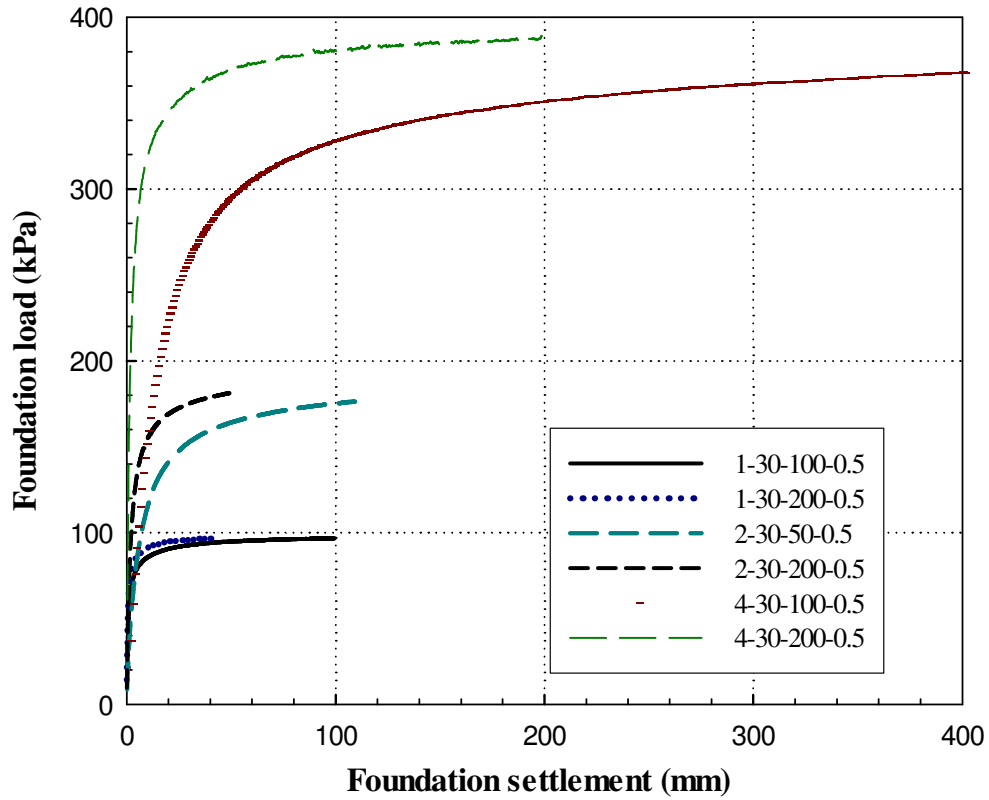


Figure 6.7: Simulated load-settlement curves of circular foundation on cohesionless soil ($\phi' = 30^\circ$, $g = 0.5$)

Based on the simulated load-settlement curves shown in Figure 6.7, average secant Young's modulus, which reflects the stiffness of the soil-foundation system, can be computed based on Equation (3.1). Similar to the modulus degradation observed for a single soil element, the average Young's modulus of soil-foundation system back calculated using Equation (3.1) was observed to decrease with the increase of the load applied on the foundation. Furthermore, the degradation of average Young's modulus E_s was normalized in terms of E_s/E_0 and q/q_{ult} , where E_0 is the small-strain stiffness; q is the footing load; q_{ult} is the ultimate bearing capacity of the foundation.

Figure 6.8 shows the computed normalized degradation of average modulus of the soil-foundation system based on the simulated load-settlement curves shown in Figure 6.7. It can be seen that, a unique normalized degradation curve of the average modulus was obtained for the three simulations on foundations resting on cohesionless soil with identical ϕ' and g , though B of the foundation and E_0 of the

soil were different. Figure 6.9 and Figure 6.11 show the simulated load-settlement curves for $\phi' = 35^\circ$ and $g = 0.25$ and $\phi' = 40^\circ$ and $g = 1.0$. Figure 6.10 and Figure 6.12 show the computed normalized degradation of average modulus of soil-foundation based on the simulated load-settlement curves, which as expected, were unique for all cases.

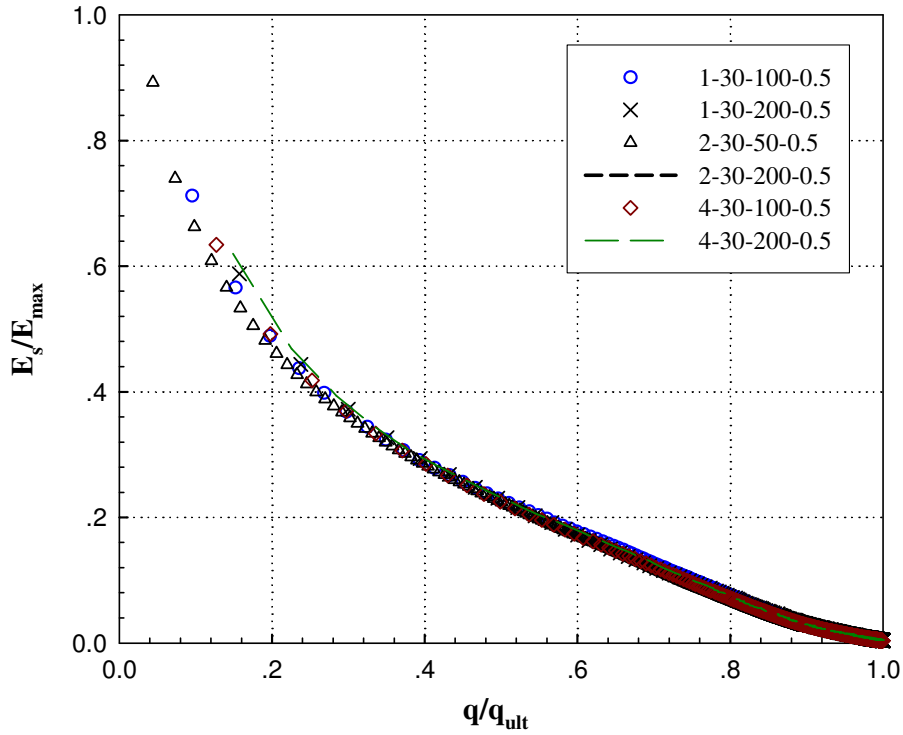


Figure 6.8: Normalized modulus degradation curves of soil-foundation system
 $(\phi' = 30^\circ, g=0.5)$

Based on the results of the parametric study, the following conclusion can be drawn: For cohesionless soil with constant ϕ' and g , a unique relationship between normalized degradation of average secant modulus of soil-foundation system and normalized modulus degradation of the soil element was obtained, regardless of the small-strain stiffness E_0 and foundation diameter B .

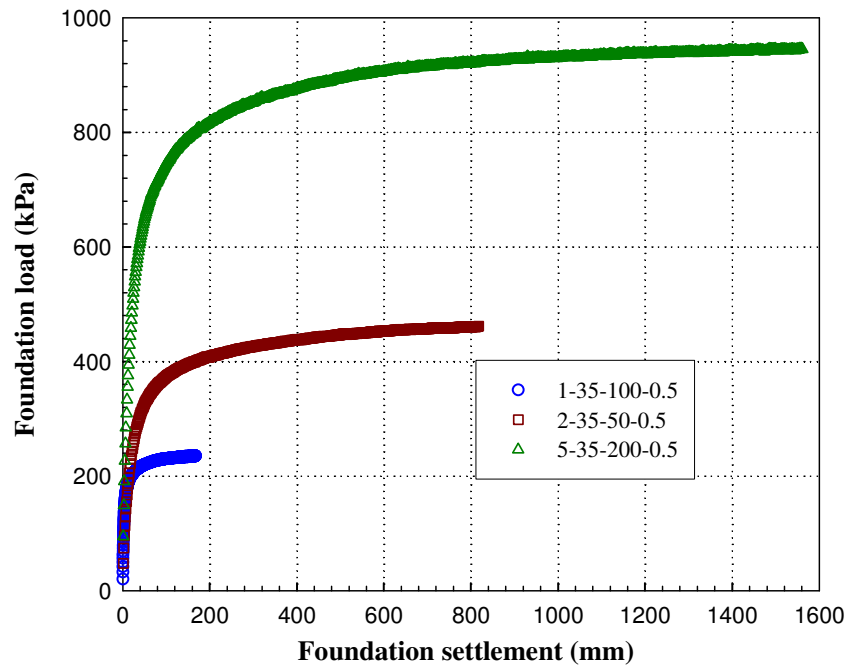


Figure 6.9: Simulated load-settlement curves of circular foundations on cohesionless soil ($\phi' = 35^\circ$, $g=0.5$)

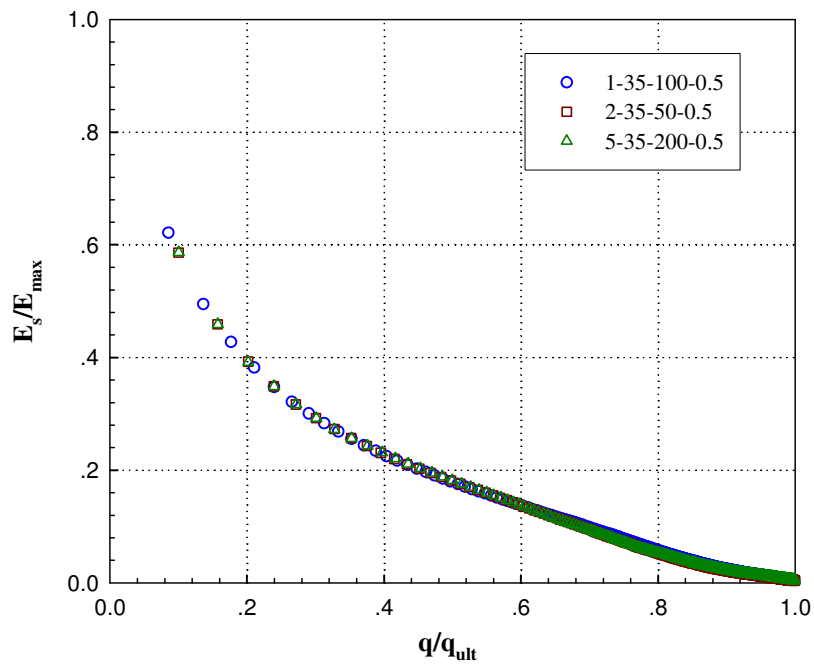


Figure 6.10: Normalized modulus degradation curves of soil-foundation system ($\phi' = 35^\circ$, $g=0.5$)

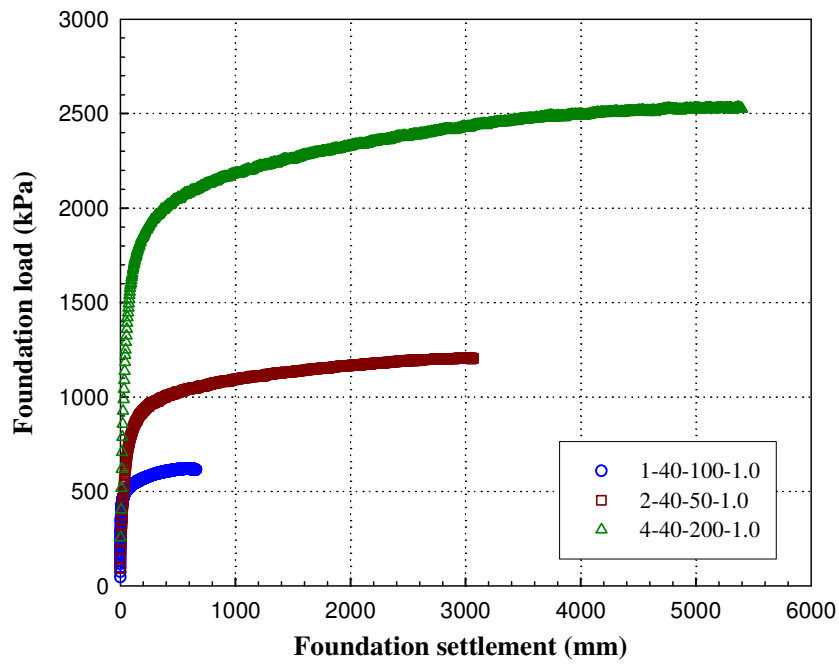


Figure 6.11: Simulated load-settlement curves of circular foundations on cohesionless soil ($\phi' = 40^\circ$, $g=1.0$)

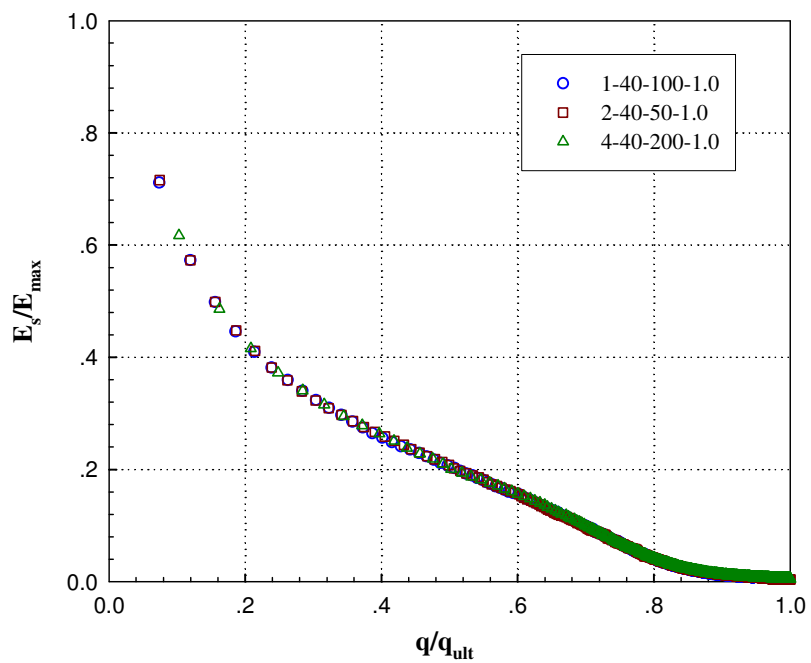


Figure 6.12: Normalized modulus degradation curves of soil-foundation system ($\phi' = 40^\circ$, $g=1.0$)

6.6 Generalized Load-settlement Behaviour of Circular Foundation and Modulus Degradation of Soil-foundation System

Subsequently, another parametric study was performed to investigate the relationship between normalized degradation of average modulus of soil-foundation system and normalized modulus degradation of the soil element. Four typical values of g were investigated, i.e., $g = 0.125, 0.25, 0.5$ and 1.0 . Three values of ϕ' were adopted, i.e., $\phi' = 30^\circ, 35^\circ$ and 40° . Constant footing width of 1m and small-strain stiffness of 100 MPa were maintained for these simulations, since they do not affect the characteristics of normalized modulus degradation.

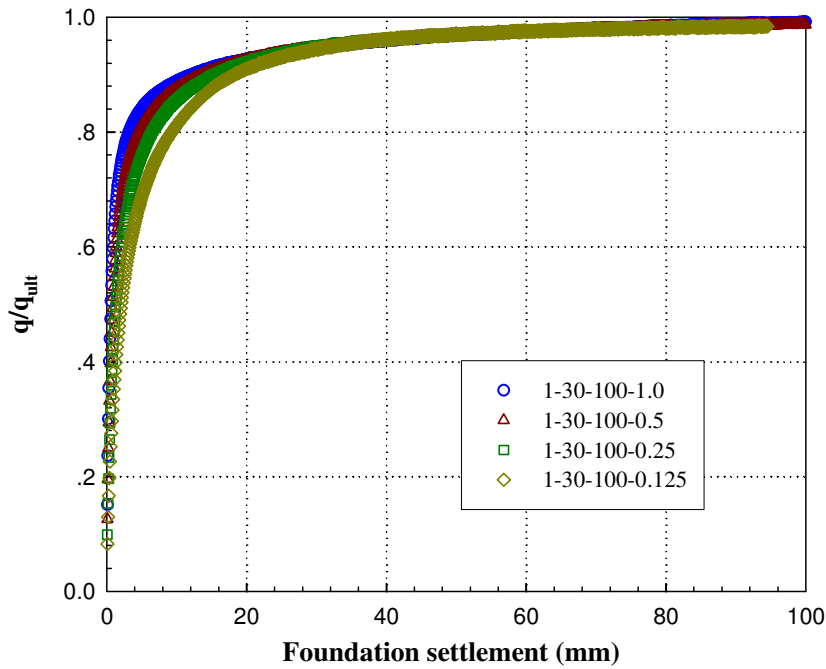


Figure 6.13: Normalized load-settlement curves of circular foundations on cohesionless soil ($\phi' = 30^\circ$)

Figures 6.14 to 6.16 show the normalized load-settlement curves obtained from the simulations for cases of $\phi' = 30^\circ, 35^\circ$ and 40° . The load-settlement curve shows some fluctuations when $\phi' = 40^\circ$. Therefore, the plotted load-settlement curves were normalized based on the maximum simulated value of the ultimate bearing capacity. The normalized load-settlement curves become less stiff as g becomes smaller.

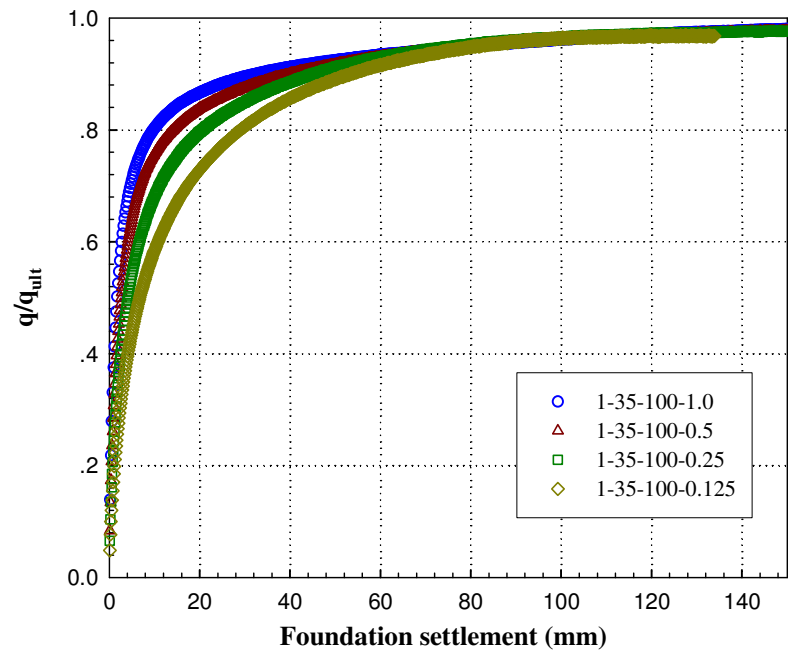


Figure 6.14: Normalized load-settlement curves of circular foundations on cohesionless soil ($\phi' = 35^\circ$)

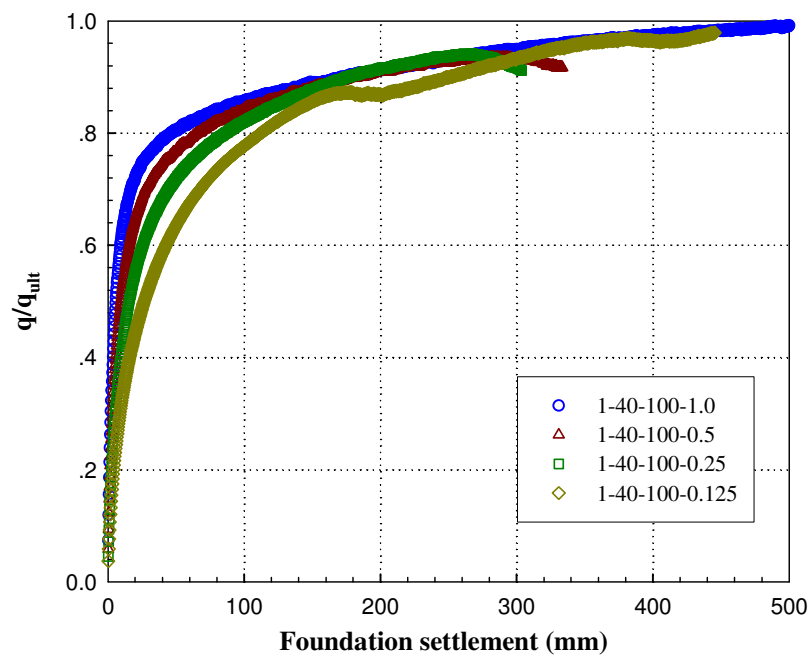


Figure 6.15: Normalized load-settlement curves of circular foundations on cohesionless soil ($\phi' = 40^\circ$)

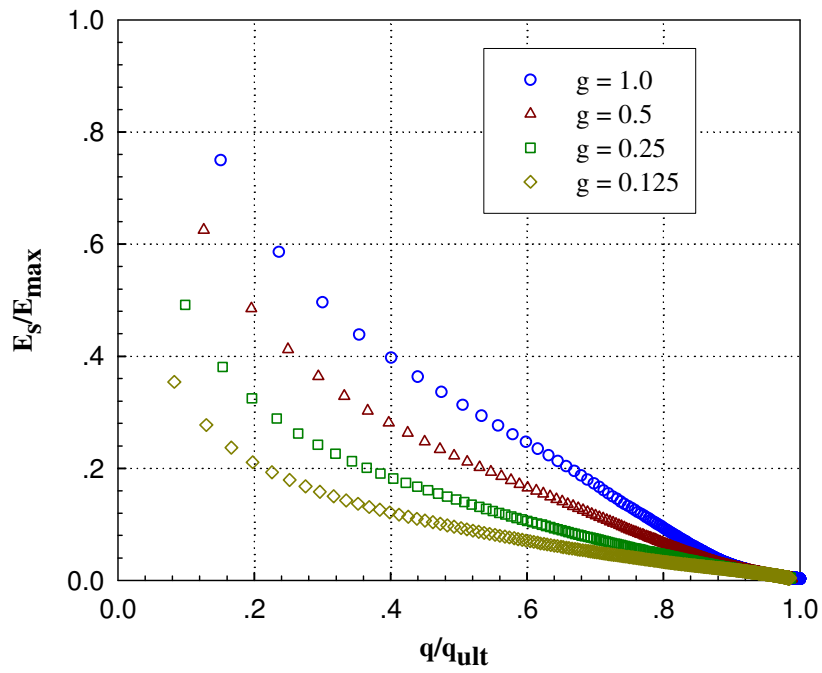


Figure 6.16: Normalized average modulus degradation curves of soil-foundation system ($\phi' = 30^\circ$)

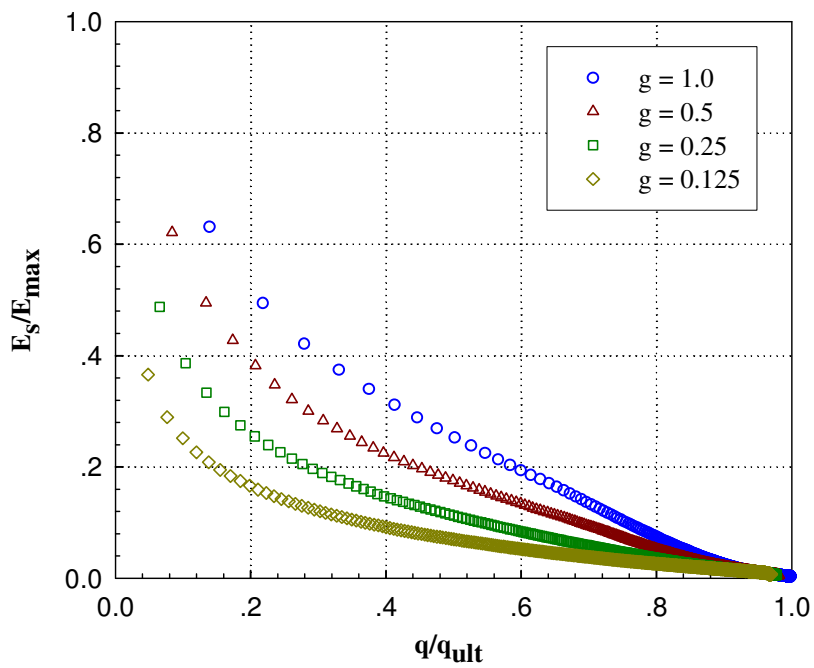


Figure 6.17: Normalized average modulus degradation curves of soil-foundation system ($\phi' = 35^\circ$)

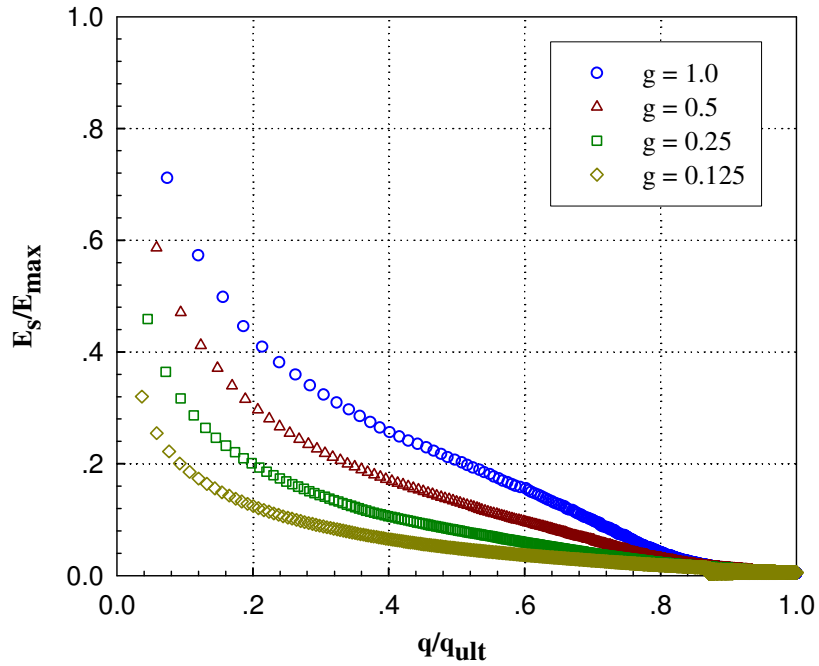


Figure 6.18: Normalized average modulus degradation curves of soil-foundation system ($\phi' = 40^\circ$)

Based on the simulated load-settlement curves, the normalized modulus degradation curves of soil mass were computed. Figure 11 to Figure 13 show the obtained normalized modulus degradation of soil mass for $\phi' = 30^\circ$, 35° and 40° . As expected, the smaller the value of g , the gentler the normalized modulus degradation curves.

Similar to the modified hyperbolic function presented by Fahey and Carter (1993), the normalized average modulus degradation of the soil-foundation system shown in Figure 6.16 to Figure 6.18, can also be described using the following expression:

$$\frac{E_s}{E_0} = 1 - f^* \left(\frac{q}{q_{ult}} \right)^{g^*} \dots\dots\dots (6.9)$$

where E_s = average secant Young's modulus of soil-foundation system; E_0 = small-strain stiffness; q = foundation load; q_{ult} = ultimate bearing capacity of the foundation; f^* and g^* = constant fitting parameters describing the degradation of the normalized average Young's modulus of soil-foundation system.

Figure 6.19 to Figure 6.21 show the curve fitting, using Equation (6.9), of the simulated normalized average modulus degradation curves of soil-foundation system $\phi' = 30^\circ, 35^\circ$ and 40° . Each figure has four plots for $g = 1.0, 0.5, 0.25$ and 0.125 . In each plot, there are two best fittings: one is to fit the whole modulus degradation curve obtained in the simulation (q/q_{ult} is up to 0.9); the other is to fit the beginning part when q/q_{ult} is up to 0.5, which corresponds to the footing load up to a safety factor of 2. The fitted curves deviate slightly from the normalized modulus degradation curves at the beginning of the loading ($q/q_{ult} < 0.2$). However, almost perfect matches can be obtained when $q/q_{ult} > 0.2$. From Figure 6.19 to Figure 6.21, it can be seen that $q/q_{ult} < 0.2$ is only a small portion in the plot. Moreover, the matched values were on the safe side by producing a slight underestimation of Young's Modulus. The matched fitting parameters f^* and g^* are also given in the figures. Table 6.4 summarizes these values.

Table 6.4: Calibrated parameters (f^* and g^*) of normalized modulus degradation of soil-foundation system

ϕ'	g	$f^*(1)$	$g^*(1)$	$f^*(2)$	$g^*(2)$
30°	0.125	1.00	0.15	1.0	0.15
	0.25	1.01	0.24		0.24
	0.5	1.02	0.40		0.38
	1	1.04	0.63		0.59
35°	0.125	1.004	0.12		0.12
	0.25	1.01	0.19		0.19
	0.5	1.02	0.31		0.30
	1	1.03	0.46		0.44
40°	0.125	1.00	0.08		0.08
	0.25	1.01	0.14		0.14
	0.5	1.02	0.23		0.22
	1	1.02	0.38		0.36

Note: $f^*(1)$ and $g^*(1)$ are fitted values by adjusting both f^* and g^* ; $f^*(2)$ and $g^*(2)$ are fitted values by setting $f^*=1.0$ and adjusting g^* only.

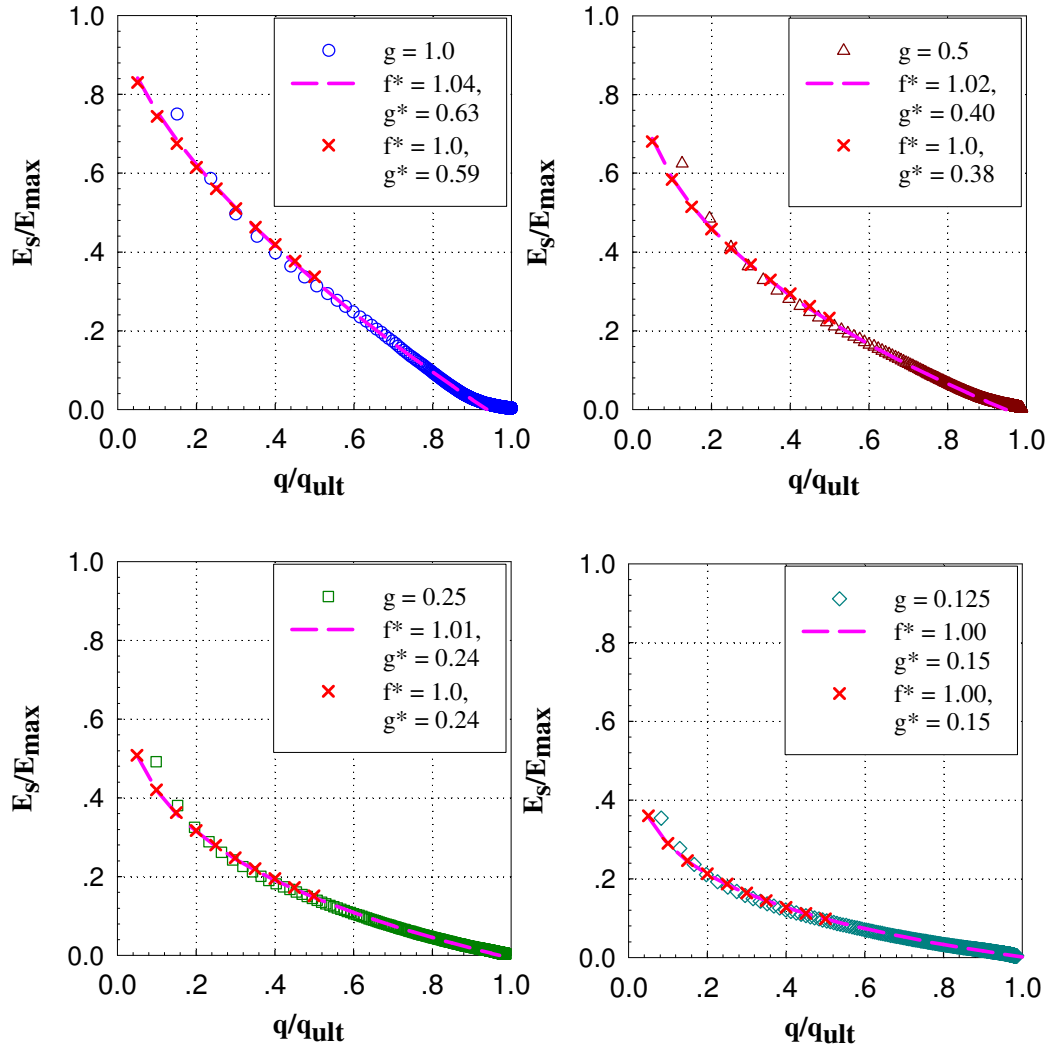


Figure 6.19: Fitted hyperbolic functions to normalized average modulus degradation curves of soil-foundation system ($\phi' = 30^\circ$)

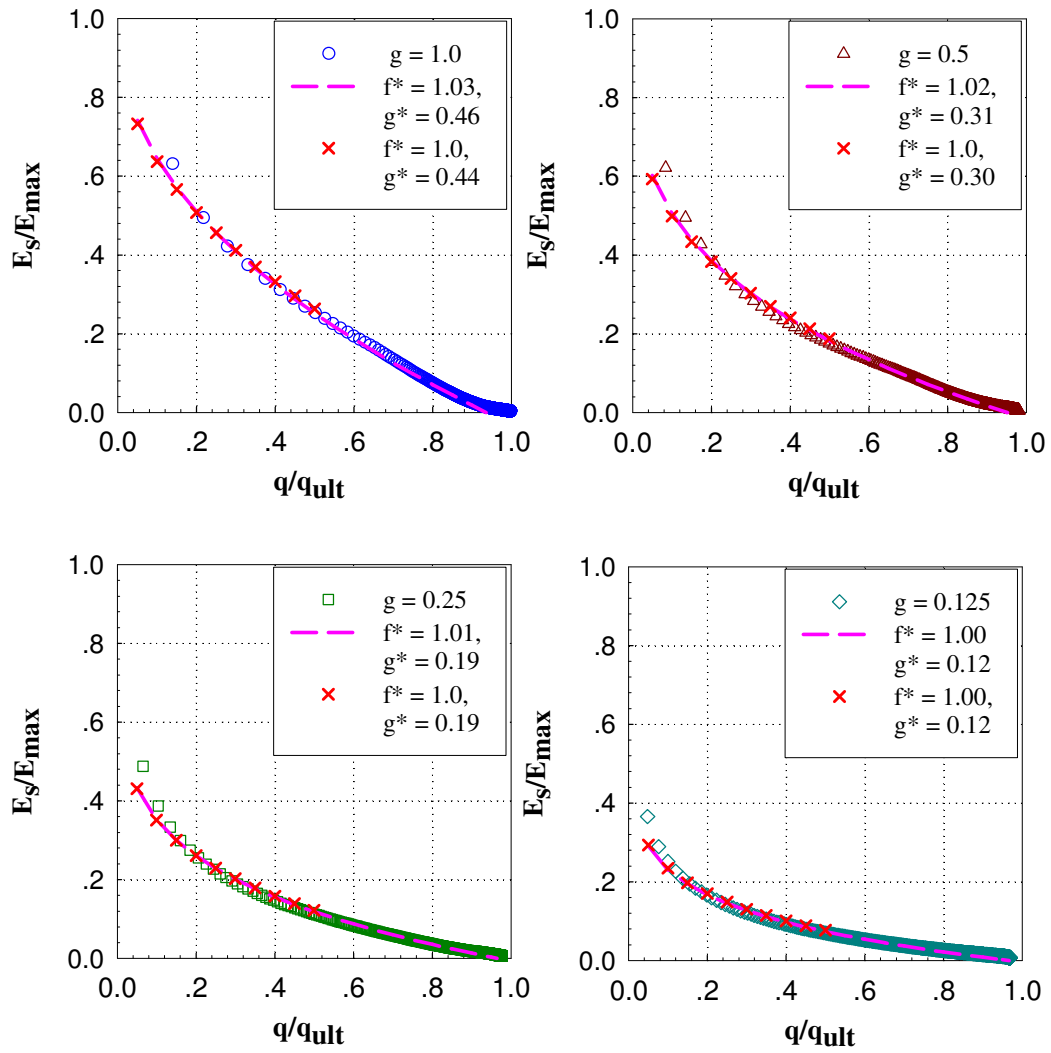


Figure 6.20 Fitted hyperbolic functions to normalized average modulus degradation curves of soil-foundation system ($\phi' = 35^\circ$)

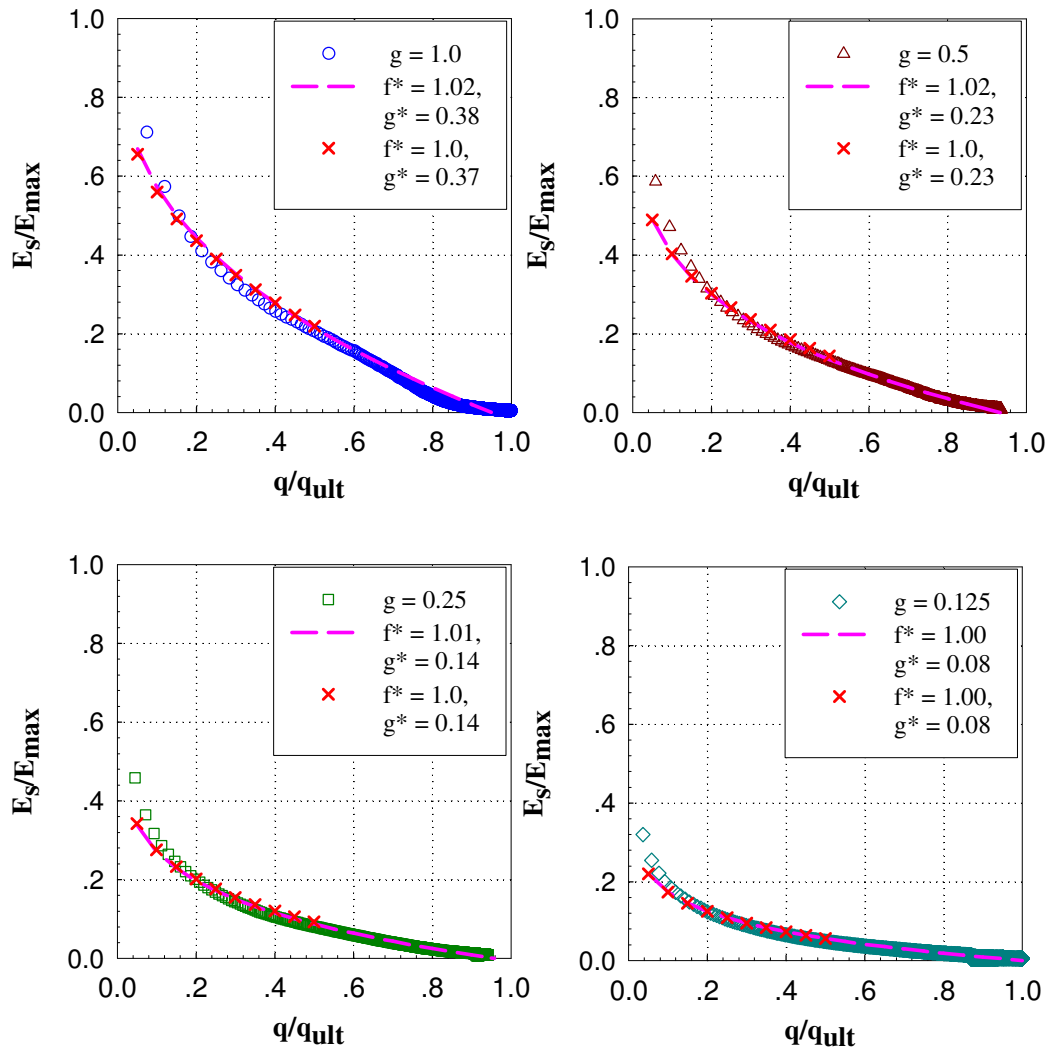


Figure 6.21: Fitted hyperbolic functions to normalized average modulus degradation curves of soil-foundation system ($\phi' = 40^\circ$)

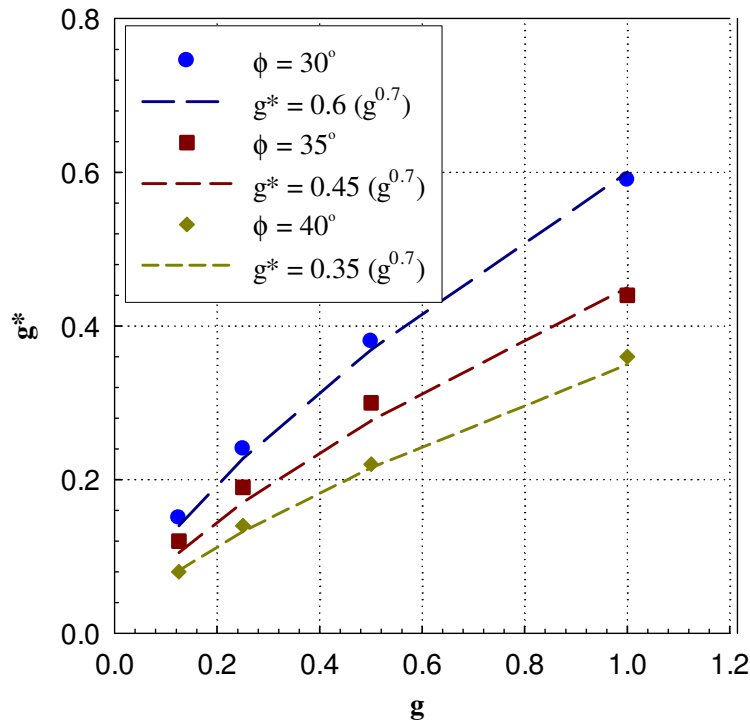


Figure 6.22: Correlation between g and g^* ($f = 0.97$, $f^* = 1.0$)

Figure 6.22 shows the correlation between g ($f=0.97$ in the simulation) and g^* ($f^*=1.0$). A power function can be used to describe the correlation between g and g^* for various ϕ' as follows:

$$g^* = \lambda \cdot (g^{0.7}) \dots\dots\dots (6.10)$$

where $\lambda = 0.6, 0.45$ and 0.35 for $\phi' = 30^\circ, 35^\circ$ and 40° .

Therefore, given the known g value obtained from soil test, estimation of g^* , which represents degradation of the stiffness of soil-foundation system, can be made based on the relationship shown in Figure 6.22 or Equation (6.10).

6.7 Approximate Closed-form Solution of Foundation Settlement Considering Modulus Degradation of Soil-foundation System

Based on Equations (3.1) and (6.9), an approximate closed-form solution for estimating displacement of circular, rigid footing considering modulus degradation of cohesionless soil can be derived. By substituting Equation (6.9) into (3.1), the following closed-form expression can be obtained:

$$s = \frac{qBI}{E_0[1 - f^* (\frac{q}{q_{ult}})^{g^*}]} \dots\dots\dots (6.11)$$

where s = footing displacement; q = footing load; B = footing width; I = displacement influence factor; E_0 = small-strain stiffness; q_{ult} = ultimate bearing capacity of the footing; f^* and g^* = fitting parameters describing the normalized average modulus degradation of soil-foundation system. For practical application, it can be assumed that $f^* = 1.0$.

In order to apply Equation (6.11), small-strain stiffness and the ultimate bearing capacity must be known. The value of g^* can be determined based on Equation (6.10) once g value is available.

6.8 Calibration of the Modulus Degradation of Soil-foundation System

Mayne (1994a) had proposed Equation (6.11) to estimate settlement of shallow foundation on cohesionless soil. The values of E_0 and q_{ult} were estimated from CPT test. However, parameters f^* and g^* were not investigated in details and Mayne (1994a) had merely suggested that $g^* = 0.3$ gave a close estimation of one of the measured load-settlement curves from Texas A&M University, U.S.A.

In this section, the normalized modulus degradation of soil-foundation system shown in Equation (6.11) was calibrated using the 14 load-settlement curves of PLT

described in Chapter 5 and given in Appendices A and B. For a given foundation on cohesionless soil, foundation width B and displacement influence factor I are constants. Using q_c from CPT, small-strain stiffness G_0 and ultimate bearing capacity were estimated using the empirical correlations discussed in Chapter 5. Measured load-settlement curves were also used to interpret the ultimate bearing capacity of shallow foundation using Decourt's (1999) method or Chin's (1971) method. Assuming ν was constant, the settlement s in Equation (6.11) depends only on f^* and g^* which therefore can be calibrated by fitting the load-settlement curve.

Interpreted G_0 from CPT and q_{ult} from PLT listed in Table 5.1 were used. Small-strain stiffness E_0 was calculated assuming that $\nu = 0.2$ and summarized in Table 6.5. Table 6.5 also lists the calibrated f^* and g^* . From Table 6.5, it can be seen that if Equation (5.10) was used to estimate G_0 , calibrated values of g^* range from 0.05 to 0.33. If Equations (5.11) and (5.12) were used to estimate G_0 , g^* ranged from 0.12 to 0.70. This implies that a higher estimated small-strain stiffness based on Equation (5.10) will give a lower g^* value. The result is consistent with that reported by Fahey (1994). Based on Equation (5.10), G_0 is about two times of the G_0 based on Equations (5.11) and (5.12). Accordingly, g^* based on G_0 from Equation (5.10) is about half of g^* based on G_0 from Equations (5.11) and (5.12). However, there is no such trend observed for f^* , as shown in Table 6.5.

Instead of a constant value of g^* as suggested by Mayne (1994a), a correlation between g^* and q_c from CPT was established. Figure 6.23 plots g^* versus the q_c from CPT. From Figure 6.23, it can be seen that for majority of tests at Changi East reclamation site and all five tests at Texas A&M University, g^* increases from 0.11 to 0.38 with q_c increasing from 2.13 MPa to 14.88 MPa. It appears a relationship exists between g^* and q_c . This is plausible since g measured in triaxial tests increases with increase of relative density of soil sample (Lee and Salgado, 1999). The value of q_c also increases with increase of relative density, as indicated in Figure 5.2. As q_c increases with effective confining stress, one may argue that q_c should also be normalized with the effective confining stress in Figure 6.23 as suggested by Lee and Salgado (1999). However, no convincing evidence shows that

it is more reasonable to correlate g^* to the normalized q_c . For practical application, q_c as shown in Figure 6.23 is adopted.

Table 6.5: Results of f^* and g^* from best matching PLT curves

Changi East (D=0.5m)	q_c (MPa)	$G_0(1)$ (MPa)	q_{ult} (kPa)	$f^*(1)$	$g^*(1)$	$G_0(2)$ (MPa)	$f^*(2)$	$g^*(2)$
Lot-1, Stage-1	14.9	49.4	1976.9	0.99	0.22	31.0	0.97	0.38
Lot-1, Stage-4	14.9	53.9	1926.6	0.95	0.30	33.6	0.94	0.58
Lot-2, Stage-1	9.0	45.5	2084.4	0.95	0.32	26.6	0.94	0.70
Lot-2, Stage-2	10.9	54.3	2011.3	0.96	0.10	26.5	0.93	0.23
Lot-2, Stage-4	12.4	50.8	2653.6	0.90	0.18	30.9	0.86	0.38
Lot-3, Stage-1	16.1	49.1	1991.0	0.98	0.32	28.9	1.00	0.66
Lot-3, Stage-2	2.3	33.2	342.6	1.00	0.14	15.1	1.00	0.33
Lot-3, Stage-4	2.1	30.1	397.2	0.99	0.06	13.5	0.97	0.15
Lot-3, Stage-5	2.5	32.9	511.6	0.98	0.05	15.2	0.96	0.11
Texas A&M University								
B=1m	7.0	52.3	1986.0	0.99	0.17	36.3	0.99	0.26
B=2.5m	8.7	68.3	1631.0	1.00	0.24	43.0	1.00	0.42
B=3m, North	9.4	72.6	1533.0	0.99	0.33	45.2	1.00	0.61
B=1.5m	5.1	52.1	2180.0	0.97	0.17	32.0	0.96	0.30
B=3m, South	5.7	62.4	1520.0	1.00	0.25	33.9	1.03	0.52

Note: $G_0(1)$ and $G_0(2)$ are small-strain stiffness based on Equation (5.10) and Equation (5.11) and (5.12); $f^*(1)$ and $g^*(1)$ are values by curve fitting using $G_0(1)$; $f^*(2)$ and $g^*(2)$ are values by curve fitting using $G_0(2)$.

However, there are two cases of very high g^* values corresponding to low q_c (Lot-2, Stage-1 and Lot-3, Stage-2) and two cases of high g^* values with high q_c (Lot-1, Stage-4 and Lot-3, Stage-1). The abnormal cases of high g^* value with low q_c may be due to underestimation of G_0 or highly over consolidation. A check of load-settlement curve of Lot-3, Stage-2 showed that the curve is much stiffer compared to others when plate load q is less than 150 kPa. This implies that a pre-stress of 150 kPa may be experienced by the soil before the PLT was conducted. However, for case of Lot-2, Stage-1, load-settlement curve is comparable with others but the q_c measured near ground surface varies greatly between Stage-1 and Stage-2, and

measurements obtained from both stages are much lower compared to other lots and stages, where PLT curves are comparable. Thus, the measured q_c for this case is suspect and G_0 interpreted from the suspect q_c is not reliable.

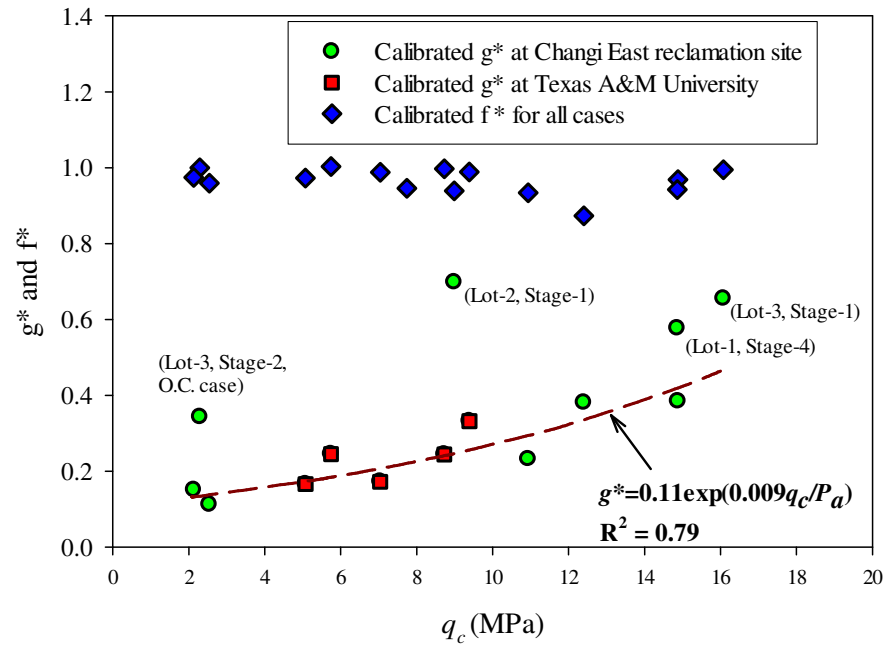


Figure 6.23: Relationship between q_c and calibrated g^*

For the case of high g^* value with high q_c at Lot-3, Stage-1, the high g^* value is reasonable considering the fact that at Lot-3, Stage-1, the soil near ground surface actually experienced cyclic loading by traffics dumping sands. For case of Lot-1, Stage-4, g^* value is slightly higher compared to the trend of the other ten cases. However, the normalized q_c profile within the depth of influence is lower compared to those from the other three stages, which may be due to disturbance caused during removal of overburden. Hence, the slightly higher g^* value is probably due to unreliable q_c within the depth of influence, so that G_0 is underestimated.

Since the four abnormal cases discussed above are not representative, they will be excluded in fitting a relationship of g^* with q_c . An exponential function was found to give a good fit. The exponential function is given by:

$$g^* = 0.1 \times e^{0.009 q_c / P_a} \dots\dots\dots (6.12)$$

where g^* = material constant controlling normalized average modulus degradation rate of soil-foundation system; q_c = tip resistance from CPT, in MPa and P_a = atmosphere pressure.

Figure 6.24 shows two comparisons between measured load-settlement curve and fitted curve for loose and medium dense sands, respectively. It can be seen that the curves obtained by adjusting both f^* and g^* normally fits the measured load-settlement curve for both loose and medium dense sand better than when f^* was set at 1.0 and g^* adjusted. The latter curve fits the measured curve well at the beginning, but not so well as the foundation load approaches q_{ult} . This is acceptable, since allowable foundation load is far less than q_{ult} in practice.

Figure 6.24 also shows the comparison between f^* - g^* best match of Equation (6.11) and Schmertmann's (1970, 1978) method. It can be seen that in application of Schmertmann's (1970, 1978) method to loose sand, it produces comparable estimation when foundation load is small. As the foundation load increase, Schmertmann's (1970, 1978) estimation underestimated the settlement. For dense sand, Schmertmann's (1970, 1978) method considerably overestimates the settlement at typical foundation working load. Apparently, it can be concluded that Equation (6.11) is more flexible and is better able to produce more accurate settlement estimation compared to Schmertmann's (1970, 1978) method.

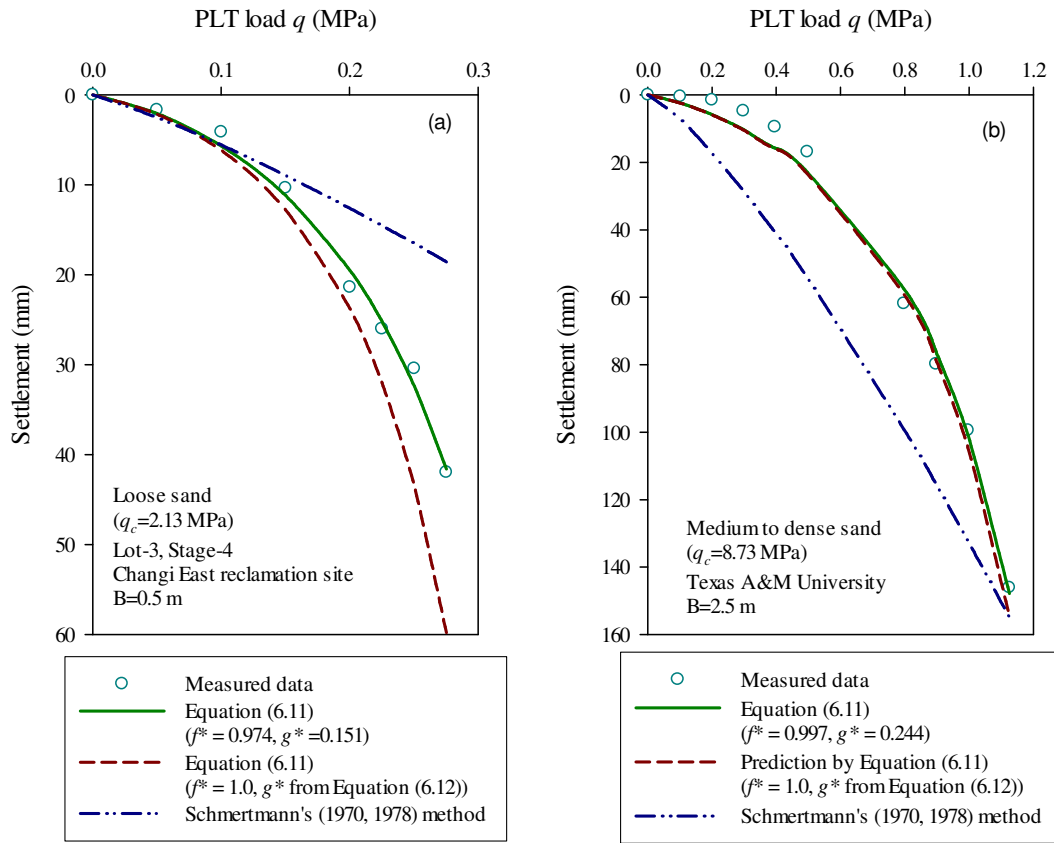


Figure 6.24: Examples of comparison of matched and measured data for (a) loose sand and (b) medium dense sand

6.9 Discussion of the Calibrated and Simulated Average Modulus Degradation of Soil-foundation System

Based on numerical simulation in this chapter, the normalized modulus degradation parameter g^* depends only on the angle of internal friction and g measured on a single soil element. Equation (6.11) can be used to estimate the range of g^* based on values of g observed from laboratory tests. For example, for the case of $\phi' = 40^\circ$, given that g increases from 0.5 to 1.0, g^* can be from 0.25 to 0.4. For highly over-consolidated soil, g can be larger than 1.0 according to the laboratory test results. Therefore, g^* could be even larger than 0.4. Similarly, for the case of $\phi' = 35^\circ$, given that g increases from 0.25 to 0.5, g^* can range from 0.19 to 0.31.

Compared with the calibrated g^* values shown in Table 6.5 and Figure 6.23, Equation (6.11) based on numerical simulation gives good estimation, based on observed g value from laboratory tests and the understanding of the stress history of the two sites. One may argue that K_0 condition assumed in the simulations in Chapter 6 may be incorrect. However, the following reasons are provided for justification: firstly, for over-consolidated site, the horizontal stress may be even larger than the vertical stress near ground surface. Secondly, for normally consolidated condition, the g^* value is expected to be lower than that given by Equation (6.11). However, one should also bear in mind the difference between the mode of loading in PLT and triaxial test. In PLT, the horizontal stress is expected to increase, compared with the constant confining pressure in triaxial test. Therefore, the increase of the effective confining stress in PLT is expected to be higher than that in triaxial test, leading to higher E_0 and secant Young's modulus, which was not considered in the numerical simulation in Chapter 6.

6.10 Proposed Modulus Degradation Method for Estimating Settlement of Shallow Foundation

Based on the studies above, a modulus degradation method for estimating settlement of shallow foundation on cohesionless soil can be described in the following steps:

- 1) Estimate small-strain stiffness G_0 within the depth of influence z_i from q_c of CPT based on Equations (5.11) and (5.12), unless there is evidence showing that Equation (5.10) is more suitable. Estimate small-strain stiffness E_0 from G_0 with an assumed ν , which can be taken as 0.2.
- 2) Estimate effective angle of internal friction ϕ' within depth of B from q_c of CPT based on Equation (5.14). Then estimate the ultimate bearing capacity of the foundation $(q_{ult})_v$ using Vesic's equation (Equation 4.1). Conservatively, the effect of embedment on $(q_{ult})_v$ is ignored. For footing width $B < 1.5\text{m}$, a corrected $(q_{ult})_{corr}$ based on Figure 5.10 is obtained.

- 3) Plot the vertical strain influence factor diagram as shown in Figure 6.25. Note that Figure 6.25 is similar to Figure 3.20.

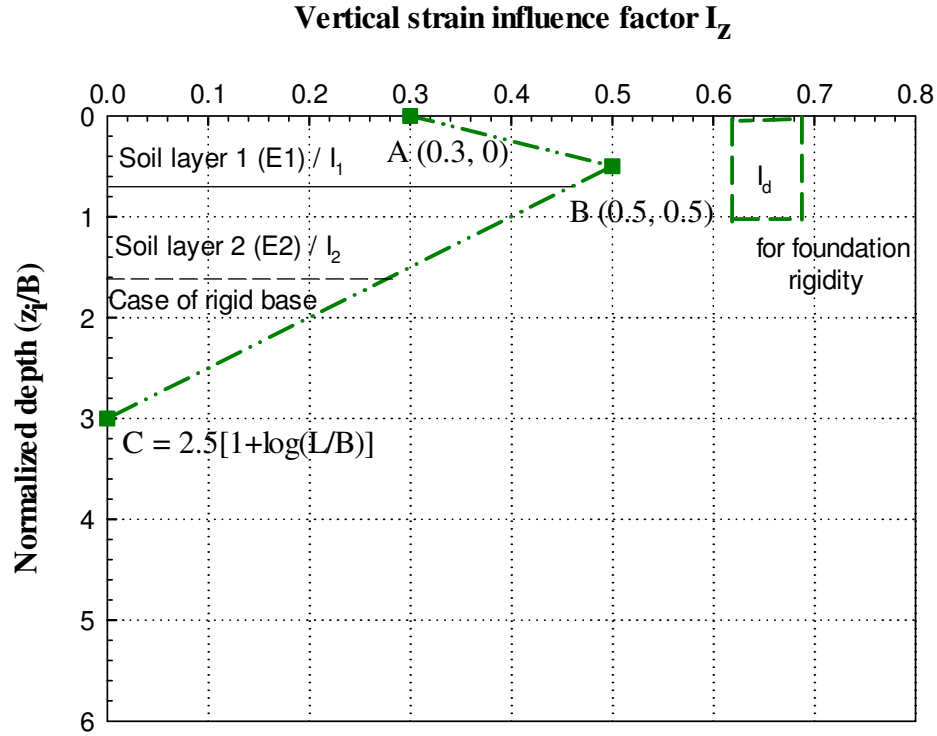


Figure 6.25: Strain influence factor diagram for modulus degradation method

- 4) Calculate average value of E_0 by considering the displacement influence factor using the following equation:

$$\frac{1}{E_{0av}} = \sum_{j=1}^n \frac{1}{E_{0j}} \frac{I_{sj}}{I_{total}} \dots \dots \dots (6.12)$$

where E_{0av} = average value of E_0 ; E_{0j} = small-strain stiffness E_0 for j^{th} soil layer; I_{sj} = displacement influence factor in j^{th} soil layer and I_{total} = total displacement influence factor.

- 5) Estimate the correction factors I_F , I_h and $I_{L/B}$. Correction factor I_F is determined from Equation (3.5). Correction factors I_h and $I_{L/B}$ are determined based on Equation (3.7) and Equation (3.9), respectively.
- 6) Estimate the normalized modulus degradation parameter g^* using Equation (6.12). For practical reason, it can be assumed that $f^* = 1.0$.
- 7) Compute the immediate settlement of shallow foundation on cohesionless soil using the following equation:

$$s = C_C C_D \frac{q \cdot B \cdot I \cdot I_F \cdot I_h \cdot I_{L/B}}{E_{0av} [1 - (\frac{q}{(q_{ult})_v})^{g^*}]} \dots\dots\dots (6.13)$$

where $I = 1$; I_F , I_h and $I_{L/B}$ = and correction factors for foundation rigidity, soil thickness and foundation shape, which are given in Equation (3.5), Equation (3.7), Equation (3.8) and (3.9). C_C and C_D are given in Equation (5.2) and Equation (5.4), respectively.

6.11 Summary

Non-linear f-g model proposed by Fahey and Carter (1993) was reviewed. Typical values of f and g were calibrated using the published laboratory test data. A non-linear elasto-plastic constitutive model of soil with f-g non-linear elastic model and MC failure criterion was established and incorporated into FLAC. The subroutine was verified by simulating triaxial test on a single-element. The load settlement behaviour of foundation on cohesionless soil was investigated numerically using FLAC.

Parametric studies showed that for idealized cohesionless soil with constant ϕ' and modulus degradation parameter g , a unique normalized degradation curve of

average secant Young's modulus of soil-foundation system can be found even though footing width B and small-strain stiffness E_0 are different. A modified hyperbolic function was used to fit the normalized degradation of average modulus. It is not surprising to observe that for foundation resting on soil with smaller values of g , more non-linear load-settlement behaviour can be obtained, i.e., small values of g^* . A power function can be used to describe the correlation between g^* and g .

Given that ϕ' , E_0 and g are known, the non-linear load-settlement behaviour of a foundation can be estimated using the correlation between g^* and g . A closed-form solution was given in this study. Furthermore, the normalized average modulus degradation of soil-foundation system was calibrated using 14 PLT results. The calibrated modulus degradation was found comparable with the numerical analysis results. Detailed procedures of the modulus degradation method for estimating foundation settlement were summarized. Illustration and evaluation of the modulus degradation method are given in Chapter 7.

Chapter 7 Illustration and Evaluation of the Two Proposed Methods

7.1 Introduction

In this chapter, calculation procedures of the two proposed methods i.e., modified Schmertmann's method and modulus degradation method, are illustrated using an example provided by Campanella et al. (2005). The two proposed methods are then further evaluated using 31 case studies from Jeyapalan and Boehm (1984). The estimated settlement by the two proposed methods are compared with the settlement estimated by Schmertmann's (1970, 1978) method. The comparison showed that a significant improvement in settlement estimation has been achieved, especially when considering the size effect of shallow foundation. Similarly, due to the intrinsic inability of CPT in measuring stress history, which is an important factor influencing settlement of shallow foundation, the estimated settlement results using the proposed methods tend to overestimate settlement for highly overconsolidated cohesionless soil. However, the error was well-controlled for all cases.

7.2 Description of the McDonald's Farm Site and the Footing

In Campanella et al.'s (2005) report, an example of estimating settlement of shallow foundation on sand deposit at McDonald's Farm, B. C. using Schmertmann's (1970, 1978) method was given. The sand deposit consists of medium dense to dense sand layer with some silt to 15 m depth. The simplified cone bearing q_c profile is given in Figure 7.1. Ground water table is at 2.0 m below the ground surface. A rigid footing with $B = 2.5$ m and $L = 30$ m (strip footing) is assumed. The depth of the footing base is $D = 2.0$ m. Footing load $q = 180$ kPa, as shown in Figure 7.1. The immediate footing settlement is calculated using Schmertmann's (1970, 1978) method and the two proposed method to illustrate the calculation procedure.

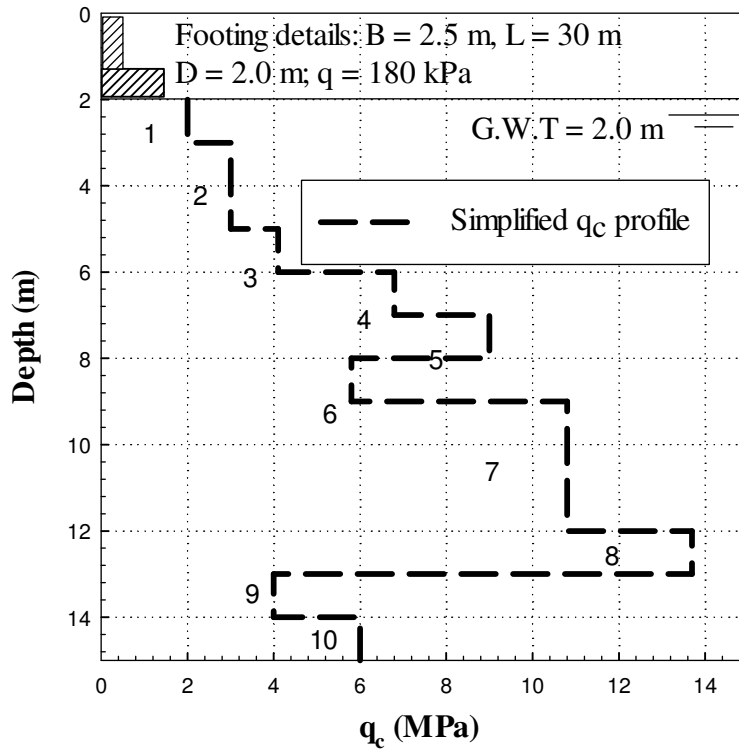


Figure 7.1: Simplified q_c profile and footing details

7.3 Application of Schmertmann's (1970, 1978) Method to Estimate Footing Settlement

First of all, the Schmertmann's (1970, 1978) method is used to estimate the settlement.

- 1) Calculate the peak strain influence factor I_{zp} . Assumed that the bulk density of the sand $\gamma = 18 \text{ kN/m}^3$ for sand above the water table and $\gamma = 20 \text{ kN/m}^3$ for sand below the water table. $\Delta q = 180 - 18 \times 2 = 144 \text{ kPa}$; σ'_{vp} (at $z = D + B/2$) = $18 \times$

$$2 + 10 \times 2.5/2 = 48.5 \text{ kPa. Hence, } I_{zp} = 0.5 + 0.1 \sqrt{\frac{\Delta q}{\sigma'_{vp}}} = 0.67;$$

- 2) Plot the vertical strain influence factor diagram for a strip footing as shown in Figure 7.2 with $I_{z0} = 0.2$, $I_{zp} = 0.67$ and $z_i/B = 4$;

- 3) Calculate C_D using Equation (5.2), $C_D = 0.875$;
- 4) Calculate $\sum_{n=1}^7 \frac{(I_z)_n (\Delta z)_n}{(E_s)_n}$ as shown in Table 7.1;
- 5) Calculate the immediate settlement s using Equation (5.1), $s = 0.875 \times 144 \times 0.248 = 31.2\text{mm}$;

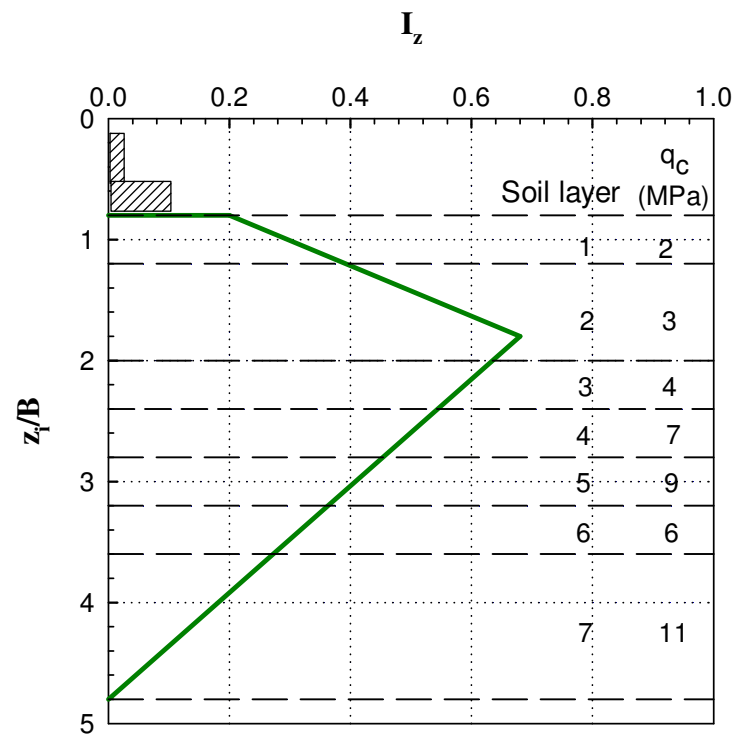


Figure 7.2: Simplified vertical strain influence factor diagram for Schmertmann's (1970, 1978) method

Table 7.1: Calculation of settlement of strip footing in sand at McDonald's Farm using Schmertmann's (1970, 1978) method

Soil layer n	$(\Delta z)_n$ (m)	$(I_z)_n$	$(q_c)_n$ (MPa)	$(E_s)_n (= 3.5q_c)$ MPa	$[(I_z)_n(\Delta z)_n]/(E_s)_n$ (m/MPa)
1	1	0.29	2	7	0.042
2	2	0.57	3	10.5	0.108
3	1	0.57	4	14	0.041
4	1	0.48	7	23.5	0.02
5	1	0.4	9	31.5	0.013
6	1	0.31	6	21	0.015
7	3	0.13	11	38.5	0.01
Sum					0.248

7.4 Application of Modified Schmertmann's Method to Estimate Footing Settlement

The modified Schmertmann's method is applied to estimate the footing settlement as follows:

- 1) Estimate small-strain stiffness G_0 within the depth of influence z_i from q_c of CPT based on Equations (5.11) and (5.12). For strip footing, the depth of influence is $5B$, i.e., 14.5m based on Figure 3.19. Based on CPT results shown in Figure 7.1, G_0 was interpreted and the profile is shown in Figure 7.3;
- 2) Estimate effective angle of internal friction ϕ' within depth of B from q_c of CPT based on Equation (5.14). The average value of ϕ' is 34° . According to Vesic's (1973, 1975) equation, $q_{ult} = \sigma'_{v0} \times N_q \times S_q \times d_q + 0.5 \times \gamma' \times B \times N_\gamma \times s_\gamma \times d_\gamma = 36.0 \times 29.4 \times 1.06 \times 1.02 + 0.5 \times 10.0 \times 2.5 \times 41.0 \times 0.97 \times 1.0 = 1641.5 \text{ kPa}$;

- 3) Calculate the peak strain influence factor I_{zp} using the average q_c within the depth of influence of 5.4 MPa as: $I_{zp} = 0.5 + (3.7 - 0.2 \times 6.0) \times (\frac{\sqrt{180/1641.5}}{1 - \sqrt{180/1641.5}})^{0.8} = 1.93$;
- 4) Plot the vertical strain influence factor diagram as shown in Figure 7.5 with $I_{z0} = 0.3$, $I_{zp} = 1.93$ and $z_i/B = 5$;

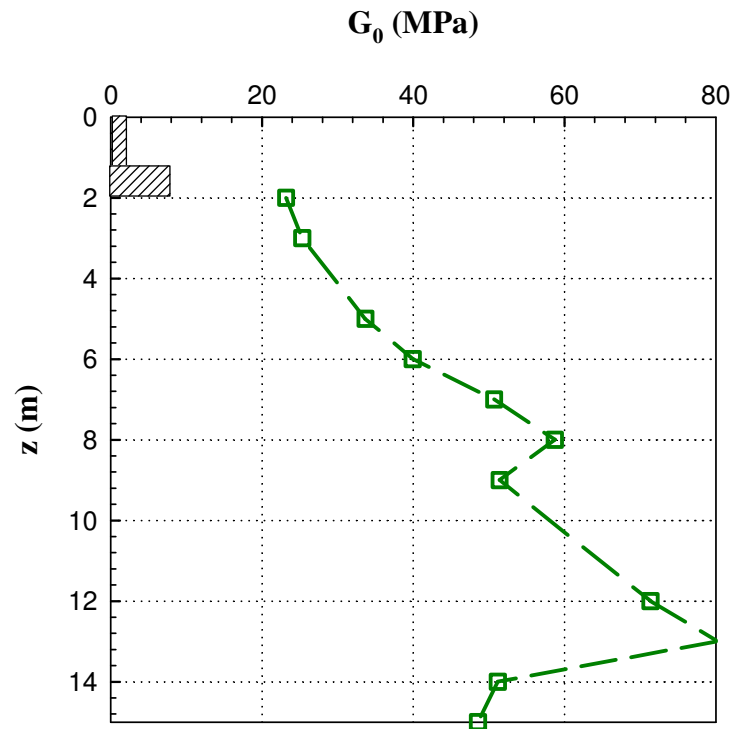


Figure 7.3: Interpreted G_0 from q_c

- 5) Calculate $\sum_{n=1}^{10} \frac{(I_z)_n (\Delta z)_n}{[2G_0(1+\nu)]_n}$ as shown in Table 7.2. Poisson's ratio $\nu = 0.2$;
- 6) Calculate C_D using Equation (5.2), $C_D = 0.875$;
- 7) Calculate immediate settlement s using Equation (5.19), $s = 0.875 \times 180 \times 0.127 = 20$ mm.

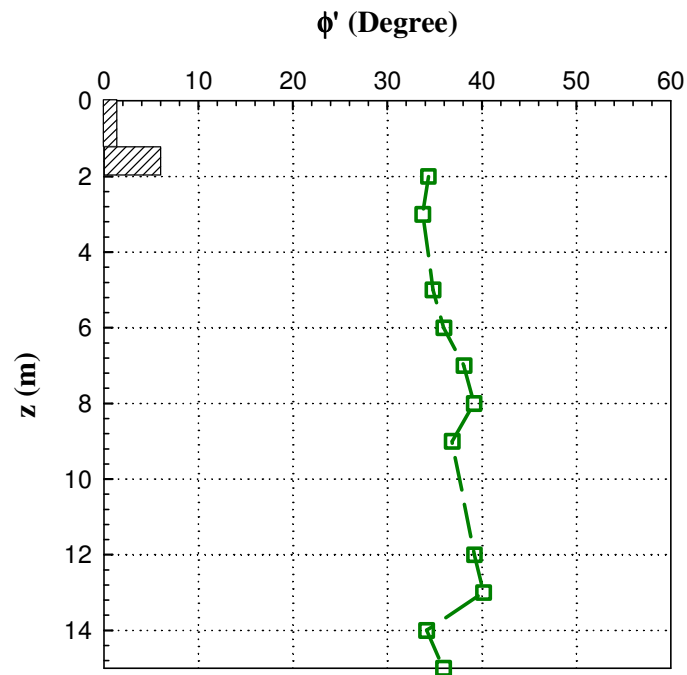


Figure 7.4: Interpreted ϕ' from q_c

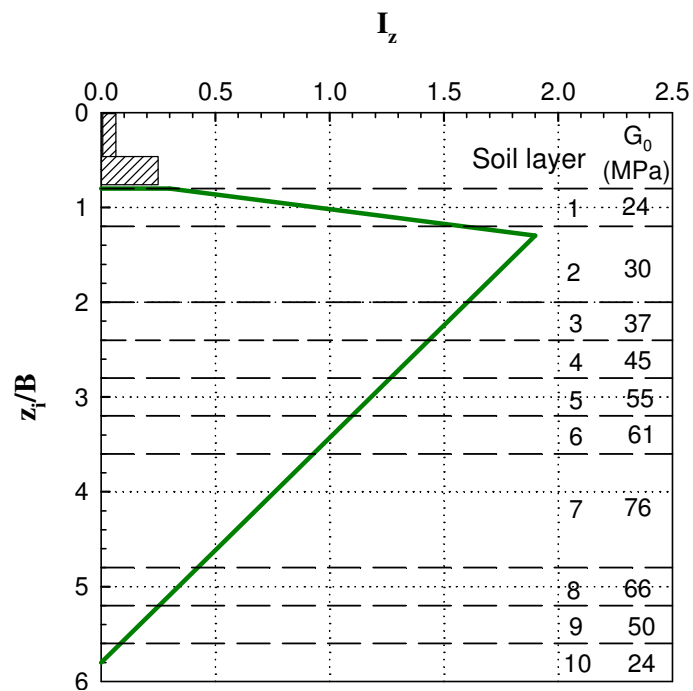


Figure 7.5: Simplified vertical strain influence factor diagram for modified Schmertmann's method

Table 7.2: Calculation of settlement of strip footing in sand at McDonald's Farm using modified Schmertmann's method

Soil layer n	$(\Delta z)_n$ (m)	$(I_z)_n$	$(q_c)_n$ (MPa)	$(G_0)_n$ (MPa)	$[(I_z)_n(\Delta z)_n]/[2G_0(1+\nu)]_n$ (m/MPa)
1	1	0.95	2	24	0.016
2	2	1.75	3	30	0.049
3	1	1.55	4	37	0.017
4	1	1.35	7	45	0.013
5	1	1.21	9	55	0.009
6	1	1.02	6	61	0.007
7	3	0.65	11	76	0.011
8	1	0.35	14	66	0.002
9	1	0.2	4	50	0.002
10	0.5	0.08	6	24	0.001
					0.127

7.5 Application of Modulus Degradation Method for Estimating Settlement of Shallow Foundation

The modulus degradation method is applied to estimate the footing settlement as follows:

- 1) Exactly same as Step 1 in Section 7.4;
- 2) Exactly same as Step 2 in Section 7.4;
- 3) Plot the simplified vertical strain influence factor diagram as shown in Figure 7.6 $I_{z0} = 0.3$, $I_{zp} = 0.5$ and $z_i/B = 5$;
- 4) Calculate the average value of G_0 considering the displacement influence factor as shown in Table 7.3; $G_{0av} = 1/0.0257 = 39$ MPa;
- 5) Calculate C_D using Equation (5.2), $C_D = 0.875$;

6) Calculate g^* using Equation (7.1), which leads to 0.172;

7) Calculate I_F , I_h and $I_{L/B}$: $I_F = \pi/4$, $I_h = 1$, $I_{L/B} = 0.5 \ln(L/B) + 1 = 2.15$

8) Calculate settlement s using Equation (6.13), which gives $s = 0.875 \times$

$$\frac{180 \times 2.5 \times 1.0}{39 \times 2.4 \times [1 - (\frac{180}{1641})^{0.172}]} \times \frac{\pi}{4} \times 2.15 = 22.5 \text{ mm.}$$

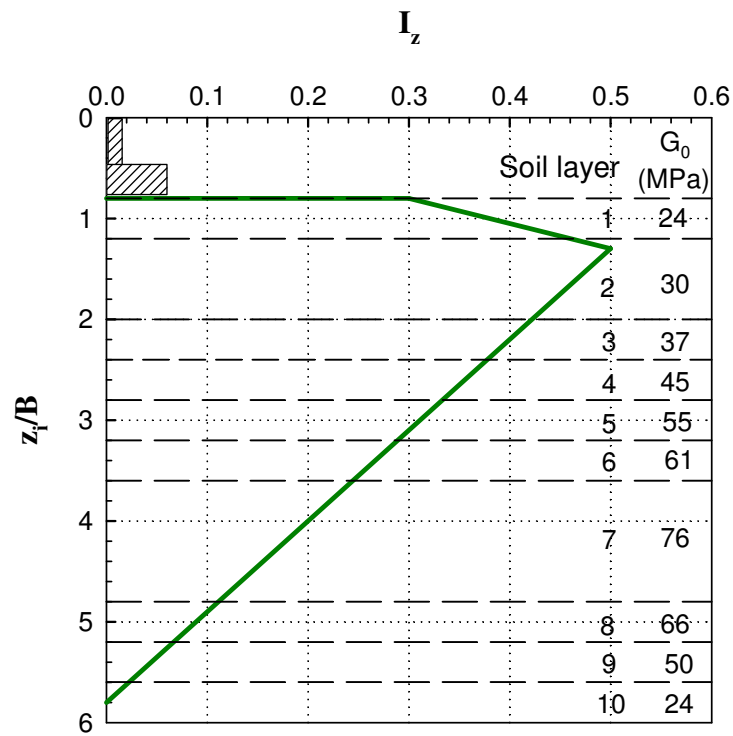


Figure 7.6: Simplified vertical strain influence factor diagram for modulus degradation method

Table 7.3: Calculation of average value of G_0 considering displacement influence factor

Soil layer n	$(\Delta z)_n$ (m)	$(I_z)_n$	$I_{si} =$ $[(I_z)_n(\Delta z)_n]$ (m)	$(G_0)_n$ (MPa)	$(I_{si})_n / ((G_0)_n I_{total})$ (1/MPa)
1	1	0.38	0.38	24	0.0048
2	2	0.46	0.92	30	0.0093
3	1	0.40	0.40	37	0.0033
4	1	0.35	0.35	45	0.0024
5	1	0.31	0.31	55	0.0017
6	1	0.26	0.26	61	0.0013
7	3	0.18	0.54	76	0.0022
8	1	0.08	0.08	66	0.0004
9	1	0.05	0.05	50	0.0003
10	0.5	0.01	0.05	24	0.0001
I_{total}			3.3	sum	0.0257

7.6 Evaluation of the Two Proposed Method

Thirty one case studies from Jeyapalan and Boehm (1984) were used to evaluate the two proposed method for estimating settlement of shallow foundation, i.e., modified Schmertmann's method and modulus degradation method. The estimated settlement were compared with those estimated by Schmertmann's (1970, 1978) method. For the 31 case studies, q_c varies from 1.8 MPa to 19.6 MPa; foundation width B varies from 0.5 m to 27.44 m; ratio of L/B varies from 1.0 to 6.76; foundation depth varies from 0 m to 0.9 m; foundation load varies from 44 kPa to 575 kPa; and measured settlement varies from 2.4 mm to 32.5 mm. It can be seen that these cases vary significantly in foundation size and in situ soil properties.

Since there were only q_c values, but not the detailed q_c profiles in the literature, the small-strain stiffness G_0 and ϕ' were interpreted using the vertical stress at depth of B. The sand was assumed to be above the ground water table. The bulk density γ of sand was assumed to be averagely 17.5 kN/m^3 if q_c from CPT was larger than 5 MPa. Otherwise, $\gamma = 15.5 \text{ kN/m}^3$. Table 7.4 summarizes the information of the 31

case studies. Table 7.5 lists the estimated settlements using three methods, i.e., Schmertmann's (1970, 1978) method, modified Schmertmann's method and modulus degradation method. It can be seen that the two proposed method generally produce much closer estimation than Schmertmann's (1970, 1978) method.

Table 7.4: Summary of the 31 case studies from Jeyapalan and Boehm (1984)

No.	B(m)	L(m)	q_c (MPa)	q (kPa)	L/B	d(m)	G_0 (Mpa)	ϕ'	q_{ult} (kPa)	Measured s (mm)
1	2.50	6.40	19.57	259	2.56	0.0	50.3	44.8	4810	3.70
2	2.50	14.02	11.74	157	5.61	0.0	41.3	42.3	3360	2.60
3	6.49	16.01	13.70	159	2.47	0.0	61.7	40.8	5982	9.10
4	6.49	16.01	13.70	215	2.47	0.0	61.7	40.8	5982	11.00
5	4.51	30.49	9.79	73	6.76	0.0	47.6	40.1	4105	3.10
6	2.99	14.33	15.66	127	4.79	0.0	49.2	43.3	4718	2.40
7	3.00	15.24	9.79	235	5.08	0.0	41.1	41.1	3169	8.50
8	8.60	15.00	17.61	149	1.74	0.0	75.1	41.3	8000	4.10
9	27.44	30.49	11.74	294	1.11	0.0	97.4	36.6	9560	37.50
10	13.11	27.44	11.74	176	2.09	0.0	74.8	38.4	7716	9.00
11	1.00	1.00	13.50	224	1.00	0.0	31.4	45.2	2223	3.60
12	0.50	2.00	10.00	336	4.00	0.0	21.8	45.4	2321	6.70
13	1.00	1.00	18.00	575	1.00	0.5	40.5	46.6	2915	4.40
14	0.50	2.00	15.00	575	4.00	0.3	27.6	47.4	3418	4.20
15	1.00	1.00	7.00	347	1.00	0.5	28.2	42.1	1238	5.50
16	0.60	0.60	1.80	131	1.00	0.3	12.3	37.1	764	6.90
17	0.60	0.60	2.20	230	1.00	0.9	15.1	38.0	1678	12.70
18	0.90	0.90	2.00	136	1.00	0.3	14.6	36.6	681	7.60
19	0.90	0.90	2.30	115	1.00	0.9	17.8	37.3	1426	6.40
20	1.20	1.20	2.70	202	1.00	0.2	17.8	37.4	685	13.00
21	1.20	1.20	3.20	274	1.00	0.9	22.3	38.2	1462	12.70
22	2.60	12.80	3.91	200	4.92	0.0	25.3	37.3	1285	12.70
23	4.57	10.00	3.91	48	2.19	0.0	31.0	35.9	1615	7.10
24	3.60	8.99	4.89	68	2.50	0.0	31.0	37.6	1705	2.40
25	6.10	6.10	3.13	44	1.00	0.0	31.5	34.2	1199	3.00
26	6.10	6.10	3.13	66	0.98	0.0	31.5	34.1	1199	6.00
27	6.10	6.10	3.13	87	1.00	0.0	31.5	34.2	1199	10.00
28	6.10	6.10	3.13	110	1.00	0.0	31.5	34.2	1199	14.80
29	6.10	6.10	3.13	131	1.00	0.0	31.5	34.2	1199	19.50
30	6.10	6.10	3.13	153	1.00	0.0	31.5	34.2	1199	25.50
31	6.10	6.10	3.13	175	1.00	0.0	31.5	34.2	1199	32.50

Table 7.5: Comparison of settlement estimations of three methods

No.	Schmertmann's (1970, 1978) method				Modified Schmertmann's method					Modulus degradation method	
	E_s (MPa)	I_{zp}	I	s_1 (mm)	m	I_{zp}	z_i/B	I_z	s_2 (mm)	g	s_3 (mm)
1	52.32	0.84	3.05	15.1	0.300	0.62	3.52	1.17	6.04	0.58	7.29
2	35.36	0.76	3.43	15.2	1.352	0.98	4.37	2.24	8.51	0.29	8.25
3	36.48	0.66	6.25	27.2	0.960	0.76	3.48	1.41	9.44	0.34	9.31
4	36.48	0.69	6.49	38.3	0.960	0.80	3.48	1.48	13.42	0.34	13.18
5	30.73	0.63	5.40	12.8	1.743	0.89	4.57	2.13	5.89	0.24	5.94
6	45.75	0.72	3.72	10.3	0.568	0.65	4.20	1.46	4.52	0.41	4.87
7	28.90	0.80	4.18	34.0	1.743	1.29	4.26	2.86	19.56	0.24	18.21
8	45.48	0.64	7.14	23.4	0.178	0.54	3.10	0.97	6.33	0.49	6.95
9	29.49	0.61	18.51	184.5	1.352	0.89	2.61	1.26	41.59	0.29	37.68
10	30.78	0.62	11.26	64.4	1.352	0.84	3.30	1.48	18.22	0.29	17.41
11	33.75	1.00	1.04	6.9	1.000	1.04	2.50	1.40	3.97	0.34	3.62
12	28.33	1.36	1.11	13.2	1.700	1.65	4.01	3.42	10.46	0.25	9.40
13	45.00	1.30	1.34	16.9	0.100	0.58	2.50	0.82	5.33	0.51	7.94
14	42.50	1.63	1.32	17.9	0.700	1.02	4.01	2.15	9.61	0.39	10.44
15	17.50	1.12	1.16	22.7	2.300	3.03	2.50	3.92	21.79	0.19	18.06
16	4.50	1.04	0.65	18.5	3.340	3.00	2.50	3.91	11.56	0.12	10.97
17	5.50	1.21	0.75	30.5	3.260	2.60	2.50	3.39	16.13	0.12	15.33
18	5.00	0.95	0.89	23.8	3.300	3.26	2.50	4.24	16.28	0.12	15.21
19	5.75	0.91	0.86	16.1	3.240	2.03	2.50	2.65	7.81	0.12	7.62
20	6.75	0.97	1.21	36.0	3.160	4.10	2.50	5.35	32.77	0.13	29.71
21	8.00	1.05	1.31	43.7	3.060	2.93	2.50	3.82	29.38	0.13	26.67
22	11.49	0.82	3.71	64.5	2.917	2.53	4.23	5.63	49.40	0.14	47.11
23	10.30	0.62	3.96	18.4	2.917	1.33	3.35	2.34	7.14	0.14	7.43
24	13.05	0.66	3.45	18.0	2.721	1.40	3.49	2.56	8.69	0.16	8.68
25	7.83	0.60	3.88	21.8	3.074	1.47	2.50	1.95	7.14	0.13	7.12
26	7.82	0.62	3.98	33.6	3.074	1.69	2.48	2.22	12.30	0.13	11.90
27	7.83	0.64	4.12	45.8	3.074	1.88	2.50	2.48	17.89	0.13	17.00
28	7.83	0.66	4.22	59.4	3.074	2.08	2.50	2.72	24.87	0.13	23.27
29	7.83	0.67	4.31	72.2	3.074	2.25	2.50	2.94	31.97	0.13	29.57
30	7.83	0.68	4.39	85.9	3.074	2.42	2.50	3.17	40.13	0.13	36.78
31	7.83	0.70	4.47	100.0	3.074	2.59	2.50	3.39	49.05	0.13	44.62

Figure 7.7 and Figure 7.8 show the comparison of the settlement estimations based on the two proposed methods and that estimated by Schmertmann's (1970, 1978) method. From Figure 7.7, it can be seen that generally, the two proposed methods

gave better estimation than Schmertmann's (1970, 1978) method, especially for those cases with larger B. Schmertmann's (1970, 1978) method tends to overestimate the settlement. Figure 7.8 shows the relationship between ratio of estimated over measured settlement (s_e/s_m) with foundation width B. It can be seen that for large foundation widths, Schmertmann et al.'s method usually significantly overestimate the settlement. However, for the two proposed methods, there is no such problem.

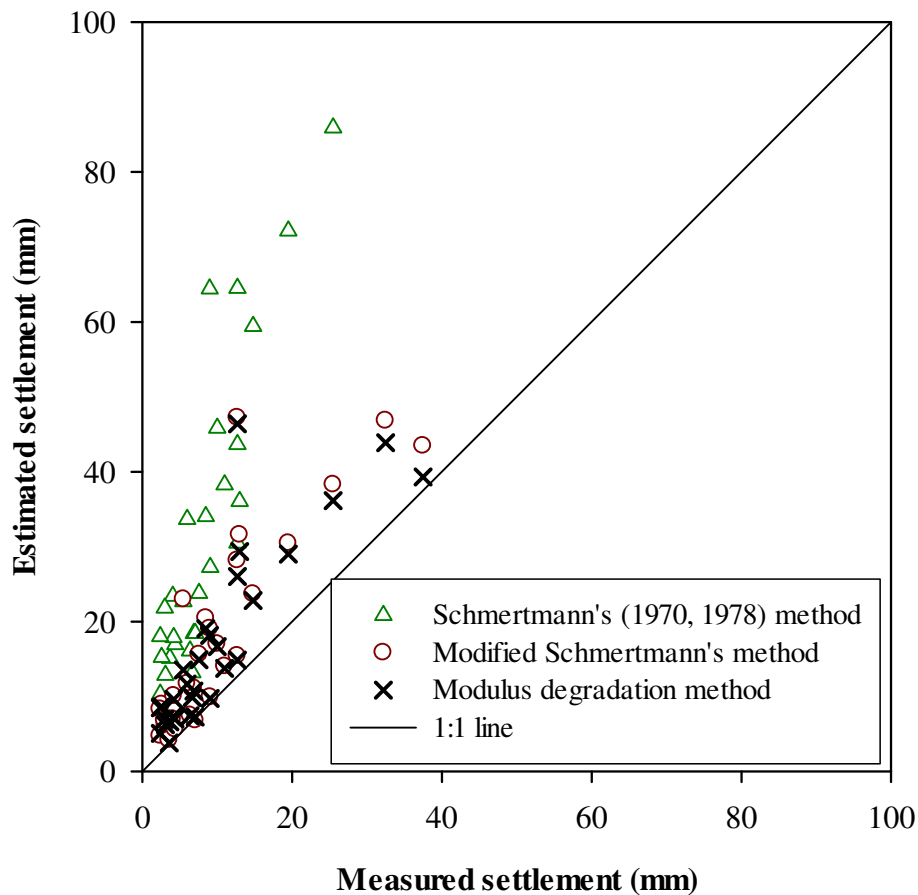


Figure 7.7: Comparison of settlement estimations from three methods

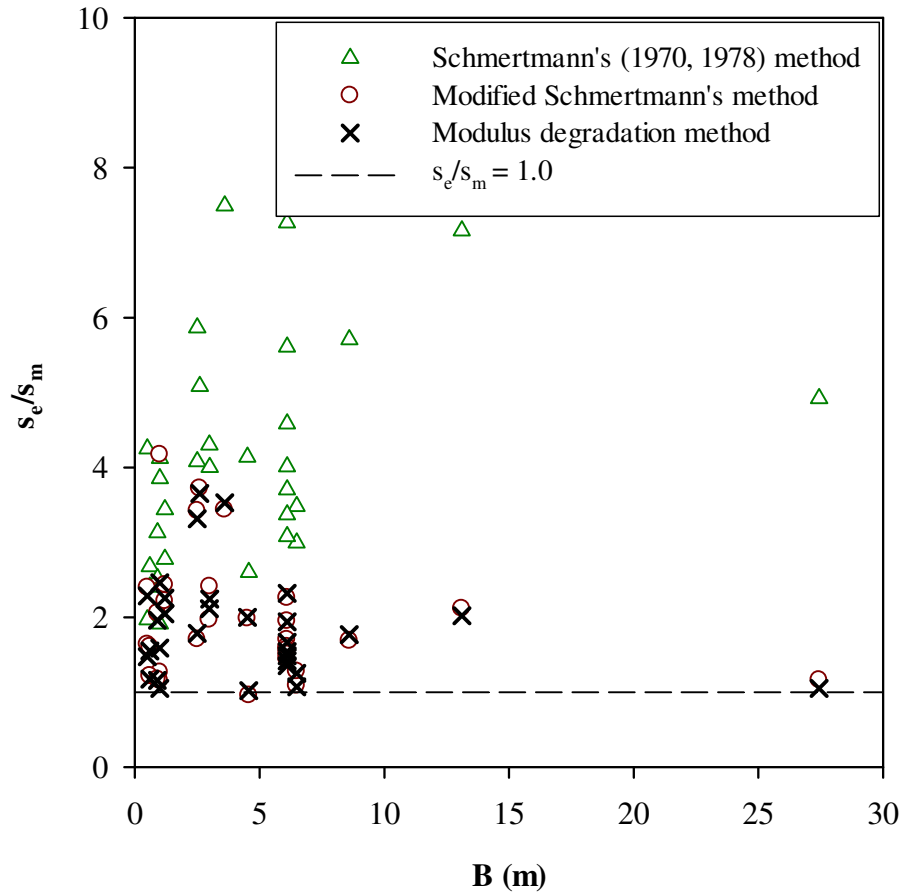


Figure 7.8: Comparison of s_e/s_m for the three methods

7.7 Discussion of the Two Proposed Methods

The two proposed methods, i.e., modified Schmertmann's method and modulus degradation method share similar characteristics, i.e., both using the small-strain stiffness in the calculation; and the mobilized loading level was adopted by both methods in the calculation.

The main difference between the two proposed methods is the way the modulus degradation is considered. For modified Schmertmann's method, a variable maximum strain influence factor I_{zp} was introduced, so that the displacement influence factor increases non-linearly with the normalized footing load. Therefore, although small-strain stiffness is not reduced in the estimation, the modulus degradation is accounted for indirectly by a variable displacement influence factor.

For modulus degradation method, the displacement influence factor is fixed once the foundation size and the soil thickness is known. The small-strain stiffness is reduced with the increase in the mobilized footing load. Conceptually, the latter method is clearer and more straightforward.

Table 7.4 compares the settlement estimations of the 31 case studies using the modified Schmertmann's method and modulus degradation method. The two methods are comparable. The modulus degradation method performed slightly better than the modified Schmertmann's method. Comparison with estimation from Schmertmann's (1970, 1978) method shows a significant improvement was achieved.

Table 7.6: Summary of the settlement estimations of three methods

Comparison	Schmertmann's (1970, 1978) method	Modified Schmertmann's method	Modulus degradation method
Range of s_e/s_m	2.0 ~ 7.5	1.04 ~ 3.96	1.01 ~ 3.78
Average s_e/s_m	4.1	1.95	1.90

7.8 Summary

The detailed calculation procedures of the proposed modified Schmertmann's method and modulus degradation method were illustrated using an example. Thirty one case studies were further used to evaluate the two methods. Although CPT is not able to reflect the stress history of in-situ soils, the proposed two methods were demonstrated to produce significant improved settlement estimations compared with Schmertmann's (1970, 1978) method. The greatest advantage of the proposed methods is that they are able to take into account the size effect of the foundation when estimating settlement, by introducing a mobilized load in the calculation. Based on the evaluation using 31 case studies, modulus degradation method gave slightly better settlement estimation than modified Schmertmann's method.

Chapter 8 Conclusions and Recommendations

8.1 Conclusions

In estimating settlement of shallow foundation on cohesionless soil, reliable determination of in situ soil modulus has the most significant effect on the accuracy of the estimation. However, the modulus of in situ soil depends not only on soil properties, but also on foundation properties. The objective of this research is to propose a practical method for estimating the settlement of shallow foundation on cohesionless soil considering the modulus degradation of soil from small-strain stiffness. Two methods to estimate settlement of shallow foundation considering modulus degradation from small-strain stiffness are proposed in this study.

Based on the results of this study, the following main conclusions can be drawn:

- 1) It is more rational to use vertical strain influence factor diagram in settlement estimation. Using FEM analyses, the effects of foundation rigidity, foundation shape, finite soil thickness, layered soil and Gibson's soil were investigated. A simplified vertical strain influence factor diagram and correction factors were proposed to account for foundation rigidity, foundation shape and finite soil thickness in settlement estimation of shallow foundation on cohesionless soil.
- 2) The scale effect of bearing capacity of shallow foundation was investigated using FDM program FLAC. Bolton's (1986) equation correlating effective peak angle of internal friction ϕ'_p to relative density of sand and mean effective stress level was adopted. The effects of associated and non-associated flow rules on the simulated results were examined. It was found that the numerical analyses produced comparable results with the observations from model

footings in centrifuge tests. However, the results of in situ PLT seem to show more significant scale effect.

- 3) Although Schmertmann's (1970, 1978) method is the most frequently used method for estimating settlement of shallow foundation on cohesionless soil, there are shortcomings and various modifications have been proposed. In this research, it was proposed to use small-strain stiffness in the calculation. To account for modulus degradation of soil and non-linear load-settlement behavior of the foundation, vertical displacement influence factor was made to increase non-linearly with the mobilized foundation load by introducing a new expression of variable peak vertical strain influence factor. The new expression was calibrated using 14 load-settlement curves from two sites. A modified Schmertmann's method for estimating settlement of shallow foundation on cohesionless soil was proposed.
- 4) Reducing soil modulus can be incorporated into the elastic solution of settlement of shallow foundation for estimating a non-linear load-settlement curve. However, it is recognized that due to the difference of the loading mode between laboratory tests and foundation, the modulus degradation curves measured in laboratory tests cannot be applied to the foundation problem directly. Therefore, numerical studies were carried out to investigate a way to make use of the modulus degradation curves measured in laboratory for estimating settlement of shallow foundation on cohesionless soil. In this research, f-g non-linear elastic model proposed by Fahey and Carter (1993) was used to describe the normalized modulus degradation of the cohesionless soil. The values of f and g were obtained from laboratory tests. To use a similar non-linear equation to model load-settlement behavior of foundation, different f^* and g^* values are needed. It was found that for a particular soil (ϕ' , $f=0.97$ and g), the parameters f^* and g^* in the modulus degradation of soil-foundation system can be obtained using the modulus degradation of soil element test in the laboratory.

- 5) The normalized average modulus degradation of soil-foundation system was calibrated using 14 load-settlement curves from two sites. The calibrated normalized modulus degradation was found comparable with the numerical results. A modulus degradation method for estimating settlement of shallow foundation on cohesionless soil was proposed.
- 6) An example was given to illustrate the calculation procedures of the two methods, followed by evaluation based on 31 case studies. It was found that significant improvement in the settlement estimation was achieved compared with settlement estimation from Schmertmann's (1970, 1978) method. The two proposed methods, i.e., modified Schmertmann's method and modulus degradation method gave comparable settlement estimation. The latter method performed slightly better based on the 31 case studies.

8.2 Recommendations for future researches

Though the objective of the research is to develop a practical method to estimate settlement of shallow foundation on cohesionless soil, two methods were proposed as it was found that modulus degradation of soil can be accounted for in the settlement estimation differently. Both methods performed better than the Schmertmann's (1970, 1978) method.

Due to time and resource constraints, not all aspects of the problem could be investigated thoroughly. The following are recommended for future researches:

- 1) Although the two proposed method showed improvement over Schmertmann's (1970, 1978) method, the methods were evaluated with a small database. More evaluation is needed to substantiate the findings in this research.

- 2) Although numerical study of the ultimate bearing capacity of shallow foundation becomes more and more accurate, the fluctuation in the simulation with large angle of internal friction and non-associated flow rule makes it difficult to determine the ultimate load. Improvement in numerical modeling in this regard is needed.
- 3) Although scale effect was investigated numerically in this research, more researches are needed to establish a practical method based on in situ tests for estimating ultimate bearing capacity of shallow foundation more accurately by considering scale effect.
- 4) Although lots of observed modulus degradation of soil element from laboratory tests based on various sands has been reported in the literature, more researches are required to better understand the factors that affect the modulus degradation.
- 5) In this research, non-linear f-g elastic model proposed by Fahey and Carter (1993) was incorporated into the built-in MC model to investigate the correlation between the normalized modulus degradation of soil element and the normalized modulus degradation of soil-foundation system. It will be interesting to evaluate other constitutive models. Moreover, K_0 was set to be unity in this study. More researches are needed to examine the effect of K_0 on the numerical results.
- 6) The current study focuses on the application of the modulus degradation of soil to shallow foundation problem. The applications of modulus degradation from small-strain stiffness to other geotechnical problems are promising and attractive.
- 7) A reliable measurement of modulus degradation of in situ cohesionless soil is still difficult. A more reliable empirical estimation of the small-strain

stiffness based on popular in situ tests, such as CPT and SPT is also greatly valuable in practice.

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Appendix A In situ test results at Changi East reclamation site, Singapore

At Changi East reclamation site, Singapore, a total of 15 PLT and 15 CPT were conducted at three lots, Lot-1, Lot-2 and Lot-3. At each lot, five PLT and five CPT were conducted at five stages, Stage-1 to Stage-5 as shown in Figure. A.1. The PLT and CPT were conducted at Stage-1 first. Then an overburden of 3m was applied and maintained for about 9 months. After that the sand was carefully removed layer by layer to a certain elevation, and PLT and CPT were conducted at this elevation. At the same elevation level, CPT was located at the centre point where PLT would be conducted. More detail descriptions of the site and in-situ tests can be found in Na et al. (2002).

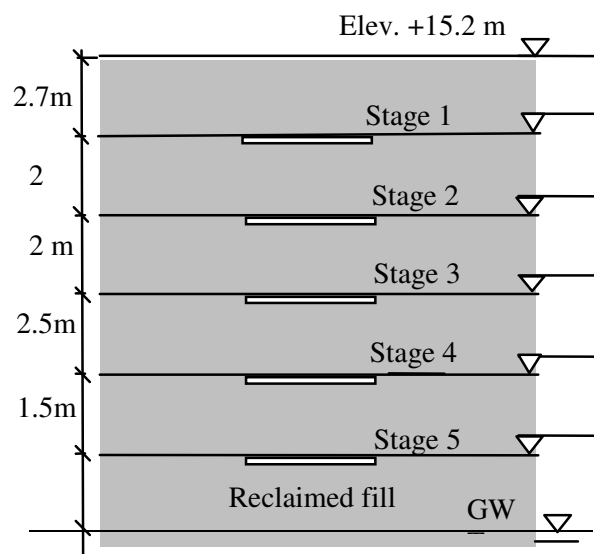


Figure A.1: Five stages of in situ tests conducted at Changi East reclamation site, Singapore

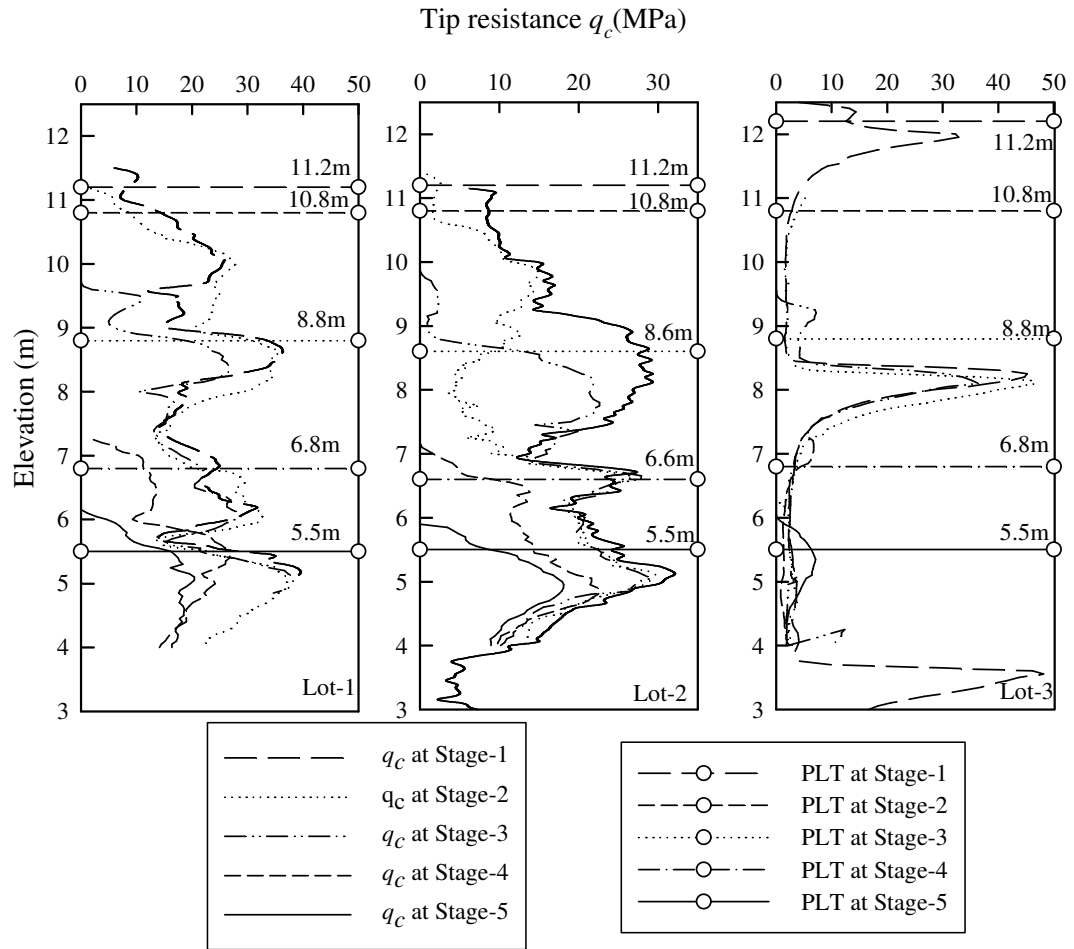


Figure A.2: CPT results at Changi East reclamation site, Singapore

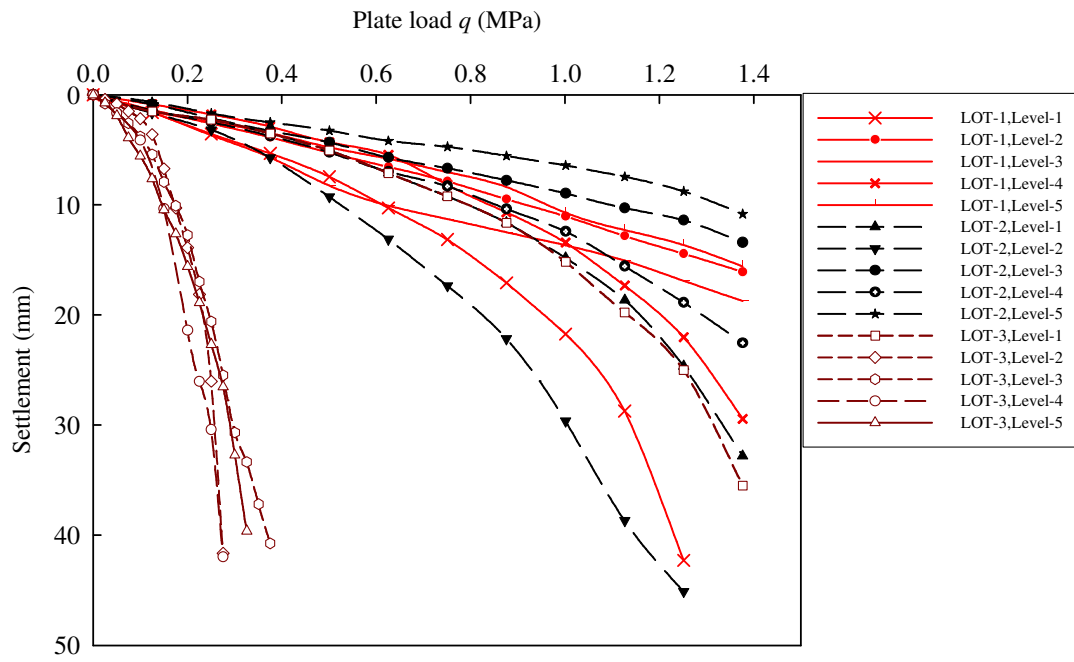


Figure A.3: PLT results at Changi East reclamation site, Singapore

Appendix B In situ test results at Texas A&M University, USA

At Texas A&M University, USA, five PLT and five CPT were conducted on an 11m thick sand layer. The layout of the CPT and PLT is shown in Figure B.1. It can be seen that CPT-2, 5, 6 and 7 were located within the area where PLT-2.5m, 3mN, 1.5m, 3mS were conducted, respectively. CPT-1 was located close to where PLT-1m was conducted. As a result, each corresponding CPT data was used to analyze load-settlement curve for each PLT.

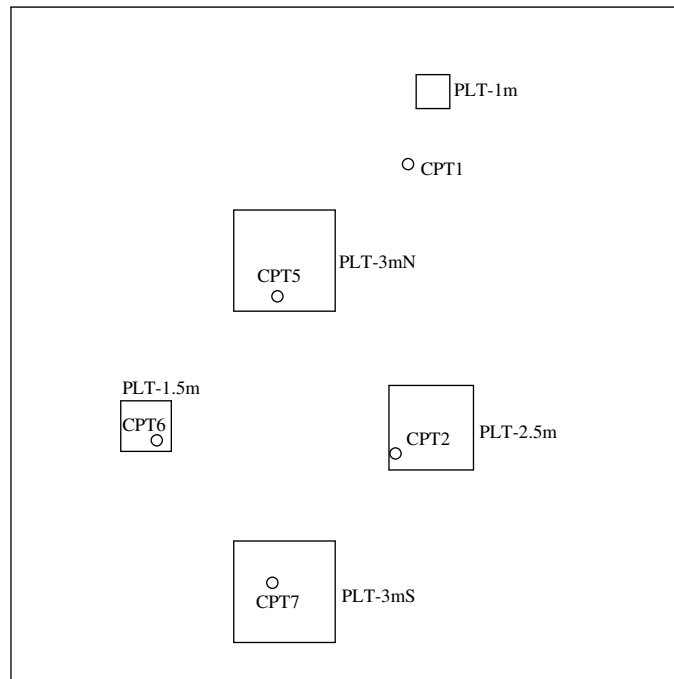


Figure B.1: Field Testing Layout at Texas A&M University, USA (after Briaud and Gibbens, 1994)

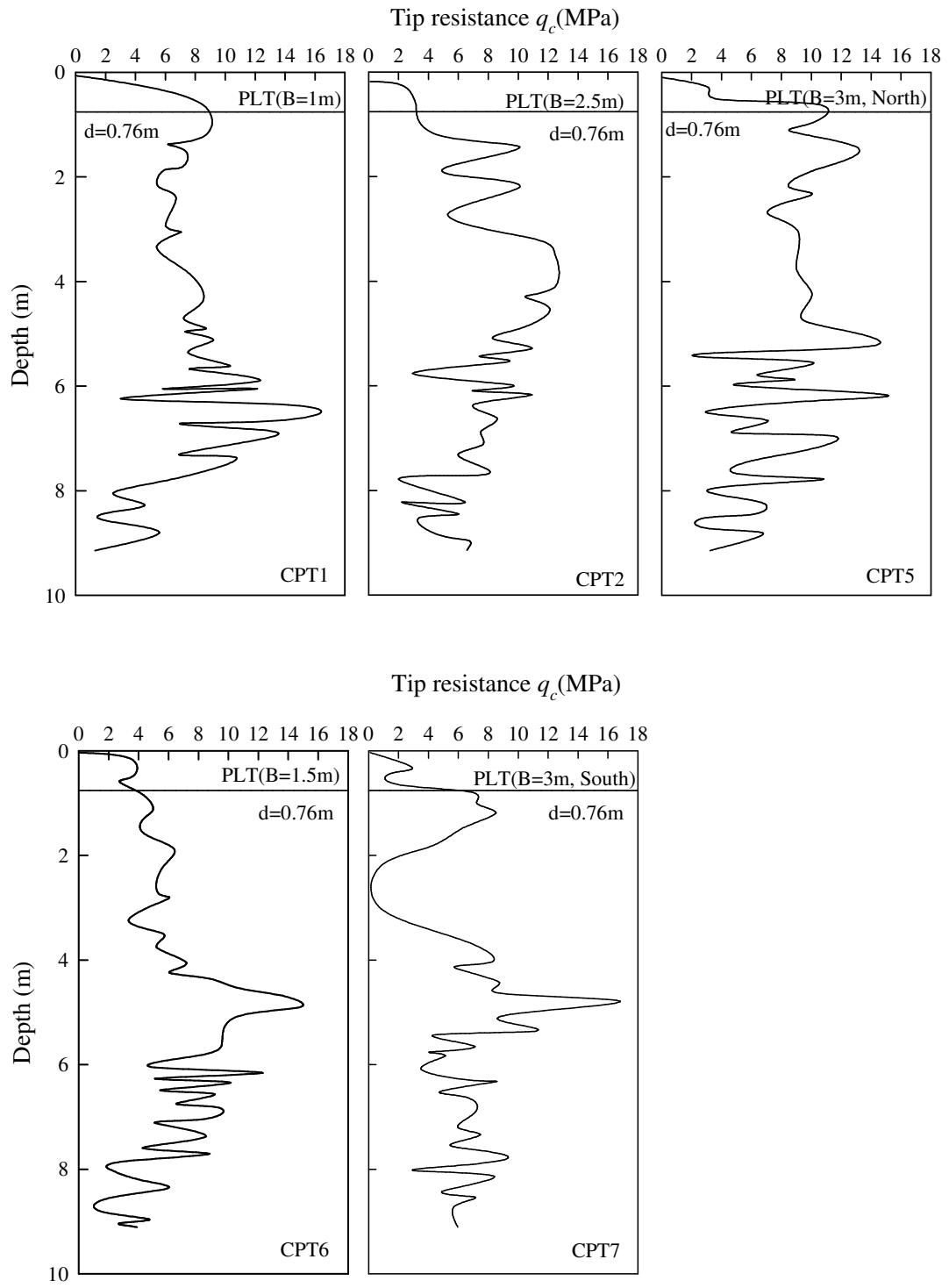


Figure B.2: CPT results at Texas A&M University, USA

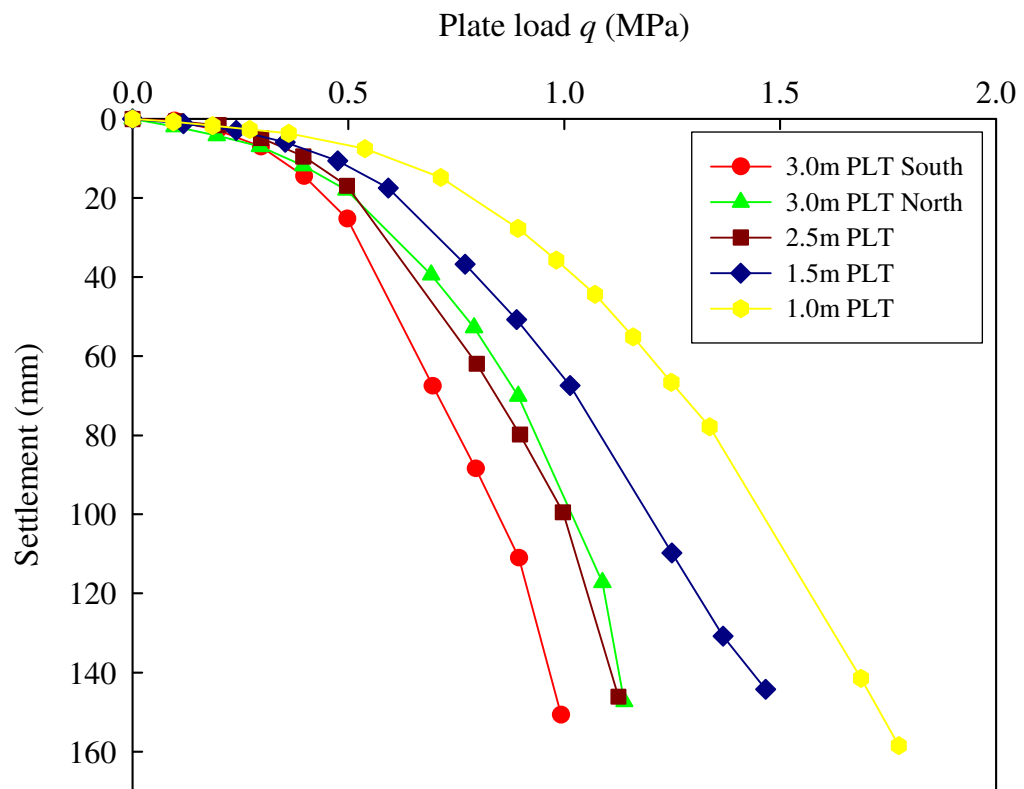


Figure B.3: PLT results at Texas A&M University, USA

Appendix C Interpretation of small-strain stiffness G_0 and internal friction angle ϕ from CPT

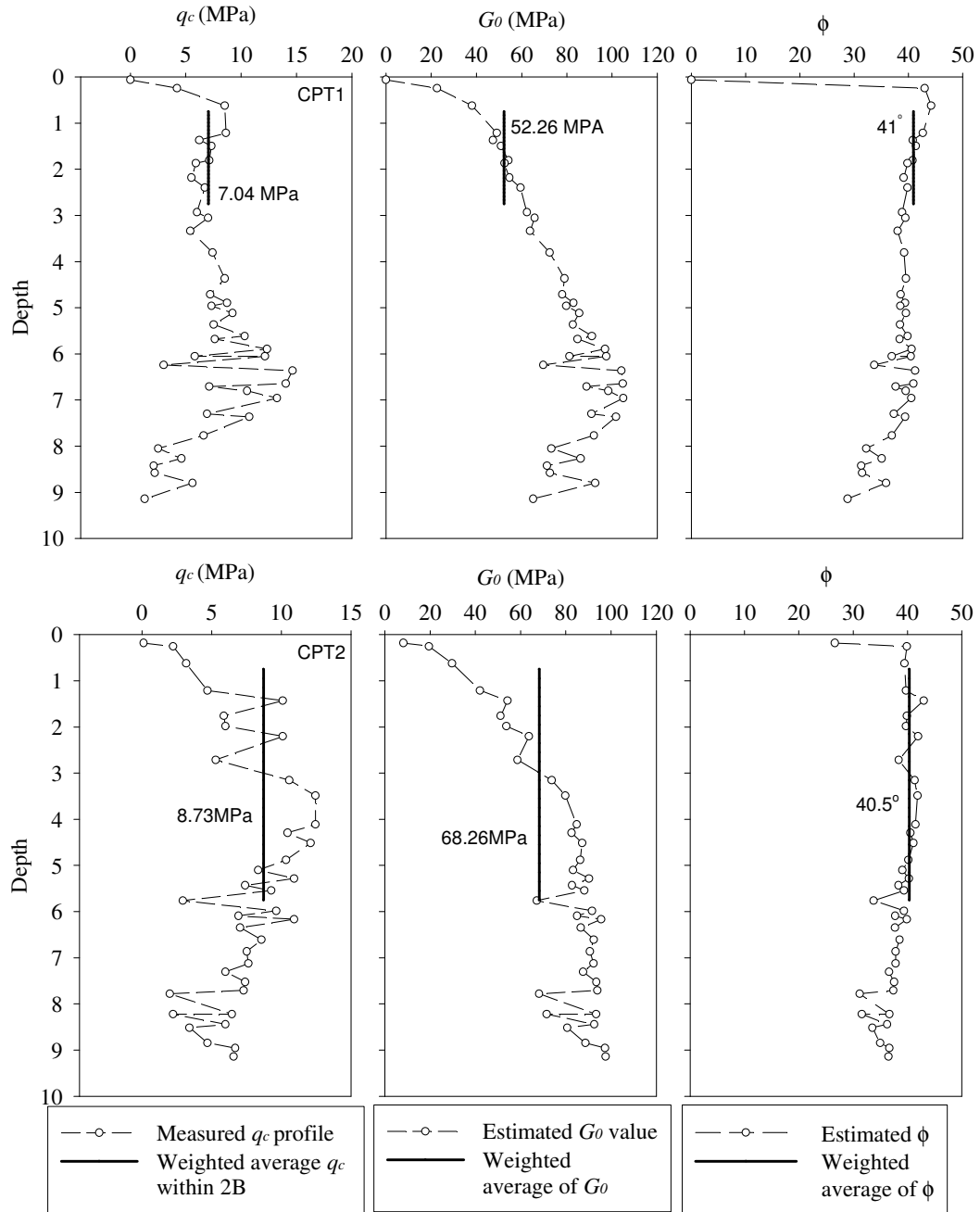


Figure C.1: Interpretation of CPT1 and CPT2 at Texas A&M University, USA

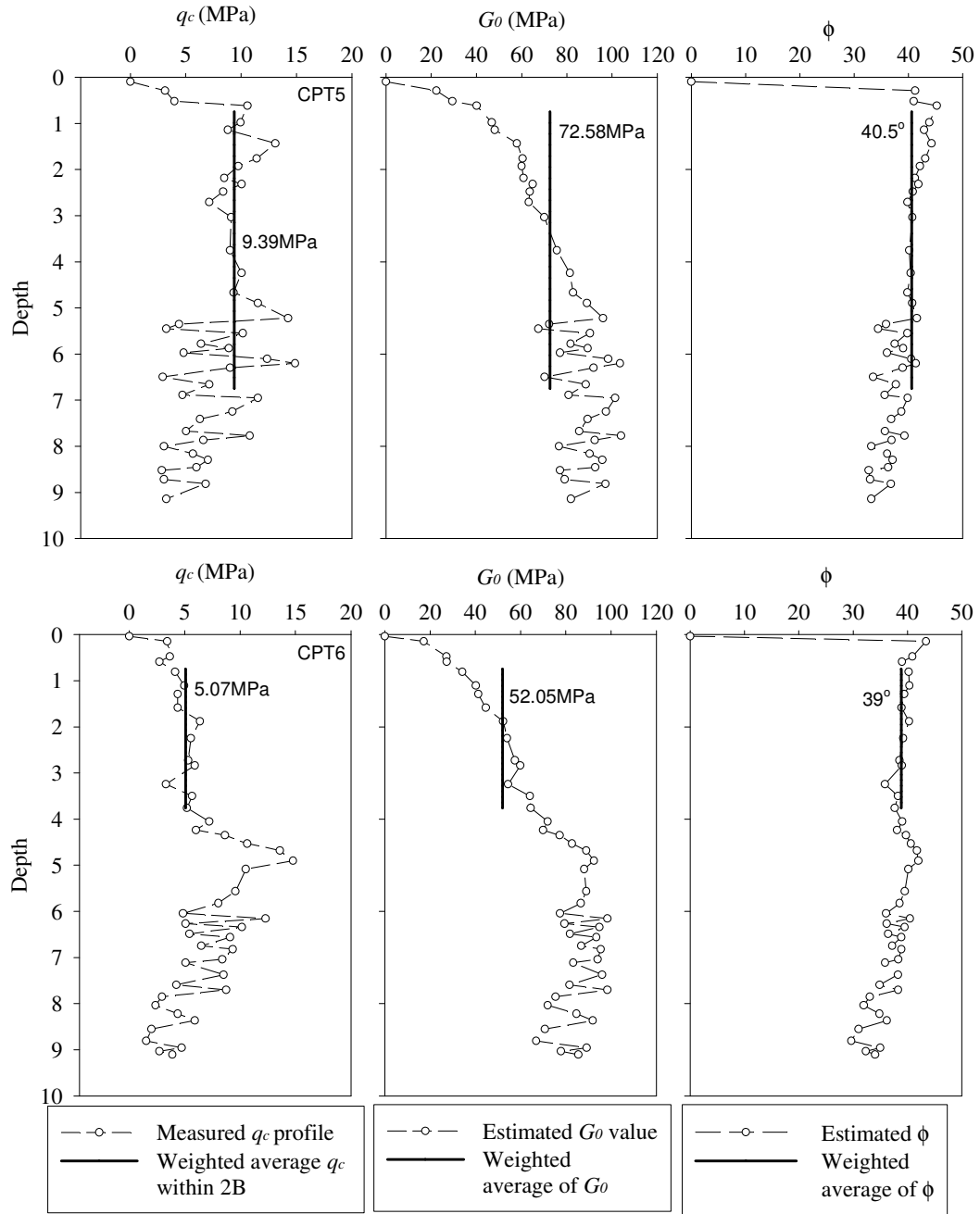


Figure C.2: Interpretation of CPT5 and CPT6 at Texas A&M University, USA

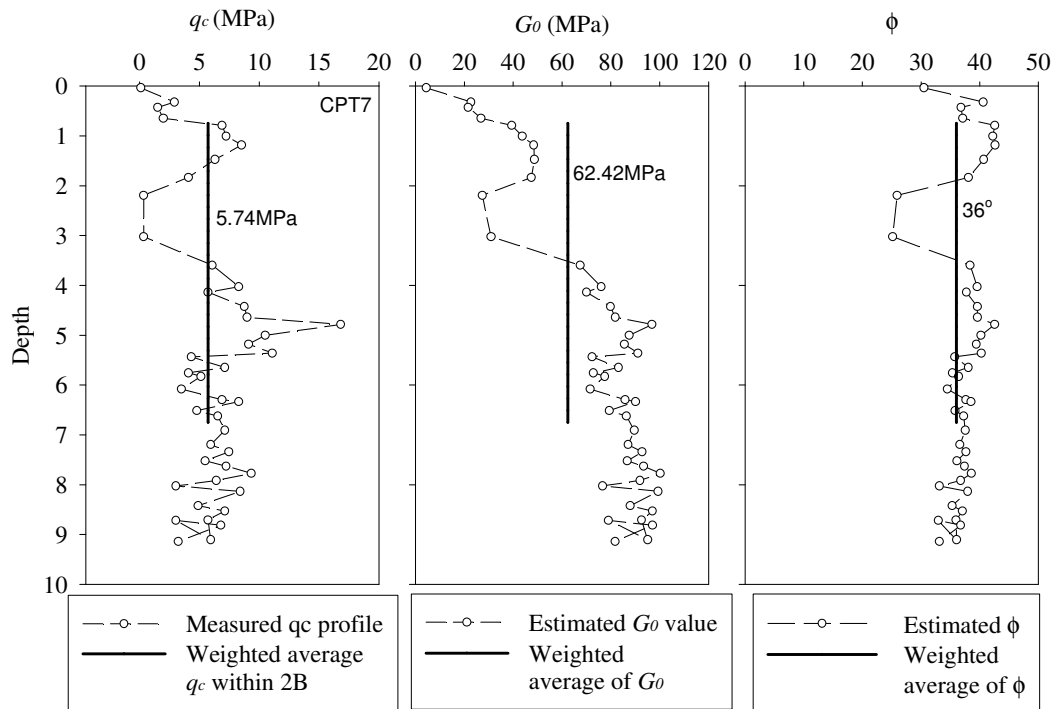


Figure C.3: Interpretation of CPT7 at Texas A&M University, USA

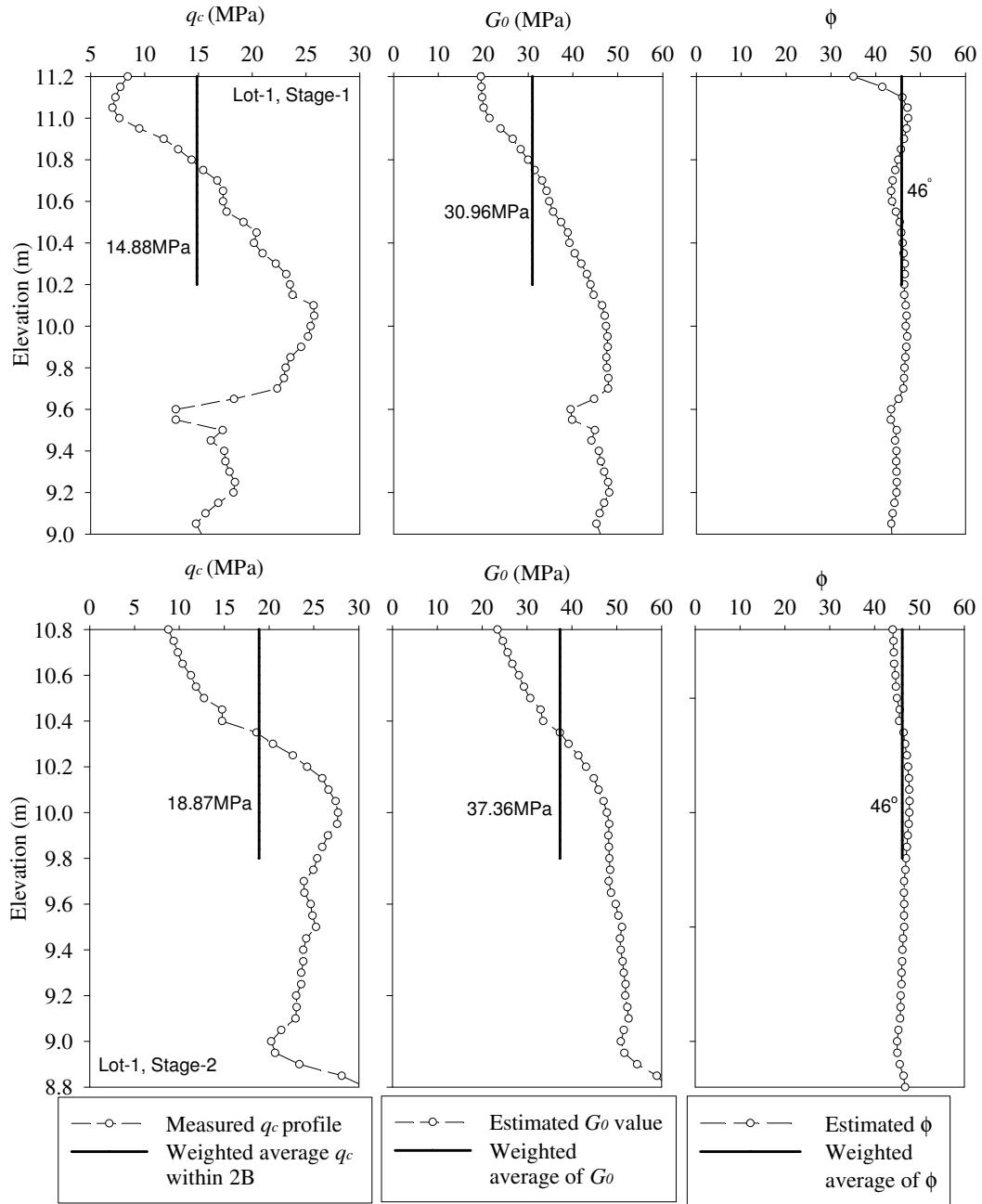


Figure C.4: Interpretation of CPT of Stage-1 and Stage-2 at Lot-1, Changi East reclamation site, Singapore

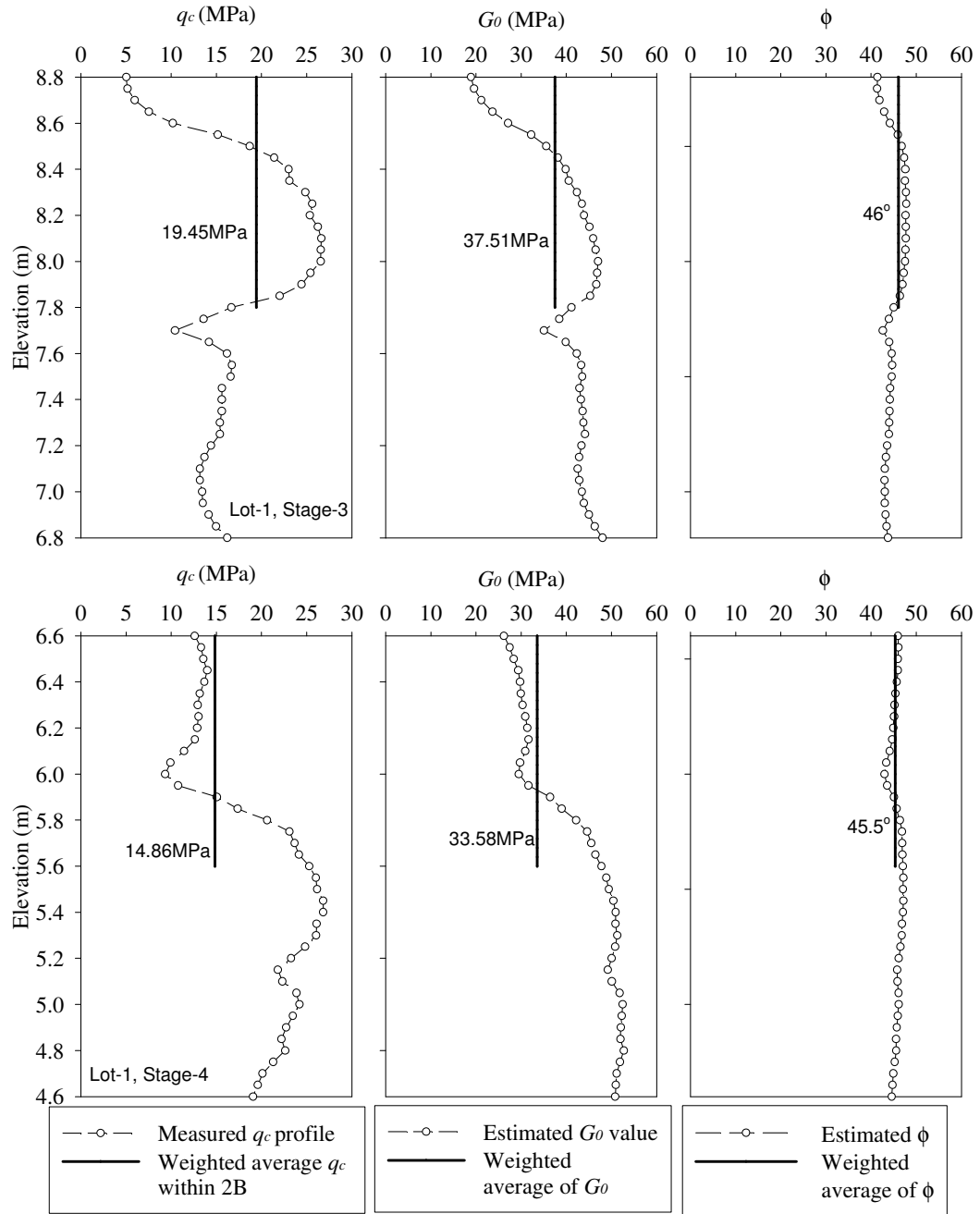


Figure C.5: Interpretation of CPT of Stage-3 and Stage-4 at Lot-1, Changi East reclamation site, Singapore

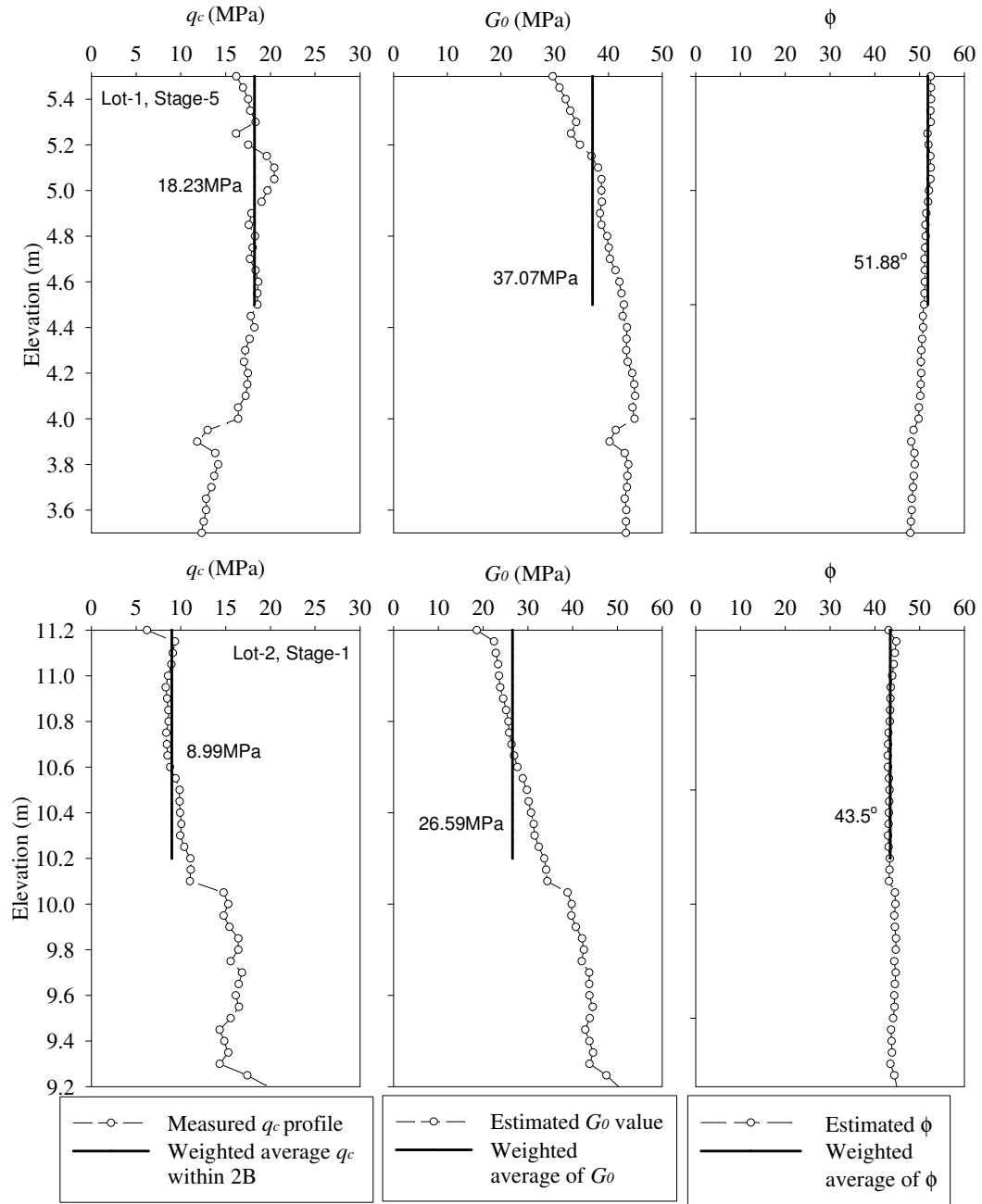


Figure C.6: Interpretation of CPT of Stage-5 at Lot-1 and Stage-1 at Lot-2, Changi East reclamation site, Singapore

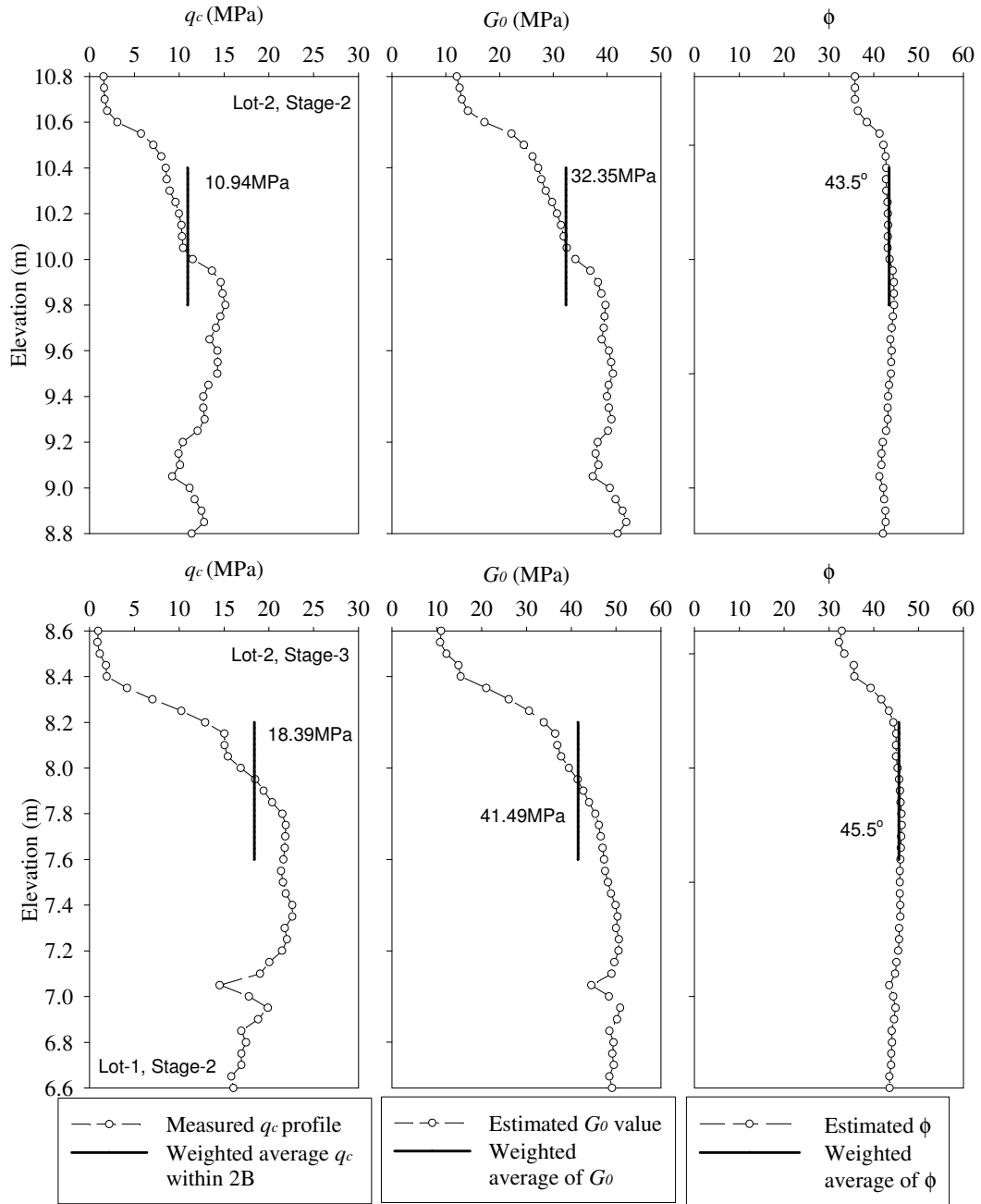


Figure C.7: Interpretation of CPT of Stage-2 and Stage-3 at Lot-2, Changi East reclamation site, Singapore

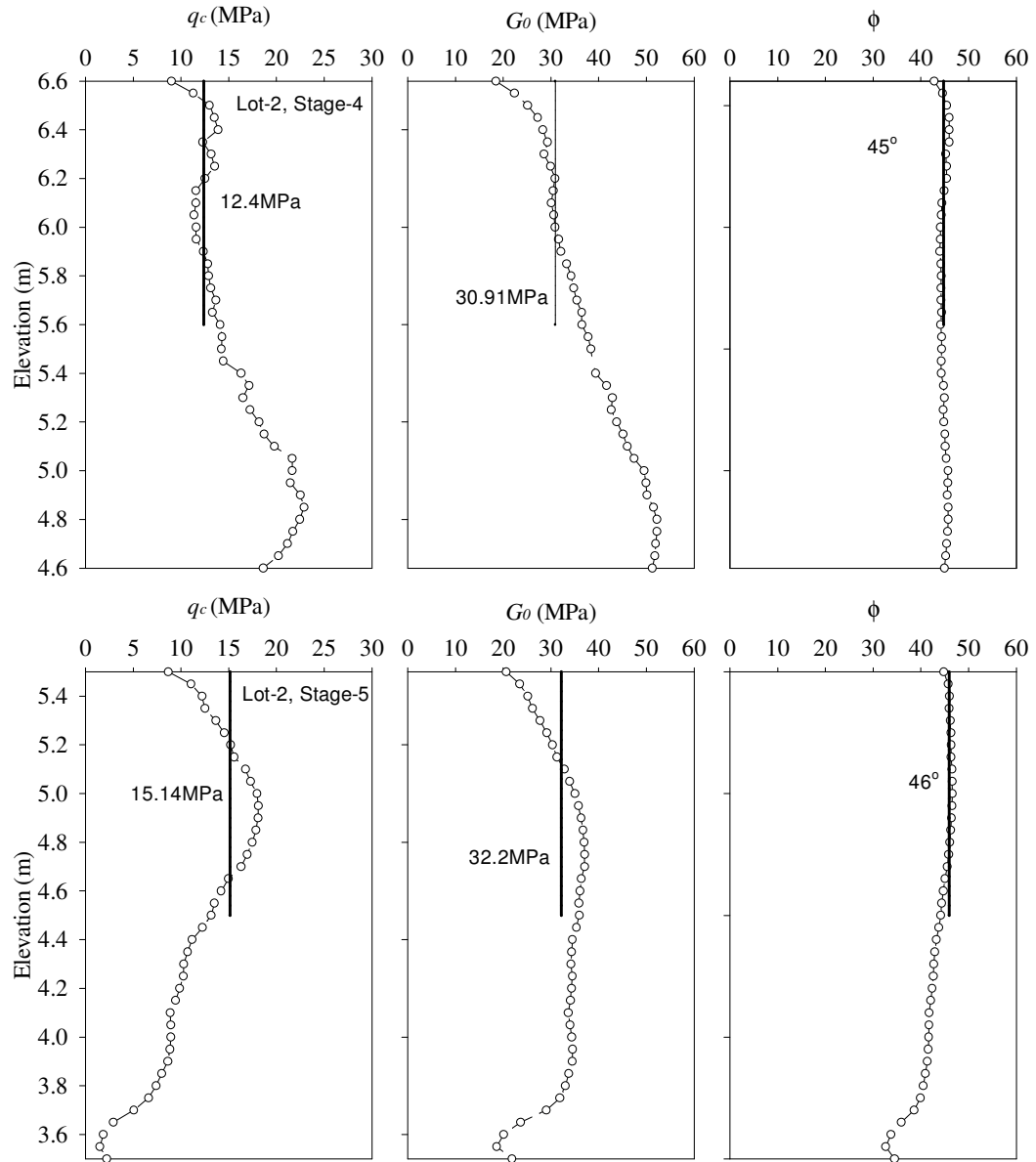


Figure C.8: Interpretation of CPT of Stage-4 and Stage-5 at Lot-2, Changi East reclamation site, Singapore

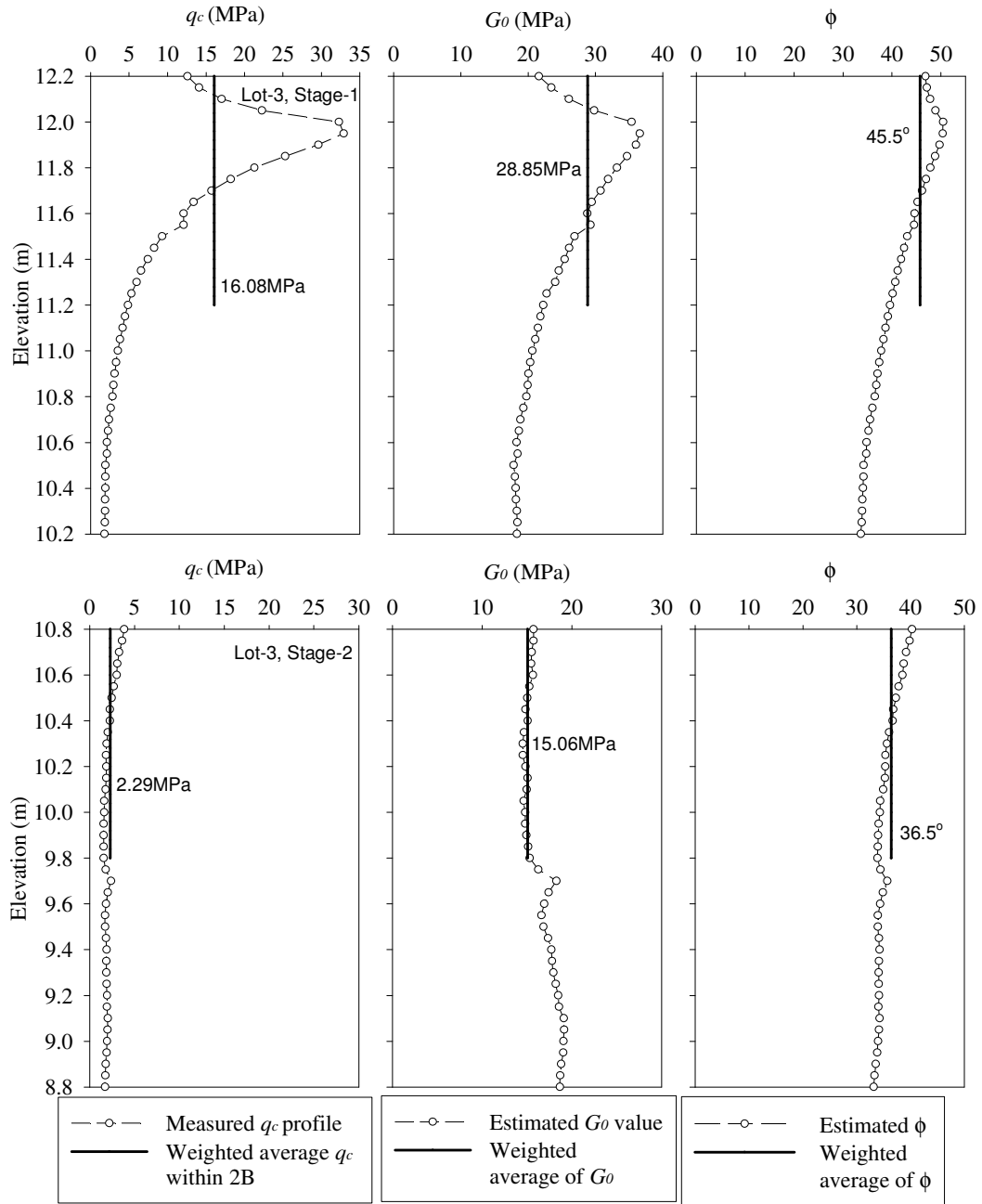


Figure C.9: Interpretation of CPT of Stage-1 and Stage-2 at Lot-3, Changi East reclamation site, Singapore

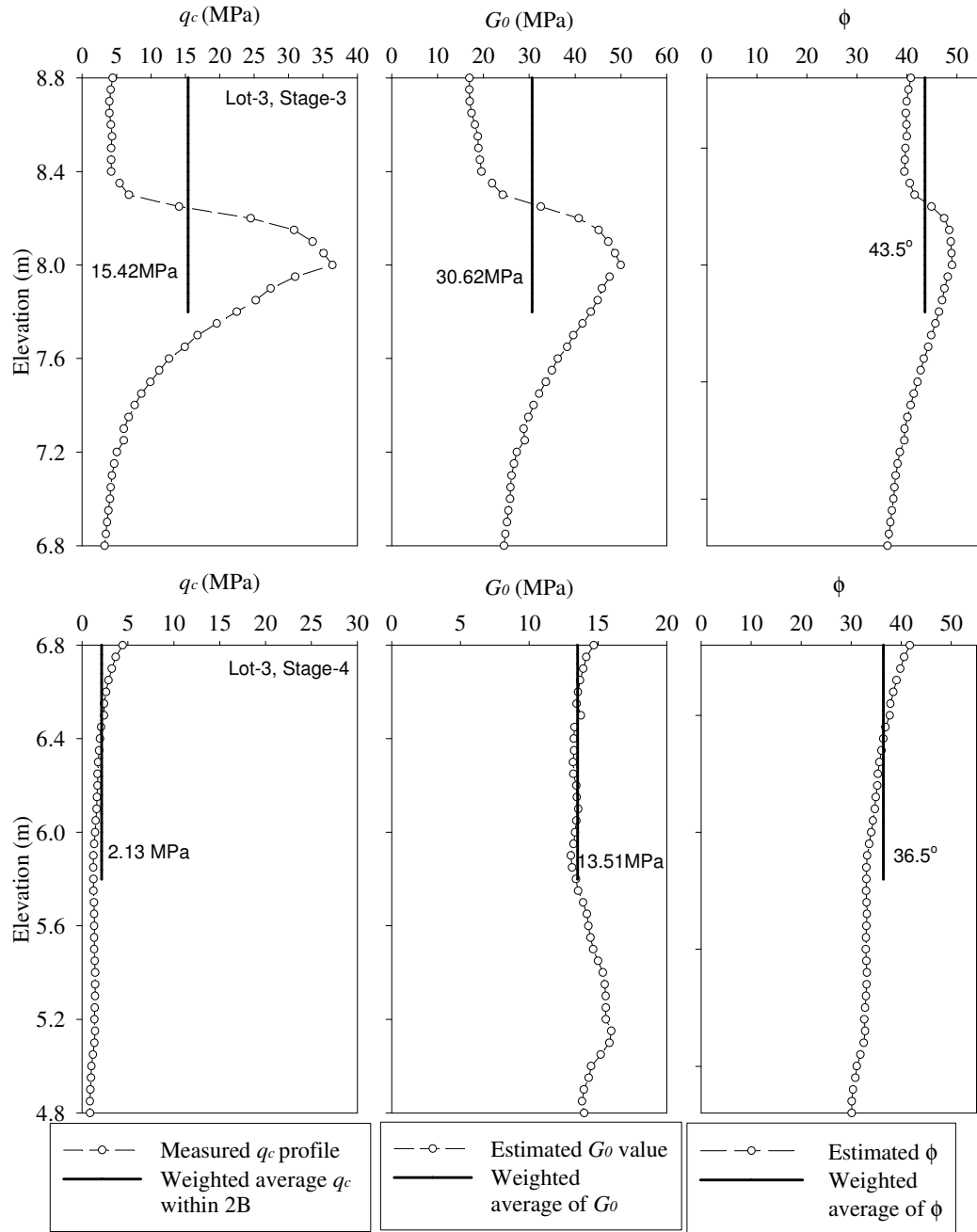


Figure C.10: Interpretation of CPT of Stage-3 and Stage-4 at Lot-3, Changi East reclamation site, Singapore

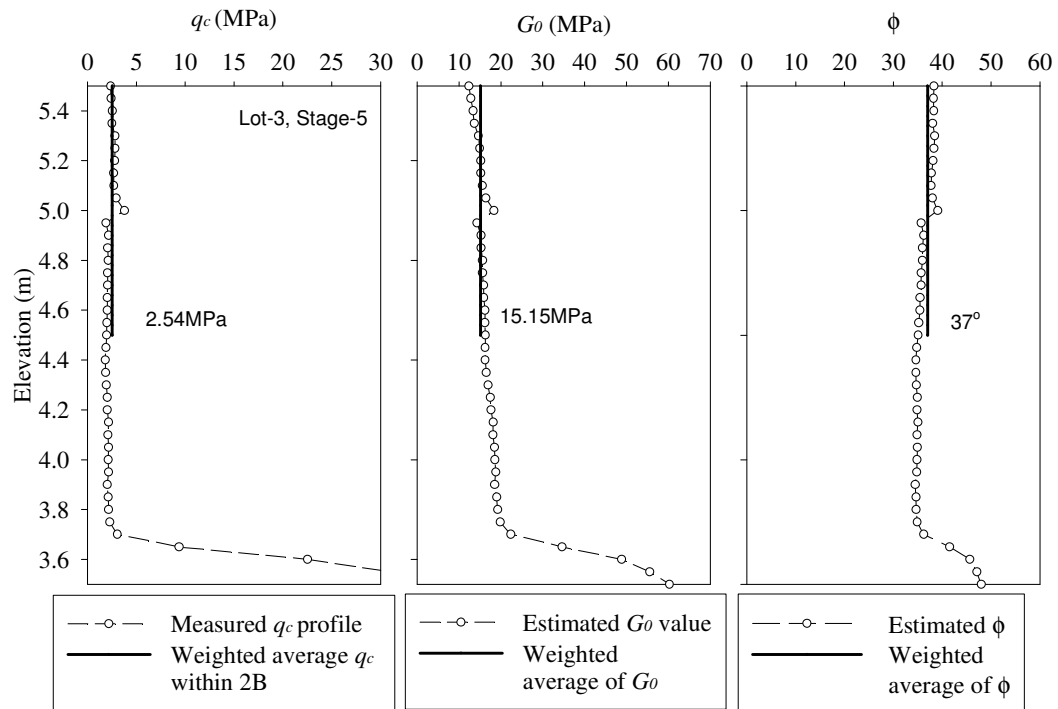


Figure C.11: Interpretation of CPT of Stage-5 at Lot-3, Changi East reclamation site, Singapore

Appendix D Interpretation of ultimate bearing capacity of footings from PLT

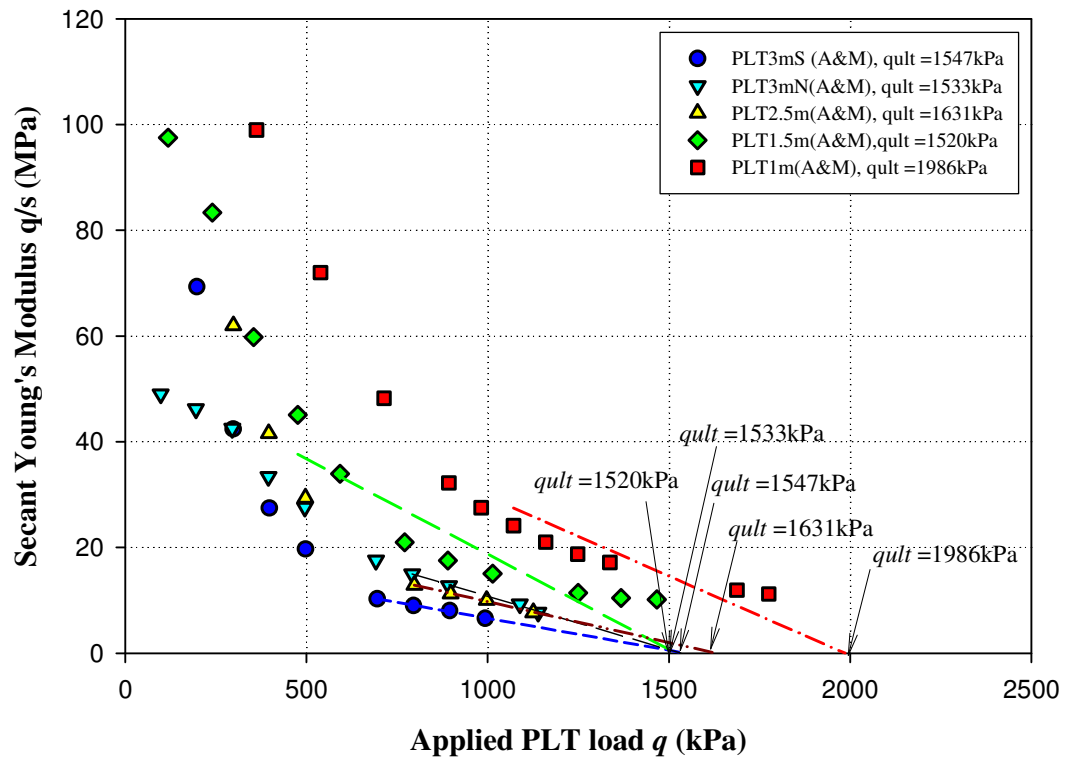


Figure D.1: Interpretation of ultimate bearing capacity from PLT using Decourt's (1999) method (Texas A&M University, USA)

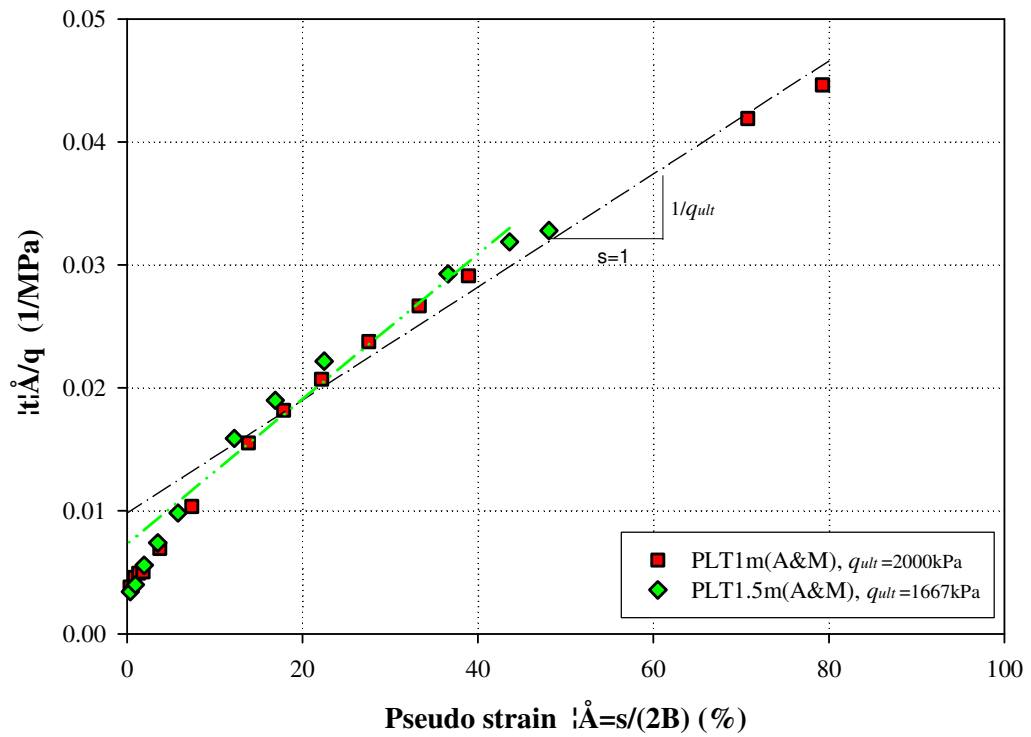
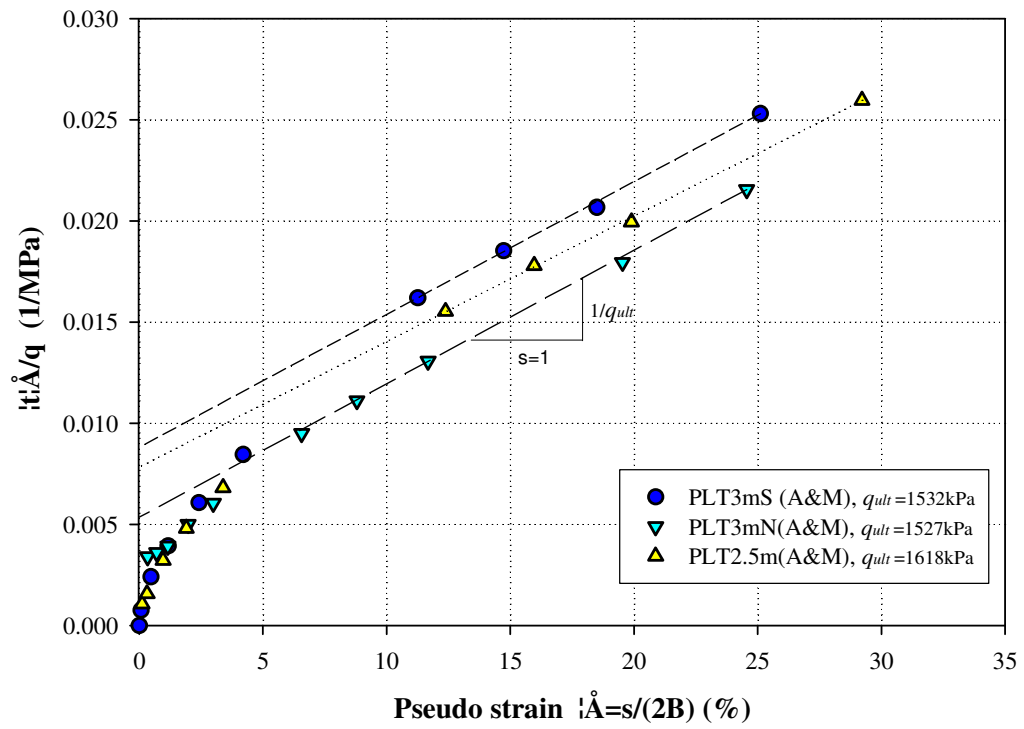


Figure D.2: Interpretation of ultimate bearing capacity from PLT using Chin's (1999) method (Texas A&M University, USA)

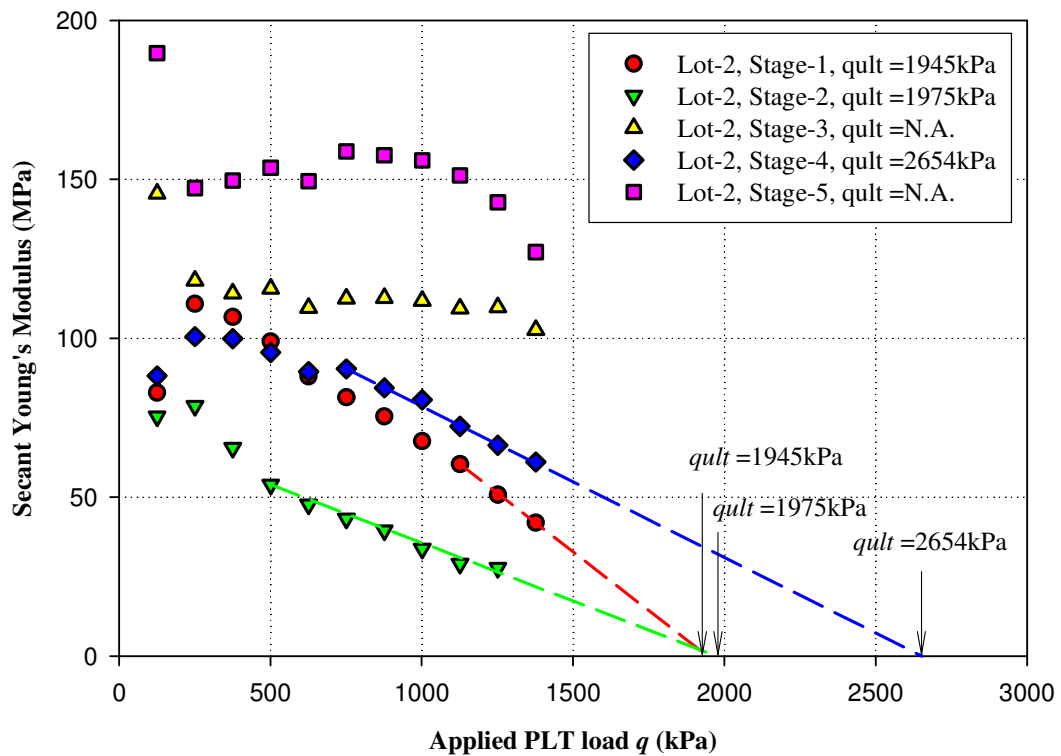
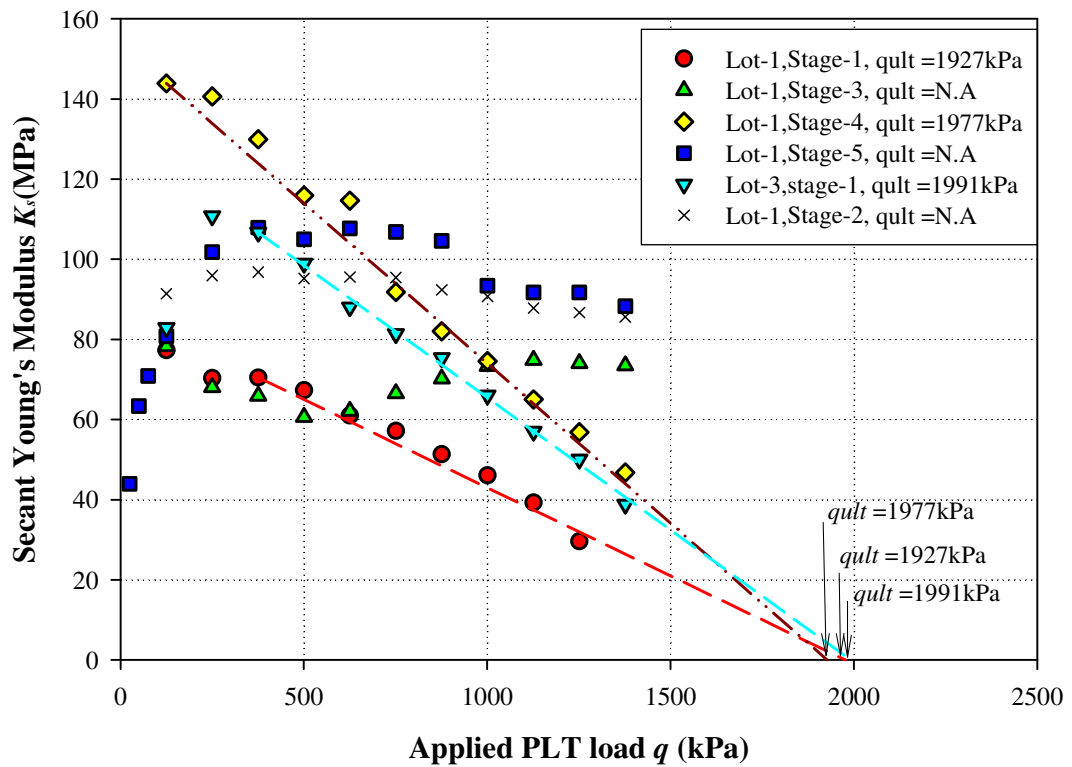


Figure D.3: Interpretation of ultimate bearing capacity from PLT (Lot-1 and Lot-2) using Decourt's (1999) method (Changi East reclamation site, Singapore)

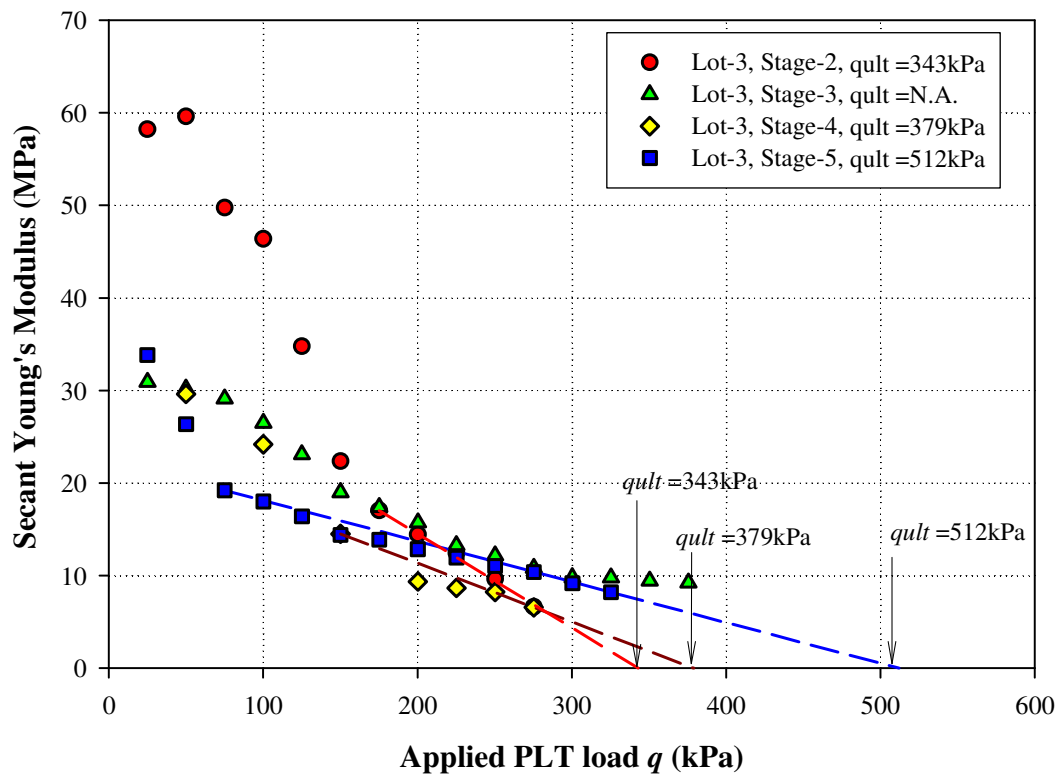


Figure D.4: Interpretation of ultimate bearing capacity from PLT (Lot-3) using Decourt's (1999) method (Changi East reclamation site, Singapore)

Appendix E Calibration of f-g Model using Laboratory Test Results

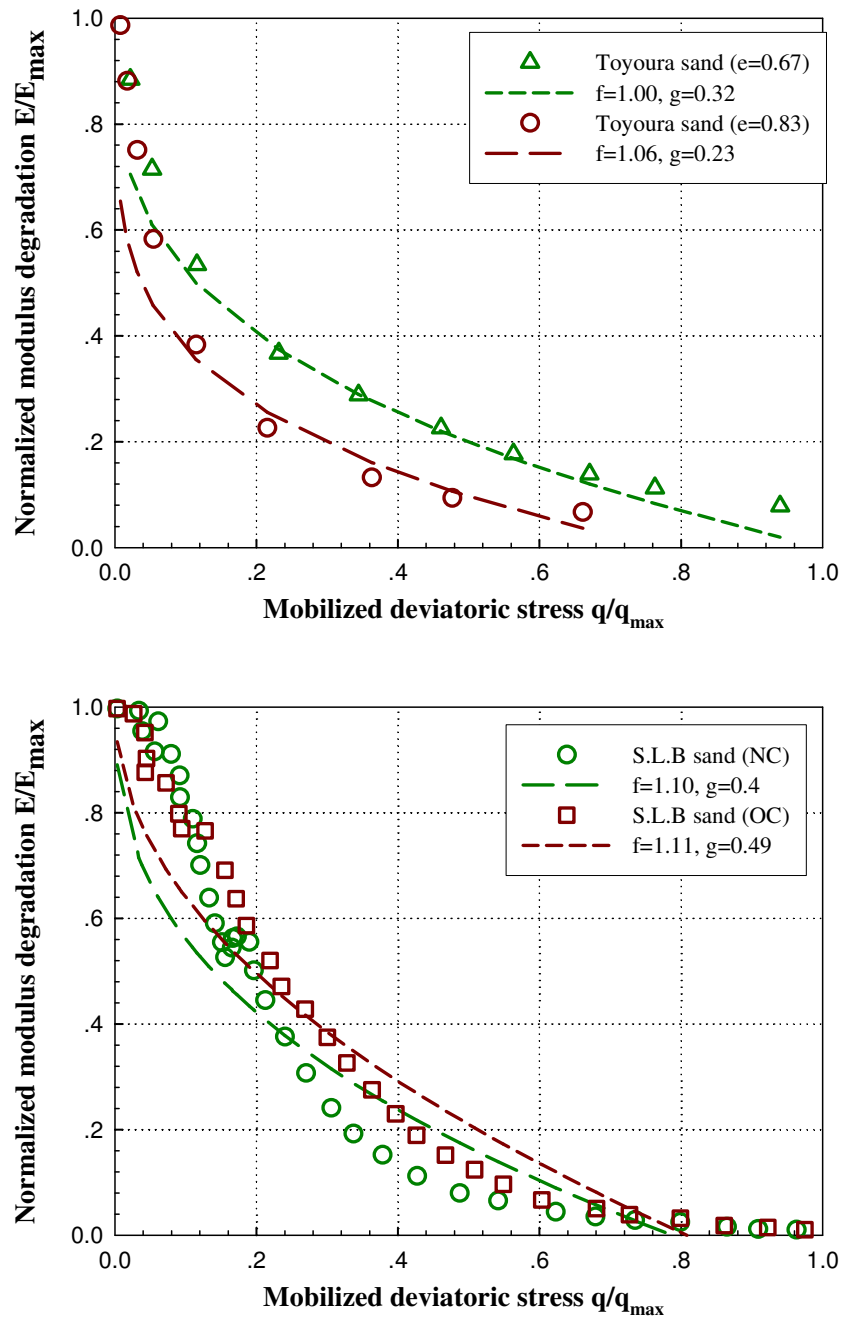


Figure E.1: Calibration of f-g model using plane strain test results

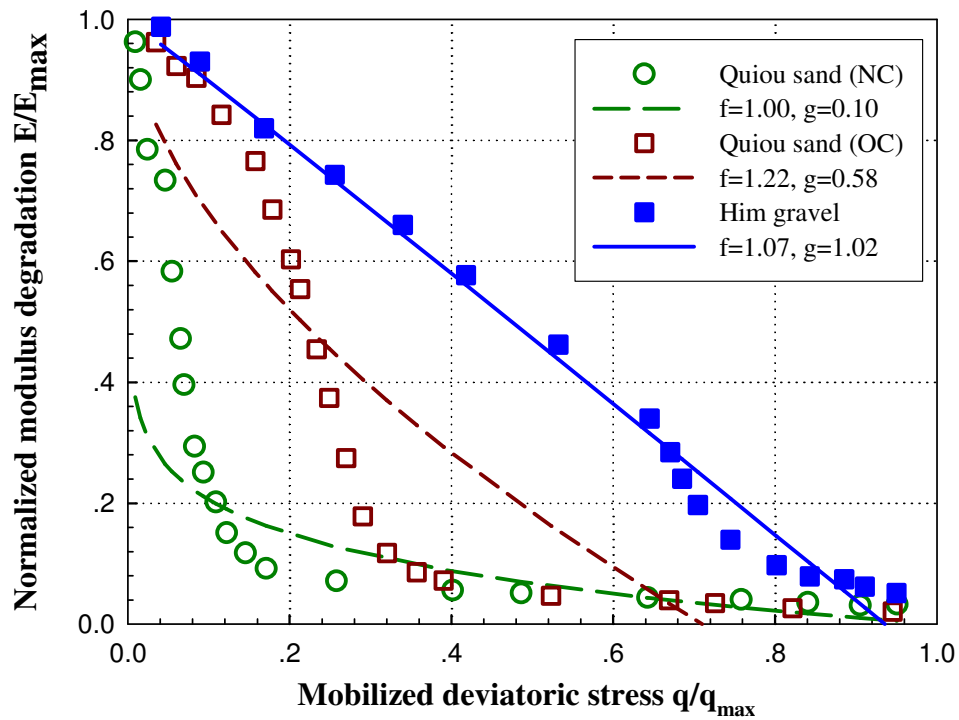
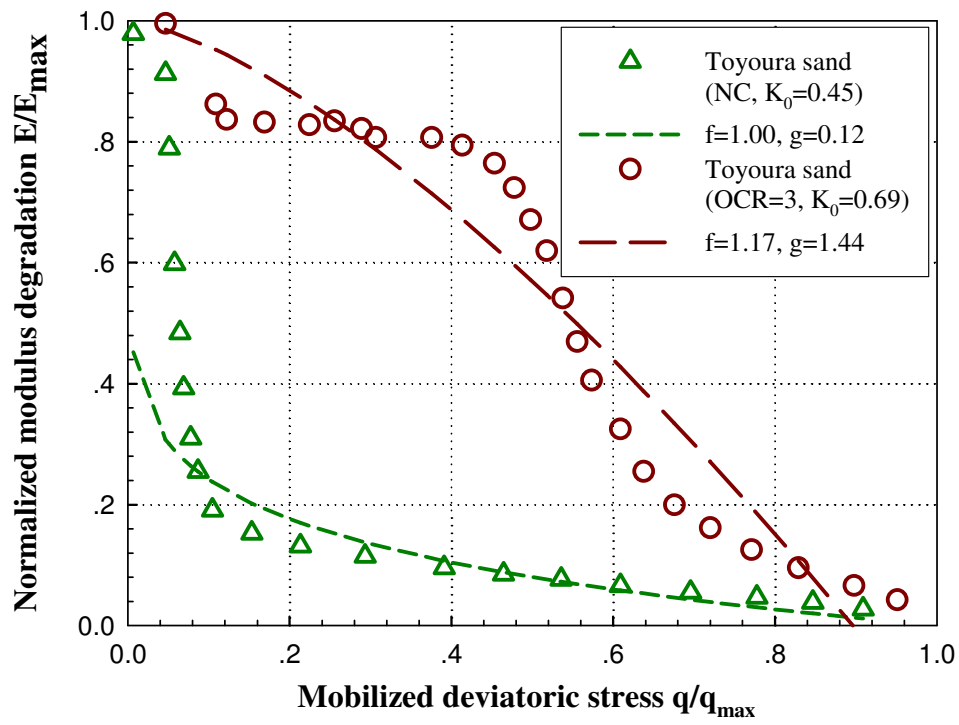


Figure E.2: Calibration of f - g model using triaxial test results

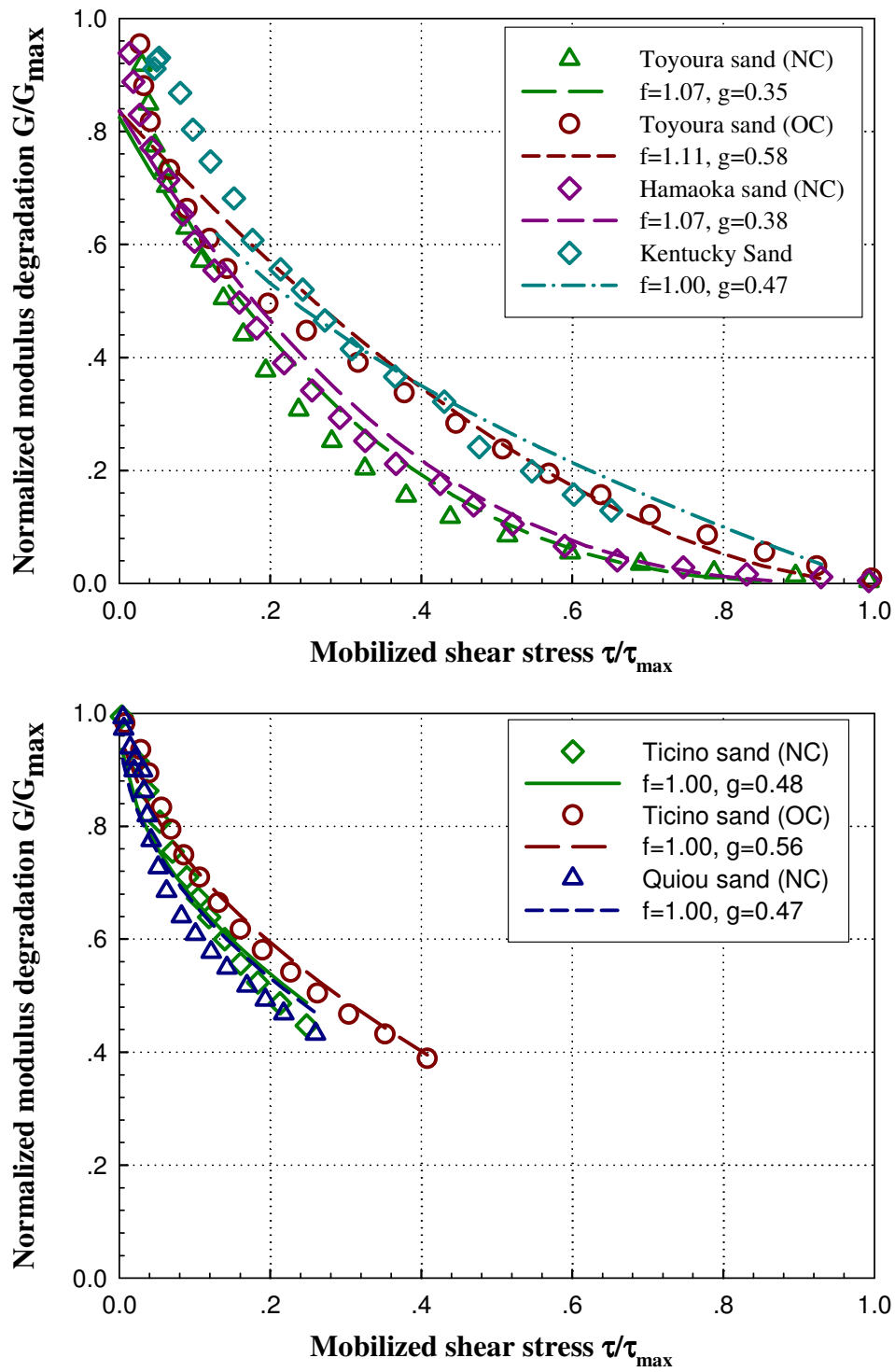


Figure E.3: Calibration of f-g model using torsional shear test results

Appendix F Subroutine of Modified MC Model

```

;Name:m_bab
;Diagram:
;FISH version of standard MC model
set echo off
def m_bab
  constitutive_model
  f_prop m_g m_k m_coh m_fricv m_dil m_ten m_rden m_ind m_fric
m_frica
  f_prop m_csnp m_nphi m_npsi m_e1 m_e2 m_x1 m_sh2 m_extrad
m_xigmam
  float $sphi $spsi $s11i $s22i $s12i $s33i $sdif $s0 $rad $s1 $s2
$s3
  float $si $sii $psdif $fs $alams $ft $alamt $cs2 $si2 $dc2 $dss
  float $apex $pdiv $anphi $bisc $tco
  int $m_err $icase

  Case_of mode
; -----
; Initialisation section
; -----
  Case 1
;
;
  $sphi = sin (m_fric * degrad)
  $spsi = sin (m_dil * degrad)
  m_nphi = (1.0 + $sphi) / (1.0 - $sphi)
  m_npsi = (1.0 + $spsi) / (1.0 - $spsi)
  m_csnp = 2.0 * m_coh * sqrt(m_nphi)
  m_e1 = m_k + 4.0 * m_g / 3.0
  m_e2 = m_k - 2.0 * m_g / 3.0
  m_x1 = m_e1 - m_e2*m_npsi + (m_e1*m_npsi - m_e2)*m_nphi
  m_sh2 = 2.0 * m_g
  if abs(m_x1) < 1e-6 * (abs(m_e1) + abs(m_e2)) then
    $m_err = 5
    nerr = 126
    error = 1
  end_if
; --- set tension to prism apex if larger than apex ---
  $apex = m_ten
  if m_fric # 0.0 then
    $apex = m_coh / tan(m_fric * degrad)
  end_if
  m_ten = min($apex,m_ten)
  Case 2
; -----
; Running section
; -----
;=====
  m_xigmam = (zs11+zs22+zs33)/3.0
; if zsub > 0.0 then
;   m_xigmam = m_xigmamm / zsub
; else
;   m_xigmam = m_xigmamm
; end_if

```

```

;
  if abs(m_xigmam) < 0.1 then
    m_extrad = 13.0
  else
    m_extrad = 3.0* (m_rden * ( 10- ln (abs(m_xigmam/1000.)))-1)
  end_if
;
;
  m_frica = m_fricv + m_extrad
;
  if m_fric = 0.0 then
    m_fric = m_frica
;
  else
;
    if m_frica > m_fric then
      m_fric = m_fric
    else
      m_fric = m_frica
    end_if
  end_if
;
  if m_fric > 45.0 then
    m_fric = 45.0
  end_if
  if m_fric < m_fricv then
    m_fric = m_fricv
  end_if
;=====
  $sphi    = sin (m_fric * degrad)
  $spsi    = sin (m_dil * degrad)
  m_nphi   = (1.0 + $sphi) / (1.0 - $sphi)
  m_npsi   = (1.0 + $spsi) / (1.0 - $spsi)
  m_csnp   = 2.0 * m_coh * sqrt(m_nphi)
  m_e1     = m_k + 4.0 * m_g / 3.0
  m_e2     = m_k - 2.0 * m_g / 3.0
  m_x1     = m_e1 - m_e2*m_npsi + (m_e1*m_npsi - m_e2)*m_nphi
  m_sh2    = 2.0 * m_g
  if abs(m_x1) < 1e-6 * (abs(m_e1) + abs(m_e2)) then
    $m_err = 5
    nerr   = 126
    error  = 1
  end_if
; --- set tension to prism apex if larger than apex ---
  $apex = m_ten
  if m_fric # 0.0 then
    $apex = m_coh / tan(m_fric * degrad)
  end_if
  m_ten = min($apex,m_ten)
;=====
;
;
  zvisc = 1.0
  if m_ind # 0.0 then
    m_ind = 2.0
  end_if
  $anphi = m_nphi
; --- get new trial stresses from old, assuming elastic increments
---
```

```

        $s11i = zs11 + (zde22 + zde33) * m_e2 + zde11 * m_e1
        $s22i = zs22 + (zde11 + zde33) * m_e2 + zde22 * m_e1
        $s33i = zs33 + (zde11 + zde22) * m_e2 + zde33 * m_e1
;-----
;m_xigammam = ($s11i+$s22i+$s33i)/3.0
;-----
        $s12i = zs12 + zde12 * m_sh2
        $sdif = $s11i - $s22i
        $s0   = 0.5 * ($s11i + $s22i)
        $rad   = 0.5 * sqrt ($sdif*$sdif + 4.0 * $s12i*$s12i)
; --- principal stresses ---
        $si    = $s0 - $rad
        $sii   = $s0 + $rad
        $psdif = $si - $sii
; --- determine case ---
        section
            if $s33i > $sii then
; --- s33 is major p.s. ---
                $icase = 3
                $s1    = $si
                $s2    = $sii
                $s3    = $s33i
                exit section
            end_if
            if $s33i < $si then
; --- s33 is minor p.s. ---
                $icase = 2
                $s1    = $s33i
                $s2    = $si
                $s3    = $sii
                exit section
            end_if
; --- s33 is intermediate ---
                $icase = 1
                $s1    = $si
                $s2    = $s33i
                $s3    = $sii
            end_section
        section
; --- shear yield criterion ---
            $fs      = $s1 - $s3 * $anphi + m_csnp
            $alams = 0.0
; --- tensile yield criterion ---
            $ft      = m_ten - $s3
            $alamt = 0.0
; --- tests for failure ---
            if $ft < 0.0 then
                $bisc = sqrt(1.0 + $anphi * $anphi) + $anphi
                $pdiv = -$ft + ($s1 - $anphi * m_ten + m_csnp) * $bisc
                if $pdiv < 0.0 then
; --- shear failure ---
                    $alams = $fs / m_x1
                    $s1 = $s1 - $alams * (m_e1 - m_e2 * m_npsi)
                    $s2 = $s2 - $alams * m_e2 * (1.0 - m_npsi)
                    $s3 = $s3 - $alams * (m_e2 - m_e1 * m_npsi)
                    m_ind = 1.0
                else
; --- tension failure ---
                    $alamt = $ft / m_e1

```

```

        $tco= $alamt * m_e2
        $s1 = $s1 + $tco
        $s2 = $s2 + $tco
        $s3 = m_ten
        m_ind = 3.0
        m_ten = 0.0
    end_if
else
    if $fs < 0.0 then
; --- shear failure ---
        $alams = $fs / m_x1
        $s1 = $s1 - $alams * (m_e1 - m_e2 * m_npsi)
        $s2 = $s2 - $alams * m_e2 * (1.0 - m_npsi)
        $s3 = $s3 - $alams * (m_e2 - m_e1 * m_npsi)
        m_ind = 1.0
    else
; --- no failure ---
        zs11 = $s11i
        zs22 = $s22i
        zs33 = $s33i
        zs12 = $s12i
        exit section
    end_if
end_if
; --- direction cosines ---
    if $psdif = 0.0 then
        $cs2 = 1.0
        $si2 = 0.0
    else
        $cs2 = $sdif / $psdif
        $si2 = 2.0 * $s12i / $psdif
    end_if
; --- resolve back to global axes ---
    case_of $icase
    case 1
        $dc2 = ($s1 - $s3) * $cs2
        $dss = $s1 + $s3
        zs11 = 0.5 * ($dss + $dc2)
        zs22 = 0.5 * ($dss - $dc2)
        zs12 = 0.5 * ($s1 - $s3) * $si2
        zs33 = $s2
    case 2
        $dc2 = ($s2 - $s3) * $cs2
        $dss = $s2 + $s3
        zs11 = 0.5 * ($dss + $dc2)
        zs22 = 0.5 * ($dss - $dc2)
        zs12 = 0.5 * ($s2 - $s3) * $si2
        zs33 = $s1
    case 3
        $dc2 = ($s1 - $s2) * $cs2
        $dss = $s1 + $s2
        zs11 = 0.5 * ($dss + $dc2)
        zs22 = 0.5 * ($dss - $dc2)
        zs12 = 0.5 * ($s1 - $s2) * $si2
        zs33 = $s3
    end_case
    zvisc = 0.0
end_section
;

```

```

    Case 3
; -----
; Return maximum modulus
; -----
    cm_max = m_k + 4.0 * m_g / 3.0
    sm_max = m_g
    Case 4
; -----
; Add thermal stresses
; -----
    ztsa = ztea * m_k
    ztsb = zteb * m_k
    ztsc = ztec * m_k
    ztsd = zted * m_k
    End_case
end
;opt m_bab
set echo on

```